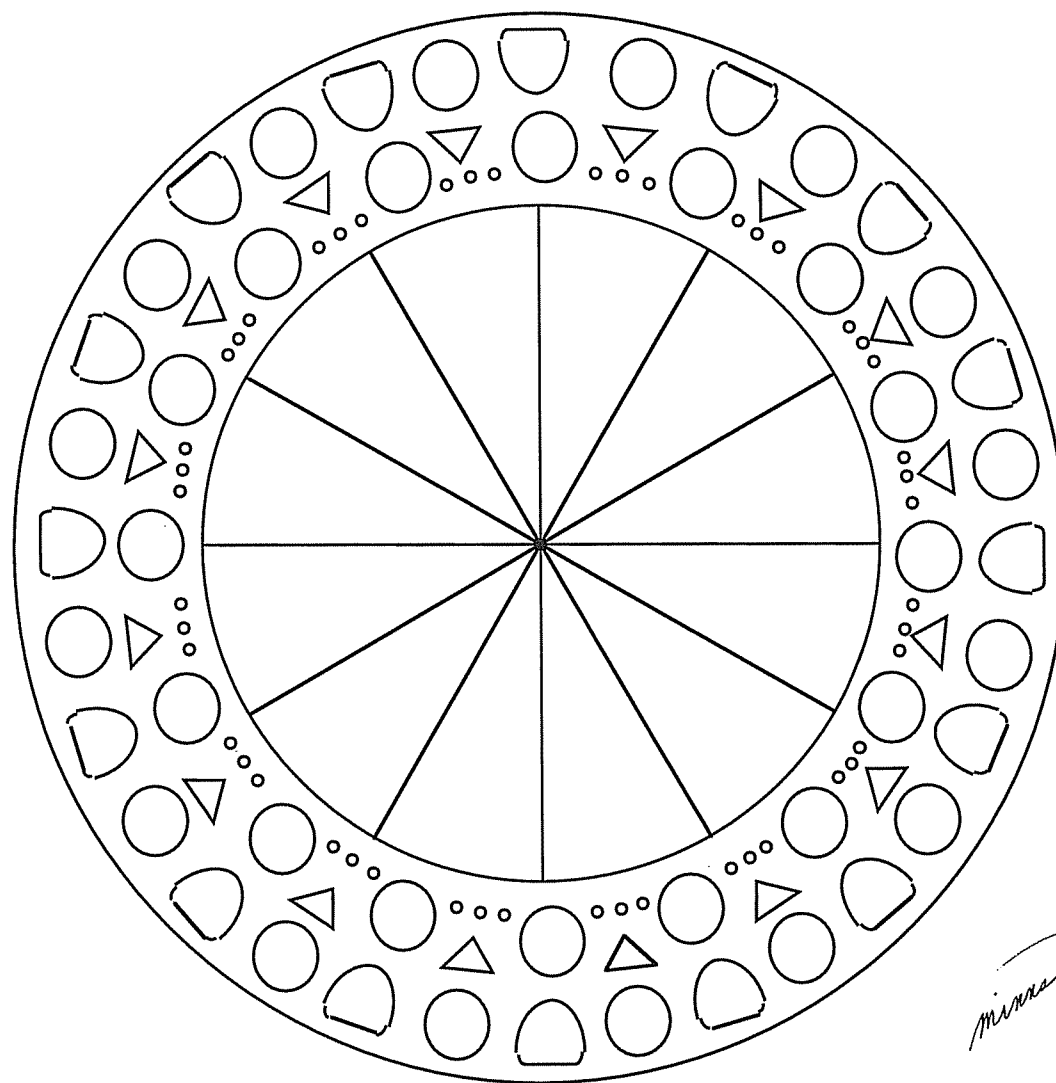


Patterns in Arithmetic

Fractions - Booklet 2 PDF

Developing Concepts and Beginning Operations

Parent/Teacher Guide



By Alysia Krafel, Susan Carpenter, and Suki Glenn

Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the

UCI Farm Elementary School

University of California, Irvine

Fractions: Booklet 2 - PDF - Developing Concepts and Beginning Operations

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The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

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Assessment Guide

Purpose The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet.

Pre-Assessment: Part 1 is review material from the last booklet and is used to determine student readiness for this booklet. Pre-Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

The Post-Assessment is the same as Pre-Assessment: Part 2 and is readministered to determine if the student learned the material that was presented in this booklet. A comparison of the score on Pre-Assessment: Part 2 to the score on the Post-Assessment will give both you and your student a sense of growth.

Prerequisites *Patterns in Arithmetic: Fractions* - Booklet 1 or previous instruction in fractions commonly taught in third-grade textbooks

Materials Fractions: Booklet 2 - Pre Assessment - Worksheets, pages 1 - 4
Score Sheets, pages vii - x in this booklet
Prism Fractions or fraction circles
Pattern blocks

If you do not have the pink or black pattern blocks in your pattern block set, make them with pattern blocks; directions are on page vi.

Note If you have just completed Fractions: Booklet 1 in the last three months and your student passed the Post-Assessment, go to Pre-Assessment: Part 2.

Instructions Allow the student to use pattern blocks, fraction circles, or Prism Fractions to complete the test. Give him at least fifteen minutes of free exploration time with the materials before the test.

Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

Do the assessment in two parts. Give Pre-Assessment: Part 1 and check it for readiness for this booklet. If he is not ready for this booklet, there is no point in giving Pre-Assessment: Part 2. If he passes all the readiness items, then give Pre-Assessment: Part 2.

After scoring the assessment, use results with the Booklet Selection Guide to place your student. The answers are in the Answer Key.

This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc.

Score sheets that match the Assessment Guide for the Pre-Assessment: Part 1, Pre-Assessment: Part 2, and the Post-Assessment follow.

All criteria in Pre-Assessment: Part 1 are from Fractions: Booklet 1

Can the student:

1. My Fractions Book: Beginning

- A. identify the one-third piece from a fraction manipulative set he is using?

2. My Fractions Book: Beginning

- A. identify the one-fourth and the one-sixth pieces in the fraction manipulative set he is using?
- B. indicate that it takes four-fourths and six-sixths to make a whole?

These items test if he understands how to use the fraction manipulative.

3. Sorting Fractions

- A. demonstrate that he understands that the two halves must be of equal size by picking out the three designs that are divided into two equal parts?

4. My Fractions Book: Beginning

- A. write the correct name of the fraction given in standard fraction notation?

5. Greater Than, Less Than, or Equal To

- A. use a greater than, less than sign to correctly show that $\frac{1}{4} < \frac{3}{4}$?
- B. choose the larger fraction from a pair of two unlike fractions with ones in the numerator?
- C. choose the larger fraction from a pair of two unlike fractions with different numbers in the numerator? This is to test if he can use the manipulative to determine the relative sizes of the fractional units given.

6. Changing Wholes: Beginning

- A. choose the correct whole given the size of the one-sixth block?

Give him pattern blocks to solve this problem. This item tests his understanding of the relationship between parts and wholes in fractions. He must choose the larger block that takes six greens to cover it.

7. Numerators Greater Than One

- A. divide the rectangle in problem a into three roughly equal parts?
- B. shade in two of the three parts in problem a?
- C. identify the fraction one-sixth shown by the graphic in problem b?
- D. use the language 'three out of four' when referring to fractions?

This is to test if he can create and interpret graphic representations of fractional units.

8. Shrinking Circles

- A. shade in half of each pizza?

Rubric points 1 point - The wholes are not the same.
 2 points - The size of the half is made by the size of the whole, or if the whole is bigger, then the half will be bigger too.
 This is to test if he understands that the whole can be defined as anything you like. Once that definition is made, all other fractional units are established by their relationship to the whole.

9. My Fractions Book: Equivalence

- A. use the manipulative to match several smaller pieces to the size of the one-half piece?
- B. list at least two of the four equivalent fractions possible?

Have him use fraction circles or Prism Fractions. This is to test if he understands that 'equal' means covering the same area and can match up a greater number of smaller pieces to cover the exact same area the larger one-half piece covers.

10. My Fractions Book: Equivalence

- A. use the manipulative to match nine-twelfths to the three-fourths?
- B. use the manipulative to match two-sixths to the one-third?

11. Equivalence Recording

- A. use the manipulative to determine how many halves would cover two wholes?
- B. extend the number pattern to fill in missing numbers?

12. My Fractions Book: Number Lines

- A. identify that the arrow points to one-third in problem a?
- B. identify that the arrow points to three-fourths in problem b?

If he gave the answer three-eighths, he counted the spaces on the whole number line, not just the spaces between the zero and the one. This is a very common error.

To interpret a number line, the student must first understand that the space between the zero and the one is the whole. Then he must know that the denominator is determined by the number of sections that the space between zero and one is divided into. Many students have difficulty with fractional number lines.

13. My Fractions Book: Number Lines

- A. use the number line to identify equivalent fractions by drawing a line from one number line to a second number line?

Make sure he does not use the fraction manipulative to answer this question. He must draw in the line.

Booklet Selection Guide based on results of Pre-Assessment: Part 1

Critical concepts: If items 1, 2A, 2B, 7A, 7B, and 7C are marked No, he should begin with Fractions: Booklet 1. Do not give Pre-Assessment: Part 2.

A score of 13 or less indicates a beginner level understanding of basic fractions concepts.

For a student of age nine or less, do the entire Fractions: Booklet 1 start to finish. Do not give Pre-Assessment: Part 2 of this assessment.

A student who is age ten to adult should do a modified Fractions: Booklet 1. You can skip items that he got correct on Pre-Assessment: Part 1. Focus on the My Fractions Book sections on equivalent fractions and number lines and shorten tracing and labeling sections in the first part of that section.

You can skip the sections on the Changing Wholes and Greater Than, Less Than, or Equal To, as these are strongly covered in Fractions: Booklet 2. Reassess on items you remediated. Then give Pre-Assessment: Part 2 and begin instruction in Fractions: Booklet 2.

14 - 18 points will allow a pass into Fractions: Booklet 2 with remediation on the items missed.

If Items 5 and 6 were missed, do not remediate these, as these topics are strongly covered in Fractions: Booklet 2.

If your student is new to this program, the My Fractions Book section of Fractions: Booklet 1 is very useful. The activities of tracing and labeling can be shortened. Focus on the drawings of equivalent fractions and number lines, which are the areas most likely to need remediation. These two concepts are not covered in Fractions: Booklet 2. Do the needed sections of Fractions: Booklet 1, retest using Pre-Assessment: Part 1, then give Pre-Assessment: Part 2, and proceed to Fractions: Booklet 2.

19 points are needed to pass into Fractions: Booklet 2 with no remediation. Give Pre-Assessment: Part 2 and proceed to Fractions: Booklet 2.

Assessment Criteria for Pre-Assessment: Part 2

All criteria in the Pre-Assessment: Part 2 are from Fractions: Booklet 2

Can the student:

1. Greater Than, Less Than, or Equal To

- A. put the fractions in order from least to greatest with a manipulative?
- B. put the fractions in order without the use of the manipulative?

This item tests to see if the student recognizes that a large number in the denominator indicates a smaller fractional piece. If he can put the first three in order, he is demonstrating he understands this. The item then tests to see if the student realizes that three-fifths is little more than a half, and four-fifths would be the largest fraction. If he uses the manipulative, he is not quite sure, but he can figure it out.

2. Changing Wholes

A. give one-sixth as the value for the green piece in problem a where the yellow block is defined as one? A No on this item indicates lack of experience with pattern blocks, or an undeveloped sense of the basic concept of fractional values.

- B. give one as the value of the blue block when the value of the yellow block is three?

A Yes on Item B indicates good understanding of this concept because the value of a single block, the yellow, is given as a number greater than one. This is a new kind of problem. He must calibrate the relationship between the blue and the yellow to realize the blue is actually the whole, or one.

3. Changing Wholes

- A. give one twenty-fourth as the value of the green piece in problem a?
- B. give one-ninth as the value of the blue piece in problem b?

C. give the correct answers on problems a and b without the use of a manipulative? A Yes on this item indicates strong understanding of this concept.

4. Addition of Like Fractions

- A. give the correct answer on two of three problems?

Fractions do not need to be simplified to be correct. Does he understand that only the numerators are

added and not the denominators? The denominators are not added because they tell only which fraction piece to use, not how many.

5. Subtraction of Like Fractions

- A. give the correct answer on two of three problems?

6. Fractions as Ratios

- A. give the denominator as fourteen, which is the total number of letters in the box?
- B. give the correct numerator as six, which is the number of capital letters in the box?

This item tests to see if he understands that the whole is determined by the total number of things in a set. The numerator is the number of a particular part of that set.

7. Fractions as Parts of Wholes - using a picture

Parts of Wholes problems require the student to understand that the whole can be defined as a set of objects or as a whole number greater than one. All fractional values are then determined in relationship to whatever the whole is.

- A. circle half of the triangles in a group of four?
- B. color in two of the six boxes to show one-third of a group of six?

These items test if a student can use a graphic to select a fraction of a group.

8. Fractions as Parts of Wholes - using a picture

- A. circle nine of the twelve letters in the box?

This item is testing to see if the student knows that he must count all the letters in the square, divide that number into four equal parts and circle three of those parts. If you see a single circle enclosing nine of the letters, you know he did this calculation in his head. You may also see evidence of four smaller divisions with three selected in a darker circle, which means he needed to visually identify the fourths and then select three of them. This indicates the graphic most likely is still needed.

9. Fractions as Parts of Wholes

This item tests to see if he can calculate the part with only one in the numerator. This requires that he knows to divide the whole number into the number of parts indicated by the denominator of the fraction, ten divided by two in the case of problem a.

- A. give the correct answer in two of the three problems?
- B. give the correct answers without evidence of a drawing or use of a manipulative?

10. Fractions as Parts of Wholes

This item tests the above concept, plus the second step needed of adding more than one group. In the first item, he must divide twenty-five by five to get one-fifth of twenty-five and then multiply that quantity by three because the numerator says there are three of the five groups needed.

- A. give the correct answer in two of the three problems?
- B. give the correct answers without evidence of a drawing or use of a manipulative?

11. Fractions as Parts of Wholes

This item tests if he can use the procedure to calculate a fraction times a whole number problem with a two-step process and a multiplication (x) sign.

- A. give the correct answer in two of the three problems?

Booklet Selection Guide based on results of Pre-Assessment: Part 2

14 or more Yes, move to Fractions: Booklet 3 - Improper and Mixed Fractions

11 - 13 Yes, remediate weak areas, retest, and move to Fractions: Booklet 3.

10 or under, with a pass on Part 1, means that this is the perfect booklet for your student.

Whenever remediation is needed, rely upon the following process, which is used throughout the *Patterns in Arithmetic* series to develop understanding of a concept.

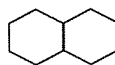
1. Introduce the concept with a manipulative. Orally discuss it. Build it. Verify it. Practice it.
Repeat the experience with a different manipulative (oral manipulative).
2. Use manipulatives to explore the concept again. This time record it with pictures (pictorial/representation). Practice it. Use worksheets.
3. Record the problem with numbers (abstract/symbolic), which links the pictorial with the abstract.
4. Practice fluency.
5. Practice for speed.

Ask questions or make statements, such as: **“Are you sure?”** or **“Build it.”** or **“What gave you the clue?”** or **“Show me how you got that.”** or **“Prove it.”** even when a student is correct. This is important to do often. Many students will ask an adult, “Am I right?” rather than answering definitively. Confidence in a student’s response must come from within. A student needs to self-check and have confidence in his or her ability and knowledge. Asking the student if he or she is right, even when correct, will encourage self-confidence and the ability to self-check.

Pattern block directions for pink or black blocks:

Pink: Glue two yellow hexagons together to make this shape.

Paint pink if you wish.



Black: Glue two blue rhombi together to make this shape.

Paint black if you wish.



Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. Teacher dialogues are in bold type and student dialogues are in regular type. The dialogue you have with your student will be unique. What’s most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **“How did you get that?”** **“Show me what you mean.”** **“Build a model of that.”** **“Tell me more so I can understand what you are saying.”**

Can the student:

1. My Fractions Book: Beginning
Yes No A. identify the one-third piece?
2. My Fractions Book: Beginning
Yes No A. identify the one-fourth and the one-sixth pieces from the fraction manipulative set he is using?
Yes No B. indicate that it takes four-fourths and six-sixths to make a whole?
3. Sorting Fractions
Yes No A. pick out the three pictures that correctly show a half?
4. My Fractions Book: Beginning
Yes No A. write the name of the fraction in words? Spelling does not count.
5. Greater Than, Less Than, or Equal To
Yes No A. use a greater than, less than sign to show that one-fourth is less than three-fourths?
Yes No B. choose the larger fraction from a pair of two unlike fractions with ones in the numerator?
Yes No C. choose the larger fraction from a pair of two unlike fractions with different numbers in the numerator?
6. Changing Whole: Beginning
Yes No A. choose the correct whole given the size of the one-sixth block?
7. Numerators Greater Than One
Yes No A. divide the rectangle in problem a into three roughly equal parts?
Yes No B. shade in two of the three parts in problem a?
Yes No C. identify the name of the fraction shown by the graphic in problem b?
Yes No D. use the language 'three out of four' when referring to fractions?
8. Shrinking Circles
Yes No A. shade in half of each pizza?
1 point The wholes are not the same.
2 points The size of the half is made by the size of the whole, or if the whole is bigger, then the half will be bigger too.
9. My Fractions Book: Equivalence
Yes No A. use the manipulative to match several smaller pieces to the size of the half piece?
Yes No B. list at least two of the four equivalent fractions possible?

