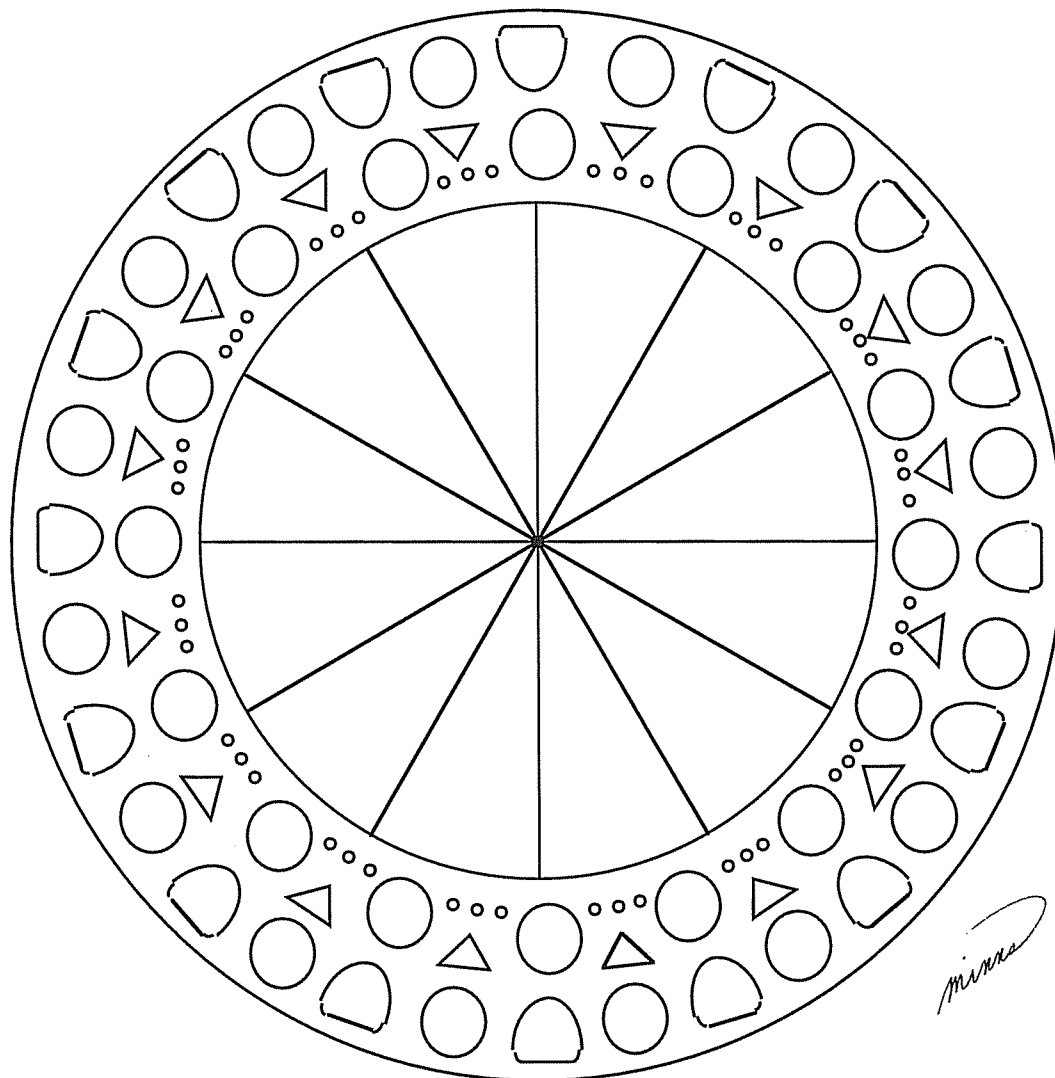


Patterns in Arithmetic
Fractions - Booklet 3 PDF
Mixed Numbers and Improper Fractions
Parent/Teacher Guide



By Alysia Krafel, Susan Carpenter, and Suki Glenn

Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Fractions: Booklet 3 - PDF - Mixed Numbers and Improper Fractions

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Acknowledgments

The knowledge, patience, and dedication of Professor Michael Butler made the UCI Farm Elementary School and this mathematics program possible. Special thanks go to Alysia Krafel and Susan Carpenter, who helped develop much of the math materials based on the teachings, ideas, and insights of Professor Butler.

For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

The books would never have been completed if the students at Chrysalis Charter School in Redding, California, under the guidance of Alysia and Paul Krafel, hadn't needed them. Thank you for your patience through all of the draft copies.

Susan Carpenter edited, added her wise words, useful suggestions, and helped make the Answer Keys a reality. Karán Founds-Benton contributed her meticulous editing skill and knowledge. Diligent and thorough copy editing was done by Zephyr Alfanash and Jacqueline Logue.

The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

Patterns in Arithmetic: Fractions - Booklet 3 PDF

Parent/Teacher Guide

©2014 Pattern Press

Published by Pattern Press
P.O. Box 2737
Fallbrook, CA 92088
(760)728-3731

www.patternpress.com

E-mail: Patternpress1@gmail.com

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ISBN 978-1-941961-02-5

Pre-Assessment: Part 1

Date _____

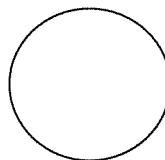
Put a question mark next to anything you do not know yet.

The student may use a manipulative for the test.

1. Shade in $\frac{3}{4}$ of this rectangle.



2. Gail cut a pizza into six slices.

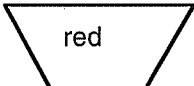
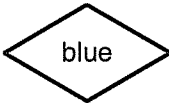


Natalia ate 1 slice. What fraction of the pizza did she eat? a. _____

Frank ate 4 slices. What fraction of the pizza did he eat? b. _____

How much of the pizza has been eaten? Number sentence. c. _____

How much of the pizza is left for Gail? Number sentence. d. _____

3. If  equals 1, what does  equal? _____

On the problems below, you may use fraction pieces. Circle any problem that you can do without using fraction pieces.

4. Fill in the missing numbers.

a. $\frac{1}{2} = \frac{\quad}{4}$

b. $\frac{1}{3} = \frac{\quad}{12}$

c. $\frac{3}{4} = \frac{\quad}{8}$

5. Write a $>$, $<$, or $=$ sign (Greater Than, Less Than, or Equal To) between these pairs of fractions.

a. $\frac{1}{2}$ $\frac{1}{3}$

b. $\frac{2}{6}$ $\frac{1}{3}$

c. $\frac{3}{4}$ $\frac{2}{3}$

6. Solve.

a. $\frac{1}{4} + \frac{2}{4} = \underline{\quad}$

b. $\frac{7}{8} - \frac{3}{8} = \underline{\quad}$

c. $7\frac{3}{4} - 3\frac{1}{4} = \underline{\quad}$

7. Solve.

a. $\frac{1}{4}$ of 12 = _____

b. $\frac{2}{3}$ of 18 = _____

c. $\frac{3}{5} \times 25 = \underline{\quad}$

Put a question mark next to anything you do not know yet.

1. Put a box around the improper fractions. Put a circle around the mixed numbers.

$$2\frac{2}{3} = \quad \frac{1}{2} = \quad \frac{3}{2} = \quad \frac{3}{4} = \quad \frac{5}{4} = \quad 3\frac{3}{4} =$$

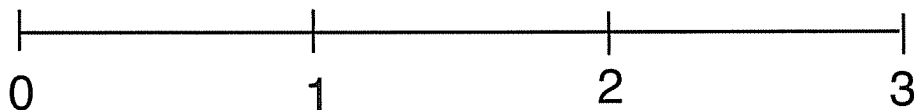
2. Change these mixed numbers to improper fractions.

$$\text{a. } 3\frac{1}{2} = \quad \text{b. } 2\frac{3}{4} = \quad \text{c. } 4\frac{5}{6} =$$

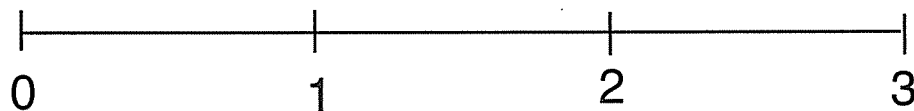
3. Change these improper fractions to mixed numbers.

$$\text{a. } \frac{7}{3} = \quad \text{b. } \frac{\quad}{\quad} = \frac{3}{2} \quad \text{c. } \frac{10}{4} =$$

4. Put $1\frac{1}{4}$ on this number line.



5. Put $\frac{11}{4}$ on this number line.



6. Solve.

$$\begin{array}{r} \text{a. } 1\frac{1}{3} \\ + 2\frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{r} \text{b. } 2\frac{2}{5} \\ + 1\frac{4}{5} \\ \hline \end{array}$$

$$\begin{array}{r} \text{c. } 11\frac{8}{9} \\ + 19\frac{8}{9} \\ \hline \end{array}$$

7. Solve.

$$\text{a. } 4 - \frac{2}{3} =$$

$$\begin{array}{r} \text{b. } 5\frac{2}{3} \\ - 2\frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{r} \text{c. } 5\frac{1}{4} \\ - 1\frac{2}{4} \\ \hline \end{array}$$

$$\begin{array}{r} \text{d. } 20\frac{1}{10} \\ - 6\frac{7}{10} \\ \hline \end{array}$$

Assessment Guide

Purpose The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Pre-Assessment: Part 1 is review material from the last booklet and is used to determine student readiness for this booklet. Pre-Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

The Post-Assessment is administered to determine if the student learned the material that was presented in this booklet. A comparison of the score on Pre-Assessment: Part 2 to the score on the Post-Assessment will give both you and your student a sense of growth. The Post-Assessment is the same as Pre-Assessment: Part 2.

Prerequisites *Patterns in Arithmetic: Fractions - Booklets 1 and 2*

Materials Fractions: Booklet 3 - Pre-Assessment: Part 1 and 2, pages 1 and 2, Post-Assessment, page 24 in this booklet and page 37 in the Student Workbook
Score Sheets, pages 7 - 9 in this booklet
Fraction pieces - Prism Fractions or fraction circles
Pattern blocks and counters of any kind

Instructions Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item. Answers are in the Answer Key. Place the manipulatives within his reach, but do not show him how to use them at this time.

As always, note how the student is solving the problem to guide your teaching. Possible stages are using manipulatives; figuring out and building understanding; making pictures; just using numbers abstractly in his head; or practicing.

Do the assessment in two parts. Give Pre-Assessment: Part 1 and check it for readiness for this booklet. If the student is not ready for this booklet, there is no point in giving Pre-Assessment: Part 2. If he passes all the readiness items, then give Pre-Assessment: Part 2.

After scoring Pre-Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

Assessment Guide This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in

the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc.

Score sheets that match the Assessment Guide for the Pre-Assessment: Part 1, Pre-Assessment: Part 2, and Post-Assessment follow.

Assessment Criteria for Pre-Assessment: Part 1

This is review material from the previous booklets and is used to do Pre-Assessment: Part 1.

Can the student:

1. Numerators Greater Than One (Fractions: Booklet 1)

A. divide the rectangle into roughly four equal sections? This demonstrates that he knows the denominator, four, to indicate that the whole is divided into four parts.

B. shade in three of the four sections indicating that he knows that the numerator, three tells him how many of the four parts are being discussed?

2. Fractions as Ratios (Fractions: Booklet 2)

A. answer correctly both problems a and b?

B. add correctly the two fractions to get $\frac{5}{6}$ in problem c and subtract the two fractions to get $\frac{1}{6}$ in problem d?

C. write the correct number sentences for problems c and d?

Does the student use the denominator six in all the problems? This shows he understands that this is the number of pieces the whole is divided into. Also, we want to know if he understands that fractions can be used in number sentences just as whole numbers can be.

3. Changing Wholes (Fractions: Booklet 2)

A. state that the blue is equal to $\frac{2}{3}$?

In Items 4 - 7, let the student know that we need to know how he is solving the problems. He can use the manipulative if he wants to. Observe if he is doing the problem without manipulatives and make a note of it by circling those items during the test. If manipulatives are used, it's a good idea to note the type of manipulative, e.g., pattern blocks, fraction pieces, or if a drawing was made.

If you are working in a large group, instruct the students to circle those items they do without using manipulatives. Tell them it is fine to use manipulatives, but we need to know if they are able to do the problems in their head or not. We are trying to decide how best to teach them, so we need good data.

4. Equivalence: Manipulative (Fractions: Booklets 1 and 2)

A. write in the correct missing number in two of the three problems?

B. No score but note: Does he do two of the three problems without the use of the fraction pieces as indicated by circled problems?

5. Greater Than, Less Than, or Equal To (Fractions: Booklet 2)

A. use the correct sign in two of the three problems?

B. No score but note: Does he do two of the three problems without the use of the fraction pieces as indicated by circled problems?

6. Addition and Subtraction of Like Fractions (Fractions: Booklet 2)

A. give the correct answer in two of the three problems?

Simplification of the answer in problem c not required.

