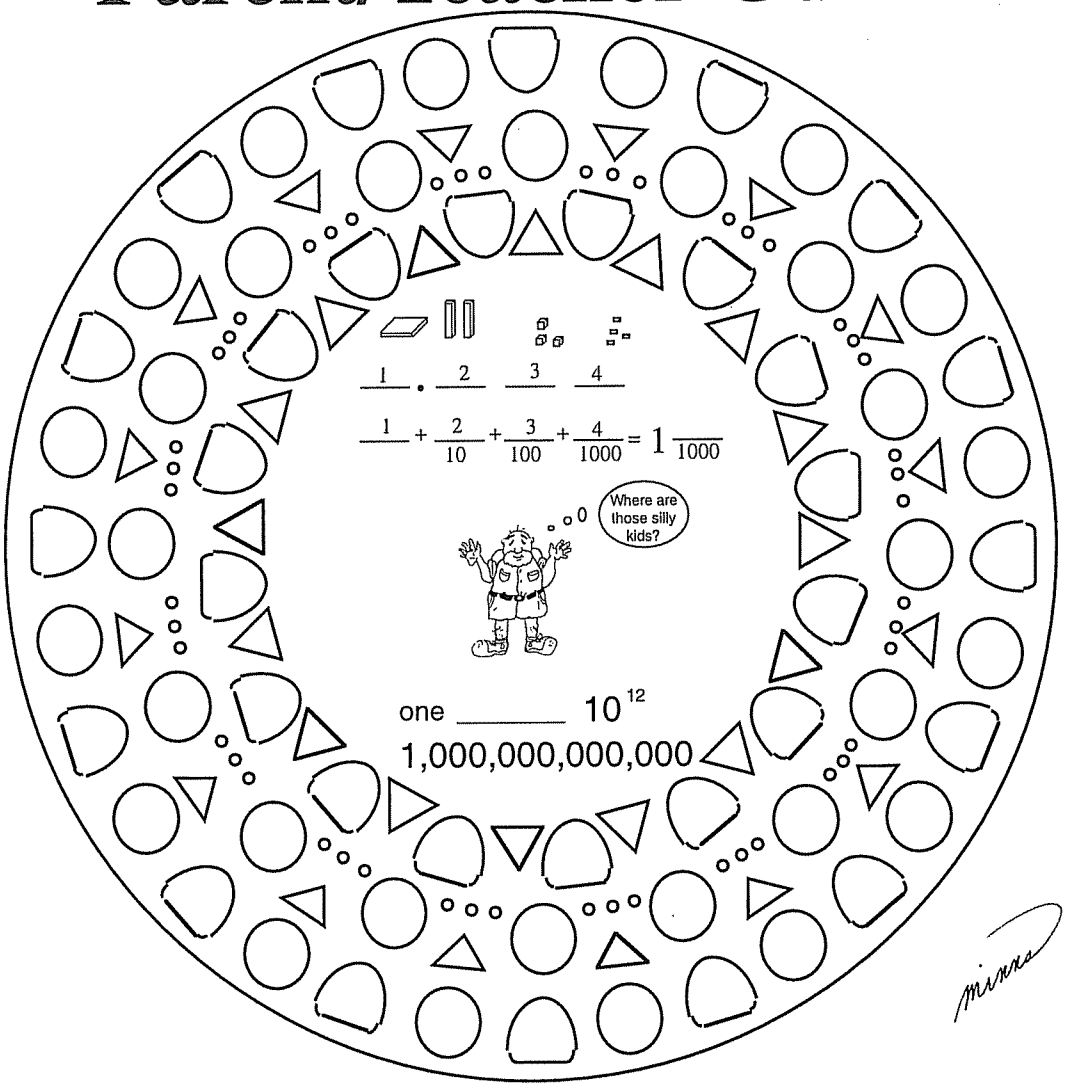
Patterns in Arithmetic

Place Value - Booklet 3 PDF

Into the Millions, Exponential Notation, and Operations Review

Parent/Teacher Guide



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Based on methods developed by Prof. Michael Butler at the
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University of California, Irvine

Place Value: Booklet 3 PDF - Into the Millions, Exponential Notation, and Operations Review

Contents

This booklet is dedicated to Jayne Moynihan. Thank you for your many contributions to education.

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The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

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Parent/Teacher Guide

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Vocabulary Words

Purpose The purpose of this lesson is to begin the math glossary the students will need for year five. Clear mathematical thinking requires clear, precise explanations. Clear, precise definitions require vocabulary that is precise. Students need to be taught mathematical vocabulary, grammar, and algebraic symbols as a second language. Without this second language, the student will have difficulty explaining patterns and procedures and understanding others who are explaining them.

Prerequisites None

Materials Vocabulary Words - Worksheets 1 and 2, pages 1 and 2
Math Journal - See *Patterns in Arithmetic:* Grade 5 - General Organization

Note Computer/Internet (optional, but wonderful)–place a bookmark at amathsdictionaryforkids.com. This site has a complete illustrated, interactive glossary of mathematical terms.

Warm Up Set up the Math Journal if you have not already done so.

Lesson Part 1 Have the student do the Vocabulary Words - Worksheet 1, down to the line on his own. This will let him know what basic words he knows and which ones he needs to learn. Instruct him to put a question mark by any word he does not know. When he is finished, have him look up on the Internet with the math dictionary the meanings of the words. You can also use a traditional dictionary. Using those definitions, he should fill in the worksheet blanks.

The words *sum*, *difference*, *product*, *quotient*, *divisor,* and *dividend* should be entered into his glossary in the Math Journal.
The lower part of the worksheet will be dealt with in Lesson Part 2 and should be done in the next session.

Lesson Part 2 Number Phrases and Mathematical Grammar
Have him enter into the Table of Contents of his Math Journal, 'Number Phrases and Mathematical Grammar' pages 31 and 32. Give him a tab for this and have him attach it to page 31. (If you are using last year's journal, your journal pages will be different. This topic is not formally addressed in year four booklets.)

On page 31 of his journal, have him copy the title 'Number Phrases and Mathematical Grammar' as the heading.

On the top line of his journal, page titled 'Number Phrases and Mathematical Grammar,' have him copy from this Vocabulary Word - Worksheet 1, 'The sum of 2 numbers is 12.' Then have him write the number sentence that matches this statement.
The sum of eight and four is twelve. $8 + 4 = 12$
Repeat this process with:
The difference between twelve and nine is three. $12 - 9 = 3$

The product of two numbers is twelve. Example: $6 \times 2 = 12$

The quotient of twelve divided by three is four. $12 \div 3 = 4$

Calculate the product of five and three. $5 \times 3 = 15$

Now for the combination statements, which generally will have question marks on them. Most students at this level have difficulty with these statements.

Have him copy this into his journal: Take the difference of five and three from the product of five and three. The number sentence that goes with this is:
$(5 \times 3) - (5 - 3) = 13$

Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **"How did you get that?" "Show me what you mean." "Build a model of that." "Tell me more so I can understand what you are saying."**

A dialogue may sound something like this:
"What does take the difference of five and three mean?" "To subtract three from five, which is two."

"What is this to be subtracted from?" "From the product of five and three."

"What does the product of five and three mean?" "It means to multiply five times three, which is 15."

"So what do you have to do first?" "Multiply the five and the three."

"So write 5 x 3 and put a grouping symbol around that. A grouping symbol is parentheses which mean, 'Do me first.'" He writes (5 x 3).

"What happens now?" "I take the difference of five and three from that."

"What symbol is used to show 'take from'?" "A minus sign."

"So write the minus sign next." He writes (5 x 3) –

"How do you show what needs to be subtracted?"

Note

Have him try both of these number sentences:
(5 x 3) - 5 - 3, which would be 15 - 5, which is ten, then minus three which is seven.
(5 x 3) – (5 – 3), which would be 15 – 2, which is thirteen.

"Does the grouping symbol for the 5 – 3 matter? If you do not use it, will it change the answer?" "Yes."
"The statement clearly says, 'Take the difference of five and three from the product of five and three,' so which number sentence is correct?" "You have to subtract the five and the three and then subtract the two from the fifteen. The second number sentence is the correct one."

Have him get in the habit of using two lines to decode these phrases. Algebra teachers require this for clarity.

The entry in the journal should look like this:
Take the difference of five and three from the product of five and three.
$(5 \times 3) - (5 - 3) =$
$15 - 2 = 13$

Repeat for the next two examples.
The quotient of twenty-four and six is how much less than the product of twenty-four and six? $(24 \times 6) - (24 \div 6) =$
$144 - 4 = 140$

Compare the product of twenty-four and six to the product of twelve and twelve.
Compare means to use the symbols <, >, or =. So the number sentence would be:
$24 \times 6 \_\_\_ 12 \times 12$
$144 = 144$

Practice

Give one problem per day of this type. Increase the complexity over time.

Here is a list of six problems to begin with. More problems like this can be found on the Internet. Have him copy these into his Math Journal in the Mathematical Phrases section, write the number sentence and solve them.

a. What is the sum of the products of three and four, and seven and five?
$(3 \times 4) + (7 \times 5) =$
$12 + 35 = 47$

Note

The insertion of the comma after the four will clarify what goes with what in the group of all those ands.

b. Take the difference of the sum of five and nine and the quotient of nine and three. $(5 + 9) - (9 \div 3) =$
$14 - 3 = 11$

c. Find the sum of one hundred and the product of eight and nine.
$100 + (8 \times 9) =$
$100 + 72 = 172$
State the problem backwards; give the number sentence and have him write the words to go with it.

d. $5 \times (9 - 4) = ?$
Answer: What is the product of five and the difference between nine and four? Twenty-five.

e. $(7 \times 6) \div (9 - 7) =$
Answer: What is the quotient of the product of seven and six, and the difference between nine and seven? Or what is the product of seven and six over the difference of nine and seven?
$(7 \times 6) \div (9 - 7) = 42 \div 2 = 21$

f. $(30 \times 4) + (2 \times 4) =$ This is the form of a distributive number sentence used for 32×4.
Answer: What is the sum of the products of thirty and four, and two and four? One hundred twenty-eight

Lesson Part 3

Mathematical Prefixes
Have him fill out Vocabulary Words - Worksheet 2, Mathematical Prefixes using whatever resources seem useful–regular or online dictionaries and talking to other people to get ideas for words with these prefixes.
Have him enter the prefixes into his Math Journal glossary. Entries should look like this:
Tri - a prefix meaning three. Words with this prefix are triple, triplets, tricycle, trillion.
See the Answer Key for suggested words.

Test for Understanding

This is where the rubber hits the road. Can he take a known procedure or algorithm and turn it into a statement using the proper vocabulary? This is how he will need to answer questions that require that he explain patterns and procedures.

Give this problem after you have been working on mathematical grammar for a month or so. (In year four, he should have studied how to multiply a fraction by a whole number, also called parts of wholes.)

Give this problem: $\frac{2}{3} \times 9 = \_\_$. **"Write out the procedure using the correct words and the number sentence that shows the order in which operations are performed."** There are two possible answers:

1. Take the quotient of the whole number and the denominator times the numerator. Or, take the product of the numerator and the quotient of the whole number and the denominator. Number sentence: $(9 \div 3) \times 2 = 6$

2. Take the product of the whole number and the numerator divided by the denominator.
Or, take the product of the whole number and the numerator over the denominator.
Or, take the quotient of the product of the whole number and the numerator and the denominator. $(9 \times 2) \div 3 = 6$

Prime Factoring: Review and Exponents and Prime Factors

Purpose

The purpose of this lesson is to further develop understanding of the concept of exponents and how they can be used in notation to list the prime factors of a number.

Prerequisites

Previous lessons on Exponents and Powers of Ten, General Math: Booklet 5, Prime Numbers: Review and Goldbach's Conjecture lessons, and Multiplication: Booklet 3 - Factoring lesson

Materials

Prime Factoring: Review - Worksheets 1 - 3, pages 3 - 5
Exponents and Prime Factors - Worksheets 1 and 2, pages 6 and 7
Tiles and cubes (smallest Cuisenaire Rod or Base Ten Cube)
A one thousand Base Ten Cube

Warm Up

Do Prime Factoring: Review - Worksheets 1 - 3, pages 3 - 5.

Lesson

An exponent is an operator. This means it tells you to do something. 5^3 means multiply five times itself three times, $5^3 = 5 \times 5 \times 5 = 125$. 5^3 does not mean 5 x 3, which is a common error students make. Use exponents only when multiplying the same numbers together over and over.

"Build a rectangle that is three tiles on each side. What shape do you get?" "A square."

"How do you calculate the area of this square?" "Three times three."

"How do you write 3 x 3 using an exponent?" "Three with a little two up high next to the three."

"3 x 3 is called factor form and 3^2 is called exponential form, or exponential notation."

"Why do you think they call 3^2 three squared?" "Because if you build any shape that is 3 x 3 or 4 x 4, you will get a square."

Hold up a one thousand Base Ten Cube. **"What is the length of this cube? The width? The height?"** "They are all ten."

"How do you calculate how many little one cubes are in this thousand cube? How do you calculate the volume of this cube?" "You multiply 10 x 10 x 10 to get 1,000." If the student does not know this, have her record the number sentence from looking at the cube again.

"How do you write that using exponential notation?" "10^3 or ten to the third power."
"Why would you call 10^3 ten cubed?" "Because when you build a shape that has

three sides all the same length you will get a cube."

Now have her begin Exponents and Prime Factors - Worksheet 1. She should be able to do most of it independently. Help when it is needed. Use the Answer Key to check that the work is done correctly.

Practice Worksheets

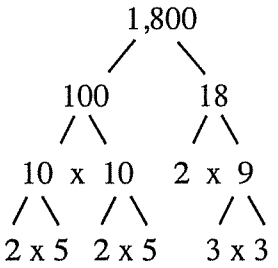
Exponents and Prime Factors - Worksheet 2

Test for Understanding

Find the Prime Factors of 1,800. Write them in exponential and factoring forms.

Answer: Factor Form = 5 x 5 x 3 x 3 x 2 x 2 x 2 or 5, 5, 3, 3, 2, 2, 2
Exponential Form $5^2, 3^2, 2^3$

Using a factor tree:



Place Value: Base Ten

Purpose

The purpose of this lesson is to review base ten place value patterns and extend them to very large numbers. Models of large place value blocks are constructed so students can see what a power of ten does to the size of the number. Students use a transparency to fit over a place value chart to help them learn how to read the names of large numbers. Students then use written names for large numbers and write them into a place value chart.

Prerequisites

Previous lessons in Base Ten Place Value up through the thousands.

