

## Fractions: Booklet 8 - Understanding Division

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### Acknowledgments

The knowledge, patience, and dedication of Professor Michael Butler made the UCI Farm Elementary School and this mathematics program possible. Special thanks go to Alysia Krafel and Susan Carpenter, who helped develop much of the math materials based on the teachings, ideas, and insights of Professor Butler.

For many years Farm School teachers, students, parents, and staff have shared their unending delight in learning. Thank you for your support and dedication.

The books would never have been completed if the students at Chrysalis Charter School in Redding, California, under the guidance of Alysia and Paul Krafel, hadn't needed them. Thank you for your patience through all of the draft copies.

Susan Carpenter edited, added her wise words, useful suggestions, and helped make the Answer Keys a reality. Karan Founds-Benton contributed her meticulous editing skill and knowledge. Diligent and thorough copy editing was done by Zephyr Alfansh and Jacqueline Logue.

The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

Patterns in Arithmetic: Fractions - Booklet 8  
Parent/Teacher Guide  
©2014 Pattern Press

Published by Pattern Press  
P.O. Box 2737  
Fallbrook, CA 92088  
(760)728-3731

www.patternpress.com  
E-mail: Patternpress1@gmail.com

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ISBN 978-1-935559-47-4

## Introduction to Division of Fractions

A teacher can teach any ten-year-old how to divide fractions by telling her the procedure and then having her practice it for about a hour. The procedure itself, however, makes no sense to most people, including most adults. The sad thing is not only are they missing out on understanding why ‘invert the divisor and multiply’ works, but more important, they are missing the chance to develop mathematical power and the delight that comes with it.

And even more sadly, they are learning that their understanding is not important. Just memorize it and confirm your answer with only the authority of an Answer Key.

To understand division of fractions, one must explore the physical process of it. Using fraction manipulatives, the student can explore what it means to solve a problem such as this:

$$\frac{3}{4} \div \frac{2}{3} =$$

What question does this problem ask? What do I need to understand to process this question? First, to approach understanding, a student must understand the basic nature of division itself. In the operations of addition, subtraction, and multiplication, the answer is a number of objects that can be counted. In division this is not the case. In division, the quotient (answer) is always an expression of the relationship between the dividend and the divisor. It is not a quantity, but a relationship. This makes division uniquely challenging to understand. I will repeat that packed sentence.

In division, the quotient (answer) is always an expression of the relationship between the dividend and the divisor. I failed eighth grade math because I could not understand what that sentence meant. So let’s look at this a minute.

Dividend  $\div$  Divisor = Quotient For example consider these three problems:

$$8 \div 2 = 4$$

$$12 \div 3 = 4$$

$$20 \div 5 = 4$$

In all three cases the quotient is four. But the number four is not a quantity. It is four groups of whatever the divisor is. In  $8 \div 2 = 4$ , the four means four groups of two. In  $12 \div 3 = 4$ , the four means four groups of three, and in  $20 \div 5 = 4$ , four groups of five. The relationship between the dividend and the divisor is called a ratio. In the problems above, it is the ratios that are equal, not the quantities of objects in the groups. All three dividends and divisors have a ratio of 4 to 1. In other words, there are four groups of the divisor in each dividend. This is hard to get your head around with whole numbers; it is even more so when working with fractions.

In looking at  $8 \div 2 = ?$  or  $\frac{3}{4} \div \frac{2}{3} = ?$ , the question is the same. That question is how many groups of the divisor, in our first case two, are in the dividend, eight?

The answer is that there are four groups of two in eight. To answer this question, one must think of the divisor, two, as a whole group. We must think of the two as the whole. Mathematicians can define a whole to be whatever they want it to be. This concept is taught as the Changing Wholes. In the second problem,  $12 \div 3$ , the divisor defines the new whole as a group of three. In  $25 \div 5$  the divisor defines one whole as a group of five. The size of the group being made, as defined by the divisor, will always be the new whole. This is easier to understand if you look at the division problems as improper fractions:  $\frac{8}{2}$ ,  $\frac{12}{3}$ ,  $\frac{20}{5}$ . When one looks at the problems as fractions, it is easy to see that the whole is two, three, and

five respectively. It is the denominator of a fraction that tells the size of the whole. The definition of the whole changes depending on what the denominator is.

