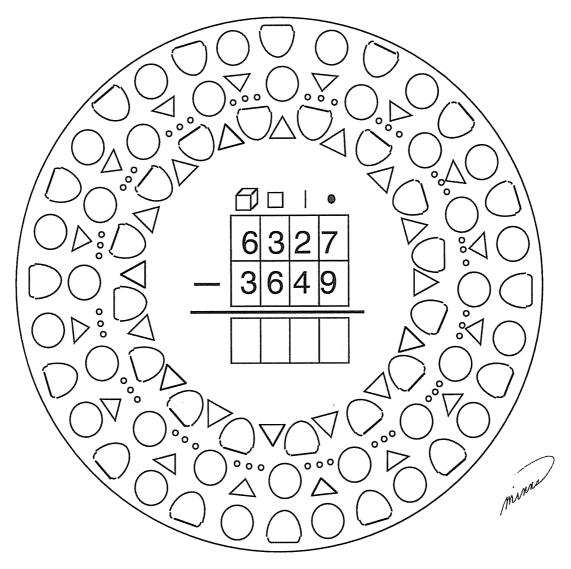
Patterns in Arithmetic Subtraction - Booklet 4 PDF

Large Numbers and Practice

Parent/Teacher Guide



By Alysia Krafel, Suki Glenn, and Susan Carpenter

Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the UCI Farm Elementary School University of California, Irvine

Subtraction: Booklet 4 PDF - Large Numbers and Practice

Contents

Pre-Assessment	1
Assessment Guide	. 4
Introduction to Nonstandard Regrouping and IOUs	11
Standard Regrouping: Review	13
Nonstandard Regrouping and IOUs: Review	
Standard Three Digits	18
Standard Three Digits with Recording	
Gus the Bus Driver	
Standard Three Digits: Assessment	31
Word Problems	
Regrouping Practice	41
Base Ten Counters Directions	43
Answer Key	
\boldsymbol{J}	

This booklet is dedicated to Diana Schmidt and Suzie Engles of the Village School in Fallbrook, California. Your constructivist teaching style broadened my teaching in so many great ways.

Suki Glenn

Acknowledgments

The knowledge, patience, and dedication of Professor Michael Butler made the UCI Farm Elementary School and this mathematics program possible. Special thanks go to Alysia Krafel and Susan Carpenter, who helped develop much of the math materials based on the teachings, ideas, and insights of Professor Butler.

For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

The books would never have been completed if the students at Chrysalis Charter School in Redding, California, under the guidance of Alysia and Paul Krafel, hadn't needed them. Thank you for your patience through all of the draft copies.

Susan Carpenter edited, added her wise words, useful suggestions, and helped make the answer keys a reality. Karán Founds-Benton contributed her meticulous editing skill and knowledge.

The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

Patterns in Arithmetic: Subtraction - Booklet 4 PDF

Parent/Teacher Guide All rights reserved.

©2014 Pattern Press Practice worksheets may be duplicated.

Published by Pattern Press

P.O. Box 2737 Fallbrook, CA 92088 (760)728-3731 Printed on recycled paper.



www.patternpress.com

E-mail: Patternpress1@gmail.com

ISBN 978-1-935559-98-6

Put a question mark next to any problem you do not know how to do.

- 1. a. There are 68 rabbits at the petting zoo. 46 have already gotten carrots for breakfast. How many rabbits have not gotten their carrots yet? _____ Write the number sentence.
 - b. Chuck laid out 653 feet of track for his train. Dave's track is 137 feet shorter than Chuck's. How long is Dave's train track? _____
 Write the number sentence. _____
 - 2. Solve each problem a different way.

b. $\boxed{53} = \boxed{30} + \boxed{20} + \boxed{3}$ $\boxed{-36} = \boxed{-20} + \boxed{-10} + \boxed{-6}$ $\boxed{=} + \boxed{+}$

b. 68 = 50 + 10 + 8 -29 = -10 + -10 + 9 = + + +

3. Solve all three problems both the expanded way and the short way.

Short way Expanded way

C.	56]= ()+()
language de la constanta de la	38]= (+(
		= +

Pre-Assessment - Part 2

Name

Date

Draw the blocks.

1. Draw the Start With number for each problem. Cross out the blocks being traded. On your drawing, Circle the Take Away number. Record the What's Left? number.

Example: 245 Start With Take Away — 168 77 What's Left?

Draw the blocks.

a. Start With 262 Take Away ____ 95 What's Left?

b. Start With 352 Take Away **-** 186 What's Left?

2. Show both the expanded way and the short way of doing each problem.

a. 234 = -128

b. 256 = -96

- C. 206 = - 99
- 3. Build and subtract. Draw pictures and trade if necessary.

a. 3 2 7 6 4 9

2 0

C.

- 4. a. Ramon had 547 CDs in his collection. He sold 429 on Amazon.com. How many CDs does he have left? _____ Write the number sentence.
 - b. In the fall Neva planted 74 camellia bushes. There are now 108 camellia bushes. How many bushes were there before? Write the number sentence.
 - c. Samuel laid out 550 feet of track for his slot cars. Ava's track is 75 feet shorter than Samuel's. How long is Ava's track? Write the number sentence.
- 5. What phrases in these word problems tell you they are subtraction problems?

Post-Assessment

Name

Date

Draw the Start With number for each problem.
 Cross out the blocks being traded.
 On your drawing, Circle the Take Away number.
 Record the What's Left? number.

Example:
Start With 245
Take Away — 168
What's Left? 77

Draw the blocks.

Draw the blocks.

a. Start With 262
Take Away _ 95
What's Left?

b. Start With 352
Take Away — 186
What's Left?

2. Show both the expanded way and the short way of doing each problem.

a. + + + = 234 - + + + = -128 + + + = -128

b. + + = 256 - + + + = = 96 + + =

c. + + = 206 - + + = - 99 + + =

3. Build and subtract. Draw pictures and trade if necessary.

a. 6 3 2 7 - 3 6 4 9 b. 7 0 2 4 -1 7 6 7

c.
- 6 0 0 0 0 0 0 1 7 6 7 4

- 4. a. Ramon had 547 CDs in his collection. He sold 429 on Amazon.com. How many CDs does he have left? _____ Write the number sentence. _____
 - b. In the fall Neva planted 74 camellia bushes. There are now 108 camellia bushes. How many bushes were there before? ______ Write the number sentence.
- 5. What phrases in these word problems tell you they are subtraction problems?

Assessment Guide

Purpose

The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. This assessment assumes one year of work in regrouping with two digit problems with understanding. Most students will be just beginning the fourth grade year and will need more work to master regrouping with understanding and be able to operate with two and three digit regrouping and zeros.

Pre-Assessment: Part 1 is review material from the last booklet and is used to determine student readiness for this booklet. Pre-Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

The Post-Assessment is administered to determine if the student learned the material that was presented in this booklet. A comparison of the score on Pre-Assessment: Part 2 to the score on the Post-Assessment will give both you and your student a sense of growth. The Post-Assessment is the same as Pre-Assessment: Part 2.

Prerequisites

Patterns in Arithmetic: Book 2 - Subtraction section and Subtraction: Booklet 3

Materials

Subtraction: Booklet 4 - Pre-Assessment and Post-Assessment, pages 1 - 3 and Pre-Assessment: Parts 1 and 2 Score Sheets, pages 8 and 9 in this booklet Base Ten Blocks

Instructions

Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

Do the assessment in two parts. Give Pre-Assessment: Part 1 and check it for readiness for this booklet. If he is not ready for this booklet, there is no point in giving Pre-Assessment: Part 2. If he passes all the readiness items, then give Pre-Assessment: Part 2.

After scoring Pre-Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion labeled with capital letters 'A,' 'B,' etc., identifies what to look for in the student work and what concept the item is assessing. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc. Score sheets that match the Assessment Guide for both Assessment: Part 1 and Part 2 follow.

Assessment Criteria for Pre-Assessment: Part 1

Can the student:

- 1. Word Problems (Patterns in Arithmetic: Book 2 Subtraction section, Subtraction: Booklet 3)
 - A. solve simple subtraction word problem (a) with no regrouping required?
 - B. solve a subtraction word problem that requires a single regrouping (b)?
 - C. write correctly the number sentence for one out of two of the problems?
- 2. Nonstandard Regrouping and IOUs (Subtraction: Booklet 3)

It is unlikely that any student who has not worked in Patterns in Arithmetic and completed Subtraction: Booklet 3 will know what to do with the circles shown in these problems. He may be able to do the short, standard notation in the box. In nonstandard subtraction (which is the general case of how subtraction is performed), the numbers can be broken up into any set of numbers as long as the sum is equal to the number on the left in the box. He can break up the number any way he wishes in Set 3 and use any number he wishes to regroup in Sets 1 and 2. It does not have to be a ten, although most students use those at this point. See the Answer Key for all the variations that are possible.

- A. write the correct answers in the boxes in two of the three sets using any method?
- B. write the same answer in boxes a as in box b in all the sets?
- C. use a number other than ten to regroup in any of the problems?
- D. regroup any number from the 50 in Set 2 to avoid creating a regrouping situation with the ten in the center circle? (See the Answer Key for a sample solution.)
 - E. use a negative number for subtraction to get at least one of the correct solutions?
 - F. break up the numbers in Set 3 two different ways?
- 3. Standard Regrouping (Subtraction: Booklet 3)

Again, not all students will know what to do with the numbers in the circles.

- A. solve correctly two of the three problems using the short, standard notation? The short notation is the standard way we all do subtraction.
- B. use correctly the base ten expanded notation in the circles in two of three problems? We are looking to see if he can use the expanded form and show the correct place value of the numbers being regrouped. In the first problem, he should cross off the 40 and write 30 above it. He should then write 10 over the 7 or put a 1 in front of the 7 to receive a Yes on this criterion.

Booklet Selection Guide based on results of Pre-Assessment: Part 1

- If the student receives a No on criterion 1A or 1C, do the Subtraction section in Patterns in Arithmetic: Book 2. Do not give Pre-Assessment: Part 2.
- If the student receives a Yes on criteria 1A and 1C but a No on criteria 1B, 2A, and 3A, begin with Subtraction: Booklet 3 or Base Ten Subtraction. Do not give Pre-Assessment: Part 2.

Subtraction: Booklet 3 teaches the general principles of expanding numbers and of using multiple methods to find solutions to subtraction problems. This booklet is a good choice for average and above-average students.

Or consider Chrysalis Charter School: Base Ten Subtraction, which uses only the base ten, standard way to break up numbers and teaches only a single method to solve subtraction problems. This booklet is intended for students who have learned standard subtraction but do not understand the place value or need review of the procedure. It is also a good choice for students who have difficulty with

Assessment Guide Patterns in Arithmetic: Subtraction - Booklet 4 5 Parent/Teacher Guide

math in general.

- sections on general subtraction (breaking up in different ways and using negative numbers). Note: Some students get confused with negative numbers and cannot get the place value implications of using this strategy. Do not allow them to use this strategy if this happens. Not everyone is ready for negative numbers at this age.
- If the student did not do *Patterns in Arithmetic*: Subtraction: Booklet 3 and receives a Yes on criteria 1A, 1B, 1C, 2A, 3A, and 3B but No on the remainder criterion of problem 2, proceed with Assessment: Part 2 and begin Subtraction: Booklet 4. If you have time and a bright student, do Subtraction: Booklet 3 to create a firm understanding of the general process of subtraction and the ability to use multiple methods to find solutions. This choice may not be right for you; it is optional.
- If the student did do *Patterns in Arithmetic*: Subtraction Booklet 3 and receives a Yes on criteria 1A, 1B, 1C, 2A, 3A, and 3B and a Yes on several of the criteria in problem 2, proceed with Assessment: Part 2, begin Subtraction: Booklet 4, and pat yourself and your student on the back for a job well done!

