

Fractions: Booklet 3 - Mixed Numbers and Improper Fractions

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The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

Patterns in Arithmetic: Fractions - Booklet 3

Parent/Teacher Guide

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Mixed Numbers to Improper Fractions

- Purpose** The purpose of this lesson is to introduce the concept of mixed numbers and improper fractions and how to change a mixed number into an improper fraction using a manipulative. The student uses the familiar fraction piece that is used to show the whole and places it with one other smaller fraction piece. The whole is then traded for the same smaller fractional pieces and the total counted up and written as an improper fraction. This simple procedure is then recorded, analyzed, and constructed into an algorithm for calculating the improper fraction from the mixed number.
- Note** While it seems much faster to just tell the student how to change a mixed number into an improper fraction,* doing so has two negative consequences. First, it prevents her from developing the skill of seeing patterns and formalizing them into mathematical procedures. This is what mathematicians do. It is where the fun is. Second, even for a student who does not easily create procedures from patterns, constructing a procedure with a teacher allows her to understand why a particular procedure works. If the procedure is simply memorized, it is likely to be filed in the brain with all of the other memorized procedures she will learn for fractions, and there are many of them. Accurate retrieval is then based solely on the skill of memory and no mathematical skill other than the multiplication tables. Many students get memorized procedures mixed up time and time again. In the long run, it is actually faster to have them construct understanding so they can reconstruct the procedure whenever needed.
- Prerequisites** Division - Booklet 1, Fractions - Booklet 1, and Fractions - Booklet 2
- Materials** Mixed Numbers to Improper Fractions - Worksheets 1 - 9, pages 3 - 11
Prism Fractions Circles (plus a paper set), or Prism Fractions Squares
Pattern blocks
- Warm Up** Make wholes with the fraction pieces. Discuss the pattern that to make one whole, the numerator and the denominator are equal. And, if you have more than one whole, the numerator will be a multiple of the denominator, for example, $\frac{2}{2}$, $\frac{4}{2}$, $\frac{6}{2}$.
- Lesson Part 1** Mixed Numbers to Improper Fractions - Worksheet 1, page 3, is an instructional page for both the student and the teacher. Have her read it aloud. Do this page together. Then do the top row of problems on Mixed Numbers to Improper Fractions - Worksheet 2, page 4, together. On the second row, have her work on the problems with you watching. If she is secure, have her finish the worksheet alone and go on to Mixed Numbers to Improper Fractions - Worksheet 3, page 5. Check answers to be sure the work is done correctly.
- Part 2** Mixed Numbers to Improper Fractions - Worksheet 4, page 6, switches the manipulative to pattern blocks and uses the changing whole. This requires her to reconstruct her understanding of the relationship between parts and wholes on every problem.
- *Multiply the denominator by the whole number and add the numerator. Place this number in the numerator and keep the denominator the same.

“If the green triangle is equal to the whole, then how many wholes would the blue rhombus be?” “Two wholes because two greens fit on it.” Record this on the worksheet.

“If the green triangle is equal to the whole, then how many wholes would the red trapezoid be?” “Three wholes because three greens fit on it.” Record this on the worksheet.

“What if the blue rhombus is equal to the whole, then what would the green triangle be?” “Only a half.” Record this on the worksheet. Finish this worksheet, check the work and stop.

Listen to what the student’s ideas are. Your job as a teacher is to figure out what her idea of the problem is. If you don’t understand what she says, work on it until you do. The student tells you her best understanding of the solution. It may or may not work in this case. Have her test the model.

- 1) It works. Test again. Will it always work? Where won’t it work?
 - 2) It doesn’t work. Test again. Where will it work? How is it different?
- Build a model, plan, test, test again. This is what learning is all about.

“Do you see any patterns that might help you figure out the new numerator without the fraction pieces?” Listen to the student’s answer. Eventually, she will figure out how many pieces are in the whole by realizing that the denominator tells how many pieces are in that whole. If there are two wholes, take the denominator and multiply it by two. Then add on whatever extra fractions there are. Do not push this concept if she does not see it herself.

Part 3

Creating an algorithm for converting a mixed number to an improper fraction is the goal of this next part of the lesson. It is done in two phases. The first phase allows for a student to construct the procedure from her own experience. The second phase leads students through the process of constructing a procedure using the blocks.