Now consider this concept with respect to a division of fractions problem.  $\frac{3}{4} \div \frac{2}{3} =$

Looking at this division problem as a fraction will make most people's heads spin.

$$\frac{\frac{3}{4}}{\frac{2}{3}}$$

For most, this approach is not immediately understandable. In this fraction, the whole is now defined as  $\frac{2}{3}$ . It is quite difficult to get your head around the idea that a fraction,  $\frac{2}{3}$ , can be defined as the whole! Try explaining that to a student!

An easier way to understand it is to ask the same question we ask for any division problem. How many groups of  $\frac{2}{3}$  are in  $\frac{3}{4}$ ?  $\frac{3}{4} \div \frac{2}{3} = \bigcirc \div \bigcirc =$

In order to answer that question, one must think of the  $\frac{2}{3}$  as a whole group. The student therefore must have a good grasp of the fact that a whole can be defined as anything, including a fraction. She must also see that since the  $\frac{3}{4}$  is larger than the  $\frac{2}{3}$ , the  $\frac{2}{3}$  will fit into the  $\frac{3}{4}$  one time with a little left over. Now, even if the procedure of invert the divisor and multiply has been memorized, she is not likely to be able to explain why the answer is more than one, in fact is one and one-eighth! Where in the world did the one-eighth come from?

To build a model of the process of division of fractions and then construct a pattern that will allow someone to calculate an answer and prove it requires patient construction with physical objects and a search for number patterns.

The pursuit of understanding of this concept is fortunately fun for most people. Given the chance to construct understanding of this whole process, most people feel invigorated with lots of 'ahas!' echoing around the classroom.

Teachers, do not be afraid to teach this unit with your students if you do not understand division of fractions. You will both learn it as you go. Trust me on this. That is how I, Alysia Krafel, learned it. These problems feel like puzzles. Have fun!

## Division of Fractions into Whole and Mixed Numbers

**Purpose** The purpose of this lesson is to begin the development of the ability to use a divisor that is a fraction. Using blocks to build models, the student will investigate division with a fractional divisor as a physical process. How many of those blocks can I fit into this space? All the quotients are whole numbers, and the patterns in the answers are obvious. Many students will see immediately that the multiplication tables are evident in the sequence of answers.

**Prerequisites** *Patterns in Arithmetic:* Division - Booklet 2 or mastery of Single Digit Division from another curriculum, for example,  $4,578 \div 8$ . Mastery of division with double digit divisors is not necessary.  
*Key to Decimals:* Booklet 3 is helpful as it demonstrates how the fractional divisors can be converted into different units using the Identity Property (the Mighty One) for ease of calculation. This may be taught concurrently.

**Materials** Division of Fractions into Whole and Mixed Numbers - Worksheets 1 - 6, pages 9 - 14  
Cuisenaire Rods  
Two sets of fraction pieces of any kind for each pair of students

**Warm Up** Division of Fractions into Whole and Mixed Numbers - Worksheet 1, page 9  
Let the student work down this page and complete the first two sets independently if possible. Require her to write out what patterns she sees.

**Math Journal Topics** 1. Edit answer to what patterns she sees.  
2. Explain why the three times table appears in the answers to the problems in Set 2, on Division of Fractions into Whole and Mixed Numbers - Worksheet 1, page 9. Require her to use the proper vocabulary.

Answer: The definition of one-third is that the whole is divided into three equal pieces. Each time another whole is added, the number of thirds increases by a factor of three (times three) each time. Or, each time another whole is added, three more pieces are added. The pattern shows that to find the total number of pieces in a given number of wholes, you multiply the number of wholes by three.

Enrichment Question: Let  $x$  be the number of wholes, and  $y$  be the number of thirds. Write the function for this relationship. Answer:  $y = 3x$

**Lesson Part 1** The lesson begins with Set 3 on Division of Fractions into Whole Numbers - Worksheet 1, Page 9. Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **“How did you get that?”** **“Show me what you mean.”** **“Build a model of that.”** **“Tell me more so I can understand what you are saying.”**

**“When writing a division problem such as  $8 \div 2 = \underline{\quad}$ , what question is being asked?”** “How many groups of two are in eight.”