10. My Fractions Book: Equivalence

- Yes No A. use the manipulative to match nine-twelfths to the three-fourths?
Yes No B. use the manipulative to match two-sixths to the one-third?

11. Equivalence: Recording

- Yes No A. use the manipulative to determine four halves cover two wholes?
Yes No B. extend the number pattern to fill in missing numbers?

12. My Fractions Book: Number Lines

- Yes No A. identify that the arrow points to one-third in problem a?
Yes No B. identify that the arrow points to three-fourths in problem b?

13. My Fractions Book: Number Lines

- Yes No A. draw a line from two-thirds on one number line to the eight-twelfths on the bottom number line?

Items Correct = _____ = _____% 72% needed to begin Fractions: Booklet 2.

Items Possible = 25 This is 18 or more Yes items.

Less than 72% (age 10 or under), review items in Fractions: Booklet 1, do additional practice in the areas missed, and retest with a pass on Pre-Assessment: Part 1. This is the perfect booklet for your student.

(Age 10 to adult see Booklet Selection Guide on page iii.)

Pre-Assessment: Part 2 Score Sheet

Name _____ Date _____

Can the student:

1. Greater Than, Less Than, or Equal To

Yes No

A. put the fractions in order from least to greatest with a manipulative?

2. Changing Wholes

Yes No

A. give one-sixth as the value for the green block in problem a?

Yes No

B. give one as the value of the blue block in problem b?

3. Changing Wholes

Yes No

A. give one twenty-fourth as the value of the green piece in problem a?

Yes No

B. give one-ninth as the value of the blue piece in problem b?

Yes No

C. give the correct answers on problems a and b without the use of a manipulative?

4. Addition of Like Fractions

Yes No

A. give the correct answer on two of three problems?

Fractions do not need to be simplified to be correct.

5. Subtraction of Like Fractions

Yes No

A. give the correct answer on two of three problems?

6. Fractions as Ratios

Yes No

A. write fourteen as the denominator?

Yes No

B. write six as the numerator?

7. Fractions as Parts of Wholes - using a picture

Yes No

A. circle two of the triangles in a group of four?

Yes No

B. color in two of the six boxes to show one-third of a group of six?

8. Fractions as Parts of Wholes - using a picture

Yes No

A. circle nine of the twelve letters in the box?

9. Fractions as Parts of Wholes - calculating with only one in the numerator

Yes No

A. give the correct answer in two of the three problems?

Yes No

B. give the correct answers without a drawing or use of a manipulative?

10. Fractions as Parts of Wholes - two step process

Yes No

A. give the correct answer in two of the three problems?

Yes No

B. give the correct answers without a drawing or use of a manipulative?

11. Fractions as Parts of Wholes - calculating with a two step process and a multiplication (x) sign

Yes No

A. give the correct answer in two of the three problems without a drawing or manipulative?

Items Correct = _____ = _____%

Items Possible = 18

Post-Assessment Score Sheet

Name _____ Date _____

Can the student:

1. Greater Than, Less Than, or Equal To

Yes No

A. put the fractions in order from least to greatest with a manipulative?

2. Changing Wholes

Yes No

A. give one-sixth as the value for the green block in problem a?

Yes No

B. give one as the value of the blue block in problem b?

3. Changing Wholes

Yes No

A. give one twenty-fourth as the value of the green piece in problem a?

Yes No

B. give one-ninth as the value of the blue piece in problem b?

Yes No

C. give the correct answers on problems a and b without the use of a manipulative?

4. Addition of Like Fractions

Yes No

A. give the correct answer on two of three problems?

Fractions do not need to be simplified to be correct.

5. Subtraction of Like Fractions

Yes No

A. give the correct answer on two of three problems?

6. Fractions as Ratios

Yes No

A. write fourteen as the denominator?

Yes No

B. write six as the numerator?

7. Fractions as Parts of Wholes - using a picture

Yes No

A. circle two of the triangles in a group of four?

Yes No

B. color in two of the six boxes to show one-third of a group of six?

8. Fractions as Parts of Wholes - using a picture

Yes No

A. circle nine of the twelve letters in the box?

9. Fractions as Parts of Wholes - calculating with only one in the numerator

Yes No

A. give the correct answer in two of the three problems?

Yes No

B. give the correct answers without a drawing or use of a manipulative?

10. Fractions as Parts of Wholes - two step process

Yes No

A. give the correct answer in two of the three problems?

Yes No

B. give the correct answers without a drawing or use of a manipulative?

11. Fractions as Parts of Wholes - calculating with a two step process and a multiplication (x) sign

Yes No

A. give the correct answer in two of the three problems without a drawing or manipulative?

Items Correct = _____ = _____% 80% needed to begin Fractions: Booklet 3

Items Possible = 18

This is 14 or more Yes items.

11 - 13 Yes, review weak areas, retest, and move to Fractions: Booklet 3.

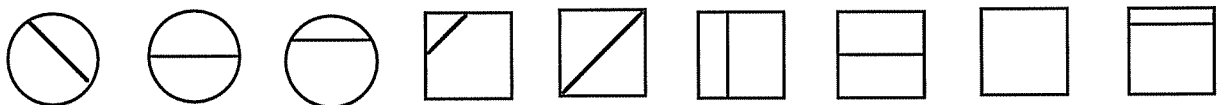
Pre-Assessment: Part 1 - Worksheet 1

Date _____


The student may use a manipulative for the test.

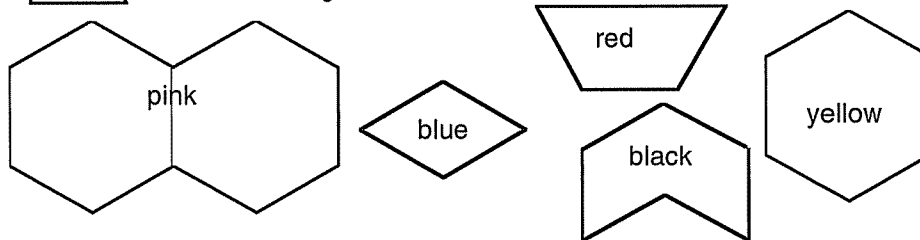
Put a question mark next to anything you do not know how to do.


1. What is the color of the $\frac{1}{3}$ piece in your set? _____
2. a. Cover the whole with fourths. How many fourths does it take to cover a whole? _____
- b. Cover the whole with sixths. How many sixths does it take to cover a whole? _____
3. Circle all of the designs below that are cut in half.



4. Write the name of this fraction in words. $\frac{2}{3}$ _____
5. a. Write in the correct < or > sign. $\frac{1}{4}$ $\frac{3}{4}$
- b. Circle the fraction that shows a larger amount. $\frac{1}{4}$ $\frac{1}{3}$
- c. Which fraction shows a larger amount $\frac{4}{5}$ or $\frac{3}{6}$? _____
- d. How do you know? _____

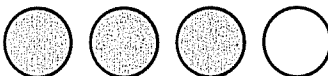
6. If the  is equal to $\frac{1}{6}$, which of these figures is equal to one whole?



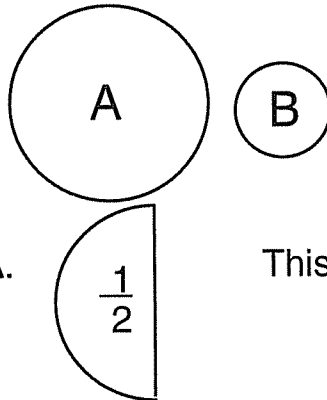
7. a. Shade in $\frac{2}{3}$ of this rectangle. 

- b. What fraction of this rectangle is not shaded? _____



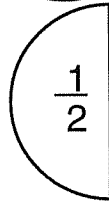
- c. ____ out of ____ of the circles are shaded. 

8. Here are two pizzas.

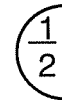


a. Shade in half of each pizza.

This is half of pizza A.



This is half of pizza B.



b. The pieces are both halves, but they are not the same size.

How can this be? _____

9. List all the fractions you can make with your set of fraction pieces that are equal to $\frac{1}{2}$. _____

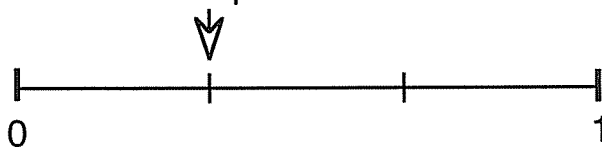
10. a. Trade three-fourths for twelfths with your fraction pieces. How many twelfths are equal to three-fourths? _____

b. How many sixths are needed to cover one-third? _____

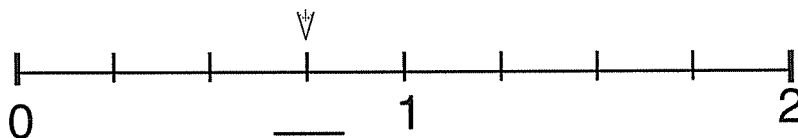
11. Fill in the missing numbers in this pattern.

$$2 = \frac{\quad}{2} = \frac{\quad}{3} = \frac{\quad}{4} = \frac{\quad}{6} = \frac{\quad}{8}$$

12. a. The arrow points to which fraction on the number line? _____

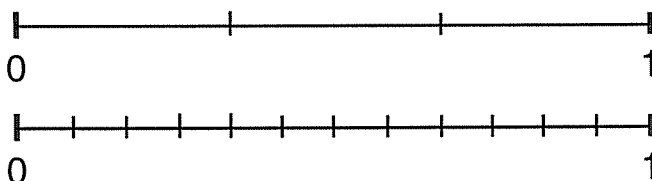


b. What fraction is the arrow pointing to? _____



13. How many twelfths are in two-thirds? _____

Draw a line on the number line to prove your answer.



Pre-Assessment: Part 2 - Worksheet 1

Date _____

Put a question mark next to anything you do not know how to do.

1. Put these fractions in order from the least to the greatest.

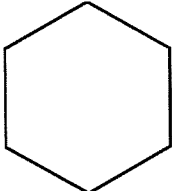

$$\frac{4}{5}$$

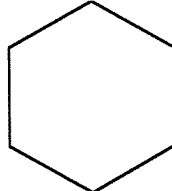
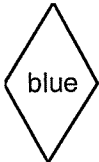
$$\frac{1}{2}$$

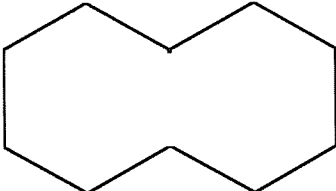

$$\frac{1}{6}$$

$$\frac{3}{5}$$

$$\frac{1}{4}$$

2. a. If  equals 1, then what does  equal? _____

b. If  equals 3, then what does  equal? _____

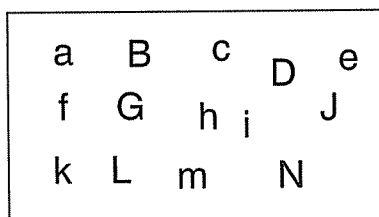
3. a. If  equals $\frac{1}{2}$, then what does  equal? _____


b. If  equals $\frac{1}{3}$, then what does  equal? _____

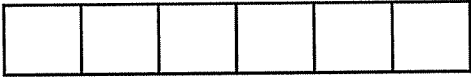
4. Solve. a. $\frac{1}{2} + \frac{1}{2} =$ b. $\frac{3}{5} + \frac{1}{5} =$ c. $\frac{6}{10} + \frac{2}{10} =$

5. Solve. a. $\frac{7}{8} - \frac{1}{8} =$ b. $\frac{2}{3} - \frac{1}{3} =$ c. $\frac{5}{5} - \frac{1}{5} =$

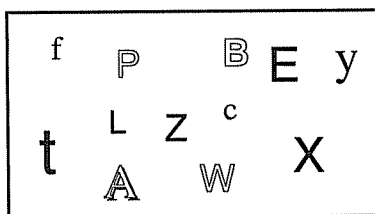
6. What fractional part of the letters are capital letters? _____



7. a. Circle $\frac{1}{2}$ of the group. 

b. Circle $\frac{1}{3}$ of the group. 

8. Circle $\frac{3}{4}$ of the set.



9. a. $\frac{1}{2}$ of 10 = b. $\frac{1}{3}$ of 15 = c. $\frac{1}{5}$ of 20 =

10. a. $\frac{3}{5}$ of 25 = b. $\frac{2}{7}$ of 21 = c. $\frac{4}{9}$ of 36 =

11. a. $\frac{3}{4}$ x 16 = b. $\frac{4}{5}$ x 30 = c. $\frac{6}{7}$ x 42 =

Free Explorations and Pattern Block Games

Purpose	The purpose of this lesson is to encourage play with the materials and gently begin formal work with manipulatives.
Prerequisites	Pre-Assessment
Materials	Free Exploration - Worksheets 1 and 2, pages 7 and 8 Pattern blocks Cuisenaire Rods Prism Factions®
Warm Up	Free Exploration - Worksheet 1 . Set out all the manipulatives (pattern blocks, Cuisenaire Rods, and Prism Factions) intended to be used for instruction on the table at once. Give the student plenty of time to play. Encourage the making of designs. Put on some music. Consider making a gallery of designs to put on the board to brighten the room. Have him trace and color one design into the workbook.
Worksheets	Free Exploration - Worksheet 2. Encourage the student to mix colors in each circle. This lets him know which colors can go together to form a complete whole. Have him trace and color to match the manipulative being used.
Games	<p>Flower Power Game, page 9</p> <p>This game can be more complex than it looks at first glance. If the players use only yellow blocks, the player to go first will always win. This encourages the second player to use a smaller block. There are multiple strategies having to do with even and odd numbers of blocks on the board as well as the ways the empty spaces can be filled.</p> <p>Predator Game, page 10</p> <p>Use yellow, red, green, and blue pattern blocks, a Predator Game board for each player, and one die. You can speed up the game with two dice. Take turns rolling the die. Take as many green blocks as rolled. Trade blue blocks and any other color block for green blocks whenever possible. When a student thinks he is done trading, he passes the die to signify the end of his turn. If he missed a trade, another player can point it out and then take the blocks that should have been traded, make the trade and place the blocks on his board. The first person to fill a Predator game board is the winner.</p>
Variation	Use a fraction die with $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$ on it. Students enjoy these games. It is beneficial to play them many times.
Test for Understanding	<p>"Describe your strategies for playing this game."</p> <p>Have the students describe difference strategies they have found playing the game over time. Look for understanding that smaller pieces allow more turns and that the smaller pieces can be fit together to cover the same area as the larger ones. Notice if he can correctly predict which piece will fit exactly into the space available for each move. If he selects the wrong piece to fit into a space several times, allow more time to play.</p>

Pattern Block Fractions: Review

Purpose	The purpose of this lesson is to review the relationship between the numerator and the denominator. This is a critical concept in fractions. The student identifies the number of parts the whole is divided into, the denominator, and the number of those parts that are shaded, the numerator. One out of four is written $\frac{1}{4}$.
Prerequisites	Free exploration of pattern blocks
Materials	Pattern Block Fractions: Review - Worksheets 1 - 4, pages 11 - 14 Pattern blocks
Note	If your set of pattern blocks does not have a pink double hexagon, glue an edge of two yellows together and paint it pink. You only need one of these. Gluing two blue rhombi together and painting them black can make thirds. Make three of these. This shape is called a <i>chevron</i> .
Warm Up	Play the Flower Power Game or the Predator Game. Or make designs with the blocks. After ten minutes or so, tell the student that designs cannot be made during the lesson. <i>Do not skip this step or she will want to only make designs during the lesson.</i>
Lesson	<p>Pattern Block Fractions: Review - Worksheet 1, page 11</p> <p>"Study the example. See if you can do the rest of the page." Watch what she does.</p> <p>Read the first sentence below the Example. The second line is read this way, "One out of four is equal to one-fourth."</p> <p>Help her only if it is needed. When help is needed, it is usually to clarify what she is to write in the blanks. Check the Answer Key to be sure it is done correctly.</p> <p>Pattern Block Fractions: Review - Worksheet 2, Page 12</p> <p>Repeat the same procedure as on Worksheet 1. On the second problem, the $\frac{3}{6}$, she may write in 3 out of 6 = $\frac{1}{2}$. "This is a good observation and is called a simplification, and it needs two steps. Write the $\frac{3}{6}$ first and then add another = sign and write the $\frac{1}{2}$."</p> <p>If she does not see the simplification, do not show it to her. This will be formally taught later in this book.</p>
Worksheets	Pattern Block Fractions: Review - Worksheet 3, pages 13
Test for Understanding	<p>Pattern Block Fractions: Review - Worksheet 4, page 14</p> <p>This is an assessment. Do not help her. If she can not do this alone, work on this concept more. Do not go on if this concept is weak.</p> <p>"Write the number $\frac{3}{4}$." The student should write it.</p> <p>"What does the four on the bottom tell you?" "The number of parts the whole is</p>

divided up into."

"What does the three on the top tell you?" "The number of parts of the whole that are shaded in. It is the number of parts you have."