A few students will be able to independently create the process at this point. They will see that multiplying the denominator by the whole number tells you how many fractional pieces you will have when converting to an improper fraction.

Do not push. If she is unable to understand Mixed Numbers to Improper Fractions - Worksheet 5, page 7, go back and do more problems with either fraction pieces or pattern blocks, or both. Do not tell her the rule.

Worksheet

Mixed Numbers to Improper Fractions - Worksheet 6, page 8, walks the teacher through the thought steps to create a procedure, an algorithm, for converting a mixed number to an improper fraction.


Do this part orally with the student after reading Worksheet 6. Write the mixed number $1\frac{3}{4}$ on a piece of paper.

“Build this amount with your fraction set. What is the name of a number that includes both a whole number and a fraction.” “A mixed number.”

In our fraction manipulative set, the whole is one color and size, and the fourths are a different color and size. Now build $1\frac{3}{4}$ so that all the pieces are the same color and size.”

This is a mixed number. $1\frac{2}{3}$ What do you think is the definition of a mixed number?

Take out fraction pieces.

Pick up a whole and $\frac{1}{4}$. 

Now cover the whole piece with more fourths.



You now have five fourths.
 This kind of fraction is written like this. $\frac{5}{4}$

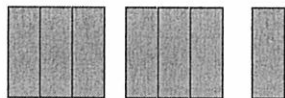
This kind of fraction is called an **Improper Fraction**.

It is called improper because the numerator (the top number) is larger than the denominator (the bottom number). Improper fractions are worth more than one.

Now take out two wholes and $\frac{1}{3}$.



Cover the two wholes with thirds.



How many thirds do you have now? _____ $2\frac{1}{3} = \frac{\quad}{3}$

What does the denominator, or bottom number, describe? _____

What does the numerator, or top number, describe? _____

Improper Fractions to Mixed Numbers

Purpose The purpose of this lesson is to provide another chance for the student to construct an algorithm from her own experience. She will construct improper fractions with the manipulative and then group them into wholes and trade them for the single whole piece. This grouping to make wholes is division and reverses the multiplication procedure used to change mixed numbers into improper fractions. This last piece of conceptual understanding should not be left out; it is a very important foundation for solving algebraic equations.

Prerequisites Fractions: Booklet 1 - Basic Concepts, Fractions: Booklet 3 - Mixed Numbers to Improper Fractions, and Mixed Numbers to Improper Fractions on Number Lines

Materials Improper Fractions to Mixed Numbers, Worksheets 1 - 15, pages 15 - 29
Fraction Circles or Prism Fractions

Note The dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **“How did you get that?” “Show me what you mean.” “Tell me more so I can understand what you are saying.”**

Warm Up Review the procedure she developed on Mixed Numbers to Improper Fractions - Worksheet 5, page 7. Solve a problem or two to make sure she is still controlling the concept, e.g., $2\frac{3}{4} = \frac{\quad}{4}$, $3\frac{3}{4} = \frac{\quad}{4}$.

“What do you do?” “I multiply the denominator by the whole number and add the numerator and put this over the denominator.”

“What do you think will be needed in order to go backwards from an improper fraction to a mixed number?” She may have no idea. If this is the case, continue the lesson below. A very astute answer would be “Division and subtraction because division reverses multiplication and subtraction reverses addition.” If she gives this answer, tell her to begin Improper Fractions to Mixed Numbers - Worksheet 1 and see if she can figure out how to get the correct answer. Build with the manipulative to confirm correctness.

Lesson Part 1 Improper Fractions to Mixed Numbers - Worksheet 1, page 15 **“Build three halves. How many wholes can you trade for the half pieces you have on the table?”** “Two halves can trade for one whole.”

“Notice that you now have two colors, one for the whole and one for the half. What do you call it when a number is made with whole numbers and a fraction?” “A mixed number.”

Improper Fractions to Mixed Numbers

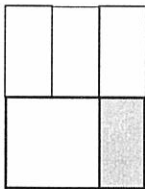
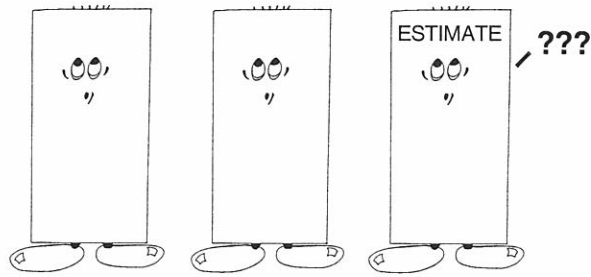
Date _____

Worksheet 1

Build with fraction pieces.