Materials

Place Value: Base Ten - Worksheets 1 and 2, pages 8 and 9
Base Ten Blocks - including the one thousand block. (If you do not have Base Ten Blocks, you can make them using beans and popsicle sticks.) See page 36 of this booklet.
2 x 3 Post-it Notes or small pieces of paper or index cards cut to appropriate size
Make a transparency of Transparency Master for Place Value: Base Ten, page 11 in this booklet. You can also use a clear sheet protector and write the numbers.
Classroom teachers will need to make a transparency of Worksheets 1 and 2 also.
Large sections of cardboard, poster board, foam core (expensive but durable), newsprint will do in a pinch.
Glue stick, paste, or tape

Optional

It is very helpful, durable, and fun to make the following:
For a ten thousand block: 4 lengths of ½ inch PVC 100 centimeters long, 8 lengths of ½ inch PVC 10 centimeters long, and 8, ½ inch corner fittings. I once made a ten thousand flat into a little garden box planter. One parent made one out of clear acrylic and silicon sealer and made it into a crayfish tank for science. Students loved it.

For a one hundred thousand flat: 8 lengths of ½ inch PVC 100 centimeters long, 4 lengths of ½ inch PVC 10 centimeters long, and 8, ½ inch corner fittings. I once made a one hundred thousand flat into a garden bed. Perfect for daffodils.

For a one million cube: 12 lengths of ¾ inch PVC pipe cut to 100 centimeters in length each. 8, ¾ in. PVC three pipe corner fittings. PVC glue. This will be used to assemble a PVC cube that will serve as the model for a one million cube. It also doubles as a wonderful play structure to throw silk scarves or light sheets over to make hidey-holes so loved by this age group.

Lesson Part 1

This fun project really gets students to see the place value patterns in 3-D. You will get a huge payoff for taking the time to do this project. Your Base Ten Block sets already have ones, tens, and hundreds. Hopefully, you have a few thousands cubes also.

Warm Up

The project is to make a Ten Thousand rod, a One Hundred Thousand flat, and a One Million cube using the instructions above. You can make these out of PVC, wood, newsprint, or plastic sheeting taped together and stuffed or blown up like

balloons. The fun is figuring out how to do it. At the very least, have the student make a ten thousand block and discover that it is a long rod shape.

"What shape would you get if you took ten, 10,000 rods and taped them together?" "A 100,000 flat."

"What shape would you get if you took ten, 100,000 flats and stacked them?" "A one million cube."

Note

Notice this cool pattern. The one block, the one thousand block, and the one million 'block' are all cubes. The ten block is a rod. So is the ten thousand block. The one hundred block is a flat. So is the one hundred thousand 'block.' She will not see this amazing pattern unless she builds it. Remember, many people are geometric thinkers. Math makes much more sense to them if they can see the shape, the geometry of the numbers.

"What is the place value pattern in geometric shapes?"

cube,	flat,	rod,	cube,	flat,	rod,	cube
million,	100 thousand,	10 thousand,	1 thousand,	100 ones,	10 ones,	1 one

"What shape would a 10 million block have?" "A rod."

"What about a 100 million block?" "A giant flat the size of a small stage!"

If you imagined a classroom or your house as a block, what number would it be? A classroom is a very flat shape. They are usually 30 x 30 square feet, or nearly so, and 10 feet tall. How many centimeter cubes would fit into that shape? To make it easier, think of a cubic yard as a one million block. It is not quite right, but it is on the same order of magnitude.

Lesson Part 2

Have Base Ten Blocks within reach of each student. The lesson will go faster if the blocks are dumped out of the containers and grouped by block size. This prevents scrabbling through the box to find the blocks needed.

"Pick up a one block. Now count out eight more. Say and write the number as you count them out." "1, 2, 3, 4, 5, 6, 7, 8, 9."

"What happens to your written number as soon as you pick up another one block?" "I will have ten."

"When you have ten ones blocks, what can you trade for?" "One ten block (rod)."

"Do that. Now write the number."

"What does one in the second row and zero in the first row mean?" "It means I have one group of ten and no ones."

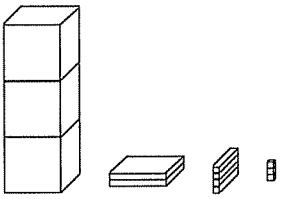
"How many tens do you need to trade for a one hundred flat?" "Ten."

"How many one hundred flats do you need for a one thousand cube?" "Ten."

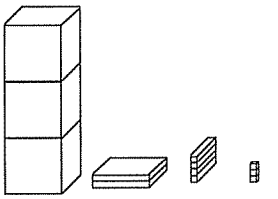
"What pattern do you notice?" "It always takes ten of a smaller block to trade for the next larger one."

"Fill that in on your worksheet under the pictures."

"Use your blocks to build three thousand, two hundred forty three."
She should build it like this:



"When you are finished, put a label just in front of the blocks to show the number of blocks of each size."



"This number has two threes in it. How do you know which size block goes with each number?" "By which place the number is in."

"So this is called place value because...?" "In place value the location of the number tells you what value the number is."

Write 1,9 4 8 on the board or on a piece of paper.

"Build this with blocks. Then read the name of the number." "One thousand, nine hundred forty-eight."

Note

Do not say 'and' at the commas. 'And' is said only at the decimal point.
Correct: One thousand, nine hundred forty-eight
Incorrect: One thousand, nine hundred *and* forty-eight
One thousand, nine hundred forty eight–there must be a hyphen between the tens and ones sections of each grouping.

"Take out these blocks. One ten thousand block (if you have one), four one thousand blocks, seven one hundred blocks, no tens blocks, and five ones blocks." Give her time to do this. You may have to repeat the instructions several times.

"Now, write the number of each size of block you have on a label and place it in front of the blocks." She should write 1 4,7 0 5.

Optional

If you have built the larger blocks, ask her to build one million, one hundred fourteen thousand, seven hundred five. Have her label it. Most students will find this great fun.

Continue with Place Value: Base Ten - Worksheet 1. Use the transparency to place the numbers over the place value graphic. Have her identify the place value of the underlined number and copy it onto her worksheet. Make sure she spells the words correctly.

Most students will not be able to be totally independent at this point. Have her finish the page on her own using the transparency, but be ready to help.

When she is finished, preview the next part of the lesson by again placing each number on the transparency onto the place value graphic. Read the name of each number to her. Emphasize how you read the number in each family group, "Forty-two..... million," then pause and give the place value of that family group. You can help her by covering up the numbers on the end, showing only the numbers in each place value group as you go.

Do not at this point ask her to read the numbers herself. You could have her read them with you a second time. She will make errors and pauses. Ignore them.
End the lesson at this point.

Lesson Part 3

Practice Pages/Math Journal
Have her make an entry in her Table of Contents for Writing Large Numbers. Place that same title on the next blank page of her journal and tab it. Leave two pages empty for this section.

Each day, write a number such as 34, 560, 891 on a piece of paper, or dictate it to her. She should copy the number down and then correctly write the name of the number. It would be thirty-four million, five hundred sixty thousand, eight hundred ninety-one. Insist on proper spelling and punctuation. Have her then read it back to you. Continue this for a month, and then periodically after that.

Note

Do not allow her to say, "five hundred ***and*** sixty," or "eight hundred ***and*** ninety-five." Remember, no ands until you get to a decimal point. Don't forget those hyphens either.

Test for Understanding

Have her use a deck of cards (no face cards) and draw between ten and fifteen cards and place them in a row. Ask her to copy the numerals on the cards in a string of numbers, such as 3 4 6 7 8 3 8 6 2 9. Then ask her to write out the name of this number and read it back to you correctly.
3,467,838,629 Three billion, four hundred sixty-seven million, eight hundred thirty-eight thousand, six hundred twenty-nine.

Transparency Master

Make a transparency at a copy center of the following page to allow the student to place each number over the graphic on the Place Value: Base 10 - Worksheet 1. Or write these numbers on a clear sheet protector. This will help her understand how to identify which place value each number is in. You can also use it to help her learn to read the names of large numbers.
Transparency Master for Place Value: Base Ten

Numbers for Place Value: Base Ten - Worksheet 1

Place over graphic on Worksheet 1 so the numbers line up in the proper place value.

42000000

27000000

174000

9350

45403402 00

80340337 0300

9347

450235

65349300

Numbers for Place Value: Base Ten - Worksheet 2

230300000

6390000090

2470000

6790000

932000000

Exponents and Multiples of Ten

Purpose The purpose of this lesson is to connect the names of place value locations with the concept of exponential notation for powers of ten and the number associated with each power. It also reinforces the sequence of the names for each location in the place value chart.

Prerequisites Previous lesson

Materials Exponents and Multiples of Ten - Worksheet, page 10
Math Journal

Warm Up Powers of Money - examples of values in 2010 US dollars
Put this in the Math Journal as Place Value: Base 10 - Powers of Money
Make a list of three things you can buy with about one dollar.
Example: A candy bar, a plastic toy, a bottle of water

Make a list of three things you can buy with about ten dollars.
Example: A book, three gallons of gas (maybe), dinner

Make a list of three things you can buy with about one hundred dollars.
Example: A week's food for one person, three bags of dog food

Make a list of three things you can buy with about one thousand dollars.
Example: A payment on an apartment or house

Make a list of three things you can buy with about ten thousand dollars.
Example: An inexpensive economy car

Make a list of three things you can buy with about one hundred thousand dollars
Example: A mobile home, a small house, ten acres of land

Make a list of three things you can buy with about one million dollars.
Example: a ranch, a really special painting

You get the idea. This is a great family activity.

Lesson **"Explain what an exponent is."** "An exponent is an operator like + , -, x, or ÷. It is an instruction to do something. It looks like this 2^3. The first number, the 2, is the base number. The ^{3} is the exponent, also called a superscript. The little three means multiply two times itself three times. 2 x 2 x 2 = 8."

"Multiply 10 x 10 x 10 x 10. What do you get?" "Ten thousand."

"You could write 10 x 10 x 10 x 10 or 10,000 a really easy way by using an ex-

ponent. Do you know how that could be done?" "10^4." If he knows, great. If not, show him.

"People use exponents, which are also called 'powers,' to write very large numbers."

Study the Exponents and Multiples of Ten worksheet.
"What patterns do you see?" "The exponents go up by one each time. The number of zeros in the number is the same as the exponent number. The first number is always one."

"What would go on the next line?" "Ten thousand 10^4 10,000."

"What comes next?" "One hundred thousand 10^5 100,000."

"You can use your place value chart to help you figure out what the names of the numbers are. Also, remember your prefixes on your vocabulary page. Do this work carefully; it is easy, but if you do it too fast you will mess it up. See if you can end up with the correct number at the end."

Math Journal/ Practice

In the glossary have the student place the words *exponents* and *powers*. He should give an example of each one. Have him use the amathsdictionary.com web site if he needs help to decide what to write.

Give him a power of ten problem like the one in the Test for Understanding once a week or so. Add it to your practice list.

Test for Understanding

"What number is ten to the ninth power (10^9)? "One followed by nine zeros. 1,000,000,000. The name of this number is one billion."

Extension

Two great books:
How Much Is a Million? (Reading Rainbow Books) by David M. Schwartz and Steven Kellogg
Powers of Ten: About the Relative Size of Things in the Universe, by Philip Morrison and Phylis Morrison

Rounding Off

Purpose

The purpose of this lesson is to extend what the student knows about rounding to the nearest ten, taught in Place Value - Booklet 2, and extend it to rounding to larger place values.

Teacher Background

There are ten digits in our base ten system:
0, 1, 2, 3 ,4 ,5 , 6, 7, 8, and 9. The halfway mark is between 4 and 5.

0, 1, 2, 3, 4 5, 6, 7, 8, 9

This is why we round up if any number ends in 5, 6, 7, 8, or 9. We are accustomed to thinking of 5 as the middle because we do not see the 0 as the first number. We always think of the first number as being 1. For the purposes of rounding, one must think of the 0 as the first number. The first five numbers are 0, 1, 2, 3, and 4. Any number ending in one of those digits will be 'rounded down.' The second five numbers are 5, 6, 7, 8, 9. Any number ending in one of those digits will be 'rounded up.'

A 'round number' is any number that uses all zeros following a single digit or set of digits other than zero. Example: 30,000 is a round number. This would be a number rounded to the ten thousands place. 33,000 is also a round number, but it is rounded to the thousands place. This distinction is hard for students to understand.

Prerequisites

Previous lessons

Materials

Rounding Off - Worksheets 1 - 3, pages 11- 13
Base Ten Blocks
Number Line Lab Sheet (after the Test for Understanding); make one copy for each student.

Warm Up

In her Math Journal, have the student enter 'Rounding' into her Table of Contents. Tab the next blank page 'Rounding' and label the heading as 'Rounding.' Enter into the journal:
There are ten digits in our base 10 system.
0, 1, 2, 3 ,4 ,5 , 6, 7, 8, and 9. The halfway mark is between 4 and 5.

0, 1, 2, 3, 4 5, 6, 7, 8, 9

The first five numbers are 0, 1, 2, 3, and 4. Any number ending in one of those digits will be 'rounded down.' The second five numbers are 5, 6, 7, 8, 9. Any number ending in one of those digits will be 'rounded up.'

In the journal, have the student sketch a number line from 0 to 100 going up in multiples of ten like the one shown on Rounding Off - Worksheet 1, page 11, except the numbers will be 0, 10, 20, and so on. Have her put a little hatch mark to show the halfway mark between 10 and 20, 20 and 30, and so on.

Have her put a point on the number line for each of the following numbers: 23, 68, 92, 45. She should put each of these numbers on her number line in her journal.

"How do you round 23 to the nearest ten using the number line?" "You look at the number line and you see if the number is before or after the halfway line. If it is before you round down to 20. If it is on or after the five, you would round up to 30."

"What are you going to do with 45?" "If the number is at the halfway mark, you round it up to the next highest ten. So 45 would round to 50."

Note

Remember, the 5 is not really the halfway mark if we include the 40 in our count. 40, 41, 42, 43, and 44 are in the first half and would be rounded down to the 40.

45, 46, 47, 48, and 49 are in the second half and would be rounded up to the 50. Have her write these explanations in her Math Journal.

In the journal, have her round each of the four numbers she put on her number line, 23, 68, 92, 45 to the nearest 10.

Lesson Part 1

Start this concept with the blocks.
Put a one hundred flat block on the table. To the right of it about 12 inches place two one hundred flats side by side. Between them build 150 with the blocks laid out side by side as well.

"Build 130 with the blocks. Is it larger or smaller than 150?" "Smaller."

"So, is 130 closer in size to the 100 block or the 200 blocks?" "Closer to the 100."

"This is what we are doing when we round numbers. We are comparing the size and deciding which round hundred it is closest to." Try another if you think your student will benefit from building these.

Now use Rounding Off - Worksheet 1 to show the exact same idea with a number line. The first row of numbers in the examples of correctly rounded numbers use the 100 to 200 number line.