Note

The dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogues you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **"How did you get that?" "Show me what you mean." "Build a model of that." "Tell me more so I can understand what you are saying."**

Recording: Prism Fractions

Purpose

The purpose of this lesson is to review the written form of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$ using a manipulative. The student, through experience, should understand the meaning of the symbol for a fraction and learn the convention of how to record it. In a fraction, the bottom number, the denominator, tells the number of equal pieces into which the whole is divided. The top number, the numerator, tells how many of those sized pieces you have. For example, one-third ($\frac{1}{3}$) means that the whole is divided into three equal pieces and you have one of those three pieces. When introducing fractions start with only 'one' in the numerator (the top number).

This lesson asks the student to make a chart of the colors and fractional units shown by each color of the Prism Fractions. If you are using fraction manipulatives other than, or in addition to, the Prism Fractions, the student will make a chart like the one in the student workbook substituting the colors. The student will refer to this chart when doing activities with the fraction pieces.

Prerequisites Free Exploration

Materials Recording: Prism Fractions, page 15
Pattern blocks
Prism Fractions and/or Prism Fractions Circles
Fraction dice or index cards with the fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$ written on them

Lesson Play the Predator Game again with the dice variation. Instead of rolling a regular die, use a fraction die. When the student rolls a $\frac{1}{2}$, or draws the $\frac{1}{2}$ card, ask, "**How do you know which block to pick up?**" He should pick up the red pattern block because it takes two reds to cover the yellow block that is labeled as the whole. Red is $\frac{1}{2}$ of the whole, which is a yellow hexagon.

Take out the Prism Fractions.

"**How do you know that the yellow piece in this set is $\frac{1}{4}$?**" "I know it is $\frac{1}{4}$ because it takes four yellows to cover the white whole."

Have him fill in all the blanks on the worksheet. If you are also using Prism Fractions Circles, make a chart similar to the one on the worksheet.

Worksheet Recording: Prism Fractions, page 15

Here is a dialogue with a student who had studied fractions for several years but was still unclear about what the numbers in a fraction mean. We were working with pattern blocks.

Teacher:

"So the blue is...?"



"What does the one mean and what does the three mean?"

"OK. And what does the one mean?"

"Three covers the whole. If you have one-third, what does the one mean?"

"One out of how many? If you have one-third, it means you have...?"

"If you have one of these (one blue block), is this a whole?"



"How much of the whole is it?"

"Your whole is how many?"

"If you're using three, how many does it take to make the whole?"

"If you have two blues? How would you write that?"

"And what does the 'two' mean?"

"And the three means?"



"If a cookie is divided into three pieces and I give you one piece, how much of the whole do you have if you have just one piece?"

"You have a third of it. One of three. Three being the total. If you have two of them?"

"A blue block is...?"

"Does the one mean how many are in the whole?"

"What number means how many are in the whole?"

"What does the one mean?"

"And the three means?"

Student:

"One-third. Is that the way you write it?" $\frac{1}{3}$



"The three means there are three blocks."

"A whole." (*She is referring to the one.*)

"One-third."

Silence

"No."

"Ah, it's a third."

"A whole is one." (*This is true but she is referring to the 1 in 1/3.*)

"Three."



"Two thirds." $\frac{2}{3}$



"Two means that there are two... Two in the whole. The whole is covered up two-thirds. No reply. (*She is still unclear.*)



"A third of it."

"You have two of the thirds."



"One-third."

"Uh, no."

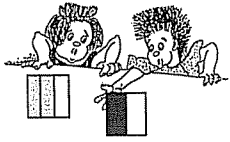
"The three. One means a third of a whole."

"You have one-third of the whole."

"That there are three in the whole."

Fraction Card Game

Purpose



The purpose of this lesson is to have fun while comparing fractional units and forming a conceptual base for equivalent fractions. This game is a variation of the old card game War. This game asks students to compare fractions, which gets to the heart of a common misunderstanding. Does the question "Which is more?" refer to the number of pieces in a fraction or to the size of a portion? For instance, comparing one-half to one-third. With one-half there are two pieces in the whole. With one-third the whole is divided into three parts, which means there are more pieces but each piece is smaller than the one-half pieces. It must be made clear that when asking to compare fractions, one is comparing the *size* of each piece not the number of pieces.

Prerequisites Free Exploration, Recording: Prism Fractions

Materials Fraction Card Game - Worksheets 1 - 4, pages 16 - 24, card stock paper, file folders or index cards, glue stick, Prism Fractions Set. Initial preparation of the cards will take some teacher time. Glue the card game pages to file folders or a piece of light cardboard. Or duplicate the pages onto card stock. Cut out the cards. A paper cutter works best for cutting evenly.

Game Rules

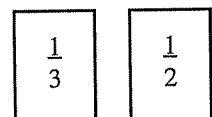
1. A player deals out all the cards, an even amount to each player. Leftover cards are placed in the center of the table face down.
2. Each player pulls the top card from his or her face-down stack and lays it on the table face up.
3. The fraction card with the larger fraction wins. The player who drew that card keeps all cards in that turn and puts them in a separate pile.
4. If both cards have equal fractions, each player deals out three cards face-down and one card face up. The larger fraction wins all. If another tie occurs, the process is repeated.
5. When a player exhausts his original stack of cards, he picks up the pile of cards he has won and plays with these cards.
6. The person who runs out of cards first is the loser.

If the players do not know which fraction is the largest, they must check with fraction pieces or blocks.

Variations

1. Play with numbered cards. Make a set of numbered cards out of 3 x 5 cards cut in half.

Example:



2. Play for the smaller fraction to win a hand.
3. Some players may like to make their own cards out of 3 x 5 cards cut in half.

Play Concentration – instructions in the Student Workbook, Concentration, page 25.

Worksheets

Fraction Card Game, Worksheets 1 - 4, pages 16 - 24

Greater Than, Less Than, or Equal To

Purpose	The purpose of this lesson is to compare and order fractions and to look for patterns. This lesson builds upon the comparisons made in the Fraction Card Game to include the greater than, less than, or equal to notation. Students will place the symbol between two fractions to make a correct statement. Students use the fraction pieces or their cards to help them determine the correct symbol to use.
Prerequisites	Fraction Card Game
Materials	Greater Than, Less Than, or Equal To - Worksheets 1 - 4, pages 26 - 29 Fraction cards
Warm Up	Play Fraction War with numbered cards. Use the fraction pieces to determine which fraction is larger if the student does not know.
Lesson	<p>Write $4 = 4$ "What does this number sentence say?" "That four is equal to four."</p> <p>Write this: $4 = 2$ "Is this a true statement?" "No."</p> <p>"What symbol could you put in to make this a true statement?" "That $4 \neq 2$ or $4 > 2$." (That four does not equal two or four is greater than two.)</p> <p>"Which statement is more precise, gives more information?" "$4 > 2$. Because it not only tells you that they are not equal, it also tells you which one is larger."</p> <p>"These symbols are used to show what mathematicians call inequalities. They are read from left to right."</p> <p>"How can you tell which symbol is for greater than and which one is for less than?" "I already learned that the 'alligator eats the larger number.' So if I am reading from the left, if the mouth of the alligator is open toward the left, it means greater than. If it is open toward the right, it means less than."</p> <p>"Study the examples at the top of Greater Than, Less Than, or Equal To - Worksheet 1. Read the statements."</p> <p>$1 > 1/2$ "One is greater than one-half."</p> <p>$1/3 < 1$ "One-third is less than one."</p> <p>$1/2 = 2/4$ "One-half is equal to two-fourths."</p>
Note	<p>Do not skip this part. It is easy for students to write in the symbols having the alligator eat the larger number, but reading it as written is more abstract, and therefore more difficult. We are building fluency with mathematical statements and vocabulary.</p> <p>Have him place the correct symbol to make a true statement as it is read from left to right. The first ones are easy. As he progresses down Worksheet 1 and onto Worksheet 2, the comparisons become more difficult. He will need to use fraction pieces.</p>

Encourage logical thinking before he reaches for the pieces.

"Look at the last problem on Worksheet 1. It compares three-fourths and three-eighths. How can you tell which one is larger without using your pieces?" "In both fractions we have three pieces because the top numbers are the same. I know that fourths are larger than eighths. So the first fraction, the three-fourths, is larger than the second one."

Worksheets

Greater Than, Less Than, or Equal To - Worksheets 2 and 3, pages 27 and 28

Test for Understanding

Worksheet 4, page 29, is an Assessment. Do not allow him to use the fraction pieces for questions 1 - 3. The answers are in the Answer Key.

Question 1: See if he fluently lists the numbers from the largest denominator to the smallest. Since all numerators are one, the only controlling factor is the size of the denominator.

Question 2: See if there is an expression of the relationships. A statement such as, "The larger the denominator, the smaller the fraction" is the target. A more specific answer would be, "Since all the numerators are equal, the larger the denominator, the smaller the fraction." **"Draw a picture to explain why this is so."**

Question 3: This question ferrets out mathematically gifted students. How much abstract understanding is there of fractional units and the concept of 'approaching one'? The concept of 'approaching one' is a very important concept used later in calculus. The idea is this: In each fraction shown, the numerator is only one less than the denominator. This means we are getting as close to one as we can get with that denominator. If the pieces are very, very small, we can get very, very close to one—.99 cents out of 100 cents, or ninety-nine one-hundredths, is closer to a full dollar than three quarters, or three-fourths, of a dollar. In both cases the numerator is only one less than the denominator. But because the ninety-nine one-hundredths has so many small pieces and you have almost all of them, it is much closer to one and therefore greater than the three-fourths. This is a very sophisticated idea. Very few children at this level will be able to think in these terms. However, if your student can, it is a mark of mathematical talent. Mathematically talented students should never be taught a rote procedure. They should construct every bit of understanding because doing so uses their talent in a creative way and thus develops it.

Many students will not be able to explain or figure out this pattern without fraction pieces. Do not force it, just notice what the student is thinking.

Most students will be unable to do this problem because this one reverses the rule used in the first problem because the numerators are not one. The largest number in the denominator is also the largest fraction because the value of the numerator is nearly equal to the denominator. Tricky. He may guess and may guess correctly, but will be unable to explain why. This is normal for this level. Just working with problems like this strengthens mathematical thinking and fluency.

Question 4: Can he use the pieces to find the answer to an unfamiliar question? Pay attention to how he goes about the work. Does he build every one? Does he build several and compare? Does he build several and compare part of the way and use the pattern to finish it? Some students will find there are too many fractions to manage. If this is the case, give him half of the fractions,—one-half, two-thirds, three-fourths, and four-fifths—and see if he can work from that.

None of these ways is incorrect. Observing will give you a clue as to how abstract his thinking is.

Question 5: See if he can spot the pattern and describe it using mathematical language.

A great answer would be: "I notice that both the numerators and the denominators go in order from lowest to highest and that the numerator is always one less than the denominator." Even more insightful is adding that each fraction gets closer and closer to one.

Changing Wholes

Purpose

The purpose of this lesson is to lay the foundation for the understanding of multiplication and division of fractions. All fractional pieces are defined in terms of the whole. The quantity that is defined as the whole can be changed. For example, with pattern blocks: If the yellow hexagon is defined as the whole, then the red trapezoid piece is one-half, the green triangle is one-sixth. But if the definition of the whole is changed to two hexagon blocks, then the one-half is one hexagon, the red trapezoid becomes one-fourth, and the green triangle is one-twelfth.

Note

This lesson introduces the very abstract idea that any quantity can be defined as a fraction of a set and that the size of the thing called the whole can change. What does not change is the relationship between the fractional unit and the whole. This lesson asks students to constantly change what is defined as the whole and then figure out the fractional value of all the parts of that whole.

The changing whole is a critical concept for understanding fractions. It is the understanding that a fraction is a relationship to a whole. This relationship is called a ratio. A half of a big pizza is not the same amount of pizza as a half of a small pizza. The fractions are equal, one-half, but the size of that half depends on the size of the whole. The unit of the fraction one-half a pizza or one-half a mile is also determined by what the whole is defined as. What makes this even trickier is that the whole may not be equal to one. I can define the whole group as six, like you would on a basketball team; one-half the team is three players. The value of a coin, or a pattern block, depends on what the whole is. The definition of the whole is changed over and over again. The student must then find the fraction of that whole by dividing it up into the correct number of equal pieces. This concept is the underpinning of ratio and proportion, which in turn underlies calculations of percentage, rates of change, and all division procedures.

This concept is also critical to understanding fractional operations in multiplication and division. For example, let's say you baked a pan of brownies to sell at a bake sale. You sell a fourth of the pan to your first customer. Then your second customer comes up and offers to buy two-thirds of what you have left. So the second customer wants to buy two-thirds of the three-fourths. To cut this up, you have to think of the three-fourths of a pan you still have as the whole in order to cut it up into thirds. Then, in order to price the brownies the second customer bought, you have to think of the original tray of brownies as the whole and the fraction of the original whole brownie pan that the two-thirds of the three-fourths was. This is the problem $\frac{2}{3} \times \frac{3}{4} = \underline{\hspace{1cm}}$. This idea is a bit of a mind bender.

Fractions are, by their nature, ratios. This means that fractions have meaning only in relationship to whatever the whole is. This is a very, very important concept in mathematics, do not skimp on this lesson.

Prerequisites

All previous lessons in the booklet

Materials

Changing Wholes - Worksheets 1 - 8, pages 30 - 38
Parts 1 and 3: Pattern blocks and crayons

Part 2: Cuisenaire Rods, a centimeter ruler, and crayons

Part 1

Changing Wholes (with pattern blocks)

Warm Up

Allow the student to choose doing a design or playing a game with the pattern blocks. Then tell him he may not make designs during the lesson unless the worksheet asks him to.

Lesson

Use Changing Wholes - Worksheet 1, page 30. It will help clarify dividing sets of things into halves.

"If you want to divide something in half, how many parts do you need to divide the whole into?" "Two."

Put down four blocks on the table. **"What would half of this group of blocks be?"**
"Two blocks."

Challenge Problem: **"What if I tell you that I have half of a group in my hand. I show you three blocks. How many blocks would be in the whole group?"** "Six."

"Why?" "Because if you have a half, that means you only have one out of the two groups. Since you showed me three blocks for one group, and I know I need two groups, I need to just *double* that number to get the whole."

Double or 'two times as much' are the words you are looking for.

A second line of questioning goes as follows:

"What is half of four?" "Two."

"How can the whole number two be called a half?" "Because it is half of four. It takes two twos to make four."

"Is the number two a fraction?" "No, it is a whole number."

"Then how can two be called a half when one-half is a fraction and two is a whole number?" "Two can be called a half if you call four a whole. It is the relationship between the two and the four that makes the one-half."

A clarifying question might be:

"So if all the money you have in the world is four dollars, and you gave me two dollars, then you would be giving me a fraction, one-half, of your money?"
"Yes."

Do Worksheet 1 together. This worksheet asks you to think of whatever is in the left hand column as half of something in the right hand column. The example shows the number 2 in the left hand column connected to the 4 in the right hand column. This is because two is half of four.

"The next one shows the red pattern block. If the red pattern block is called a half, then what would be the whole?" "The yellow pattern block."

"Why?" "Because it takes two red blocks to make the yellow."

"What is the trapezoid half of?" "The hexagon." Have him draw the line connecting those two shapes.

"What would you connect a 20 to?" "The 40, because 20 is half of 40."

"Finish the rest of the page by yourself and then use the Answer Key to be sure you did them all correctly."

Worksheet

Changing Wholes - Worksheet 2, page 31

There are three groups of hexagons on this page. The top one defines the whole as one yellow. The student is asked to write down the fractional value of each color block when the whole is one yellow. Have him record those on the worksheet at the top. This should be easy, as these relationships have been covered before.

The middle problem defines the whole as two yellows. This means that the fractional values of the same blocks used in the first example will change. In the first problem, the red block is one-half. In the second problem, that same red block is now one-fourth.

"Color the double hexagon pink. We have decided that two yellows make a pink. What if we now say that the pink is equal to one? Then the pink is the whole. Now which block is the one-half?" "The yellow because it takes two yellows to make the pink."

"How could you figure out what fraction of the whole the red block is?" "I could put reds on to cover the pink and count them up. Or I could just see them in my mind. I already know that two reds cover a yellow, so it would take four reds to cover two yellows."

Note

Pay attention to which strategy he uses. If he gives you the second answer, it means the concept is becoming abstractly understood. This is what we are working towards.

Have him finish the page alone if possible.

Test for Understanding

Watch to see what he does on the last problem. This last problem defines the smallest block as the whole. He has not seen this before. If he does it alone, that's great. He gets it. If he needs help, have him put greens on top of the other blocks. You can even write the number 1 on the block and have him count. He already knows it takes three greens to cover a red, but the concept of the whole is what is shifting.

Note

If he calls the green one-third when he covers the red, it means he is confused about what the whole is. We have defined the green as being one whole. This means the red is equal to three. Tell him this. You can even write the number 1 on each block to help. Show him how the green is only one-third if the red is declared the one. If this confuses him, do more of this kind of problem.

Encourage him to build each one to make a clear picture in his mind. Watch him do the other two blocks.

Check the Answer Key immediately to be sure he is doing them correctly.

Challenge What fraction of the whole is represented by the orange square? The answer is at the end of the lesson. No peeking!!!!

Worksheet Changing Wholes - Worksheet 3, page 32

Test for Understanding Watch him solve Worksheet 4, page 33. If he can do the top problem alone, he understands this concept.

The bottom one is a bit of a challenge. If he can solve the challenge problem on the bottom of this page, he is demonstrating strong understanding and problem solving skills.

Stop. Check the work to see that it is done correctly.