Draw a picture.

Record the mixed numbers.



$$\frac{3}{2} = 1\frac{1}{2}$$

$$\frac{4}{2} =$$

$$\frac{5}{2} =$$

$$\frac{3}{3} =$$

$$\frac{4}{3} =$$

$$\frac{5}{3} =$$

$$\frac{6}{3} =$$

$$\frac{4}{4} =$$

$$\frac{5}{4} =$$

$$\frac{6}{4} =$$

$$\frac{7}{4} =$$

$$\frac{8}{4} =$$

$$\frac{5}{5} =$$

$$\frac{7}{5} =$$

$$\frac{10}{5} =$$

$$\frac{11}{6} =$$

$$\frac{10}{10} = 1$$

$$\frac{20}{10} = 2$$

$$\frac{7}{3} = 2\frac{2}{3}$$

Make your own.

$$\frac{\quad}{\quad} =$$

$$\frac{\quad}{\quad} =$$

$$\frac{\quad}{\quad} =$$

$$\frac{\quad}{\quad} =$$

Improper Fractions to Mixed Numbers

Worksheet 3

1. What improper fraction is being shown in this drawing? _____
Each rectangle stands for one whole.



2. What mixed number is being shown in the drawing above? _____

3. Draw a picture of this fraction. $\frac{12}{8}$



4. Draw a picture of each improper fraction below.
Then change these improper fractions into mixed numbers.

$$\frac{9}{4} =$$

$$\frac{15}{8} =$$

$$\frac{22}{3} =$$

$$\frac{17}{5} =$$

Improper Fractions to Mixed Numbers

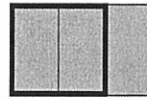
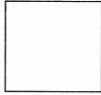
Worksheet 4

Date _____

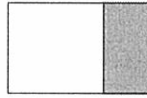
Build with fraction pieces.

Name these fractions.

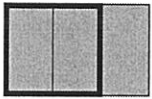
This is one.



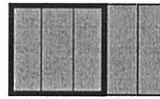
3 reds = $\frac{3}{2}$
improper fraction



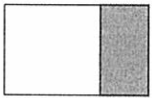
1 white and 1 red = $1\frac{1}{2}$
mixed number



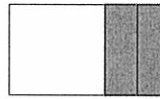
improper fraction



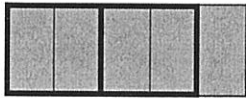
improper fraction



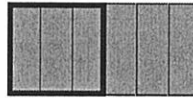
mixed number



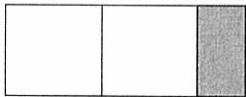
mixed number



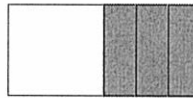
improper fraction



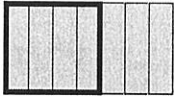
improper fraction



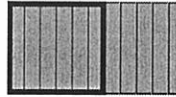
mixed number



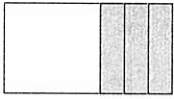
mixed number



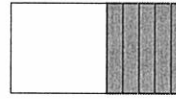
improper fraction



improper fraction



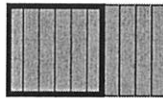
mixed number



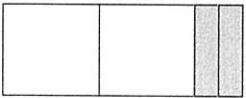
mixed number



improper fraction



improper fraction

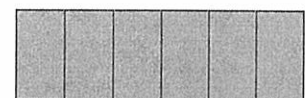
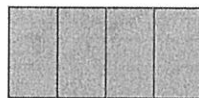
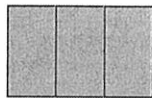
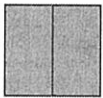


mixed number



mixed number

Record these fractions.




$\frac{1}{2}$ _____


Make your own.

Fractions - Booklet 3



Pre-Assessment - Part 1

The student may use a manipulative for the test.

1. Shade in $\frac{3}{4}$ of this rectangle. 

2. Gail cut a pizza into six slices. 

Natalia ate 1 slice. What fraction of the pizza did she eat? a. $\frac{1}{6}$
 Frank ate 4 slices. What fraction of the pizza did he eat? b. $\frac{4}{6}$
 How much of the pizza has been eaten? Number sentence. c. $\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$
 How much of the pizza is left for Gail? Number sentence. d. $\frac{6}{6} - \frac{5}{6} = \frac{1}{6}$

3. If  equals 1, what does  equal? $\frac{2}{3}$

On the problems below you may use fraction pieces. Circle any problem you can do without using fraction pieces.