Note

Number lines do not work for all students. Some will benefit from building for a while. If you encounter any confusion or difficulty, have her build it with the blocks and then use the number line for the same problem.

The rest of page 11 uses larger numbers. Have her use the number lines on the lab sheet to work with larger numbers.

Have her find the approximate place on the number line where the target number would be and see which round hundred it is closest to.

"Tell me what your rule is for rounding a number to the nearest hundred." Have her explain it before she writes it. She will get better writing it that way.

Answer: I look at the number in the tens place. If the number is less than 5 then I

keep the number in the hundreds place the same as it was, and add two zeros to the right. If the number in the tens place is 5 or more, then I increase the number in the hundreds place by one and follow it with two zeros.

Have her write a first draft of this explanation on the worksheet. Edit it and have her copy the corrected explanation into her Math Journal.

Note

There is a challenge problem on the worksheet, 1,262. If she can not figure this out, leave it for now. This complication will be dealt with on Rounding Off - Worksheet 3, page 13.

Require that she write the explanation clearly using the correct vocabulary. For example, use words such as 'to the right of' instead of 'behind.'

Rounding Off - Worksheet 2 teaches the concept of 'Rounding to the Greatest Place.' In general when you are asked to estimate the answer to a problem, you will round to the greatest place. This kind of rounding is the most useful.

Math Journal

Round Number and Rounding. Have her enter these two words into her glossary. She can look them up in a dictionary to get the formal definitions, which will help her later.

End the session here.

Lesson Part 2

Rounding Off - Worksheet 2, page 12 The worksheet guides the student through the process of rounding to the nearest thousand using a number line.

When given this instruction, 'Graph the number on the number line,' it is asking you to place a point on the number line and label it. The worksheet asks her to graph 6,723 on the number line. She should put a dot just after the hatch mark between the 6,000 and 7,000 because 6,723 is fairly close to 6,500. She should write 6,723 (with a sharp pencil) above her dot.

She should do this for each number on the chart. The chart uses the example of 6,723 to show her how to fill out the chart. Use the Answer Key to help you if needed.

Supervise her as she fills out the chart. Correct her work immediately.

Rounding to the Greatest Place

This is what you do when you are in the grocery store and you keep a mental tab of about how much money you need to cover the cost of the items in your basket. This skill is used all the time when estimating answers especially in division and multiplication with decimals. A place value error is major. Rounding to the greatest place and estimating the place value of the answer, as a general habit will alert her to 'this answer can't be right' kind of thinking. Mathematicians do this as a habit of mind.

When she completes the worksheet, direct her to Patterns in Sums - Column Addition, page 14 of the Student Workbook. Have her round each number in the problem to the greatest place and write it to the right of the problem. Then total up the rounded numbers. Then do the actual sum. The rounded sum and the actual sum should be in the same place value and fairly close. This is the symbol for approximately ≈. It is sort of like wiggling your hand when you talk about something you are estimating.

Math Journal/ Practice

Put this into the Symbols section of the glossary.

Practice: Go to the workbook, Addition: Review, page 24, and have her round each number to get an approximate answer. Then have her do the addition problem. You can give one row each day to keep it from getting tedious.

Rounding Answers:

532 ≈ 500	374 ≈ 400	836 ≈ 800	912 ≈ 900
+346 ≈ 300	+259 ≈ 300	+376 ≈ 400	+788 ≈ 800
878 ≈ 800	633 ≈ 700	1,239 ≈ 1,200	1,700 ≈ 1,700

Repeat this same process on Multiplication: Review - Regrouping Practice, page 41. In this case, do not have her round the multiplier. Leave it. We only round the multiplier when we are working with two-digit numbers. These estimates will be further from the mark, but the answer and the estimate will be in the same place value range and close enough to identify a wildly incorrect answer.

You can also bracket answers using the upper and lower rounded numbers. The product of the brackets in the first problem, 165 x 3, will be between 100 x 3 = 300 and 200 x 3 = 600. The answer must fall in that range. If it does not, a place value error is likely.

165 ≈ 200	249 ≈ 200	142 ≈ 100
x 3 x 3	x 3 x 3	x 6 x 6
495 ≈ 600	747 ≈ 600	852 ≈ 600
range 300 – 600	range 600 – 900	range 600 – 1,200

Now have her look at the center problem in the bottom row of page 41. The student is being asked to identify the error.

Have her round the 357 to 300 x 2 and 400 times 2. The range the product should be in is 600 to 800.

There is no way that answer can be 7,014. The fact that it is so out of range tells her that the error is probably a place value error. Something got recorded in the wrong column.

In this case, the error is that when 7 x 2 was multiplied, the 14 was written down without carrying the one into the tens place. Then 5 x 2 was multiplied putting the 0 in the hundreds place and carrying the one over to the 3 and multiplying 2 x 3 + 1 to get the 7.

Get in the habit of having her identify what kinds of errors she is making when she misses a problem.

Math Journal Have her write about rounding to the greatest place and how it is used to estimate answers.

Lesson Part 3 **"Sometimes it is necessary to round numbers not to the greatest place, but to a specific place value. You will see an example of why at the end of this lesson."**

On her Rounding page in her journal, have her write 4,5 7 3.
"Round it to the nearest thousand." She already knows how to do this. Have her put a √ over the number 4. It is the 'target' number. √
4,5 7 3
Now underline the number to the right of the checked number.
"Why do you need to look at the 5 to know how to round to the nearest thousand?" "Because it will tell me if the number is closer to 4,000 or closer to 5,000."
√
"Which is it?" "Five thousand." Have her write 4,5 7 3 ≈ 5,000 in her journal.
Now we will round this same number to the nearest hundred. Have her write 'nearest hundred' on the next line of her journal, followed by the number 4,573. This time the target number is the number in the hundreds place. **"Put a √ over the number in the hundreds place and underline the number to the right of the checked number."**
√
4,5 7 3 ≈ 4,600

Now this is tricky. She must round the number in the hundreds place, using the underlined number, the 7. This causes the 5 to go up to a 6. Then the first number, the 4, stays the same. The last two numbers become zeros.
√
Repeat this process for the nearest ten. 4,5 7 3 ≈ 4,570
The 7 stays the same because 3 is less than five. The 3 turns into a 0 and the 45 remains.

At the top of Rounding Off - Worksheet 3, the steps you just taught are written out. Have her fill in the blanks. Use the Answer Key to help you. Then do the first two problems in each column with her. Then allow her to finish independently. Check the work as soon as she finishes.

Math Journal After completing Rounding Off, page 13, have her copy into her journal the procedure she developed at the top of page 13 for rounding to a specific place.
After completing Rounding Off - Worksheet 3, assign column one on Subtraction Review, page 25.

Have her round to the greatest place if the two numbers being worked with are in the same place value. The third problem in the set is an example of this.

$$\begin{array}{r} 318 \approx 300 \\ \underline{-\ 115} \approx \underline{-100} \\ 203 \quad 200 \end{array}$$

If the numbers in the problem are in different place values, the estimate will be more accurate if the numbers are rounded to the specific place value of the smaller number.

Compare these two estimations: The first is rounded to the greatest place, the second rounded to the place value of the smaller number.

$$\begin{array}{rr} 137 \approx 100 & 137 \approx 140 \\ \underline{-\ 25} \approx \underline{-30} & \underline{-\ 25} \approx \underline{-30} \\ 112 \approx 70 & 112 \approx 110 \end{array}$$

greatest place round to nearest ten

Here is another example:

$$\begin{array}{rr} 1{,}456 \approx 1{,}000 & 1{,}465 \approx 1{,}500 \\ \underline{-\ 285} \approx \underline{-300} & \underline{-\ 285} \approx \underline{-\ 300} \\ 1{,}171 \approx 700 & 1{,}171 \approx 1{,}200 \end{array}$$

greatest place round to nearest hundred

This is why there are two different kinds of rounding. When working with addition and subtraction, when numbers being worked with have different place values, it is usually best to round to the place value of the lower valued number.

Remember, she should round both numbers first, then subtract the rounded numbers to get the estimated answer. A common error students make here is to subtract the numbers and then round the answer. If her subtraction is too wobbly to complete the harder problems, let her use a calculator to get the point of how rounding can help spot errors.

Test for Understanding

Have her copy this question into her Math Journal in the Rounding section and write the answer there.

For this problem, use both rounding to the greatest place and rounding to a place value you think would give the best estimate. Explain which kind of rounding would give the best estimate to the answer.

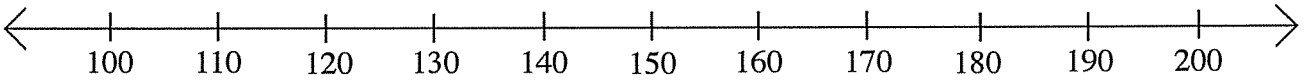
4,423 – 117 =

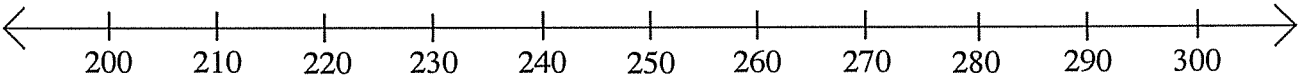
Answer: 4,306 Rounding to greatest place 4,000 - 100 = 3,900

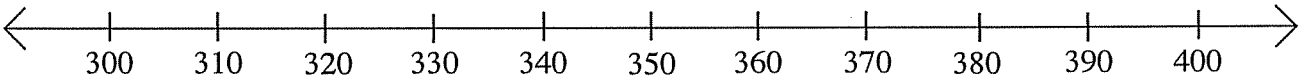
Rounding to the nearest hundred 4,400 - 100 = 4,300

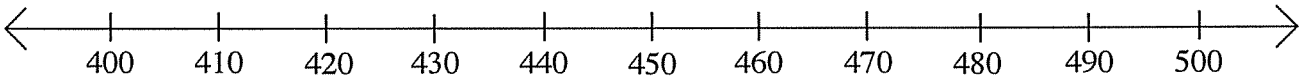
It would be best to round to the nearest 100, not to the greatest place.

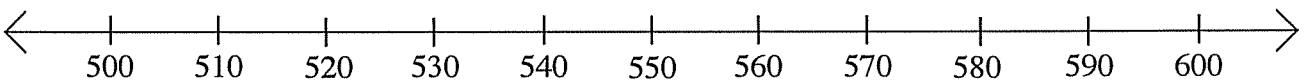
Number Line Lab Sheet

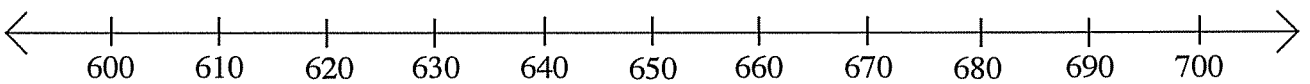


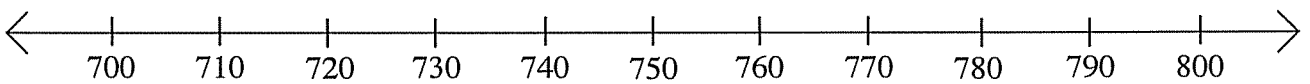


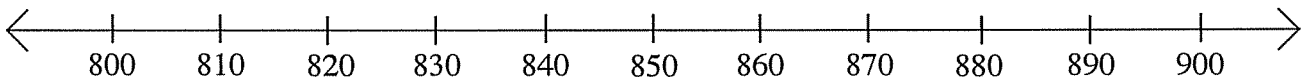


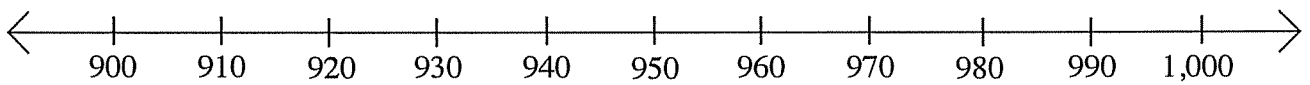


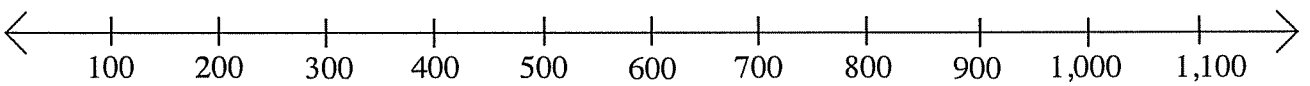












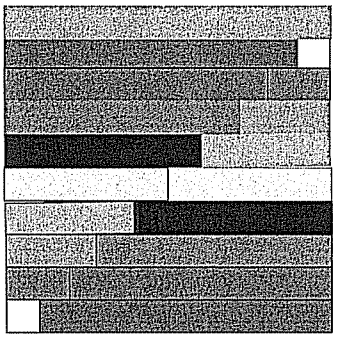
Patterns in Sums: Column Addition

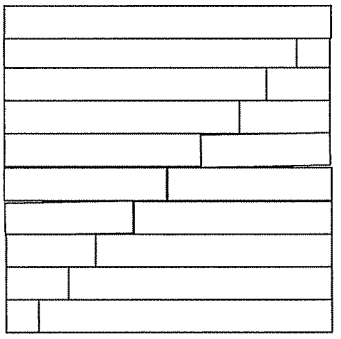
Purpose The purpose of this lesson is to review the strategies of using doubles and sums of ten to facilitate accurate addition of long columns of numbers. It teaches students another way to add other than counting on their fingers or on dots.

Prerequisites Mastery of addition with regrouping

Materials Patterns in Sums: Column Addition, page 14
Cuisenaire Rods
Colored pencils

Warm Up Have the student place an orange rod (10 cm) horizontally on the table. Directly below it the blue rod (9) is placed, followed by the white rod (1). The blue plus the white are equal to the orange. The next pair in the pattern is the brown (8) rod and the red (2) rod, which are also equal in length to the orange rod. Have her continue to build this pattern and end at the white (1) plus the blue (9) Cuisenaire Rod.





This makes a visual representation for all the sums of ten. Even though sums equal to ten are easy for most students at this level, the physical representation of this equality will help her understand the pattern she is using to make the addition easier and more accurate. Most students enjoy this activity also.

Have her list all the combinations of two numbers whose sum is ten. She can copy it right off the rods.

Lesson Patterns in Sums: Column Addition, page 14
Write large on a piece of paper this first single column (the ones column only) addition problem shown on the worksheet example. Have her place the correct rod next to each number. Build it in the order the numbers are seen on the worksheet:

7 - black rod
4 - purple rod
4 - purple rod
3 - light green rod
1 - white rod
9 - blue rod

Have her pick up the rods and make as many tens as she can. The black and the light green make a ten. The blue and the white make a ten, and the two purples

make a double equal to eight.