Lesson Changing Wholes (with Cuisenaire Rods)

Part 2

Changing Wholes - Worksheet 5, page 34, changes not only the definition of the whole, but also switches the manipulative. This requires him to reconstruct his understanding of the concept.

The top problem shows a rectangle that is twelve centimeters long. Have him measure the length and width of the box. Have him color it in a color not used by the Cuisenaire Rods.

"How many red rods fit in the whole rectangle?" "Six."

"If the rectangle is equal to one, then what is the value of one red rod?" "One sixth because it takes six equal blocks to fill the whole."

Watch him do the rest of the problems. Help only if needed. Have him color each whole with a unique color or design. Then figure out the fractional value of each block.

Stop. Check the work. Make more problems using the centimeter grid paper if needed.

Lesson Changing Wholes (with pattern blocks and mixed color wholes)

Part 3

This section is difficult for many students. It requires him to realize that in order to determine what the fractional value of a block is, he must change the entire whole to a single color.

Most students do the first problem with little help because the shape is now so familiar. The second problem is a stumper for many. There is a little assist with question 4, but some students do not know how to use this clue.

Do not help too quickly. Let them puzzle it out.

Have the student do Changing Wholes - Worksheets 6 and 7, pages 35 and 36, alone.

**Test for
Understanding**

Changing Wholes - Worksheet 8, page 37, is an assessment. Have him do this alone. Watch to see if he uses the blocks (concrete processing), if he draws lines in to show the units of the smaller blocks (representational processing), or if he simply visualizes the parts (abstract processing).

He may use more than one kind of processing as the problems get harder.

This assessment looks at two parts of the changing whole concept.

First, can he shift his definition of the whole from one figure to the next?

Second, does he realize that in order to determine the fractional unit of the whole, the figure must be covered with a block of a single shape or color?

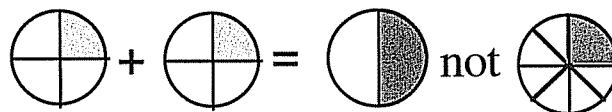
Note

If it looks like the student is struggling and needs help, then assess the difficulty. Reteach any parts he is unclear about. Keep moving on with the booklet, but keep practicing this idea with problems you make up together. The concept of the changing whole shows up over and over as you are working with fractions.

Addition of Like Fractions

Purpose	The purpose of this lesson is to construct an algorithm (a procedure) and test it using concrete manipulatives to verify correctness. Like fractions have the same denominators. The apparent simplicity of the addition of like fractions allows the student to build, look for a pattern, and then create an abstract mathematical algorithm. In doing this she is building mathematical fluency and self-confidence in her ability to understand the numbers and the equations. Using a fraction manipulative, the student will easily discover that when like fractions are added, the numerators are added but the denominators are not. The exploration of why this is so allows the student to further her understanding of what a denominator is. This is a major topic.
Prerequisites	Greater Than, Less Than, or Equal To mastered and Changing Wholes developing or mastered
Materials	Addition of Like Fractions - Worksheets 1 - 3, pages 39 - 41 Fraction manipulative
Warm Up	Have her build various fractions. Do a few greater than, less than problems. Then give a few easy whole number addition problems such as $3 + 4 = \underline{\quad}$.
Lesson	"If whole numbers such as $3 + 4$ can be added, do you think fractions can be added?" Answer is unpredictable.

"What would be the sum of one-fourth plus one-fourth?" Many students will say two-eighths. A few will know it is one-half from previous experience. Even if she says one-half, have her explain how she got that answer.



"Build it with your pieces."



Pick up two eighths and place them next to the two fourths pieces she took out. She can see that the sum is definitely not two eighths. Give her the next problem. Then go back and see if she can identify that the two fourths are then traded for the one-half.

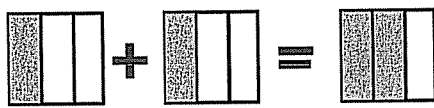
"What is one red block plus one red block?" "Two red blocks."

By using the color name of the block instead of the numerical name, it is very clear that you do not add the kind of block it is, only the number of blocks you have of each color.

Note	The word <i>denominator</i> has the root word 'nom' in it. In Latin, <i>nom</i> means, 'name.' The denominator names the kind or size of piece being used. The word <i>numerator</i> obviously has the root for the word <i>number</i> . It is the numerator that tells you what to add. Later lessons deal with adding two blocks of different sizes by trading to make all the blocks the same color. This is called common denominators. She is not ready
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for this yet, but she will build her understanding of addition and subtraction with fractions from this simple lesson.

Give another easy fraction problem such as $\frac{1}{3} + \frac{1}{3} = \underline{\hspace{1cm}}$. Have her predict the answer, then build it. Again use the color of the pieces to show that one green plus one green (or whatever color your thirds are) is equal to two greens.



$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Note

As usual, do not tell her the pattern. Let her figure it out on her own.

Worksheet

Give Addition of Like Fractions - Worksheet 1, page 39. This worksheet asks her to guess and then build, recording the results of her constructions. As she works, you may hear her talking about the patterns she sees. The worksheet will ask her to describe the patterns she sees. This pattern is then constructed into a procedure that works for all problems of this type.

Finish the worksheet and then check.

Worksheets

Addition of Like Fractions - Worksheets 2 and 3, pages 40 and 41

Test for Understanding

"So, in the procedure you figured out, you add the numerators but not the denominators. Why do you think you do not add the denominators?"

"Because the denominator is not a quantity, it is just the size/color of the block. The denominator just tells you which block to use. The denominator is the number of equal parts into which the whole is divided."

Subtraction of Like Fractions

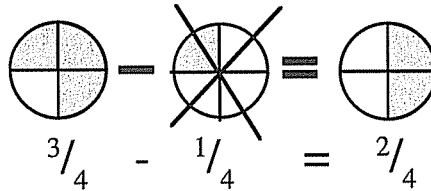
Purpose The purpose of this lesson is to generalize what the student learned in the Addition of Like Fractions lesson to subtraction of like fractions. This allows the student to combine changing wholes with operations involving like fractions.

Prerequisites Addition of Like Fractions and Changing Wholes.

Materials Subtraction of Like Fractions - Worksheets 1 - 5, pages 42 - 46
Fraction manipulative

Lesson This lesson is a repeat of the addition lesson.

"What would be the difference of three-fourths minus one-fourth? Guess first then build it."



"What did you find out?" "That it works the same way as addition except that you subtract the numerators instead of adding them."

"Do you think the pattern you figured out for addition of like fractions will work for subtraction also?" "Yes, because now you are just taking away a block."

Worksheets Do Subtraction of Like Fractions -Worksheet 1 together. Then give Subtraction of Like Fractions - Worksheets 2 - 4, pages 43 - 45.

Test for Understanding Subtraction of Like Fractions - Worksheet 5, page 46, is an assessment. See notes below.

"Which block is an eighteenth?" "The green one because the whole is three yellows. There are six greens in each yellow, so the eighteenth came from 3×6 ."

Watch to see how she does the problems. Does she build them or does she use the procedure?

"How do you know your answer is correct? Can you prove it?"

Extension After Worksheet 5 is completed, ask her to build the answer to:

$$\frac{18}{18} - \frac{6}{18} = \underline{\hspace{2cm}} \quad \text{She will get the answer } \frac{12}{18}.$$

"Can you trade that twelve-eighteenths for a fewer number of larger blocks?"
"Yes, I can trade the twelve greens for two yellows." **"What fraction would you have then?"** "Two-thirds."

"What does that tell you about twelve-eighteenths and two-thirds?" "They must be equal."

Fractions as Ratios

Purpose	The purpose of this lesson is to introduce a new way to look at fractions,—as ratios, or relationships between parts and wholes of groups. In past lessons, the student sees the whole and gives the fraction of that whole. With ratios, the parts can also be compared to each other, although this will come later. This concept is used when working with units of measurement such as miles per hour or when looking at probability and statistics. Example: Two out of three people like chocolate better than vanilla. The teacher must prepare materials for Part 2.
Prerequisites	All lessons up to this point
Materials	Fractions as Ratios - Worksheets 1 and 2, pages 47 and 48 Cuisenaire Rods
Part 1 Warm Up	<p>Take out the Cuisenaire Rods and decide which rod is going to stand for each person in the family.</p> <p>Go through Fractions as Ratios - Worksheet 1 with your student.</p> <p>"How do you know which number to put in the denominator of your fraction?" "It will be eight because these are questions about the fraction of the whole family. The whole family is eight people."</p> <p>Check your answers in the Answer Key to be sure you are doing them correctly.</p>
Worksheet	<p>Fractions as Ratios - Worksheet 2, page 48</p> <p>Before beginning this page, have him circle the tomato plants with red, the cucumber plants with blue, and the pepper plants with orange. He can also build 'plants' out of Cuisenaire Rods if the drawing is confusing.</p>
Part 2	Fractions as Ratios: Egg Carton Problems - Worksheets 1 and 2, pages 49 and 50 A dozen hard-boiled eggs, food dye, or paper cutouts to simulate the colored eggs
Materials	Fractions as Ratios: Egg Carton Problems - Worksheet 1. Prepare the eggs as instructed. Have him follow the directions. Allow the eggs to dry before dyeing the second color. Have him complete the bottom of the worksheet alone. Check his work.
Lesson	Do Fractions as Ratios: Egg Carton Problems - Worksheet 2 together. The wording 'the ratio of eggs with red on them to the total' looks like a fraction and acts like one. We are introducing a new language for thinking about fractions.
Extension	<p>If the work goes easily, then try this extension. Warning: This next part is really tricky conceptually. When we compare parts (eggs with red on them) to the total number of eggs in the carton, the denominator is still the whole of the set. In this case, the whole of the set is a dozen. But, ratios can also be used to compare the parts to each other.</p> <p>In this new case, the old concept of the denominator is changed. When you are comparing parts to parts, you write the fraction with the first quantity, eggs with red</p>

on them, in the numerator and the second quantity, eggs with green on them, in the denominator. You then can use the fraction to calculate probabilities and equations. We use this frequently in percent later.

"What is the ratio of eggs with red on them to eggs with green on them?"

"It is six to two or 6:2 or $\frac{6}{2}$."

Note

Do not have him calculate a reduction or simplification of the ratio at this time. Yes, you and I can see that 6:2 is equal to 3:1. But he will not see the connection even if you build it. The fraction looks like halves, but it isn't; it is the number of eggs with green on them.

"What is the ratio of eggs with green on them to eggs with red on them?"

"It is two to six or 2:6 or $\frac{2}{6}$."

This aspect of ratios can be confusing to a student at this level. By changing the positions of the colors green and red, we change the way the ratio looks.

"What is the ratio of eggs with purple on them to eggs with blue on them?" "Nine to three because there are nine eggs with purple on them and only three with blue on them."

"Do you think it changes the ratio if some of the eggs have both purple and blue on them?" "No, because we are just comparing the number of times each color appears."

Now we can use this to figure out harder problems.

Test for Understanding

Use a set of anything you have around, for example, silverware on a table setting, and ask, **"What is the ratio of forks to the total number of silverware pieces on the table? Or, what fraction of the silverware are forks?"** He should be able to answer this question easily. He should count all the silverware pieces first (including serving spoons) and then the forks. The fork number is the numerator and the total number is the denominator. *Stop.* Check the work to see that it is done correctly.

Challenge

(Most students will not be ready for this at this time, but you never know!) Try it yourself. Ask the question, if he does not understand what you are asking, drop it. If he does, he is abstracting very quickly and may have a talent for this sort of thing.

"If I want the ratio of eggs with red to eggs with green to stay the same, 6 to 2, how many eggs with red should I make if I have eight eggs with green on them right now?" "Twenty-four because the ratio was 6:2 red to green. If I go from two eggs with green to eight eggs with green, I have increased the number of eggs with green by a factor of four. I have four times more eggs with green. So if I had six eggs with red and increased it by a factor of four also, I would end up with twenty-four eggs with red."

This is called equivalent fractions, or equivalent ratios, which are called proportions. There is lots more on this concept in middle school math.

Fractions as Parts of Wholes

Purpose This lesson's purpose is to lay the foundation for multiplication of fractions. Students parse out given fractions of the whole. Taking a fraction of something feels like division, yet it is multiplication by a fraction. The student also has another chance to look for patterns and develop an algorithm for working with problems of this type.

Prerequisites Fractions as Ratios, multiplication and division facts up through the five times tables

Materials Beans or counters of any kind

Warm Up Part 1 Begin with twelve counters. **"Let us call these twelve a whole set. Give me one-half of the set. How do you know it is half?"** Put them all back together. Have the student give you one-third of the counters. This seems easy to an adult but can be difficult for a student. The story form seems to help. See the dialogue below. Repeat with one-fourth, one-sixth, and one-twelfth.

Make up a story using counters. **"A mama bunny had twelve baby bunnies. The mama bunny wanted to go to the lettuce patch, but she could take only half of the bunnies with her. How many bunnies could she take? How many piles would you have with halves? If she could take only one-third of the bunnies, how many bunnies could she take? How many piles would you have with thirds?"** Have the student use the counters to represent the bunnies.

If this was easy for the student, change the amount of the whole to eight and then ten.

Parts of Wholes

A conversation with Suki and two students.

Each person has twelve buttons.

"Let's do a story about a mama chinchilla that had twelve baby chinchillas.

The mama chinchilla could take only half of her babies to the raisin patch to eat raisins.

How many could she take? How many would half be?"

"If she could take only a third of her babies, how many would that be?"

"So, how could you prove that?"

"How would you figure it out with the buttons?"

"Yes, a third of all twelve of the babies."

"How many piles would you have?"

"When you had twelve and you divided it in half, how many piles did you have?"

(Kirstin had just gotten a chinchilla for Christmas. Laura is the other student.)

They each have twelve counters.

⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙⊙

K & L "Six." *(They each divide the counters into two piles of six.)*

⊙⊙⊙⊙⊙⊙ ⊙⊙⊙⊙⊙⊙

L "Three." K "Yeah, three."

(They each divide the counters in four piles of three.) ⊙⊙⊙ ⊙⊙⊙ ⊙⊙⊙ ⊙⊙⊙

K "A third of all of the babies?"

K "That would be three."

L "A third of twelve is three. All in three's or something? There would be three in each pile." *(This is a common misconception.)*

L "Four piles. So, she could only take three."

K & L "Two."

"Two piles. Remember when we went back to two? One of two, one-half. Now, in a half you have two piles. In a third, how many piles will you have?"

(They divide the counters into two piles.)

⊙⊙⊙⊙⊙⊙ ⊙⊙⊙⊙⊙⊙

L "Three. For thirds you should have three piles." ⊙⊙⊙⊙ ⊙⊙⊙⊙ ⊙⊙⊙⊙

"Is that true? If we had a pie and we were going to cut it into thirds, one for each of us, how many pieces would we have?"

K "Three."

L "You'd have three pieces."

"Start with all of the baby chinchillas. Divide them into thirds."

They divide the counters into three piles.

⊙⊙⊙⊙ ⊙⊙⊙⊙ ⊙⊙⊙⊙

"And how many piles?"

K & L "Three."

"So if the numbers are arranged in thirds, you'd have how many in each pile?"

L "OK, I've got three piles. There are four in each pile."

K "Four."

"Now the mama chinchilla, she could take only a fourth of her babies."

K "She could only take three." (Said confidently.)

"How many in each pile?"

K & L "Three."

"How many piles?"

K & L "Four."

We repeated this the following day with twelve again, and then changed the whole to eighteen.

Note

Taking a fraction of a group requires most students to change their concept of the divisor in division. When most students solve $12 \div 3$, they think about making groups of three from the twelve blocks. If you did the lesson Monkeys on the Roof in Division: Booklet 1, this would be the definition of the divisor. When we work with parts of wholes, the divisor is not the size of the groups but the number of groups. When we think of a third of something, we are not making groups of three, but we are making three groups. If we have twelve objects and we make groups of three, we have divided the whole into fourths. If we make groups of four, we have made thirds. This complexity concerning division is a source of confusion for many students. Do not simply tell them to divide to save time. The meaning of divisor and dividend is critical for understanding fractions.

Part 2

Materials

Cuisenaire Rods, crayons, and a piece of paper folded into thirds (a letter fold) Do a few fractions as ratio problems.

Warm Up

Put down two one dollar bills.

"Can you pick up one-half of the money?" She should pick up one dollar bill.

"How did you know to pick up a single dollar?" "There are two dollars. One half means to pick up one out of two, so I picked up one out of the two."

"So you divided all the things we had into two identical groups?" "Yes."

Put down six identical objects.

"How many of these would you pick up if I asked you to move one-half of the objects over to there?" (Pointing to a place a foot or so away from the pile.) "I would pick up three because I would need to make two equal groups and pick up one of the groups. Six divided into two groups gives three in each group. So I would move one group of three."

"What fraction is shown by my folded paper?" "Thirds."

"Can you arrange the six objects so that there are equal numbers of blocks in each section?" "I would put two blocks in each section because I would need to make three equal groups. Thirds means the whole is divided into three equal parts."

"What would you pick up if I asked you to move one-third of the blocks off the paper?" "I would move two blocks. Six divided into three groups gives two in each group. So I would give you one group of two."

"So two is one-third of six?" "Yes."

Move all the blocks off of the paper. Then add three more to the pile for a group of nine.

"Use the paper to divide this group of blocks into thirds. How many will be in each third?" "Three, because nine divided by three is three."