4. Fill in the missing numbers.
 a. $\frac{1}{2} = \frac{2}{4}$ b. $\frac{1}{3} = \frac{4}{12}$ c. $\frac{3}{4} = \frac{6}{8}$

5. Write a <, >, or = sign (Greater Than, Less Than, or Equal To) between these pairs of fractions.

a. $\frac{1}{2} > \frac{1}{3}$ b. $\frac{2}{6} = \frac{1}{3}$ c. $\frac{3}{4} > \frac{2}{3}$

6. Solve.

a. $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$ b. $\frac{7}{8} - \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$ c. $7\frac{3}{4} - 3\frac{1}{4} = 3\frac{2}{4} = 3\frac{1}{2}$

7. Solve.

a. $\frac{1}{4}$ of 12 = 3 b. $\frac{2}{3}$ of 18 = 12 c. $\frac{3}{5} \times 25 = 15$

1

Pre-Assessment - Part 2

1. Put a box around the improper fractions. Put a circle around the mixed numbers.

$2\frac{2}{3} =$ (circled) $\frac{1}{2} =$ $\frac{3}{2} =$ (boxed) $\frac{3}{4} =$ $\frac{5}{4} =$ (boxed) $3\frac{3}{4} =$ (circled)

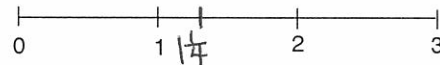
2. Change these mixed numbers to improper fractions.

a. $3\frac{1}{2} = \frac{7}{2}$ b. $2\frac{3}{4} = \frac{11}{4}$ c. $4\frac{5}{6} = \frac{29}{6}$

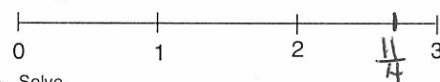
3. Change these improper fractions to mixed numbers.

a. $\frac{7}{3} = 2\frac{1}{3}$ b. $\frac{12}{2} = \frac{3}{2}$ c. $\frac{10}{4} = 2\frac{2}{4} = 2\frac{1}{2}$

4. Put $1\frac{1}{4}$ on this number



5. Put $\frac{11}{4}$ on this number



6. Solve.

a. $1\frac{1}{3} + 2\frac{2}{3} = 3\frac{3}{3} = 4$ b. $2\frac{2}{5} + 1\frac{4}{5} = 3\frac{6}{5} = 4\frac{1}{5}$ c. $11\frac{8}{9} + 19\frac{8}{9} = 30\frac{16}{9} = 31\frac{7}{9}$

7. Solve.


a. $4 - \frac{2}{3} = 3\frac{1}{3}$ b. $5\frac{2}{3} - 2\frac{1}{3} = 3\frac{1}{3}$ c. $\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$ d. $\frac{19}{10} - \frac{6}{10} = \frac{13}{10}$

2

Mixed Numbers to Improper Fractions - Worksheet 1

This is a mixed number $1\frac{2}{3}$. What do you think is the definition of a mixed number? A whole number and a fraction.

Take out Fraction pieces.

Pick up a whole and $\frac{1}{4}$. 

Now cover the whole piece with more fourths.



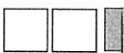
You now have five fourths.

This kind of fraction is written like this. $\frac{5}{4}$

This kind of fraction is called an **Improper Fraction**.

It is called improper because the numerator (the top number) is larger than the denominator (the bottom number). Improper fractions are worth more than one.

Now take out two wholes and $\frac{1}{3}$.



Cover the two wholes with thirds.



How many thirds do you have now? 7 $2\frac{1}{3} = \frac{7}{3}$

Write nine thirds as an improper fraction. $\frac{9}{3}$

What does the denominator or bottom number describe? How many equal parts into which the whole is divided.

What does the numerator or top number describe? How many parts of the whole there are in the fraction.

3

Mixed Numbers to Improper Fractions - Worksheet 2

Build with Fraction pieces. Then record the improper fractions.



$1\frac{1}{2} = \frac{3}{2}$

Look for patterns.