"Look at the example on the worksheet. Which number pairs added up made 10?" "The black seven and the light green three make ten, and the blue nine and white one make ten."

"Use your pencil to trace how those number pairs are linked with the curved lines with a ten written beside the line."

"How were the two fours marked on the worksheet?" "As eight because they are a double."

By linking tens and doubles, you can add columns of numbers more accurately.

"Now look at the problem to the right of the example. Look at only the ones column. You see the numbers 4, 8, 7, 2, 3, and 6. Which pairs make ten?" "Four and six, eight and two, and seven and three."

"Link those pairs with a curved line."

"What is the sum of the numbers in the ones column?" "Thirty."

"Record that, and carry the three into the tens place."

"It is very crowded in the tens column, so many people use colored pencils to circle pairs. Each pair is in a different color. Try that. Which pairs did you find?" "Eight and two on the top, one and nine, and then seven and three, plus the carried three."

"So what is the sum of this column?" "10 + 10 + 10 + 3, or 33."

"Record that and carry the three into the hundreds place."

"Repeat the same procedure on the numbers in the hundreds place. What pairs do you find?" "The three and the seven at the top, the two fives which make twenty, and then the last three, so the sum of this column is twenty-three."

"What is the sum of the entire problem?" "2,330."

Repeat with the other problems. It works best if you do one or two of these per day for a week or so.

Practice Worksheets

None

Test for Understanding

None

Decimal Place Value

Purpose

The purpose of this lesson is to review decimal place value. This lesson is intended as a review, not a first introduction.

Prerequisites

Manipulative work with decimal place value, building decimal numbers with Base Ten Blocks, Key to Decimals - Book 1, *Patterns in Arithmetic*: Fractions - Booklet 2 - the Changing Whole, and Place Value: Booklet 2
Free exploration with Base Ten Blocks

Note

This lesson moves too fast for a first introduction to decimals and building decimal numbers. Review Key to Decimals - Book 1

Materials

Decimal Place Value: Worksheets 1 - 7, pages 15 - 21
Base Ten Blocks with decimal chips (The chips are very thin so that ten of them equal the volume of a one centimeter cube. They can be purchased from Nasco.)
A round counter or bean to stand for the decimal point

Warm Up

Have the student play with the blocks for at least ten minutes before beginning the lesson. Leave the blocks as she has them on the table and begin asking questions in a light voice.

"How many tiny chips does it take to be the same volume as the small 1 centimeter cube? Build it to be sure." "It takes ten chips to equal one small cube."

"How many small cubes does it take to be the same volume as the rod? Build it to be sure." "It takes ten small cubes to equal one rod."

"How many rods does it take to be the same volume as the flat? Build it to be sure." "It takes ten rods to equal one flat."

"What pattern do you notice?" "They all trade at ten, or it takes ten of the smaller blocks to equal the next larger block."

"Why do you think we call our numbering system Base Ten?" "We trade up whenever we have ten of a smaller unit."

"Clean up whatever buildings or designs you have made." Have her put the blocks back onto the tray or in whatever container you have placed them in. Let her know that the lesson is going to begin and that she should not make buildings or designs during the lesson.

Lesson Part 1

Begin the lesson with a single small cube on the table in front of her. Right now, we are defining the value of this small cube as one.

"How many small cubes does it take to equal one rod?" "Ten."

Write this in front of her: 1 __ _____ = 10 **"Write that as a number sentence."** "1 x 10 = 10."

"How many rods does it take to equal one flat?" "Ten."

"What is the value of the flat?" "One hundred."

"Write that as a number sentence." "10 x 10 = 100."

Go to Decimal Place Value - Worksheet 1 - Place Value Patterns, page 15, and work on this worksheet together.

Embedded Test for Understanding

Do not tell her what to do on the third row. Watch to see if she can extend the pattern to fill in the decimal numbers on the third row. This is an embedded assessment of her grasp of decimal place value. If she has difficulty, skip it and go to the next worksheets, then come back to it after you have completed Decimal Place Value - Worksheet 3.

Check the worksheet with the Answer Key to be sure it was completed correctly. Then continue on with this Changing Whole Oral Exercise.

Lesson Part 2

"You are used to defining the small cube as one. What if I define the rod as equal to one. Then what would be the value of the large flat?" "It would be equal to ten because it takes ten rods to equal it."

"What would the value of the large cube be?" "One hundred."

"What if I change the definition and define the flat to be one? Then what is the rod worth?" "One tenth."

"The small cube?" "One hundredth."

"The thin chip?" "One thousandth."

"The large cube?" "Ten."

Have her turn to Decimal Place Value - Worksheet 2, page 16.

"Study the top section of the worksheet." Give her time to do this.

Point out that the flat is defined as 1. Have her build the number shown in the picture with the blocks: 1.147. Her block building should look like the picture on the worksheet.

"In the decimal notation, look at the 1s. There are two ones. How do you know what this 1 (point to the number to the right of the decimal point) stands for?"

"I know it is tenths because that one is in the first place to the right of the decimal point. That is the one tenths place."

"This is why mathematicians call it 'place' value. The location of the number, its place tells us what it is worth. These numbers can be written using decimal notation, or fractional notation. This page will have you do both."

"Look at the fractional notation in the example. At the end, see the mixed number. Why is the denominator one thousand?" "Because you can only have one denominator in a mixed fraction, so you use the smallest unit or the largest number."

Note

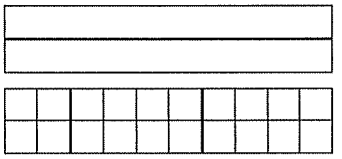
This concept is difficult for many students. Notice that when we read the number 234, we say two hundred thirty-four, but we do not say the unit. What is it two hundred thirty-four of? It is two hundred thirty-four ones. The general rule is that you use the denominator of the smallest unit to write the mixed number. Writing the number as a mixed fraction helps students understand how to read decimal numbers.

Do the first two problems on the worksheet together.
Have her build the number, and read the name of the number, then write the decimal and fractional notation. If she has difficulty reading the decimal number from the blocks, have her write out the fractional notation and the mixed fraction at the end and then read that, then go back and read it from the blocks or the decimal notation. Have her finish the worksheet on her own and then check it with the Answer Key. End the session.

Lesson Part 3

Have her complete Decimal Place Value - Worksheet 3 on her own. Check it immediately.

We are again defining the flat as equal to one.



"Build two tenths." Answer: two rods

"Cover the rods with small cubes."

"When I asked you to build two tenths, you built two rods. Now you have covered those rods with small cubes. How many small cubes does it take to cover two rods?" "Twenty."

"What is the value of those twenty small cubes?" "Twenty hundredths."

Worksheet

Take out Decimal Place Value - Worksheet 4 - Decimal Equivalence

Note

Do not use the calculation method of finding equal fractions on this exercise. Many students have not mastered that concept at this point. We want her to construct it with the blocks to get a firm physical grasp of these equalities.

Embedded Test for Understanding

"Fill in the first equation on the page. You have just built it." $\frac{2}{10} = \frac{20}{100}$ Work down the left hand column of the worksheet.

"Take out two rods. How many chips would it take to cover these two rods?" Let her use her logic. Wait and give her time to think. It will take 200 chips and no one wants to build this. She may be able to figure this out on her own. Have her explain how she knows. If you have a class, let each group come up with a solution and have one person from each group explain their logic to the class.

Optional

Have her write this explanation into her Math Journal.

Important

If you have a class, do not allow the faster students to give the slower ones the answers. It is very important that students have the time to construct understanding here. Sort the students into ability groups to do this work.

Logic and building sequence may go something like this:

"How many small one hundredths cubes will cover the one tenth rod?" "Ten."

"Build this."

"Remove one small cube off one end of the rod."

"How many thousandths chips do you need to replace that one hundredth small cube you just removed?" "Ten."

Build that. Place the little stack of ten thousandths chips on one end of the rod.

"How many thousandth chips are here?" Point to the little pile of chips on the end of the rod. "Ten."

"How many thousandth chips would you need to trade for this next little cube?" "Ten."

"So how many chips would there need to be to trade all the little cubes for chips?" "Hundred."

"So how many chips would there need to be to cover two rods?" "Two hundred."

"If two rods are two tenths, then how many thousandths chips would be needed to cover the two rods?" "Two hundred."

Fill in the equal fraction on the worksheet.

Use a similar strategy for each of the next problems. There are four sets of equal fractions here. If she is having difficulty, do one set each day for the next three

days. Give her time to construct each one so she develops control of this idea.

Worksheets

Decimal Place Value - Worksheets 5 - 7, pages 19 - 21

Test for Understanding

There are two embedded assessments in the lesson itself. Watch to see if she can complete Decimal Place Value - Worksheet 6 without assistance.

Give these three problems for a final test.

1. Write these words on a piece of paper: thirteen and four hundred fifty-two thousandths. Ask her to build this with blocks.
Answer: One large cube, three flats, a decimal bean, four rods, five small cubes, and two chips

2. Write the name of this number: 367.408. Answer: Three hundred sixty-seven and four hundred eight thousandths.

3. Write the above number as a mixed fraction. Answer: $367\ \frac{408}{1000}$

Addition and Subtraction Review

Purpose

The purpose of this group of worksheets is to serve as a warm-up and review at the beginning of a new school year after a long summer break. The worksheets are also used by the lessons on rounding for estimation.

Prerequisites

Previous instruction in addition and subtraction with regrouping

Materials

Gus the Bus, page 22
Addition Puzzle, page 23
Addition: Review, page 24
Subtraction: Review, page 25

Most students will not need instruction on these worksheets with the exception of the directions. Before you assign any of these pages, check the Answer Key so you understand what the student is to do.

Press her to work quickly and accurately. Do not let her dawdle.
Assign one column per day. She will get the most out of frequent short-burst practices than a tedious whole page practice only a single time.
If you find she can not do a certain type of problem, reteach that concept.

Practice Worksheets

You may want to have her work on the speed of her fact recall in addition and subtraction.

Test for Understanding

Check for accuracy and fluency.

Regrouping with Different Units

Purpose

The purpose of this series of lessons is to have the student reconstruct understanding of the general concept of regrouping using units other than base ten units and to review basic units of measurement. He will do problems with units of time and customary units of length, weight, and capacity. Then he will work with metric units of length and weight and see the connection between these and the base ten system.

These lessons do not have to be taught in daily sequence. I liked to use one per week just to keep the students' minds thinking.

Prerequisites

Mastery of subtraction with regrouping and telling time on a clock to one minute. Familiarity with units of length (inches and feet, centimeters and millimeters), weight (pounds and ounces, kilograms, grams, and milligrams), capacity (gallons, quarts, and cups, liters and milliliters). These lessons are a good way to review these units once they have been learned but are not appropriate for teaching these measurement units initially.

Materials

Regrouping with Different Units - Worksheets 1 - 11, pages 26 - 36
A clock with moveable hands
A yardstick and a meter stick*
Clean plastic gallon jug, a quart jug, and a one cup measuring cup*
A two liter plastic soda bottle*
A 10 cc syringe (which can be gotten from a feedstore or vet office)
Metric measuring cups (available through math/science supply houses)
Base Ten Blocks and chips*
Make kilogram and gram weights with water.* Measure out one half of a one liter soda bottle into a gallon Ziploc bag. That will be about one kilogram. If you want to be precise, measure out one hundred, ten cc syringe loads of water into your bag. One cc of water weighs one gram. You can also buy metric weight sets from any math supply house.

*Optional but helpful if the student has not mastered all these units.

Warm Up

Units of Time

Using a clock with moveable hands, ask the student to set the clock to read:
12:00, 12:15, 12:30, 12:45

2:00, 2:05, 2:04, 8:04

3:30, 3:25, 3:23, 3 :17

If he is unable to do these sequences, these worksheets will be too hard. Remediate time-telling before proceeding.

Lesson Part 1

Regrouping with Different Units: Time - Worksheet 1, shows a pair of clocks in

each row. The first clock has the hands drawn in. The student is asked to advance the clock a certain amount of time and then draw in the hands as they would look after that time had passed.

Study the Example problem. Have him set his clock at 2:04. The addition problem to the right asks him to advance the clock thirty minutes. He should also add the thirty minutes to the 2:04 to get 2:34. He then draws in the hands on the clock on the worksheet.

Note

Look at the Answer Key to see how to fill out this worksheet.

Problem 1: Read the clock on the left. It says 8:04. Set your clock to match this. Now ask him to advance the clock forty-five minutes. Give him time to work this out.
Write 8:04 under the 'Write and solve the problem.' Then add forty-five minutes onto 8:04 to get 8:49. That is what he should have set his clock at. He will realize it is easier to do the addition problem than it is to move the hands on the clock.

Finish the worksheet in the same manner. Have him move his clock from the start time to the end time. This will assist him in drawing in the hands of the clock.

Practice Worksheets

Regrouping with Different Units - Worksheets 2 and 3 Worksheet 3 has him subtract minutes using the clock and arithmetic. There is no regrouping on this worksheet.

Units of Time: Part 2

Warm Up

Do two subtraction problems. 129 – 35 = __ Answer: 94
Have him make up one that requires regrouping in both places.

Lesson Part 2

Begin with Regrouping with Different Units - Worksheet 4, page 29

These problems can be quite difficult for students. A clock will not help here. We are doing just straight arithmetic, and we are doing subtraction.

Problems 1 and 2 are not difficult as the number of minutes in the top number is larger than the number of minutes being taken away. This changes in Problem 3.

Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **"How did you get that?" "Show me what you mean." "Build a model of that." "Tell me more so I can understand what you are saying."**

The dialogue may go something like this:
"How can you take thirty-five minutes away from twenty-nine minutes?" Hint: How did you do 129 - 35 in the warm-up? "I regrouped in the tens place and got more tens from the one hundreds place."

"Where can you get more minutes to make the twenty-nine minutes larger?" "I could get it from the hour."

"How many minutes are in an hour?" "Sixty."

"How many minutes would you have if you added that sixty minutes to the twenty-nine minutes?" "Eighty-nine minutes."

"How would you record that?" "Write the eighty-nine minutes above the twenty-nine and cross off the twenty-nine."

"How many hours would you have now since you used one to regroup to sixty minutes?" "I would be down to seven hours. So I should cross off the eight and write seven."

"Can you now subtract thirty-five minutes from eighty-nine minutes?" "Yes, I would have fifty-four minutes left."

"How many hours left?" "7 – 3 equals 4 hours left."

Problem 4 works the same way except you are subtracting minutes and seconds instead of hours and minutes, but the process is the same. He must regroup one minute into sixty seconds and add it to the thirty minutes. When he subtracts the minutes, 55 – 28, he will have to use standard base ten regrouping. This change of bases is what makes these problems tricky.