"What if there were fifteen blocks? Then how many would be in each third?" "Five, because fifteen divided into three equal groups is five in each group."

Note

If the student is unable to do this task, work on this concept before continuing. For example, fold a paper into fourths or use blocks to figure out what one-fourth of twenty-four is. One fourth of twenty-four is six in each section. If instead she makes groups of four and calls them fourths, ask, **"How many fourths make a whole?"** "Four." Put four fourths (of the twenty-four) together. **"Does this equal one whole?"** "No." **"It only equals sixteen. The whole is twenty-four. Try again. Prove that the group is one-fourth."** When she is working with twenty-four objects, a group of four blocks would be a sixth of the total. Making groups of four makes sixths and making groups of six makes fourths. Try using egg cartons and fractions of a dozen to clarify this. Do this in the grocery store as well. This concept will be needed when she learns how to simplify fractions.

Worksheet

Fractions as Parts of Wholes - Worksheet 1, page 51

Lesson Part 1

"Look at Fractions as Parts of Wholes - Worksheet 1. There are four groups. How many figures are in each group?" "Six."

"Color in half of the figures." She colors in three figures.

Do the same with the other shaped groups in this top section.

"Does it change the number of figures you color if I change the shapes or objects?" "No, it would only change if you changed the number of objects in a group."

"Now try the next set of figures . This next one asks you to color in one-third of the squares. How do you do that?" "I need to count all the squares in the set; there are twelve. Then I need to divide the twelve squares into three equal groups and color in one of those groups."

"How many squares did you color in this time?" "Four."

As she fills in the rest of the page, watch to see that she first counts all the figures in the whole group.

Be sure you have her go back now and make her own using different numbers.

Test for Understanding

Watch to see what she does with the last row of hexagons. This figure changes the number of shapes in the whole. It asks her to color in one-fourth of the hexagons. Watch to see if she first counts the number of hexagons and then divides that number into four equal sections of which she colors one. If she gets stuck, give her twenty-four blocks and a paper folded into fourths.

Worksheet

Do Fractions as Parts of Wholes - Worksheet 2, page 52, top half. The top of Worksheet 2 asks her to write out her thinking from when she worked with the hexagons. See if she can complete the top part on her own. Help only if needed.

"What was the first step?" "To count all the hexagons in the group."

"What was the second step?" "To divide the twenty-four hexagons into four equal groups. Twenty-four divided into four groups is six in each group. So I colored in six hexagons."

As she finishes the worksheet, be sure she draws lines in the rows of stars showing the groups and not just fill in the number answer on the end of the line, even if the work is easy. The visual image is important to develop so she will be able to do more difficult problems. Be sure she writes clear answers to the questions and can explain how she is getting the answers. Check the work immediately.

Note

If she has difficulty figuring out how many stars to color, as always, pair it with a manipulative. If she is working with sixths, fold the paper into sixths and give her blocks to stand for the stars. Have her put the blocks on the paper so that each section has the same number of blocks. Work towards her creating the manipulative she needs.

Worksheets

Fractions as Parts of Wholes - Worksheets 3 - 5, pages 53 - 55

These worksheets restate the partitioning of twelve with a new manipulative, Cuisenaire Rods.

Fractions as Parts of Wholes - Worksheet 3, page 53 On this worksheet, both the whole and the manipulative are changed, so she has to reconstruct the idea again.

Have her construct the whole out of twelve white cubes.

“Which block will cover half of the whole?” “The dark green.” Have her place the dark green block over the picture to check. Then have her color in the picture.

“How many white blocks are half of the whole twelve?” “Six.”

Finish the top half of the page in this way. Most students will whiz through this.

Most of the problems on the worksheet have the bars between the fractions darkened in. The last problem does not.

Have her put in the lines to show the fractional parts.

Practice Worksheets

Fractions as Parts of Wholes - Worksheets 4 and 5, pages 54 and 55

Worksheet 4 has no bars to show fractional parts. Halfway down the page, there is a prompt for her to check her work against what was done on the page before. Make sure she uses that.

If she needs help, have her use the colors on Worksheet 3 to help her know which color rod to use. Have her draw in the bars between the rods. Have her use the colors that match the colors of the rods.

Worksheet 5 shows the whole twelve blocks on the top row and then shows the new fractional part in a simplified way. To get the answer, she must use both bars. The last three problems require her to draw the simpler fraction. On the second one, for example, she would use the purple rod to measure out the thirds and put dark lines between the rods on the picture. Then she shades the required amount and gives the answer using the top blocks.

Check the Answer Key immediately. Watch for the confusion between number of blocks in each group and the number of groups. It is the number of groups that determines the fraction.

Be sure she colors in the worksheets to match the colors of the rods. Check the work with the Answer Key. This is a good stopping point.

Lesson Part 2

Developing a Procedure for Calculation

Fractions as Parts of Wholes - Worksheets 6 and 7, pages 56 and 57

It is important that the student uses blocks to solve these problems. She should get out fifteen blocks and group them into fifths only once. All of the problems in that set can be solved by looking at the blocks arranged in five equal groups.

Hopefully, she will notice the connection between this process and simple division facts. She may begin to verbalize methods to predict the answers. If she begins to do the work without using the blocks, ask her how she is getting the answers. Verify answers with blocks. Be sure she fills in all the answers to the questions in clear language.

Test for Understanding

As you work down on Fractions as Parts of Wholes - Worksheet 7 you are testing for understanding and the ability to move from the blocks to an abstract way of thinking about the numbers. Use restatement to focus the words.

1. **"Look at the last five problems on Worksheet 7. What is $\frac{1}{5}$ of twenty?"** "Four."
"How did you figure out what is $\frac{1}{5}$ of twenty?" "I divide twenty by five because I need to cut twenty into five equal groups."
"So taking one-fifth of twenty is the same as dividing by five? One-fifth of twenty equals $20 \div 5$?" "Yes."

2. **"How did you get two-fifths of twenty?"** I know one-fifth of twenty is equal to four, so two-fifths would be twice that, or eight. I multiply the four by two to get eight."

"So first you divided twenty by five to get one-fifth of twenty, which is equal to four, and then multiplied by two to get the eight?" "Yes."

Write that process down as a number sentence. $20 \div 5 = 4$ $4 \times 2 = 8$

"Can you write the process as an equation?"

If not, the teacher writes this: $\longrightarrow (20 \div 5) \times 2 = 8$

Help her if she needs it.

"Why do you think I put parentheses in the number sentence?"

"To make sure that I divide the twenty by five first, before I multiply."

"What would happen if I multiplied five times two first and then divided?

Try it." $20 \div 5 \times 2$ "You would get $20 \div 10 = 2$, which is the wrong answer."

At the bottom of Fractions as Parts of Wholes - Worksheet 7, **"How do you figure out what is four-fifths of twenty?"**

Step 1: Divide the twenty into five equal groups. There are four in each group.

Step 2: Multiply the four in each group by the four given in the numerator.

Don't let her carelessly write numbers in there for answers.

If she has no idea how to use multiplication or division to help, do more manipulative practice before moving on to Worksheet 8. Help her put her process into words.

Fractions as Parts of Wholes - Worksheet 8 guides the transition from the manipulative way of doing the problems to an abstract algorithm used in multiplication of fractions.

"What patterns did you find in your work on pages 56 and 57, Worksheets 6 and 7?" Answers will vary. See the Answer Key for some possible answers.

"We are now going to work on calculating the answers to problems like these

without using the blocks or a drawing. You are going to construct the algorithm to make it faster."

"Look at the top question on Fractions as Parts of Wholes - Worksheet 8. How do you solve $\frac{3}{4}$ of 40 = ____?"

Step 1: "Divide the forty by four, which is ten."

"What number told you to divide by four?" "The number in the denominator."

Step 2: "Multiply the ten by three to get thirty."

"What number told you to multiply by three?" "The number in the numerator."

"What is the first step of figuring out what three-fourths of twenty is?"
"Divide the twenty by the four."

"Where do you write the first step in the number sentence?" "Inside the first set of parentheses." Have her write the four in the blank after the $20 \div$ ____.

"Where do you write the second step in the number sentence?" "After the times sign."

"What is the second step here?" "To multiply by three because we need three of the four groups." Have her write in the three under the words 'second step' and then fill in the answer.

Have her finish the rest of the worksheet alone. Check the work immediately with the Answer Key.

Parts of Wholes as Multiplication of Fractions

Purpose	The purpose of this lesson is to connect the concept of taking a fraction of a quantity, parts of wholes, to the written operation of multiplication by a fraction. The essential understanding here is that multiplying by a fraction is a division by the denominator followed by a multiplication of the numerator. Multiplication by a fraction can also mean adding a fraction several times.
Prerequisites	Fractions as Parts of Wholes and Addition of Like Fractions
Materials	Parts of Wholes as Multiplication of Fractions - Worksheet 1, page 59
Warm Up	Review Parts of Wholes by giving a problem such as one-third of twenty-one. Have the student build it and write it as a number sentence as was done on Parts of Wholes - Worksheet 8. Do several of these types of problems.
Lesson	<p>Give Parts of Wholes as Multiplication of Fractions - Worksheet 1. Go over the material in the box at the top of the worksheet that introduces the x sign as a replacement for the word 'of.' Do the first problem. This material is already familiar from the last lesson.</p> <p>Do the first half of the worksheet with him. Check your answers.</p> <p>Then, have him complete Parts of Wholes as Multiplication of Fractions - Worksheet 1.</p>
Note	Do not teach the standard multiplication procedure at this point. The student needs to do the next booklet to understand improper fractions and the relationship between fractions and division to process this. Two-thirds of nine can also mean two-thirds added nine times which gives eighteen-thirds, an improper fraction. This lovely connection occurs in the next booklet.
Worksheets	Parts of Wholes as Multiplication of Fractions, Worksheets 2 - 4, pages 60 - 62
Test for Understanding	Parts of Wholes as Multiplication of Fractions - Worksheet 3 is an assessment in the form of word problems. If he understands the concept of a fraction of a whole, and the changing nature of what is defined as the whole, he will be able to do these word problems easily.

Fractional Parts of Sets

Purpose The purpose of this lesson is to develop an understanding of fractional parts of a set. When fractions describe parts of a set of objects, only the number of objects is regarded, not the size of the objects. This is in contrast to one whole divided into fractional parts where each piece has to be equal in size.

Prerequisites Changing Wholes and Parts of Wholes

Materials Groups or sets of objects that are not equal in size, e.g., a pencil cup holding writing tools, stuffed animals, silverware, pieces of clothing such as socks, or family members. There is no Student Worksheet.

Lesson Review sorting a group of objects in different ways and naming the groups, e.g., pencil cup sorted into pencils, pens, scissors, erasers, markers, long shapes, short shapes, black objects, non-black objects.

Initially use more and less in the comparison of the sorted groups. Are there more black objects or non-black ones? Then, return to a concrete set, sort, and discuss it.

"How many objects are in the whole group?" "Sixteen in the pencil cup."

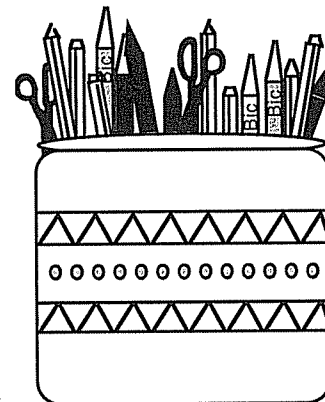
"Are they all of equal size?" "No."

"How many of the whole group are pencils?" "Eight."

"What part of the whole are pencils?" "Eight out of sixteen or eight-sixteenths."

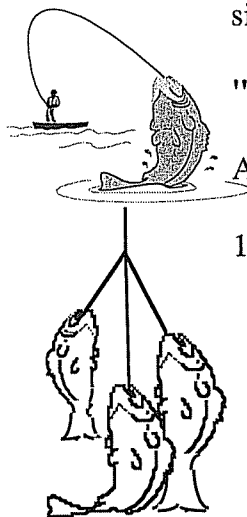
"What part of the whole are not pencils?" "Eight out of sixteen or eight-sixteenths."

"What part of the whole are black?" "Six out of sixteen or six-sixteenths."



Ask these types of questions:

1. Casey caught three trout and his dad caught only two.
 - A. "How many trout were caught in total?" "Five."
 - B. "What fraction of the fish did Casey catch?" "Three-fifths."
 - C. "What fraction of the fish did his dad catch?" "Two-fifths."
 - D. "Do you think the weight of Casey's fish is three-fifths of the weight of all the fish?" _____ " This is an open ended question as we don't know the weight of the fish."



Fractions - Booklet 2

Pre-Assessment: Part 1 - Worksheet 1

- What is the color of the $\frac{1}{3}$ piece in your set? orange AWV
- a. Cover the whole with fourths. How many fourths does it take to cover a whole? 4
b. Cover the whole with sixths. How many sixths does it take to cover a whole? 6
- Circle all of the designs below which are cut in half.
- Write the name of this fraction in words. $\frac{2}{3}$ two-thirds
- a. Write in the correct < or > sign. $\frac{1}{4} < \frac{3}{4}$
b. Circle the fraction which shows a larger amount. $\frac{1}{4}$ $\frac{1}{3}$
c. Which fraction shows a larger amount $\frac{4}{5}$ or $\frac{3}{6}$?
d. How do you know? Fifths are larger than sixths and there are more of them.
- If the is equal to $\frac{1}{6}$, which of these figures is equal to one whole?
- a. Shade in $\frac{2}{3}$ of this rectangle.
- b. What fraction of this rectangle is not shaded? $\frac{1}{6}$
- c. 3 out of 4 of the circles are shaded.

1

Pre-Assessment: Part 1 - Worksheet 2

- Here are two pizzas. a. Shade in half of each pizza.
This is half of pizza A. This is half of pizza B.
b. The pieces are both halves, but they are not the same size.
How can this be? Each is $\frac{1}{2}$ of a different size whole.
- List all the fractions you can make with your set of fraction pieces that equal to $\frac{1}{2}$. $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$
- a. Trade three-fourths for twelfths with your fraction pieces. How many twelfths are equal to three-fourths? 9
b. How many sixths are needed to cover one-third? 2
- Fill in the missing numbers in this pattern. $2 = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = \frac{10}{5} = \frac{12}{6}$
- a. The arrow points to which fraction on the number line. $\frac{1}{3}$

b. What fraction is the arrow pointing to? $\frac{3}{4}$
- How many twelfths are in two-thirds? 8
Draw a line on the number line to prove your answer.

2

Pre-Assessment: Part 2 - Worksheet 1

- Put these fractions in order from the least to the greatest.

$\frac{4}{5}$ $\frac{1}{2}$ $\frac{1}{6}$ $\frac{3}{5}$ $\frac{1}{4}$
 $\frac{1}{6}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{5}$ $\frac{4}{5}$

- a. If equals 1, then what does equal? $\frac{1}{6}$
b. If equals 3, then what does equal? 1
- a. If equals $\frac{1}{2}$, then what does equal? $\frac{1}{24}$
b. If equals $\frac{1}{3}$, then what does equal? $\frac{1}{9}$
- Solve. a. $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$ or 1 b. $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$ c. $\frac{6}{10} + \frac{2}{10} = \frac{8}{10}$ or $\frac{4}{5}$
- Solve. a. $\frac{7}{8} - \frac{1}{8} = \frac{6}{8}$ b. $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ c. $\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$

3

Pre-Assessment: Part 2 - Worksheet 2

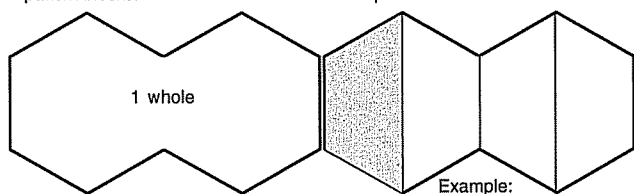
- What fractional part of the letters are capital letters? $\frac{6}{14}$
- a. Circle $\frac{1}{2}$ of the group.
b. Circle $\frac{1}{3}$ of the group.
- Circle $\frac{3}{4}$ of the set.
- a. $\frac{1}{2}$ of 10 = 5 b. $\frac{1}{3}$ of 15 = 5 c. $\frac{1}{5}$ of 20 = 4
- a. $\frac{3}{5}$ of 25 = 15 b. $\frac{2}{7}$ of 21 = 6 c. $\frac{4}{9}$ of 36 = 16
- a. $\frac{3}{4} \times 16 = 12$ b. $\frac{4}{5} \times 30 = 24$ c. $\frac{6}{7} \times 42 = 36$

4

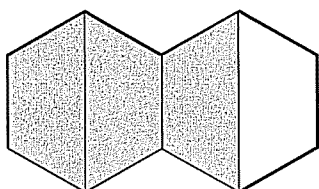
Pattern Block Fractions: Review - Worksheet 1

Cover with four equal pattern blocks.

Each block is one of four equal parts or $\frac{1}{4}$ of the shape.



Example:
How many parts are colored? $\frac{1}{4}$ out of 4



$\frac{3}{4}$ out of 4 = $\frac{3}{4}$

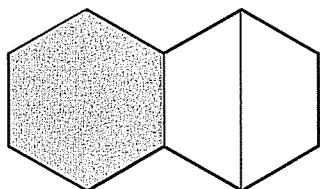


How many parts are colored? 3 out of 4

Shade this one.