$1\frac{2}{3} = \frac{5}{3}$

$2\frac{1}{4} = \frac{9}{4}$

$3\frac{2}{3} = \frac{11}{3}$

$2\frac{2}{5} = \frac{12}{5}$

$2\frac{3}{4} = \frac{11}{4}$

$1\frac{1}{3} = \frac{4}{3}$

$1\frac{3}{8} = \frac{11}{8}$

$3\frac{1}{6} = \frac{19}{6}$

$3\frac{1}{4} = \frac{13}{4}$

$2\frac{5}{6} = \frac{17}{6}$

$1\frac{5}{12} = \frac{17}{12}$

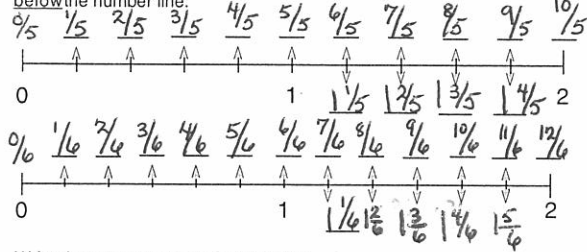
$1\frac{6}{8} = \frac{14}{8}$

4

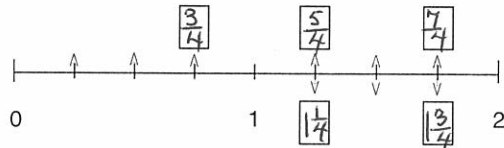
Mixed Numbers and Improper Fractions on Number Lines - Worksheet 2

Remember, count the spaces to get the denominator.

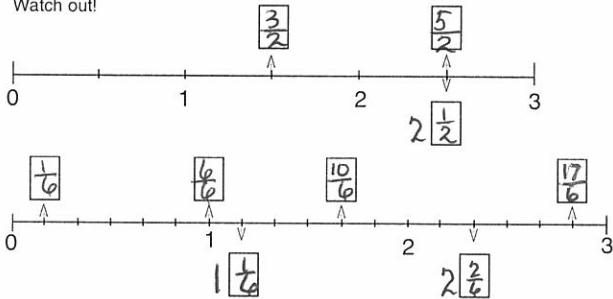
Write the fractions above the number line. Write the mixed numbers below the number line.



Write the correct number in each box.



Watch out!

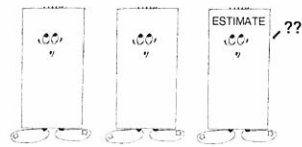



13

Improper Fractions to Mixed Numbers - Worksheet 1

Build with fraction pieces. Draw a picture.

Record the mixed number.



 $\frac{3}{2} = 1\frac{1}{2}$ $\frac{4}{2} = 2$ $\frac{5}{2} = 2\frac{1}{2}$ $\frac{3}{3} = 1$

$\frac{4}{3} = 1\frac{1}{3}$ $\frac{5}{3} = 1\frac{2}{3}$ $\frac{6}{3} = 2$ $\frac{4}{4} = 1$

$\frac{5}{4} = 1\frac{1}{4}$ $\frac{6}{4} = 1\frac{2}{4}$ $\frac{7}{4} = 1\frac{3}{4}$ $\frac{8}{4} = 2$

$\frac{5}{5} = 1$ $\frac{7}{5} = 1\frac{2}{5}$ $\frac{10}{5} = 2$ $\frac{11}{6} = 1\frac{5}{6}$

$\frac{10}{10} = 1$ $\frac{20}{10} = 2$ $\frac{8}{3} = 2\frac{2}{3}$

15

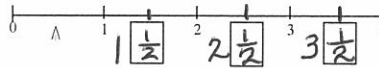
Mixed Numbers and Improper Fractions on Number Lines - Worksheet 3

Use your acetate inch rulers to help you.

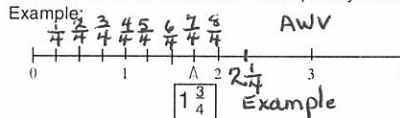
Rulers are Number Lines. When we read a line distance, we might say "It's 2 inches long." This line is 3 inches long. Draw and label the 1 and 2 inch marks.