Assessment Criteria for Pre-Assessment: Part 2

Can the student:

- 1. Standard Regrouping with a drawing or blocks (Subtraction: Booklet 4)
- If you are working with a single student and you wish to use Base Ten Blocks instead of a drawing to check understanding of regrouping, you may do this. You are checking to see if the student understands the place value of the numbers he is regrouping rather than simply using a memorized short notation without understanding. Direct his attention to the example problem, but do not explain how to do it. Note: Most students will naturally start in the hundreds place and work backwards. Allow them to do this.
- A. demonstrate understanding with a drawing or with blocks how regrouping from the hundreds place works in both problems? In other words, does he cross off a one hundred square and show the trade of the one hundred for ten tens by drawing ten lines in the tens row?
- B. demonstrate understanding with a drawing or blocks how regrouping from the tens place works in both problems? Crosses off a ten and draws in ten ones in the ones row?
 - C. get the correct answer on both problems?
- 2. Standard Subtraction: Three Digits with Recording (Subtraction: Booklet 4) See the Answer Key for the correct formats for each problem.
- A. write the correct expanded numbers in the circles for all numbers above the line in all three problems?
- B. show the correct place value of the numbers being regrouped in problem a, which involves a single regroup from the tens place?
- C. show the correct place value of the numbers being regrouped in problem b, which involves a double regroup from both the tens and hundreds places?
- D. show the correct place value of the numbers being regrouped in problem c, which involves a double regroup from the tens place and the hundreds place over a zero?
 - E. correctly use the short form notation on two of the three problems?
 - F. write the correct answer on two of the three problems?

Patterns in Arithmetic: Subtraction - Booklet 4
Parent/Teacher Guide

- 3. Regrouping Practice (Subtraction: Booklet 4)
 - A. obtain the correct answer on problem a using the short notation?
 - B. regroup correctly over one zero in problem b?
 - C. regroup correctly over four zeros in problem c?
- 4. Word Problems (Subtraction: Booklet 4)
 - A. write the correct number sentence in two of the three problems?
 - B. obtain the correct answer for two of the three problems?
 - C. identify two of these three phrases: 'What's Left?'; 'how much more?'; 'shorter than'?

Booklet Selection Guide based on results of Pre-Assessment: Part 2

If the student scores 12 Yes marks on Pre-Assessment: Part 2, he does not need to work in this booklet except for practice if needed. Remediate any areas where he scored a No and move on. If he scores between 9 and 11 Yes marks, do only the sections that apply to his No marks. Any score of 8 or below indicates this booklet is appropriate.

Pre-Assessment: Part 1 Score Sheet

Mama	Data
Name	Date

Can the student:

- 1. Word Problems (Patterns in Arithmetic: Book 2, Subtraction: Booklet 3)
 - Yes No A. solve simple subtraction word problem (a) with no regrouping?
 - Yes No B. solve a subtraction word problem that requires a single regrouping?
 - Yes No C. write correctly one of the two number sentences?
- 2. Nonstandard Regrouping and IOUs (Subtraction: Booklet 3)
 - Yes No A. write the correct answers in two of the three problems?
 - Yes No B. write the same answer in box a as in box b in all the sets?
 - Yes No C. use a number other than ten to regroup in any of the problems?
 - Yes No D. regroup any number from the 50 in Set 2?
 - Yes No E. know how to use a negative number for subtraction?
 - Yes No F. break up the numbers in Set 3 two different ways?
- 3. Standard Regrouping (Subtraction: Booklet 3)
 - Yes No A. solve correctly two of the three problems using the short notation?
 - Yes No B. use correctly the expanded notation in two of three problems?

A % score is not used in this assessment. Place the student according to the criteria of Yes and No discussed in the Booklet Selection Guide.

Dr	·o_ A c	2692	sment: Part 2 Score Sheet	Name	Date
	n the			1 (41114)	
			Regrouping - with a drawing or block	cs (Subtraction: Book	let 4)
1.	Yes	No	A. demonstrate understanding of he off a ten and draws in ten ones in	grouping from the hund ow regrouping from the	Ireds place in both problems? e tens place works? Crosses
	Yes	No	C. get the correct answer on both pr		STOOTOINS.
			Subtraction: Three Digits with Recor	rding (Subtraction: Bo	oklet 4) See the Answer Key
fo			ct formats for each problem.		** * * * * * * * * * * * * * * * * * * *
			A. write the correct expanded numbin all three problems?		
	Yes	No	B. show the correct place value of t involves a single regroup from t		ouped in problem a, which
	Yes	No	C. show the correct place value of t involves a double regroup from	he numbers being regr	ouped in problem b, which lreds places?
	Yes	No	D. show the correct place value of t involves a double regroup from	he numbers being regr	ouped in problem c, which
	Yes	No			
	Yes	No	F. write the correct answer on two	of the three problems?	
3.	Regi	roupi	ing Practice (working with large num	bers and zeros)	
			A. obtain the correct answer on pro		notation?
			B. regroup correctly over one zero		
	Yes	No	C. regroup correctly over four zero	s in problem c?	
4.			oblems (Subtraction: Booklet 4)		
			A. write the correct number sentence		
			B. obtain the correct answer for tw C. identify two of these three phrase		

Post-As	ssessm	ent Score Sheet	Name	Date
Can the				
1. Stand	ard Reg	grouping - with a drawing or b	olocks (Subtraction: Bo	ooklet 4)
Yes	No A.	demonstrate understanding of	of regrouping from the h	undreds place in both problems?
Yes	No B.	demonstrate understanding of a ten and draws in ten on		the tens place works? Crosses th problems?
Yes	No C.	get the correct answer on bo	th problems?	
			decording (Subtraction:	Booklet 4) See the Answer Key
		ormats for each problem.		
Yes	No A.	write the correct expanded n in all three problems?	numbers into the circles	for all numbers above the line
Yes	No B.	show the correct place value involves a single regroup from		egrouped in problem a, which
Yes	No C		of the numbers being re	egrouped in problem b, which undreds places?
Yes	No D.	show the correct place value	of the numbers being r	egrouped in problem c, which he hundreds place over a zero?
Yes	No E	use the standard short form		
		write the correct answer on		-
3. Regro	ouping l	Practice (working with large i	numbers and zeros)	
		obtain the correct answer on		ort notation?
Yes	No B.	regroup correctly over one z	ero in problem b?	
Yes	No C.	regroup correctly over four a	zeros in problem c?	
4. Word	Proble	ms (Subtraction: Booklet 4)		

Yes No A. write the correct number sentence in two of the three problems?

Yes No C. identify two of these three phrases, 'What's left?'; 'How much more?'; 'Shorter than'?

Yes No B. obtain the correct answer for two of the three problems?

Number	of 'Yes'	scores		=	 <i>7</i> c
Number	Possible		15		

Introduction to Nonstandard Regrouping and IOUs

The purpose is to learn how to use negative numbers in a standard subtraction **Purpose**

problem to offer students yet another way to quickly solve problems.

Prerequisites Fluency in circle subtraction and nonstandard regrouping

Materials Black and red board markers

Red colored pencils

Paper money if a manipulative is needed

Number line

Tell this familiar story: "You are in the store with your mom or dad. You have Lesson

> three dollars in your pocket. You see a very cool notebook that you would really like to have. The notebook costs five dollars. What would you say to

your parent?" "Mom, can I borrow two dollars to buy this notebook?"

Then, have a discussion about paying back loans.

Continue: "Your mom says yes and loans you the two dollars. "How much money do you have now?" "Five dollars."

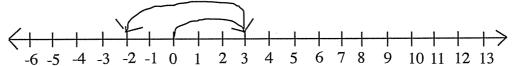
"After buying the notebook, how much money do you have?" Most students will say, "Zero." Correct them and say, "No, you have less than zero. You are in debt and have an IOU of two dollars. You are in the hole by two dollars. You have minus two dollars. You are in the red."

Language Arts Connection: These idioms for debt-'IOUs,' 'in the hole,' and 'in the red'-all mean we owe money. Do not skip this at this point. Discuss all these idioms. Students have heard these and often know what they mean. Also put the word "debt" on their spelling lists.

History: In the old days before computers, people used to keep track of money on long sheets of paper called ledgers. The ledgers were written in black. If there was a debt or if you spent more money than you had, you would write the debt in red ink so you would be sure to see it. Many computer programs still print these debts in red ink so you will see them. So 'in the red' now means that you have spent more money than you have and you are in debt and must pay it back.

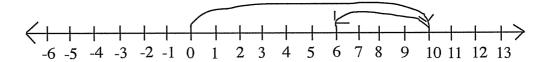
Continue: "So you owe your mom how much?" "Two dollars." Write this on the board. Have \$3 Spend \$5 = owe \$2. Write the 2 in red. Write this: \$3 - \$5 = -\$2Again, write the 2 in red.

Use a number line to show negative numbers. Draw the first line going from 0 to the 3 in black. Draw the second line going from the 3 to the -2 in red. Discuss that a debt is shown as a negative number. "Is a negative number greater than or less than zero?" "Less than." "Is being in the red a good thing?" "No."

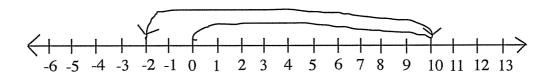


Do these problems on the board. You can do them orally or have students copy them onto a piece of paper. All negative numbers should be written in red. Have the students use a number line to prove the solutions.

A person has \$10 and spends \$4, which leaves \$6.



A person has \$10 and spends \$12, which leaves <u>\$-2</u>.



Standard Regrouping: Review

Purpose

The purpose is to review standard subtraction using both expanded notation and standard notation. It assumes students have been on summer vacation or had a break between Level 3 and Level 4 math. This lesson is not only a warm-up review, but also an assessment.

Prerequisites

Patterns in Arithmetic: Subtraction - Booklet 3 or previous instruction in subtraction with regrouping

Materials

Standard Regrouping: Review - Worksheets 1 and 2, pages 1 and 2

Base Ten Blocks or bean sticks

Money: a dollar bill, ten dimes, and ten pennies

Lesson

Take out a one dollar bill and hand it to your student. Lay some dimes and pennies on the table.

"Please give me twenty-nine cents out of this bill."

Wait: See if your student can work through the making change process. He should know that he must trade the bill for ten dimes or nine dimes and ten pennies. He then can give you the twenty-nine cents. If your student cannot do this problem, consider using Patterns in Arithmetic: Subtraction Booklet 3.

Worksheet

Take out Standard Regrouping - Worksheet 1 and the Base Ten Blocks. Review what each block stands for. The small cube equals one, the rod equals ten, the flat equals one hundred, and the large cube equals one thousand.

Remember that when you are doing a subtraction problem with blocks, you build only the top number, or the number we call the 'Start With' number. The number being subtracted is removed from the Start With number. What is left is the answer. Be sure the students understand this. This is a convention for building subtraction problems.

Note

Standard Regrouping: Review - Worksheet 1 has an example problem recorded on the top right hand side of the page. The numbers in the square show the standard or short notation, 76. Here the place value is shown only by which column the number is in. Seven in the second column means seventy. This notation is much more abstract, and students need to be reminded what it all means.

The numbers in the circles show the expanded notation. Notice that the regrouping is different in the standard notation from that of the expanded notation.

Problem 1

"Build seventy-four and take away fifty-three."

Watch: They should build seven tens and four ones, remove or take away five tens and three ones. They will have two tens and 1 one left over.

Patterns in Arithmetic: Subtraction - Booklet 4

Standard Regrouping: Review 13 Parent/Teacher Guide

So 74-53=21. This will remind the student how the blocks work.

Next: Look at the recording space for Problem 1.

In the circles on the top row have the student write 70 in the left circle and 4 in the right circle. There will be a + in between. 74 = 70 + 4

"What is the difference between the way the number is written in the square and the way it is written in the circles?" "In the square, the column the number is written in shows the place value of the number. The 7 in the second column means it's in the tens place, so it means tens, or seventy. In the circles the 7 tens are written as seventy with no ones next to it."

Explain to the student that the way the number in the square is written is called standard notation. 'Notation' is a math word for 'writing.' It is called standard because that is the way everyone on the planet writes seventy-four. The numbers in the circles with the tens separated from the ones with a + sign in between is called expanded notation. The number of zeros after the number tells the place value of the number.

"Write the expanded notation for 53 in the circles. You will see a little – sign in the circles; that is because this is the take away number."

-53 = -50 + -3 The – sign here means it is the number being taken away.

"Now do the subtraction. Record the answer in both standard and expanded notation."

Watch: He should write 21 = 20 + 1 for the answer.

Next do Problem 2. These problems are very easy. Make sure the student understands the notation.

Now do the example problem with blocks only.

"Can you do 76 – 48 with the blocks?"

Watch: The student will build the seventy-six with seven tens and six ones. Now to take away the four tens and eight ones, he must do some trading. Most students start with the ones. We have six ones and we need to take away eight. Most students will pick up a ten and then trade it for ones. They then remove the eight ones from the sixteen ones they now have from the trade. They then take away the four tens from the six remaining tens. This leaves behind two tens and eight ones, or twenty-eight.