7. Parts of Wholes (Fractions: Booklet 2)

- A. give the correct answer to both problems a and b?
- B. give the correct answer to problem c?
- C. No score but note: Does he do two of the three problems without the use of counters as indicated by circled problems?

Extra Items (extra point for not using a manipulative)

- A. give the correct answers on two of three problems in each set without the use of any manipulatives on Items 4, 5, 6, and 7? If so, he gets a total of one point.

Booklet Selection Guide based on results of Pre-Assessment: Part 1

If the student has a No on Items 1A, 1B, and 2A, begin with Fractions: Booklet 1. Do not give Pre-Assessment: Part 2 of this assessment. There is much to lose by not developing basic concepts.

If the student has less than 6 points or a Yes on Items 1A, 1B, and 2A, 2B, but a No on three of the four items 4A, 5A, 6A, and 7A, even with the manipulatives available, begin with Fractions: Booklet 2.

Critical concepts for Fractions: Booklet 3 are concepts 2A, 4A,* and 6A. If your student has 7 or more points but missed any of the above items (2A, 4A, and 6A), remediate these using manipulatives before moving on with this booklet. These items test the understanding of the denominator. The denominator is the driving concept behind the equivalence of improper and mixed numbers. After you have finished reteaching any of the above concepts, retest and then give Part 2 of this assessment.

*It is not necessary for a student to be able to calculate equivalence at this point. If he can build it with the manipulatives, that is sufficient. Calculating equivalence is taught in Fractions: Booklet 4.

If you are working with a class, students with weak areas can be remediated concurrently with this booklet while reteaching from Fractions: Booklet 2 as long as the student's total score is at least 6. A score lower than 6 indicates that basic concepts are not in place.

A score of 9 or more points indicates good background knowledge in concepts presented up to this point. Proceed with Part 2 of this assessment.

Assessment Criteria for Pre-Assessment: Part 2

Do not use manipulatives for this part of the test. He can use drawings to help if needed.

Can the student:

1. Mixed Numbers to Improper Fractions and Improper Fractions to Mixed Numbers
 - A. place a box around both improper fractions in the list?
 - B. place a circle around both mixed numbers in the list?
2. Mixed Numbers to Improper Fractions
 - A. change correctly two of the three mixed numbers to improper fractions?
 - B. obtain the correct answers without the use of a drawing?
3. Improper Fractions to Mixed Numbers
 - A. change correctly two of the three improper fractions to mixed numbers?
 - B. obtain the correct answers without the use of a drawing?

4. Mixed Numbers and Improper Fractions on Number Lines
 - A. locate correctly the mixed number $1\frac{1}{4}$ on the number line?
5. Mixed Numbers and Improper Fractions on Number Lines
 - A. locate correctly $\frac{11}{4}$ on the number line?

6. Addition of Mixed Numbers with Like Denominators

- A. add correctly the fractions in two out of the three problems?

This is to see if the student knows to add only the numerators and not the denominators. Then, check to see if he knows to regroup the improper fractions in the answer to a whole number and combine that with the sum of the whole numbers given in the problem.

B. regroup the improper fractions to create whole numbers and combine them with the sum of the whole numbers in the problem? Give a Yes if he attempted this, even if he made an arithmetic error.

- C. obtain the correct final sum in two of the three problems?

7. Mixed Numbers Meet Subtraction

- A. get the correct answer on both problems a and b?
- B. get the correct answer on problem c?
- C. get the correct answer on problem d?

In the first two problems, the denominators are the same. The student only has to regroup the fraction and the whole number which increases the size of the top fraction so subtraction can take place. A few students will use negative numbers to solve these and not regroup. This is acceptable.

Booklet Selection Guide based on results of Pre-Assessment: Part 2

A student who scores 11 Yes points or more can move to Fractions: Booklet 4. Reteach any weak areas.

If a student scores Yes on Items 1 - 5, and scores No on Items 6 and 7, you can skip the early part of this booklet and begin with the Addition of Mixed Numbers with Like Denominators, or Mixed Numbers Meet Subtraction. A score of 8 or less indicates that this booklet is just right.

Whenever remediation is needed, rely upon the following process, which is used throughout the *Patterns in Arithmetic* series to develop understanding of a concept.

1. Introduce the concept with a manipulative. Orally discuss it. Build it. Verify it. Practice it.
Repeat the experience with a different manipulative (oral manipulative).
2. Use manipulatives to explore the concept again. This time record it with pictures (pictorial/representation). Practice it. Use worksheets.
3. Record the problem with numbers (abstract/symbolic), which links the pictorial with the abstract.
4. Practice fluency.
5. Practice for speed.

Ask questions or make statements, such as: “Are you sure?” or “Build it.” or “What gave you the clue?” or “Show me how you got that.” or “Prove it.” even when a student is correct. This is important to do often. Many students will ask an adult, “Am I right?” rather than answering definitively. Confidence in a student’s response must come from within. A student needs to self-check and have confidence in his or her ability and knowledge. Asking the student if he or she is right, even when correct, will encourage self-confidence and the ability to self-check.

Pre-Assessment: Part 1 Score Sheet

Name _____ Date _____

Can the student:

1. Numerators Greater Than One (Fractions: Booklet 1)

Yes No A. break the rectangle into roughly four equal sections?

Yes No B. shade in three of the four sections?

2. Fractions as Ratios (Fractions: Booklet 2)

Yes No A. answer correctly both problems a and b?

Yes No B. supply the correct answer for both problems c and d?

Yes No C. write the correct number sentences for problems c and d?

3. Changing Wholes (Fractions: Booklet 2)

Yes No A. state that the blue is equal to $\frac{2}{3}$?

4. Equivalence: Manipulative (Fractions: Booklets 1 and 2)

Yes No A. write the correct missing number in two of the three problems?

No score: Yes No B. correctly do two of the three problems without the use of the fractions pieces?

5. Greater Than, Less Than, or Equal To (Fractions: Booklet 2)

Yes No A. use the correct sign in two of the three problems?

No score: Yes No B. correctly do two of the three problems without the use of the fractions pieces?

6. Addition and Subtraction of Like Fractions (Fractions: Booklet 2)

Yes No A. give the correct answer in two of the three problems?

No score: Yes No B. correctly do two of the three problems without the use of the fractions pieces?

7. Parts of Wholes (Fractions: Booklet 2)

Yes No A. give the correct answer to both problems a and b?

Yes No B. give the correct answer to problem c?

No score: Yes No C. correctly do two of the three problems without the use of the counters?

Extra Item (extra point for not using a manipulative)

Yes No A. give the correct answers on two of three problems in each set without the use of any manipulative on Items 4, 5, 6, and 7? If so, he gets a total of one point.

Items Correct = _____ = _____%

Items Possible = 12

Pre-Assessment: Part 2 Score Sheet

Name _____ Date _____

Can the student:

1. Mixed Numbers to Improper Fractions and Improper Fractions to Mixed Numbers
Yes No A. place a box around both improper fractions in the list?
Yes No B. place a circle around both mixed numbers in the list?
2. Mixed Numbers to Improper Fractions
Yes No A. change correctly two of the three mixed numbers to improper fractions?
Yes No B. obtain the correct answers without the use of a drawing?
3. Improper Fractions to Mixed Numbers
Yes No A. change correctly two of the three improper fractions to mixed numbers?
Yes No B. obtain the correct answers without the use of a drawing?
4. Mixed Numbers and Improper Fractions on Number Lines
Yes No A. locate correctly the mixed number $1\frac{1}{4}$ on the number line?
5. Mixed Numbers and Improper Fractions on Number Lines
Yes No A. locate correctly $\frac{11}{4}$ on the number line?
6. Addition of Mixed Numbers with Like Denominators
Yes No A. add the fractions correctly in two of the three problems?
Yes No B. regroup the improper fractions to create whole numbers?
Yes No C. obtain the correct final sum in two of the three problems?
7. Mixed Numbers Meet Subtraction
Yes No A. get the correct answer on both problems a and b?
Yes No B. get the correct answer on problem c?
Yes No C. get the correct answer on problem d?

Items Correct = _____ = _____ %
Items Possible = 14

Score of 11 needed to pass to the next booklet.
Less than 72% (10 or under)—review items in
Fractions: Booklet 1 or 2, do additional practice in
the areas missed, and retest with a pass on Part 1. A
score of 8 or less indicates this is the right booklet.

Post-Assessment Score Sheet

Name _____ Date _____

Can the student:

1. Mixed Numbers to Improper Fractions and Improper Fractions to Mixed Numbers
Yes No A. place a box around both improper fractions in the list?
Yes No B. place a circle around both mixed numbers in the list?
2. Mixed Numbers to Improper Fractions
Yes No A. change correctly two of the three mixed numbers to improper fractions?
Yes No B. obtain the correct answers without the use of a drawing?
3. Improper Fractions to Mixed Numbers
Yes No A. change correctly two of the three improper fractions to mixed numbers?
Yes No B. obtain the correct answers without the use of a drawing?
4. Mixed Numbers and Improper Fractions on Number Lines
Yes No A. locate correctly the mixed number $1\frac{1}{4}$ on the number line?
5. Mixed Numbers and Improper Fractions on Number Lines
Yes No A. locate correctly $\frac{11}{4}$ on the number line?
6. Addition of Mixed Numbers with Like Denominators
Yes No A. add the fractions correctly in two of the three problems?
Yes No B. regroup the improper fractions to create whole numbers?
Yes No C. obtain the correct final sum in two of the three problems?
7. Mixed Numbers Meet Subtraction
Yes No A. get the correct answer on both problems a and b?
Yes No B. get the correct answer on problem c?
Yes No C. get the correct answer on problem d?

Items Correct = _____ = _____ % Score of 11 needed to pass to Fractions: Booklet 4.
Items Possible = 14 This is 11 or more Yes items.
8 - 10 Yes—review weak areas, retest, and move to
Fractions: Booklet 4.

Mixed Numbers to Improper Fractions

Purpose	The purpose of this lesson is to introduce the concept of mixed numbers and improper fractions and how to change a mixed number into an improper fraction using a manipulative. The student uses the familiar fraction piece that is used to show the whole and places it with one other smaller fraction piece. The whole is then traded for the same smaller fractional pieces and the total counted up and written as an improper fraction. This simple procedure is then recorded, analyzed, and constructed into an algorithm for calculating the improper fraction from the mixed number.
Note	While it seems much faster to just tell the student how to change a mixed number into an improper fraction,* doing so has two negative consequences. First, it prevents her from developing the skill of seeing patterns and formalizing them into mathematical procedures. This is what mathematicians do. It is where the fun is. Second, even for a student who does not easily create procedures from patterns, constructing a procedure with a teacher allows her to understand why a particular procedure works. If the procedure is simply memorized, it is likely to be filed in the brain with all of the other memorized procedures she will learn for fractions, and there are many of them. Accurate retrieval is then based solely on the skill of memory and no mathematical skill other than the multiplication tables. Many students get memorized procedures mixed up time and time again. In the long run, it is actually faster to have them construct understanding so they can reconstruct the procedure whenever needed.
Prerequisites	Division - Booklet 1, Fractions - Booklet 1, and Fractions - Booklet 2
Materials	Mixed Numbers to Improper Fractions - Worksheets 1 - 9, pages 3 - 11 Prism Fractions Circles (plus a paper set), or Prism Fractions Squares Pattern blocks
Warm Up	Make wholes with the fraction pieces. Discuss the pattern that to make one whole, the numerator and the denominator are equal. And, if you have more than one whole, the numerator will be a multiple of the denominator, for example, $\frac{2}{2}$, $\frac{4}{2}$, $\frac{6}{2}$.
Lesson Part 1	Mixed Numbers to Improper Fractions - Worksheet 1, page 3, is an instructional page for both the student and the teacher. Have her read it aloud. Do this page together. Then do the top row of problems on Mixed Numbers to Improper Fractions - Worksheet 2, page 4, together. On the second row, have her work on the problems with you watching. If she is secure, have her finish the worksheet alone and go on to Mixed Numbers to Improper Fractions - Worksheet 3, page 5. Check answers to be sure the work is done correctly.
Part 2	Mixed Numbers to Improper Fractions - Worksheet 4, page 6, switches the manipulative to pattern blocks and uses the changing whole. This requires her to reconstruct her understanding of the relationship between parts and wholes on every problem. *Multiply the denominator by the whole number and add the numerator. Place this number in the numerator and keep the denominator the same.

“If the green triangle is equal to the whole, then how many wholes would the blue rhombus be?” “Two wholes because two greens fit on it.” Record this on the worksheet.

“If the green triangle is equal to the whole, then how many wholes would the red trapezoid be?” “Three wholes because three greens fit on it.” Record this on the worksheet.

“What if the blue rhombus is equal to the whole, then what would the green triangle be?” “Only a half.” Record this on the worksheet. Finish this worksheet, check the work and stop.

