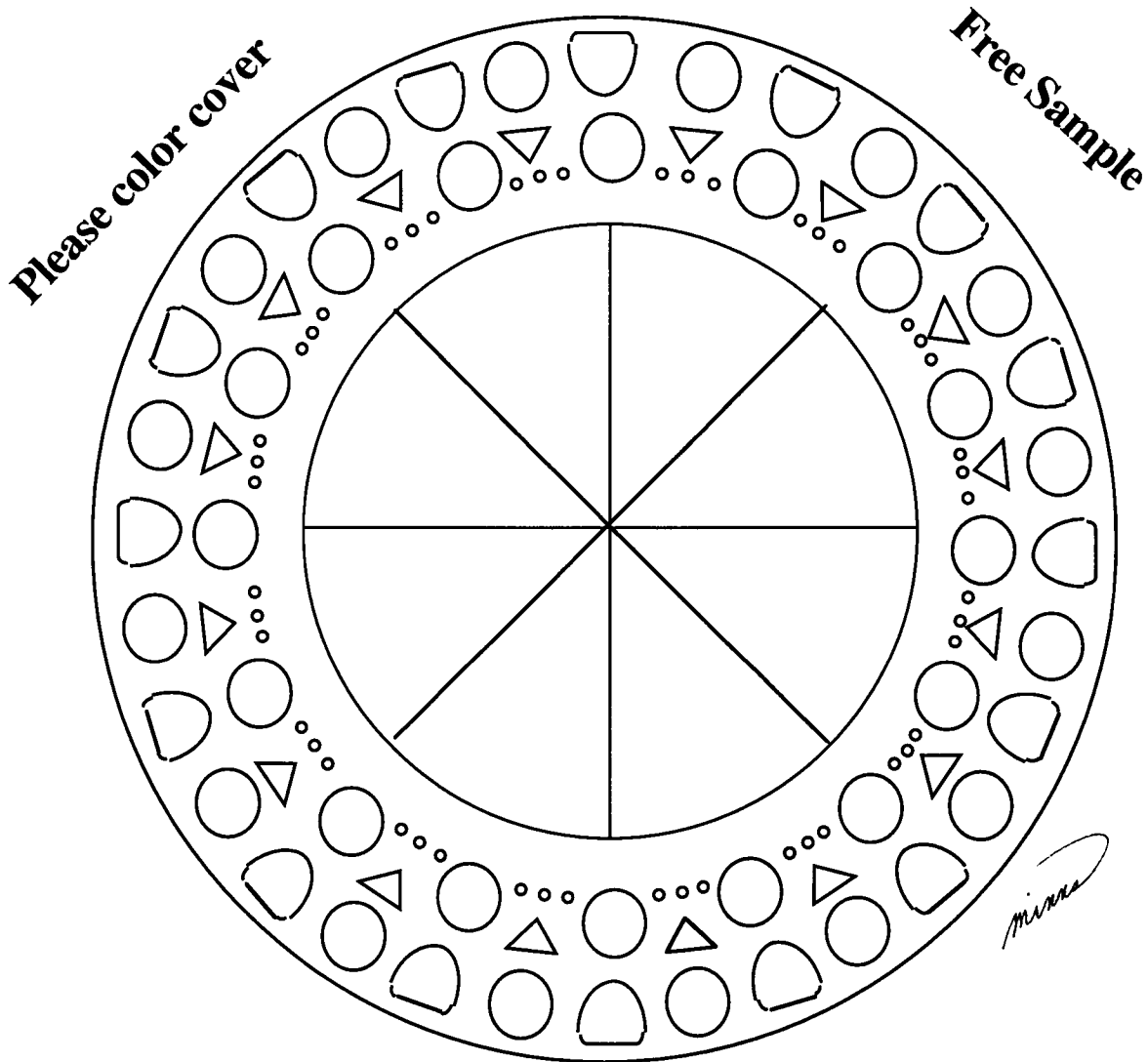# Fractions: Booklet 1- Basic Concepts
Sample Lessons
Teacher Guide and Student Workbook
Third Grade

Patterns in Arithmetic

By Alysia Krafel, Suki Glenn, and Susan Carpenter

Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Fractions: Booklet 1 - Basic Concepts
Contents

Materials needed:

Free Exploration
Prism Fractions® Paper Set - Circles (free downloads) Two choices:
1. Black Line Master that students color and then cut out.
2. Colored set that just needs to be printed on heavy paper and then cut out.
Pattern blocks
Optional - Prism Fractions® Circle Plastic Set (purchase from www.patternpress.com)
Optional - Prism Fractions® Paper Set - 10 pages of squares and 10 pages of circles printed on heavy colored paper (download free sample or puchaase from www.patternpress.com)

My Fractions Book: Beginning
My Fractions Book: Beginning, page 3 - instructions are also in the Student Workbook
Eight sheets of 11 x 17 inch paper
Prism Fractions® Paper Set - Circles or Prism Fractions® Plastic Circle Set

Fractions Guessing Game
Prism Fractions® Paper Set - Circles or Prism Fractions® Plastic Circle Set
Fractions Guessing Game - Student Workbook page 7

Numerators Greater Than One
Numerators Greater Than One - Worksheets 1 - 3, pages 8 - 10
Pattern blocks

Shrinking Circles
Shrinking Circles - Worksheets 1 and 2, pages 11 and 12

Prism Fractions Paper Set - Circles Instructions
Print on heavy paper–24 pound works well.
Carefully cut the fraction circles along the outside lines. In just one circle of each fractional unit from halves to eighths cut the circles into individual units along the inside lines. The fraction pieces from tenths to sixteenths are very small and harder to use if cut, so leave these uncut.

This is a free sample of a larger product from Patterns in Arithmetic: Fractions - Booklet 1 - Basic Concepts.

Free Exploration, or How Not To Be a Nag

by Alysia Krafel,

I recall an incident years ago when my six-year-old nephew was learning to set the table. Dinner was almost ready and there he was popping his fists down onto fork tines to flip them into the air. The stage was set for an unpleasant confrontation. I see now that the problem is one of time frames. We needed him to get the job done; he needed to explore. When we press a child to use a tool seriously and with skill before we have provided fiddle time, we set him up to be seen as a behavior problem and ourselves to be seen as nags. If a child can do his necessary playing and exploring with materials when he is not under pressure or expectations, before he needs to use them as we wish him to use them, not only will he learn the new skill faster and with more understanding, he will learn it with more delight (or at least with less complaining). The first association will be of joy and competence rather than frustration and forced anxiety.

Learning to use math manipulatives is like learning to use other tools. An adult will look at a screwdriver and use it to drive screws. A child will roll it on the floor, listening intently as the grooves in the plastic handle thump. She will roll it over her tongue, poke it into cracks, bang it on the table, spin it, or balance it on her finger. When finally satisfied with the investigation, she will watch you screw in the screws and then try it herself. And so it is with all of our doodads.

Play, unhindered exploration, is very important to children. At the Farm School, we came to realize that this self-directed learning style was not an impediment but a boon to our aims. We found that children who had enough time to fiddle with the plexiglass tiles came to their first fractions lessons knowing that the smaller the pieces there were, the more of them they had; that you could cover the black ($^1/2$) tile[1] with the yellow ($^1/4$) tile or orange ($^1/6$) pieces, but you couldn't with the clear ones ($^1/3$). When they began to work with the formal fractions lessons, they brought into action all that their senses already knew about these tiles. As soon as the labels "halves" and "fourths" were attached to the colored tiles, they knew that $^1/2 = ^2/4$. They found the patterns very quickly and mastered the materials faster than the previous children who had not been allowed fiddle time.

The problem the teachers at the Farm School had encountered was that we wanted to teach x amount of material in a certain time frame. When the children needed three weeks of fiddle time with the fraction tiles, we experienced that as a delay, a delay that made us nervous. We just had to teach and have the children master fractions by the end of the school term. The solution was to introduce the materials long, long before (in many cases years before) we intended to use them as teaching tools. After ample free exploration time, not only did the children not resist the use of the materials in the specific way that the formal lessons demanded, they eagerly attended to the new way to interact with the now familiar materials. The new work was for them a continuation of the old play. The experience did not feel all that different to them. Since the struggle between teacher and child had been eased, the adversarial relationship became a co-worker relationship. The instruction then proceeded easily. The children and the teachers learned together to truly understand the world of numbers and enjoyed doing it.

