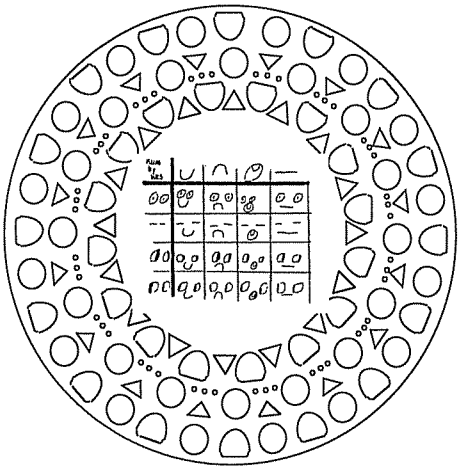
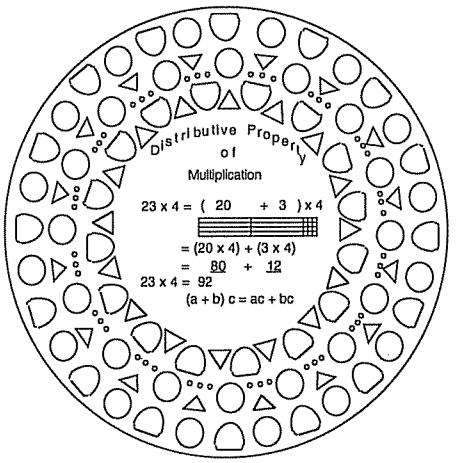
Patterns in Arithmetic

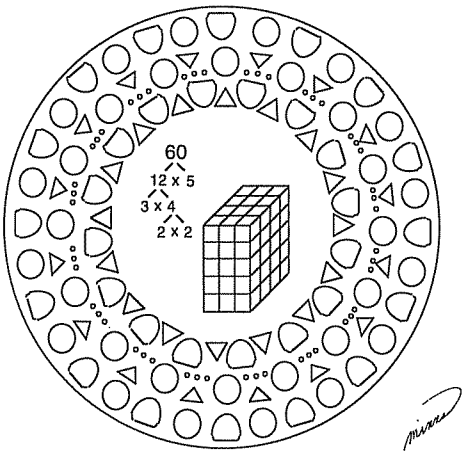
Multiplication Placement PDF

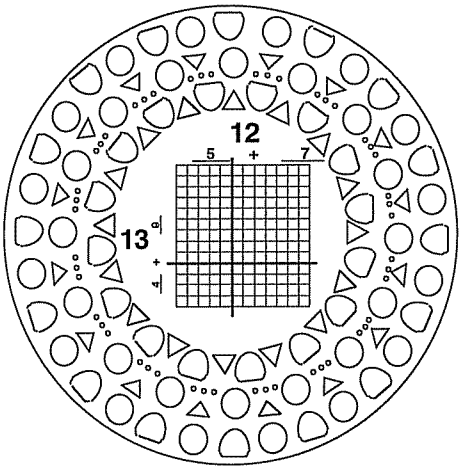
Parent/Teacher Guide

Booklet 1 - Basic Concepts
Booklet 2 - Beginning Long Multiplication and Basics of Distribution
Booklet 3 - Properties and Factoring
Booklet 4 - Working with Large Numbers and Decimals









By Alysia Krafel, Susan Carpenter, and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Multiplication Placement PDF

Page

Student Placement Assessment worksheets are located at the end of each Assessment Guide

Patterns in Arithmetic: Multiplication Placement PDF
Parent/Teacher Guide

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E-mail: Patternpress1@gmail.com

Multiplication: Booklet 1 Placement Assessment Guide

Purpose The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Assessment: Part 1 is review material from *Patterns in Arithmetic:* Book 2. Since this is the first booklet of multiplication, Part 1 will assess a student coming from any other math program. Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

Prerequisites Basic addition with regrouping

Materials Assessment: Part 1, page 4, Assessment: Part 2, page 6
Score sheets, pages 4 and 5
Cuisenaire Rods
Centimeter ruler

Instructions Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

Do the assessment in two parts. Give Assessment: Part 1 and check it for readiness for this booklet. If he is not ready for this booklet, there is no point in giving Assessment: Part 2. If he passes all the readiness items, then give Assessment: Part 2.

After scoring Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

Assessment Guide This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc.
Score sheets that match the Assessment Guide for both Assessment: Part 1 and Part 2 follow.

Assessment Criteria for Assessment: Part 1
Can the student:
1. Addition Number Sentences (*Patterns in Arithmetic: Books 1 and 2*)
    A. give the correct answer for both addition number sentences in problems a and b?

B. give the correct answer to the addition problem c, which requires regrouping?

Assessment Criteria for Assessment: Part 2

Can the student:

1. Multiplication Concept (Multiplication: Booklet 1)

A. identify a multiplication problem from a picture? Must get both a and b correct for a Yes. In the picture there are five groups of four blocks. This represents 4 x 5 = 20. A student new to this program may label the problem as 5 x 4 = 20. This is acceptable. The number sentence will be defined in the first lesson of the booklet.

2. Multiplication Concept (Multiplication: Booklet 1)

A. draw a picture of a multiplication problem? The picture should show four groups of three, or for a new student three groups of four is acceptable. If he draws three blocks and then four blocks, he can not demonstrate that he knows that multiplication is repeated addition of the same number.

B. show that multiplication is repeated addition of the same number? 3 x 4 = 3 + 3 + 3 + 3 or 4 + 4 + 4? Count it a No if he writes 3, 6, 9, 12, etc.

3. Building Tables and Associative Property (Multiplication: Booklet 1)

A. correctly solve two of three multiplication fact problems (a, b, and c) using any method?

B. demonstrate that he has memorized the multiplication tables as he does problems a, b, and c? Observe him while he does problems a, b, and c. For a Yes, he should write the answers quickly without using a manipulative–counting on his fingers, skip counting, or using addition on the side. If he gets a Yes on 3A and a No on 3B, make a note of what strategy he is using to get the answers.

C. multiply three numbers together correctly in one of the two problems (d or e)?

D. demonstrate that he knows that multiplying by a 0 (e) will cause the answer to be zero regardless of what the other numbers are?

4. Multiplication by Addition (Multiplication: Booklet 1)

A. correctly solve two of the three larger problems?

B. use repeated addition? Give a Yes if the repeated strategy is used even if the answer is incorrect.

C. use the standard multiplication procedure? Give a Yes if the standard strategy is used even if the answer is incorrect. The standard procedure for this kind of problem is taught in Multiplication: Booklet 2.

5. Building Word Problems (Multiplication: Booklet 1)

A. formulate a multiplication word problem for a given multiplication fact?

6. Beginning Factoring (Multiplication: Booklet 1)

A. show correctly the factors of fifteen?

B. show correctly one of the factor pairs for twenty-four? Give a bonus point if he showed more than one factor pair for twenty-four.

Booklet Selection Guide based on the results of the assessment:

Assessment: Part 1

Item 1A must be a Yes for the student to begin this booklet. He must be able to interpret a number sentence and get the correct answer. If this item is a No, begin with *Patterns in Arithmetic: Book 1.*

If Item 1A is a Yes and Item 1B is a No, begin with *Patterns in Arithmetic: Book 2.*

If Items 1A and 1B are Yes, then have him complete Assessment: Part 2.

Assessment: Part 2

If the page is full of question marks, this is the correct booklet to start him in.

If Item 3B or 4A are a Yes but Items 1, 2, and 4B are No, then the student has had exposure to multiplication and has memorized the facts but can not demonstrate understanding. Again, this booklet is appropriate.

If he correctly answered Items 1A, 2A, 3A, and 4A and B but missed Items 4C, 4D, 5, or 6, you should teach only the needed lessons and move on to Multiplication: Booklet 2. If he is new to this program, give ample free exploration of Cuisenaire Rods before beginning the lesson on Trains.

If he used the standard procedure to answer problems a, b, and c in Item 4 and gave the correct answers and has a Yes on Items 1 - 3, move on to Multiplication: Booklet 2. Consider doing the Building Tables section of Multiplication: Booklet 1 as an investigation of patterns.

Assessment - Part 1 Name _____ Date _____

Put a question mark next to any problem you do not know how to do.