So 76 - 48 = 28.

Note

If your student begins with the tens, consider allowing him to do this. He then regroups from the answer. He removes four tens to get three tens left and then regroups one of the tens to take out the eight ones. This is one good solution, which works well for those who are good in mental math. A student who does this may be using the IOU strategy. There is nothing wrong with this; it just is messy when writing it down. You decide if you want to prohibit this or not.

"How is this problem different from the first two problems?" "This problem asks you to take away more ones than you start with. This is a 'borrowing,' or 'trading,' or 'regrouping' problem."

"Do Problem 3 with blocks only, 53 - 35 =___."

If you encounter any uncertain work, do problems 4 and 5 from Standard Regrouping: Review - Worksheet 1 using only blocks.

If after two or three more blocks-only problems your student is not fluent and comfortable, go back to Subtraction: Booklet 3. This is not the correct book for teaching regrouping.

Review of Recording

Redo 76 - 48 with the blocks. Study the example problem.

"Why is the seventy in the circle crossed off with a sixty shown above?" "Because you took one of the tens from the tens place to trade for ones, so now you only have six tens, or sixty."

"What does that little six above the seven mean in the square?" "It means that there are now only six tens left after you traded the ones."

"Why doesn't that little six in the square have a zero after it?" "Because in standard notation, or the short way, you can tell the six is a sixty because it is in the second column."

"What does that little one next to the six in the circle stand for?" "It stands for the ten ones you got when you traded the ten for ones."

Do problems 4 and 5. Then have the student make up his or her own problems. This step is important because it allows the student to demonstrate he understands the attributes of the problems on the page and can duplicate them with new numbers. Tell him that at least one of the made-up problems must be a regrouping problem. Students tend to resist doing the Make Your Own problems, but insist on it. It gives students the power to create and test understanding for themselves.

Worksheet

Assign Standard Regrouping: Review - Worksheet 2. Have the student check his work with a calculator.

Test for Understanding

Use the Make Your Own problems as an assessment. Watch as the student does the work. Can he write a regrouping problem? Does he use blocks or not? Using blocks is fine, but not using them shows more confidence. Is he fluent and easy with both notations? Are the answers correct?

The answers to all of these questions should be 'yes' before going on. If there is still difficulty, give more problems of this type. If there is considerable difficulty, consider Subtraction: Booklet 3.

Nonstandard Regrouping and IOUs: Review

Purpose

This lesson is optional. It is designed for students who have completed *Patterns* in *Arithmetic*: Subtraction Booklet 3.

This is an assessment and a review of the general concept of subtraction. There are many, many ways to subtract two numbers. The standard way we all learned in school is simply the preferred way for doing written problems with large numbers.

Many other ways actually work better with small numbers.

The general principle of subtraction, indeed all arithmetic procedures, is to break up the large numbers into smaller ones, do the small subtractions and add up the differences (the answers). It is not true that you cannot take a large number from a smaller one; you just get a negative number as a result.

Negative numbers can be problematic when you have large numbers. They do not mix well with standard notation unless you are very, very careful. This lesson is a review of those principles.

Note

Some teacher study is needed before this lesson is taught to a student.

Prerequisites

Patterns in Arithmetic: Subtraction - Book 2 or a student who is strong in mathematics and has mastered the material in the previous lesson of Nonstandard Regrouping and IOUs.

Materials

Nonstandard Regrouping and IOUs: Review - Worksheets 1 - 3, pages 3 - 5 Pencil with an eraser Any kind of counter *other* than Base Ten Blocks; Unifix cubes work well.

Background for the Teacher

Study the example problems at the top of Nonstandard Regrouping and IOUs: Review - Worksheet 1, page 3.

In both problems A and B, the numbers are broken up in the same way. Note that both problems use expanded notation but not the standard expanded notation. The standard expanded notation for forty-seven would be 40 + 7. Forty-seven can be expanded or 'broken up' in our common math language many ways.

In both situations, we have a number on the top that is smaller than the take-away number below it. In this case, we start with seven and take away eight.

In the standard method, regroup by taking a ten and add it to the seven to make seventeen. However, it is not required that a ten is taken. Any number will do. In example A, you see a common solution. The seven is only one less than eight, so one was taken off of the thirty to make it twenty-nine and added the one to the seven to make it eight. Many students will do this to 'zero-out' the columns: 8-8 is 0. Then taking ten from twenty-nine to get nineteen is easy. If your student is new to this book, it may surprise him that you can do this and still get the correct answer.

Patterns in Arithmetic: Subtraction - Booklet 4
Parent/Teacher Guide

This is not the only way. Any number can be taken from the thirty. Some students would take a three to make the seven into a ten. Others would use the more standard approach and move a ten. Moving units of five is also common.

What is remarkable is that most people do not know that this nonstandard regrouping is allowed. They think that the standard way is *the* way, in other words the *only* way. I would never have known this either if not for Professor Michael Butler.

In example B, we are required to use an IOU. This is simply a negative number. Students have experienced wanting something at the store and not having enough money and wanting to borrow it from the Bank of Parents. They say, "I'll pay you back when we get home!" So we call this have-seven-spend-eight an IOU. I Owe You a dollar. You are 'in the hole a buck.' You have less than no money; you are in debt. It is not uncommon for people to think that if you are in debt, you have no money. They have a hard time understanding that you have less than no money.

So eight can be taken from seven. The answer is simply -1. You then take that minus one out of the twenty at the end. You have to pay back your IOU at the end. Many students like this method for easy two digit problems because it is easy to do in your head. However, on larger problems with negative numbers in the tens place, students make frequent place value errors. For strong math students, this little bit of struggle is good for them.

There are literally hundreds of ways to subtract. Some ways are easier than others, and the method you choose needs to fit the problem at hand.

Worksheets

Finish Nonstandard Regrouping and IOUs: Review - Worksheets 1, 2, and 3, pages 3, 4, and 5.

Test for Understanding

Give this problem: 345 - 186 and ask the student to solve it with an IOU.

What is tricky here is that if you write the problem in standard short notation like I did above, you have to be very careful with the place value in the tens place.

The student must remember that the - 4 is actually a - 40. So mentally he must do 200 - 40 is 160. Then 160 - 1 is 159. If he gives you the answer 195, you know he is making a place value error.

Note

Some students will give the answer two hundred because they think that 5-6 is 0, and that 4-8 is 0. If this problem persists, drop the IOU solution and do not allow the student to solve subtraction problems this way. This is a developmental problem that needs time to resolve. Meanwhile, use regrouping instead of negative numbers.

Patterns in Arithmetic: Subtraction - Booklet 4 Parent/Teacher Guide

Standard Three Digits

Purpose

The purpose of this series of worksheets is to rework the standard procedure for subtraction with regrouping in two places using Base Ten Blocks and drawings. There are six worksheets with an assessment in the middle. These lessons will uncover conceptual problems students may have on regrouping independent of the standard notation. Students begin with a drawing of the Start With number and they circle the drawings of the numbers being taken away. They use Xs to show trades or regrouping of the numbers. They record the work with drawings.

Note

Some teacher study is needed before this lesson is taught to a student.

Prerequisites

Review section of this book. Student must have completed *Patterns in Arithmetic:* Subtraction - Booklet 3. A student new to this program should have completed, at a minimum, subtraction with regrouping with single trades; regrouping in only one place. Some students new to *Patterns in Arithmetic* may know how to do subtraction but may not understand the concept of regrouping. This is especially true for students who learned subtraction using only the short notation and not expanded notation, which shows the place value of numbers being regrouped. This is a serious issue and should be remediated.

If your student did not do Subtraction - Booklet 3, ignore the IOU problems. IOUs use negative numbers and are a wonderful way to do subtraction mentally. If you are interested, read Introduction to Nonstandard Regrouping and IOUs and Nonstandard Regrouping and IOUs: Review.

Materials

Standard Three Digits - Worksheets 1 and 2, pages 6 and 7 Base Ten Blocks or bean sticks
Sharp pencil with an eraser

Warm up Review

Begin with the blocks only.

"Remember that when you are doing a subtraction problem with blocks, you only build the top number; the number we call the 'Start With' number. The number being subtracted is removed from the Start With number. What is left is the answer. Build seventy-six and take away fifty-four."

"How do you do this problem?" "Build seven tens and six ones and remove or take away five tens and four ones. There will be two tens and two ones left over. So 76 - 54 = 22."

"Now build seventy-six and take away fifty-nine." Write 76 - 59 =____ on a piece of paper so the student can see it.

Wait: The student should build the **seventy-six**. To take away the five tens and nine ones, she must do some trading. Most students start with the ones. There are six ones and nine need to be removed. Most students will pick up a ten, trade it for ones and then remove the nine ones from the sixteen ones they now have from the trade. This leaves behind one ten and seven ones, or seventeen.

If you encounter a student who does not know how to do this problem, go back to Subtraction - Booklet 3.

Note

If your students begin with the tens, consider allowing them to do this. Some students then regroup from the answer, which is a good solution that works well for those who are good in mental math. A student who does this may be using the IOU strategy. There is nothing wrong with this; it is just messy when you go to write it down. You decide if you want to prohibit this or not. If you have a strong student, let her sort it out.

If you encounter any uncertain work, do a few more two-digit problems like the one just given. Then consider continuing this lesson the next day.

Lesson Part 1

This is the building step only.

"Build four hundred twenty-three with the blocks. Now take away two hundred fifty-six." Write 423 - 256 = ____ on a piece of paper so the student can see it.

Wait and watch: This problem requires two trades. Having students do all the trades and then do the subtractions reduces errors. But, some people like to trade and then immediately subtract. It does not matter as long as the student does not get confused or leave out parts.

If the student needs assistance, here is a conversation you may have.

"You are hesitating. What seems hard?" "I do not have enough ones to take away six." Or "The number is too big."

"You have four hundred twenty-three in there. That is enough ones."

"How do you get pennies out of a dime?" "You get change, you trade. So I can get one of the tens and trade it for ones. I now have thirteen ones and only one ten."

"Do you have enough tens to take away five tens?" "No, but I can trade in some hundreds."

"Now you have enough ones and tens to do your take away. What number is the What's Left? number?" "One hundred sixty-seven."

"Look at the first problem on Standard Three Digits - Worksheet 1. Build your Start With number. What is it?" "Three hundred fifty-three."

"Find the number under the words 'Take Away.' Do not concern yourself with the word 'circle' at this time. What number is written there?" "One hundred eighty-four."

"Take away one hundred eighty-four out of the three hundred fifty-three you have built."

Wait: Do not assist unless it is needed.

The student will see there is not enough ones to take away four. She will take one

Patterns in Arithmetic: Subtraction - Booklet 4 Standard Three Digits
Parent/Teacher Guide 19

ten and trade it for ones. She now has thirteen ones. Now she will go to the tens. She will see she needs eight tens and she has only four. She will trade one of the hundreds for ten tens. She will now have fourteen tens and two one hundreds. Now she can do all the take aways. She will have one hundred sixty-nine left.

"Record this answer in the What's Left? box on the worksheet."

Do the rest of the page, using only the blocks, recording only the answer in the What's Left? box.

Check the answers.

Lesson Part 2

Standard Three Digits - Worksheets 1 - 3. Drawing and recording step.

Now drawings are used to show the blocks and record what is going on with the numbers. It is important that the student uses the blocks at the same time she is drawing and recording with numbers.

Look at the Example problem on Standard Three Digits - Worksheet 1, page 6.

The Start With number is four hundred twenty-three. The Take Away number is two hundred fifty-six. It appears in the Circle box.

Use the same talk that was used in Part 1, but do the drawing activity in the box labeled 'Draw here.'

Talk and Build		Draw
"You do not have enough ones	to take away	six."
"So you take one of the tens and	ones." X out one ten (from the left)	
	000	•••••• Draw in ten dots next to the three ones.
"You now have thirteen ones ar	nd only one te	X out one hundred
"Continuing to the tens, you ne you now only have one. So trac a one hundred in for ten tens."		Draw tens sticks next to the tens.
"You now have eleven tens and Now you can easily take away t Take away your six ones."	three one hu he 256.	ndreds. Circle six ones.
"How many are not circled?"	"Seven."	Draw seven dots on the right side of the What's Left? box.
		What's Left? •••••

Standard Three Digits **20**

Patterns in Arithmetic: Subtraction - Booklet 4
Parent/Teacher Guide

"Take away five tens." "How many are not circled?" "Six."	Circle five tens.
	Draw six sticks to the left of your seven dots in the What's Left? box. What's Left?
"Take away your two hundreds."	Circle two hundreds.
"How many are not circled?" "One."	Draw one square to the left of your six sticks in the What's Left? box.
What's	Left?