My life suddenly changed when I had a baby. When giving her a bath one day, I had the idea to give her as a toy one thing she would later use as a tool, a cup. In the tub, she could pour, drink (or sputter), spill, bang, and splash to her heart's content without undoing whatever housework managed to get done. She would talk into the cup and put it on her head. Doing these things at the table with a cup filled with milk would have caused an uproar just as fiddling with colored tiles during a fractions lesson did. She had had the cup as a toy many months before we ever put milk into it for

[1] As used in *Fraction Tiles* by Lee Jenkins and Peggy McLean

her to drink at the table. Interestingly enough, when given milk in the cup at the table, she reacted in the same way as the children in the fractions lesson did. Since pouring liquid from the cup was old hat and drinking milk from it like Mommy and Daddy was new, she tried very hard to do it right because that's what she was ready to do, that was the new fun. She wouldn't pour milk out at the table and say "oh no" when she spilled it. Later, when she showed an interest, spoons and bowls were added to the toy shelf and to the bath set. While in the tub recently, she filled her bowl with water, bent down, and began to blow bubbles into it. She dumped out the water and put the bowl on her head and said, "Hat." I thought to myself, "I'm glad that was warm water on her naked body instead of hot tomato soup on her white sweater!"
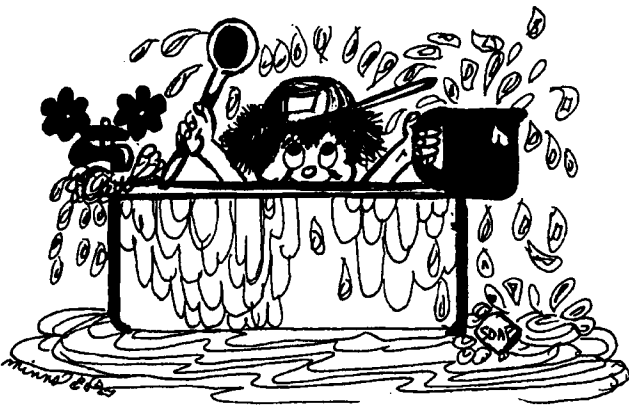
An unintended side effect of all of this was that she became competent with a cup at fourteen months and with a spoon and bowl by sixteen.

Having done all of that, I was surprised recently to find myself remaking that same mistake of not allowing exploration as a parent as I had as a beginning teacher. When my daughter put eggs and Cheerios in her cup of milk, I said, "No put food in cup! No put eggs in milk!" I was expecting her to be like me, to use tools as tools, food as food, instead of tools and food as toys. Resisting my constant scolding, she persisted in this behavior. Obviously, she needed to explore how things, like scrambled eggs and Cherrios, would interact with milk in her cup. My choice

was either to press her to "act properly at the table," triggering tears and tantrums, or to allow her to explore with the knowledge (or at least great hope) that when she was satisfied, she would stop. This is always a difficult decision for me to make. One doesn't want to be so permissive as to develop a totally undisciplined child. On the other hand, one doesn't want to be a repressive nag either. Since food couldn't be a toy anywhere else but at the table, I gave in on unconventional mixing, squashing food through fingers, and food painting but drew the line on throwing. So with a certain amount of plaintive sighing, I watched her put my lovingly prepared, warm eggs into her milk, got out the dish towels, and added them to her toy box. To my delight, she ate the cold, milky eggs out of her cup with the spoon without as much mess as one would expect! My husband realized that the current fascination was food and liquid together. Now we put the liquid in the bowl with the food to begin with, and everyone is happy. She certainly is not like me.

I wonder how many things children must learn, like setting the table, feeding the cats, making the beds and so on, would move more easily if we parents allowed the child to play with the new materials long before we or society demanded that they use them properly and with skill.

Student Worksheets

Free Exploration - Worksheets 1 and 2, pages 1 and 2

My Fractions Book: Beginning and What Fraction Is This?

Purpose The purpose of this lesson is to concretely discover, understand, and represent basic fractions using a manipulative. The process will first be with Fraction Circles, page 9, then with pattern blocks, and, finally, with number lines.

Prerequisites Free Exploration with the Prism Fraction Circle Set and pattern blocks.

Materials My Fractions Book: Beginning, page 3 - instructions are also in the Student Workbook
Eight sheets of 11 x 17 inch paper
Prism Fractions® Circle Set
Pencil
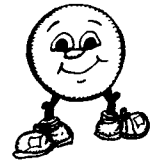
Directions Each student makes a book and learns the names of most common fractions, draws fractions, and writes fractions with numbers and words. Have her work on the book about twenty minutes per day. She needs to be neat and use a sharp pencil! This book will be used for at least a year.

Stack eight sheets of 11 x 17 inch paper.
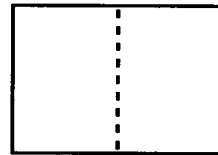
Fold in half to find the middle and then staple in the fold.

She writes her name and My Fractions Book on the cover. Decorate.
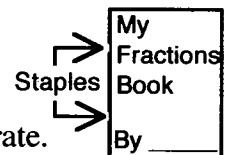
Page 1 Begin with fraction circles. Trace the whole piece. Label one whole and record the fraction in words and then in numerals.

Leave about one-fourth of the page blank on all of the pages.
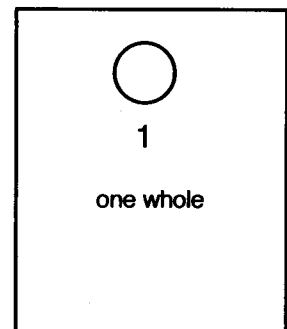
Page 2 Trace the whole piece. Place two halves on the whole and draw on the whole what this looks like.
Record one whole = how many halves in words and then in numerals.

Trace around one half piece and record the fraction in words and in numerals. Color the fraction drawings the same color as the fraction circle pieces.

Page 3 Trace the whole piece. Place the thirds on the whole and draw on the whole what this looks like.

Record one whole = how many thirds in words and then in numerals.

Trace around one-third piece and record the fraction in words and in numerals.

$$1 = \frac{3}{3}$$

one whole = three thirds

$$\frac{1}{3}$$
one third

$$\frac{2}{3}$$
two thirds

Trace around two-thirds and record the fraction in words and in numerals. Color the fraction drawings the same color as the fraction circle pieces. You may need more than one page.

Repeat with fourths, fifths, sixths, eighths, tenths, and twelfths. Draw and label all of the fractions. For example, one-fourth, two-fourths, three-fourths.

What Fraction Is This?

Give the worksheets and these directions: What fraction of the circle is shaded? Guess first. Then use the fraction pieces to measure.

Worksheets   What Fraction Is This? - Worksheets 1 - 3, pages 4 - 6
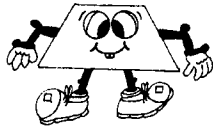
Numerators Greater Than One

$$\frac{2}{3} \quad \begin{array}{l}\text{numerator} \\ \text{denominator}\end{array}$$

Purpose

The purpose of this lesson is to introduce fractions with numerators (the top number) greater than one with a manipulative using the language 'two out of.'

Prerequisites

My Fractions Book: Fraction Circles, What Fraction Is This? - Worksheets 1 - 3, pages 4 - 6, and Fraction Guessing Game, page 7

Materials

Numerators Greater Than One - Worksheet 1, page 8
Pattern blocks

Lesson

Give each student Numerators Greater Than One - Worksheet 1 and a set of pattern blocks. **"Cover the first hexagon with three equal pattern blocks. Color 'one out of three.'"** Notice that 'one out of three' is written in mathematical notation as $\underline{1}$.
3

"Record on the lines provided." $\begin{array}{c}1 \\ \text{out of} = \frac{1}{3} \\ 3\end{array}$

"Color 'two out of three' blocks in the next picture."
"How do you think this would be recorded?"
"<u>Two</u> out of <u>three</u>." $\begin{array}{c}2 \\ \text{out of} = \frac{2}{3} \\ 3\end{array}$
"Why?"
"Because two of the three blocks are colored."