**“The worksheet asks, ‘How many fourths are in one?’ What division problem is that?”** “One divided by one-fourth.”

Have her study the next two problems and then continue the pattern to complete Set 3. Move on to the next page.

## Note

One dark green rod is defined as the whole, or one. The rectangle on the right that has six long rectangles in it is equal to six wholes. Some students will interpret the large rectangle as a whole as was done in the Changing Wholes exercises. Make sure she understands that the whole, one, is a single dark green rod.

Division of Fractions into Whole and Mixed Numbers - Worksheet 2, page 10

**“If the dark green rod is one, then what color rod is equal to one-third?”** “The red rod.”

**“If the dark green rod is one, then what color rod is equal to two-thirds?”** “One purple rod or two red rods.”

**“Record this on the line on the left side of the worksheet.”**

Critical Concept: She must see the two-thirds in her mind as a single piece or a single group. Many students have difficulty looking at two blocks and thinking of it as one group. They see two groups because they see two pieces. In this exercise, using the purple rod helps students overcome this perceptual problem. She is counting how many purple rods fit into the big rectangle. The attachment of the name of the block, two-thirds, and the size of the rectangle, six wholes, happens after she understands that she is trying to see the divisor in the division problem.

**“How many groups of one-third are in six?”** It is important that she builds this, even though she can easily use her multiplication pattern. “Eighteen.”

**“How many groups of two-thirds are in six?”** *Wait.* Let her work this out. “Nine.”

## Test for

## Understanding/ Math Journal Question

**“The first quotient is eighteen; the second one is nine. Explain why it makes sense that the quotient of the second problem is half of the first.”** “It makes sense because we doubled the size of the divisor, which halves the size of the quotient.”

Continue in this way to finish the worksheet. Do not help unless assistance is needed. Check the answers immediately. End the session.

## Note

Cuisenaire Rods or fraction pieces can be used for this exercise. If you use the Cuisenaire Rods, use the dark green rod as the whole and the purple rod for two-thirds, or tape two red rods together. If fraction pieces are used, it is important that two

**Division of Fractions into Whole and Mixed Numbers - Worksheet 1**

Use the fraction pieces to find or prove the answers. Look for patterns.

1. How many  $\frac{1}{2}$ s in 1? \_\_\_\_\_

How many  $\frac{1}{2}$ s in 2? \_\_\_\_\_

How many  $\frac{1}{2}$ s in 3? \_\_\_\_\_

How many  $\frac{1}{2}$ s in 4? \_\_\_\_\_

How many  $\frac{1}{2}$ s in 5? \_\_\_\_\_

How many  $\frac{1}{2}$ s in 6? \_\_\_\_\_

2. How many  $\frac{1}{3}$ s in 1? \_\_\_\_\_

How many  $\frac{1}{3}$ s in 2? \_\_\_\_\_

How many  $\frac{1}{3}$ s in 3? \_\_\_\_\_

How many  $\frac{1}{3}$ s in 4? \_\_\_\_\_

How many  $\frac{1}{3}$ s in 5? \_\_\_\_\_

How many  $\frac{1}{3}$ s in 6? \_\_\_\_\_

What patterns do you see?

3. How many  $\frac{1}{4}$ s in 1? \_\_\_\_\_

How many  $\frac{1}{4}$ s in 2? \_\_\_\_\_

How many  $\frac{1}{4}$ s in 3? \_\_\_\_\_

How many  $\frac{1}{4}$ s in 4? \_\_\_\_\_

How many  $\frac{1}{4}$ s in 5? \_\_\_\_\_

How many  $\frac{1}{4}$ s in 6? \_\_\_\_\_

Write as a division problem.

$$1 \div \frac{1}{4} =$$

$$2 \div \frac{1}{4} =$$

$$3 \div \frac{1}{4} =$$

4

5

6

## Division of Fractions into Whole and Mixed Numbers - Worksheet 2

1
---

If the dark green rod is equal to one,

then what rod is  $\frac{1}{3}$ ? \_\_\_\_\_

then what rod is  $\frac{2}{3}$ ? \_\_\_\_\_


$$6 \div \frac{1}{3} =$$

$$6 \div \frac{2}{3} =$$

1
---

Try trading.