1. Solve. a. 4 + 5 + 6 = ___

b. 4 + 4 + 4 + 4 + 4 = _____

c.
$$\begin{array}{r} 13 \\ 13 \\ 13 \\ +\ 13 \\ \hline \end{array}$$

Assessment - Part 1 Score Sheet

Part 1

Can the student:

1. Addition Number Sentences (Books 1 and 2)

Yes No A. give the correct answer for both addition number sentences in problems a and b?

Yes No B. give the correct answer to the addition problem c, which requires regrouping?

Notes

Assessment: Part 2 Score Sheet Name _____ Date _____

Can the student:

1. Multiplication Concept (Multiplication: Booklet 1)
   - Yes No A. identify at least one multiplication problem from a picture and give the correct number sentence?

2. Multiplication Concept (Multiplication: Booklet 1)
   - Yes No A. draw correctly a picture of a multiplication problem?
   - Yes No B. show that multiplication is repeated addition of the same number?

3. Building Tables and Associative Property (Multiplication: Booklet 1)
   - Yes No A. correctly solve two of three multiplication fact problems (a, b, and c) using any method?
   - Yes No B. demonstrate that he has memorized the multiplication tables as he does problems a, b, and c? See note in criterion 3B on page 2.
   - Yes No C. multiply three numbers together correctly in one of the two problems (d or e)?
   - Yes No D. demonstrate that he knows that multiplying by a zero (e) will cause the answer to be zero regardless of what the other numbers are?

4. Multiplication as Addition (Multiplication: Booklet 1)
   - Yes No A. solve correctly two of the three larger problems?
   - Yes No B. use repeated addition? Give a Yes if the repeated addition strategy is used even if the answer is incorrect.
   - Yes No C. use the standard multiplication procedure? Give a Yes if the standard strategy is used even if the answer is incorrect. This procedure is not taught in this booklet. We are testing for previous memorization of the procedure.

5. Building Word Problems (Multiplication: Booklet 1)
   - Yes No A. formulate a multiplication word problem for a given multiplication fact?

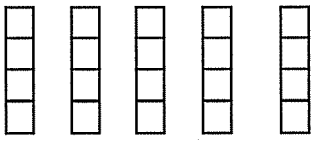
6. Beginning Factoring (Multiplication: Booklet 1)
   - Yes No A. show correctly the factors of fifteen?
   - Yes No B. show correctly one of the factor pairs for twenty-four?
   - Bonus point: Give an extra point if he writes more than one correct factor pair for twenty-four.

12 points possible Score _____ Placement: _____

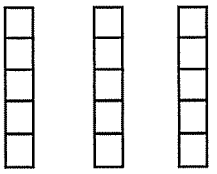
Assessment: Part 2 Name _____ Date _____

Put a question mark next to any problem you do not know how to do.

1. What multiplication problem is shown in each picture?



a. ___ x ___ = ___



b. ___ x ___ = ___

2. a. Draw a picture of 3 x 4.

b. Show 3 x 4 as an addition problem.

3. Solve these problems.

a. 3 x 2

b. 6 x 4

c. 7 x 3

d. 2 x 3 x 4 = _____

e. 5 x 7 x 0 = _____

4. Use addition to solve.

a. 14 x 5

b. 25 x 4

c. 32 x 6

5. Write a word problem that uses the problem 6 x 3 = _____.

_____

_____

_____

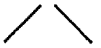
_____

6. Factor these numbers.

a. 15

___ x ___

b. 24



___ x ___

Multiplication: Booklet 2 Placement Assessment Guide

Purpose The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Assessment: Part 1 is review material from *Patterns in Arithmetic:* Multiplication: Booklet 1. Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

Prerequisites Multiplication: Booklet 1

Materials
Assessment: Part 1 and Part 2, pages 13 and 14
Score sheets, pages 11 and 12
Cuisenaire Rods
Meter stick

Instructions Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

Do the assessment in two parts. Give Assessment: Part 1 and check it for readiness for this booklet. The answers are in the Answer Key. If he is not ready for this booklet, there is no point in giving Assessment: Part 2. If he passes all the readiness items, then give Assessment: Part 2.

After scoring Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

Assessment Guide This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc.

Score sheets that match the Assessment Guide for both Assessment: Part 1 and Part 2 follow.

Assessment Criteria for Part 1
Can the student:

1. Multiplication Concept (Multiplication: Booklet 1)
    A. identify a multiplication problem from a drawing?
    B. record the correct product?

2. Multiplication Concept (Multiplication: Booklet 1)
    A. draw a picture of a given multiplication number sentence?
    B. record the correct product?

3. Multiplication by Addition (Multiplication: Booklet 1)
    A. show 23 x 4 as 23 + 23 + 23 + 23 = 92?

If he shows the problem as 23 x 4 and solves it with the standard procedure, mark this item a No.

    B. give the correct answer?

4. Building Times Tables (Multiplication: Booklet 1)
    A. skip count by fours to forty? There may be a few errors.
    B. insert the correct numbers?

The items in criterion 5 are broken into lower times tables 2 - 5 for problems a, b, and c and into upper times tables 6 - 9 for problems d, e, and f.

5. Building Times Tables (Multiplication: Booklet 1)
    A. give the correct answers for two of three lower times table problems a, b, and c?
    B. give the correct answers for two of three upper times table problems d, e, and f?

6. Factoring: Recording (Multiplication: Booklet 1)
    A. list three of the four pairs of factors for twenty-four?
    B. use the Cuisenaire Rods to do it ? No score on this item. Just a note.

Booklet Selection Guide

Use the score sheet to record the results.

Readiness for Multiplication: Booklet 2 is based on results of Assessment: Part 1. If the student's score is 7 or less, don't give Assessment: Part 2 and begin with Multiplication: Booklet 1. If the student's score is 8 or more, give Assessment: Part 2.

Assessment Criteria for Assessment: Part 2

Can the student:

1. Breaking Up Times
    A. write the correct answer for problem a?
    B. fill in line b correctly?
    C. fill in line c correctly?
    D. fill in line d correctly?
    E. fill in the correct product at e?

Note the procedure used by the student in problem a. Did he use repeated addition or the standard multiplication procedure? This format for a multiplication problem may be unfamiliar to you. If so, simply use the Answer Key to determine if your student wrote in the correct numbers or not. After you teach this section in the booklet, it will make sense to you.

2. Box Multiplication
    A. break up the sixty-three in the parentheses?
    Example: 63 x 8 = (30 + 20 + 10 + 3) x 8

The way the sixty-three is broken up will vary. Check to see if the addition is correct.

B. draw the same number of sections in the box as the number of addition terms he wrote in the parentheses? For example: If he wrote 63 x 8 = (30 + 20 + 10 + 3) x 8–in the parentheses he used four numbers to break up the sixty-three. The box should be divided into four sections also.

C. write the correct partial product in each section of the box? For example: If he wrote 63 x 8 = (30 + 20 + 10 + 3) x 8–in the parentheses on the top row, he would multiply 30 x 8 in his head and write 240 in the first section of the box. The second section would contain 20 x 8, or 160; the third section 10 x 8, or 80, and the last section 3 x 8, or 24. The numbers will vary, but the pattern should be the same.

D. write the correct product of 63 x 8 on the last line?

3. Expanded Tables
   A. break the thirty-eight into 30 + 8 in the parentheses on the top line, line a?
   B. write in the correct little multiplication problems on line b?
   C. write in the correct partial products on line c?
   D. write in the correct final product on line d?

4. Distributive Property
   A. identify the number four as the number being distributed in problem 3?

5. Expanded Multiplication and Short Notation
   A. use the long, expanded notation correctly in problem a?
   B. give the correct answer to problem a? Note any place value errors.
   C. use the short notation correctly on problem b?
   D. give the correct answer on problem b?

6. Short Notation
   A. give the correct answer to problem a?
   B. give the correct answer to problem b?

Booklet Selection Guide (based on results of Assessment: Part 2)

If the student scores 80% or better on Assessment: Part 2–this is 16 or more Yes items–move on to Multiplication: Booklet 3.

If the student gets a Yes on the following items: 1E, 2D, 3D, 5C, 6A, 6B, and a No on items 3A, 3B, 3C, 4A, and 5A, you have a student who knows how to multiply but does not understand it. You have two choices.