"Record it as a fraction." $\frac{2}{3}$
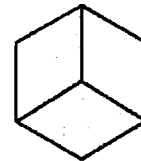
"In the fraction two-thirds, what does the denominator, the bottom number, mean?" "There are three parts in the whole."

"What does the numerator, the top number, mean?" "Two. You have two parts of that whole."
Complete the worksheet independently, then give Numerators Greater Than One - Worksheet 2.

Practice Worksheets

Numerators Greater Than One - Worksheets 2 and 3, pages 9 and 10
Shrinking Circles - Worksheets 1 and 2, pages 11 and 12

Here is a dialogue with a student who had studied fractions for several years but was still unclear about what the numbers in a fraction mean. We were working with pattern blocks.

Teacher:	**Student:**
"So the blue is...?"	"One third. Is that the way you write it?" $\frac{1}{3}$
"What does the one mean and what does the three mean?"	"The three means there are three blocks."
"OK. And what does the one mean?"	"A whole." (*She is referring to the one in the numerator.*)
"Three covers the whole. If you have one third, what does the one mean?"	"One third."
"One out of how many? If you have one third, it means you have...?"	Silence
"If you have one of these (one blue block), is this a whole?"	"No."
"How much of a whole is it?"	"Ah, it's a third."
"Your whole is how many?"	"A whole is one." (*This is true but she is referring to the 1 in 1/3.*)
"If you're using three, how many does it take to make the whole?"	"Three."
"If you have two blues, how would you write that?"	"Two-thirds." $\frac{2}{3}$
"And what does the 'two' mean?"	"Two means that there's two... Two in the whole. The whole is covered up by two-thirds."
"And the three means?" (*She is still unclear.*)	No reply.
"If a cookie is divided into three pieces and I give you one piece, how much of the whole do you have if you have just one piece?"	"A third of it."
"You have a third of it. One of three, three being the total. If you have two of them?"	"You have two of the thirds."
"A blue block is...?"	"One-third."
"Does the one mean how many are in the whole?"	"Uh, no."
"What number means how many in the whole?"	"The three. One means a third of a whole."
"What does the one mean?"	"You have one of the whole."
"And the three means?"	"That there are three in the whole."
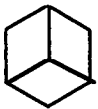
Answer Key

What Fraction Is This? - Worksheet 1

What fraction of the circle is shaded? Guess first. Then use your fraction pieces to measure.
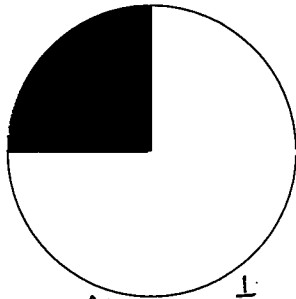
Guess _AWV_ Actual __1/4__   Guess _____ Actual __1/3__
Answer will vary
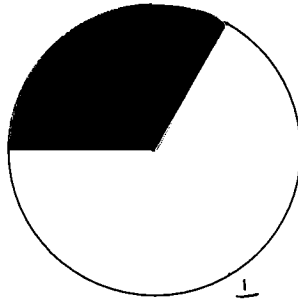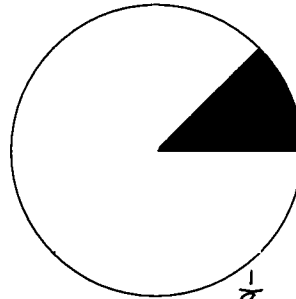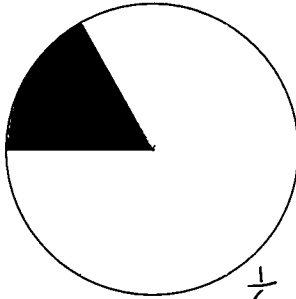
Guess _____ Actual __1/8__   Guess _____ Actual __1/6__

3

What Fraction Is This? - Worksheet 2

What fraction of the circle is shaded? Guess first. Then use your fraction pieces to measure.
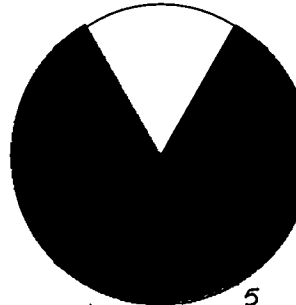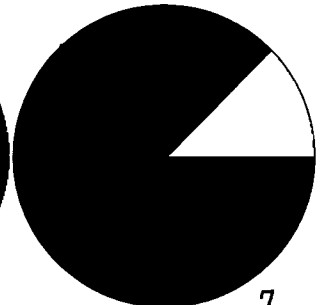
Guess _AWV_ Actual __5/6__   Guess _____ Actual __7/8__
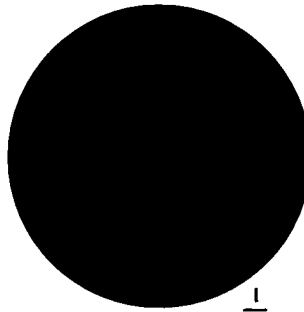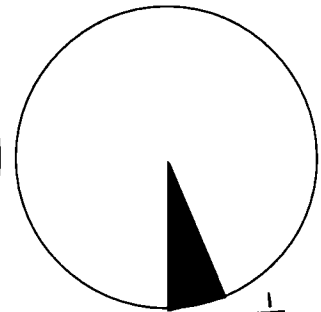
Guess _____ Actual __1__   Guess _____ Actual __1/16__

4

What Fraction Is This? - Worksheet 3

What fraction of the circle is shaded? Challenge! These will take more than one piece. There is more than one correct answer
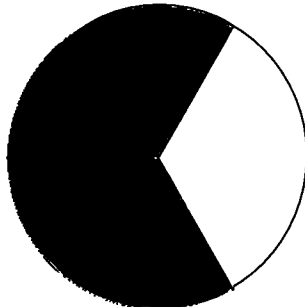
There are 3 correct answers.
__2/3__ or __4/6__ or __8/12__
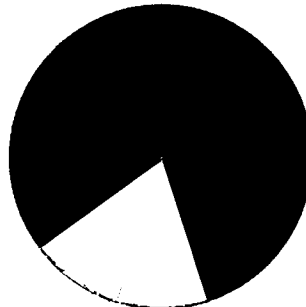
There are 2 correct answers.
__4/5__ or __8/10__

Challenge!
What fraction of the circle
shaded? __0__
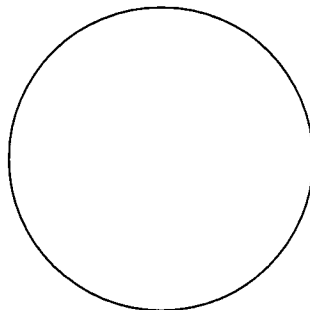
5

Numerators Greater Than One - Worksheet 1

out of $\frac{1}{3}$ = $\frac{1}{3}$

Cover with three equal pattern blocks.
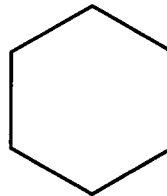
Each block is one of three equal parts $\frac{1}{3}$ of the shape.

Color one block.
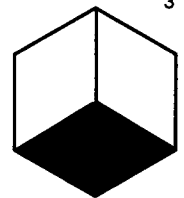
How many parts are colored?
__1__ out of __3__

out of $\frac{1}{3}$ = $\frac{1}{3}$

Color two blocks.
How many parts are colored?
__2__ out of __3__
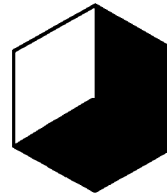
out of $\frac{2}{3}$ = $\frac{2}{3}$

Cover with three pattern blocks.
Color $\frac{1}{3}$ one color and $\frac{2}{3}$ another color.
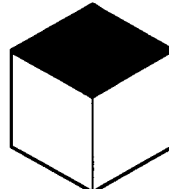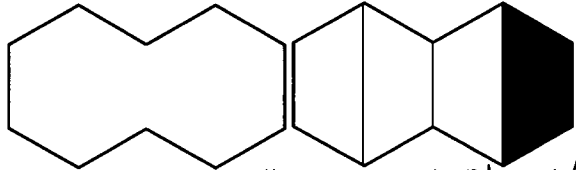
8

Numerators Greater Than One - Worksheet 2

1 out of = $\frac{1}{4}$
4
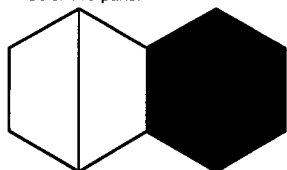
Cover with four equal pattern blocks.