If the brown rod is equal to one,

then which rods are equal to  $\frac{1}{4}$ ? \_\_\_\_\_ or \_\_\_\_\_.

then which rods are equal to  $\frac{2}{4}$ ? \_\_\_\_\_ or \_\_\_\_\_.

then which rods are equal to  $\frac{3}{4}$ ? \_\_\_\_\_ or \_\_\_\_\_.


$$8 \div \frac{1}{4} =$$

$$8 \div \frac{2}{4} =$$

Challenge

$$8 \div \frac{3}{4} =$$

## Fractions as Divisors: Whole Number Quotients

**Purpose** The purpose of this lesson is to model the physical process of division with a fraction as a divisor and to teach students to make representational drawings of that process.

**Prerequisites** Previous lessons

**Materials** Fractions as Divisors: Whole Number Quotients - Worksheets 1 - 3, pages 15 - 17  
Paper Square Prism Fractions (Do not use fraction circles for this lesson as they are too difficult for students to draw accurately.)  
Colored pencils

**Warm Up** Have the student explore the paper square Prism Fractions for as long as she needs. Give her a chance to figure out how to manipulate and then draw a division with a fractional divisor on her own.

**“Pick up the  $\frac{1}{2}$  and  $\frac{1}{4}$  pieces. How many groups of  $\frac{1}{4}$  are in  $\frac{1}{2}$ ?”** *Wait.* Give her time to figure out how to model this. “There are two groups of  $\frac{1}{4}$  in  $\frac{1}{2}$ .”

**“How did you arrive at that answer?”** Have her explain her idea to you or to her classmates.

**Math Journal** **“Pick up the  $\frac{1}{2}$  and  $\frac{1}{4}$  pieces. How many groups of  $\frac{1}{4}$  are in  $\frac{1}{2}$ ? Draw the pieces. Use the colored pencils to help you make a clear drawing. Explain how you decided on your answer.”**

Answers will vary. You are looking to see if she counts the number of one-fourth pieces that cover or match up with the one-half piece. Listen to hear if she uses the concept of equivalent fractions to justify her answer, such as one-half equals two-fourths. This would be an indicator of developing understanding.

Examine how she drew the problem. If the drawing does not make sense to you, have her redraw it while she explains her thinking.

**Lesson** Fractions as Divisors: Whole Number Quotients - Worksheet 1, page 15  
Read the top box together. Discuss it to be sure she understands how the problem,  $\frac{1}{2} \div \frac{1}{4}$ , was drawn this time, and how it compares with the drawing from the Warm Up. Have her color the page to match the color of the pieces she is using.

Continue reading the worksheet together. Use both the pieces and drawings as you work.

Watch her solve problems 1 - 4 at the bottom of the page. Check the answers for problems 5 - 8, page 15 and correct them. Check these answers. Make sure the answers are correct before proceeding.



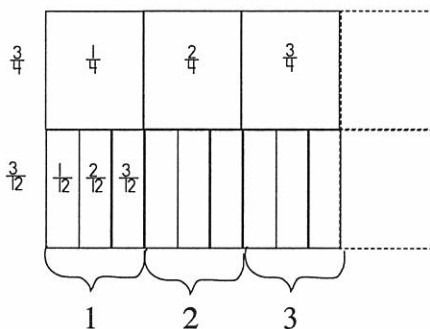
**Practice Worksheets**

Fractions as Divisors: Whole Number Quotients - Worksheet 2, page 16. Correct it.

**Test for Understanding**

Use Fractions as Divisors: Whole Number Quotients - Worksheet 3, page 17, as a test. You are looking for correct answers and clear drawings that show the dividend, the divisor, and the quotient. Require her to use the colored pencils to support her in making the drawings clear.

Example:  $\frac{3}{4} \div \frac{3}{12} = \underline{3}$



## Fractions as Divisors: Whole Number Quotients - Worksheet 1

Use fraction pieces to help you figure out the answers.  
All the answers will be whole numbers.