Listen to what the student’s ideas are. Your job as a teacher is to figure out what her idea of the problem is. If you don’t understand what she says, work on it until you do. The student tells you her best understanding of the solution. It may or may not work in this case. Have her test the model.

- 1) It works. Test again. Will it always work? Where won’t it work?
 - 2) It doesn’t work. Test again. Where will it work? How is it different?
- Build a model, plan, test, test again. This is what learning is all about.

“Do you see any patterns that might help you figure out the new numerator without the fraction pieces?” Listen to the student’s answer. Eventually, she will figure out how many pieces are in the whole by realizing that the denominator tells how many pieces are in that whole. If there are two wholes, take the denominator and multiply it by two. Then add on whatever extra fractions there are. Do not push this concept if she does not see it herself.

Part 3

Creating an algorithm for converting a mixed number to an improper fraction is the goal of this next part of the lesson. It is done in two phases. The first phase allows for a student to construct the procedure from her own experience. The second phase leads students through the process of constructing a procedure using the blocks.

A few students will be able to independently create the process at this point. They will see that multiplying the denominator by the whole number tells you how many fractional pieces you will have when converting to an improper fraction.

Do not push. If she is unable to understand Mixed Numbers to Improper Fractions - Worksheet 5, page 7, go back and do more problems with either fraction pieces or pattern blocks, or both. Do not tell her the rule.

Worksheet

Mixed Numbers to Improper Fractions - Worksheet 6, page 8, walks the teacher through the thought steps to create a procedure, an algorithm, for converting a mixed number to an improper fraction.

Do this part orally with the student after reading Worksheet 6. Write the mixed number $1\frac{3}{4}$ on a piece of paper.







“Build this amount with your fraction set. What is the name of a number that includes both a whole number and a fraction.” “A mixed number.”

In our fraction manipulative set, the whole is one color and size, and the fourths are a different color and size. Now build $1\frac{3}{4}$ so that all the pieces are the same color and size.”

“How many fourths are in the fractional part?” (Point to the three-fourths.)
“Three.” “How many fourths are in the whole?” “Four.”

Then have her change the fraction to $\frac{7}{4}$ using the Prism Fractions Circles or Prism Fractions Squares.

She will have this:



Trade  for this  or trade   for this  

“How many fourths are in the one and three-fourths?” “Seven.”

“Write it.” $1\frac{3}{4} = \frac{7}{4}$. **“You changed a mixed number to an equal improper fraction. Excellent thinking.”**

“Now add on another whole ($2\frac{3}{4} = \frac{11}{4}$). How many fourths will be in the two wholes?” **“Eight because there are four fourths in each whole. Four two times is eight.”**

“What multiplication problem gives you the eight?” **“ 2×4 .”**

Trade  for this: 

“Where did the two come from?” **“From the two wholes.”**

“Where did the four come from?” **“We are working with fourths.”**

“Where does it tell you we are working with fourths?” **“From the bottom number, the denominator, of the fraction.”**

“How can you figure out how many fourths would be equal to two and three-fourths?” **“From the two wholes, I get eight fourths, plus there were already three there, so I have a total of eleven-fourths.”**

“How do you write eleven-fourths?” $\frac{11}{4}$.

Make up a few problems and have her change them. Now have her do Mixed Numbers to Improper Fractions - Worksheet 5, page 7. Have her write out the procedure using the correct words. If she has difficulty, do the whole process again.

Worksheet Mixed Numbers to Improper Fractions - Worksheet 7, page 9

Test for Understanding Mixed Numbers to Improper Fractions: Assessment - Worksheets 8 and 9, pages 10 and 11, are Tests for Understanding. Have her do Worksheet 8 alone. Correct it and remediate if needed. If Mixed Numbers to Improper Fractions - Worksheet 8 is done correctly, have her try Mixed Numbers to Improper Fractions - Worksheet 9 - Challenge. Assist only if needed. Her questions will tell you what she understands and what she does not understand. Have her use the fraction pieces to help solve the problems on this page. Answers are in the Answer Key.

Mixed Numbers and Improper Fractions on Number Lines

Purpose	The purpose of this lesson is to represent fractional quantities on number lines. Number lines are primarily used to put numbers in order of distance from zero. Fractional number lines are especially useful in measurement.
Prerequisites	<i>Patterns in Arithmetic:</i> Fractions - Booklet 1, Basic Concepts; My Fractions Booklet from Fractions - Booklet 1 is helpful. Or use Whole Numbers as Fractions: Number Lines and Numerators Greater Than One: Number Lines, pages 15 and 16, in this booklet, to review before this lesson.
Materials	Mixed Numbers and Improper Fractions on Number Lines - Worksheets 1 - 3, pages 12 - 14 Ruler
Note	If you are new to this series, do a lesson on fraction number lines before you do the lesson on mixed number number lines. See the next page for the lesson My Fractions Book: Number Lines, and pages 15 and 16.
Warm Up	<p>If you have a My Fractions Booklet that was made while doing Fractions - Booklet 1, use it to review fraction number lines.</p> <p>For review, use Whole Numbers as Fractions: Number Lines and Numerators Greater Than One: Number Lines, pages 15 and 16, in this booklet, for this lesson. Watch to see if he remembers to count the divisions between zero and one to determine the denominator of the fraction being shown on the line. If he writes three-eighths in the box on the top problem on page 12, he needs review on this topic. Let him find and correct his error by asking appropriate questions. The answers to Whole Numbers as Fractions: Number Lines and Numerators Greater Than One: Number Lines are on page 23 of this booklet.</p>
Lesson	Study the example on Mixed Numbers and Improper Fractions on Number Lines - Worksheet 1 together, then do the problem at the bottom of the page. Have him write in the missing numbers. Check the work with the Answer Key to be sure he is doing the work correctly.
Worksheets	Mixed Numbers and Improper Fractions on Number Lines - Worksheets 2 and 3, pages 13 and 14
Test for Understanding	Watch him do the line measurement at the bottom of Mixed Numbers and Improper Fractions on Number Lines - Worksheet 2. “How do you know which fraction to use?” “The denominators are sixths because there are six spaces between the lines. I count how many one-sixth spaces it is from zero to determine the numerator.”

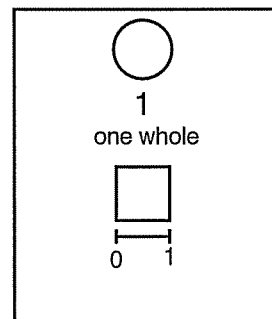
My Fractions Book: Number Lines

Purpose The purpose of this lesson is to represent fractions by points or distance on a number line.

Prerequisites Free exploration with Prism Fractions Square Set, string, tape measure, or rope

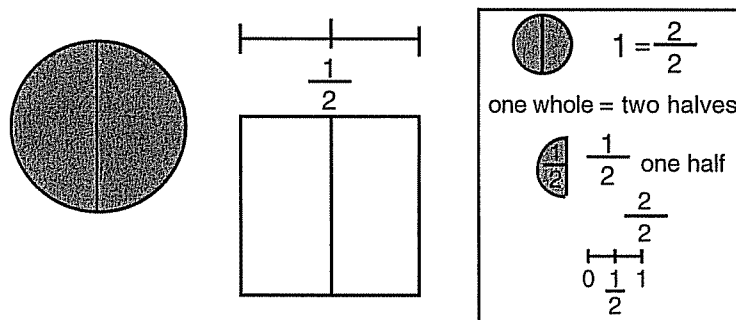
Materials Student's My Fractions Book (from Fractions: Booklet 1) or blank sheets of paper
Prism Fractions Square Set
Student review, pages 15 and 16 in this booklet Answers are on page 23 of this booklet.

Lesson First, find the page in My Fractions Book that shows the 1 = one whole fraction circle. Find the square that represents one whole and place it on the page. Trace the top edge of the square. Remove the square. Draw an end bar on each end of this line segment. This line will now equal one. Label under the left end bar the numeral 0. Label under the right end bar the numeral 1.



Draw a number line in My Fractions Book at the bottom of each whole number page.

Find the Prism Fractions square that shows halves. Check the edge of it against the line segment equal to one whole. **"Is it the same in length?"** Now go to the page of halves in My Fractions Book. Place the square on the page and trace the top edge of it and mark the middle point. Remove the square. Make it into a line segment with end bars. Label 0 and 1 with numerals. **"What fraction describes the distance from zero to the middle point? Label it underneath the point."**

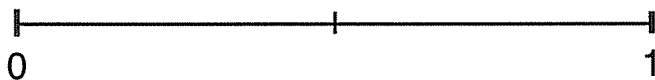


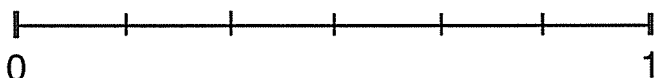
Note The space between zero and the point of one-half is equal to one-half of the distance of the whole. Start at zero and say, **"Zero."** Trace the distance (space) on the line with your finger. As you reach each number say, **"One-half, two-halves."**

Label $\frac{2}{2}$ above the end bar marked 1. Students frequently confuse the spaces with the points and label the end of the space. Throughout these number line exercises, focus on the spaces created by the division of the whole.

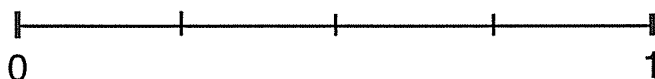
Continue this process of creating number lines for each set of fractions in My Fractions Book. The Prism Fractions Squares pieces will keep the length of an edge equal to one and have equal spacing for the fractional parts.

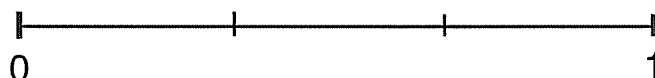
The line segment is divided into how many sections?

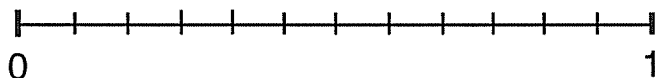




Count the spaces.

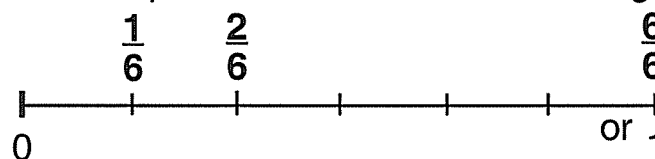






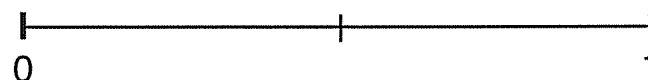
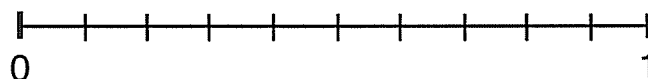
What fraction describes the distance along each line segment?

Write it in place on all the above line segments. For example:



The line segment is
divided into how
many sections?

Write the fraction that describes each point.



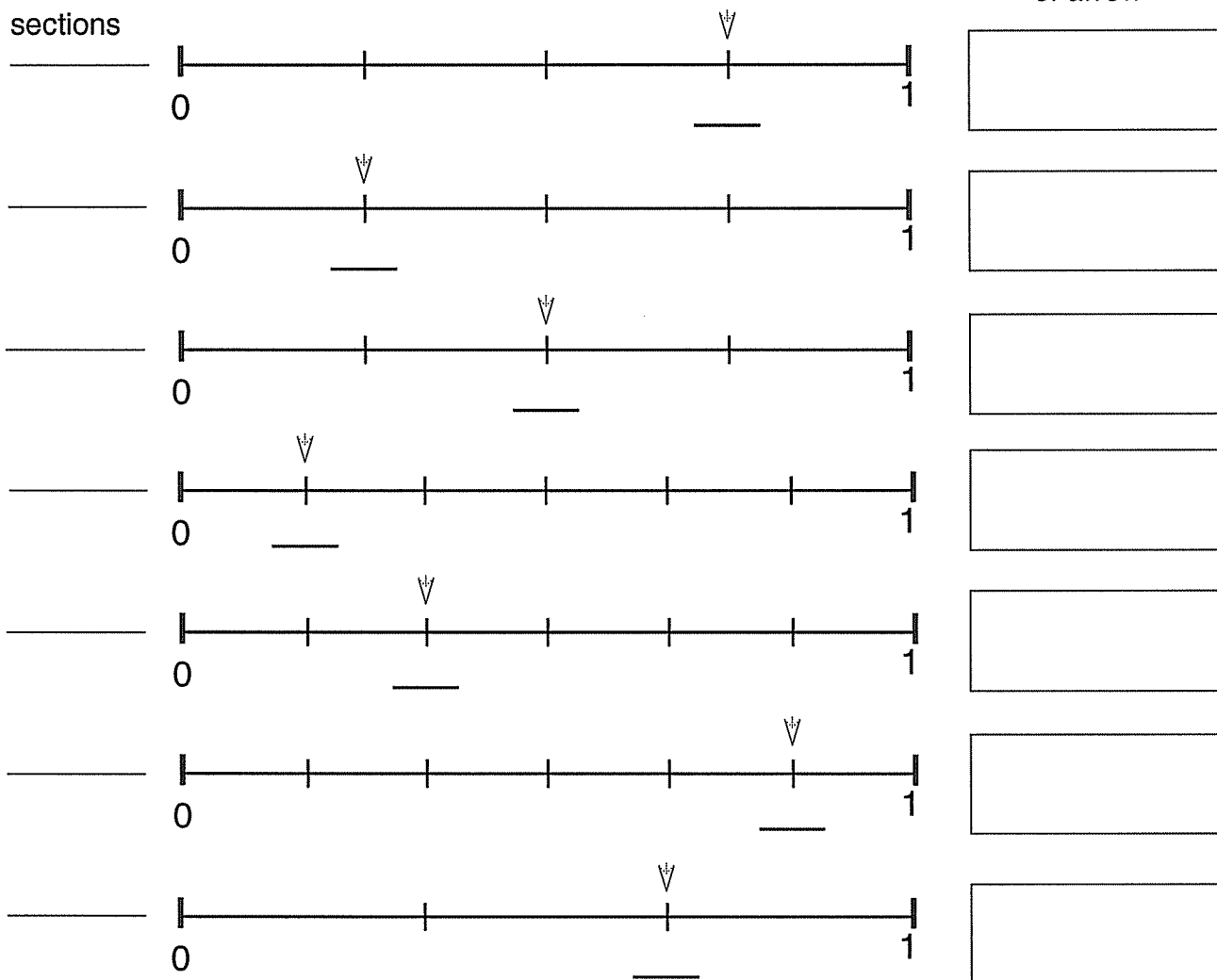
Numerators Greater Than One: Number Lines