Each block is one of four equal parts $\frac{1}{4}$ of the shape Color one
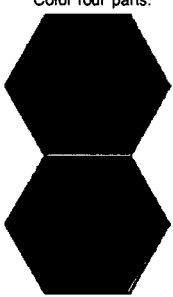
How many parts are colored? _1_ out of _4_

1 out of = $\frac{1}{4}$
4

Color two parts.

$\frac{2}{4}$ out of = $\frac{2}{4}$
4
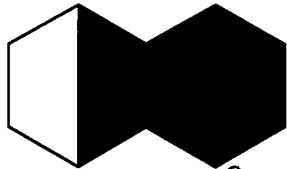
How many parts are colored? _2_ out of _4_

Color four parts.

How many parts are colored?
_4_ out of _4_
out of = $\frac{4}{4}$
4

Color three parts.

$\frac{3}{4}$ out of = $\frac{3}{4}$
4

How many parts are colored? _3_ out of _4_

9

Numerators Greater Than One - Worksheet 3

Wholey Cow

One fraction is mixed up.
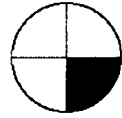
"My name is $\frac{1}{2}$."

"My name is $\frac{2}{3}$."

"My name is $\frac{1}{4}$."

"My name is $\frac{6}{6}$."

"My name is $\frac{4}{8}$."

1. Which fraction above is up?

Will the real one please stand up? _____

2. Draw $\frac{3}{4}$.

3. Draw $\frac{3}{6}$.

4. Draw $\frac{4}{4}$.

10
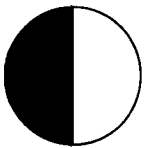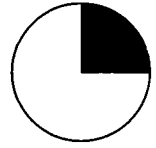
Shrinking Circles - Worksheet 1
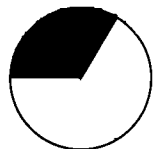
Shade in $\frac{1}{2}$
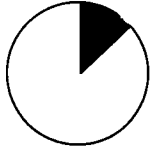
Shade in $\frac{1}{4}$
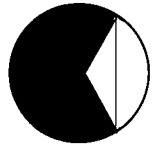
Shade in $\frac{1}{3}$

Shade in $\frac{1}{8}$
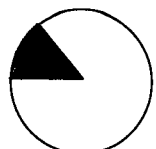
Shade in $\frac{2}{3}$
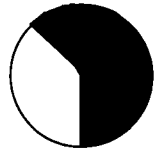
Shade in $\frac{3}{4}$

Challenge!

Shade in $\frac{1}{6}$
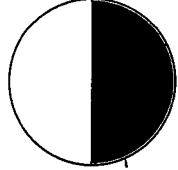
Shade in $\frac{5}{8}$
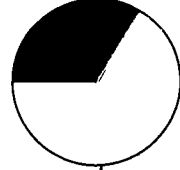
11

Shrinking Circles - Worksheet 2
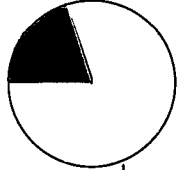
$\frac{1}{2}$

$\frac{1}{3}$

$\frac{1}{5}$

$\frac{3}{4}$

$\frac{2}{3}$

$\frac{5}{6}$

$\frac{1}{10}$

$\frac{5}{8}$

$\frac{1}{10}$

12
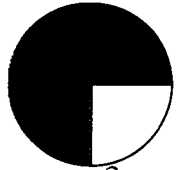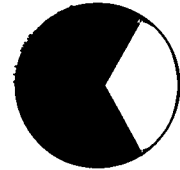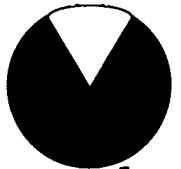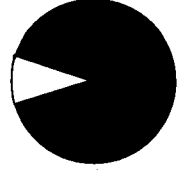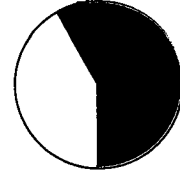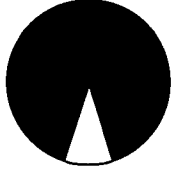
Answer Key: Fractions - Booklet 1

Fractions: Booklet 1- Basic Concepts
Student Workbook
Sample Worksheets

Please color cover
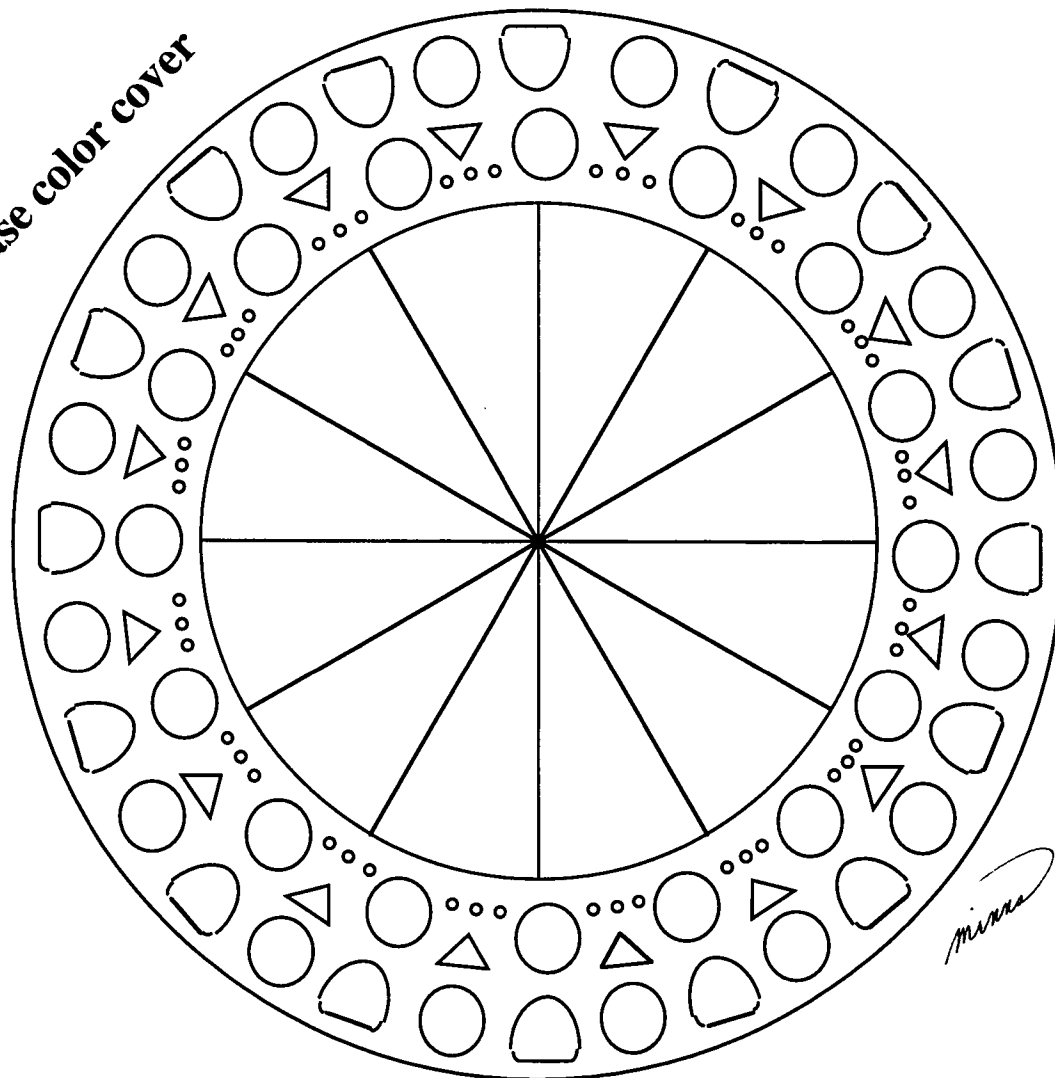
Patterns in Arithmetic

By Alysia Krafel, Suki Glenn, and Susan Carpenter

Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Free Exploration

Freely explore and build with these fraction materials:

Pattern Blocks
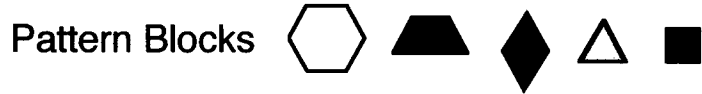
Prism Fractions® Circle Paper or plastic set (no sixteenths)
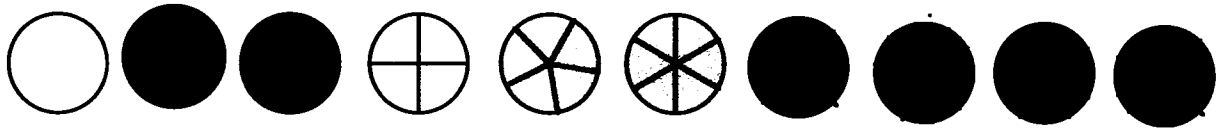
Make many designs using all the materials. Draw your favorite design here.