Note

A common error is for him to regroup one minute from the fifty-six minutes and record the new number of seconds as one hundred thirty. This is a place value error. One minute is not equal to one hundred seconds: instead, it is equal to sixty seconds. So instead of just putting a one (base ten) in front of the three, he must add sixty to thirty to get ninety.

Lesson Part 3

Have him finish the worksheet and use the Answer Key to check his work.

Military Time (or the twenty-four hour clock)
Regrouping with Different Units - Worksheet 5, page 30

It is much, much easier to add and subtract time using Military Time. In Military Time, there is no a.m. and p.m. to cause confusion and arrive on the battlefield twelve hours too early or too late. So after 12:00 p.m., they just keep going up to 13:00, said as thirteen hundred hours. 2:00 p.m. is 14:00.

To convert 8:34 p.m. to Military Time, look on the clock figure. You will see that eight lines up with twenty. So 8:34 p.m. equals 20:34 Military Time.
In problems 1 - 5, the student must change the given times to Military Time and then subtract, or add twelve to the p.m. hours. This is much easier because you can just use standard base ten subtraction. Have him complete the worksheet and check his answers in the Answer Key.

Warm Up

Working with Yards, Feet, and Inches
Using a yardstick, have the student measure how many feet are in a yard by putting a mark on the stick where each foot is if it is not marked on the ruler.
How many feet in a yard? Answer: Three.
How many inches in a foot? Answer: Twelve.
How many inches in a yard? Answer: Thirty-six.

Let him count if he is not sure and then reason out how he could have figured it out with multiplication.

Have him fill out the top of Regrouping with Different Units - Worksheet 6 with these unit conversions.

Lesson Part 4

The first problem has no regrouping. The second requires a regrouping of feet back to inches in order to subtract.

Problem 3 is harder.
"You are short inches here. You have seven and need to take away eleven. Where will you get more inches?" "I will have to regroup a foot."

"How many feet will be left then?" "Four."

"How many inches will you get when you regroup the foot?" "Twelve."

"When you do that, how many inches will you have then?" "Nineteen."
Record those trades. See example below.

"You are now short feet here. You have four left and need to take away five. Where will you get more feet?" "I will have to regroup a yard."

"How many yards will be left then?" "Nine."

"How many feet will you get when you regroup the yard?" "Three."

"When you do that, how many feet will you have then?" "Seven."

"Now you can subtract. What will be the final answer?"

The finished problem will look like this:

		7 ft.	
	9 yds.	~~4 ft.~~	19 inches
	~~10 yds.~~	~~5 ft.~~	~~7 inches~~
	- 3 yds.	5 ft.	11 inches
Answer:	6 yds.	2 ft.	8 inches

Do problem 8 together. A long string may help with this problem. Use scissors and cut as you go to confirm the answer he gets from the arithmetic. Finish the page.

Warm Up

Working with Gallons, Quarts, and Cups

Using a gallon jug, have the student measure how many quarts are in a gallon.
How many cups in a quart? Answer: Four.
How many cups are in a gallon? Answer: Sixteen. Let him measure if he is not sure and then reason out how he could have figured it out with multiplication.

Have him fill out the top of Regrouping with Different Unit - Worksheet 7 with these unit conversions.

Lesson Part 5

The first problem is two gallons, two quarts, and three cups. He must take away first two cups. No problem, one cup is left. Then he must take away three quarts from two quarts. There is a problem here.

"Where will you get more quarts?" "I will have to regroup a gallon."

"How many gallons will be left then?" "One."

"How many quarts will you get when you regroup the gallon?" "Four."

"When you do that, how many quarts will you have then?" "Six."

"What will be the final answer?" "Zero gallons, three quarts, and one cup."

In problem 2, he must regroup both cups and quarts. The finished problem will look like this:

		4 qts.	
	3 gal.	~~0 qts.~~	7 cups
	~~4 gal.~~	~~1 qt.~~	~~3 cups~~
	- 1 gal.	2 qts.	4 cups
Answer:	2 gal.	2 qts.	3 cups

Here is a little song a home school parent named Karen Faller made up for her daughter Selena. It goes to the tune of Happy Birthday to You.

Two cups in a pint
Two pints in a quart,
Four quarts in a gallon,
That's all you need to know.

Practice Worksheets

Finish this page Regrouping with Different Units - Worksheet 7, page 32.
Regrouping with Different Units - Worksheet 8, page 33, works with pounds and ounces.

He may not know how many ounces are in a pound. He can measure this at the grocery store. The process of regrouping is the same except the conversion factor is different. He will now have to regroup pounds to sixteen ounces.

Regrouping with Different Units - Worksheet 9, page 34, works with meters, centimeters, and millimeters. You will need a meter stick, centimeter cubes, and the chips from your Base Ten Blocks. To model millimeters, place the chips on their thin edge. Each chip is one millimeter wide. There are one hundred centimeters in a meter. Those itty-bitty lines between the centimeters are millimeters. There are ten millimeters in a centimeter and one thousand millimeters in a meter.

Regrouping with Different Units - Worksheet 10, page 35, works with liters and milliliters.

Regrouping with Different Units - Worksheet 11, page 36, works with kilograms and milligrams.

Test for Understanding

On all the worksheets, the Test for Understanding is the ability to do the last problem on each worksheet independently.

"Why is the metric system easier to use than the system of customary weights used in England and the United States?"

Possible Answer: The metric system units all work with the base ten system and allow standard decimal procedures to be used to convert units. The customary system uses different bases for each unit. Not only does each measurement system, like yards, feet, and inches, not use base ten, the units internal to the system, like 3 feet to a yard, and then 12 inches to a foot, are not consistent. Not only that, parts of units use fractions, which require even more arithmetic. This makes them hard to convert as anyone who has ever built or cooked anything knows. How many tablespoons are in a cup? You get the picture. See if you can get your student to see it too.

Extension

The reason the United States and England do not use the metric system is that it was developed on the orders of Napoleon, whom the English hated. The French scientists and mathematicians were asked to develop an internally consistent measurement system based on the Earth. So they used the distance from the North Pole to the South Pole as the base for measures of distance, and the weight and volume of water to create the units of weight, volume, and temperature. All the units are connected, easy to convert, and use the base ten system of numeration. Decimals are used for fractional units, not fractions, which are beastly to convert.
All scientists in the world use the metric system. Students in the UK and the US must learn both systems.

Multiplication and Division Review

Purpose

The purpose of this group of worksheets is to provide brush up and practice after a long summer break.

Prerequisites

Patterns in Arithmetic: Multiplication - Booklet 2, Division - Booklet 2, or similar previous instruction in single digit multiplication and division.

Materials

Families of Facts - Worksheets 1 - 3, pages 37 - 39
Multiplication Review: Expanded Numbers, page 40
Multiplication Review: Regrouping Practice, page 41
Division Puzzle - Worksheets 1 and 2, pages 42 and 43
Make Your Own Puzzle, page 44
Multiplication and Division Word Problems, page 45

Be sure you study the Answer Key for these worksheets so you can give your student correct directions.

Lesson

Families of Facts: You may need to do the first set with the student to remind her how these work. The algebraic notation type of problem is taught in *Patterns in Arithmetic*: General Math - Booklet 5 in the lesson on Algebraic Thinking.

Multiplication Review: Expanded Practice: If you are new to this program, this worksheet may not make sense. How to do this type of worksheet is taught in both *Patterns in Arithmetic*: Multiplication - Booklets 2 and 3.

Multiplication Review: Regrouping Practice reviews the standard multiplication procedure. The bottom asks her to identify errors students make in calculation.

Multiplication and Division Word Problems are all two and three step problems. Many students will have difficulty with this worksheet. The Answer Key shows an order of number sentences that can be used to solve the problems.

Most students at this level need to work on the multiplication and division fact tables.

Test for Understanding

Simply speed and accuracy.

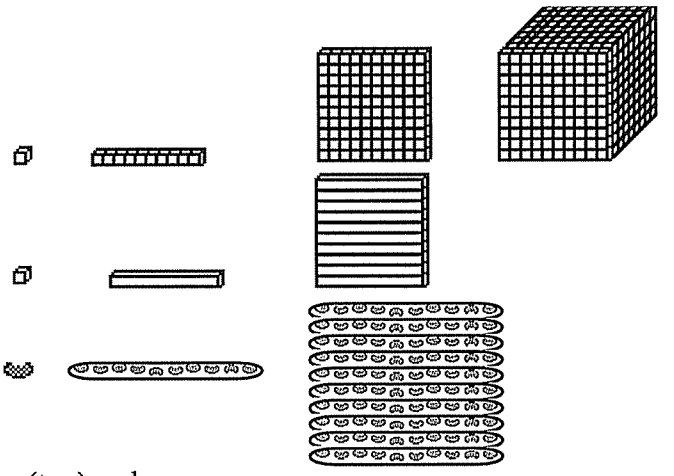
Base Ten Materials

Base Ten Counters

Base Ten Blocks

Cuisenaire Rods

Bean Sticks



Cuisenaire Rods

Use the white (one) and the orange (ten) rods.
For 100 use ten orange rods rubber banded or glued together. DAP™ Fun-Tak Reusable Adhesive can be used to temporarily hold rafts together.

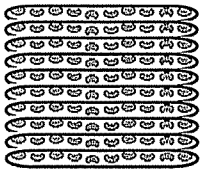
Inexpensive paper cubes representing a thousand can be purchased from ETA Cuisenaire.
ETA-Cuisenaire (800) 445-5985 www.etacuisenaire.com
Or cut out and fold a 1,000 cube using 10 x 10 x 10 centimeter dimension. Be precise. This is a good art project.

Base Ten Bean Sticks

An inexpensive alternative is to make base ten bean sticks.

Materials: small beans, popsicle sticks, glue

Activity: Glue ten beans to a popsicle stick. Make about twenty of these.



Rafts can be made to represent 100 with ten bean sticks and rubber bands, or glue ten sticks to a cardboard square.

Patterns in Arithmetic

Place Value: Booklet 3

Into the Millions, Exponential Notation,
and Operations Review

Answer Key for the Student Workbook

By Suki Glenn, Susan Carpenter, and Alysia Krafel

Answer Key Legend

AWV = answer(s) will vary
BUWV = break up will vary
OWV = order will vary

Pattern Blocks
r = red trapezoid
g = green triangle
y = yellow hexagon
o = orange square
b = blue parallelogram
t = tan rhombus

Cuisenaire Rods
1 w = white
2 r = red
3 lg = light green
4 p = purple
5 y = yellow
6 dg = dark green
7 bk = black
8 bn = brown
9 bl = blue
10 o = orange

Note: Some items and pages are left out of the answer key.
1) Some pages in which the answers are open-ended or will vary.
2) Make your own problems. Since students create their own problems and solutions, these sections give valuable information about the level of confidence and competence. It can be a useful source of curriculum for other students.
3) Practice pages.
4) Workboards.
5) The answers are in the Parent/Teacher Guide.

Patterns in Arithmetic: Place Value - Booklet 3
Answer Key for the Student Workbook

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Place Value - Booklet 3

Vocabulary Words - Worksheet 1

Sum is the answer to a addition problem.

Difference is the answer to a subtraction problem.

Product is the answer to a multiplication problem.

Quotient is the answer to a division problem.

Food is the answer to a awv problem.

_____ is the answer to a _____ problem.

The divisor is the size of the group being made.

The dividend is the amount being divided. The amount started with.

A Prime Number is a whole number > one whose only two whole number factors are it self and one.

A factor is numbers multiplied together to get another number. factor x factor = product

The sum of 2 numbers is 12.
What could these numbers be? 7+5, 2+10, 3+9, 4+8, 6+6, 1+11, 12+0

The difference between 12 and 9 is 3. 12 - 9 = 3.

The product of 2 numbers is 12.
What could these numbers be? 6x2, 3x4, 1x12

The quotient of 12 divided by 3 is 4. 12 ÷ 3 = 4

Calculate the product of 5 and 3. 15 5x3

Take the difference between 5 and 3 from the product of 5 and 3. 13

(5x3) - (5-3)
15 - 2 = 13

The quotient of 24 and 6 is how much less than the product of 24 and 6? 140

(24x6) - (24÷6)
144 - 4 = 140

Compare the product of 24 and 6 to the product of 12 and 12. What do you notice? They are equal, 144

24x6 = 144 12x12 = 144

1

Vocabulary Words - Worksheet 2

Prefixes

What do these prefixes mean? List at least three words using the prefix.

tri- = three tricycle, triangle, triplet, trident

bi- = two bisect, binoculars, bicycle

quadri- = four quadtrangle, quadrilateral, quadrant

octa- or octo- = eight octagon, octave, octopus

quint- = five quintuplets, quintet, quintillion

hex- = six hexagon, hexapod, hexagram

dec- or deca- = ten decade, decimals, decagon

2

Prime Factoring: Review - Worksheet 1

Find the prime factors of all the composite numbers from 4 to 16. List the prime factors in the box. Remember to put them in order from largest to smallest. Put commas between the prime factors.

4 → 2,2 (2x2)

6 → 3,2 (x)

8 → 2,2,2 (4x2, 2x2)

9 → 3,3 (x)

10 → 5,2 (5x2)

14 → 7,2 (7x2)

15 → 5,3 (5x3)

16 → 2,2,2,2 (2x8, 4x2, 2x2) or (4x4, 2x2, 2x2)

3

Prime Factoring: Review - Worksheet 2

Continue factoring. Remember to put the prime factors in order from largest to smallest. Put commas between the prime factors.
Example:

24 → 3,2,2,2 (2x12, 4x3, 2x2)

30 → 5,3,2 (2x15, 3x5)

20 → 5,2,2 (5x4, 2x2)

22 → 11,2 (2x11)

25 → 5,5 (5x5)

32 → 2,2,2,2,2 (2x16, 2x8, 2x4, 2x2)

27 → 3,3,3 (3x9, 3x3)

4

Prime Factoring: Review - Worksheet 3

Continue factoring. Remember to put the prime factors in order from largest to smallest. Put commas between the prime factors.

48 → 24 x 2 → 12 x 2 → 4 x 3 → 2 x 2 → **3,2,2,2,2**

56 → 14 x 4 → 7 x 2, 2 x 2 → **7,2,2,2**

21 → 3 x 7 → **7,3**

36 → 4 x 9 → 2 x 2, 3 x 3 → **3,3,2,2**

81 → 9 x 9 → 3 x 3, 3 x 3 → **3,3,3,3**

50 → 25 x 2 → 5 x 5 → **5,5,2**

Make your own. AWV

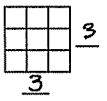
64 → 16 x 4 → 8 x 2, 2 x 2 → 2 x 4 → 2 x 2 → **2,2,2,2,2,2**

__ x __ → **__**

Exponents and Prime Factors - Worksheet 1

Build this square with tiles.

The sides of this square are 3 units. Find the area and write the area in exponential form.