Date _____

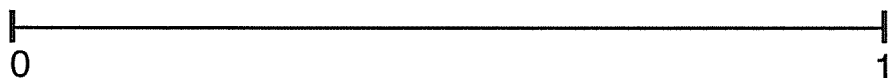
Write the fraction that describes the distance each point is along the number line.
Write the fractional part of the line shown at the arrow.

Number of
sections

Location
of arrow



Draw an X where $\frac{1}{3}$ is on the number line.



Draw a ☆ where $\frac{1}{2}$ is on the number line.



Make your own.



Improper Fractions to Mixed Numbers

Purpose	The purpose of this lesson is to provide another chance for the student to construct an algorithm from her own experience. She will construct improper fractions with the manipulative and then group them into wholes and trade them for the single whole piece. This grouping to make wholes is division and reverses the multiplication procedure used to change mixed numbers into improper fractions. This last piece of conceptual understanding should not be left out; it is a very important foundation for solving algebraic equations.
Prerequisites	Fractions: Booklet 1 - Basic Concepts, Fractions: Booklet 3 - Mixed Numbers to Improper Fractions, and Mixed Numbers to Improper Fractions on Number Lines
Materials	Improper Fractions to Mixed Numbers, Worksheets 1 - 15, pages 15 - 29 Fraction Circles or Prism Fractions
Note	The dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: "How did you get that?" "Show me what you mean." "Tell me more so I can understand what you are saying."
Warm Up	<p>Review the procedure she developed on Mixed Numbers to Improper Fractions - Worksheet 5, page 7. Solve a problem or two to make sure she is still controlling the concept, e.g., $2\frac{3}{4} = \frac{\quad}{\frac{11}{4}}$, $3\frac{3}{4} = \frac{\quad}{\frac{5}{4}}$.</p> <p>"What do you do?" "I multiply the denominator by the whole number and add the numerator and put this over the denominator."</p> <p>"What do you think will be needed in order to go backwards from an improper fraction to a mixed number?" She may have no idea. If this is the case, continue the lesson below. A very astute answer would be "Division and subtraction because division reverses multiplication and subtraction reverses addition." If she gives this answer, tell her to begin Improper Fractions to Mixed Numbers - Worksheet 1 and see if she can figure out how to get the correct answer. Build with the manipulative to confirm correctness.</p>
Lesson Part 1	<p>Improper Fractions to Mixed Numbers - Worksheet 1, page 15 "Build three halves. How many wholes can you trade for the half pieces you have on the table?" "Two halves can trade for one whole."</p> <p>"Notice that you now have two colors, one for the whole and one for the half. What do you call it when a number is made with whole numbers and a fraction?" "A mixed number."</p>

“Look at the second problem. It is four halves. What whole number is that?” “Two wholes, because I can make two groups of two with the halves to make two wholes.”

“The next problem adds another half. How many wholes can you make now?” “I can still make two wholes, but there will be one-half left over.”

“What mixed number comes out?” “Two and a half.” Watch her do two or three problems. Then let her finish alone.

Check the work. Do not tell her to divide. As always, she can discover this on her own.

Worksheets Improper Fractions to Mixed Numbers - Worksheets 2 - 5, pages 16 - 19

Lesson Part 2 Improper Fractions to Mixed Numbers - Worksheet 6, page 20, encourages her to write out the procedure for calculating converting an improper fraction to a mixed number based on her experience. If she can not do it yet, listen to what her ideas are. Your role as a teacher is to figure out what her idea of the problem is. If you don't understand what she says, work on it until you do. The student tells you her best understanding of the solution. It may or may not work in this case. Have her test the model.

1) It works. Test again. Will it always work? Where won't it work?

2) It doesn't work. Test again. Where will it work? How is it different?

Build a model, plan, test, test again. This is what learning is all about.

Worksheets Improper Fractions to Mixed Numbers - Worksheet 7, page 21, is an explanation page.

On Improper Fractions to Mixed Numbers - Worksheet 8, page 22, problem 3, she is asked to show 5 dollars and 3 quarters as a mixed number with no decimal. The answer is $5\frac{3}{4}$ dollars.

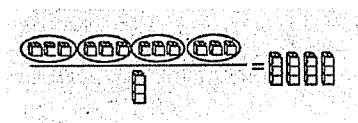
Lesson Part 3 Improper Fractions to Mixed Numbers - Worksheet 9 page 23, Fractions as Division Problems

Do this worksheet together.

dividend	=	quotient
divisor		(number of groups made)

Key to understanding fractions as division is the understanding of the meaning of the positions of the numbers in a division problem. The numeral under the line, the divisor, is an operator. It gives the instructions to make groups of a certain size out of the number of objects you start with. The number on top of the line, the dividend, is the number of objects you start with. The answer, the quotient, represents the *number of groups* that are made. It is important that the student understands the difference between the number in the dividend and the number in the quotient. The first is the number of individual objects, the second is a number of groups of objects. It is important to have the student clearly articulate these differences in order to establish a good level of understanding.

The student is to draw a picture of the blocks in the problem $\frac{12}{3}$. Have her explain what she is doing while doing the problem.



Practice Worksheets

Read Improper Fractions to Mixed Numbers - Worksheet 10, page 24, together if needed.

Improper Fractions to Mixed Numbers - Worksheets 11 - 14, pages 25 - 28
Worksheet 14, page 28, is a blank page for more practice as needed.

Test for Understanding

1. Do the assessment on Improper Fractions to Mixed Numbers: Assessment - Worksheet 15, page 29.

2. **“What two operations do you use to solve problem set 1?”** “They are multiplication and addition.”

“Why do you multiply the four and the five?” “Because the denominator tells you that there are five pieces in each whole. The four tells you there are four wholes. To figure out how many fifths there will be, you take fifths, four times, or 4×5 .”

“Four times five is twenty. How come the new numerator is twenty-two?”
“Because you have to add on the two fifths that were there with the four wholes.”

“So you multiply, then add. When you go to reverse this process, to go from $4\frac{2}{5}$ back to $4\frac{2}{5}$, what procedures do you use?” “Division.”

“You also used addition when you changed the mixed number to the improper fraction; where do you reverse that with subtraction?” “When you find the remainder of the division problem, you subtract.”

Addition of Mixed Numbers with Like Denominators

Purpose	The purpose of this lesson is to combine changing improper fractions to mixed numbers and addition of fractions with like denominators.
Prerequisites	Fractions: Booklet 2, Addition of Like Fractions Fractions: Booklet 3, Lessons 1 - 4, pages 10 - 19 in this booklet
Materials	Addition of Mixed Numbers with Like Denominators - Worksheets 1 and 2, pages 30 and 31 Prism Fractions Circles or Squares
Warm Up	Review converting mixed numbers to improper fractions and back again.
Lesson	<p>Ask the student to do the first problem at the top of Addition of Mixed Numbers with Like Denominators - Worksheet 1 and the second two problems at the top of Addition of Mixed Numbers with Like Denominators - Worksheet 2. Watch.</p> <p>See if he can figure out, on his own, how to handle the sum of the fractions that are improper. These need to be converted back to a mixed number and added again to the sum of the wholes.</p> <p>Some students will figure this out and some will not. Many will leave the sum as a whole number and an improper fraction. If this happens, prompt him to simplify the answer by showing the whole thing as one mixed number with no improper fractions. Go back now and do all of Addition of Mixed Numbers with Like Denominators - Worksheet 1.</p>
Worksheet	Addition of Mixed Numbers with Like Denominators - Worksheet 2, page 31

Mixed Numbers Meet Subtraction

Purpose	The purpose of this lesson is to combine changing improper fractions to mixed numbers, subtraction of fractions with regrouping, and subtraction of like fractions. Using a manipulative, the student will see that sometimes you need to take away a fraction from a whole number, which requires him to regroup or trade wholes into fractions.
Prerequisites	Subtraction of Like Fractions in Fractions - Booklet 2, previous lessons in this booklet
Materials	Mixed Numbers Meet Subtraction - Worksheets 1 and 2, pages 32 and 33 Pattern blocks Prism Fractions Circles or Squares Give a few subtraction problems with like denominators, e.g., $\frac{5}{8} - \frac{3}{8} = \underline{\hspace{1cm}}$, $\frac{3}{4} - \frac{1}{4} = \underline{\hspace{1cm}}$.
Lesson	<p>Review converting mixed numbers to improper fractions and back again, and addition of mixed numbers. Also give a regrouping subtraction problem such as $35 - 18 = \underline{\hspace{2cm}}$. “Explain how you solved this problem.”</p> <p>“I started with the thirty and traded it for two tens and ten ones and added that ten ones to the five to get fifteen. Then I crossed out the 3 and wrote 2, which stands for twenty. And then I subtracted the eight from the fifteen and the ten from the twenty.” Working with fractions uses the same idea.</p> <p>Offer as a challenge problem, “Two take away one-half.” See if he can work it out. Give him enough time. Ask him to let you know if he becomes stuck. Watch to see if he regroups the two into one and two halves. “One and one-half.” “Now, build two. Take away three-fourths.” “One and one-fourth.”</p> <p>Give him a slice of bread. “Give me half of the bread.” He will probably tear the slice in half and hand one over.</p> <p>Then, give him a one-dollar bill. “Give me half of the dollar.” “I need to make change.” This means he knows he can ‘break up’ the one-dollar bill into smaller pieces of total equal value so he can give you the one-half.</p> <p>“Why didn’t you just tear the dollar bill in half?” “Because it would destroy the value of the money, so I traded the dollar bill for quarters.”</p> <p>“So, you traded the one-dollar bill for four quarters and then gave me half of the quarters because you knew you could break up the dollar bill into parts.”</p> <p>Put a yellow pattern block hexagon in front of him.</p> <p>“Take away one-half of the hexagon.” He will almost certainly trade the yellow hexagon for two red trapezoids and give you one.</p>
Worksheets	Do Mixed Numbers Meet Subtraction - Worksheet 1, page 32, together. What is happening is that the whole piece is being regrouped into whatever fractional form is needed. Have him build the first number and then regroup and take away. Mixed Numbers Meet Subtraction - Worksheet 2, page 33 Check the work.