Use fraction circles. On each circle, place different pieces to make a complete circle. Color to match the pieces.
Example:
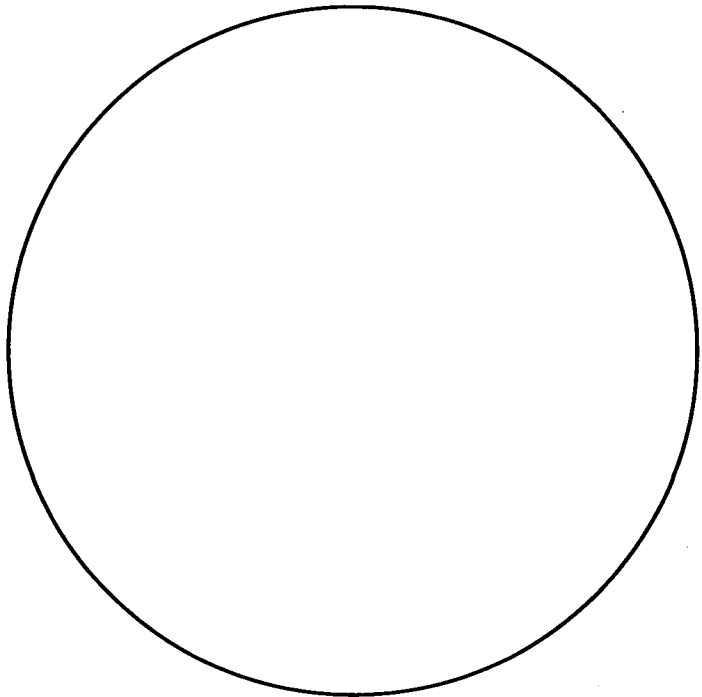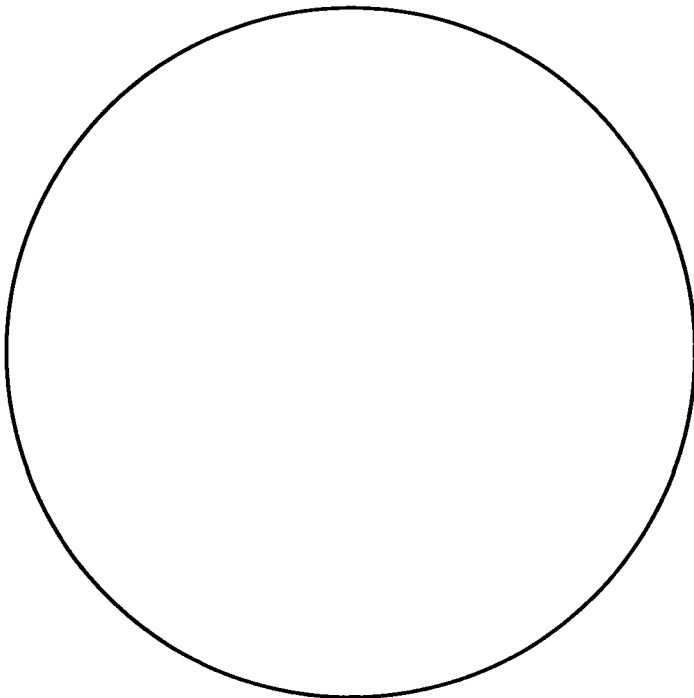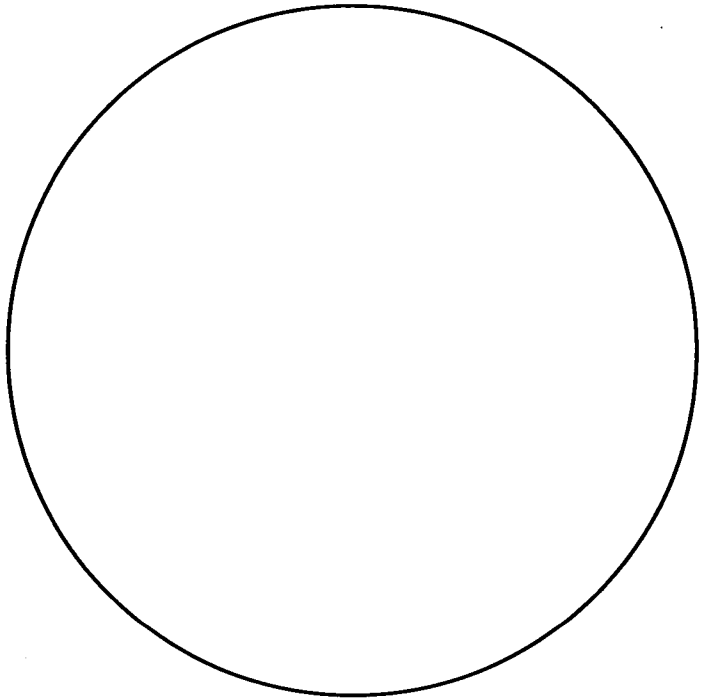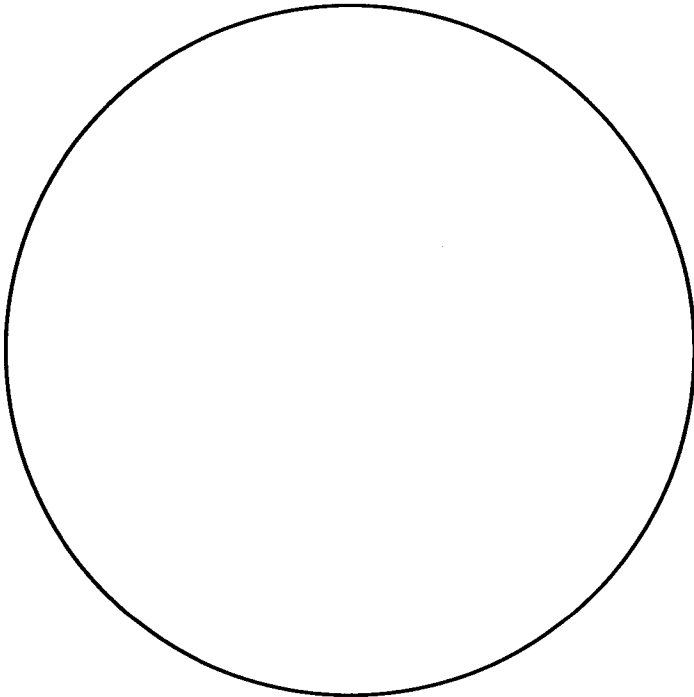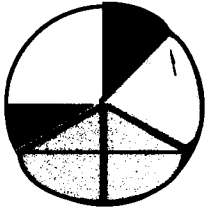
My Fractions Book: Beginning

Make a book. You will learn the names of most common fractions, draw fractions, and write fractions with numbers and words. Work about 20 minutes per day on this project. Be neat and use a sharp pencil! This book will be used for at least a year. Have fun!

Stack eight sheets of 11 x 17 inch paper.

Fold in half to find the middle and then staple in the fold.

Staples

My Fractions Book

By_____

Write your name and My Fractions Book on the cover. Decorate.

Page 1 Begin with Fraction Circles. Trace the whole piece. Label one whole and record the fraction in words and then in numerals.