Example Problem:

The problem  $\frac{1}{2} \div \frac{1}{4} = \underline{\quad}$  asks, "How many groups of  $\frac{1}{4}$  are in  $\frac{1}{2}$ ?"

Put the  $\frac{1}{2}$  piece on the table like this:



Now put the  $\frac{1}{4}$  piece like this:



You can see that it will take two  $\frac{1}{4}$  pieces to fit under the  $\frac{1}{2}$  piece.

So the answer to the question, "How many groups of  $\frac{1}{4}$  are in  $\frac{1}{2}$ ?" is 2.

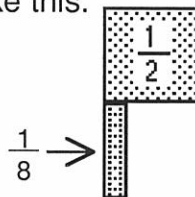
There are 2 groups of  $\frac{1}{4}$  in  $\frac{1}{2}$ .

Now you try one.

The problem  $\frac{1}{2} \div \frac{1}{8} = \underline{\quad}$  asks, "How many groups of  $\frac{1}{8}$  are in  $\frac{1}{2}$ ?"

Take out a  $\frac{1}{2}$  piece and several  $\frac{1}{8}$  pieces.

Put them on the table like this.



How many groups of  $\frac{1}{8}$  fit under the  $\frac{1}{2}$ ?

Therefore,  $\frac{1}{2} \div \frac{1}{8} = \underline{\quad}$ .

Solve these problems. Use fraction pieces to find the answers.  
Record the answers on the line.

1.  $\frac{1}{2} \div \frac{1}{6} = \underline{\quad}$     2.  $\frac{1}{2} \div \frac{1}{12} = \underline{\quad}$     3.  $\frac{1}{2} \div \frac{1}{10} = \underline{\quad}$     4.  $\frac{1}{2} \div \frac{1}{16} = \underline{\quad}$

5.  $\frac{3}{4} \div \frac{1}{8} = \underline{\quad}$     6.  $\frac{2}{3} \div \frac{1}{12} = \underline{\quad}$     7.  $\frac{5}{8} \div \frac{1}{16} = \underline{\quad}$     8.  $\frac{3}{5} \div \frac{1}{10} = \underline{\quad}$

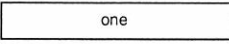
## Division of Fractions into Whole and Mixed Numbers - Worksheet 1

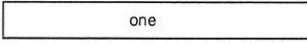
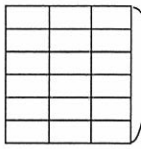
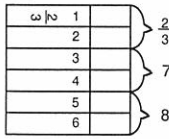
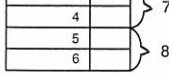
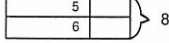
Use the fraction pieces to help you find or prove the answers. Look for patterns.

- How many  $\frac{1}{2}$ s in 1? 2 Two times table.  
 How many  $\frac{1}{2}$ s in 2? 4  
 How many  $\frac{1}{2}$ s in 3? 6 Counting by twos.  
 How many  $\frac{1}{2}$ s in 4? 8  
 How many  $\frac{1}{2}$ s in 5? 10  
 How many  $\frac{1}{2}$ s in 6? 12
- How many  $\frac{1}{3}$ s in 1? 3 What patterns do you see?  
 How many  $\frac{1}{3}$ s in 2? 6 Three times table.  
 How many  $\frac{1}{3}$ s in 3? 9 Counting by threes.  
 How many  $\frac{1}{3}$ s in 4? 12 Multiply the  
 How many  $\frac{1}{3}$ s in 5? 15 denominator by the  
 How many  $\frac{1}{3}$ s in 6? 18 whole number.
- How many  $\frac{1}{4}$ s in 1? 4 Write as a division problem.  
 How many  $\frac{1}{4}$ s in 2? 8  $1 \div \frac{1}{4} = 4$   
 How many  $\frac{1}{4}$ s in 3? 12  $2 \div \frac{1}{4} = 8$   
 How many  $\frac{1}{4}$ s in 4? 16  $3 \div \frac{1}{4} = 12$   
 How many  $\frac{1}{4}$ s in 5? 20  $4 \div \frac{1}{4} = 16$   
 How many  $\frac{1}{4}$ s in 6? 24  $5 \div \frac{1}{4} = 20$   
 $6 \div \frac{1}{4} = 24$