Subtraction of Mixed Numbers with Regrouping

Purpose	The purpose of this lesson is to add to the complexity of subtracting a large fraction from a mixed number. In this lesson the fraction attached to the whole is less than the larger fraction to be taken away. This lesson allows the student to do some original thinking and use what she knows to find a solution to a new kind of problem.
Prerequisites	Fractions: Booklet 2, all the previous lessons in this booklet
Materials	Subtraction of Mixed Numbers with Regrouping - Worksheet 1, page 34 Prism Fractions Circles or Squares
Warm Up	Do the top two problems of Subtraction of Mixed Numbers with Regrouping - Worksheet 1. Just watch and see what she does with the third problem. It is an excellent test for understanding.
Lesson	<p>What you are looking for is the understanding that the three wholes can be traded for some extra thirds. Several things can happen here.</p> <ol style="list-style-type: none">1) She may trade all the wholes for thirds, remove two-thirds and a whole or five thirds and reassemble the pieces into a mixed number. If she does this, accept the answer and ask, “How many trades did you make? Can you do it with fewer trades? What is the fewest number of trades you can make?”2) She may trade only one whole for thirds and then remove two-thirds and then remove a whole. This is the response that moves towards the standard procedure.3) She may do it in her head. Ask her to clarify which of the above two <p>Any of these solutions are reasonable. Have her complete the page.</p> <p>If she can't solve the problem, make the problem a little simpler and more like the problems in the last lesson, Mixed Numbers Meet Subtraction. “Build one and one-third and remove two-thirds.” Hopefully this will remind her that she can trade wholes to get more of the fractional parts she needs.</p> <p>“Where can you get more thirds so you can remove two-thirds from the one and one-third?”</p> <p>“Can you break up the one whole to get more thirds?” “I can trade the one whole for three-thirds, and then I can take two away.”</p> <p>Then try the larger problem again. Have her build three and one-third. Then have her trade to get more thirds, followed by removing the one and two-thirds. Always do all of the trades before you do the subtraction. Stop when Subtraction of Mixed Numbers with Regrouping - Worksheet 1 is completed.</p>

Lesson 2

The next day, do Subtraction of Mixed Numbers with Regrouping - Worksheet 2, page 35.

Worksheet

Subtraction of Mixed Numbers with Regrouping - Worksheet 3, page 36

Check for accuracy and remediate any difficulties.

Test for Understanding

Post-Assessment, page 37.

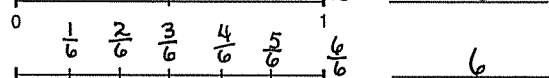
Answers are for pages 15 and 16 in this booklet.

Whole Numbers as Fractions: Number Lines

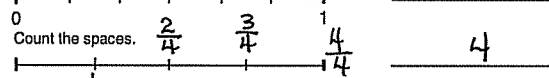
The line segment is divided into how many sections?



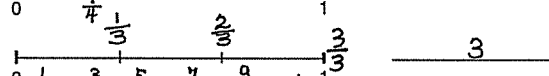
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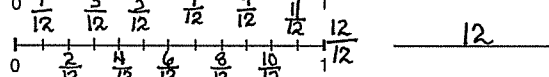
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4



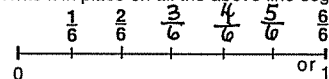
3



12

What fraction describes the distance along each line segment?

Write it in place on all the above line segments. For example:



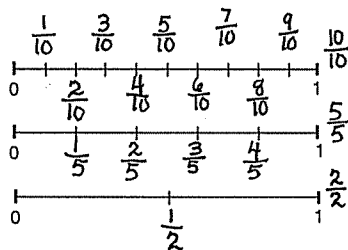
The line segment is divided into how many sections?

10

5

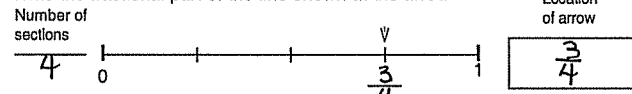
2

Write the fraction that describes each point.



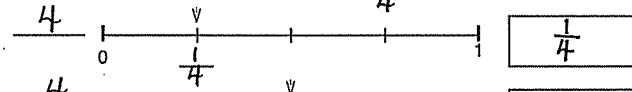
Numerators Greater Than One: Number Lines

Write the fraction that describes the distance each point is along the number. Write the fractional part of the line shown at the arrow.

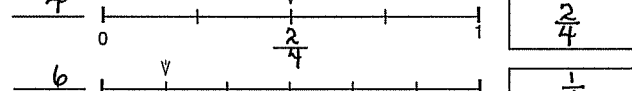


Location of arrow

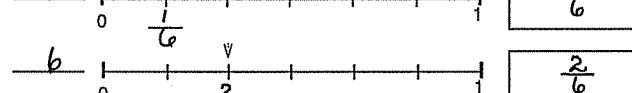
$\frac{3}{4}$



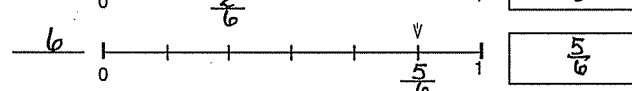
$\frac{1}{4}$



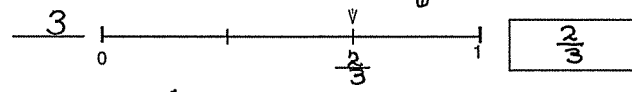
$\frac{2}{4}$



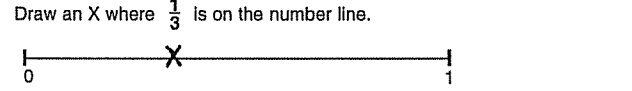
$\frac{1}{6}$



$\frac{2}{6}$



$\frac{5}{6}$



$\frac{2}{3}$

Draw an X where $\frac{1}{3}$ is on the number line.



Draw a ☆ where $\frac{1}{2}$ is on the number line.



Make your own.



1. Put a box around the improper fractions. Put a circle around the mixed numbers.

$$2\frac{2}{3} = \quad \frac{1}{2} = \quad \frac{3}{2} = \quad \frac{3}{4} = \quad \frac{5}{4} = \quad 3\frac{3}{4} =$$

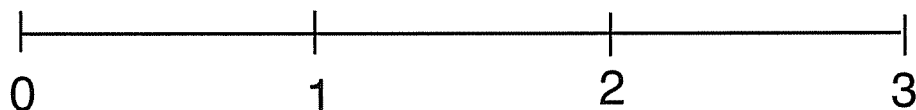
2. Change these mixed numbers to improper fractions.

$$\text{a. } 3\frac{1}{2} = \quad \text{b. } 2\frac{3}{4} = \quad \text{c. } 4\frac{5}{6} =$$

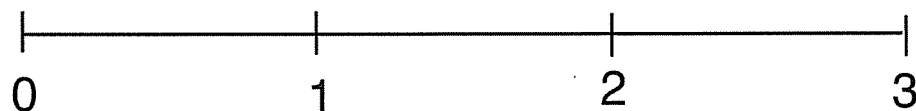
3. Change these improper fractions to mixed numbers.

$$\text{a. } \frac{7}{3} = \quad \text{b. } \frac{\quad}{\quad} = \frac{3}{2} \quad \text{c. } \frac{10}{4} =$$

4. Put $1\frac{1}{4}$ on this number line.



5. Put $\frac{11}{4}$ on this number line.



6. Solve.

$$\begin{array}{r} \text{a. } 1\frac{1}{3} \\ + 2\frac{2}{3} \\ \hline \end{array} \quad \begin{array}{r} \text{b. } 2\frac{2}{5} \\ + 1\frac{4}{5} \\ \hline \end{array} \quad \begin{array}{r} \text{c. } 11\frac{8}{9} \\ + 19\frac{8}{9} \\ \hline \end{array}$$

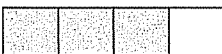
7. Solve.


$$\begin{array}{r} \text{a. } 4 - \frac{2}{3} = \\ \hline \end{array} \quad \begin{array}{r} \text{b. } 5\frac{2}{3} \\ - 2\frac{1}{3} \\ \hline \end{array} \quad \begin{array}{r} \text{c. } 5\frac{1}{4} \\ - 1\frac{2}{4} \\ \hline \end{array} \quad \begin{array}{r} \text{d. } 20\frac{1}{10} \\ - 6\frac{7}{10} \\ \hline \end{array}$$

Fractions - Booklet 3



Pre-Assessment - Part 1

The student may use a manipulative for the test.

1. Shade in $\frac{3}{4}$ of this rectangle. 

2. Gail cut a pizza into six slices. 

Natalia ate 1 slice. What fraction of the pizza did she eat? a. $\frac{1}{6}$
Frank ate 4 slices. What fraction of the pizza did he eat? b. $\frac{4}{6}$
How much of the pizza has been eaten? Number sentence. c. $\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$
How much of the pizza is left for Gail? Number sentence. d. $\frac{6}{6} - \frac{5}{6} = \frac{1}{6}$

3. If  equals 1, what does  equal? $\frac{2}{3}$

On the problems below you may use fraction pieces. Circle any problem you can do without using fraction pieces.

4. Fill in the missing numbers.
a. $\frac{1}{2} = \frac{2}{4}$ b. $\frac{1}{3} = \frac{4}{12}$ c. $\frac{3}{4} = \frac{6}{8}$

5. Write a $<$, $>$, or $=$ sign (Greater Than, Less Than, or Equal To) between these pairs of fractions.

a. $\frac{1}{2} > \frac{1}{3}$ b. $\frac{2}{6} = \frac{1}{3}$ c. $\frac{3}{4} > \frac{2}{3}$

6. Solve.

a. $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$ b. $\frac{7}{8} - \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$ c. $7\frac{3}{4} - 3\frac{1}{4} = 3\frac{2}{4} = 3\frac{1}{2}$

7. Solve.

a. $\frac{1}{4}$ of 12 = 3 b. $\frac{2}{3}$ of 18 = 12 c. $\frac{3}{5} \times 25 = 15$

1

Pre-Assessment - Part 2

1. Put a box around the improper fractions. Put a circle around the mixed numbers.

$2\frac{2}{3} =$ $\frac{1}{2} =$ $\frac{3}{2} =$ $\frac{3}{4} =$ $\frac{5}{4} =$ $3\frac{3}{4} =$

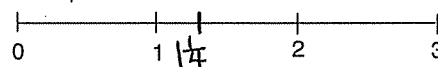
2. Change these mixed numbers to improper fractions.

a. $3\frac{1}{2} = \frac{7}{2}$ b. $2\frac{3}{4} = \frac{11}{4}$ c. $4\frac{5}{6} = \frac{29}{6}$

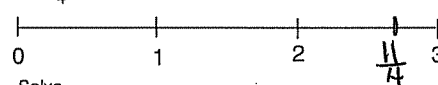
3. Change these improper fractions to mixed numbers.

a. $\frac{7}{3} = 2\frac{1}{3}$ b. $\frac{11}{2} = 5\frac{1}{2}$ c. $\frac{10}{4} = 2\frac{2}{4} = 2\frac{1}{2}$

4. Put $1\frac{1}{4}$ on this number



5. Put $\frac{11}{4}$ on this number



6. Solve.

a. $1\frac{1}{3} + 2\frac{2}{3} = 3\frac{3}{3} = 4$ b. $2\frac{2}{5} + 1\frac{4}{5} = 3\frac{6}{5} = 4\frac{1}{5}$ c. $11\frac{8}{9} + 19\frac{8}{9} = 30\frac{16}{9} = 31\frac{7}{9}$

7. Solve.


a. $4 - \frac{2}{3} = 3\frac{1}{3}$ b. $5\frac{2}{3} - 2\frac{1}{3} = 3\frac{1}{3}$ c. $\frac{4}{5} - \frac{1}{4} = \frac{16}{20} - \frac{5}{20} = \frac{11}{20}$ d. $20\frac{1}{10} - 6\frac{7}{10} = 13\frac{4}{10}$

2

Mixed Numbers to Improper Fractions - Worksheet 1

This is a mixed number $1\frac{2}{3}$. What do you think is the definition of a mixed number? A whole number and a fraction.

Take out Fraction pieces.

Pick up a whole and $\frac{1}{4}$. 

Now cover the whole piece with more fourths.



You now have five fourths.

This kind of fraction is written like this. $\frac{5}{4}$

This kind of fraction is called an **Improper Fraction**.

It is called improper because the numerator (the top number) is larger than the denominator (the bottom number). Improper fractions are worth more than one.

Now take out two wholes and $\frac{1}{3}$.



Cover the two wholes with thirds.



How many thirds do you have now? 7 $2\frac{1}{3} = \frac{7}{3}$

Write nine thirds as an improper fraction. $\frac{9}{3}$

What does the denominator or bottom number describe? How many equal parts into which the whole is divided.

What does the numerator or top number describe? How many parts of the whole there are in the fraction.

3

Mixed Numbers to Improper Fractions - Worksheet 2

Build with Fraction pieces. Then record the improper fractions.



$1\frac{1}{2} = \frac{3}{2}$

Look for patterns.