First, you could take the time to do Multiplication: Booklet 2, which is what you should do if you have an eight- or nine-year-old student. Follow Multiplication: Booklet 2 with Multiplication: Booklet 3, which is on factoring and prime numbers.

Second, you could begin Multiplication: Booklet 4, which teaches the same concept with problems like 35 x 27. This may be a good choice for a strong fifth-grader.

Multiplication: Booklet 3 can be taught concurrently with Multiplication: Booklet 4.

Whenever remediation is needed, rely upon the following process, which is used throughout the *Patterns in Arithmetic* series to develop understanding of a concept.

1. Introduce the concept with a manipulative. Orally discuss it. Build it. Verify it. Practice it. Repeat the experience with a different manipulative (oral manipulative).
2. Use manipulatives to explore the concept again. This time record it with pictures (pictorial/representation). Practice it. Use worksheets.
3. Record the problem with numbers (abstract/symbolic), which links the pictorial with the abstract.
4. Practice fluency.
5. Practice for speed.

Begin each lesson with a warm-up and review. Always end a lesson with a success before the student is tired. It is best to end while the student is still enjoying the lesson.

Ask questions or make statements such as, **"Are you sure?"** or **"Build it,"** or **"What gave you the clue?"** or **"Show me how you got that,"** or **"Prove it,"** even when a student is correct. This is important to do often. Many students will ask an adult, "Am I right?" rather than answer definitively. Confidence in a student's response must come from within. A student needs to self-check and have confidence in his or her ability and knowledge. Asking the student if he or she is right, even when correct, will encourage self-confidence and the ability to self-check.

Please note that the dialogues in most lessons are idealized with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as, **"How did you get that? "Show me what you mean," "Build a model of that," "Tell me more."**

Assessment: Part 1 Score Sheet Name _____ Date _____

Can the student:

1. Multiplication Concept (Multiplication: Booklet 1)
   - Yes No A. identify a multiplication problem from a drawing?
   - Yes No B. record the correct product?

2. Multiplication Concept (Multiplication: Booklet 1)
   - Yes No A. draw a picture of a given multiplication number sentence?
   - Yes No B. record the correct product?

3. Multiplication by Addition (Multiplication: Booklet 1)
   - Yes No A. show 23 x 4 as 23 + 23 + 23 + 23?
   - Yes No B. give the correct answer?

4. Building Times Tables (Multiplication: Booklet 1)
   - Yes No A. skip count by fours to forty? There may be a few errors.
   - Yes No B. insert the correct numbers?

5. Building Times Tables (Multiplication: Booklet 1)
   - Yes No A. give the correct answers for two of three on a, b, and c?
   - Yes No B. give the correct answers for two of three on d, e, and f?

6. Factoring: Recording (Multiplication: Booklet 1)
   - Yes No A. list three of the four pairs of factors for twenty-four?
   - B. use the Cuisenaire Rods to do it? No score on this item.

Items Correct = _____ = _____% 72% needed to begin Booklet 2
Items Possible = 11 This is 8 or more Yes items

Assessment: Part 2 Score Sheet Name _____ Date _____

Can the student:

1. Breaking Up Times
   - Yes No A. write the correct answer for problem a?
   - Yes No B. fill in line b correctly?
   - Yes No C. fill in line c correctly?
   - Yes No D. fill in line d correctly?
   - Yes No E. fill in the correct product at e?

2. Box Multiplication
   - Yes No A. break up the sixty-three in the parentheses?
   - Yes No B. draw the correct number of sections in the box?
   - Yes No C. write the correct partial product in each section of the box?
   - Yes No D. write the correct product of 63 x 8 on the last line?

3. Expanded Tables
   - Yes No A. break the 38 into 30 + 8 in the parentheses on line a?
   - Yes No B. write in the correct little multiplication problems on line b?
   - Yes No C. write in the correct partial products on line c?
   - Yes No D. write in the correct final product on line d?

4. Distributive Property
   - Yes No A. identify the number four as the number being distributed?

5. Expanded Multiplication and Short Notation
   - Yes No A. use the correct long, expanded notation in problem a?
   - Yes No B. give the correct answer to problem a?
   - Yes No C. use the correct short notation on problem b?
   - Yes No D. give the correct answer on problem b?

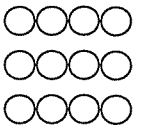
6. Short Notation
   - Yes No A. give the correct answer to problem a?
   - Yes No B. give the correct answer to problem b?

Items Correct = _____ = _____% 80% needed to move on to Multiplication: Booklet 3
Items Possible = 20 This is 16 or more Yes items.

Assessment - Part 1 Name _____ Date _____

Put a question mark next to any problem you do not know how to do.

1. What multiplication problem is shown in this drawing?



_____ x _____ = _____

2. Draw a picture like the one above for 7 x 3 = _____.

3. Write this multiplication problem as an addition problem and solve: 23 x 4.

4. Skip count by fours from 0 to 40. _____ _____ _____ _____ _____ _____

_____ _____ _____ _____ _____

5. Solve these problems.

a. 6 x 4

b. 5 x 5

c. 9 x 3

d. 7 x 6

e. 8 x 8

f. 9 x 6

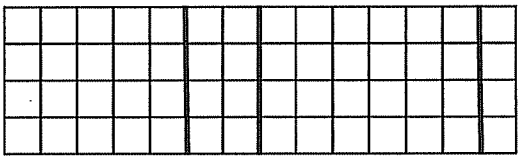
6. Write all multiplication problems that equal 24.
You may use Cuisenaire Rods and a meter stick if you like.
Hint: There are more than two.

Assessment - Part 2 Date _____

1. Fill in the missing numbers.

   a. 14
   x 4

   b. 14 x 4 = (___ + ___ + ___ + ___) x 4



   c. (__ x __) + (__ x __) + (__ x __) (__ x __)

   e. _____ = d. _____ + _____ + _____ + _____

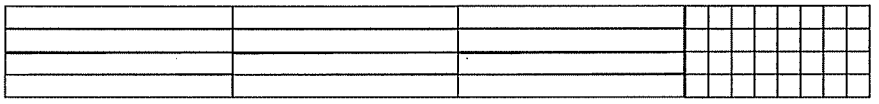
2. Show how you would break up the 63. Label the partial products in the box. Show the final product below the box on the line.

   63 x 8 = ( ) x 8

   63 x 8 = _____

3. Break up the 38 into tens and ones; fill in the missing numbers.

   a. 38 x 4 = (_____ + _____) x 4



   = b. (__ x __) + (__ x __)

   d. _____ = c. _____ + _____

4. In problem 3, what number is being distributed? _____

5. Solve these problems the Long way and the Short way.

   a. Long way
   1,439
   x 5

   b. Short way
   1,439
   x 5

6. Solve. a. 138
   x 6

   b. 2,375
   x 4

Multiplication: Booklet 3 Placement Assessment Guide

Purpose The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Assessment: Part 1 is review material from *Patterns in Arithmetic:* Multiplication: Booklet 2. Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

Prerequisites *Patterns in Arithmetic:* Multiplication - Booklet 2

Materials Assessment: Part 1, page 20, and Part 2, pages 21 and 22
Score sheets, pages 18 and 19
Cuisenaire Rods

Instructions Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

Note Do the assessment in two parts. Give Assessment: Part 1 and check it for readiness for this booklet. The answers are in the Answer Key. If he is not ready for this booklet, there is no point in giving Assessment: Part 2. If he passes all the readiness items, then give Assessment: Part 2.

Assessment Guide This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc. Score sheets that match the Assessment Guide for both Assessment: Part 1 and Part 2 follow.

Assessment Criteria for Assessment: Part 1
Can the student:
1. Concept (Multiplication: Booklets 1 and 2)
   A. identify correctly the addition number sentence associated with the picture in three of the four problems?
   B. identify correctly the multiplication number sentence associated with the picture in three of the four problems?

If he identifies the problem in item 1a as 5 x 4 instead of 4 x 5, it means he is not familiar with the convention used in these books.

The *Patterns in Arithmetic* series always places the size of the group first, then the operations sign. The second number tells the number of groups. Reteach this convention but give him a Yes on the score sheet.