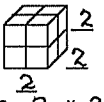


Area = 3 x 3 or 3^2 equals 9 square units

3^2 is read "3 to the second power or 3 to the power of two or three squared."

Build this prism with cubes.

The sides of this cube are 2 units. Find the volume and write the volume in exponential form.



Volume = 2 x 2 x 2 or 2^3 ← exponential form equals 8 cubic units

2^3 is read "2 to the third power or 2 to the power of three or two cubed."

Write the numeral in exponential form.

1. 4 squared 4^2
2. 3 to the fourth power 3^4
3. 5 cubed 5^3
4. 2 to the power of 5 2^5
5. 3 to the power of four 3^4
6. 10 cubed 10^3

Write the words for each expression.

7. 5^2 five squared*
8. 6^3 six to the fourth power
9. 8^4 eight to the fourth power
10. 10^3 ten cubed

* five to the second power

Write in factor form and calculate.

11. 5^3 5 x 5 x 5 = 125
12. 4^2 4x4 = 16
13. 3^2 3x3 = 9
14. 2^4 2x2x2x2 = 16
15. 6^4 6x6x6x6 = 1,296
16. 8^3 8x8x8 = 512
17. 2^6 2x2x2x2x2x2 = 64
18. 10^6 10x10x10x10x10x10 = 1,000,000

Write each number in factor form and exponential form with a base of 10.

19. 100 = 10 x 10 = 10^2
20. 1,000 = 10 x 10 x 10 = 10^3
21. 10,000 = 10 x 10 x 10 x 10 = 10^4
22. 1,000,000 = 10 x 10 x 10 x 10 x 10 x 10 = 10^6

Exponents and Prime Factors - Worksheet 2

The prime factors of 8 are 2, 2, 2.

8 → 2 x 4 → 2 x 2

Another way of writing 2 x 2 x 2 is with an exponent.

$2 \times 2 \times 2 = 2^3$ ← exponent

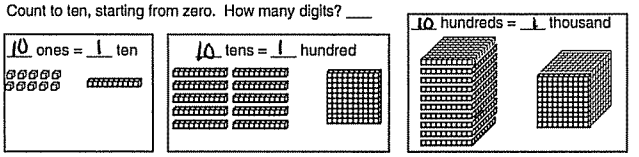
What does the exponent 3 mean in 2^3? multiply 2 3 times or 2x2x2

List the prime factors of each number in prime factor form and exponential form.

1. 12 → (2) x 6 → (3) x (2)
   Prime factor form 3,2,2
   Exponential form $2^2 \times 3$
2. 18 → 3 x 6 → 2 x 3
   Prime factor form 3,3,2
   Exponential form $3^2 \times 2$
3. 24 → 2 x 12 → 3 x 4 → 2 x 2
   Prime factor form 3,2,2,2
   Exponential form $2^3 \times 3$
4. 36 → 3 x 12 → 3 x 4 → 2 x 2
   Prime factor form 3,3,2,2
   Exponential form $2^2 \times 3^2$
5. 40 → 2 x 20 → 10 x 2 → 5 x 2
   Prime factor form 5,2,2,2
   Exponential form $2^3 \times 5$
6. 48 → 4 x 12 → 2 x 2, 3 x 4 → 2 x 2
   Prime factor form 3,2,2,2,2
   Exponential form $2^4 \times 3$
7. 45 → 5 x 9 → 3 x 3
   Prime factor form 5,3,3
   Exponential form $3^2 \times 5$
8. 27 → 3 x 9 → 3 x 3
   Prime factor form 3,3,3
   Exponential form 3^3
9. 32 → 2 x 16 → 4 x 4 → 2 x 2, 2 x 2
   Prime factor form 2,2,2,2,2
   Exponential form 2^5

Place Value: Base Ten - Worksheet 1

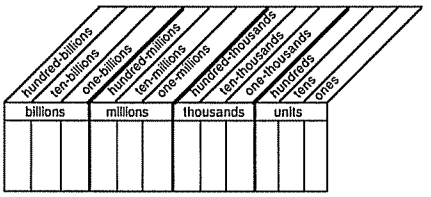
Count to ten, starting from zero. How many digits? __



Each time you count out ten of one size of block, you trade it for 1 of the next larger block.

This chart shows the names of the first twelve places. The place in which a digit is located determines its place value.

A 5 in the hundreds' place means 500.
A 5 in the thousands' place means 5,000.



Give the place value name and value of each underlined digit.

1. 4**2**,000,000 — one million, 2,000,000
2. **2**7,000,000 — ten millions, 20,000,000
3. 1**7**4,000 — ten thousands, 70,000
4. **9**,350 — one thousand, 9,000
5. **4**,540,340,200 — one billion, 4,000,000,000
6. **8**03,403,370,300 — hundred billion, 800,000,000,000
7. 9,3**4**7 — tens, 40
8. 4**5**0,235 — hundred thousand, 400,000
9. 6**5**,349,300 — one million, 5,000,000

Place Value: Base Ten - Worksheet 2

Write the numerals for the words on this
1. three hundred eighty-two
2. four thousand five hundred sixty-three
3. nine million one hundred thousand
4. fifteen billion nine million
5. four trillion, nine hundred billion
6. two million five
7. thirty million sixty thousand two hundred
8. sixty thousand three hundred twenty-one
9. two trillion, three hundred fifty billion
10. Make your own. AWV

	TRILLION			BILLION			MILLION			THOUSAND			UNITS		
1.													3	8	2
2.												4	5	6	3
3.									9	1	0	0	0	0	0
4.					1	5	0	0	9	0	0	0	0	0	0
5.			4	9	0	0	0	0	0	0	0	0	0	0	0
6.									2	0	0	0	0	0	5
7.								3	0	0	6	0	2	0	0
8.											6	0	3	2	1
9.			2	3	5	0	0	0	0	0	0	0	0	0	0
10.															

Write the word names for the numerals.

11. 230,300,000 two hundred thirty million, three hundred thousand
12. 6,390,000,090 six billion, three hundred ninety million, ninety
13. 2,470,000 two million, four hundred seventy thousand
14. 67,900,000 sixty-seven million, nine hundred thousand
15. 9,320,000,000 nine billion, three hundred twenty million

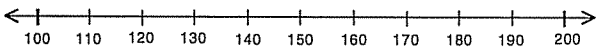
9

Exponents and Multiples of Ten

ten	10^1	10
one hundred	10^2	100
one thousand	10^3	1,000
ten thousand	10^4	10,000
one hundred thousand	10^5	100,000
one million	10^6	1,000,000
ten million	10^7	10,000,000
one hundred million	10^8	100,000,000
one billion	10^9	1,000,000,000
ten billion	10^{10}	10,000,000,000
one hundred billion	10^{11}	100,000,000,000
one trillion	10^{12}	1,000,000,000,000
ten trillion	10^{13}	10,000,000,000,000
one hundred trillion	10^{14}	100,000,000,000,000
one quadrillion	10^{15}	1,000,000,000,000,000
ten quadrillion	10^{16}	10,000,000,000,000,000
one hundred quadrillion	10^{17}	100,000,000,000,000,000
one quintillion	10^{18}	1,000,000,000,000,000,000
ten quintillion	10^{19}	10,000,000,000,000,000,000
one hundred quintillion	10^{20}	100,000,000,000,000,000,000
one hextillion	10^{21}	1,000,000,000,000,000,000,000

10

Rounding Off - Worksheet 1



Use the number line above. Put your finger on the 130. Is your finger closer to 100 or to 200? ____ Put your finger on the 180. Is your finger closer to 100 or to 200? ____

These numbers are correctly rounded off to the nearest hundred.

180 → 200 140 → 100 150 → 200

615 → 600 342 → 300 265 → 300

These numbers are incorrectly rounded off to the nearest hundred. ↛ means does not

425 ↛ 500 612 ↛ 610 458 ↛ 400

782 ↛ 600 225 ↛ 100 175 ↛ 300

Which of these are correctly rounded off to the nearest hundred? Circle the correct ones. Write the correct answer to problems that are not correct.

264 → 300 125 → 100 483 → 500

362 → 360 (400) 865 → 900 728 → 800 (700)

Round off these numbers to the nearest hundred. AWV

952 → 1,000 678 → 700 ____ → 700

854 → 900 245 → 200 ____ → 500

827 → 800 1,262 → 1,300 ____ → ____

What is the rule for rounding off to the nearest hundred?
Go to the hundreds place and look to the digit to the right. If that digit is five or greater, round up to the higher number. If the digit to the right is less than five, change the remaining digits to zeros.

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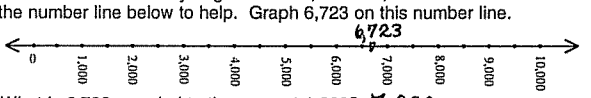
Rounding Off - Worksheet 2

'About how many' is another way of estimating answers. Rounding off is a way to find out 'about how many'. For example, 6,723 dogs live in my town. To tell about how many dogs there are, round 6,723 to the nearest thousand. the number line below to help. Graph 6,723 on this number line.

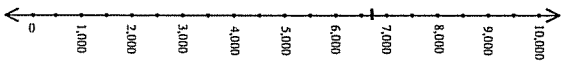


What is 6,723 rounded to the nearest 1,000? 7,000

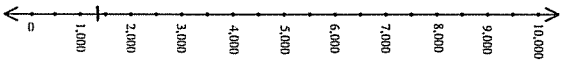
So about 7,000 dogs live in my town.

Graph each number on the number line. Then fill in the chart.

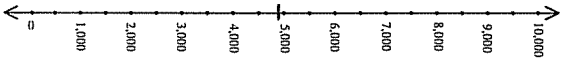
6,836



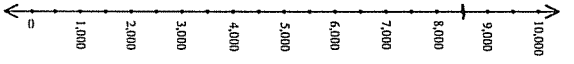
1,436



4,927



8,500



Fill in the chart to round each number to the nearest thousand.

	comes between			halfway number	rounds to
6,723	6,000	and	7,000	6,500	7,000
6,836	6,000	and	7,000	6,500	7,000
1,436	1,000	and	2,000	1,500	1,000
4,927	4,000	and	5,000	4,500	5,000
8,500	8,000	and	9,000	8,500	9,000

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Rounding Off - Worksheet 3

Rounding to the greatest place means to look at the place value of the highest number. For example, in the number 825, the highest value number is the 8 in hundreds place. So 825 rounds to 800. This Kind of rounding is used to estimate answers to arithmetic problems.

825	→ 800		6,493	→ 6,000	
67	→ 70		821	→ 800	
+ 1,250	→ 1,300		+ 4,658	→ 5,000	
2,142	2,170	approximate	11,972	11,800	approximate
actual sum	estimated sum about 2,000		actual sum	estimated sum about 12,000	

Rule for rounding to any place Fill in the blanks.

1. Identify the digit in the place to be rounded. Put a check mark above it. ✓
2. Look at the digit to the right of that number and underline it.
   a) If that digit is 5 or greater, increase the ✓ digit by 1 and change all the digits to the right of the ✓ digit to zero.
   b) If that digit is less then 5, the ✓ digit stays the same and all the digits to the right of the ✓ digit are changed to zero.

Put a ✓ over the digit in the place being rounded. Round the number to the place value indicated.

nearest ten	nearest hundred	nearest thousand
52 50	275 300	8,734 9,000
234 230	961 1,000	6,072 6,000
682 680	7,941 7,900	10,634 11,000
3,714 3,710	5,342 5,300	37,243 37,000
1,267 1,270	15,350 15,400	348,850 349,000

Patterns in Sums: Column Addition

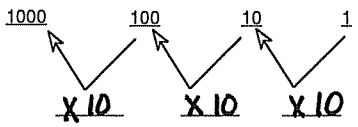
27 + 34 + 84 + 43 + 61 + 79 = 328	784 + 528 + 517 + 272 + 133 + 96 = 2,330	690 + 806 + 354 + 496 + 358 = 2,704
427 + 469 + 676 + 483 + 231 = 2,286	338 + 876 + 352 + 444 + 155 = 2,165	529 + 783 + 527 + 723 + 381 = 2,943
526 + 484 + 350 + 653 = 2,013	685 + 175 + 680 + 932 = 2,472	264 + 630 + 856 + 275 = 2,025
285 + 571 + 175 + 835 + 534 = 2,400	564 + 684 + 472 + 476 + 796 = 2,992	2,434 + 5,618 + 8,252 + 2,290 + 3,676 = 22,270

Decimal Place Value - Worksheet 1

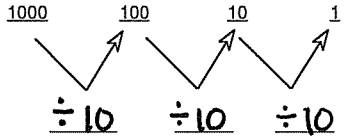
If I were to write the number **4,357**, what does the **5** mean? 5 tens or 50

How do you know it is **50** and not **5**, **5** hundred or **5** thousand? place in number

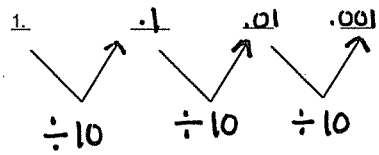
By where it is, it's place in the number. This is what we mean by Place Value.



What do you have to do to change?:
- 1 into 10? X 10
- 10 into 100? X 10
- 100 into 1000? X 10



What if we start at 1000 and go backwards?:
- 1000 into 100? ÷ 10
- 100 into 10? ÷ 10
- 10 into 1? ÷ 10



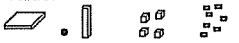
What if we start at 1 and continue backwards? Fill in the blanks with numerals. Draw in the arrows.

Decimal Place Value - Worksheet 2

This block is one. ▱

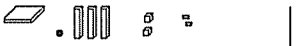
Build with Base Ten Blocks.

Write the value.



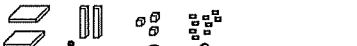
Decimal 1 . 1 4 7

Fraction $\frac{1}{1} + \frac{1}{10} + \frac{4}{100} + \frac{7}{1000} = 1\frac{147}{1000}$



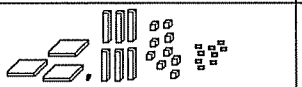
1 . 3 2 2

$1 + \frac{3}{10} + \frac{2}{100} + \frac{2}{1000} = 1\frac{322}{1000}$



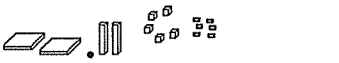
2 . 2 3 9

$2 + \frac{2}{10} + \frac{3}{100} + \frac{9}{1000} = 2\frac{239}{1000}$



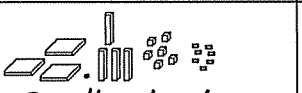
3 . 6 9 9

$3 + \frac{6}{10} + \frac{9}{100} + \frac{9}{1000} = 3\frac{699}{1000}$



2 . 2 5 6

$2 + \frac{2}{10} + \frac{5}{100} + \frac{6}{1000} = 2\frac{256}{1000}$



3 . 4 6 6

$3 + \frac{4}{10} + \frac{6}{100} + \frac{6}{1000} = 3\frac{466}{1000}$

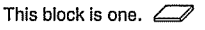
4 . 5 3 9

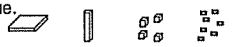
$4 + \frac{5}{10} + \frac{3}{100} + \frac{9}{1000} = 4\frac{539}{1000}$

Make your own. Draw the blocks.