$1\frac{2}{3} = \frac{5}{3}$

$2\frac{1}{4} = \frac{9}{4}$

$3\frac{2}{3} = \frac{11}{3}$

$2\frac{2}{5} = \frac{12}{5}$

$2\frac{3}{4} = \frac{11}{4}$

$1\frac{1}{3} = \frac{4}{3}$

$1\frac{3}{8} = \frac{11}{8}$

$3\frac{1}{6} = \frac{19}{6}$

$3\frac{1}{4} = \frac{13}{4}$

$2\frac{5}{6} = \frac{17}{6}$

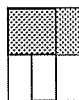
$1\frac{5}{12} = \frac{17}{12}$

$1\frac{6}{8} = \frac{14}{8}$

4

Mixed Numbers to Improper Fractions - Worksheet 3

Build with Fraction pieces.
Then record the improper fractions.



$$1\frac{1}{2} = \frac{3}{2}$$

Look for patterns.

$$2\frac{2}{3} = \frac{8}{3}$$

$$2\frac{3}{4} = \frac{11}{4}$$

$$2\frac{3}{8} = \frac{19}{8}$$

$$2\frac{4}{5} = \frac{14}{5}$$

$$3\frac{3}{4} = \frac{15}{4}$$

$$4\frac{2}{3} = \frac{14}{3}$$

$$2\frac{5}{8} = \frac{21}{8}$$

$$2\frac{3}{6} = \frac{15}{6}$$

$$1\frac{3}{4} = \frac{7}{4}$$

$$1\frac{2}{6} = \frac{8}{6}$$

$$2\frac{4}{12} = \frac{28}{12}$$

$$2\frac{1}{8} = \frac{17}{8}$$

5

Mixed Numbers to Improper Fractions - Worksheet 5

Patterns are used to create algorithms for changing mixed numbers to improper fractions. An algorithm is a math recipe.

$$2\frac{3}{4} = \frac{\quad}{4}$$

Fill in the missing numerator (top number) on the problem above.

Have you discovered an easy way to figure out what the missing number is without using the fraction pieces?

If so, please explain how you do it below. Using math words will help you explain. Here are some words which might be helpful.

add subtract multiply divide sum product
numerator denominator

AWV

Multiply the denominator times the whole number. Add the numerator and place the sum over the denominator.

Check to see if your pattern works on these problems.
Does the pattern work the same way every time?

$$3\frac{2}{5} = \frac{17}{5}$$

$$4\frac{2}{3} = \frac{14}{3}$$

7

Mixed Numbers to Improper Fractions - Worksheet 4

If this equals one, then is 2.

If this equals one, then is 3.

If this equals one, then is 1/2.

If this equals one, then is 1 1/2.

If this equals one, then is 2 1/2.

If this equals one, then is 1/3.

If this equals one, then is 2/3.

If this equals one, then is 1 1/3.

If this equals one, then is 1 2/3.

If this is 1/3, then draw 1.

If this is 1/3, then draw 1 1/3

If this is 1/3, then draw 2/3

If this is 1/4, then draw 1.

If this is 1/4, then draw 1 3/4

If this is 1/4, what is this? 1 2/4
Guess first _____

6

Mixed Numbers to Improper Fractions - Worksheet 6 Teacher Instructions

Use patterns to create algorithms for changing mixed numbers to improper fractions

$$3\frac{4}{5} = \frac{19}{5}$$

How do you figure out the missing numerator of the fraction?

Fill in the missing words.

You are going to think in your head, "How many fifths are in one whole?"

There are 5. So then, how many fifths will be in three wholes? 15

If you used your times tables you would do the problem 5 x 3 = 15

You got the answer fifteen. Fifteen whats? fifteen fifths

Now we know there are fifteen fifths in the whole number three.

What do we do with the 4 in the numerator? add it to the fifteen

You probably realized that you add the 4 onto the fifteen. This gives you a total of nineteen fifths.

Challenge: $2\frac{4}{7} = \frac{18}{7}$

How do you know what the denominator of the improper fraction will be?

The same one is kept.

Practice: Make up some mixed numbers and change them to improper fractions

8

Mixed Numbers to Improper Fractions - Worksheet 7

Change the mixed numbers to improper fractions using the calculation method you developed.

$$3\frac{2}{3} = \frac{11}{3} \quad 2\frac{3}{4} = \frac{11}{4} \quad 2\frac{5}{8} = \frac{21}{8}$$

$$2\frac{3}{5} = \frac{13}{5} \quad 3\frac{3}{4} = \frac{15}{4} \quad 1\frac{1}{3} = \frac{4}{3}$$

$$2\frac{3}{8} = \frac{19}{8} \quad 2\frac{5}{6} = \frac{17}{6} \quad 4\frac{3}{4} = \frac{19}{4}$$

$$1\frac{4}{6} = \frac{10}{6} \quad 2\frac{5}{12} = \frac{29}{12} \quad 2\frac{3}{8} = \frac{19}{8}$$

$$3\frac{1}{8} = \frac{25}{8} \quad 3\frac{1}{6} = \frac{19}{6} \quad 5\frac{1}{4} = \frac{21}{4}$$

$$2\frac{7}{8} = \frac{23}{8} \quad 3\frac{5}{8} = \frac{29}{8} \quad 3\frac{1}{4} = \frac{13}{4}$$

9

Mixed Numbers to Improper Fractions - Worksheet 9 Challenge!

In the recipe for baking combread, it says to thoroughly mix oil and milk in a cup. The container holds exactly one cup of liquid. If $\frac{2}{3}$ cup of oil and $\frac{2}{3}$ cup of milk are mixed, will the cup overflow? yes Defend your answer.

$\frac{2}{3}$ cup + $\frac{2}{3}$ c is more than 1 cup. It is $1\frac{1}{3}$ cups

How much over or under, a little or a lot? a little over

What if $\frac{1}{3}$ cup of milk and $\frac{1}{3}$ cup of oil are mixed, will the cup overflow? no

How much over or under, a little or a lot? under a little

What if $\frac{4}{4}$ cup of milk and $\frac{3}{4}$ cup of oil are mixed, will the cup overflow? yes

What size cup would be needed? 2 cups

Drawing a picture will help you solve these problems.

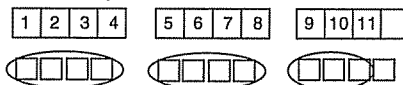
11

Mixed Numbers to Improper Fractions: Assessment Worksheet 8 - Show You Know

- What is an improper fraction? The numerator is larger than the denominator and it equals more than one.
- What is a mixed number? When there is both a whole number and a fraction.

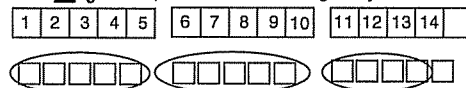
- Change $2\frac{3}{4}$ to an improper fraction. $\frac{11}{4}$ (2×4) + 3 = 11

- Draw a picture of $2\frac{3}{4}$ and prove with the drawing that your answer is correct.



- Change this mixed number into an improper fraction. $2\frac{4}{5} = \frac{14}{5}$

- Draw a picture of $2\frac{4}{5}$ and prove with the drawing that your answer is correct.



- You are having pizza for dinner. You have 3 and $\frac{1}{2}$ pizzas. You want to give each person $\frac{1}{4}$ of a family-sized pizza for their dinner. How many people will this feed? 14

Once you cut the pizza up, do you still have 3 $\frac{1}{2}$ pizzas? Prove it.

Yes, you can break a number up into smaller parts and the total doesn't change.



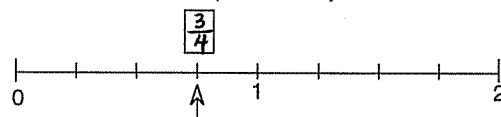
You want to give each person $\frac{1}{3}$ of a pizza and you have 10 people. How many pizzas would you need to order? 4

Would you have any left over? yes If so, how much? $\frac{2}{3}$

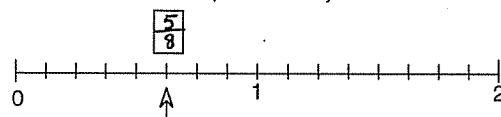
10

Mixed Numbers and Improper Fractions on Number Lines - Worksheet 1 Review

What fraction does the arrow point to? Put your answer in the box.



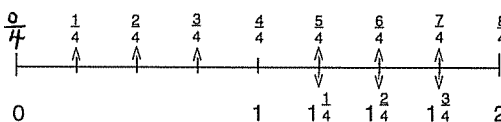
What fraction does the arrow point to? Put your answer in the box.



Mixed numbers and improper fractions can be shown on number lines.

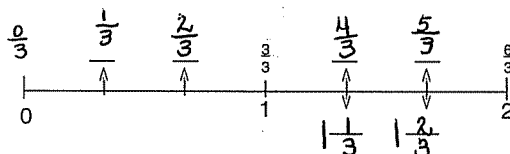
Focus your attention on the part of the number line between the 1 and 2.

Write the fractions above the number line.



Write the mixed numbers below the number line.

Write in the fractions and mixed numbers on the lines. What is the fractional unit? thirds

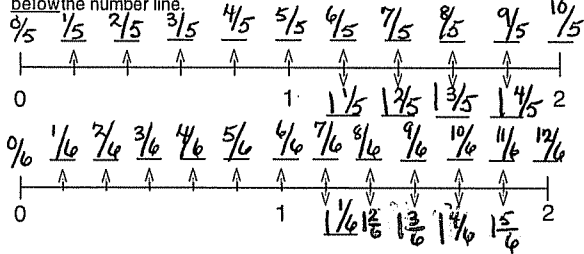


12

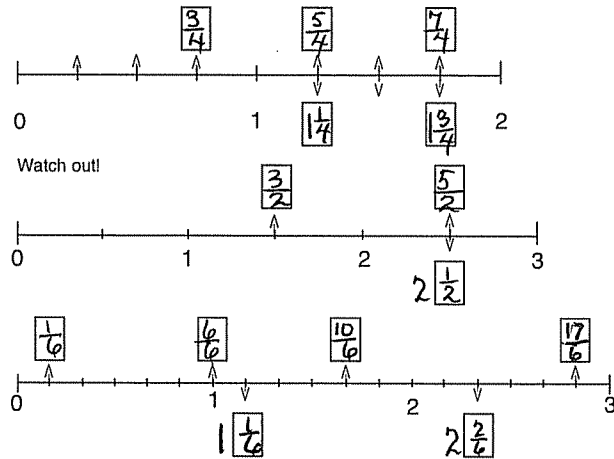
Mixed Numbers and Improper Fractions on Number Lines - Worksheet 2

Remember, count the spaces to get the denominator.

Write the fractions above the number line. Write the mixed numbers below the number line.



Write the correct number in each box.

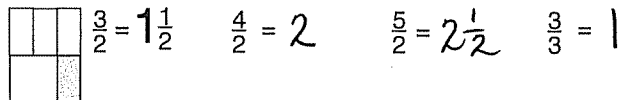
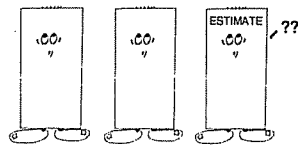


Watch out!

13

Improper Fractions to Mixed Numbers - Worksheet 1

Build with fraction pieces.
Draw a picture.
Record the mixed number.



$$\frac{4}{3} = 1\frac{1}{3} \quad \frac{5}{2} = 2\frac{1}{2} \quad \frac{6}{3} = 2 \quad \frac{4}{4} = 1$$

$$\frac{5}{4} = 1\frac{1}{4} \quad \frac{6}{4} = 1\frac{2}{4} \quad \frac{7}{4} = 1\frac{3}{4} \quad \frac{8}{4} = 2$$

$$\frac{5}{5} = 1 \quad \frac{7}{5} = 1\frac{2}{5} \quad \frac{10}{5} = 2 \quad \frac{11}{6} = 1\frac{5}{6}$$

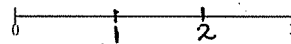
$$\frac{10}{10} = 1 \quad \frac{20}{10} = 2 \quad \frac{8}{3} = 2\frac{2}{3}$$

15

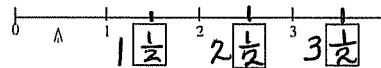
Mixed Numbers and Improper Fractions on Number Lines - Worksheet 3

Use your acetate inch rulers to help you.

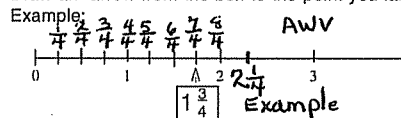
Rulers are Number Lines. When we read a line distance, we might say "It's 2 inches long." This line is 3 inches long. Draw and label the 1 and 2 inch marks.



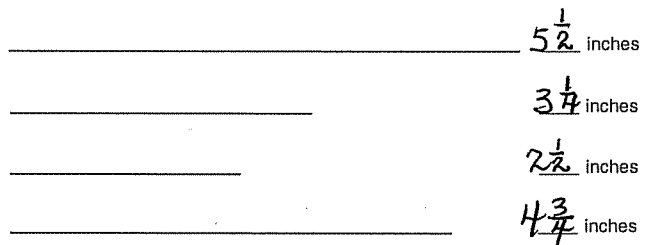
This line is 4 inches long. Draw in the $\frac{1}{2}$ inch marks. Write the mixed fraction below the line.



Draw in the $\frac{1}{4}$ inch marks. Label one mixed number for each one inch section. Draw an arrow from the box to the point you labeled.



How long is each line?



14

Improper Fractions to Mixed Numbers - Worksheet 2

Do not require students to simplify fractions.