2 out of 4

$\frac{2}{4}$ out of 4 = $\frac{2}{4}$

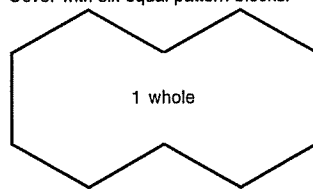


11

Pattern Block Fractions: Review - Worksheet 2

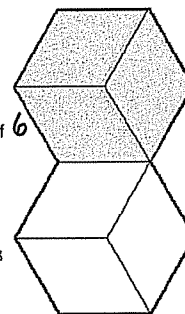
Cover with six equal pattern blocks.

Each block is one of six equal parts or $\frac{1}{6}$ of the shape.



How many parts are colored? 3 out of 6

$\frac{3}{6}$ out of 6 = $\frac{3}{6}$



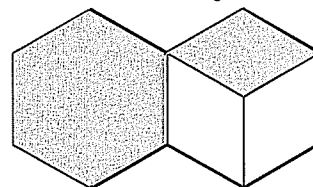
How many parts are colored? 3 out of 6

$\frac{1}{6}$ out of 6 = $\frac{1}{6}$

Shade this one.

4 out of 6

$\frac{4}{6}$ out of 6 = $\frac{4}{6}$

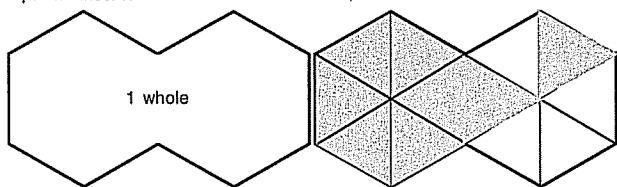


12

Pattern Block Fractions: Review - Worksheet 3

Cover with twelve equal pattern blocks.

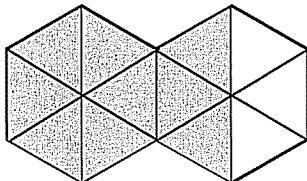
Each block is one of twelve equal parts or $\frac{1}{12}$ of the shape.



How many parts are colored? 8 out of 12

How many parts are colored? 9 out of 12

$\frac{8}{12}$ out of 12 = $\frac{8}{12}$



$\frac{9}{12}$ out of 12 = $\frac{9}{12}$

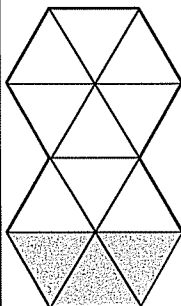
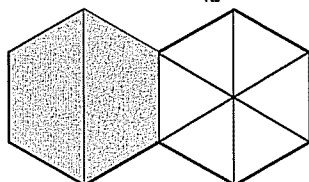
How many parts are colored? 3 out of 12

$\frac{3}{12}$ out of 12 = $\frac{3}{12}$

Shade this one.

6 out of 12

$\frac{6}{12}$ out of 12 = $\frac{6}{12}$

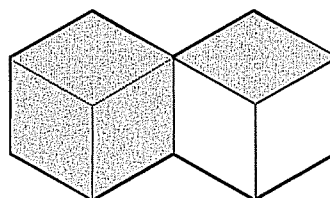


13

Pattern Block Fractions: Review - Worksheet 4

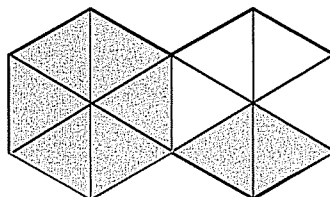
What fraction is colored?

4 out of 6 $\frac{4}{6}$ out of 6 = $\frac{4}{6}$



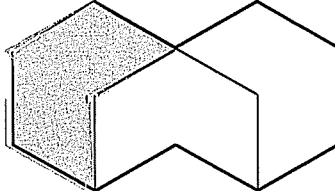
What fraction is colored?

8 out of 12 $\frac{8}{12}$ out of 12 = $\frac{8}{12}$



What fraction is colored?

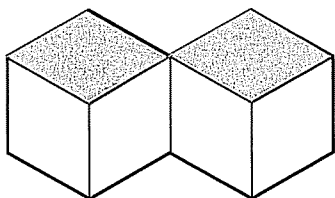
1 out of 3 $\frac{1}{3}$ out of 3 = $\frac{1}{3}$



Shade this one.

2 out of 6

$\frac{2}{6}$ out of 6 = $\frac{2}{6}$



14

Recording: Prism Fractions



Refresh your memory as to the colors in Prism Fractions.

If 1 white is 1 whole.

1 red piece is $\frac{1}{2}$ of the whole.

1 orange piece is $\frac{1}{3}$ of the whole.

1 yellow piece is $\frac{1}{4}$ of the whole.

1 green piece is $\frac{1}{5}$ of the whole.

1 blue piece is $\frac{1}{6}$ of the whole.

1 indigo piece is $\frac{1}{8}$ of the whole.

1 violet piece is $\frac{1}{10}$ of the whole.

1 black piece is $\frac{1}{12}$ of the whole.
or gray

15

Greater Than, Lesser Than or Equal To Worksheet 1

$$1 > \frac{1}{2} \quad \frac{1}{3} < 1 \quad \frac{1}{2} = \frac{2}{4}$$

Show the relationship by putting in $<$ or $>$ or $=$ between the fractions. Use Fraction pieces to prove your answers.

$$\frac{1}{2} < 1$$

$$\frac{1}{3} < 1$$

$$\frac{1}{4} < 1$$

$$\frac{1}{2} > \frac{1}{3}$$

$$\frac{1}{4} > \frac{1}{8}$$

$$\frac{2}{4} = \frac{1}{2}$$

$$\frac{1}{6} < \frac{1}{4}$$

$$\frac{1}{3} > \frac{1}{4}$$

$$\frac{1}{8} > \frac{1}{12}$$

$$\frac{1}{2} > \frac{1}{6}$$

$$\frac{1}{3} = \frac{2}{6}$$

$$\frac{1}{3} < \frac{1}{2}$$

$$\frac{2}{3} > \frac{1}{2}$$

$$\frac{2}{3} > \frac{3}{8}$$

$$\frac{3}{4} > \frac{3}{8}$$

26

Greater Than, Lesser Than or Equal To Worksheet 2

$$1 > \frac{1}{2} \quad \frac{1}{3} < 1 \quad \frac{1}{2} = \frac{2}{4}$$

Show the relationship by putting in $<$ or $>$ or $=$ between the fractions. Use Fraction pieces to prove your answers.

$$1 > \frac{3}{4}$$

$$\frac{2}{3} < 1$$

$$\frac{4}{5} < 1$$

$$\frac{1}{4} < \frac{3}{8}$$

$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{2}{3} < \frac{3}{4}$$

$$\frac{3}{8} < \frac{3}{4}$$

$$\frac{2}{3} < \frac{4}{5}$$

$$\frac{1}{6} < \frac{1}{3}$$

$$\frac{2}{5} < \frac{3}{4}$$

$$\frac{3}{6} < \frac{3}{4}$$

$$\frac{4}{12} > \frac{2}{6}$$

$$\frac{3}{4} > \frac{2}{3}$$

$$\frac{2}{3} > \frac{5}{8}$$

$$\frac{3}{4} > \frac{5}{8}$$

27

Greater Than, Lesser Than or Equal To Worksheet 3

Put fractions into the blanks to make the number sentence true. No zeros allowed.

$$\frac{1}{2} > \frac{1}{3}$$

$$\frac{1}{2} > \frac{1}{4}$$

$$1 > \frac{1}{2}$$

$$1 > \frac{1}{3}$$

$$\frac{1}{3} > \frac{1}{4}$$

$$\frac{2}{3} > \frac{1}{4}$$

$$\frac{1}{3} < \frac{1}{2}$$

$$\frac{1}{3} > \frac{3}{4}$$

Do the same with these number sentences.

$$\frac{1}{2} < \frac{2}{2}$$

$$\frac{1}{2} < 1$$

$$\frac{1}{3} < \frac{1}{2}$$

$$\frac{1}{6} < \frac{1}{2}$$

$$\frac{1}{2} < \frac{1}{3}$$

$$\frac{1}{3} < \frac{1}{4}$$

$$1 < \frac{1}{4}$$

$$\frac{1}{2} < \frac{1}{4}$$

Do the same with these number sentences.

$$\frac{2}{6} < \frac{5}{6}$$

$$\frac{5}{6} > \frac{1}{6}$$

$$\frac{1}{6} > \frac{1}{10}$$

$$\frac{1}{10} < \frac{1}{6}$$

Do the same with these number sentences.

$$\frac{1}{2} = \frac{2}{4}$$

$$\frac{1}{3} = \frac{2}{6}$$

$$\frac{1}{4} = \frac{2}{8}$$

$$\frac{2}{8} = \frac{1}{4}$$

Put the symbol in the box to show the relationship.

$$\frac{1}{4} \boxed{>} \frac{1}{6}$$

$$\frac{1}{2} \boxed{=} \frac{6}{12}$$

$$\frac{1}{3} \boxed{>} \frac{1}{4}$$

$$\frac{1}{3} \boxed{=} \frac{3}{9}$$

$$\frac{2}{6} \boxed{<} \frac{3}{6}$$

$$\frac{1}{2} \boxed{>} \frac{1}{6}$$

$$\frac{1}{3} \boxed{<} \frac{1}{2}$$

$$\frac{1}{4} \boxed{<} \frac{1}{2}$$

$$\frac{3}{4} \boxed{=} \frac{9}{12}$$

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Greater Than, Lesser Than or Equal To: Assessment Worksheet 4

1. Put these fractions in order from smallest to largest.

$$\frac{1}{3}, \frac{1}{2}, \frac{1}{10}, \frac{1}{5}, \frac{1}{4}, \frac{1}{12}$$

$$\frac{1}{12}, \frac{1}{10}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$$

2. How do you know which fraction is the smallest?

The denominator is larger and the pieces are smaller.

3. Put these fractions in order from smallest to largest. Do not use Fraction pieces. Just reason it out.

$$\frac{9}{10}, \frac{2}{3}, \frac{4}{5}, \frac{3}{4}, \frac{7}{8}, \frac{5}{6}, \frac{1}{2}$$

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}$$

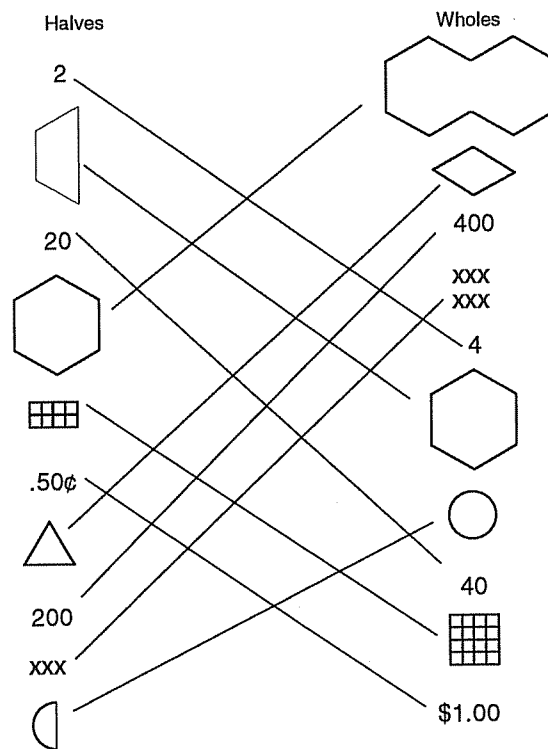
4. Now check with your fraction pieces. Do not erase what you did above. Write the order here again even if you got it right.

5. What pattern do you notice about the numbers in the fraction list.

Numerator increases by 1 and so does the denominator. Each fraction is a little larger than the previous one and all the numerators are one.

Changing Wholes - Worksheet 1

Connect these halves with their wholes.



Make your own.

The bigger the half, the bigger the whole.

30

Changing Wholes - Worksheet 2

Use pattern blocks.

If 1 yellow = 1,
then 1 red = $\frac{1}{2}$
1 blue = $\frac{1}{3}$
1 green = $\frac{1}{6}$

If 2 yellows = 1,
then 1 red = $\frac{1}{4}$
1 blue = $\frac{1}{6}$
1 green = $\frac{1}{12}$
1 yellow = $\frac{1}{2}$

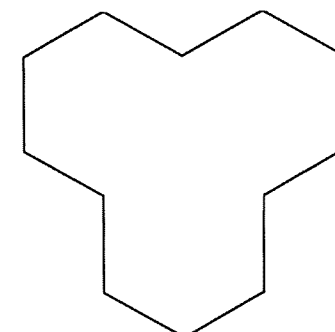
If 3 yellows = 1,
then 1 red = $\frac{1}{3}$
1 blue = $\frac{1}{6}$
1 green = $\frac{1}{18}$
1 yellow = $\frac{1}{9}$



If 1 green = 1,
then 1 red = 3 or $\frac{3}{1}$
1 blue = 2 or $\frac{2}{1}$
1 yellow = 6 or $\frac{6}{1}$

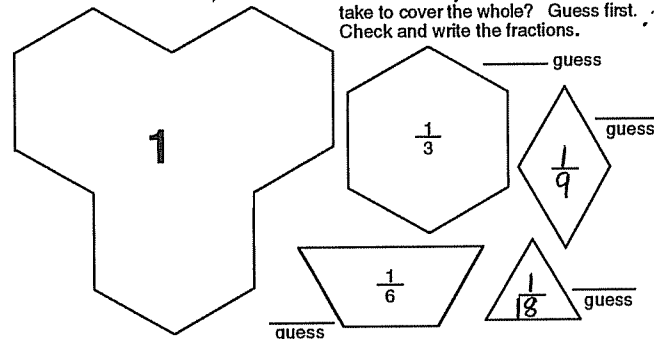
Changing Wholes - Worksheet 3

Make a design. Trace and color.

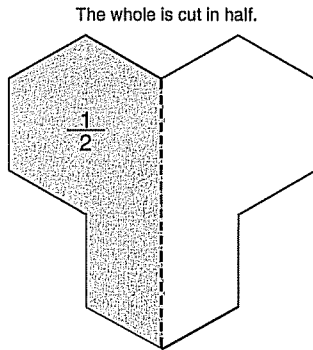
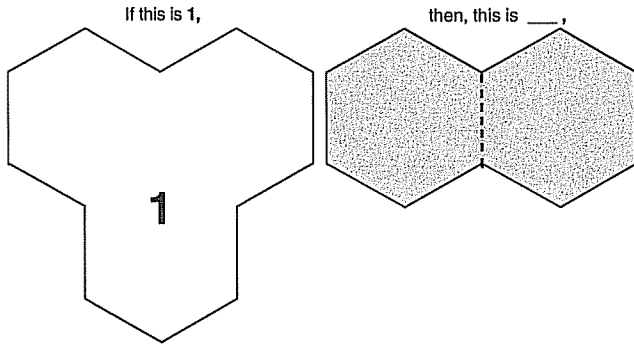


If this is 1,

then how many of each block does it take to cover the whole? Guess first. Check and write the fractions.



Changing Wholes - Worksheet 4



How many reds fit on the half? 3

$$\frac{1}{2} = \frac{3}{6}$$

How many greens fit on the half? 9

$$\frac{1}{2} = \frac{9}{18}$$

Challenge!

How many blues fit on the half? 4 blues + 1 green

$$\frac{1}{2} = \frac{4.5}{9}$$

33

Changing Wholes - Worksheet 5

If this = 1, twelve

then the red rod = $\frac{1}{6}$
 the light green rod = $\frac{1}{4}$
 the dark green rod = $\frac{1}{2}$
 1 purple rod = $\frac{1}{3}$
 1 white rod = $\frac{1}{12}$

If this = 1, twenty-four

then the red rod = $\frac{1}{12}$
 the light green rod = $\frac{1}{8}$
 the dark green rod = $\frac{1}{4}$
 1 purple rod = $\frac{1}{6}$
 1 white rod = $\frac{1}{24}$

If this = 1, six

then the red rod = $\frac{1}{3}$
 the light green rod = $\frac{1}{2}$
 the dark green rod = $\frac{1}{1}$
 1 white rod = $\frac{1}{6}$

If this = 1, sixteen

then the brown rod = $\frac{1}{2}$
 the purple rod = $\frac{1}{4}$
 the red rod = $\frac{1}{8}$
 1 white rod = $\frac{1}{16}$

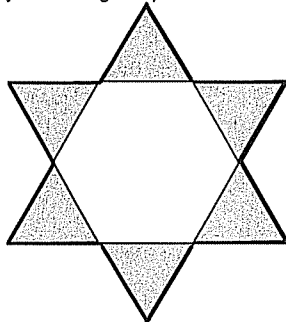
If this = 1, thirty-two

then the brown rod = $\frac{1}{4}$
 the purple rod = $\frac{1}{8}$
 the red rod = $\frac{1}{16}$
 1 white rod = $\frac{1}{32}$

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Changing Wholes - Worksheet 6

Teacher: Please read the guide before doing this activity
 Build this figure with yellow and green pattern blocks.

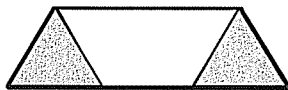


1) Is there more green wood or yellow wood?

2) What fraction is yellow? $\frac{1}{2}$

3) What fraction is green? $\frac{1}{2}$

Build this figure with red and green pattern blocks.