C. write the correct answer to three of the four problems?

2. Concept (Multiplication: Booklets 1 and 2)
The three may be identified as the number of groups or the number in each group. Whichever one is used for the three, the other one must be used for the seven.

A. identify the correct meaning of the three?

B. identify the correct meaning of the seven?

C. identify twenty-one as the total? The words 'the answer' are not sufficient to score a Yes on this item.

3. Standard Area (Multiplication: Booklets 1 and 2)

A. give the correct area on problem a, which gives a picture?

B. give the correct area on problem b, which gives only the numbers?

Note: If he gives the answer as 20 on 3a or 24 on 3b, he has area confused with perimeter. The answers should also be given in square units. If the numbers are correct, but he leaves off the square units, remediate this while you are teaching the lesson on area in Multiplication: Booklet 3.

Booklet Selection Guide based on results of Assessment: Part 1

If he gets a No on 1A, 1B, 2A, and 2B, he is not ready for this booklet. It is possible to know the multiplication table and not understand the concept or what the multiplication number sentence means. Begin with Booklet 1 to build his conceptual understanding of multiplication and the multiplication number sentences. If he knows his multiplication tables, you can focus on the first part of Multiplication: Booklet 1 and move onto Booklet 2, hitting the parts he does not know. Use the assessments in Multiplication: Booklet 2 to determine what he does and does not understand about single digit multiplication.

If he gets a Yes on 1A, 1B but gets a No on 2A, 2B, or 3A, he is not quite ready for this booklet; begin with Multiplication: Booklet 2. If Multiplication: Booklet 2 has been completed, remediate and retest. Then give Assessment: Part 2 and begin this booklet. Multiplication: Booklets 2 and 3 can be taught concurrently once the number sentence is understood.
If he gets a Yes on six or more of the items in Assessment: Part 1, give Assessment: Part 2.

Assessment Criteria for Assessment: Part 2

Can the student:

1. Associative Property (Multiplication: Booklet 3)

A. write the numbers into the parentheses on at least one problem?

B. fill in all three problems with the numbers in different orders?

C. fill in the correct numbers on the second line of two of the three problems?

D. give the answer sixty on any of the problems?

This group of items tests if he knows that the numbers can be placed in any order of combination and the product will be unchanged. The goal is for him to recognize what associating is and to be able to use the parentheses to show all the orders in which these three numbers can be multiplied.

2. Volume Puzzles (Multiplication: Booklet 3)

A. determine the correct volume of a rectangular solid with a picture showing the cubes (a)?

B. determine the correct volume of a rectangular solid with a picture with only numbers and no cubes shown (b)?

C. determine correctly the volume of a rectangular solid given only the length, width, and height (c)?

3. Discovering Prime Numbers (Multiplication: Booklet 3)

A. explain and/or show an example of a factor?

2 Points: A factor is a number that results in a product (answer) when multiplied by another factor. Example: $2 \times 3 = 6$. Two and three are factors of six.

1 Point: A factor is a number used to make an answer such as $2 \times 3 = 6$.

4. Discovering Prime Numbers (Multiplication: Booklet 3)

A. explain what a prime number is?

2 Points: A number that has only itself and one as factors.

1 Point: A number that can only be made one way (referring to the rectangular arrays drawn in the lesson).

5. Discovering Prime Numbers (Multiplication: Booklet 3)

A. list eight of the first ten prime numbers?

If he includes 9, 15, 25, or 27 in this list, it indicates he thinks that all odd numbers are prime.

6. Prime Factoring (Multiplication: Booklet 3)

A. prime factor correctly two of the three numbers?

B. place the prime factors from largest to smallest in the box?

C. realize thirty-seven can not be factored?

7. Factoring by Tens (Multiplication: Booklet 3)

A. factor both of the numbers correctly by factors of ten?

8. Expanded Tables (Multiplication: Booklet 3)

A. give the correct answers to three of the four problems?

B. explain that the number forty-eight will always occur as the first two numbers in the product but the number of zeros after the forty-eight will change? Or that the place value of the forty-eight will change?

C. extend that pattern to get forty-eight followed by five zeros? If he used addition to get the answer, or the standard multiplication format, give a No. To get a Yes on this item, just the answer should be written down. You might also see little dots above the zeros where he counted the number of them.

9. Using Factoring with Expanded Tables (Multiplication: Booklet 3)

A. give the correct product at the bottom?

B. show that the four hundred is factored into 4×100 in the first factoring line?

C. show the reordering of the factors to combine the 4×6 in the second factoring line?

Assessment: Part 1 Score Sheet

Name _____ Date _____

Can the student:

1. Concept (Multiplication: Booklets 1 and 2)
   - Yes No A. identify correctly the addition number sentence associated with the picture in three of the four problems?
   - Yes No B. identify correctly the multiplication number sentence associated with the picture in three of the four problems?
   - Yes No C. write the correct answer to three of the four problems?

2. Concept (Multiplication: Booklets 1 and 2)
   - Yes No A. identify the correct meaning of the three?
   - Yes No B. identify the correct meaning of the seven?
   - Yes No C. identify twenty-one as the total?

Note: The words 'the answer' are not sufficient to score a Yes on this last item.

3. Standard Area (Multiplication: Booklets 1 and 2)
   - Yes No A. give the correct area on problem a, which gives a picture?
   - Yes No B. give the correct area on problem b, which gives only the numbers?

Items Correct = _____ = _____%
Items Possible = 8

Assessment: Part 2 Score Sheet Name _____ Date _____

Can the student:

1. Associative Property (Multiplication: Booklet 3)
   - Yes No A. write the numbers into the parentheses on at least one problem?
   - Yes No B. fill in all three problems with the numbers in different orders?
   - Yes No C. fill in the correct numbers on the second line of two of the three problems?
   - Yes No D. give the answer sixty on any of the problems?

2. Volume Puzzles (Multiplication: Booklet 3)
   - Yes No A. determine the correct volume of a rectangular solid with a picture showing the cubes (a)?
   - Yes No B. determine the correct volume of a rectangular solid with a picture with only numbers and no cubes shown (b)?
   - Yes No C. determine the correct volume of a rectangular solid given only the length, width, and height (c)?

3. Discovering Prime Numbers (Multiplication: Booklet 3)
   - 1 Pt. 2 Pts. A. explain and/or show with an example what a factor is?

4. Discovering Prime Numbers (Multiplication: Booklet 3)
   - 1 Pt. 2 Pts. A. explain what a prime number is?

5. Discovering Prime Numbers (Multiplication: Booklet 3)
   - Yes No A. list eight of the first ten prime numbers?

6. Prime Factoring (Multiplication: Booklet 3)
   - Yes No A. prime factor correctly two of the three numbers?
   - Yes No B. place the prime factors from largest to smallest in the box?
   - Yes No C. realize thirty-seven can not be factored?

7. Factoring by Tens (Multiplication: Booklet 3)
   - Yes No A. correctly factor both of the numbers by factors of ten?

8. Expanded Tables (Multiplication: Booklet 3)
   - Yes No A. give the correct answers to three of the four problems?
   - Yes No B. explain the pattern?
   - Yes No C. extend that pattern to get forty-eight followed by five zeros?

9. Using Factoring with Expanded Tables (Multiplication: Booklet 3)
   - Yes No A. give the correct product at the bottom?
   - Yes No B. show that the four hundred is factored into 4 x 100 in the first factoring line?
   - Yes No C. show the reordering of the factors to combine the 4 x 6 in the second factoring line?

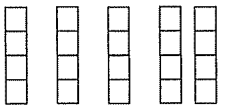
Items Correct = _____ = _____% 17 Yes points to pass, or 80%

Items Possible = 22

Assessment: Part 1 Date _____

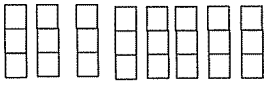
Put a question mark next to any problem you do not know how to do.

1. Look at the picture. Write the addition and multiplication number sentence.

a. 

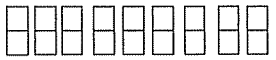
__ + __ + __ + __ + __ = _____ Addition

__ x __ = _____ Multiplication

b. 