9

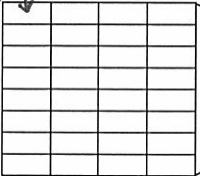
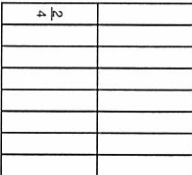
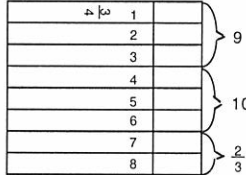
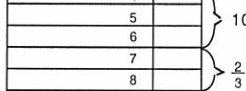
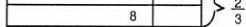
## Division of Fractions into Whole and Mixed Numbers - Worksheet 2

one   $6 \div \frac{1}{3} =$   
 If the dark green rod is equal to one,  
 then what rod is  $\frac{1}{3}$ ? red  
 then what rod is  $\frac{2}{3}$ ? 2 reds or purple

one  Try trading.   $6 \div \frac{2}{3} =$   
  $\frac{2}{3}$   
 7  
 8

If the brown is equal to one,  
 then what rod is  $\frac{1}{4}$ ? red or 2 whites  
 then what rod is  $\frac{2}{4}$ ? 2 reds or 1 purple  
 then what rod is  $\frac{3}{4}$ ? 3 reds or 1 dark green

$8 \div \frac{1}{4} = 32$   
 $8 \div \frac{2}{4} = 16$   
 $8 \div \frac{3}{4} = 10 \frac{2}{3}$

 8  
 10  
 9  
 10  
  $\frac{2}{3}$

## Division of Fractions into Whole and Mixed Numbers - Worksheet 3


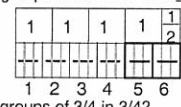

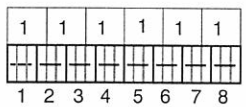

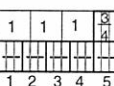

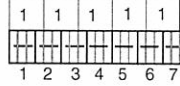
Use any fraction set to help you find the answers. Look for patterns.

- Build 1  $\frac{1}{3}$  with fraction pieces.  
 How many groups of  $\frac{2}{3}$  are in 1  $\frac{1}{3}$ ? 2
- Build 2 with fraction pieces.  
 How many groups of  $\frac{2}{3}$  are in 2? 3
- Build 2  $\frac{1}{3}$  with fraction pieces.  
 How many groups of  $\frac{2}{3}$  are in 2  $\frac{1}{3}$ ?  $3 \frac{1}{2}$
- Build 3  $\frac{1}{3}$  with fraction pieces.  
 How many groups of  $\frac{2}{3}$  are in 3  $\frac{1}{3}$ ? 5
- What number has 6 groups of  $\frac{2}{3}$  in it? 4
- What number has 9 groups of  $\frac{2}{3}$  in it? 6
- Guess!  
 How many groups of  $\frac{2}{3}$  do you think will be in 5  $\frac{1}{3}$ ? AWV  
 Now try it. How many groups of  $\frac{2}{3}$  are in 5  $\frac{1}{3}$ ? 8
- Guess again!  
 How many groups of  $\frac{2}{3}$  do you think will be in 4  $\frac{2}{3}$ ? AWV  
 Now try it. How many groups of  $\frac{2}{3}$  are in 4  $\frac{2}{3}$ ? 7

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## Division of Fractions into Whole and Mixed Numbers - Worksheet 4

Use the  $\frac{1}{4}$  fraction pieces.