4) How many green blocks would cover the entire figure? 5

5) Is there more green wood or red wood? red

6) What fraction is red? $\frac{2}{5}$

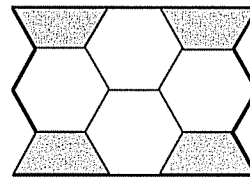
7) What fraction is green? $\frac{3}{5}$

8) Circle the larger fraction: $\frac{2}{5}$ $\frac{3}{5}$

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Changing Wholes - Worksheet 7

Build this figure with yellow and red pattern blocks.



1) Is there more red wood or more yellow wood? yellow

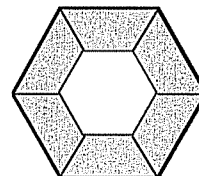
2) How many red blocks would the entire figure? 12

3) What fraction of the figure is red? $\frac{4}{12}$

4) What fraction is yellow? $\frac{8}{12}$

5) Record on the line the two fractions.
 $\frac{8}{12}$ is greater than $\frac{4}{12}$
 (fraction) (fraction)

Build this figure with yellow and red pattern blocks.



6) Is there more red wood or more yellow wood? red

7) What fraction is red? $\frac{3}{6}$

8) What fraction is yellow? $\frac{3}{6}$

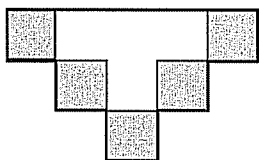
9) Record on the line the two fractions.
 $\frac{3}{6}$ is greater than $\frac{3}{6}$
 (fraction) (fraction)

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Changing Wholes - Worksheet 8

Build the figures with pattern blocks to help answer these questions.

- 1) Is more of the figure shaded or more unshaded? more shaded



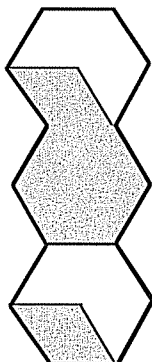
- 2) What fraction of the whole is shaded? $\frac{7}{10}$

- 3) What fraction of the whole is unshaded? $\frac{3}{10}$

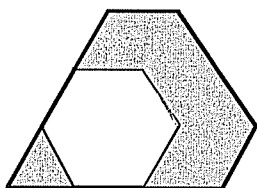
- 4) Is more of the figure shaded or more unshaded? shaded

- 5) What fraction of the whole is shaded? $\frac{5}{9}$

- 6) What fraction of the whole is unshaded? $\frac{4}{9}$



- 7) Is more of the figure shaded or more unshaded? shaded



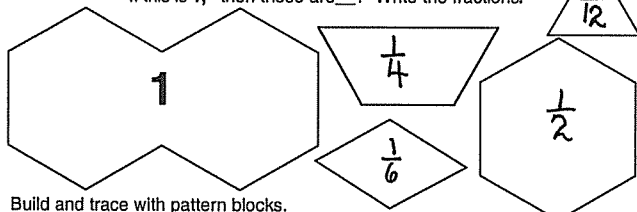
- 8) What fraction of the whole is shaded? $\frac{8}{14}$

- 9) What fraction of the whole is unshaded? $\frac{6}{14}$

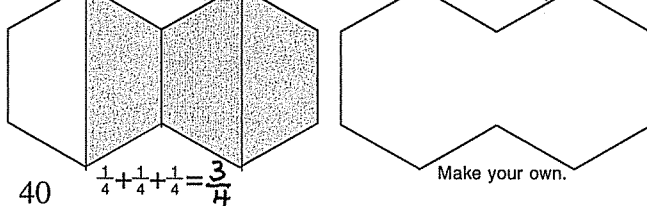
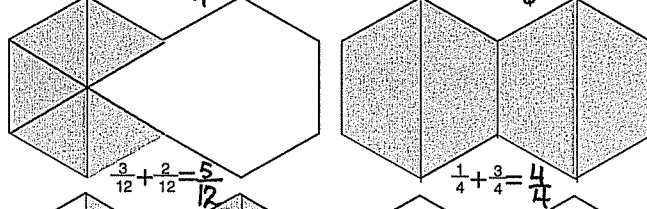
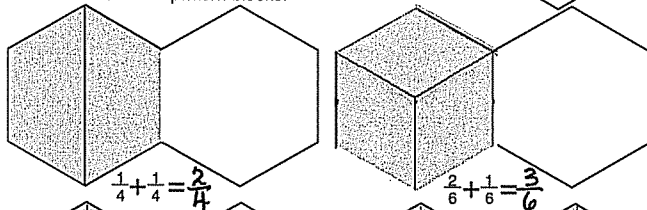
37

Addition of Like Fractions - Worksheet 2

If this is 1, then these are __? Write the fractions.



Build and trace with pattern blocks.



40

Addition of Like Fractions - Worksheet 1

Guess the answers to these fraction problems.

Then build each problem with fraction pieces.

Look for patterns. Record the correct answer on the check line.

Guess	Check	Guess	Check
$\frac{1}{4} + \frac{1}{4} =$ —	<u>$\frac{2}{4}$</u>	$\frac{1}{12} + \frac{4}{12} =$ —	<u>$\frac{5}{12}$</u>
$\frac{1}{3} + \frac{1}{3} =$ —	<u>$\frac{2}{3}$</u>	$\frac{1}{8} + \frac{2}{8} =$ —	<u>$\frac{3}{8}$</u>
$\frac{1}{6} + \frac{1}{6} =$ —	<u>$\frac{2}{6}$</u>	$\frac{1}{5} + \frac{1}{5} =$ —	<u>$\frac{2}{5}$</u>

What patterns did you find? Both fractions have the same denominators in the addend in all problems.

Write a rule for adding like fractions. Add the top numbers (numerators) not the bottom numbers (denominators).

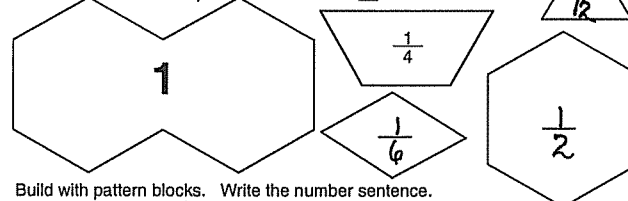
Why are these problems called "like" fractions? The denominators are the same in both addends.

$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$	$\frac{2}{12} + \frac{5}{12} = \frac{7}{12}$	$\frac{4}{6} + \frac{1}{6} = \frac{5}{6}$	$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$	$\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$
$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$	$\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$	$\frac{1}{3} + \frac{2}{3} = \frac{3}{3}$	$\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$	$\frac{2}{8} + \frac{5}{8} = \frac{7}{8}$

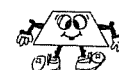
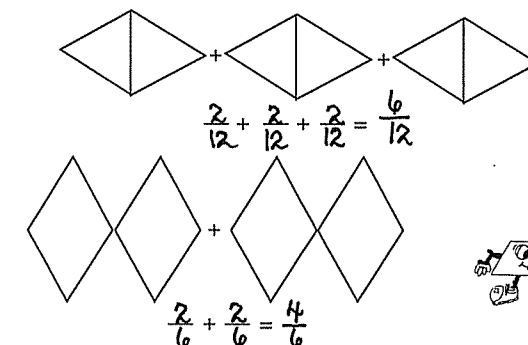
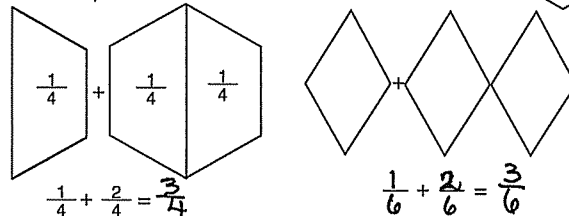
39

Addition of Like Fractions - Worksheet 3

If this is 1, then these are __? Write the fractions.



Build with pattern blocks. Write the number sentence.



41

Subtraction of Like Fractions - Worksheet 1

Guess the answers to these fraction problems.
Then build each problem with fraction pieces.
Look for patterns. Record the correct answer on the check line.

Guess	Check	Guess	Check
$\frac{3}{4} - \frac{1}{4} = \underline{\quad}$	$\frac{2}{4}$	$\frac{8}{12} - \frac{3}{12} = \underline{\quad}$	$\frac{5}{12}$
$\frac{2}{3} - \frac{1}{3} = \underline{\quad}$	$\frac{1}{3}$	$\frac{7}{8} - \frac{2}{8} = \underline{\quad}$	$\frac{5}{8}$
$\frac{5}{6} - \frac{2}{6} = \underline{\quad}$	$\frac{3}{6}$	$\frac{4}{5} - \frac{3}{5} = \underline{\quad}$	$\frac{1}{5}$

What patterns did you find? Both fractions in a problem have the same denominator.

Write a rule for subtracting like fractions. Why are these fractions called "like" fractions?
Take away the second numerator from the first numerator. Denominator stay the same.
Solve these problems using the rule you figured out. Build and check with fraction pieces.

$$\begin{array}{r} \frac{5}{8} \\ - \frac{1}{8} \\ \hline \frac{4}{8} \end{array} \quad \begin{array}{r} \frac{7}{12} \\ - \frac{3}{12} \\ \hline \frac{4}{12} \end{array} \quad \begin{array}{r} \frac{4}{6} \\ - \frac{1}{6} \\ \hline \frac{3}{6} \end{array} \quad \begin{array}{r} \frac{3}{5} \\ - \frac{1}{5} \\ \hline \frac{2}{5} \end{array} \quad \begin{array}{r} \frac{4}{6} \\ - \frac{2}{6} \\ \hline \frac{2}{6} \end{array}$$

$$\begin{array}{r} \frac{11}{12} \\ - \frac{3}{12} \\ \hline \frac{8}{12} \end{array} \quad \begin{array}{r} \frac{7}{8} \\ - \frac{3}{8} \\ \hline \frac{4}{8} \end{array} \quad \begin{array}{r} \frac{2}{3} \\ - \frac{1}{3} \\ \hline \frac{1}{3} \end{array} \quad \begin{array}{r} \frac{8}{10} \\ - \frac{4}{10} \\ \hline \frac{4}{10} \end{array} \quad \begin{array}{r} \frac{3}{8} \\ - \frac{3}{8} \\ \hline 0 \end{array}$$

42

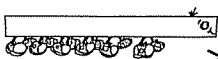
Subtraction of Like Fractions - Worksheet 3

Build these problems with Fraction pieces and find the answers. Choose three problems and draw a picture of the solution.

$$\begin{array}{r} \frac{6}{6} \\ - \frac{1}{6} \\ \hline \frac{5}{6} \end{array} \quad \begin{array}{r} \frac{8}{8} \\ - \frac{2}{8} \\ \hline \frac{6}{8} \end{array} \quad \begin{array}{r} \frac{12}{12} \\ - \frac{4}{12} \\ \hline \frac{8}{12} \end{array} \quad \begin{array}{r} \frac{5}{5} \\ - \frac{2}{5} \\ \hline \frac{3}{5} \end{array} \quad \begin{array}{r} \frac{10}{10} \\ - \frac{3}{10} \\ \hline \frac{7}{10} \end{array}$$

$$\begin{array}{r} \frac{5}{6} \\ - \frac{2}{6} \\ \hline \frac{3}{6} \end{array} \quad \begin{array}{r} \frac{8}{8} \\ - \frac{7}{8} \\ \hline \frac{1}{8} \end{array} \quad \begin{array}{r} \frac{12}{12} \\ - \frac{8}{12} \\ \hline \frac{4}{12} \end{array} \quad \begin{array}{r} \frac{5}{8} \\ - \frac{2}{8} \\ \hline \frac{3}{8} \end{array} \quad \begin{array}{r} \frac{8}{10} \\ - \frac{1}{10} \\ \hline \frac{7}{10} \end{array}$$

$$\begin{array}{r} \frac{5}{8} \\ - \frac{4}{8} \\ \hline \frac{1}{8} \end{array} \quad \begin{array}{r} \frac{11}{12} \\ - \frac{9}{12} \\ \hline \frac{2}{12} \end{array} \quad \begin{array}{r} \frac{8}{12} \\ - \frac{6}{12} \\ \hline \frac{2}{12} \end{array} \quad \begin{array}{r} \frac{4}{5} \\ - \frac{3}{5} \\ \hline \frac{1}{5} \end{array} \quad \begin{array}{r} \frac{2}{3} \\ - \frac{1}{3} \\ \hline \frac{1}{3} \end{array}$$



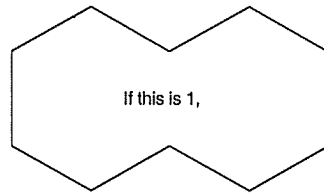
Have you found a pattern that makes it easier?

Just subtract the numerators.

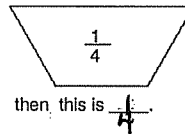
44

Subtraction of Like Fractions - Worksheet 2

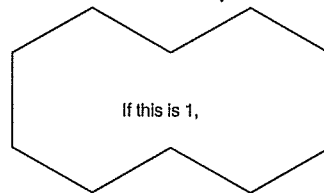
Use pattern blocks to find the answers to these problems.



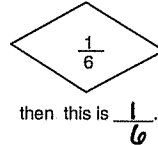
If this is 1,



then this is $\frac{1}{4}$.



If this is 1,



then this is $\frac{1}{6}$.

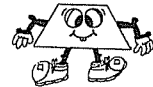
$$\begin{array}{l} \frac{4}{4} - \frac{3}{4} = \frac{1}{4} \\ \frac{4}{4} - \frac{1}{4} = \frac{3}{4} \\ \frac{4}{4} - \frac{2}{4} = \frac{2}{4} \\ \frac{3}{4} - \frac{1}{4} = \frac{2}{4} \end{array}$$



Make your own

AWV

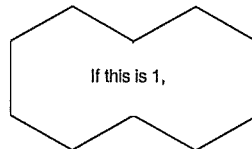
$$\begin{array}{l} \frac{6}{6} - \frac{5}{6} = \frac{1}{6} \quad \frac{6}{6} - \frac{3}{6} = \frac{3}{6} \\ \frac{6}{6} - \frac{1}{6} = \frac{5}{6} \quad \frac{5}{6} - \frac{5}{6} = 0 \\ \frac{5}{6} - \frac{4}{6} = \frac{1}{6} \quad \frac{4}{6} - \frac{2}{6} = \frac{2}{6} \end{array}$$



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Subtraction of Like Fractions - Worksheet 4

Use pattern blocks to find the answers to these problems.



If this is 1,



then this is $\frac{1}{12}$.

$$\begin{array}{l} \frac{9}{12} - \frac{5}{12} = \frac{4}{12} \\ \frac{11}{12} - \frac{6}{12} = \frac{5}{12} \end{array}$$

$$\begin{array}{l} \frac{12}{12} - \frac{4}{12} = \frac{8}{12} \\ \frac{12}{12} - \frac{11}{12} = \frac{1}{12} \\ \frac{12}{12} - \frac{3}{12} = \frac{9}{12} \\ \frac{6}{12} - \frac{2}{12} = \frac{4}{12} \end{array}$$

Challenge!

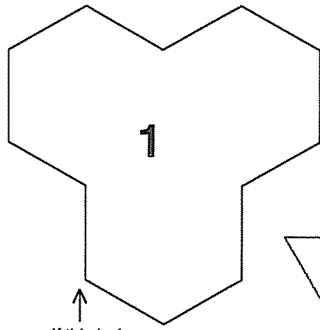
$$\begin{array}{l} 1 - \frac{1}{2} = \frac{1}{2} \quad 2 - \frac{1}{2} = 1\frac{1}{2} \quad 3 - \frac{1}{2} = 2\frac{1}{2} \\ 1 - \frac{1}{3} = \frac{2}{3} \quad 2 - \frac{1}{3} = 1\frac{2}{3} \quad 3 - \frac{1}{3} = 2\frac{2}{3} \\ 1 - \frac{1}{6} = \frac{5}{6} \quad 2 - \frac{1}{6} = 1\frac{5}{6} \quad 3 - \frac{1}{6} = 2\frac{5}{6} \end{array}$$

What patterns did you find? In each set, the fractional part of the answer is the same, while the whole number part increases by one.

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Subtraction of Like Fractions - Worksheet 5

This is 1.



How many of each block does it take to cover the whole? Guess first. Write the fractions.


guess

guess

guess

guess

If this is 1,



then, this is $\frac{1}{18}$.

$\frac{18}{18} - \frac{1}{18} = \frac{17}{18}$

$\frac{18}{18} - \frac{6}{18} = \frac{12}{18}$

$\frac{18}{18} - \frac{8}{18} = \frac{10}{18}$

$\frac{18}{18} - \frac{12}{18} = \frac{6}{18}$

$\frac{18}{18} - \frac{3}{18} = \frac{15}{18}$

$\frac{18}{18} - \frac{18}{18} = 0$

$\frac{16}{18} - \frac{6}{18} = \frac{10}{18}$

$\frac{17}{18} - \frac{11}{18} = \frac{6}{18}$

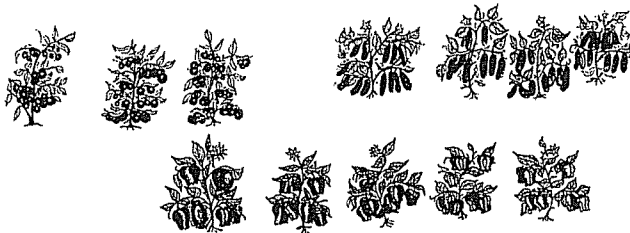
$\frac{12}{18} - \frac{6}{18} = \frac{6}{18}$

$\frac{12}{18} - \frac{0}{18} = \frac{12}{18}$

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Fractions as Ratios - Worksheet 2

Suki planted a garden. There were three tomato plants, four cucumber plants, and five green pepper plants.