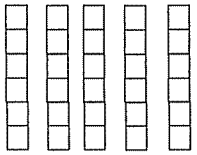
__ + = _____ Addition

__ x __ = _____ Multiplication

c. 

__ + = _____ Addition

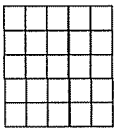
__ x __ = _____ Multiplication

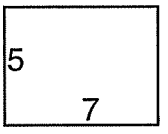
d. 

__ + = _____ Addition

__ x __ = _____ Multiplication

2. Here is a multiplication number sentence: 3 x 7 = 21.
   Write the meaning of each number in the multiplication number sentence.
   a. The 3 means the _____.
   b. The 7 means the _____.
   c. The 21 means the _____.

3. a. What is the area of this rectangle? _____ 

   b. What is the area of this rectangle? _____ 

Assessment: Part 2 - Worksheet 1 Date _____

Put a question mark next to any problem you do not know how to do.

1. Show all the ways of associating (grouping) these three numbers: 2, 5, 6

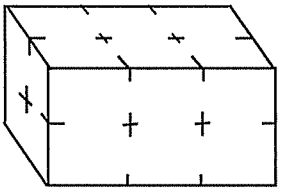
(___ x ___) x ___ = ___ x ___ = ___

(___ x ___) x ___ = ___ x ___ = ___

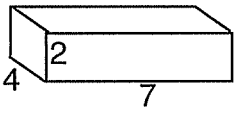
(___ x ___) x ___ = ___ x ___ = ___

2. Find the volume of these rectangular solids.
Length (L) Width (W) Height (H)

a. Volume _____



b. Volume _____



c. Volume _____

L = 3; W = 6; H = 2

3. What is a Factor? Use numbers to help you explain.

_____

_____

_____

4. What is a Prime Number?

_____

_____

_____

5. List the first ten Prime Numbers starting with 2.

2 ___ ___ ___ ___ ___ ___ ___ ___ ___

Assessment: Part 2 - Worksheet 2 Date _____

6. Prime Factor 24, 50, and 37. Record the Prime Factors in the boxes.

a. ☐

24

/ \

__ x __

b. ☐

50

/ \

__ x __

c. ☐

37

/ \

__ x __

7. Factor 50 and 80 by tens.

By tens

a. 50

/ \

__ x __

By tens

b. 80

/ \

__ x __

8. a. Solve these problems.

6	60	600	6,000
x 8	x 8	x 8	x 8

b. What patterns do you notice in the answers to these problems?

_____

_____

_____

c. Use the pattern to find the answer to 600,000 x 8. _____

9. Use factoring by tens to show how to get the product of 400 x 6.

400 x 6

/ \

__ x __ x __

(__ x __) x __

__ x __

_____

Multiplication: Booklet 4 Placement Assessment Guide

Purpose

The purpose is to determine if this booklet is the correct starting place. Assessment: Part 1 assesses the fundamental knowledge necessary for success in this booklet.

Assessment: Part 1 is review material from the last booklet and is used to determine student readiness for this booklet. Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

Prerequisites

Patterns in Arithmetic: Multiplication - Booklet 3, or previous instruction in the basic concepts of the Distributive Property of Multiplication as it applies to single digit multiplication, for example 345 x 6. Also, the student must have had work on the use of the Expanded Tables, for example 3 x 4 =12, 30 x 4 = 120, and 300 x 4 =1,200. And Multiplication: Booklet 3 on Factoring, or other instruction in prime numbers and prime factoring.

Note

Knowing the standard procedure for multiplication we all learned is not the same thing as understanding the Distributive Property of Multiplication. Understanding this property is key to understanding both multiplication and division. See Introduction to the Distributive Property of Multiplication in Multiplication - Booklet 4.

Materials

Assessment: Part 1, page 31, and Part 2, pages 31 and 32
Score sheets, pages 28 and 29
Cuisenaire Rods
Colored pencils or crayons

Instructions

Do the assessment in two parts. Give Assessment: Part 1 and use the Booklet Selection Guide to determine readiness for this booklet. If the student is not ready for this booklet, there is no point in giving Assessment: Part 2. If he passes all the readiness items, then give Assessment: Part 2.

Assessment Guide

After scoring Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc.

Score sheets that match the Assessment Guide for both Assessment: Part 1 and Part 2 follow.

Assessment Criteria for Assessment: Part 1

Can the student

1. Short Notation (Multiplication: Booklet 2)

A. use the standard short notation for multiplication and get two of three answers correct? You are looking to see if the student knows the standard short procedure for multiplication. If he uses addition, you can assume he does not know how to use the multiplication procedure. Students who have completed Multiplication: Booklet 2 may do the problems the long way. Point out the instruction to use the short way. He can use the long way on item 2.

2. Expanded Multiplication (Multiplication: Booklet 2)

A. write the partial products on each line in two of the three problems? Partial products for problem a would be twenty-four and one hundred eighty. The student may write twenty-four and eighteen and still get a Yes on this item. It means he knows what to do but made a place value error, which is common at this point. You are looking to see if he knows what to do, even if there is an error in calculation.

B. write the correct partial products with the correct place value in two of the three problems? Partial products for problem a would be twenty-four and one hundred eighty. The order he writes them in is not important.

C. sum the partial products to get a final answer in two of the three problems? The answer may be incorrect if the student used eighteen instead of one hundred eighty. If he summed his partial products he gets a Yes on this item. It shows he knew to add to complete the process.

D. sum the partial products and get the correct answer in two of the three problems?

3. Expanded Tables and Breaking Up Times (Multiplication: Booklet 2)

A. break up the fifty-seven into 50 + 7 on the top line and fill in x 3 correctly?

B. show the correct little multiplication problems on the second line?

C. show the partial products on the third line? Give a Yes even if he made a multiplication error. It shows he knows what to do.

D. add to get the final product? Give a Yes if the addition was done even if there is an arithmetic error. He may need to review his tables.

E. get the correct final product?

4. Discovering Prime Numbers (Multiplication: Booklet 3)

A. explain and/or show with an example what a factor is?

2 Points: A factor is a number that results in a product (answer) when multiplied by another factor. Example: 2 x 3 = 6. Two and three are factors of six.

1 Point: A factor is a number used to make an answer such as 2 x 3 = 6.

5. Discovering Prime Numbers (Multiplication: Booklet 3)

A. explain what a prime number is?

2 Points: A number that has only itself and one as factors.

1 Point: A number that can only be made one way (referring to the rectangular arrays drawn in the lesson).

6. Prime Factoring (Multiplication: Booklet 3)

A. correctly prime factor 120?

B. place the prime factors in order from largest to smallest in the box?

7. Factoring by Tens (Multiplication: Booklet 3)
    A. correctly factor 120 by factors of ten? Answer can be 12 x 10 x 10 or 12 x 100

8. Using Factoring with Expanded Tables (Multiplication: Booklet 3)
    A. give the correct product at the bottom as 6,300?
    B. show that the 700 x 9 is factored into (7 x 100) x 9 in the second line?
    C. show the reordering of the factors to combine the (7 x 9) x 100 in the third line?
    D. show 63 x 100 in the fourth line?

Booklet Selection Guide based on results of Assessment: Part 1

To begin Multiplication: Booklet 4, the student must receive a Yes on either 1A or 2A – 2D. If both of these are a No, the student is not ready for Multiplication: Booklet 4, and should begin in Multiplication: Booklet 2. Do not give Assessment: Part 2.

If the student received 15 or more Yes marks on Assessment: Part 1, give Assessment: Part 2 and begin Multiplication: Booklet 4.

For the students who did *Patterns in Arithmetic*: Multiplication - Booklets 2 and 3 previously:

a. If he receives between 12 and 14 Yes marks, give Assessment: Part 2 and remediate the places that are weak using the pages in the early sections of Multiplication: Booklet 4. You may also want to reteach items in the earlier booklets. Begin Multiplication: Booklet 4 carefully.

b. If the score is 11 or less, go back to Booklet 2 and reteach the Expanded Multiplication using Base Ten Blocks and tens and ones notation only and move to the short notation and practice. Allow him to use the zero pattern to find answers and reteach factoring by tens. Reteach prime numbers and prime factoring. In Multiplication: Booklet 4, teach only the tens and ones way of breaking up the numbers. Skip the nonstandard ways of breaking up numbers. Consider skipping the factoring and let him use the zero patterns to get partial products. Do not go to the short notation until he can do it the long way with two digit numbers times two digit numbers. Then you can teach the short way for longer problems.