Build with Fraction pieces. Record the mixed number.

$$\frac{6}{3} = 2 \quad \frac{7}{3} = 2\frac{1}{3} \quad \frac{8}{3} = 2\frac{2}{3} \quad \frac{9}{3} = 3$$

$$\frac{7}{6} = 1\frac{1}{6} \quad \frac{9}{6} = 1\frac{3}{6} \quad \frac{11}{6} = 1\frac{5}{6} \quad \frac{13}{6} = 2\frac{1}{6}$$

$$\frac{12}{8} = 1\frac{3}{8} \quad \frac{15}{8} = 1\frac{7}{8} \quad \frac{19}{8} = 2\frac{3}{8} \quad \frac{23}{8} = 2\frac{7}{8}$$

$$\frac{13}{12} = 1\frac{1}{12} \quad \frac{18}{12} = 1\frac{6}{12} \quad \frac{23}{12} = 1\frac{11}{12} \quad \frac{28}{12} = 2\frac{4}{12}$$

16

Improper Fractions to Mixed Numbers - Worksheet 3

1. What improper fraction is being shown in this drawing? $\frac{7}{5}$
Each rectangle stands for one whole.



2. What mixed number is being shown in the drawing above? $1\frac{2}{5}$

3. Draw a picture of this fraction. $\frac{12}{8}$



4. Draw a picture of each improper fraction below.
Then change these improper fractions into mixed numbers.

$\frac{9}{4} = 2\frac{1}{4}$

$\frac{15}{8} = 1\frac{7}{8}$

$\frac{22}{3} = 7\frac{1}{3}$

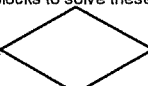



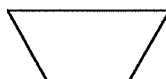
$\frac{17}{5} = 3\frac{2}{5}$

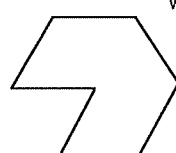
17

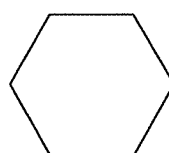
Improper Fractions to Mixed Numbers - Worksheet 5

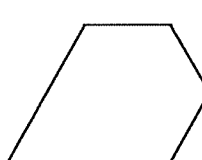
Use pattern blocks to solve these problems.

If this is one,  then  this is $\frac{1}{2}$.

 This takes 3 greens to cover which equals $\frac{3}{2}$.
It also can be covered with 1 blue and 1 green which equals $1\frac{1}{2}$.

 This takes 5 greens to cover which equals $\frac{5}{2}$.
It also can be covered with 2 blue(s) and 1 green(s) which equals $2\frac{1}{2}$.

 This takes 6 greens to cover which equals $\frac{6}{2}$.
It also can be covered with 3 blue(s) and 0 green(s) which equals 3.


 This takes 7 greens to cover which equals $\frac{7}{2}$.
It also can be covered with 3 blue(s) and 1 green(s) which equals $3\frac{1}{2}$.

19

Improper Fractions to Mixed Numbers - Worksheet 4

Build with fraction pieces.

Name these fractions.

This is one. 



$\frac{3}{2}$

improper fraction



$1\frac{1}{2}$

mixed number



$\frac{5}{2}$

improper fraction



$2\frac{1}{2}$

mixed number



$\frac{7}{4}$

improper fraction



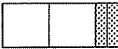
$1\frac{3}{4}$

mixed number



$\frac{10}{4}$

improper fraction



$2\frac{2}{4}$

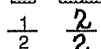
mixed number

Record these fractions.

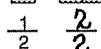


$\frac{1}{2}$

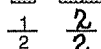
Make your own.



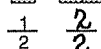
$\frac{2}{2}$



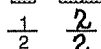
$\frac{3}{2}$



$\frac{4}{2}$



$\frac{5}{2}$



$\frac{6}{2}$

18

Improper Fractions to Mixed Numbers - Worksheet 6

Using patterns to create the algorithm for converting an improper fraction to a mixed number:

Change these improper fractions to mixed numbers:

$\frac{5}{2} = 2\frac{1}{2}$

$\frac{6}{4} = 1\frac{2}{4}$

Did you find a fast way to calculate how to change an improper fraction to a mixed number?

If so, explain how you did one of the problems above.

It will be easier to explain your pattern if you use math language. Here are some words that will help: product sum denominator numerator

Drawing a picture will also help you explain your pattern.

If you have 5 halves,
how many wholes can you
make? Divide 5 by 2,
which equals 2 wholes and
1 left over. The left over
part is half (1/2).

20

Improper Fractions to Mixed Numbers - Worksheet 7

Just to be sure, let's go through this procedure again. We are creating a way to calculate the mixed number that is equal to a given improper fraction.

$$\frac{11}{4} = 2 \frac{3}{4}$$

Use the problem above to fill in the blanks below. To change this improper fraction of $\frac{11}{4}$ into the mixed number $2 \frac{3}{4}$, you think about how many fourths are in one whole. That would mean you are taking your 11 fourths and grouping them into wholes. When we make equal groups out of a number are we adding? subtracting? multiplying? or dividing? Circle one.

So you start dividing the 11 fourths into groups of 4. You will get 2 groups of 4 with a remainder of 3.

So you now know that you have 2 wholes and 3 pieces left over.

What do you do with the remainder of 3? Make it into a fraction.

You think that these 3 remaining pieces are fourths. Three pieces are not enough to make another whole. So we are left with the fraction of $\frac{3}{4}$.

So the mixed number that is equal to the 11 fourths is 2 wholes and 3 fourths. Write this in numerals. $2 \frac{3}{4}$

Eleven-fourths is equal to two and three-fourths.

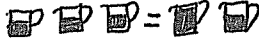
Now, test yourself. Try these problems. Check right away to be sure you are doing it correctly.

$$\frac{13}{4} = 3 \frac{1}{4} \quad \frac{11}{5} = 2 \frac{1}{5} \quad \frac{19}{3} = 6 \frac{1}{3}$$

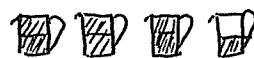
21

Improper Fractions to Mixed Numbers - Worksheet 8

1. A recipe calls for $\frac{1}{2}$ cup of butter. You want to triple the recipe.

How many cups of butter will you need? $1 \frac{1}{2}$ or $\frac{3}{2}$
Draw a picture of your measuring cup filled with butter. 

2. If you had three and one-half cups of cookie dough and you want to give each child half a cup, how many children could you serve? 7 children
A drawing will help you solve this problem.

 There are 7 half cups

3. You have a 5 dollar bill and 3 quarters.
Write this amount as a mixed number. Do not use a decimal. \$ $5 \frac{3}{4}$

4. You need to wash your clothes at a coin-operated laundromat. You can only use quarters in the washing machines. Rewrite the mixed number from problem 3, and change it into an improper fraction to show how many quarters you will end up with.

What denominator do you use for quarters? 4 $5 \frac{3}{4} = \frac{23}{4}$

5. Two step problem.

Each load of laundry costs 75¢. If you do as many loads as you can with the money you have, how much money will you have left over? 2 quarters.
What do you have to figure out first? How many $\frac{3}{4}$ are in 23 fourths.

Then what? There are 7 three fourths in 23 fourths.

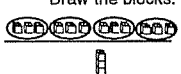
6. Challenge!! What fraction of your original amount do you have left?

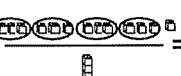
$$\frac{2}{23}$$


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Improper Fractions to Mixed Numbers - Worksheet 9

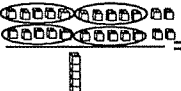
Show these fractions as division problems. $\frac{\text{dividend}}{\text{divisor}} = \text{quotient (number of groups made)}$
Show all remainders as fractions.

$\frac{12}{3} = 12 \div 3 = 3$ 4
Draw the blocks.  Quotient (Answer) 4

$\frac{13}{3} = 13 \div 3 = 3$ $4 \frac{1}{3}$
 Quotient (Answer) $4 \frac{1}{3}$

$\frac{15}{6} = 15 \div 6 = 6$ $2 \frac{3}{6}$
 Quotient (Answer) $2 \frac{3}{6}$

$\frac{11}{4} = 11 \div 4 = 4$ $2 \frac{3}{4}$
 Quotient (Answer) $2 \frac{3}{4}$

$\frac{24}{5} = 24 \div 5 = 5$ $4 \frac{4}{5}$
 Quotient (Answer) $4 \frac{4}{5}$

Don't draw this one.

$\frac{100}{10} = 100 \div 10 = 10$ 10

23

Improper Fractions to Mixed Numbers - Worksheet 10

Using Division to Change Improper Fractions to Mixed Numbers

$$\frac{11}{4} = 2 \frac{3}{4}$$

The improper fraction $\frac{11}{4}$ is a division problem which can be read

as $11 \div 4$. The answer to $11 \div 4$ is 2 with a remainder of 3.

$$\begin{array}{r} 2 \text{ r } 3 \\ 4 \overline{) 11} \\ \underline{- 8} \\ 3 \end{array}$$

When we are working with fractions, we do something new with the remainder. In a division problem, the divisor (the 4) is the same thing as a denominator in fractions. $11 \div 4$ is the same thing as $\frac{11}{4}$.

So when we are working with fractions, we think of the 4 in the division problem as fourths. We are working with 11 fourths. So when we divide, we get 2 wholes and with three fourths left over.

$$\begin{array}{r} 2 \text{ r } 3 \\ 4 \overline{) 11} \\ \underline{- 8} \\ 3 \end{array} = 4 \overline{) \frac{11}{4}} \begin{array}{r} 2 \text{ r } 3 \\ \underline{- 8} \\ 3 \end{array}$$

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Improper Fractions to Mixed Numbers - Worksheet 11

Use division to change improper fractions to mixed numbers.

$$\frac{11}{4} = 4 \overline{) \begin{array}{r} 11 \\ -8 \\ \hline 3 \end{array}} \quad \frac{11}{4} = 2 \frac{3}{4}$$

Sometimes we want to show the remainder as a fraction. This is how you do that. Now you try. Use division to change the improper fraction to a mixed number.

$$\frac{14}{5} = 5 \overline{) \begin{array}{r} 14 \\ -10 \\ \hline 4 \end{array}} \quad \frac{14}{5} = 2 \frac{4}{5}$$

$$\frac{17}{4} = 4 \overline{) \begin{array}{r} 17 \\ -16 \\ \hline 1 \end{array}} \quad \frac{17}{4} = 4 \frac{1}{4}$$

$$\frac{21}{4} = 4 \overline{) \begin{array}{r} 21 \\ -20 \\ \hline 1 \end{array}} \quad \frac{21}{4} = 5 \frac{1}{4}$$

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Improper Fractions to Mixed Numbers - Worksheet 12

Use division to change improper fractions to mixed numbers.

$$\frac{16}{5} = 5 \overline{) \begin{array}{r} 16 \\ -15 \\ \hline 1 \end{array}} \quad \frac{16}{5} = 3 \frac{1}{5}$$

$$\frac{17}{3} = 3 \overline{) \begin{array}{r} 17 \\ -15 \\ \hline 2 \end{array}} \quad \frac{17}{3} = 5 \frac{2}{3}$$

$$\frac{25}{4} = 4 \overline{) \begin{array}{r} 25 \\ -24 \\ \hline 1 \end{array}} \quad \frac{25}{4} = 6 \frac{1}{4}$$

$$\frac{21}{6} = 6 \overline{) \begin{array}{r} 21 \\ -18 \\ \hline 3 \end{array}} \quad \frac{21}{6} = 3 \frac{3}{6}$$

$$\frac{14}{4} = 4 \overline{) \begin{array}{r} 14 \\ -12 \\ \hline 2 \end{array}} \quad \frac{14}{4} = 3 \frac{2}{4}$$

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Improper Fractions to Mixed Numbers - Worksheet 13

Use division to change improper fractions to mixed numbers.

$$\frac{20}{3} = 3 \overline{) \begin{array}{r} 20 \\ -18 \\ \hline 2 \end{array}} \quad \frac{20}{3} = 6 \frac{2}{3}$$

$$\frac{19}{5} = 5 \overline{) \begin{array}{r} 19 \\ -15 \\ \hline 4 \end{array}} \quad \frac{19}{5} = 3 \frac{4}{5}$$

$$\frac{22}{3} = 3 \overline{) \begin{array}{r} 22 \\ -21 \\ \hline 1 \end{array}} \quad \frac{22}{3} = 7 \frac{1}{3}$$

$$\frac{26}{4} = 4 \overline{) \begin{array}{r} 26 \\ -24 \\ \hline 2 \end{array}} \quad \frac{26}{4} = 4 \frac{2}{4}$$

Make your own.

$$\frac{\text{AWV}}{\text{---}} = \overline{) \begin{array}{r} \text{---} \\ - \text{---} \\ \hline \text{---} \end{array}} \quad \text{---} = \text{---} \frac{\text{---}}{\text{---}}$$

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Improper Fractions to Mixed Numbers - Assessment Worksheet 15

1. Change these mixed numbers to improper fractions.

$$\text{a. } 4 \frac{2}{5} = \frac{22}{5} \quad \text{b. } 3 \frac{3}{4} = \frac{15}{4} \quad \text{c. } 2 \frac{7}{8} = \frac{23}{8}$$

2. Explain in words how you change mixed numbers to improper fractions.

Multiply the whole number by the denominator and add the numerator of the fraction to that product. Place the sum over the original denominator.