"How much do you have left?" "One hundred sixty-seven."

"Does that match what you got yesterday?" "Yes."

Do Problems 1, 3, and 5. Check the work. Skip problems 2 and 4.

If your student did Subtraction - Booklet 2, have him now do problems 2 and 4. Assist only if needed.

Possible student thought process. "Three take away four is negative one. Record that under the numbers with the circles around them. Fifty take away eighty is negative thirty. Record that. Three hundred take away one hundred is two hundred. Record that. Now do 200 - 30 - 1 in my head. Two hundred take away thirty is one seventy. One hundred seventy take away one is one hundred sixty-nine."

"Does that answer match the one you got in problem 1?" "Yes."

The same talk pattern is used for problem 4.

Worksheets

Do Standard Three Digits - Worksheet 2, page 7, in the same way. Many students will still need you to work with them on this. Some will be independent. Require them to make the drawings. Some students will not need the blocks.

Do the IOU problems only if the student did Subtraction - Booklet 3.

Patterns in Arithmetic: Subtraction - Booklet 4 Standard Three Digits 21

Test for Understanding

Standard Three Digits: Assessment - Worksheet 3 Make the blocks available but do not require the student to use them. It is important that you watch your student while she does this test. Require her to draw. Many students will resist this because it takes too much time. Tell these students they will be able to drop the drawings soon. But right now it is important that they can show that they understand the place values while they are regrouping.

Watch to see if the student uses the blocks. If she does, allow her to use the blocks for two problems and then see if she can do the problem with just the drawings. What you are looking for is sureness about the correctness of the answer and the accuracy of the trades. Does she know the difference between a trade shown by an X and a take-away shown by a circle?

Scoring Rubric

Here is a list of things to watch for to decide what is the best next step. Give one point for each item for each problem. Each problem has four possible points.

- 1. Student does not use the blocks.
- 2. Does the drawings easily and correctly showing all trades.
- 3. Gets the correct answer.
- 4. The answer matches the drawing.

Score of 12 - 16 indicates strong understanding of the lesson. The student will need a few more short lessons on recording with numbers but will probably be independent after that. It is not a problem if students use the blocks; it just tells you how secure they feel with the process. When they are secure, they will generally drop the blocks.

Score of 5-11 indicates the need for you to work with the student in the upcoming pages until you see a more accurate response. Check to see if the drawings are causing the problem. If a student has difficulty, it does not mean that you cannot go on to the next page, but it does mean that you need to work with them while they do it.

Score of 4 with correct answers but no drawings or incorrect drawings:

If the student does only the standard, short notation and does not do the drawings, require her to go back and do those drawings. If she is unable to do so, assume she does not understand the drawings, or she does not understand the place value. To tell the difference, use the blocks.

It is important to tell the difference between unwilling and unable.

If she can move through the process and show the trades and write the answer with a number, assume it is the drawings causing the difficulty. Representations of physical objects are a barrier for a few students. Work with them with a tracing of the blocks on a large paper to help them see the connection between the blocks and the pictures.

Practice Worksheets

If she cannot do the trades with the blocks, assume the problem is with the concept itself. Go back to Subtraction - Booklet 3, or make up problems with only one trade like we did in the first lesson. Work forward from there until the student can do problems independently. Then resume with Standard Three Digits - Worksheets 4 - 7, pages 9 - 12. Continue to practice with one worksheet per day.

At the top of some of the pages it asks the student to create problems that do or do not require regrouping. This is an embedded assessment. Can the student recognize the attributes of a subtraction problem that requires regrouping?

If the student shows hesitation, make a set of cards where half the problems need regrouping and half do not. Talk about how to tell the difference.

Standard Three Digits with Recording

Purpose

The purpose is to review both the short and expanded notations for the standard subtraction procedure with regrouping. Students draw, record using the expanded notation, and then do the same step using the standard, short notation. Drawing and using the expanded notation ensures the student understands the place value and conceptual base behind regrouping as well as the notation and what it means.

Note

Some teacher study is needed before this lesson is taught to a student.

Prerequisites

Part 1: Score of five or more on the assessment in the last lesson, Standard Three Digits - Subtraction.

Part 2: Rounding to the nearest hundred (this topic is covered in Place Value: Booklet 2).

Materials

Standard Three Digits with Recording - Worksheets 1 - 6, pages 13 - 18

Base Ten Blocks
Sharp pencil with an eraser

Warm Up

This is an embedded assessment.

"Choose a Start With number. Draw it. Choose a Take Away number that requires regrouping in both the ones and the tens places. Draw your way though the problem as you did in the last lesson."

Watch to see if the student has a sure sense of what causes a problem to be a regrouping problem. He should know that the entire top number must be larger than the bottom number, but that numbers in the tens place or ones place or both would be smaller than those of the Take Away number.

Example: 758 - 169 Note that the top number is larger than the bottom number but that in both the ones place and the tens place the top number is smaller than the bottom number requiring a regrouping to subtract.

If you encounter any difficulty, do a second problem. For some students, the recording process clears up confusions. So do not be afraid to go forward with the recording.

Lesson Part 1

Have the student take out Standard Three Digits with Recording - Worksheet 1, page 13. Most students will not need the blocks at this point, but if they do, incorporate the movement of the blocks just before you draw. Do it one-for-one just like you did in the last lesson.

"Study the example. Point to the drawing of the subtraction problem 265 - 97. Point to the expanded recording of the trading and subtraction. Point to the short recording of those same trades and subtractions."

"Look at problem 1. What is the Start With number?" "Two hundred thirty-four."

Patterns in Arithmetic: Subtraction - Booklet 4
Parent/Teacher Guide

"The writer of this book chose the Take Away number. She chose 85. We are going to do what is called a one-for-one recording. This means that we will do a regrouping on the drawing, record it in the expanded form, and then record that same regrouping in the short form. We are doing this to make sure you understand what you are doing when you use the shorter, faster way to write down these kinds of problems."

"Write 85 in the box that says 'Circle.'"

Note

If you get confused as to how to write this, check the Answer Key. It will show you what the writing should look like.

"How do you record the -85 in the expanded notation?" "- 80 + - 5 in the circles in the middle."

"How do you record the -85 in the short notation?" "85 under the 34 on the right side of the paper."

"Examine the problem to see where the regrouping will be needed." (IOUs and nonstandard regrouping are not allowed here.)

"Begin with the ones. Do you need to regroup here? "Yes."

"On the drawing, cross out one ten." (Do not draw the tens dots yet.)

"How many tens do you have now?" "Two."

"In the expanded notation in the middle, cross out the 30 and above it write 20. Where did the 20 come from?" "From having two tens left, 2 tens = 20."

"On the short notation at the right, cross out the three and above it write two in the tens place."

"What does this two stand for?" "Two tens."

"Now draw the ten dots for the ones you traded."

"How many ones do you have now?" "Fourteen."

"In the expanded recording section, write a ten above the four and circle them both to show the fourteen ones."

"In the standard short recording section, write a small one just to the left of the four."

"What does this little one stand for?" "One ten."

"How many ones does that little one and the big four show you have after the trade?" "Fourteen ones."

"Why did you do this?" "Because I need more ones so I can subtract the five."

Patterns in Arithmetic: Subtraction - Booklet 4 Parent/Teacher Guide

Note

Some students will want to do the subtraction of the five from the fourteen at this point. If you want to do that, follow the instructions for the 'Circling and Take Away' section written later in this lesson. It does not really matter if you subtract as soon as you regroup, but I have found doing this increases errors.

"Now move to the tens. Do you need to regroup here?" "Yes."

"In the drawing, cross off one hundred. How many hundreds squares do you have left?" "One." (Do not draw the ten sticks yet.)

"What should you do now in the expanded recording section?" "Cross out the 200 and above it write 100."

"Why did you write the 100 above the 200?" "Because I used one of the hundreds to trade for tens and now I have only 100 left."

"How should you record this in the standard short recording?" "Cross out the two in the hundreds place and above it write one."

"What does this one stand for?" "One hundred."

"Now draw the ten sticks."

"How many tens do you have now?" "Twelve."

"How many tens did you just bring in from the trade?" "Ten."

"How much are these ten tens worth?" "One hundred."

"Record that one hundred in the expanded section above the twenty already there and circle it. Why do you think it was circled?" "To show that I have one hundred twenty-in the tens place."

Do not allow him to put a one in front of the twenty. It hides the regrouping.

"Where did one hundred twenty come from?" "From the trades. I have twelve tens in my picture. Twelve tens is worth one hundred twenty."

"How should you record this in standard short recording?" "Write a small one next to the two, I wrote in the tens place."

"What does this little one stand for?" "One hundred."

"But it says twelve. How can that be a one hundred when it looks like a ten?"
"Because it is in the tens place. It means twelve tens. Twelve tens is one hundred twenty. That is why the little one is really a one hundred."

"Why did you do this?" "Because I need more tens so I can subtract the eight tens."

"How many hundreds are left?" "One hundred."

Circling and Take Away

"Now you are ready to do the subtraction. Remember, you draw first, then record in the expanded notation, and then record in the standard short notation. See if you can do this by yourself. Let me know if you get stuck."

Student begins with the ones.

Drawing: Circles five dots. Counts how many are not circled.

Counts how many are not circled and draws nine dots in the What's Left? box.

Expanded recording: Subtract five from fourteen and records What's Left? below the line directly under the circled five. Student should write 9.

Standard short recording: Subtract five from fourteen and record What's Left? below the line directly under the five. Student should write 9 in the ones place.

Continues with the tens.

Drawing: Circles eight sticks. Counts how many are not circled. Draws four sticks in the What's Left? box.

Expanded recording: Subtracts eighty from one hundred twenty and records forty below the line directly under the -80 in the circle.

Standard short recording: Subtracts eight tens from twelve tens and records a 4 below the line directly under the 8 in the tens place.

Finish: The student draws a one hundred square in the What's Left? box, writes 100 + next to the 40, and a 1 in the hundreds place in the short form.

Check to see if the student recorded + signs between the numbers in the expanded notation section.

"When you write the 40 under the line, what do you need to write in between the 40 and the 9? "A plus sign."

"Why do you have to write a plus sign there?" "If you do not, the 'What's Left? will look like 409. You must write 40 +9 to make it forty-nine."

"Where else do you need a + sign?" "Between the one hundred and the forty in the expanded notation space."

"Do you need to write a plus sign in the short notation too?" "No."

"Why not?" "Because I am using the short notation. The eight in the second column means eighty. When one writes 89, the 80 and the 9 are automatically added. You do not need to write the plus sign."

"What is the answer?" "One hundred forty-nine."

"Does your drawing, expanded notation, and short notation match?" "Yes."

"Do problem 2 alone, doing all the steps." Watch. Help only if needed.

Patterns in Arithmetic: Subtraction - Booklet 4 Standard Three Digits with Recording Parent/Teacher Guide 27

Test for Understanding

Problem 3 is an embedded assessment. Watch to see if the student can fill in the numbers, draw, do the subtractions, and record. Check the Answer Key to make sure all parts are filled in correctly.

End the session at this point.

Worksheets

Standard Three Digits with Recording - Worksheet 2, page 14

Lesson Part 2

Review rounding to the nearest hundred.

Have the student take out Standard Three Digits with Recording - Worksheet 3.

"Read the instructions and study the example." Wait. In the example, 341 rounded to 300. Notice where it is recorded. Then, 243 rounded to 200. Then the subtraction was done.

"Before we do the subtraction, we are going to estimate 'about what' the answer should be. Estimation and reckoning allow us to know if our final answer is in the right place value. It helps us spot big mistakes."

"Is one hundred the answer to 341 - 243?" "No, it is the rounded answer."

"Look at the list of approximate answers. It says a 900 b 500 c 250 d 150 e 75 f 25. Which one do you think will be closest to the real answer?" "It will be closer to the 75."

"But the rounded answer is in the hundreds; why not choose 150?"

Ask him to explain his thinking.