For students new to *Patterns in Arithmetic*:

a. If the student receives a Yes on 1A and 3E and a No on criteria in section 2A - 2D and 3A - D, strongly consider beginning with Multiplication: Booklet 2 in the section on Breaking Up Times. The short delay here will pay off in the long run. There is a review in Multiplication: Booklet 4, but it is not a good place to teach the concept of distribution. But if time is a factor and your student is in fifth grade or higher, you can begin the teaching of distribution in Multiplication: Booklet 4, but the results will not be as strong.

b. If the student receives a Yes on 2A - 2D, 3A - D, and No on 5 - 8, begin with Multiplication: Booklet 3 on prime numbers, prime factoring, and factoring in general. These are important tools students need in Multiplication: Booklet 4 and in fractions.

Assessment Criteria for Assessment: Part 2

All criteria in this section come from Multiplication: Booklet 4.
Can the student:

1. Multiplying Factors of Ten
    A. get the correct product?

B. give a clear and correct explanation of how he got his answer?

3 points: I factored out the four from the one hundred and the three from the ten. Then I rearranged the factors and combined the 4 x 3 to get twelve, and the 100 x 10 to get one thousand. Then I multiplied 12 x 1,000 to get twelve thousand.

2 points: I took the four off of the one hundred and the three off the ten. Then I multiplied the 4 x 3 and added the three zeros from the one hundred and the ten.

1 point: I multiplied 4 x 3 and added three zeros.

2. Factoring by Tens

A. use a factor tree to show how the final product is obtained? Give a Yes even if the student did not use factors of ten but other sets of factors including prime factors.

B. use numbers in the factor tree that are factors of ten? The student should not use any numbers other than four, seven, ten, and one hundred in the factoring.

C. obtain the correct final product? The correct answer may appear without the factor tree being filled in.

3. Factoring by Tens

A. explain how the adding the zeros trick works?

3 points: It works because of factoring. The Associative Property of Multiplication allows the rearrangement of the factors in any combination. So you multiply 9 x 5 and then the 100 x 100. The four zeros come from multiplying 100 x 100 to get 10,000, which has the four zeros.

2 points: It works because of factoring. So you multiply 9 x 5. The four zeros come from multiplying 100 x 100.

1 point: The four zeros come from multiplying the hundreds.

If he gets a Yes on criterion 4a and only 1 point on 4b, followed by No on criteria 5A, 5B, and 6A, it means he memorized the trick with no understanding.

4. Breaking Up Times

Most students will be unable to do this item on Assessment: Part 2.

A. fill in the numbers 10 + 2 and 10 + 3 on the diagram?

B. fill in the top line of the distribution number sentence correctly?

C. show all the little multiplication problems correctly in the second line?

D. obtain the correct partial products in the third line?

E. write in the correct answer on the dark line? This answer may be correctly done using a memorized short procedure. Give credit for a correct answer with a Yes on this criterion.

5. Using Arrow Patterns to Multiply Tens and Ones

Most students will be unable to do this item on the Assessment. We are looking to see if the student understands the arrow pattern (distribution) tested by the use of nonstandard numbers. The nonstandard numbers will confuse a student who has only memorized the short procedure.

A. break up the thirteen using nonstandard method of nine and four instead of ten and three?

B. draw in the arrow pattern correctly?

C. write all the small multiplication problems and partial products correctly?

D. obtain the correct final answer using this nonstandard procedure only?

6. Breaking Up Times

A. explain why it makes sense that the problems would have the same answers?

2 points: Even though the problems were broken up differently, the answers will still be the same because it is the same problem both times.

1 point: Yes, because it is the same problem both times.

7. Breaking Up Times: Long Way

A. write down all the correct little multiplication problems on the lines in the left hand column? Order is not important.

B. show the correct place value of each partial product in the right hand column?

C. obtain the correct answer?

8. Standard Way

This problem is solved using the procedure we all learned in school. If the student can get the correct answer for this item and not for criteria 4 and 7, then he can do the problem but does not understand the procedure.

A. use the standard procedure? Even if the final answer is incorrect, does it appear that he is using the correct procedure?

B. place a zero or placeholder in the second row of the partial products below the line?

C. obtain the correct answer?

9. Short Notation

A. give a clear explanation as to why there is a zero in the second row of the partial product?

3 points: The zero is there because the four is in the tens place, so you are really multiplying forty times five, which is two hundred, which has a zero in the ones place.

2 points: The zero is there because you are multiplying by a number in the tens place.

1 point: Because it's tens.

0 points: It is a placeholder. Or, you have to do it to get the right answer.

Booklet Selection Guide based on results of Assessment: Part 2

If he scores 24 or more points, he does not need to do this book.
If he scores 21 - 23 points, remediate the weak answers and move on.
If he scores 20 or less, this is the correct booklet for the student.

Assessment: Part 1 Score Sheet Name _____ Date _____

Can the student:

1. Short Notation (Multiplication: Booklet 2)
   - Yes No A. use the standard short notation and get two of three answers correct?

2. Expanded Multiplication (Multiplication: Booklet 2)
   - Yes No A. write the partial products on each line in two of the three problems?
   - Yes No B. write the correct partial products with the correct place value in two of the three problems?
   - Yes No C. sum the partial products to get a final answer in two of the three problems?
   - Yes No D. sum the partial products and get the correct answer in two of the three problems?

3. Expanded Tables and Breaking Up Times (Multiplication: Booklet 2)
   - Yes No A. fill in the top line correctly?
   - Yes No B. show the correct little multiplication problems on the second line?
   - Yes No C. show the partial products on the third line?
   - Yes No D. add to get the final product?
   - Yes No E. get the correct final product?

4. Discovering Prime Numbers (Multiplication: Booklet 3)
   - 2, 1, 0 points A. explain and/or show with an example what a factor is?

5. Discovering Prime Numbers (Multiplication: Booklet 3)
   - 2, 1, 0 points A. explain what a prime number is?

6. Prime Factoring (Multiplication: Booklet 3)
   - Yes No A. prime factor 120 correctly?
   - Yes No B. place the prime factors in order in the box?

7. Factoring by Tens (Multiplication: Booklet 3)
   - Yes No A. factor 120 by factors of ten correctly?

8. Using Factoring with Expanded Tables (Multiplication: Booklet 3)
   - Yes No A. give the correct product at the bottom as 6,300?
   - Yes No B. write (7 x 100) x 9 in the second line?
   - Yes No C. show the reordering of the factors in the third line?
   - Yes No D. show 63 x 100 in the fourth line?

Items Correct = ____ = _____%

Items Possible = 21

Assessment: Part 2 Score Sheet Name _____ Date _____

Can the student:

1. Multiplying Factors of Ten
   - Yes No A. get the correct product?
   - 3, 2, 1, 0 points B. give a clear and correct explanation of how he got his answer?

2. Factoring by Tens
   - Yes No A. use a factor tree to show how the final product is obtained?
   - Yes No B. use numbers in the factor tree that are factors of ten?
   - Yes No C. obtain the correct final product?

3. Factoring by Tens
   - 3, 2, 1, 0 points A. explain how the adding the zeros trick works?

4. Breaking Up Times
   - Yes No A. fill in the numbers 10 + 2 and 10 + 3 on the diagram?
   - Yes No B. fill in the top line of the distribution number sentence correctly?
   - Yes No C. show all the little multiplication problems correctly?
   - Yes No D. obtain the correct partial products in the third line?
   - Yes No E. write in the correct answer on the dark line?

5. Using Arrow Patterns to Multiply Tens and Ones
   - Yes No A. break up the thirteen using the nonstandard method?
   - Yes No B. draw in the arrow pattern correctly?
   - Yes No C. write all the multiplication problems and partial products correctly?
   - Yes No D. obtain the correct answer using the nonstandard procedure only?

6. Breaking Up Times
   - 2, 1, 0 points A. explain why the problems would have the same answers?

7. Breaking Up Times: Long Way
   - Yes No A. write correctly little multiplication problems in the left hand column?
   - Yes No B. show the correct place value of each partial product in the right hand column?
   - Yes No C. obtain the correct answer?