1.273

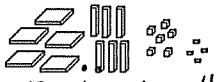
Decimal Place Value - Worksheet 3

This block is one. 

Write the value. 

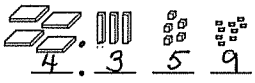
Decimal $\frac{1}{1}.\frac{1}{}\ \frac{4}{}\ \frac{7}{}$

Fraction $\frac{1}{1}+\frac{1}{10}+\frac{4}{100}+\frac{7}{1000}$



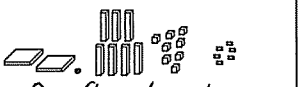
$5.6\ 6\ 4$

$5+\frac{6}{10}+\frac{6}{100}+\frac{4}{1000}=5\frac{664}{1,000}$



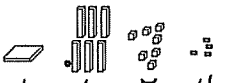
$4.3\ 5\ 9$

$4+\frac{3}{10}+\frac{5}{100}+\frac{9}{1000}=4\frac{359}{1,000}$



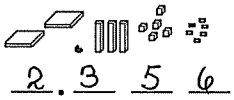
$2.8\ 6\ 6$

$2+\frac{8}{10}+\frac{6}{100}+\frac{6}{1000}=2\frac{866}{1,000}$



$1.6\ 7\ 4$

$1+\frac{6}{10}+\frac{7}{100}+\frac{4}{1000}=1\frac{674}{1,000}$



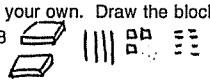
$2.3\ 5\ 6$

$2+\frac{3}{10}+\frac{5}{100}+\frac{6}{1000}=2\frac{356}{1,000}$

$3.4\ 0\ 7$

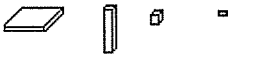
$3+\frac{4}{10}+\frac{0}{100}+\frac{7}{1000}=3\frac{407}{1,000}$

Make your own. Draw the blocks.

2.438 

Decimal Place Value - Worksheet 4

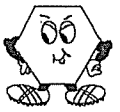
Decimal Equivalence

1 $\frac{1}{10}$ $\frac{1}{100}$ $\frac{1}{1000}$ 

$\frac{2}{10}=\frac{20}{100}$	$\frac{5}{10}=\frac{50}{100}$
$\frac{2}{10}=\frac{200}{1000}$	$\frac{5}{10}=\frac{500}{1000}$
$\frac{2}{100}=\frac{20}{1000}$	$\frac{5}{100}=\frac{50}{1000}$
$\frac{7}{10}=\frac{70}{100}$	$\frac{4}{10}=\frac{40}{100}$
$\frac{7}{10}=\frac{700}{1000}$	$\frac{4}{10}=\frac{400}{1000}$
$\frac{7}{100}=\frac{70}{1000}$	$\frac{4}{100}=\frac{40}{1000}$

Decimal Place Value - Worksheet 5

Find the decimal numerals that match the word. Write the letter of the matching words on the line above the numeral. Find the answer to this riddle.

What is a polygon? 

D six hundredths

R four and twelve thousandths

E nine

A two and two tenths

R seventeen hundredths

T five and five hundredths

D sixty-five thousandths

A ninety-nine hundredths

P two and two hundredths

A sixty-one and four tenths

O one and six tenths

A	D	E	A	D	
2.2	.06	9	.99	.065	
P	A	R	R	O	T
2.02	61.4	4.012	.17	1.6	5.05

Decimal Place Value - Worksheet 6

The number 1.3426 is made of:
1 one unit, 3 tenths, 4 hundredths, 2 thousandths, 6 ten thousandths

ones	tenths	hundredths	thousandths	ten thousandths
1	3	4	2	6

Fill in the place value chart.

decimal point ↓

	ten thousands	thousands	hundreds	tens	ones	tenths	hundredths	thousandths	ten thousandths
5.2318					5	2	3	1	8
34.582				3	4	5	8	2	
203.846			2	0	3	8	4	6	
18352.8	1	8	3	5	2	8			
0.8475						8	4	7	5
30483.6	3	0	4	8	3	6			
39.4908				3	9	4	9	0	8

Write the place name for each underlined digit.

45.3<u>9</u>4 thousandths

239.0<u>4</u>9 hundreds

4903.9<u>4</u>7 hundredths

293.<u>8</u>562 ones (units)

23.<u>0</u>987 tenths

5.9<u>4</u>83 hundredths

28<u>9</u>.09 tens

2.33<u>9</u>4 ten thousandths

1.<u>9</u>836 tenths

23<u>4</u>.87 ones or units

Decimal Place Value - Worksheet 7

Sometimes decimals are used to show how long a line is. When measuring with an inch ruler, fractions of inches are used. A line might be 3 ½ inches long. When using the metric system, decimal fractions are used to show parts of a unit, for example, 4.5 centimeters.

Use a centimeter ruler and a sharp pencil to complete this exercise.

Example: Draw a line that is exactly 5 centimeters and 3 millimeters long.

You can say this line is 53 millimeter long. Or you can say the line is 5.3 centimeters long.

How many millimeters are in a centimeter? 10

What fraction of a centimeter is one millimeter? $\frac{1}{10}$

1. Draw a line that is 10 centimeters and 7 millimeters long.
Measure the student's line.

This line is 107 mm or 10.7 cm Challenge! .107 meters

2. A line that is 1 meter long is 100 centimeters long and 1,000 millimeters long.

3. What fraction of a meter is a centimeter? $\frac{1}{100}$ Write this number as a decimal. 1 cm = .01 meter

4. Draw a line that is 12 centimeters and 6 millimeters long.
Measure the line.

This line is 126 mm or 12.6 cm or .126 meters long.

5. 12 mm = .012 meters

6. 45.7 cm = 457 mm

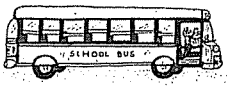
7. 157 centimeters = 1.57 meters

8. 89 mm = 8.9 cm

9. 999 mm = .999 meters

10. 3.4 meters = 340 cm

Gus the Bus Driver



1. Gus, the bus driver, picks up 48 children at the first stop. Then he picks up 21 more at the second stop. At the next stop 18 children get on the bus. How many children are there now? 87
Write the number sentence. 48 + 21 + 18 = 87

2. At the next stop 36 children get off the bus. How many are left on the bus? 51 Write a number sentence. 87 - 36 = 51

3. At the next stop 27 children get off the bus and 16 get on the bus.
How many are left on the bus? 40
Write the number sentence. (51 - 27) + 16 = 40

4. At the next stop 18 children get off the bus and 24 get on.
How many are left on the bus? 46
Write the number sentence. (40 - 18) + 24 = 46

5. At the next stop 27 children are left on the bus after 19 children get off. How many children get off the bus? 19
Write the number sentence. 46 - 27 = 19

6. If Gus, the bus driver, starts with 104 children, and 39 children get off at one stop, 45 get off at the next stop, how many children will get off at the last stop? 20
Write the number sentence. (104 - 39) - 45 = 20
or 39 + 45 = 84 104 - 84 = 20

Addition Puzzle

If the sum has this number in the hundreds place: Color the space:

4 →

5 →

6 →

7 →

187 + 276 = 463	258 + 487 = 745	373 + 163 = 536	398 + 244 = 642
586 + 156 = 742	199 + 205 = 404	587 + 94 = 681	286 + 237 = 523
329 + 295 = 624	168 + 395 = 563	439 + 308 = 747	151 + 259 = 410
377 + 147 = 524	473 + 150 = 623	182 + 258 = 440	238 + 485 = 723

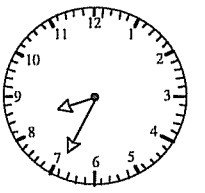
Addition: Review

532 + 346 = 878	374 + 259 = 633	836 + 376 = 1,212	912 + 788 = 1,700
9,832 + 4,376 = 14,208	7,647 + 2,567 = 10,214	2,356 + 3,984 = 6,340	8,632 + 532 = 9,164
5,684 + 4,698 = 10,382	9,630 + 4,872 = 14,502	3,456 + 7,456 = 10,912	8,469 + 4,829 = 13,298
23,418 + 3,987 = 27,405	92,435 + 3,058 = 95,493	34,562 + 90,768 = 125,330	23,456 + 95,837 = 119,293
43,678 + 49,586 = 93,264	96,547 + 3,948 = 100,495	34,529 + 24,938 = 59,467	34,864 + 59,658 = 94,522
67,328 + 24,878 = 92,206	78,413 + 4,438 = 82,851	985,762 + 654,784 + 987,945 = 2,628,491	567,834 + 874,563 + 456,298 = 1,898,695

Subtraction Review

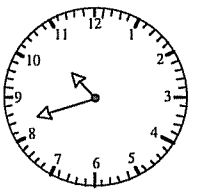
28 − 4 = 24	38 − 17 = 21	38 − 15 = 23	43 − 27 = 16	60 − 15 = 45
137 − 25 = 112	149 − 136 = 13	237 − 123 = 114	154 − 38 = 116	200 − 178 = 22
318 − 15 = 303	427 − 219 = 208	262 − 93 = 169	543 − 175 = 368	300 − 168 = 132
1.456 − .025 = 1.431	2.464 − .232 = 2.232	4.423 − .117 = 4.306	3.453 − .379 = 3.074	6.000 − .785 = 5.215
12,452 − 321 = 12,131	23,586 − 1,477 = 22,109	15,850 − 12,682 = 3,168	71,000 − 16,495 = 54,505	

Regrouping with Different Units: Time - Worksheet 1



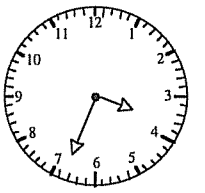
Show 30 minutes earlier. Draw the clock hands. This is how you would record it.

8:34
− :30
8:04



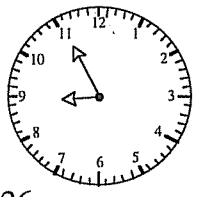
Show 30 minutes earlier. Draw the clock hands. Write and solve the problem.

10:42
− :30
10:12



Show 25 minutes earlier. Draw the clock hands. Write and solve the problem.

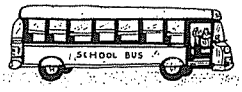
3:34
−:25
3:09



Show 35 minutes earlier. Draw the clock hands. Write and solve the problem.

8:55
− :35
8:20

Regrouping with Different Units: Time - Worksheet 2



1. Gus, the bus driver, picks up 36 students at the first stop. Then he picks up 14 more at the second stop. At the next stop 24 students get on the bus. How many students are there now? 74
   Write the number sentence. 36 + 14 + 24 = 74

2. Gus starts picking up students at 6:20 am. He has to get to the second stop in 15 mintues. What time do the children at the second stop have to be at the bus stop? 6:35
   Write a number sentence 6:20 + :15 = 6:35

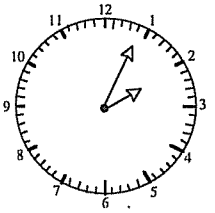
3. He has to get to the third stop in 10 mintues. What time do the students at the third stop have to be at the bus stop? 6:45
   Write a number sentence 6:35 + :10 = 6:45

4. If he gets to the fourth stop by 7:03, how long did it take him to get there? 18 minutes
   Write a number sentence 7:03 − 6:45 = 18

5. What time is it now? 7:03
   Write a number sentence 7:03

6. It takes Gus 9 minutes to get to the next stop. Five students race to bus stop and get there at 7: 14. Are they on time if he waits 3 minutes at the stop? yes
   Write a number sentence 7:03 + :09 = 7:12 + :03 = 7:15

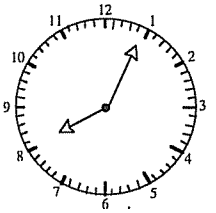
Regrouping with Different Units: Time - Worksheet 3



Example:

2:04
+ :30
2:34

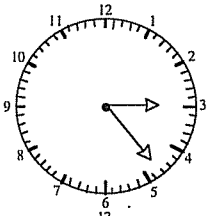
Show 30 minutes later. Draw the clock hands.



Show 45 minutes later. Draw the clock hands.

Write and solve the problem.

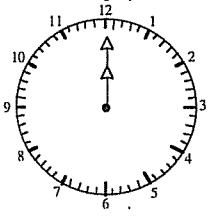
8:04
+:45
8:49



Show 25 minutes later. Draw the clock hands.

Write and solve the problem.

3:23
+:25
3:48



Show 35 minutes later. Draw the clock hands.

Write and solve the problem.

12:00
:35
12:35

Regrouping with Different Units: Time - Worksheet 4

Write the answer in simplest form.

1. 4 hours 45 minutes
   - 1 hour 23 minutes
   3 22

2. 7 hours 43 minutes
   - 4 hours 25 minutes
   3 18

3. 8 hours 29 minutes
   - 3 hours 35 minutes
   4 54

4. 56 Minutes and 30 seconds
   - 28 Minutes and 40 seconds
   27 50

5. 11 hours ___ minutes
   - 7 hours 35 minutes
   4 hours 10 minutes

6. 8 hours 14 minutes
   - 5 hours 35 minutes
   2 hours 39 minutes

7. A small turkey cooks for 2 hours and 15 minutes. It has been in for 1 hour and 50 minutes. How much longer should it bake? 25 minutes

Write the equation and answer. 2 hours 15 minutes
1 50
25 minutes

29

Regrouping with Different Units: Yards, Feet and Inches - Worksheet 6

1 foot = 12 inches
1 yard = 3 feet = 36 inches
Write the answer in simplest

1. 5 yards 2 feet 8 inches
   - 2 yards 1 foot 5 inches
   3 1 3

2. 6 yards 6 feet 2 inches
   - 2 yards 4 feet 8 inches
   4 1 6

3. 10 yards 5 feet 7 inches
   - 3 yards 5 feet 11 inches
   6 2 8

4. 7 yards 1 foot 6 inches
   - 4 yards 2 feet 8 inches
   2 1 10

5. 4 yards 2 feet 3 inches
   - 1 yards 4 feet 10 inches
   2 0 5

Write the equation and answer.

6. A truck is 9 feet 6 inches high. The bottom of the overpass is 12 feet 9 inches. How much clearance will be between the truck and the overpass? 3' 3"
   12'9" = 9'6" 3 feet 3 inches

7. Fred threw a football 20 yards 1 foot and 5 inches. Lynn threw a football 15 yards 2 feet and 10 inches. How much farther did Fred throw the football?
   20 yards 1 foot 5 inches
   -15 4 yd 2 1 ft 7 in.