Leave about 1/4 of the page blank on all of the pages.

1

one whole

Page 2 Trace the whole piece. Place two halves on the whole and draw on the whole what this looks like.
Record one whole = how many halves in words and then in numerals.
Trace around one-half piece and record the fraction in words and in numerals. Color the fraction drawings the same color as the fraction circle pieces.

$1 = \dfrac{2}{2}$

one whole = two halves

$\dfrac{1}{2}$ one half

Page 3 Trace the whole piece. Place the thirds on the whole and draw on the whole what this looks like.
Record one whole = how many thirds in words and then in numerals.
Trace around one-third piece and record the fraction in words and in numerals.
Trace around two-thirds and record the fraction in words and in numerals. Color the fraction drawings the same color as the fraction circle pieces. You may need more than one page.

$1 = \dfrac{3}{3}$

one whole = three thirds

$\dfrac{1}{3}$ one third

$\dfrac{2}{3}$ two thirds

Repeat with fourths, fifths, sixths, eighths, tenths, and twelfths. Draw and label all of the fractions. For example, one-fourth, two-fourths, three-fourths.

What Fraction Is This?   Date_____

Worksheet 1

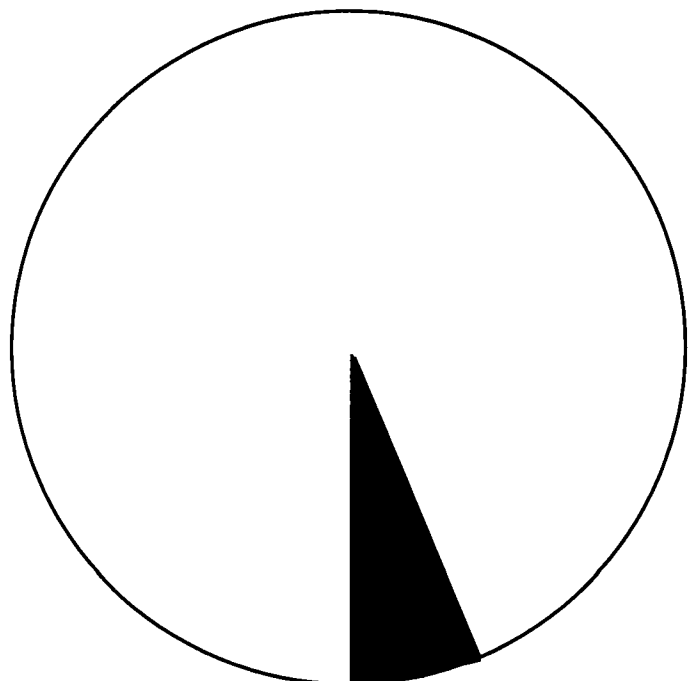What fraction of the circle is shaded? Guess first. Then use your fraction
pieces to measure.
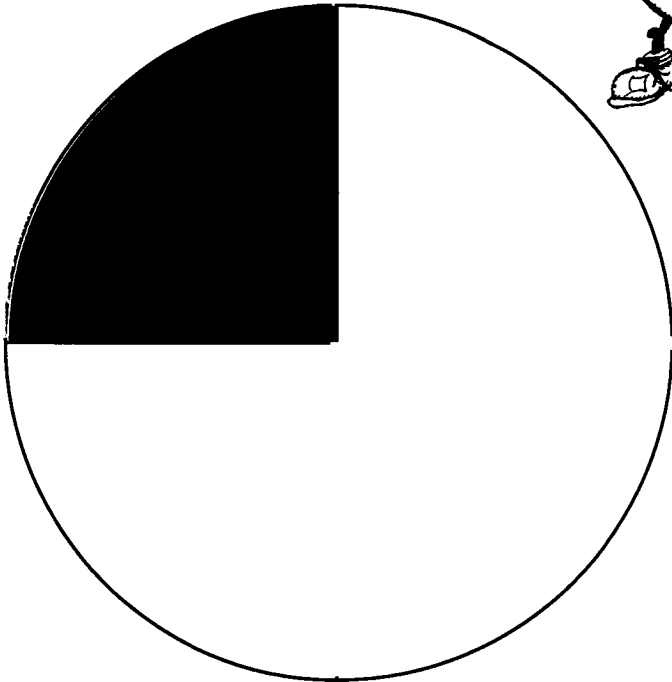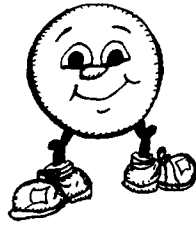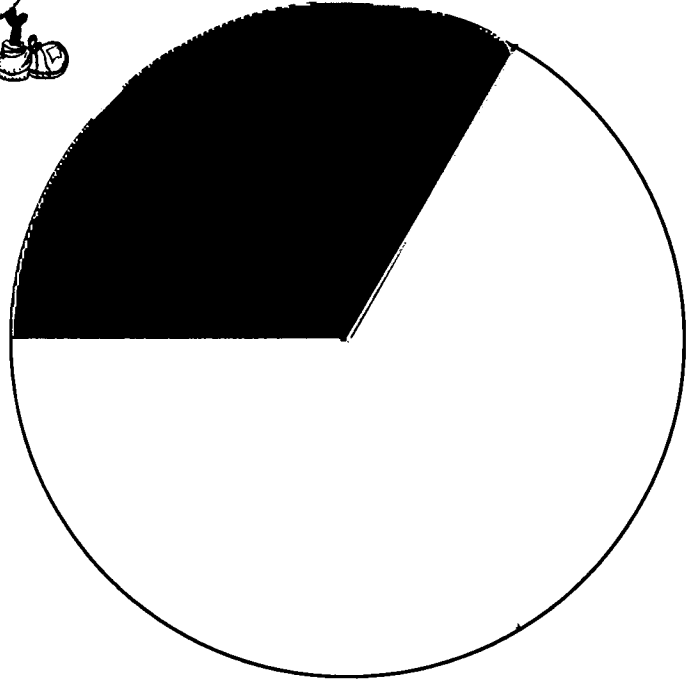
Guess _____   Actual_____   Guess _____   Actual_____
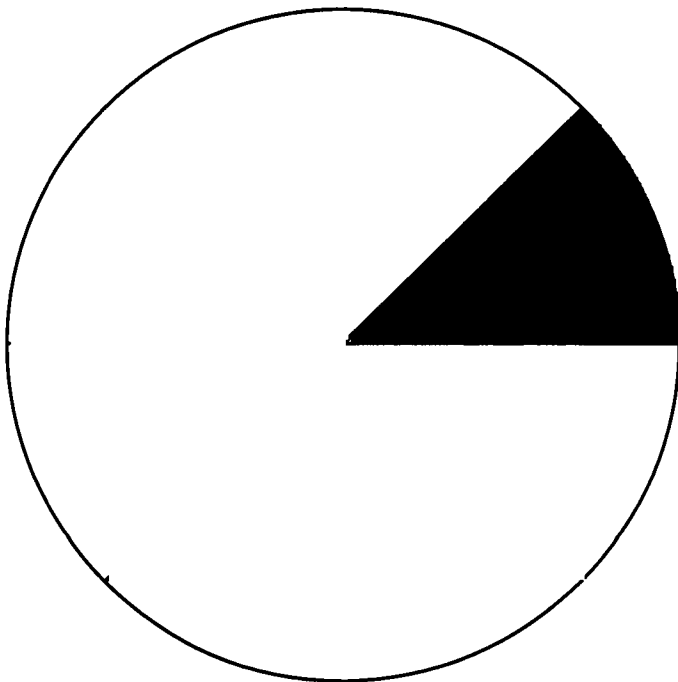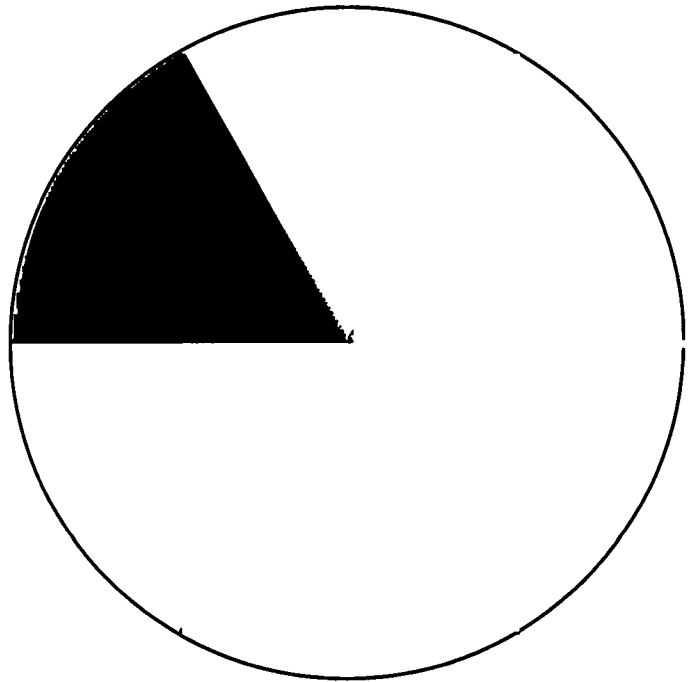
Guess _____   Actual_____   Guess _____   Actual_____