8. Standard Way
   - Yes No A. use the standard procedure?
   - Yes No B. place a zero or placeholder in the second row below the line?
   - Yes No C. obtain the correct answer?

9. Short Notation
   - 3, 2, 1, 0 points A. give a clear explanation as to why there is a zero in the second row?

Items Correct = _____ = _____%

Items Possible = 30

Assessment: Part 1 Name _____ Date _____

Put a question mark next to any problem you don't know how to do.

1. Solve the Short Way.

a. 34 x 6 b. 258 x 3 c. 1,735 x 4

2. Solve the Long Way.

a. 34 x 6 b. 258 x 3 c. 1,735 x 4

3. Break up the 57 using tens and ones. Fill in all the blanks.

57 x 3 = (_____ + _____) x ____

= (____ x ____) + (____ x ____)

_____ = _____ + _____

4. What is a factor? You may use a numerical example to help you explain.

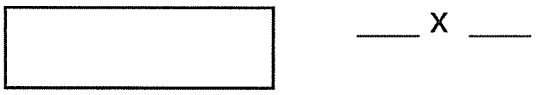
_____

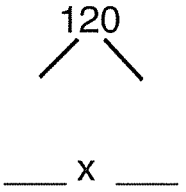
_____

5. What is a prime number? _____.

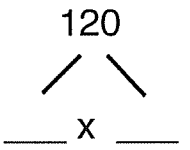
_____

6. a. Prime factor 120.
   b. Record the prime factors in this box.





7. Factor 120 into factors of ten.



8. Use factoring by tens to show how to get the product of 700 x 9.

700 x 9

___ x ___ x ___

(___ x ___) x ___

___ x ___

_____

Name _____ Date _____

Assessment: Part 2 - Worksheet 1

1. a. Solve. 400 x 30 = _____

   b. Explain how you got your answer. _____

   _____

   _____

2. Use factoring by tens to show how to get the answer to 40 x 700.

   40 x 700

   (___x___) x (___ x ___)

   (___x___) x (___ x ___)

   _____ x _____ = _____

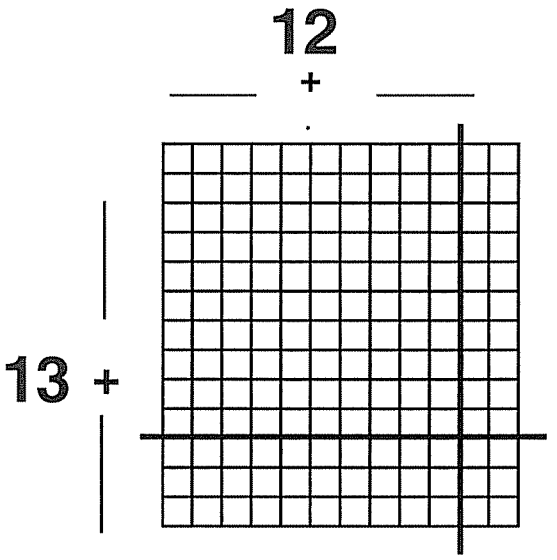
3. There is a trick to solving a problem such as 900 x 500. Multiply 9 x 5 and add four zeros. Explain why this trick works.

   _____

   _____

   _____

4. Use this diagram to help you fill in the distribution number sentences.



___ x ___ = (___ +___) x (___ + ___)

= (___x___) + (___x___) + (___x___) + (___x___)

_____ = ____ + ____ + ____ + ____

Assessment: Part 2 - Worksheet 2 Name _____ Date _____

5. Show the distribution pattern with arrows between the numbers.
   Use this diagram to help.

$$12 = 5 + 7$$

$$\times 13 = 9 + 4$$

$$4 \times 7 = \_\_\_\_$$

Write the little problems here.

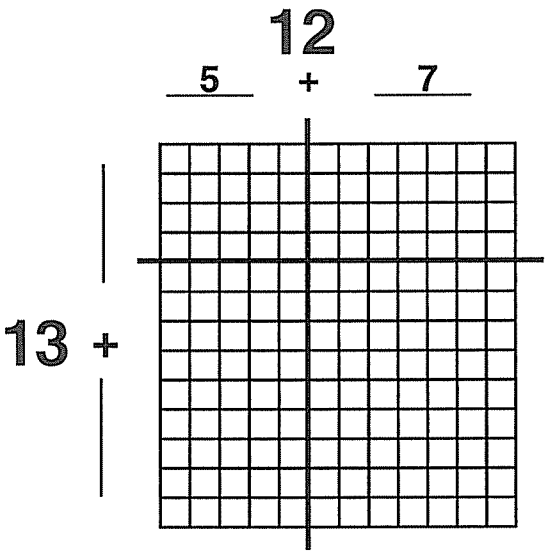
$$\_\_\_\_ = \_\_\_\_$$

$$\_\_\_\_ = \_\_\_\_$$

$$\_\_\_\_ = \_\_\_\_$$

$$\_\_\_\_ = \_\_\_\_$$

Final answer. $= \_\_\_\_$



6. You should have gotten the same answer for problems 4 and 5. Explain why.

_____

_____

7. Solve this problem the long way using the arrow pattern.

$$\begin{array}{r} 235 \\ \times\ 46 \\ \hline \end{array}$$

little multiplication problems		partial products
_____	=	_____
_____	=	_____
_____	=	_____
_____	=	_____
_____	=	_____
_____	=	_____

$= \_\_\_\_$

8. Solve the same problem the short way.

$$\begin{array}{r} 235 \\ \times\ 46 \\ \hline \end{array}$$

9. Why do you have to put a zero in the ones place in the second row of the partial product in problem 8?

_____

_____

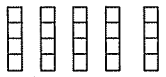
Multiplication: Booklet 1

Assessment - Part 1

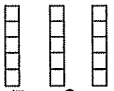
1. Solve. a. 4 + 5 + 6 = 15 b. 4 + 4 + 4 + 4 + 4 + 4 = 20 c. 13 + 13 + 13 + 13 = 52

Assessment - Part 2

1. What multiplication problem is shown in each picture?

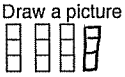


a. 4 x 5 = 20



b. 5 x 3 = 15

2. a. Draw a picture of 3 x 4.



b. Show 3 x 4 as an addition problem.

3 + 3 + 3 + 3 = 12

3. Solve these problems.

a. 3 x 2 = 6 b. 6 x 4 = 24 c. 7 x 3 = 21 d. 2 x 3 x 4 = 24 e. 5 x 7 x 0 = 35

4. Use addition to solve.

a. 14 x 5 = 14 + 14 + 14 + 14 + 14 = 70

b. 25 x 4 = 25 + 25 + 25 + 25 = 100

c. 32 x 6 = 32 + 32 + 32 + 32 + 32 + 32 = 192

5. Write a word problem that uses the problem 6 x 3 = 18.

AWV = Answer will vary.

6. Factor these numbers.

a. 15 = 3 x 5 or 15 x 1

b. 24 = 4 x 6, 3 x 8, 2 x 12, 24 x 1

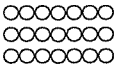
Multiplication: Booklet 2

Assessment - Part 1

1. What multiplication problem is shown in this drawing?

4 x 3 = 12

2. Draw a picture like the one above for 7 x 3 = 21



3. Write this multiplication problem as an addition problem: 23 x 4.

23 + 23 + 23 + 23 = 92

4. Skip count by fours from 0 to 40. 0 4 8 12 16 20 24 28 32 36 40

5. Solve these problems.

a. 6 x 4 = 24 b. 5 x 5 = 25 c. 9 x 3 = 27

d. 7 x 6 = 42 e. 8 x 8 = 64 f. 9 x 6 = 54

6. Write all multiplication problems that equal 24.
You may use Cuisenaire Rods and a meter stick if you like.
Hint: There are more than two.