8. Wendy has lace 3 yards 1 foot and 6 inches long. The blouse she is making needs 1 foot and 8 inches of lace.
   Will she have enough lace? yes The skirt needs 2 yards and 10 inches of lace.
   Will there be any lace left over? yes How much? 2 feet
   A matching srunchy needs 18 inches of lace. Will she have enough lace for the srunchy? yes

   2 3yd 43 2ft. +12 6in.
   -2 1 18
   2 0

   1ft 8in
   +2yd 0ft. 10in
   2 1 18

31

Regrouping with Different Units: Pounds and Ounces Worksheet 8

1 pound = 16 ounces Write the answer in simplest form.

1. 22 pounds 8 ounces
   - 13 pounds 6 ounces
   9 2

2. 23 pounds 1 ounces
   - 2 pounds 8 ounces
   20 9

3. 34 pounds 3 ounces
   - 22 pounds 15 ounces
   11 4

4. 32 pounds 4 ounces
   - 17 pounds 13 ounces
   14 7

5. 28 pounds 4 ounces
   - 19 pounds 9 ounces
   8 11

6. Make your own. AWV
   ___ pounds ___
   - ___ pounds ___
   5 pounds 0 ounces
   -2 6
   2 lb. 10 oz.

Write the equation and answer.

7. Gracie makes the best butter cookies. She has 5 pounds of butter. Her giant cookie recipe calls for 2 pounds and 6 ounces of butter
   How much butter will she have left over? 2 pounds 10 ounces
   Does she have enough butter to make a double recipe? yes

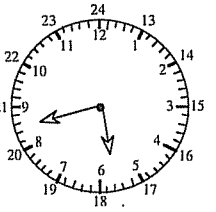
33

Regrouping with Different Units: Military Time - Worksheet 5

How many hours and minutes are between sunrise at 6:00 a.m. and sunset at 5:00 p.m.?
A good way to figure this out is to convert afternoon times to military or 24 hour time.

5: 00 P.M. = 17:00
Then subtract 6:00 a.m.
11 hours
Convert these times:
8:34 p.m.= 20:34
10:56 p.m. 22:56
Solve.

17:00
- 6:00
11:00



1. 4:46 p.m. = 16:46
   - 9:29 a.m. = - 9:29
   7:17

2. 10: 00 p.m. = 22:00
   - 2:30 a.m. = - 2:30
   19:30

3. 3:56 p.m. = 15:56
   - 4:49 a.m. = - 4:49
   11:07

4. 11:44 p.m. = 23:44
   - 4: 37 a.m. = - 4:37
   19:07

5. Kaitlin and Lindsay wanted to watch the meteor shower at 1:25 a.m. They went to bed at 5:45 p.m. How many hours of sleep did they get? 8:20
   17:45
   - 1:25
   8:20

30

Regrouping with Different Units: Gallons, Quarts and Cups - Worksheet 7

1 gallon = 4 quarts 1 quart 4 cups
Write the answer in simplest form.

1. 2 gallons 2 quarts 3 cups
   - 1 gallons 3 quarts 2
   3 1

2. 4 gallons 1 quart 3 cups
   - 1 gallons 2 quarts 4
   2 2 3

3. 8 gallons 2 quarts 1 cups
   - 2 gallons 5 quarts 3
   5 0 2

4. 3 gallons 3 quarts 2 cups
   - 2 gallons 3 quarts 4
   0 3 2

5. 5 gallons 3 quarts 1 cups
   - 1 gallons 3 quarts 3
   3 3 2

6. Make your own
   __ gallons __ quarts __
   - __ gallons __ quarts __
   2 gal. 1 qt. 0 cups
   1 2 2
   2 qt. 2 cups

Write the equation and answer.

7. Kate is making hot cocoa for a slumber party. She has 2 gallons plus one quart of milk. She mixed up 6 quarts and 2 cups of hot cocoa. How much milk did she have left? 2 quarts 2 cups

32

Regrouping with Different Units: Meters, Centimeters and Millimeters - Worksheet 9

1 meter = 100 centimeters = 1,000 millimeters
Write the answer in simplest form.

1. 9 cm 4 mm
   - 3 cm 9 mm
   5 5

2. 6 m 12 cm 3 mm
   - 3 m 23 cm 9 mm
   2 88 4

3. 8 m 63 cm 2 mm
   - 2 m 25 cm 7 mm
   6 36 5

4. 57 m 36 cm 5 mm
   - 25 m 80 cm 9 mm
   31 55 6

5. 224 m 78 cm 3 mm
   - 113 m 94 cm 6 mm
   110 83 7

___ m ___ cm ___ mm AWV
- ___ m ___ cm ___ mm

Write the equation and answer.

7. Eric ran 524 meters, 30 centimeters and 5 millimeters onTuesday. The next day he was tired and only ran 498 meters, 92 centimeters and 9 millimeters. How much farther did he run on Tuesday? 25 m 37 cm 6 mm
   524 m 30 cm 5 mm
   -498 92 9
   25 m 37 cm 6 mm

34

Regrouping with Different Units: Liters and Milliliters Worksheet 10

You need a 2 liter soda bottle. This is a metric measure. (I wonder why we measure soda bottles in metric and hardly anything else here in the U.S.?)

1 liter is half of a bottle of soda. 1 liter is also the size of the one thousand block in your Base Ten sets. A liter is the basic unit of metric measure of volume or capacity.

Liters are divided into fractions of 1 thousandth of a liter. These units are called milliliters or ml for short.

If a liter is the same as the one thousand block in size, what size block would be the same as a milliliter? 1cm the smallest cube

A milliliter is also called a cubic centimeter or cc. We use cc+-*s to measure the amount of liquid in a shot! The thing that holds the medicine is called a syringe.

So one cc is the same as one ml Now lets try some regrouping problems!

1. 5 liters 200 mls + 2 liters 590 mls + 6 liters 700 mls = 14 L490 ml
2. Change the amount in the answer above to be expressed in milliliters only. 14,490 mls
3. 1 milliliter of water weighs one gram. How many grams are in 4.7 mls? Be careful. This one is tricky! 4.7grams
4. 150 liters 200 ml - 75 liters 450 ml = 74 liters 750 mls
5. 15,000 mls - 7.5 liters is 7,500 mls

Regrouping with Different Units: Kilograms, Grams, and Milligrams - Worksheet 11

1 kilogram = 1000
1 gram = 1000
Write the answer in simplest

1.
8 kg 532 gm 433 mg
- 3 kg 421 gm 232 mg
5 111 201

2.
47 kg 245 gm 381 mg
- 2 kg 723 gm 932 mg
44 521 449

3. Make your own. AWV

_____kilograms _____ grams ___ milligrams
- _____kilograms _____ grams ___milligrams

4.
24 kilogram 462 grams 321 milligram
- 9 kilogram 873 grams 325 milligrams
14 588 996

5.
34 kilogram 53 grams 2 milligrams
- 28 kilogram 97 grams 4 milligrams
5 955 998

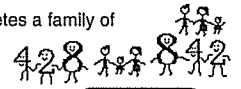
Write the equation and answer

6. Curious George bought 2.2 kilograms of bananas. How many grams is that? 2,200 He ate three bananas that weighed a total 880 grams. How much did the bananas he didn't eat weigh? 1320 gm
7. Marmaduke weighed 43 kilograms 234 grams. He lost 5 kilograms and 497 grams. What does he weigh now? 37kg 737gm
8. Dina, the kitten, weighs 1.5 kilograms. How many grams is that? 1,500 Her friend Bosco, the grown up cat with big blue eyes, weighs 5.9 kilograms. How many grams is that? 5,900 How much heavier is Bosco? 4,4 kilograms 4,400 grams

6. 2,200 − 880 = 1,320
7. 43 kg 234 gm − 5 497 = 37kg 737gm.
8. 5,900 − 1,500 = 4,400 ; 5.9 − 1.5 = 4.4

Families of Facts - Worksheet 1

Fill in the blank space with a number that completes a family of
Use only the three numbers in the box.
to make four number sentences.



32 64 2
32 × 2 = 64
2 × 32 = 64
64 ÷ 32 = 2
64 ÷ 2 = 32

16 64 4
16 × 4 = 64
4 × 16 = 64
64 ÷ 16 = 4
64 ÷ 4 = 16

8 72 9
8 × 9 = 72
9 × 8 = 72
72 ÷ 8 = 9
72 ÷ 9 = 8

8 128 16
16 × 8 = 128
8 × 16 = 128
128 ÷ 16 = 8
128 ÷ 8 = 16

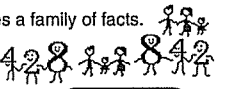
2 56 28
2 × 28 = 56
28 × 2 = 56
56 ÷ 2 = 28
56 ÷ 28 = 2

a b c
a × b = c
b × a = c
c ÷ a = b
c ÷ b = a

or
56 × 2 = 112
2 × 56 = 112
112 ÷ 56 = 2
112 ÷ 2 = 56

Families of Facts - Worksheet 2

Fill in the blank space with a number that completes a family of facts.
Use only the three numbers in the box
to make four number sentences.



12 24 2
2 × 12 = 24
12 × 2 = 24
24 ÷ 2 = 12
24 ÷ 12 = 2

8 24 j
8 × j = 24
j × 8 = 24
24 ÷ 8 = j
24 ÷ j = 8

6 24 4
6 × 4 = 24
4 × 6 = 24
24 ÷ 6 = 4
24 ÷ 4 = 6

Make your own.

P 48 Q
P × Q = 48
Q × P = 48
48 ÷ P = Q
48 ÷ Q = P

3 48 k
3 × 48 = k
48 × 3 = k
k ÷ 3 = 48
k ÷ 48 = 3

AWV
◯ × ◯ = ◯
◯ × ◯ = ◯
◯ ÷ ◯ = ◯
◯ ÷ ◯ = ◯

or 6 × 24 = 144
24 × 6 = 144
144 ÷ 6 = 24
144 ÷ 24 = 6

Families of Facts - Worksheet 3
Missing Numbers

9	9	8
3 | 27	7 | 63	3 | 24
8	7	9
7 | 56	5 | 35	2 | 18
7	7	3
4 | 28	8 | 56	3 | 9
8	8	4
5 | 40	6 | 48	5 | 20
8	6	9
7 | 56	6 | 36	6 | 54
5	8	9
7 | 35	4 | 32	8 | 72

Multiplication Review: Expanded Numbers
Worksheet 1

78 x 6 = (70 + 8) x 6

70 x 6 = 420	8 x 6 = 48

78 x 6 = 468

97 x 5 = (90 + 7) x 5

450 +	35

97 x 5 = 485

58 x 8 = (50 + 8) x 8

400	64

58 x 8 = 464

125 x 4 = (100 + 20 + 5) x 4

400	80	20

125 x 4 = 500

129 x 8 = (100 + 20 + 9) x 8

800	160	72

129 x 8 = 1,032

Multiplication Review: Regrouping Practice - Worksheet 2

165 X 3 = 495	249 X 3 = 747	142 X 6 = 852
376 X 2 = 752	175 X 2 = 350	365 X 2 = 730
294 X 3 = 882	197 X 3 = 591	237 X 4 = 948

What did these students do wrong? Fix and record your reasoning.

1 145 X 3 325 435	11 357 X 2 7014 714	22 267 X 3 791 801
Did not regroup in tens or hundreds place.	Did not regroup in hundreds place. Wrote hundreds in the thousands place.	Did not regroup in tens or hundreds place.

Division Puzzle - Worksheet 1

Solve these problems. If the quotient is: Color the space:

21 →
12 →
13 →
14 →
15 →

Correct answers make a design.

21 6 | 126	14 8 | 112	12 7 | 84	13 5 | 65
14 9 | 126	15 5 | 75	15 9 | 135	12 6 | 72
13 4 | 52	15 8 | 120	15 7 | 105	21 9 | 189
12 8 | 96	13 7 | 91	21 5 | 105	14 7 | 98

Division Practice - Worksheet 2

Solve these problems.
If the remainder is:

Color the space:

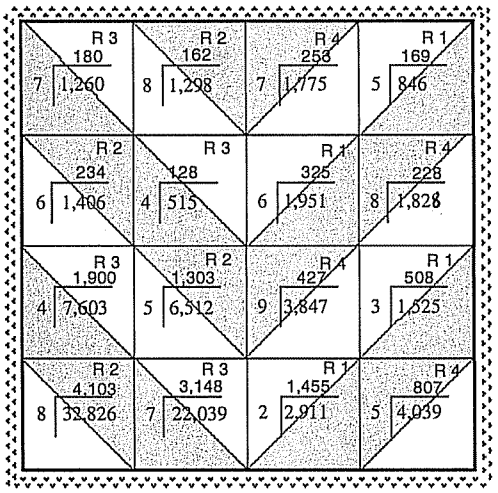
1 → 

2 → 

3 → 

4 → 

Correct answers make a design.



Multiplication and Division Word Problems

When doing word problems, get in the habit of reading the problem at least twice before beginning. Underline all numbers with their units. Circle the question. Decide what operations will be needed. Write a number sentence and then solve the problem.

1. A group of 153 students is going on a field trip to a museum. The bus can hold 98 students. The rest of the students will be transported in mini-vans, which carry 7 students each. (How many mini-vans) will the group need?

Write the two number sentences needed to solve this problem. Then solve it.
Number Sentence $153 - 98 = 55$
Number Sentence $55 \div 7 = 7^{R6}$
Answer: 8 mini-vans
number unit

2. The admission price is $6.00 per student and $8 for each adult mini van driver. The teacher and the bus driver did not have to pay. 17 students did not have to pay either because their parents were members of the museum. Plus the museum gave the teacher 10 free passes for students who could not pay for admission. (How much money) will the group need to pay for their admission to the museum?
Number Sentence $153 - 27 = 126$
Number Sentence $126 \times 6 = 756$
Number Sentence $8 \times 8 = 64$
Answer: $820
$756 + 64 = 820$

153 − 27 = 126 | 126 × 6 = 756 | 756 + 64 = $820 | $8 × 8 = 64 drivers

3. At the museum, the tour guides divided the students into 8 groups. (How many students) were in each group?
Number Sentence $153 \div 8 = 19^{r1}$
Answer: 19 students and one group with 20 students

4. The entrance of the museum had a beautiful tile floor. It had 8 sections each in a different color. The guides used the colored sections to help the students get into groups. Each section had 9 rows of 125 tiles. (How many tiles) were there in all?
Number Sentence $125 \times 9 = 1,125$
Number Sentence $1,125 \times 8 = 9,000$
Answer: 9,000 tiles

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