- Write as a division problem.  
 Example:  $3 \div \frac{3}{4}$  or  $\frac{3}{4} \overline{)3}$
- How many groups of  $\frac{3}{4}$  in 3? 4  

  - How many groups of  $\frac{3}{4}$  in 4  $\frac{1}{2}$ ? 6  
  $\frac{3}{4} \overline{)4 \frac{1}{2}}$   $4 \frac{1}{2} \div \frac{3}{4}$
  - How many groups of  $\frac{3}{4}$  in  $\frac{3}{4}$ ? 1  
  $\frac{3}{4} \overline{)\frac{3}{4}}$   $\frac{3}{4} \div \frac{3}{4}$
  - How many groups of  $\frac{3}{4}$  in 6? 8  
 $6 \div \frac{3}{4}$   $\frac{3}{4} \overline{)6}$  
  - How many groups of  $\frac{3}{4}$  in 1  $\frac{1}{2}$ ? 2  
 $1 \frac{1}{2} \div \frac{3}{4}$   $\frac{3}{4} \overline{)1 \frac{1}{2}}$  
  - How many groups of  $\frac{3}{4}$  in 3  $\frac{3}{4}$ ? 5  
 $3 \frac{3}{4} \div \frac{3}{4}$   $\frac{3}{4} \overline{)3 \frac{3}{4}}$  
  - How many groups of  $\frac{3}{4}$  in 2  $\frac{1}{4}$ ? 3  
 $2 \frac{1}{4} \div \frac{3}{4}$   $\frac{3}{4} \overline{)2 \frac{1}{4}}$    $\frac{1}{4}$
  - How many groups of  $\frac{3}{4}$  in 5  $\frac{1}{4}$ ? 7  
 $5 \frac{1}{4} \div \frac{3}{4}$   $\frac{3}{4} \overline{)5 \frac{1}{4}}$    $\frac{1}{4}$

## Division of Fractions into Whole and Mixed Numbers - Worksheet 5

On Division of Fractions into Whole and Mixed Numbers - Worksheet 1 you found a pattern that would allow you to calculate the answers to a kind of division problem with fractions. Explain how you can find the answer to this problem without having to use the fraction pieces.

How many groups of  $\frac{1}{6}$  are in 3? or  $3 \div \frac{1}{6} =$  18

Now check your answer with the fraction pieces. Does your pattern work? Yes  
Use your pattern on these problems.

- How many groups of  $\frac{1}{5}$  are in 4?  $4 \div \frac{1}{5} = 20$
- How many groups of  $\frac{1}{6}$  are in 4?  $4 \div \frac{1}{6} = 24$
- How many groups of  $\frac{1}{8}$  are in 3?  $3 \div \frac{1}{8} = 24$
- How many groups of  $\frac{1}{10}$  are in 3?  $3 \div \frac{1}{10} = 30$
- How many groups of  $\frac{1}{12}$  are in 2?  $2 \div \frac{1}{12} = 24$
- How many groups of  $\frac{1}{16}$  are in 2?  $2 \div \frac{1}{16} = 32$
- Why are there more 16ths in 2 than 12ths in 2? 16<sup>th</sup> are smaller than 12<sup>th</sup>, so more would fit into a whole.
- In a division problem, this number is called a divisor.  
 $3 \div \frac{1}{12} = ?$

What will happen to the answer of a division problem if the size of the divisor is doubled and the dividend remains constant? Answer: It will be cut in half.  
Extend the pattern.

$16 \div 1 = 16$     $16 \div 2 = 8$     $16 \div 4 = 4$     $16 \div 8 = 2$

What pattern do you see in the answers? The quotient halves.

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## Fractions as Divisors: Whole Number Quotients - Worksheet 1

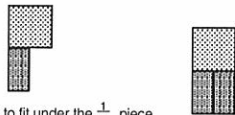
Use fraction pieces to help you figure out the answers.  
All the answers will be whole numbers.

Example Problem:

This problem  $\frac{1}{2} \div \frac{1}{4} =$  asks you, "How many groups of  $\frac{1}{4}$  are in  $\frac{1}{2}$ ?"

Put the  $\frac{1}{2}$  piece on the table like this:

Now put the  $\frac{1}{4}$  piece like this:



You can see that it will take two  $\frac{1}{4}$  pieces to fit under the  $\frac{1}{2}$  piece.

So the answer to the question, "How many groups of  $\frac{1}{4}$  are in  $\frac{1}{2}$ ?" is 2.

There are 2 groups of  $\frac{1}{4}$  in  $\frac{1}{2}$ .

Now you try one.

The problem  $\frac{1}{2} \div \frac{1}{8} =$  asks, "How many groups of  $\frac{1}{8}$  are in  $\frac{1}{2}$ ?"