Note

The rounded answer is not in this list because that number is already available to the student from the rounding that was just done. This list allows the student to 'reckon' the final answer based on what the rounding indicated and how far off from the real number the rounding was. In the example problem, the rounded answer is one hundred. But how close to one hundred will the real answer be? It will be very close because you can see that 341 and 243 are nearly exactly one hundred apart. The student is presented with the question, will it be closer to one hundred fifty or to seventy-five? The student needs to then think, "Will the answer be more or less than one hundred?" The answer is a bit less. So the closest estimate is seventy-five. We are doing mental calisthenics here.

Test for Understanding

"Will this problem require regrouping?" "Yes."

Understanding "How do you know?" "The top numbers in the ones and in the tens places are smaller than the bottom numbers in those places."

"Do the problem now using the expanded notation. Tell me when you get the answer." "Answer = 98."

"What is the answer to the question to the right of the problem?" "e. 75."

"Now do problems B through G. Let me know if you need help." Watch. Help only if needed.

Worksheets

Standard Three Digits with Recording - Worksheets 3 and 4, pages 15 and 16

Before doing Standard Three Digits with Recording - Worksheets 5 and 6, pages 17 and 18, discuss the arrow to the target (instructions below).

Arrow to the Target: Have the student use a combination of rounding and reckoning to estimate the answer. No fair doing the problem in your head then putting the answer down as your estimate. Example: On Standard Three Digits with Recording - Worksheet 5, the first problem is 344 - 165. Student thinks, "From rounding I would estimate the answer is about one hundred. Reckoning on top of that I see that three hundred forty-four rounded way down and one hundred sixty-five rounded way up so the distance between the numbers is probably closer to two hundred than one hundred, so I will estimate one hundred seventy-five. That is what I would put in the box. Honestly, I didn't look the answer up first. Then doing the problem, I see the answer is one hundred seventy-nine. Wow, I was really close. So I will draw my arrow on the bull's-eye. I was so on target!"

If you are off by more than fifty, put the arrow about a centimeter away from the target. If you are off by more than one hundred, put the arrow about an inch from the target. You get the idea.

Check all work as soon as it is completed.

Test for Understanding

Do the assessment. Look for scoring notes on the lesson plan for the next lesson.

29

Gus the Bus Driver

Purpose

The purpose is to develop skill in multistep word problems. This lesson uses a familiar image of people getting on and off a bus. Gus the bus driver has to keep track of how many people get on the bus, how many people get off at each stop, and how many people are on the bus as he goes between stops. The famous Fisher-Price yellow bus was the inspiration for this set of problems.

Prerequisites

Basic addition and subtraction in two digits with regrouping

Materials

Gus the Bus Driver - Worksheet 1, page 21

Toy bus (optional)

Counters of any kind are helpful. A yellow piece of paper can stand for the bus and form a workboard for the student to move counters off and on.

Lesson

Take out the bus and tell the story of Gus the bus driver. Get creative! Explain that school bus drivers have to keep track of how many people are on the bus at all times. Gus the bus driver has to keep track of how many people get on the bus, how many people get off at each stop, and how many people are on the bus as he goes between stops. This is required so that if there is an accident the rescue workers know how many people they are looking for.

"Load the bus! How many children are on the bus at the end of the third stop?" "Fifty-two."

"What number sentence did you write to get fifty-two?" "17 + 12 + 23 = 52."

"What happens next?" "Children get off."

"What number sentence did you write to find how many are on the bus in problem 2?" "52 - 16 = 36."

"In problem 3, there are people on the bus, then some get off and some get on. What number sentence could you write to show people getting on and then people getting off?" "I have to add and then subtract. Or I could subtract and then add."

"Does the order you do that in matter? Will it change the answer?" "Let me try it."

"36-18+9 is 27. 36+9-18 is also 27, so, no, the order does not matter."

Keep going on like this until you finish both pages. Some students will use the cubes and take them on and off the yellow paper. If they do this, make sure they write the number sentence, as many will forget to do it. Some students will use only numbers. *Check* accuracy with the Answer Key.

Then have the students write a three-problem sequence for each other on a separate page.

Extension

Redo the entire page with a series of keystrokes on a calculator.

Standard Three Digits: Assessment

Purpose

The purpose is to give a formal assessment of subtraction and regrouping. The results of this assessment will guide the teacher in planning for further lessons and remediation of low areas, and determine the amount of practice needed.

Prerequisites

Completion of lessons up to this point

Materials

Standard Three Digits: Assessment - Worksheets 1 and 2, pages 19 and 20

Note Part 1

If you are working with a group, be sure everyone understands they must do only the top row and then stop and wait. Tell the students that they will have thirty seconds to do as many of the problems in the top row as they can. Repeat this same instruction for the second row. When they finish, instruct them to make dark Xs over the problems they did not complete. Check. This will prevent them from filling in the numbers later when you are not looking. Temptation to fudge on speed tests is overwhelming for some students.

Note Part 2

Draw the student's attention to the estimation requirement. Remind her that she must do this estimation before beginning the calculation. Estimates should be given in round numbers to nearest ten or nearest hundred. During the test, observe to see if that is what the student is doing. This section is not timed. Do pay attention to about how long it takes the student to complete the work. The average time for this section is fifteen minutes. Some students have taken as long as thirty minutes. Time longer than that indicates a problem.

Note Part 3

This section is quite difficult. Let the student know that in advance. This section often results in questions from the student. Before starting this section, draw the student's attention to problems 12, 13, and 14.

Tell her that each problem shows an error another student had made in calculation. The student is to identify the error, explain the error in writing below the problem, and then solve the problem correctly. If she did not do IOU problems during the instruction, she should skip problem 14. On the word problems, just tell her to do the best she can.

Using the Results Score the test using the Answer Key and the score sheet on page 34 in this booklet.

The goal is for the student to have 80% or better on each section. We define 80% as a pass. This would be normally thought of as a B, and one can pass a student at 70%.

Do so with care, as accurate subtraction is fundamental. Any student not at 80% or better on a part needs weekly practice of twenty or so problems.

A student at 80% or more can drop down to a problem a week just to keep things fluid.

Lesson Part 1

You may find the student is fluent on the differences below ten and not fluent on the differences in the teens. This is common and is a main cause for slow calculation in

Standard Three Digits: Assessment Patterns in Arithmetic: Subtraction - Booklet 4 31 Parent/Teacher Guide

subtraction. Many students at this level still count backwards or use their fingers on the teen subtractions. While this is preferable to getting the wrong answer, it is slow and inefficient. Remediate this problem with flash cards, speed drills, and games you can find on the Internet.

Part 2

The student needs to have 80% or better in this part, and if she does not have that score, she needs to practice until she does. Continue with the Gus the Bus Driver problems, but do not go into calculations into the thousands until she has fluency in two and three digit problems.

Use the following guide to tell if the student needs work on the different parts.

Estimation—22 points or better is a pass Accuracy—58 points or better is a pass

If the student missed the last problem, give more practice working problems with double zeros. Use the blocks and have her watch for patterns in the regrouping numbers.

Part 3

This section is a Test for Understanding.

Word problems are very difficult if there isn't a complete understanding of a process. There is a difference between doing isolated problems on a page and using them in a real world problem setting. This book has not strongly addressed word problems up until this point. The reason this is included is to identify very strong students. They will not need as much help in the word problem sections and will need less practice. Strong results here are a small indicator of mathematical prowess. Our experience is that most students will not pass this section.

The word problems are difficult and require fluent understanding not only of subtraction but also the relationship between addition and subtraction. Do the Gus the Bus Driver section and then reassess. Many students at this level need a considerable amount of practice with multistep problems. They are easy to make up. Students can also make up problems for each other. Give mixed practice where students must decide if they should add or subtract. There are many sources for practice on this topic.

Error identification: Does the student understand the process well enough to spot common errors?

In problem 12, the student did not regroup in the tens place, a very common error. The student just took the difference between the numbers. Most students will spot this one.

Problem 13 addresses a common notation error when students have been doing expanded notation and the standard short notation. In the tens place, the student used a ten instead of a one in front of the five in the tens place. The student thought there was 105 in the tens place. 105 - 8 is 97. The student wrote the 97 disregarding place value, so the answer is really out of whack. Many students will have difficulty isolating the specific error but can identify that some terrible error

happened in the tens place, or will say the answer is too large.

Problem 14 addresses a common error in IOUs. The student disregarded the place value in the tens place and added the -3 to the -7 for a total of -10, which was subtracted from the 200 to be 190. But the difference in the tens is not -3, but -30. The student mentally did 200 - 10 instead of 200 - 30 - 7.

If you notice persistent errors of this kind, tell the student to stop using IOUs. Students who continue to use them correctly are the ones who are good at mental math.

Parent/Teacher Guide

Subtraction Test Score Sheet			Name _	
			Date _	
Part 1				% correct
Differences under ten	Give 10%:	for eacl	n correct answer	
Differences in teens	Give 10%:	for eacl	n correct answer	***************************************
			TOTAL FOR PART 1	***************************************
Part 2				
Estimates should be in	n round numb	ers. Gi	ve 4% points for each	
estimate in the correct	t rounded zone	e.		
The student may have	e estimated to	nearest	ten on larger problems.	Give credit for this
Give 8% points for co	orrect answers	on eac	h of the first seven proble	ms
Give 12% points for o	correct answer	on the	last problem.	
			TOTAL FOR PART 2	-
Part 3				
Word Problems	Correct proce	edure	Give 20% for each corn	rectly
			selected procedure	
	Correct answ	er er	Give 13% points for ea	ch
			TOTAL Word Proble	ms
Error Identification w	rithout IOU pr	oblem	12	
Correct identification	of the error.	Give	40% for each	Personal Security Control of the Con
Correct solution of the problem		Give	10% for each	***************************************
		TOT	AL Error Identification	MATERIAL PROPERTY AND ADMINISTRATION ADMINISTRATION ADMINISTRATION AND
Error Identification w	rith IOU probl	em 14		
Correct identification	of the error.	Give	26% for each	***************************************
Correct solution of th	e problem	Give	7% for each	
		ТОТ	AL Error Identification	

Word Problems

Purpose

This lesson works on one-step subtractions problems working with 'Take Away,' 'What's Left?,' 'Difference Between,' and 'How Much More?' problems. The important part of the lesson is to help students connect English language to mathematical language. The inability to do this is what causes most people to have difficulty with word problems. Then the lesson also works with a variable, blank space, challenging students to determine from language alone if the problem is an addition or a subtraction problem.

Note

Teacher study is needed before Parts 2 and 3.

Prerequisites

Gus the Bus Driver, basic addition and subtraction skill

Materials

Word Problems - Worksheets 1 - 3, pages 22 - 24

Base Ten Blocks for both parts Cuisenaire Rods for Part 2

Lesson Part 1

It is important for students to have a strategy for approaching word problems.

First, read the problem at least twice.

Second, underline important numbers and important language, such as 'each'; 'altogether'; 'what's left?'; 'how much more?' etc.

"In subtraction there are two different kinds of problems. One is 'Take Away, What's Left?' The other kind is 'Difference Between' or 'How Much More?' kind of a problem that compares the sizes of two numbers. Each one has specific language. Read problem 1. What kind of subtraction problem is this one?" "It is a 'Take Away, What's Left?' problem."

"Which words are the 'Take Away' language?" "Ate seventy-eight.' They ate the cookies and they want to know 'What's Left?'"

"Underline your Start With number and your 'ate seventy-eight' and the word 'left.' What number sentence did you write to go with this problem?" 90 - 78 = 12."

"Twelve what?" "Twelve cookies."

"Where do you write the twelve cookies?" "On the line after the question mark."

Problems 1 through 4 and 6 are obvious 'Take Away, What's Left?' problems.

Problem 5 looks like a 'Difference Between' or 'How Much More' problem, but it is actually a 'Take Away, What's Left?' problem with some key words left out.

"Read problem five. How is this problem different from the first four we did?" "It does not have the word 'left' in the question."

"Did she get rid of any boxes? Did she give any away?" "Delivering them is

Patterns in Arithmetic: Subtraction - Booklet 4 Word Problems

like giving them away."

"Does she have any boxes left to deliver?" "Yes."

"Where could you write in the word 'left' so the question would still make sense?" "Between the words 'have' and 'to.'"

"What about problem six?" "It is an easy 'Take Away, What's Left?' problem."