1 x 24 = 24

2 x 12 = 24

3 x 8 = 24

4 x 6 = 24

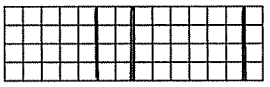
Multiplication: Booklet 2

Assessment - Part 2

1. Fill in the missing numbers.

a. 14 x 4 = 56

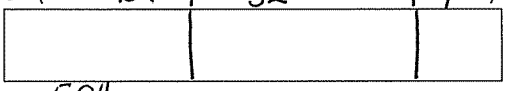
b. 14 x 4 = (5 + 2 + 6 + 1) x 4



e. 56 = d. c. (5 x 4) + (2 x 4) + (6 x 4) + (1 x 4)
20 + 8 + 24 + 4

2. Show how you would break up the 63. Label the partial products in the box. Show the final product below the box on the line. Example:

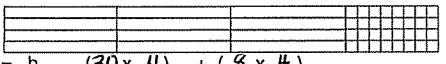
63 x 8 = (24 + 32 + 7) x 8



63 x 8 = 504

3. Break up the 38 into tens and ones; fill in the missing numbers.

a. 38 x 4 = (30 + 8) x 4



d. 152 = b. (30 x 4) + (8 x 4) = c. 120 + 32

4. In problem 3, what number is being distributed? 4

5. Solve these problems the Long way and the Short way.

a. Long way: 1,439 x 5 = 45 + 150 + 2,000 + 5,000 = 7,195

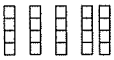
b. Short way: 1,439 x 5 = 7,195

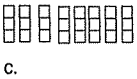
6. Solve. a. 138 x 6 = 828 b. 2,375 x 4 = 9,500

Multiplication: Booklet 3

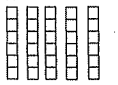
Assessment - Part 1

1. Look at the picture. Write the addition and multiplication number sentence.

a. 
5 + 5 + 5 + 5 + 5 = 25 Addition
5 x 5 = 25 Multiplication

b. 
3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24 Addition
3 x 8 = 24 Multiplication

c. 
2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 18 Addition
2 x 9 = 18 Multiplication

d. 
6 + 6 + 6 + 6 + 6 = 30 Addition
6 x 5 = 30 Multiplication

2. Here is a multiplication number sentence: 3 x 7 = 21.
Write the meaning of each number in the multiplication number sentence.
a. The 3 means the number of blocks in each group.
b. The 7 means the number of times each group is made.
c. The 21 means the total number of blocks.

3. a. What is the area of this rectangle? 25 sq. units

b. What is the area of this rectangle? 35 sq units

Multiplication: Booklet 3

Assessment - Part 2 - Worksheet 1

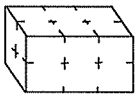
1. Show all the ways of associating (grouping) these three numbers: 2, 5, 6

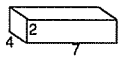
$(2 \times 5) \times 6 = 10 \times 6 = 60$ $\quad (6 \times 2) \times 5 = 12 \times 5 = 60$ $\quad (6 \times 5) \times 2 = 30 \times 2 = 60$

2. Find the volume of these rectangular solids.

Length (L)	Width (W)	Height (H)
3	2	2
7	4	2
3	6	2

a. Volume 12 cu units b. Volume 56 cu units c. Volume 36 cu units





L = 3; W = 6; H = 2

3. What is a factor? Use numbers to help you explain.

A factor is a number that results in a product (answer) when multiplied by another factor.

4. What is a Prime number?

A number that has only itself and one as factors.

5. List the first ten Prime Numbers starting with 2.

2 3 5 7 11 13 17 19 23 29

Multiplication: Booklet 3

Assessment - Part 2 - Worksheet 2

6. Prime Factor 24, 50, and 37. Record the Prime Factors in the boxes.

a. [3, 2, 2, 2] — 24 → 2 × 12 → 3 × 4 → 2 × 2

b. [5, 5, 2] — 50 → 5 × 10 → 2 × 5

c. [37, 1] — 37 → 37 × 1

7. Factor 50 and 80 by tens.

By tens

a. 50 → 5 × 10

By tens

b. 80 → 8 × 10

8. a. Solve these problems.

6	60	600	6,000
× 8	× 8	× 8	× 8
48	480	4,800	48,000

b. What patterns do you notice in the answers to these problems?

Forty-eight is always the first two numbers in the product. The number of zeros after the forty-eight changes. The place value of forty-eight changes.

c. Use the pattern to find the answer to 600,000 x 8. 4,800,000

9. Use factoring by tens to show how to get the product of 400 x 6.

400 x 6

4 x 100 x 6

(4 x 6) x 100

24 x 100

2,400

Multiplication: Booklet 4

Assessment: Part 1 - Worksheet 1

1. Solve the Short Way.

a. 34	b. 258	c. 1,735
× 6	× 3	× 4
204	774	6,940

2 Solve the Long Way.

a. 34	b. 258	c. 1,735
× 6	× 3	× 4
24	24	20
180	150	120
204	600	2800
	774	4,000
		6,940

3. Break up the 57 using tens and ones. Fill in all the blanks.

57 x 3 = (50 + 7) x 3

= (50 x 3) + (7 x 3)

171 = 150 + 21

4. What is a factor? You may use a numerical example to help you explain.

2 x 3 are factors of 6. A factor is a number that results in a product when multiplied by another factor.

5. What is a Prime Number? A number that has only one and itself as factors.

6. a. Prime Factor 120.
b. Record the Prime Factors in this box.

120 → 2 × 60 → 6 × 10 → 3 × 2 × 2 × 5

[5, 3, 2, 2, 2]

7. Factor 120 into factors of ten.

120 → 100 × 12 or 12 × 10 × 10

8. Use factoring by tens to show how to get the product of 700 x 9.

700 x 9

7 x 100 x 9

(7 x 9) x 100

63 x 100

6,300

Multiplication: Booklet 4

Assessment: Part 2 - Worksheet 1

1. a. Solve. 400 x 30 = 12,000

b. Explain how you got your answer. Factor 4 from the 100 and 3 from the 10, rearrange the factors and combine the 4x3=12 and the 100x10 to get 1,000. Multiply 12 x 1,000=12,000.

2. Use factoring by tens to show how to get the answer to 40 x 700.

40 x 700

(4 x 10) x (7 x 100)

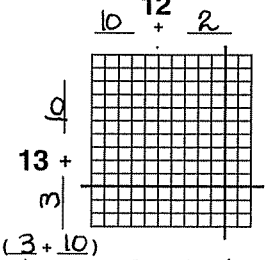
(4 x 7) x (10 x 100)

28 x 1,000 = 28,000

3. There is a trick to solve a problem such as 900 x 500. Multiply 9 x 5 and add four zeros. Explain why this trick works.

Associative Property allows the rearrangement of factors. Multiply 9x5 and 100x100. The four zeros come from multiplying 100 x 100 = 10,000, which has four zeros.

4. Use this diagram to help you fill in the distribution number sentences.



12 x 13 = (10 + 2) x (3 + 10)

= (3 x 2) + (3 x 10) + (10 x 2) + (10 x 10)

= 6 + 30 + 20 + 100

156

Multiplication: Booklet 4

Assessment: Part 2 - Worksheet 2

5. Show the distribution pattern with arrows between the numbers. Use this diagram to help.

12 = 5 + 7

x 13 = 9 + 4

Write the little problems here.

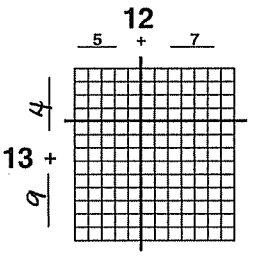
4 x 7 = 28

4x5 = 20

9x7 = 63

9x5 = 45

Final answer = 156



6. You should have gotten the same answer for problems 4 and 5. Explain why.

Even though the problems are broken up differently, the answers will be the same because it is the same problem.

7. Solve this problem the long way using the arrow pattern.

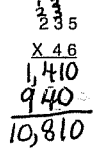
235

X 46

little multiplication problems		partial products
5x6	=	30
30x6	=	180
200x6	=	1,200
5x40	=	200
30x40	=	1,200
200x40	=	8,000

= 10,810

8. Solve the same problem the short way.



9. Why do you have to put a zero in the ones place in the second row of the product in problem 8?

Because the four is in the tens place, so the multiplication problem is 40x5=200 which has a zero in the ones place.

Patterns in Arithmetic: Multiplication Placement PDF
Parent/Teacher Guide

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