What Fraction Is This?
Worksheet 2

Date_____

What fraction of the circle is shaded? Guess first. Then use your fraction pieces to measure.

Guess _____   Actual_____   Guess _____   Actual_____

Guess _____   Actual_____   Guess _____   Actual_____

Worksheet 3

What fraction of the circle is shaded? Challenge! These will take more than one piece. There is more than one correct answer.
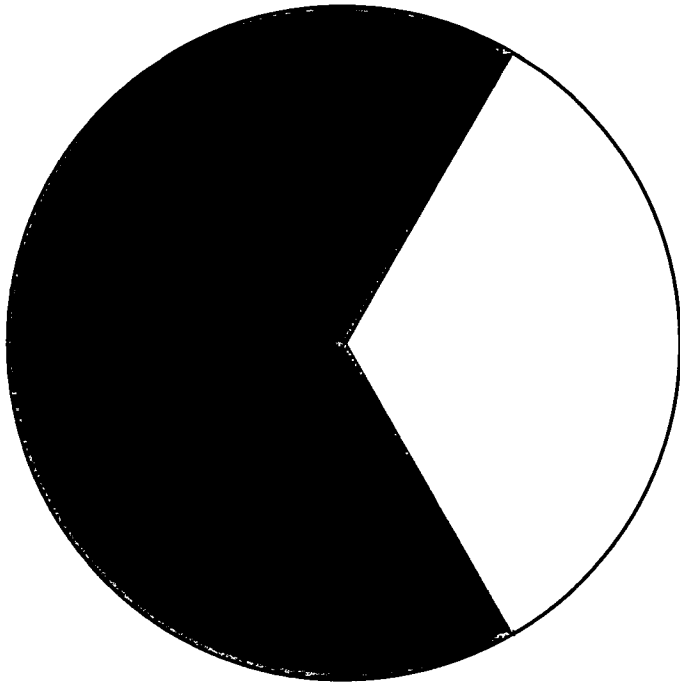
There are 3 correct answers.

____ or ____ or ____
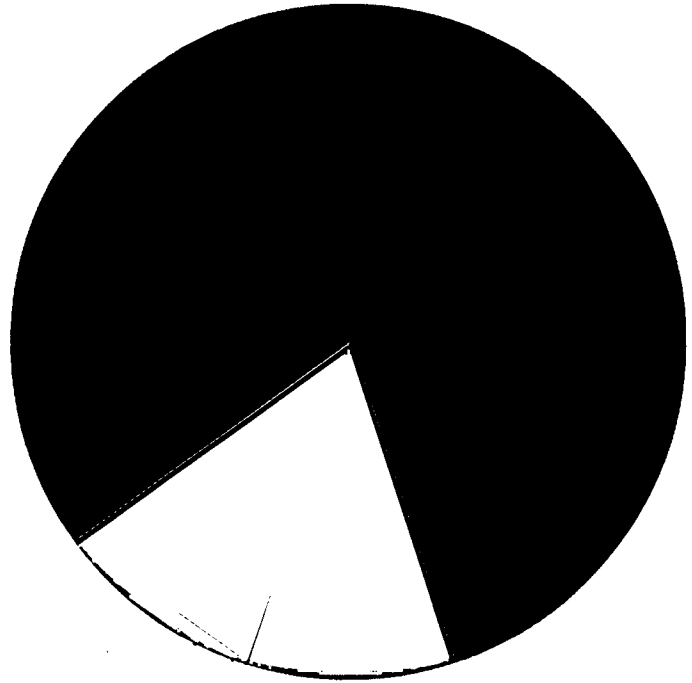
There are 2 correct answers.

____ or ____

Challenge!
What fraction of the circle is
shaded? _____

Fraction Guessing Game

Use fraction circles.

one
whole
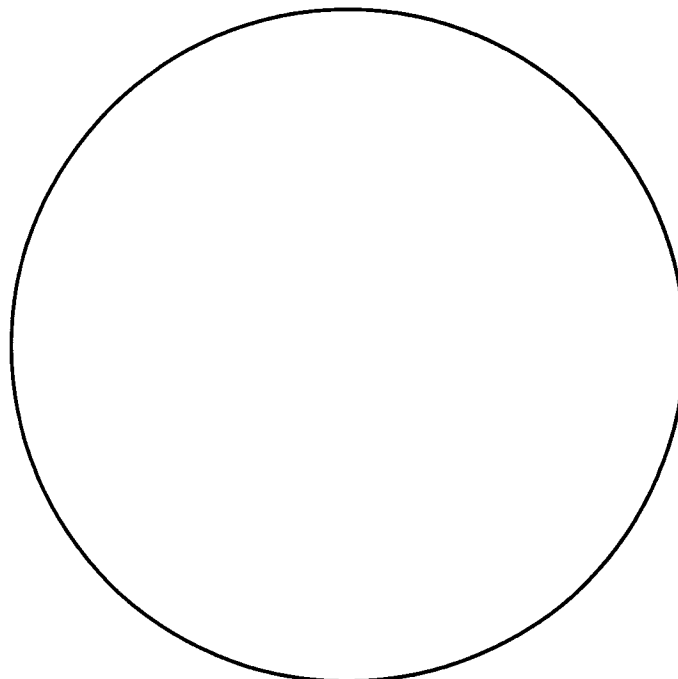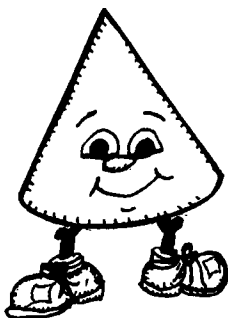
Date_____
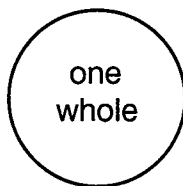
Player 1 builds and then draws a picture of a fraction with a numerator greater than one. Player 2 guesses the name of the fraction. Verify answers with fraction pieces. Players take turns drawing and guessing.

Example:
Player 1 draws: Player 2 guesses 3/4.
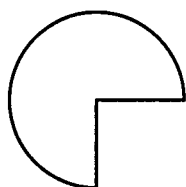
Player 2 draws: Player 1 guesses 5/6.
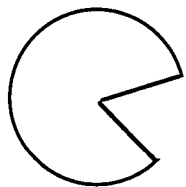
Variation: Play again with Prism Fractions Squares.

one
whole

Example:
Player 1 draws: Player 2 guesses 2/3.
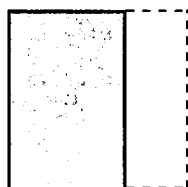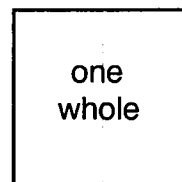
Numerators Greater Than One
Worksheet 1

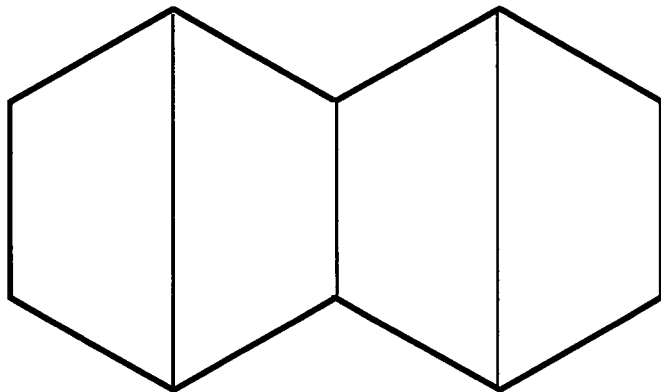$$\text{out of} \frac{1}{3} = \frac{1}{3}$$

Cover with three equal
pattern blocks.
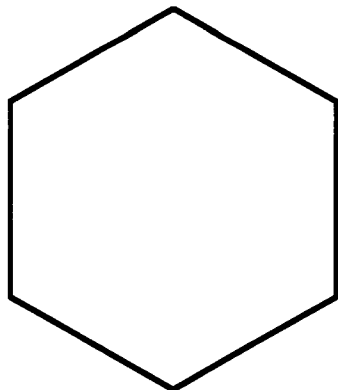
Each block is one of three equal parts or $\frac{1}{3}$
of the shape.