Check the accuracy of the subtractions, and number sentences and that the units are labeled. You have to say if it is twelve cars or twelve cookies.

Stop here. The next part is long and hard for most students.

Lesson Part 2

'Difference Between' is a comparison of the size or space between two numbers.

We use subtraction to find differences between, but children generally do not understand why. These do not seem like subtraction problems because nothing is being taken away. We are actually using the Families of Facts in this situation. Subtraction is used to find the missing number. This is a tricky concept for many students at this level.

Teach the 'covering' strategy to create a visual demonstration of 'Difference Between.' You are going to build the larger number on the bottom and then build the smaller number directly on top of the larger one using Cuisenaire Rods and Base Ten Blocks. The uncovered space is the 'Difference Between.' Do not tell the student this. Set up the situation so she can see it. The problem is done with manipulatives first. Then the number sentences are written. Some students will be able to see the number sentences right away; some will need to build to get answers all down the page and then be led back through to find the sentences. Expect variation on this. Give as little assistance as possible.

Take out an orange rod and lay it on the table. Take a dark green rod and place it directly on top of the orange rod. Line up the left hand ends of both rods so they are flush with each other.

"How much longer is the orange rod than the dark green rod?" "Four."

"Four what?" "Four centimeters longer."

"How can you prove that the orange is four centimeters longer?" "I can put on four whites after the dark green until it is the same length as the orange. Or, I can fit a purple rod in on top of the orange and fill the hole."

Note

What you are modeling is that 6+4=10, so the difference between, four, is obtained by the inverse statement of 4 = 10 - 6. This is why we use subtraction. This is very, very abstract! Do not explain this to students. They will not get it yet.

Just use the rods and let her develop the concept first.

"What do you have to do to show how much longer a brown rod is than a

yellow rod?" "I can fit a light green rod in the space between the end of the yellow rod and the end of the brown rod. Light green is three centimeters long, so brown is three centimeters longer than the yellow."

"Build a twenty-four centimeter train. On top of it build a fifteen-centimeter train. How much longer is the twenty-four train?" "Nine centimeters. The blue rod fits in between the end of my fifteen train and the end of my twenty-four train."

Take out the Base Ten Blocks. Put down a one hundred flat. Now put four ten rods on top of the flat one hundred block. (It is helpful if you can use different colors for the bottom number and the top number.)

"How much more is one hundred than forty?" "Sixty."

"How do you know the answer is sixty?" "It is the space on the one hundred flat that is not covered." Or rarely, "Because one hundred minus forty equals sixty."

"Can you show me where the sixty is on the blocks?" "Fit six tens blocks into the empty space on top of the hundred." Or, points to the empty space.

"Can you use your blocks to find out what the difference between one hundred seventy-two and one hundred thirty-seven is?"

Wait. This will take a bit of time. Several strategies are possible. You may see students counting empty space, or you may see a trade of tens for ones.

Build one hundred seventy-two on the bottom and cover it with one hundred thirty-seven on the top. Count up what fits in the empty space.

"The answer is thirty-five."

"What would happen if you put the thirty-five into the empty space on top of the one hundred seventy-two? (Use another color if you can.) "You would cover the whole one hundred seventy-two."

"Can you find the answer to problem 1 on Word Problems - Worksheet 2 by using a covering strategy?" "Maybe," or, "I don't need to, I can see the difference is thirteen."

"Explain your strategy." Listen to the response.

Use this next sequence only if the student has difficulty.

"What number do you build on the bottom?" "The fifty-six."

"What number do you build on top of it?" "The forty-three."

"Can you see how much more the fifty-six is than the forty-three?" "Yes, the empty space is thirteen."

Do not push the number sentence unless the student sees it.

Patterns in Arithmetic: Subtraction - Booklet 4 Word Problems
Parent/Teacher Guide 37

"Can you see the number sentence you could write to get thirteen as an answer?" There are several possible answers. "No." "56 - 43 = 13." Or, "43 + ___ = 56."

Finish all the problems with a covering strategy.

To get the number sentence part go back to the problems we did with the rods in the beginning of the lesson. Some students will not need this. But if you suspect the student can write the number sentences because she has been shown before, then use the following questions to probe for understanding.

"How much longer is the orange rod than the dark green rod?" "Four centimeters."

"What number is the orange rod?" "Ten."

"What number is the dark green rod?" "Six."

"What number sentence could you write using a ten and a six that would give four as an answer?" "10 - 6 = 4." (Write those numbers down in front of the student in that order.)

Give another one.

"How much longer is a brown rod than a yellow rod?" "Three centimeters."

"What number is the brown rod?" "Eight."

"What number is the yellow rod?" "Five."

"What number sentence could you write using an eight and a five that would give three as an answer?" "8 - 5 = 3." (Write those numbers down in front of the student in that order.)

"What pattern do you see in these two number sentences?" "They both are subtraction number sentences."

"Go back and see if this pattern works on the problems you solved with the blocks." Problem 2 would be 125 - 97 = 28. Have the student do every one.

"So how do you solve 'Difference Between' or 'How Much More' problems?" "You subtract the smaller number from the larger one."

"Go back and write all the number sentences in now and do the subtractions to check if you pattern works every time."

Lesson Part 3

This is a lesson in algebraic thinking. The twist for students is that there are no numbers given. Many students have difficulty accepting that you can do math without numerals.

The big idea here is that the pattern of what kind of problem is being presented, addition or subtraction, is in the language of the problem, not the numbers. The

language of an addition problem will be the same no matter what the numbers are. Students have to test this over and over to be sure it is really true that the numbers do not change the pattern. They do change the value of the answer, but not the method by which that answer was gotten. This is a very big idea.

Begin with oral problems like this.

"Lucy had some cats. Then she got more cats. How many cats does she have altogether? Is this an addition problem or a subtraction problem?" "It's an adding problem."

"How do you know?" "She had some and then she got more. She did not lose any. So she has the old cats plus she has the new cats. That is adding to find out how many she has altogether. 'Altogether' tells me it is an adding problem."

"Lucy has some cats. John has some cats too. How many more cats does John have than Lucy? Is this an addition or a subtraction problem?" "It is a subtraction problem because it is a 'Difference Between.'"

"What language tells you it is a subtraction problem?" "The words how 'many more.'"

Many students will get stuck on this one because they cannot see that the numbers really do not matter in identifying that it is a subtraction problem. They want to be sure John had more to begin with, so they think they have to have the numbers.

If the student is struggling, have her choose numbers to put in the blanks and write number sentences.

"Lucy has five cats. John has nine cats. How many more cats does John have than Lucy?" "Here he has four more, 9 - 5 = 4."

"Lucy has ten cats. John has fifteen cats. How many more cats does John have than Lucy?" "Here he has five more, 15 - 10 = 5."

"Compare your two number sentences. What pattern do you see? How are they the same? How are they different?"

Some students will say the numbers matter because Lucy's number could be higher than John's. "If that were the case, how would the question sentence be different?" "It could be that the question would use the word 'less' rather than 'more.' Or it could say, how many more does Lucy have?"

"What language tells you it is a subtraction problem?" "The words 'how many more.'"

Worksheet

Do Word Problems - Worksheet 3, page 24. If a student has difficulty, have her use numbers that would make sense in the problem and identify it that way. Have the student underline the important words in the question that identifies the kind of problem it is.

Patterns in Arithmetic: Subtraction - Booklet 4

The key words are:

Addition	Subtraction

how many left how many altogether

in all how much more

does she have [in all] how much less

Practice

A little practice, such as one problem per day, will be more effective than a whole worksheet in a single day.

The best practice is using problems the student writes for herself.

Write a 'Difference Between' problem. Write an addition problem with four numbers.

In a home-school setting, make up problems relevant to the student's life.

In a class, have students make up problems for each other.

Test for

Field trips: Go to a cemetery. Find the grave of a young person. Ask the student, Understanding "How many years did this person live?" or "How old was this person when he died?"

> Grocery Store: If I buy two pounds of oranges and two pounds of apples, how much money will I need? Which is more expensive, bananas or grapes?

> Give real world problems and see if the student can solve them. If not, keep at it until she can.

Regrouping Practice

Purpose This lesson is an embedded assessment. The teacher should not do any instruction

other than to assign the pages and then give a lesson on checking subtraction problems with addition and have the student use this to check his own work.

Prerequisites All the lessons in this book

Materials Regrouping Practice - Worksheets 1 - 4, pages 25 - 28

Base Ten Blocks

Lesson Assign the pages and watch the student while he works.

Then, teach a lesson on checking. This concept is important for many reasons. The most important is the idea of inverse relationships and Families of Facts.

Give the problem 15 - 8 =__. Have the student use some ones blocks to build it.

"What will be the result if I push the blocks that were left together with the blocks I took away?" "You will end up with what you started with."

'Prove it." "Take fifteen blocks and take away eight. Remove the eight. Then count what is left and get seven. If I push the 'What's Left?' number, the seven, back with the 'Take Away' number, the eight, I will get back my 'Start With' number, which is fifteen."

Another way to do it is with covering. Build fifteen, put eight on top, and see that the other number must be seven.

"Does that always work? Try a bigger number. What happens if you regrouped while you subtracted? Do it with the first problem on Regrouping Practice - Worksheet 1." "Take 3,132 blocks and take away 1,621. You have to regroup and get 1,511 left. If I push the What's Left? number, the 1,511, back with the Take Away number, the 1,621, I will get back my Start With number, but I will have to regroup again to get it back the way I started."

"Why would it make sense that the Take Away number plus the What's Left? number would have to add up to the Start With number." "Because the Take Away number is part of the Start With number and the What's Left? number is the other part. If you put those two parts together, you have to end up with what you started with."

"How can you do it without the blocks?" "I could just add the 1,511 and the 1,621. The sum would be 3,132."

"What would it mean if you added those two numbers and the sum was not 3,132?" "It would mean that I made a mistake somewhere."

"Check all the problems you did on these three worksheets and see if you made any mistakes."

Patterns in Arithmetic: Subtraction - Booklet 4 Regrouping Practice
Parent/Teacher Guide 41

If the student did not get 80% of the problems correct during independent work, he must keep practicing until he does. Have him make up three to five problems per day for himself.

Use addition to check the work.

Test for Understanding

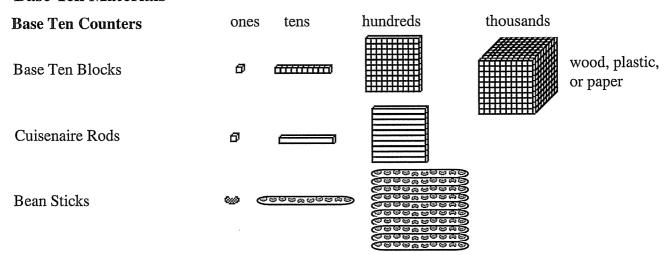
Give three problems with answers. Make two problems with errors in them. Have the student locate the incorrect answers and use the addition answer to locate the subtraction error.

Practice Worksheets

Facts Review - Worksheets 1 and 2, pages 29 and 30

Patterns in Arithmetic: Subtraction - Booklet 4
Parent/Teacher Guide

Base Ten Materials



Cuisenaire Rods

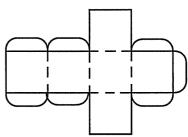
Use the white (one) and the orange (ten) rods.

For one hundred use ten orange rods rubber banded or glued together. DAPTM Fun-Tak Reusable Adhesive can be used to temporarily hold rafts together.

Inexpensive paper cubes representing a thousand can be purchased from ETA Cuisenaire. ETA-Cuisenaire (800) 445-5985 www.etacuisenaire.com

Instructions for making thousands cubes Materials

Poster board Centimeter grid paper Paper glue or paste



Cut out precisely six 10×10 centimeter squares from the grid paper. These should be the same size as the one hundred flat Base Ten Block. Arrange the six cut out squares on the poster board and paste them down so the grid paper will be on the outside. Leave room for tabs. Draw in tabs like the illustration. Cut out the figure including the tabs. Score along the dotted lines with a ruler and pen. Flip it over, and fold it into a cube, and glue the tabs to hold it together.

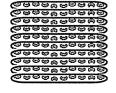
Base Ten Bean Sticks



An inexpensive alternative is to make Base Ten Bean Sticks.

Materials: small beans, Popsicle sticks, glue

Activity: Glue ten beans to a Popsicle stick. Make about twenty of these.