Circle the tomato plants with red, the cucumber plants with blue, and the pepper plants with orange.

- How many plants are in the whole garden? $\underline{12}$
- What fraction of the whole are the tomato plants? $\underline{\frac{3}{12}}$
- What fraction of the whole are the pepper plants? $\underline{\frac{5}{12}}$
- What fraction of the whole are the cucumber plants? $\underline{\frac{4}{12}}$
- What fraction of the plants are tomatoes and peppers? $\underline{\frac{8}{12}}$
- What fraction of the plants have green fruits? $\underline{\frac{9}{12}}$
- What fraction of the plants will produce seeds? $\underline{\frac{12}{12}}$
- What fraction of these plants will grow in the snow? $\underline{0}$

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Fractions as Ratios - Worksheet 1



Dad Bridget Sally Grandma Grandpa Tommy Joe Mom

This whole family is a group. The whole family went to the county fair. They all went to see the ferris wheel. Grandma and Grandpa didn't want to go on the ferris wheel.

How many tickets did they buy to get into the fair? $\underline{8}$

Tommy is what fraction of the family? $\underline{\frac{1}{8}}$

What fraction of the family are children? $\underline{\frac{4}{8}}$

What fraction of the family didn't want to go on the ferris wheel? $\underline{\frac{2}{8}}$

What fraction of the family went on the ferris wheel? $\underline{\frac{6}{8}}$

What fraction of the family are adults? $\underline{\frac{4}{8}}$

What fraction of the family are female? $\underline{\frac{4}{8}}$

What fraction of the family are males? $\underline{\frac{4}{8}}$

Make your own. **AWV**

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Fractions as Ratios: Egg Carton Problems - Worksheet 1

Ratios are used to compare the parts and wholes of groups of things. For example, how much is a half of a dozen donuts? $\underline{6}$



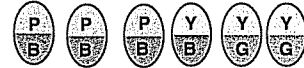
Instructions: Hard-boil a dozen white eggs. Mix up red, green, blue, and yellow food coloring dye in water. Add one teaspoon of white vinegar for each cup of water. Drawn and cut out eggs can also be used, but it is not as much fun.

Dye Patterns



Dye the bottom half of six eggs red.

Dye the bottom half of four eggs blue.



Dye the bottom half of two eggs green.

Dye the top half of nine eggs purple (mix red and blue). Use six eggs with bottoms and three eggs with blue bottoms.

Dye the top half of three eggs yellow. Use one egg with a blue bottom two eggs with green bottoms.

You have twelve colored eggs. Answer these questions.

How many of the eggs have red on them? $\underline{6}$

How many of the eggs have blue on them? $\underline{4}$

How many of the eggs have purple on them? $\underline{9}$

How many of the eggs have green on them? $\underline{2}$

How many of the eggs have yellow on them? $\underline{3}$

How many of the eggs are purple and red? $\underline{6}$

How many of the eggs are purple and blue? $\underline{3}$

How many of the eggs are yellow and blue? $\underline{1}$

How many of the eggs are yellow and green? $\underline{2}$

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Fractions as Ratios: Egg Carton Problems - Worksheet 2

Any of these numbers can be written as ratios.

The ratio of eggs with red on them to the total number of eggs is 6 eggs with red
12 total number of eggs.

Another way of asking the same question is: What fraction of the eggs have red on them? 6 eggs 6/12

What is the ratio of eggs with blue on them to the total number of eggs? 4:12

What is the ratio of eggs with green on them to the total number of eggs? 2:12

What is the ratio of eggs with yellow on them to the total number of eggs? 3:12

What is the ratio of eggs with blue and purple on them to the total number of eggs? 3:12

What is the ratio of eggs with yellow and purple on them to the total number of eggs? 0:12

What is the ratio of eggs with yellow and green on them to the total number of eggs? 2:12

What is the ratio of eggs with yellow and blue on them to the total number of eggs? 1:12

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Fractions as Parts of Wholes - Worksheet 2

At the bottom of Worksheet 1 there is a long row of
You are asked to color in $\frac{1}{4}$ of the hexagons.

What is the first step you had to do before you could divide that row of hexagons into fourths? Count the total number of hexagons.

Once you knew that number, how did you figure out how many hexagons were in $\frac{1}{4}$ of the row? Divide 24 by 4.
This gave me 6.

Color the fractional part.

Color $\frac{1}{6}$ ★★★★★★

Color $\frac{1}{12}$ ★★★★★★

Color $\frac{2}{12}$ ★★★★★★

How many stars are $\frac{1}{4}$ of the row of stars? 3
★★★★

So, how many stars would there be in three of those fourths? 9

★★★★★★★★

How did you get the answer? I divided the 12 into 4 groups of 3, then I took 3 of those groups to make 9 stars in all.

Fractions as Parts of Wholes - Worksheet 3

Build with Cuisenaire Rods.
Cover each with one color train.
Darken the edge of the fraction. Color the fractions.

Darken halves
Color $\frac{1}{2}$ of 12

Darken fourths
Color $\frac{1}{4}$ of 12

Darken sixths
Color $\frac{1}{6}$ of 12

Darken twelfths
Color $\frac{1}{12}$ of 12

Darken thirds
Color $\frac{1}{3}$ of 12

Darken halves
Color $\frac{1}{2}$ of 18

What color rod is $\frac{1}{2}$ of 18? blue

Darken thirds
Color $\frac{1}{3}$ of 18

What color rod is $\frac{1}{3}$ of 18? dark green

Darken sixths
Color $\frac{1}{6}$ of 18

What color rod is $\frac{1}{6}$ of 18? light green

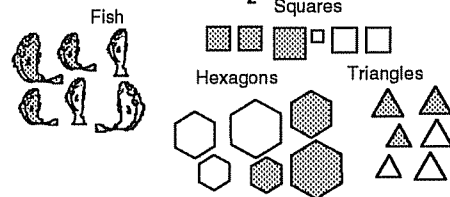
Darken eighteenths
Color $\frac{1}{18}$ of 18

What color rod is $\frac{1}{18}$ of 18? white



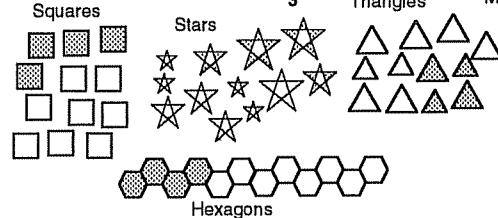
Fractions as Parts of Wholes - Worksheet 1

Color one half of each group. $\frac{1}{2}$



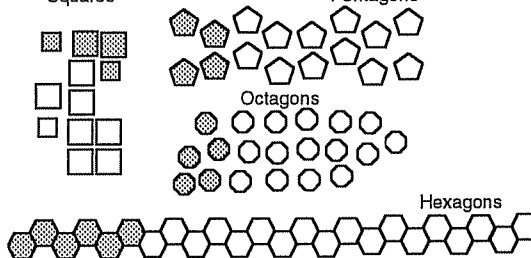
Make your own.

Color one third of each group. $\frac{1}{3}$



Make your own.

Color one fourth of each group. $\frac{1}{4}$



Make your own.

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Fractions as Parts of Wholes - Worksheet 4

Build with Cuisenaire Rods. Cover each with one color train. Cover the fractions the same color as the rod. Draw the lines for the fractional units. Color using the same color as the rod.

Color $\frac{1}{2}$ of 12

Color $\frac{1}{4}$ of 12

Color $\frac{5}{6}$ of 12

Color $\frac{2}{12}$ of 12

Color $\frac{1}{3}$ of 12

Color $\frac{1}{2}$ of 18

Color $\frac{2}{18}$ of 18

Color $\frac{1}{3}$ of 18

Color $\frac{2}{18}$ of 18

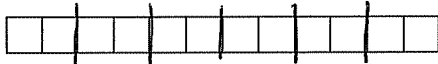
53

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Fractions as Parts of Wholes - Worksheet 5

Build with Cuisenaire Rods. Cover each with one color train. Cover the fractions the same color as the rod. Draw the lines for the fractional units. Color using the same color as the rod.

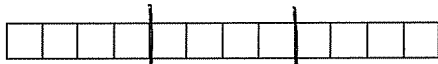
$$\frac{1}{6} \text{ of } 12 = 2$$



Make with red sixths.

$$\frac{5}{6} \text{ of } 12 = 10$$

$$\frac{1}{3} \text{ of } 12 = 4$$



Make with purple thirds.

$$\frac{2}{3} \text{ of } 12 = 8$$



$$\frac{1}{4} \text{ of } 12 = 3$$



Make with light green fourths.

$$\frac{3}{4} \text{ of } 12 = 9$$



$$\frac{1}{2} \text{ of } 12 = 6$$



Make with halves.



Note to the teacher. A source of confusion is that groups of three = fourths groups of four = thirds.

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Fractions as Parts of Wholes - Worksheet 7

Review.

$$\frac{1}{5} \text{ of } 15 = 3$$

$$\frac{2}{5} \text{ of } 15 = 6$$

$$\frac{3}{5} \text{ of } 15 = 9$$

How can you figure out what $\frac{4}{5}$ of 15 is if you know what $\frac{1}{5}$ of fifteen is? Multiply 3×4 , to get the numerator = 12/15

Take out 14 blocks. Arrange the blocks into sevenths. Use the blocks to get the answer to this set of problems.

$$\frac{1}{7} \text{ of } 14 = 2$$

$$\frac{4}{7} \text{ of } 14 = 8$$

$$\frac{2}{7} \text{ of } 14 = 4$$

$$\frac{5}{7} \text{ of } 14 = 10$$

$$\frac{3}{7} \text{ of } 14 = 6$$

$$\frac{6}{7} \text{ of } 14 = 12$$

Take out 20 blocks. Use the blocks to solve these problems.

$$\frac{1}{2} \text{ of } 20 = 10$$

$$\frac{1}{5} \text{ of } 20 = 4$$

$$\frac{1}{4} \text{ of } 20 = 5$$

$$\frac{2}{5} \text{ of } 20 = 8$$

$$\frac{2}{4} \text{ of } 20 = 10$$

$$\frac{3}{5} \text{ of } 20 = 12$$

$$\frac{3}{4} \text{ of } 20 = 15$$

$$\frac{4}{5} \text{ of } 20 = 16$$

Explain in words how you find out what $\frac{4}{5}$ of 20 is.

Step 1 Divide 20 by 5.

Step 2 Multiply that quotient by 4.

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Fractions as Parts of Wholes - Worksheet 6

Take out twelve blocks.

Group the blocks into fourths. Draw what the blocks look like now.



How do you know how many blocks go in each fourth? Divide 12 by 4.

$\frac{3}{4}$ of 12 is 9. How do you get the nine? You take 3 of the groups.

USE BLOCKS to find the fractional parts of fifteen.

$$\frac{1}{3} \text{ of } 15 = 5$$

$$\frac{2}{5} \text{ of } 15 = 6$$

$$\frac{2}{3} \text{ of } 15 = 10$$

$$\frac{3}{5} \text{ of } 15 = 9$$

$$\frac{1}{5} \text{ of } 15 = 3$$

$$\frac{4}{5} \text{ of } 15 = 12$$

USE BLOCKS to find the fractional parts of eight.

$$\frac{1}{4} \text{ of } 8 = 2$$

$$\frac{1}{8} \text{ of } 8 = 1$$

$$\frac{1}{2} \text{ of } 8 = 4$$

$$\frac{3}{8} \text{ of } 8 = 3$$

$$\frac{3}{4} \text{ of } 8 = 6$$

$$\frac{5}{8} \text{ of } 8 = 5$$

USE BLOCKS to find the fractional parts of sixteen.

$$\frac{1}{2} \text{ of } 16 = 8$$

$$\frac{1}{8} \text{ of } 16 = 2$$

$$\frac{1}{4} \text{ of } 16 = 4$$

$$\frac{1}{16} \text{ of } 16 = 1$$

$$\frac{3}{4} \text{ of } 16 = 12$$

$$\frac{3}{16} \text{ of } 16 = 3$$

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Fractions as Parts of Wholes - Worksheet 8

Explain in words how to figure out what $\frac{3}{4}$ of 40 is.

Step 1 Divide 40 by 4 to get 10.

Step 2 Multiply 10 by 3 to get 30.

$\frac{3}{4}$ of 20 is the same as $\frac{3}{4} \times 20$. The word "of" changes to a times sign.

These two steps can be written as a number sentence. The first step has () around it.

$$\text{So } \frac{3}{4} \text{ of } 20 = (\text{First Step } 20 \div 4) \times 3 \text{ (Second) } = 15 \text{ (Answer)}$$

Solve.

$$\frac{1}{4} \text{ of } 20 = (20 \div 4) \times 1 = 5$$

$$\frac{1}{2} \text{ of } 20 = (20 \div 2) \times 1 = 10$$

$$\frac{1}{5} \text{ of } 20 = (20 \div 5) \times 1 = 4$$

$$\frac{2}{5} \text{ of } 20 = (20 \div 5) \times 2 = 8$$

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Parts of Wholes as Multiplication of Fractions - Worksheet 1

Calculate these problems. Draw a picture only if you need to.

In fractions, the "of" is written with a X sign.

$\frac{1}{3}$	of	9	=	$\frac{1}{3}$	X	9
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Write the procedure with a number sentence.

$$\frac{2}{3} \text{ of } 9 = (9 \div 3) \times 2 = 6$$

$$\frac{1}{4} \text{ of } 8 = (8 \div 4) \times 1 = 2$$

$$\frac{3}{4} \text{ of } 8 = (8 \div 4) \times 3 = 6$$

$$\frac{1}{5} \text{ of } 25 = (25 \div 5) \times 1 = 5$$

$$\frac{3}{5} \text{ of } 25 = (25 \div 5) \times 3 = 15$$

$$\frac{5}{6} \text{ of } 24 = (24 \div 6) \times 5 = 20$$

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Parts of Wholes as Multiplication of Fractions - Worksheet 3

1. Susan has eight feet of ribbon. She used $\frac{3}{4}$ of it for a project. $8 - 4 = 2$
How much does she have left? 2 feet $8 \times \frac{3}{4} = \frac{24}{4} = 6$

Write a number sentence for how much she used. \uparrow

2. Wendy weighs 60 pounds on Earth. She would weigh only $\frac{1}{6}$ of that on the moon. Write a number sentence for how much she would weigh on the moon. $60 \times \frac{1}{6} = \frac{60}{6} = 10$ pounds

How much would she weigh on the moon? 10 pounds

3. Tom weighs 180 pounds on Earth. Write a number sentence for how much he would weigh on the moon. $180 \times \frac{1}{6} = \frac{180}{6} = 30$

How much would he weigh on the moon? 30 pounds

4. Annie, a dog, weighs 20 pounds on Earth. She would weigh $\frac{2}{5}$ of that on Venus. Write a number sentence for how much she would weigh on Venus. $20 \times \frac{2}{5} = \frac{40}{5} = 8$

How much would she weigh on Venus? 8 pounds

5. Write your own problem. Include the number sentence and answer

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Parts of Wholes as Multiplication of Fractions - Worksheet 2

Practice. Write the letter of the problem above the number that is equal to the answer

I	S	N	'	T	T	H	I	S	E	A	S	Y	N	O	W	?
9	20	12	-	10	10	16	9	25	30	8	25	20	12	18	6	-

$$T = \frac{2}{3} \times 15 = 10$$

$$N = \frac{3}{5} \times 20 = 12$$

$$O = \frac{3}{4} \times 24 = 18$$

$$S = \frac{5}{6} \times 30 = 25$$

$$I = \frac{3}{7} \times 21 = 9$$

$$H = \frac{2}{3} \times 24 = 16$$

$$W = \frac{2}{9} \times 27 = 6$$

$$A = \frac{4}{10} \times 20 = 8$$

$$E = \frac{3}{4} \times 40 = 30$$

$$Y = \frac{1}{5} \times 100 = 20$$

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Parts of Wholes as Multiplication of Fractions - Worksheet 4

$$\frac{1}{2} \times 2 \text{ can also mean } \frac{1}{2} \text{ added } 2 \text{ times. } \frac{1}{2} + \frac{1}{2} = \frac{2}{2}$$

This is called iterative addition.

Write these problems as iterative addition problems and solve.

$$\begin{aligned} \frac{1}{4} \times 2 &= \frac{2}{4} & \frac{1}{4} + \frac{1}{4} &= \frac{2}{4} \\ \frac{1}{4} \times 4 &= \frac{4}{4} & \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} &= \frac{4}{4} \\ \frac{1}{6} \times 5 &= \frac{5}{6} & \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} &= \frac{5}{6} \\ \frac{2}{3} \times 4 &= \frac{8}{3} & \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} &= \frac{8}{3} \\ \frac{3}{4} \times 6 &= \frac{18}{4} & \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} &= \frac{18}{4} \\ \frac{5}{6} \times 3 &= \frac{15}{6} & \frac{5}{6} + \frac{5}{6} + \frac{5}{6} &= \frac{15}{6} \end{aligned}$$

Post-Assessment is the same as Pre-Assessment: Part 2

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