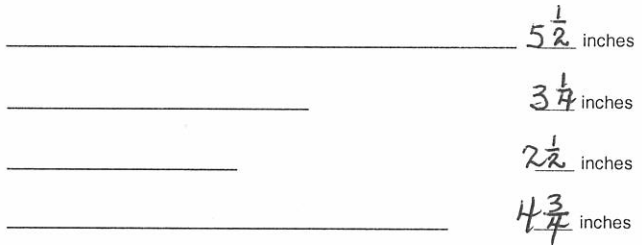
This line is 4 inches long. Draw in the $\frac{1}{2}$ inch marks. Write the mixed fraction below the line.



Draw in the $\frac{1}{4}$ inch marks. Label one mixed number for each one inch section. Draw an arrow from the box to the point you labeled.



How long is each line?



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Improper Fractions to Mixed Numbers - Worksheet 2

Do not require students to simplify fractions.

Build with Fraction pieces. Record the mixed number.

$\frac{6}{3} = 2$ $\frac{7}{3} = 2\frac{1}{3}$ $\frac{8}{3} = 2\frac{2}{3}$ $\frac{9}{3} = 3$

$\frac{7}{6} = 1\frac{1}{6}$ $\frac{9}{6} = 1\frac{3}{6}$ $\frac{11}{6} = 1\frac{5}{6}$ $\frac{13}{6} = 2\frac{1}{6}$

$\frac{12}{8} = 1\frac{4}{8}$ $\frac{15}{8} = 1\frac{7}{8}$ $\frac{19}{8} = 2\frac{3}{8}$ $\frac{23}{8} = 2\frac{7}{8}$

$\frac{13}{12} = 1\frac{1}{12}$ $\frac{18}{12} = 1\frac{6}{12}$ $\frac{23}{12} = 1\frac{11}{12}$ $\frac{28}{12} = 2\frac{4}{12}$

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Improper Fractions to Mixed Numbers - Worksheet 3

1. What improper fraction is being shown in this drawing?
Each rectangle stands for one whole.



$$\frac{7}{5}$$

2. What mixed number is being shown in the drawing above?

$$1\frac{2}{5}$$

3. Draw a picture of this fraction. $\frac{12}{8}$



4. Draw a picture of each improper fraction below.
Then change these improper fractions into mixed numbers.

$$\frac{9}{4} = 2\frac{1}{4}$$

$$\frac{15}{8} = 1\frac{7}{8}$$

$$\frac{22}{3} = 7\frac{1}{3}$$

$$\frac{17}{5} = 3\frac{2}{5}$$

Improper Fractions to Mixed Numbers - Worksheet 4

Build with fraction pieces.

Name these fractions.

This is one.



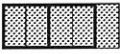
$$\frac{3}{2}$$

improper fraction



$$1\frac{1}{2}$$

mixed number



$$\frac{5}{2}$$

improper fraction



$$2\frac{1}{2}$$

mixed number



$$\frac{7}{4}$$

improper fraction



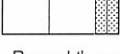
$$1\frac{3}{4}$$

mixed number



$$\frac{10}{4}$$

improper fraction



$$2\frac{3}{4}$$

mixed number

Record these fractions.



Make your own.

Improper Fractions to Mixed Numbers - Worksheet 5

Use pattern blocks to solve these problems.

If this is one, then this is $\frac{1}{2}$.

This takes 3 greens to cover which equals $\frac{3}{2}$.
It also can be covered with 1 blue and 1 green which equals $1\frac{1}{2}$.

This takes 5 greens to cover which equals $\frac{5}{2}$.
It also can be covered with 2 blue(s) and 1 green(s) which equals $2\frac{1}{2}$.

This takes 6 greens to cover which equals $\frac{6}{2}$.
It also can be covered with 3 blue(s) and 0 green(s) which equals 3.

This takes 7 greens to cover which equals $\frac{7}{2}$.
It also can be covered with 3 blue(s) and 1 green(s) which equals $2\frac{1}{2}$.

Improper Fractions to Mixed Numbers - Worksheet 6

Using patterns to create the algorithm for converting an improper fraction to a mixed number

Change these improper fractions to mixed numbers:

$$\frac{5}{2} = 2\frac{1}{2} \quad \frac{6}{4} = 1\frac{2}{4}$$

Did you find a fast way to calculate how to change an improper fraction to a mixed number?
If so, explain how you did one of the problems above.

It will be easier to explain your pattern if you use math language. Here are some words that will help: product sum denominator numerator

Drawing a picture will also help you explain your pattern.

If you have 5 halves, how many wholes can you make? Divide 5 by 2, which equals 2 wholes and 1 left over. The leftover part is half (1/2).