Rafts can be made to represent 100 with ten bean sticks and rubber bands or glue ten sticks to a cardboard square.

Patterns in Arithmetic: Subtraction - Booklet 4

Parent/Teacher Guide

Patterns in Arithmetic

Subtraction: Booklet 4

Large Numbers and Fluency

Answer Key for the Student Workbook

By Suki Glenn and Susan Carpenter

Answer Key Legend

AWV = answer(s) will vary BUWV = break up will vary OWV = order will vary

Pattern Blocks

r = red trapezoid

g = green triangle

y = yellow hexagon

o = orange square

b = blue parallelogram

t = tan rhombus

Cuisenaire Rods

1 w = white

2 r = red

 $3 \lg = light green$

4 p = purple

5 y = yellow

6 dg = dark green

7 bk = black

8 bn = brown

9 bl = blue

10 o = orange

Note: Some items and pages are left out of the answer key.

- 1) Some pages in which the answers are open-ended or will vary.
- 2) Make your own problems. Since students create their own problems and solutions, these sections give valuable information about the level of confidence and competence. It can be a useful source of curriculum for other students.
- 3) Practice pages.
- 4) Workboards.
- 5) The answers are in the Parent/Teacher Guide.

Patterns in Arithmetic: Subtraction - Booklet 4 Answer Key for the Student Workbook

©2013 Pattern Press

All rights reserved.

Printed on recycled paper.

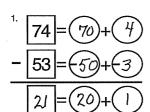
Published by Pattern Press

P.O. Box 2737 Fallbrook, CA 92088

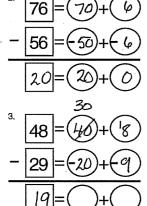
ISBN 978-1-935559-35-1

Subtraction - Booklet 4

Standard Regrouping: Review - Worksheet 1



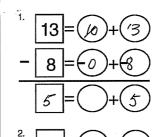
Example:
$$\frac{6}{10} = \frac{60}{10} + \frac{6}{10} = \frac{60}{10} = \frac{60}{10} + \frac{6}{10} = \frac{60}{10} = \frac{60}{1$$

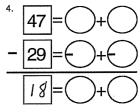


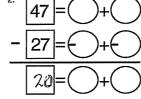
$$\begin{array}{c|c}
4. & 53 = \cancel{8}0 + \cancel{3} \\
- & 35 = \cancel{3}0 + \cancel{5} \\
\hline
\cancel{8} = \cancel{0} + \cancel{8}
\end{array}$$

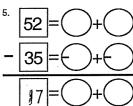
$$\begin{array}{c|c}
5. & 74 = 76 + 4 \\
\hline
-19 = 10 + 9 \\
\hline
55 = 60 + 5
\end{array}$$

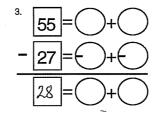
Standard Regrouping: Review - Worksheet 2











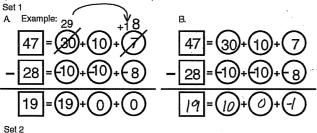
0

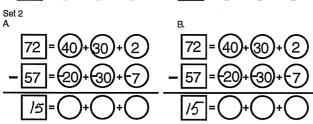
2

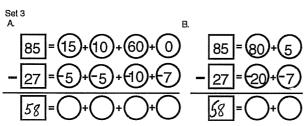
1

Nonstandard Regrouping and IOUS: Review -Worksheet 1

In each set there is an A and a B problem. Use regrouping to solve all the A problems. Use IOUs to solve all the B problems.



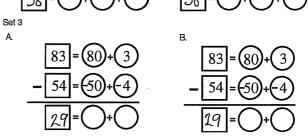




Nonstandard Regrouping and IOUS: Review -Worksheet 2

In each set there is an A and a B problem. Use regrouping to solve all the A problems. Use IOUs to solve all the B problems. Set 1

A 51 37 37 Set 2

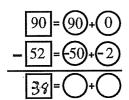


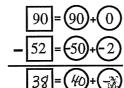
A.

Nonstandard Regrouping and IOUS: Review -Worksheet 3

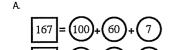
In each set there is an A and a B problem. Use regrouping to solve all the A problems. Use IOUs to solve all the B problems.

Set 1





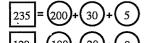
Set 2



В.

$$\begin{array}{c|c}
\hline
167 &= & 100 \\
\hline
- & 68 &= & 50 \\
\hline
+ & 10 \\
\hline
+ & 8
\end{array}$$

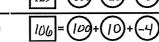
Set 3



В.

8

106



6

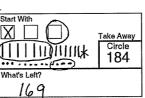
Standard Three Digits - Worksheet 1

Example:

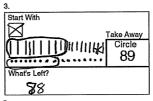
Start With	Take Away
*	Circle 256
What's Left?	

Put an X over the pictures of the blocks that you use in the trading.

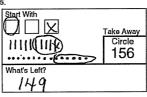
Use an IOU. Watch your place value.



353 = (300 + (50) + (3 184= (100)+(80)+



4. Use an IOU. 177=(100)+(70)+(7 89= E <u>88</u> = 100 + -10 + -2



5

Standard Three Digits - Worksheet 2

Start With

	Take Away
	Circle 238
What's Left?	

Use an IOU.

$$326 = 300 + 20 + 6$$

$$-238 = 200 + 30 + 8$$

$$8^{2} = 100 + -10 + -2$$

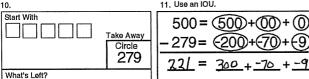
୫୫

Start With Take Away Circle 163

9. Use an IOU.

	<u> </u>
- 163=	(100+60+3
37=	100 + -60 +-3

What's Left? 37



221

Z. Make your own.	
Start With	
	Circle
What's Left?	

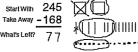
11, Use an IOU.

10.	USO ATTIOU.		
	= <) +()+0
<u> </u>	= (<u></u> +E	<u>)+O</u>
_	=	+	+

Standard Three Digits: Assessment - Worksheet 3

Draw the Start With number for each problem. On your drawing Circle the Take Away number.
Write the What's Left? number under the line.

start With 245



Example:

Start With 264 🗆 🖾 What's Left? 169

Start With 138 Take Away - 69 69



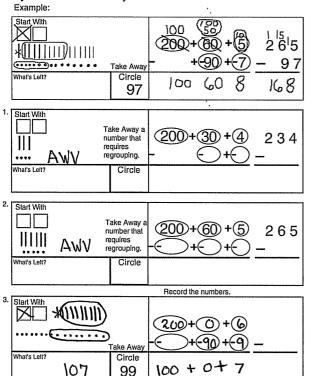
Start With 352 11 Take Away -186 ([[]]]]]]] What's Left? 166

start With 403 (

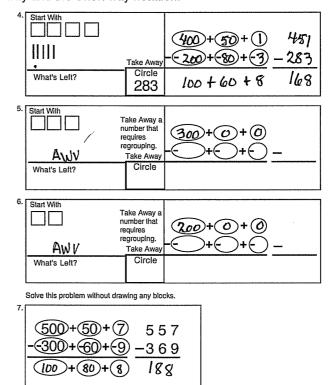
Do any two problems above as an IOU if you can.

Standard Three Digits - Worksheet 4	Standard Three Digits - Worksheet 5			
Regrouping in Two Places	Write in the numbers.			
Example: Start With Take Away Circle 256 What's Left?	4. Start With Take Away a number that doesn't require regrouping. AWV Circle			
What's Left?				
	5. Start With			
1. Start With	Take Away Circle 37 What's Left? 75			
40	6. Start With			
2 Start With	Take Away Circle 156 - + + + -			
289 ———				
3. Start With Take Away Circle 379 - + +	7. Start With			
What's Left?	What's Left? 278++			
288+_+_	10			
9	i il)			
Standard Three Digits - Worksheet 6	Standard Three Digits - Worksheet 7			
Standard Three Digits - Worksheet 6 Write in the numbers.	Standard Three Digits - Worksheet 7 Write in the numbers.			
Standard Three Digits - Worksheet 6 Write in the numbers. 8. Start With Take Away a number that requires regrouping Circle	Standard Three Digits - Worksheet 7 Write in the numbers. 12. Start With Take Away Circle			
Standard Three Digits - Worksheet 6 Write in the numbers. 8. Start With Take Away a number that requires regrouping that requires regretaring reg	Standard Three Digits - Worksheet 7 Write in the numbers. 12. Start With Take Away Circle 162 What I of 2			
Standard Three Digits - Worksheet 6 Write in the numbers. 8. Start With Take Away a number that requires regrouping Circle Circle	Standard Three Digits - Worksheet 7 Write in the numbers. 12. Start With Take Away Circle 162 162			
Standard Three Digits - Worksheet 6 Write in the numbers. 8. Start With Take Away a number that requires regrouping What's Left? 9. Start With Take Away a number that requires regrouping.	Standard Three Digits - Worksheet 7 Write in the numbers. 12. Start With Take Away Clrcle 162 What's Left? 138 Take Away Clrcle 162 Take Away Clrcle 162 Take Away Clrcle			
Standard Three Digits - Worksheet 6 Write in the numbers. 8. Start With	Standard Three Digits - Worksheet 7 Write in the numbers. 12. Start With Take Away Clrcle 162 What's Left? 13. Start With Take Away Clrcle 347 Take Away Clrcle 347			
Standard Three Digits - Worksheet 6 Write in the numbers. 8. Start With Take Away a number that requires regrouping What's Left? 9. Start With Take Away a number that requires regrouping.	Standard Three Digits - Worksheet 7 Write in the numbers. 12. Start With Take Away What's Left? 138 Take Away Circle 162 + + + + Circle 162 Take Away Circle 347 Take Away Circle 347			
Standard Three Digits - Worksheet 6 Write in the numbers. 8. Start With Take Away a number that requires regrouping	Standard Three Digits - Worksheet 7 Write in the numbers. 12. Start With Take Away Clrcle 162 What's Left? Take Away Clrcle 347 Take Away Clrcle 347			
Standard Three Digits - Worksheet 6 Write in the numbers. 8. Start With Take Away a number that requires regrouping Take Away a number that requires regrouping Take Away a number that requires regrouping.	Standard Three Digits - Worksheet 7 Write in the numbers. 12. Start With Take Away Circle 162 What's Left? 138 Take Away Circle 347			
Standard Three Digits - Worksheet 6 Write in the numbers. 8. Start With Take Away a number that requires regrouping Take Away a number that requires regrouping. What's Left? 10. Start With Take Away a number that requires regrouping.	Standard Three Digits - Worksheet 7 Write in the numbers. 12. Start With Take Away Circle 162 H 13. Start With Take Away Circle 347 What's Left? Take Away Circle 347 Take Away Circle 347 Take Away Circle			
Standard Three Digits - Worksheet 6 Write in the numbers. 8. Start With	Standard Three Digits - Worksheet 7 12. Start With			
Standard Three Digits - Worksheet 6 Write in the numbers. 8. Start With Take Away a number that requires regrouping Take Away a number that requires regrouping. 9. Start With Take Away a number that requires regrouping. AWV What's Left? 10. Start With Take Away a number that requires regrouping. What's Left? Take Away Take Away Circle 67 What's Left? Take Away Circle 67 Take Away Take Away Circle 67 Take Away	Standard Three Digits - Worksheet 7 12. Start With			
Standard Three Digits - Worksheet 6 8. Start With Take Away a number that requires regrouping	Standard Three Digits - Worksheet 7 12. Start With			

Standard Three Digits with Recording - Worksheet 1 Draw and record each problem using both the Expanded way and the Short way notation.

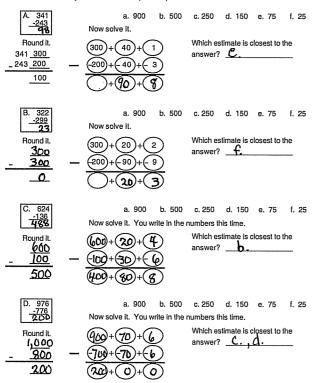


Standard Three Digits with Recording - Worksheet 2 Draw and record each problem using both the Expanded way and the Short way notation.