Color one block.
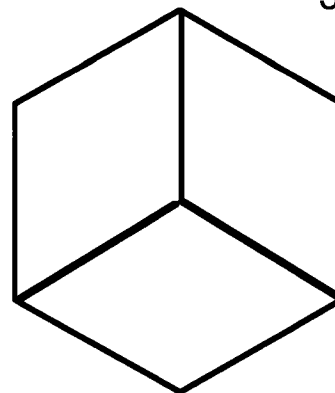
How many parts
are colored?
_____ out of _____

$$\text{out of} \frac{1}{3} = \frac{\quad}{\quad}$$

Color two blocks.

How many parts
are colored?
_____ out of _____

$$\text{out of} \frac{2}{3} = \frac{\quad}{\quad}$$
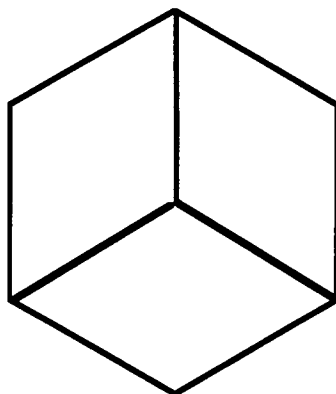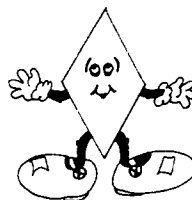
Cover with three pattern blocks.

Color $\frac{1}{3}$ one color and $\frac{2}{3}$ another color.
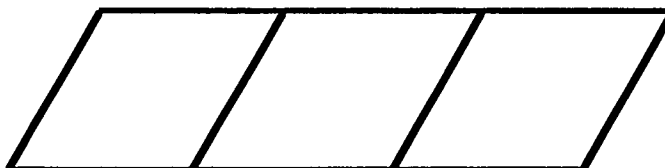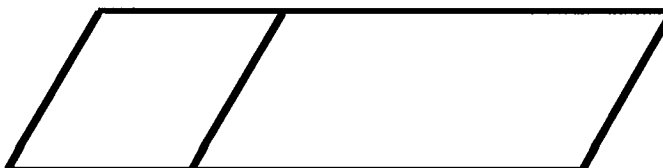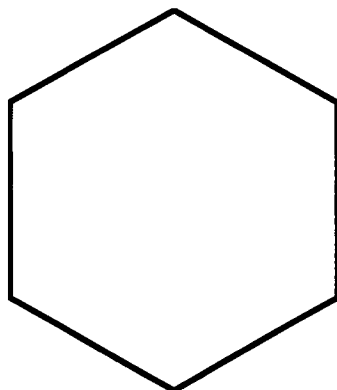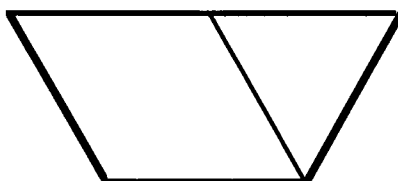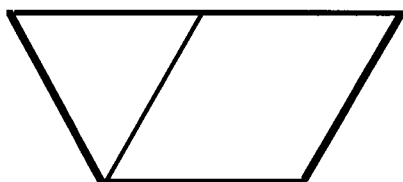
Numerators Greater Than One
Worksheet 2

$\dfrac{1}{4}$ out of $=\dfrac{1}{4}$

Date_____
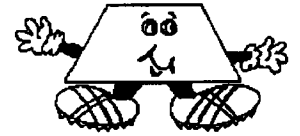
Cover with four equal
pattern blocks.

Each block is one of four equal parts or $\dfrac{1}{4}$
of the shape. Color one part.
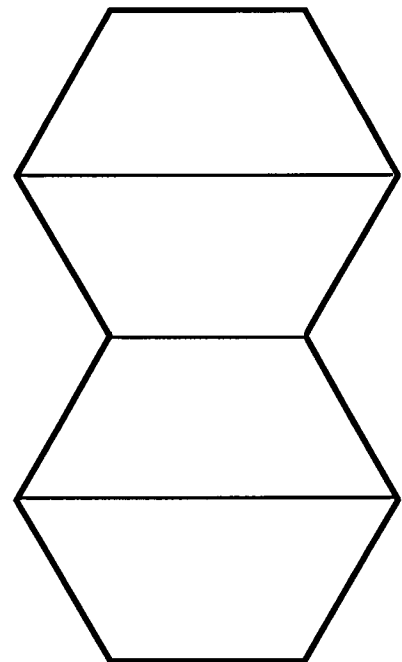
How many parts are colored? ___ out of ___

$\overline{}$ out of $=\dfrac{\overline{}}{4}$

Color two parts.

out of $=\dfrac{\overline{}}{4}$

How many parts are colored? ___ out of ___

Color four parts.

Color three parts.

out of $=\dfrac{\overline{}}{4}$

How many parts are colored? ___ out of ___

How many parts
are colored?
____ out of _____

out of $=\dfrac{\overline{}}{4}$

Date_____

Wholey Cow

One fraction is mixed up.

"My name is $\frac{1}{2}$."

Will the real one please stand up? _____

"My name is $\frac{2}{3}$."

"My name is $\frac{1}{4}$."
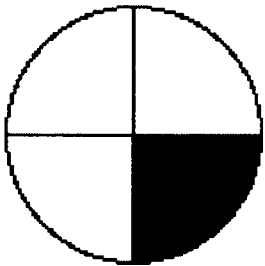
2. Draw $\frac{3}{4}$.

"My name is $\frac{6}{6}$."

3. Draw $\frac{3}{6}$.

4. Draw $\frac{4}{4}$.

"My name is $\frac{4}{8}$."

1. Which fraction above is mixed up?

_____

Date_____

Oh no!

Yup... Honey, I shrunk the circles!
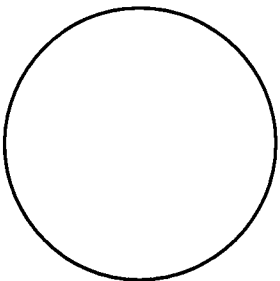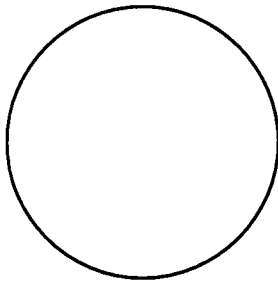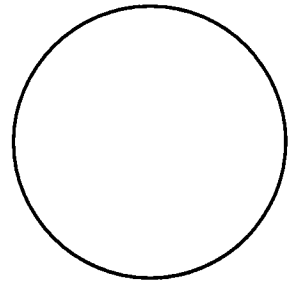
Shade in $\frac{1}{2}$.

Shade in $\frac{1}{4}$.

Shade in $\frac{1}{3}$.

Shade in $\frac{1}{8}$.

Shade in $\frac{2}{3}$.

Shade in $\frac{3}{4}$.

Shade in $\frac{1}{6}$.

Challenge!

Shade in $\frac{5}{8}$.

Make your own.

Shade in ___ .

Shrinking Circles - Worksheet 2
Challenge!
Estimate which fraction each shaded part of the circle represents.

_____   _____   _____

_____   _____   _____

_____   _____   _____