3. Change these improper fractions to mixed numbers.

$$\text{a. } \frac{15}{6} = 2 \frac{3}{6} \quad \text{b. } \frac{23}{3} = 7 \frac{2}{3} \quad \text{c. } \frac{27}{5} = 5 \frac{2}{5}$$

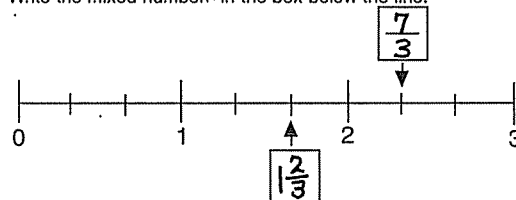
4. Explain in words how you change improper fractions to mixed numbers.

Divide the denominator into the numerator. This is the whole number. The remainder is the numerator for the fraction. The denominator stays the same.

5. The arrow points to two fractions on the number line.

Write the improper fraction in the box above the line.

Write the mixed number: in the box below the line.



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Addition of Mixed Numbers with Like Denominators - Worksheet 1

Solve.

$$\begin{array}{r} 1 \frac{1}{4} \\ + 1 \frac{2}{4} \\ \hline 2 \frac{3}{4} \end{array}$$

Solve.

$$\begin{array}{r} 1 \frac{3}{5} \\ + 2 \frac{1}{5} \\ \hline 3 \frac{4}{5} \end{array}$$

$$\begin{array}{r} 4 \frac{5}{7} \\ + 5 \frac{1}{7} \\ \hline 9 \frac{6}{7} \end{array}$$

$$\begin{array}{r} 26 \frac{1}{6} \\ + 39 \frac{3}{6} \\ \hline 65 \frac{4}{6} \end{array}$$

Sometimes when adding fractions the sum will be an improper fraction. When this happens the improper fraction is changed to a mixed number and then added again.

Examples:

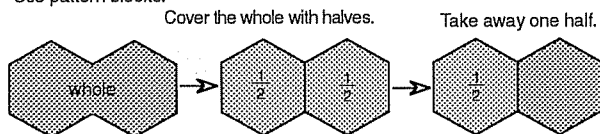
$$\begin{array}{r} 1 \frac{1}{4} \\ + 2 \frac{3}{4} \\ \hline 3 \frac{4}{4} \\ \downarrow \\ 3 + 1 = 4 \end{array}$$

$$\begin{array}{r} 1 \frac{2}{4} \\ + 2 \frac{3}{4} \\ \hline 3 \frac{5}{4} \\ \downarrow \\ 3 + 1 \frac{1}{4} = 4 \frac{1}{4} \end{array}$$

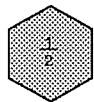
30

Mixed Numbers Meet Subtraction - Worksheet 1

Use pattern blocks.



What is left?



Record the number sentence.

$$1 - \frac{1}{2} = \underline{\quad}$$

$$\begin{array}{r} 1 \\ - \frac{1}{2} \\ \hline \frac{1}{2} \end{array}$$

Solve. Use pattern blocks if you need to.

$$1 - \frac{1}{3} = \frac{2}{3}$$

$$\begin{array}{r} 1 \\ - \frac{1}{3} \\ \hline \frac{2}{3} \end{array}$$

$$1 - \frac{3}{4} = \frac{1}{4}$$

$$\begin{array}{r} 1 \\ - \frac{3}{4} \\ \hline \frac{1}{4} \end{array}$$

$$1 - \frac{5}{6} = \frac{1}{6}$$

$$\begin{array}{r} 1 \\ - \frac{5}{6} \\ \hline \frac{1}{6} \end{array}$$

$$1 - \frac{5}{12} = \frac{7}{12}$$

$$\begin{array}{r} 1 \\ - \frac{5}{12} \\ \hline \frac{7}{12} \end{array}$$

$$2 - 1 \frac{1}{2} = \frac{1}{2}$$

$$\begin{array}{r} 2 \\ - 1 \frac{1}{2} \\ \hline \frac{1}{2} \end{array}$$

$$2 - 1 \frac{3}{4} = \frac{1}{4}$$

$$\begin{array}{r} 2 \\ - 1 \frac{3}{4} \\ \hline \frac{1}{4} \end{array}$$

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Addition of Mixed Numbers with Like Denominators - Worksheet 2

Solve.

$$\begin{array}{r} 3 \frac{4}{5} \\ + 2 \frac{1}{5} \\ \hline 5 \frac{5}{5} \\ \downarrow \\ 5 + 1 = 6 \end{array}$$

$$\begin{array}{r} 6 \frac{2}{3} \\ + 2 \frac{2}{3} \\ \hline 8 \frac{4}{3} \\ \downarrow \\ 8 + 1 \frac{1}{3} = 9 \frac{1}{3} \end{array}$$

$$\begin{array}{r} 4 \frac{5}{6} \\ + 3 \frac{5}{6} \\ \hline 7 \frac{10}{6} \\ \downarrow \\ 7 + 1 \frac{4}{6} = 8 \frac{4}{6} \end{array}$$

$$\begin{array}{r} 7 \frac{1}{3} \\ + 2 \frac{2}{3} \\ \hline 9 \frac{3}{3} \\ \downarrow \\ 9 + 1 = 10 \end{array}$$

$$\begin{array}{r} 7 \frac{10}{6} \\ \downarrow \\ 7 + 1 \frac{4}{6} = 8 \frac{4}{6} \end{array}$$

$$\begin{array}{r} 15 \frac{5}{3} \\ \downarrow \\ 15 + 1 \frac{2}{3} = 16 \frac{2}{3} \end{array}$$

Make your own.

$$\begin{array}{r} + \underline{\quad} \\ - \underline{\quad} \\ \hline \downarrow \\ \underline{\quad} + \underline{\quad} = \end{array}$$

Challenge.

$$\begin{array}{r} 11 \frac{8}{9} \\ + 19 \frac{8}{9} \\ \hline 30 + \frac{16}{9} = 30 + 1 \frac{7}{9} = 31 \frac{7}{9} \end{array}$$

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Mixed Numbers Meet Subtraction - Worksheet 2

$$2 - \frac{3}{4} = 1 \frac{1}{4}$$

$$1 - \frac{3}{5} = \frac{2}{5}$$

$$1 - \frac{3}{4} = \frac{1}{4}$$

$$2 - \frac{1}{4} = 1 \frac{3}{4}$$

$$3 - \frac{3}{4} = 2 \frac{1}{4}$$

$$4 - \frac{2}{3} = 3 \frac{1}{3}$$

$$\begin{array}{r} 4 \frac{2}{3} \\ - \frac{2}{3} \\ \hline 4 \end{array}$$

$$\begin{array}{r} 3 \frac{2}{3} \\ - \frac{1}{3} \\ \hline 3 \frac{1}{3} \end{array}$$

$$\begin{array}{r} 2 \frac{5}{6} \\ - \frac{1}{6} \\ \hline 2 \frac{4}{6} \end{array}$$

$$\begin{array}{r} 4 \frac{4}{5} \\ - \frac{3}{5} \\ \hline 4 \frac{1}{5} \end{array}$$

$$\begin{array}{r} 4 \frac{4}{5} \\ - 1 \frac{1}{5} \\ \hline 3 \frac{3}{5} \end{array}$$

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Subtraction of Mixed Numbers with Regrouping Worksheet 1

When subtracting sometimes this situation can happen.

What makes this problem more difficult?
 $\frac{2}{3}$ is greater than $\frac{1}{3}$

How do you solve this type of problem?
Regroup or borrow from the whole number.

Try solving this problem with the same method. Use fraction pieces if you need them.

Solve. If you get stuck and need some help, see the next page and then solve these.

$$\begin{array}{r} 2\frac{5}{4} \\ - 3\frac{3}{4} \\ \hline 2\frac{2}{4} \\ 2\frac{3}{8} \\ - \frac{5}{8} \\ \hline 2\frac{6}{8} \end{array}$$

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Subtraction of Mixed Numbers with Regrouping Worksheet 3

$$\begin{array}{r} 6\frac{1}{4} \\ - 3\frac{3}{4} \\ \hline 2\frac{2}{4} \end{array}$$

$$\begin{array}{r} 7\frac{1}{4} \\ - 2\frac{3}{4} \\ \hline 4\frac{2}{4} \end{array}$$

Hard ones. Watch out!

$$\begin{array}{r} 24\frac{1}{5} \\ - 17\frac{3}{5} \\ \hline 6\frac{2}{5} \end{array}$$

Challenge!

$$\begin{array}{r} 30\frac{1}{2} \\ - 16\frac{3}{4} \\ \hline 13\frac{3}{4} \end{array}$$

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Subtraction of Mixed Numbers with Regrouping Worksheet 2

Think of how you solve this problem with whole numbers.

$$\begin{array}{r} 32 \\ - 18 \\ \hline \end{array}$$

You regroup or trade. You take one ten from the thirty and change it into ones, which is added to the two ones to make it twelve. Do it.

Trading can be done with fractions, ones are used instead of tens.

$$\begin{array}{r} 4\frac{1}{3} \\ - 1\frac{2}{3} \\ \hline \end{array}$$

Take a one from the 4 and give it to the fraction.

$$\begin{array}{r} 3\frac{1}{3} \rightarrow 3\frac{1}{3} \\ - 1\frac{2}{3} \\ \hline \end{array}$$

But now the mixed number $1\frac{1}{3}$ needs to be changed to thirds.

$$1\frac{1}{3} = \frac{4}{3}$$

Now the subtraction can be done easily.

Solve it now

Remember to subtract the whole numbers too.

$$\begin{array}{r} 3\frac{4}{3} \\ - 1\frac{2}{3} \\ \hline 2\frac{2}{3} \end{array}$$

$$\begin{array}{r} 1\frac{1}{3} = 1\frac{4}{3} \\ - \frac{2}{3} = \frac{2}{3} \\ \hline 1\frac{2}{3} \end{array}$$

$$\begin{array}{r} 1\frac{1}{4} = \frac{5}{4} \\ - \frac{3}{4} \\ \hline \frac{2}{4} \end{array}$$

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Post-Assessment

1. Put a box around the improper fractions. Put a circle around the mixed numbers.

$$\boxed{2\frac{2}{3}} = \frac{1}{2} = \boxed{\frac{3}{2}} = \frac{3}{4} = \boxed{\frac{5}{4}} = \boxed{3\frac{3}{4}}$$

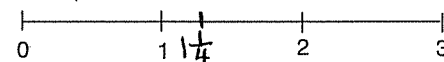
2. Change these mixed numbers to improper fractions.

$$a. 3\frac{1}{2} = \frac{7}{2} \quad b. 2\frac{3}{4} = \frac{11}{4} \quad c. 4\frac{5}{6} = \frac{29}{6}$$

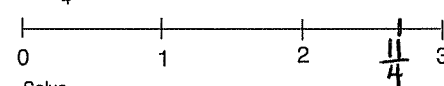
3. Change these improper fractions to mixed numbers.

$$a. \frac{7}{3} = 2\frac{1}{3} \quad b. \frac{1}{2} = \frac{3}{2} \quad c. \frac{10}{4} = 2\frac{2}{4} = 2\frac{1}{2}$$

4. Put $1\frac{1}{4}$ on this number



5. Put $\frac{11}{4}$ on this number



6. Solve.

$$a. 1\frac{1}{3} + 2\frac{2}{3} = 3\frac{3}{3} = 4 \quad b. 2\frac{2}{5} + 1\frac{4}{5} = 3\frac{6}{5} = 4\frac{1}{5} \quad c. 11\frac{8}{9} + 19\frac{8}{9} = 30\frac{16}{9} = 31\frac{7}{9}$$

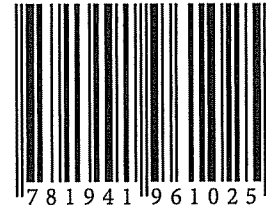
7. Solve.

$$a. 4 - \frac{2}{3} = 3\frac{1}{3} \quad b. 5\frac{2}{3} - 2\frac{1}{3} = 3\frac{1}{3} \quad c. 5\frac{1}{4} - 1\frac{2}{4} = 3\frac{3}{4} \quad d. 20\frac{1}{10} - 6\frac{7}{10} = 13\frac{4}{10}$$

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Patterns in Arithmetic: Fractions - Booklet 3 PDF
Parent/Teacher Guide
Mixed Numbers and Improper Fractions
ISBN 978-1-941961-02-5

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