Take out a  $\frac{1}{2}$  piece and several  $\frac{1}{8}$  pieces.

Put them on the table like this:



How many groups of  $\frac{1}{8}$  fit under the  $\frac{1}{2}$ ?

Therefore,  $\frac{1}{2} \div \frac{1}{8} = 4$ .

Solve these problems. Use fraction pieces to find the answers. Record the answers on the line.

1.  $\frac{1}{2} \div \frac{1}{6} = 3$    2.  $\frac{1}{2} \div \frac{1}{12} = 6$    3.  $\frac{1}{2} \div \frac{1}{10} = 5$    4.  $\frac{1}{2} \div \frac{1}{16} = 8$

5.  $\frac{3}{4} \div \frac{1}{8} = 6$    6.  $\frac{2}{3} \div \frac{1}{12} = 8$    7.  $\frac{5}{8} \div \frac{1}{16} = 10$    8.  $\frac{3}{5} \div \frac{1}{10} = 6$

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## Division of Fractions into Whole and Mixed Numbers - Worksheet 6

Here are two division problems. What will happen to the answer when the divisor is doubled?

$18 \div 3 = 6$   
 $18 \div 6 = 3$

Do you think this pattern will be true for fractions also? Yes  
Try it.

Use fraction pieces to make a group of  $\frac{1}{3}$ .

How many groups of  $\frac{1}{3}$  are in 2? 6

Now make a group of  $\frac{2}{3}$ .

How many groups of  $\frac{2}{3}$  are in 2? 3

What happened to the answer when you doubled the size of the group you were making? It was cut in half. The larger the divisor, the smaller the quotient.

Use your patterns to predict the answers to each set of problems. Then use fraction pieces to check if your pattern worked.

Set 1   Predict   Check

$2 \div \frac{1}{6} = 12$

$2 \div \frac{2}{6} = 6$

$2 \div \frac{4}{6} = 3$

Challenge Set

Fill in the missing fractions.

$2 \div \frac{1}{5} = 10$  and  $2 \div \frac{2}{5} = 5$

Set 2

The size of the divisor is tripled then doubled.  
What will happen to the quotients?

Predict   Check

$3 \div \frac{1}{8} = 24$

$3 \div \frac{3}{8} = 8$

$3 \div \frac{6}{8} = 4$

Was your prediction correct? Yes

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## Fractions as Divisors: Whole Number Quotients - Worksheet 2

Use fraction pieces to solve these problems. Record your answers.

$\frac{1}{4} \div \frac{1}{8} = 2$     $\frac{3}{4} \div \frac{1}{8} = 6$     $\frac{3}{4} \div \frac{1}{12} = 9$     $\frac{1}{4} \div \frac{1}{12} = 3$

5.  $\frac{3}{8} \div \frac{1}{16} = 6$     $\frac{7}{12} \div \frac{1}{12} = 7$     $\frac{1}{4} \div \frac{1}{16} = 4$     $\frac{3}{4} \div \frac{1}{16} = 12$

Sometimes the group you are making will have more than one piece in it.

Look at this problem.  $\frac{3}{4} \div \frac{3}{8} =$  The question is, "How many groups of  $\frac{3}{8}$  are in  $\frac{3}{4}$ ?"

Build  $\frac{3}{4}$  Now build a group of  $\frac{3}{8}$

Put your pieces like this: Now build another group of  $\frac{3}{8}$

There are 2 groups of  $\frac{3}{8}$  in  $\frac{3}{4}$ .

Use your fraction pieces to solve these problems. Record your answers.

9.  $\frac{3}{8} \div \frac{3}{8} = 1$    10.  $\frac{3}{8} \div \frac{3}{16} = 2$    11.  $\frac{3}{8} \div \frac{1}{8} = 3$    12.  $\frac{5}{6} \div \frac{2}{12} = 5$

13.  $\frac{3}{4} \div \frac{3}{16} = 4$    14.  $\frac{7}{8} \div \frac{2}{16} = 7$    15.  $\frac{3}{4} \div \frac{3}{8} = 2$    16.  $\frac{5}{6} \div \frac{1}{12} = 10$

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