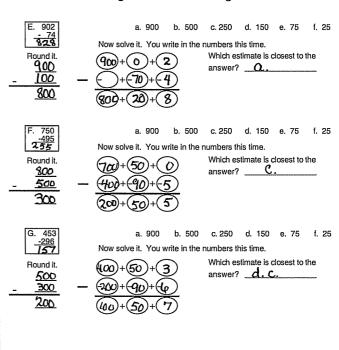


Standard Three Digits with Recording - Worksheet 3

Look at the problem in the box. Round the numbers in the subtraction problem to the nearest hundred. Look at the list of possible answers that are written next to the problem in the box. Circle the estimate that seems the most reasonable. Then do the problem with blocks. Record your work on the expanded problem.



Standard Three Digits with Recording - Worksheet 4



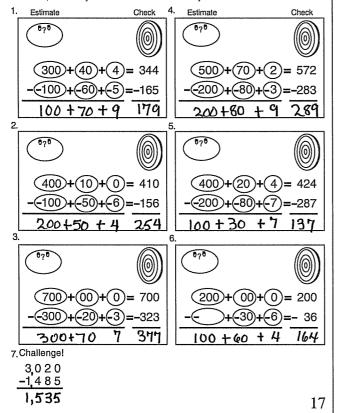
H. Write a hard subtraction word problem whose answer is around 350. AWV

13

14

Standard Three Digits with Recording - Worksheet 5

Estimate the answer before you solve each problem. Solve each problem the expanded way and the short way. Draw an arrow to the target. The closer the answer is to your estimate, the closer you draw the arrow to the buil's-eye.



Standard Three Digits: Assessment Worksheet 1

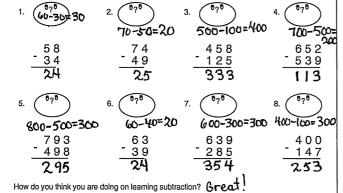
Part 1 Speed and Accuracy

You have 30 seconds to solve as many of these subtraction problems as you can.

You have 30 seconds to solve as many of these subtraction problems as you can

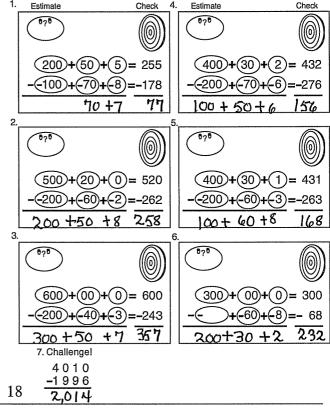
Part 2 Accuracy

First, estimate the answer. Then, solve the problem.



Standard Three Digits with Recording - Worksheet 6

Estimate the answer before you solve each problem. Solve each problem the expanded way and the short way. Draw an arrow to the target. The closer the answer is to your estimate, the closer you draw the arrow to the built's-eye.



Standard Three Digits: Assessment Worksheet 2

Part 3 Please solve these word problems.

- 9. The National Park Service ordered 579 tons of sand to fill in a beach. The sand company delivered 694 tons of sand. _______tons How much more sand was delivered than was ordered? _ 694-579=115
- 10. If you spent \$8.45 on a new toy and still have \$2.69 left, how much money did you start with? \$ 11.14 8.45+2.69=11.14
- 11. When Alysia went to the store she had \$250. When she left the shoe store she had

Figure out what is wrong in each of these problems. Explain the mistake. Then fix it.

did not regroup in the lov's and los places.

14. What went wrong with this IOU problem's solution?

$$\begin{array}{c} 3 & 5 & 2 \\ -1 & 8 & 9 \\ \hline 2 & -3 & -7 & = 190 \end{array}$$

Gus the Bus Driver





- 1. Gus, the bus driver, picks up 17 children at the first stop. Then he picks up 12 more at the second stop. At the next stop 23 children get on the bus. How many children are there now? 52 children Write the number sentence. 17 + 12 + 23 = 52
- 2. At the next stop 16 children get off the bus. How many are left on the bus? 36 children Write a number sentence. 52 - 16 = 36
- 3. At the next stop 18 children get off the bus and 9 get on the bus. How many are left on the bus? 27 children Write the number sentence. 36-18+9=27
- 4. At the next stop 20 children get off the bus and 24 get on. How many are left on the bus? 31 children Write the number sentence. 27-20 +24= 31
- 8_ children 5. At the next stop 23 children are left on the bus after_ off. How many children get off the bus? g children Write the number sentence. 31-23=8
- 6. If Gus, the bus driver, starts with 53 children, and 19 children get off at one stop, 18 get off at the next stop, how many children will get off at the last stop? Ile children Write the number sentence. 53 - 19 - 18 = 16or 53-37=16

Word Problems - Worksheet 1

- 1. Nancy baked 90 cookies for a class party. The class ate 78. How many cookies are left? 12 cookies

 Write the number sentence. Write the number sentence.
- 2. There are 68 rabbits at the petting zoo. 46 got carrots for breakfast. How many rabbits are left to get carrots? 22. rab bits

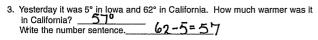
 Wite the number sentence. 68-46 = 22
- 3. George had 48 marbles. He gave 23 away to his friends. How many does he have left to play with? 25 marbles Write the number sentence. 48-23-25
- 4. Ramon had 547 CDs in his collection. He sold 429 on Amazon.com. How many CDs does he have left? \\\ \(\mathbb{CDS} \) Write the number sentence. 547-429=119
- 5. Mary sold 324 boxes of Girl Scout cookies. She delivered 238 boxes. How many more boxes does she have to deliver? <u>Slaboxes</u>
 Write the number sentence. <u>324-238=86</u> Write the number sentence._
- 6. Caitlin bought 139 apples for the school's apple bobbing contest. Only 88 were successfully caught. How many apples were left over? 51 apples Write the number sentence. 139 - 88 = 51
- 7. What phrase in these word problems tell you it is a subtraction problem? How many more?, left?

22

21

Word Problems - Worksheet 2

- 1. Drew has 56 Hot Wheel cars. Eric has 43. Who has more? Drew How many more? 13 cars 56-43= 13 Write the number sentence.
- 2. Catie has 97 stufed animals in her collection. Susan has 125 animals. How many more animals does Susan have? Wite the number sentence. 125-97=28



4. In the fall Neva planted 74 camellia bushes. There are now 108 camellia

How many bushes were there before? $\frac{34}{74}$ bushes Write the number sentence. 109 - 74 = 34



- There were 137 hobby collections at the county fair this year Last year there were 172. How many more were there last year? 35 hobby collections Write the number sentence. 172-137=35
- 6. Tony laid out 650 feet of track for his train. Jeremy's track is 65 feet shorter than Tony's. How long is Jeremy's train track? 585 feet shorter than Tony's. How long is Jeremy's train track? 585 feet shorter than Tony's. How long is Jeremy's train track?
- 7. What phrases in these word problems tell you they are subtraction problems? How many more? Shorter? How much more?

Word Problems - Worksheet 3

Decide whether to add or subtract to find the answer to these problems. Write the operation on the line under each word problem.

1. Ron has ____ jawbreakers and Harry has ____. How many more jawbreakers does Harry have?
Subtraction

2. Brianna has _ outfits for her Barbie doll. She got more for her birthday. How many does she have all together?

3. Sierra, the cat, caught _ Tiger caught ____ mice. How many more mice did Sierra catch?

<u>Subtraction</u>



4. There were rainy days in December. In January there were rainy days. How many more

rainy days in January?



5. One tomato bush had _

6. At the fair there were _ _ caramel apples in the morning. At the end of the day there were ____ left. How

many were sold?



7. Drew scored ___ points in the soccer game today. Yesterday he scored ____ points. How many more did he score today? _subtraction

8. Yesterday the nine chickens laid ____ eggs. Today they laid ____ eggs. How many eggs did they lay

addition

9. Make up a subtraction word prob-

AWV			

10. Make up an addition word problem.

 VWA	 _
	_
	-



Regrouping Practice - Worksheet 1

Build and subtract. Draw pictures and trade if necessary.

Regrouping Practice - Worksheet 2

Build and subtract. Draw pictures and trade if necessary.

	G			0	_	<u>_</u>) <u>_</u>]	8	
1.	3	2	8	0	5.	4	3	2	0	
	 1	5	8	1		 1	8	7	6	
	1	6	9	9		2	4	4	4	

25

Regrouping Practice - Worksheet 3

Build and subtract. Draw pictures and trade if necessary.

Before the storm the pier was 1075 feet long. After the storm it was only 793 feet long. How many feet of the pier were lost in the storm?

10. Sally James died in 1793. She was 95 years old. In what year was she born? $4 \varphi 9$

Regrouping Practice - Worksheet 4

Test your subtraction skills. What operation do you use to check a difference? addition After subtracting, cover up the top number and check your answer. Example:

27

Facts Review - Worksheet 1

7	8	5	9	8	7
- 3	- 8	- 4	<u>- 6</u>	- 7	- 1
- 4	O		3	2	6
8	8	9	8	9	8
- 2	- 0	<u>-4</u>	- 4	<u>- 2</u>	- 6
(e	8	5	4	7	2
7 <u>- 4</u> 3	6 <u>- 1</u> 5	9 - 3 6	9 - 7 2	7 - 6	9 <u>- 1</u> 8
4	3	7	8	5	6
- 0	- 3	- 0	- 3	- 3	- 3
H	O	7	5	2	3
5	6	4	6	3	6
- 2	- 4	- 2	<u>- 5</u>	- 2	- 2
3	2	2	l		4
7 - <u>5</u> 2	7 - 2 5	8 - 5 3	9 <u>- 8</u> 1	9 - 5 H	6 <u>- 0</u>
29					

Facts Review - Worksheet 1

2	8	13	18	10	12
- 2	- 1	-5	<u>- 6</u>	- <u>5</u>	- <u>7</u>
0	7	12	<i>IH</i>	5	5
17	18	15	13	18	17
- 3	- 9	<u>- 4</u>	<u>- 6</u>	- 7	- 12
1件	9		13]	5
18	16	15	13	27	14
- 5	<u>- 12</u>	- 8	- 8	- 26	- 7
13	6	7	6	\	7
14	13	27	10	15	16
- 10	<u>- 13</u>	- 0	<u>- 2</u>	<u>- 7</u>	<u>- 13</u>
4	<i>O</i>	27	8	12	3
14	20	14	16	22	11
- 8	<u>- 10</u>	- 6	<u>- 5</u>	- 11	- 8
6	10	- 8	I \	1(3
12	16	20	11	19	17
- 6	- 8	- 9	- 7	- 5	<u>- 14</u>
¢	8	11	4	4	3

Pre-Assessment - Part 1

- 1. a. There are 68 rabbits at the petting zoo. 46 have already gotten carrots for breakfast. How many rabbits have not gotten their carrots yet? 22.rabbits

 Write the number sentence. 68-46=22
 - b. Chuck laid out 653 feet of track for his train. Dave's track is 137 feet shorter than Chuck's. How long is Dave's train track? 516 feet
 Write the number sentence. 653-137=516

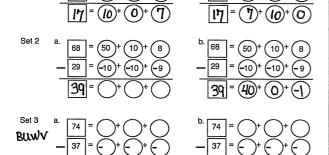
3

53

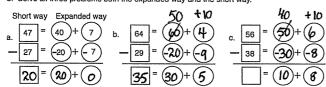
(30)

2. Solve each problem a different way Set 1

53



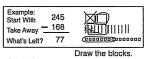
3. Solve all three problems both the expanded way and the short way.



Post-Assessment is the same as Pre-Assessment - Part 2 i

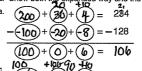
Pre-Assessment - Part 2

- Draw the Start With number for each problem. Cross out the blocks being traded.
 On your drawing, Circle the Take Away number. Record the What's Left? number.
- Draw the blocks a. Start With 262 Take Away - 95 What's Left? 167 (00000000000000

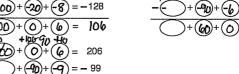


30

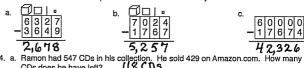
- Start With 352 Take Away - 186
- 2. Show both the expanded way and the short way of



(00)+(0)+(7)=107



3. Build and subtract. Draw pictures and trade if necessary.



- c. Samuel laid out 550 feet of track for his slot cars. Ava's track is 75 feet shorter than Samuel's. How long is Ava's track?

 Write the number sentence. 550- 15 11/5
- What phrases in these word problems tell you they are subtraction problems? Shorter than how many before? how many how many left?

Patterns in Arithmetic: Subtraction - Booklet 4

Parent/Teacher Guide

Large Numbers and Practice

ISBN 978-1-935559-98-6