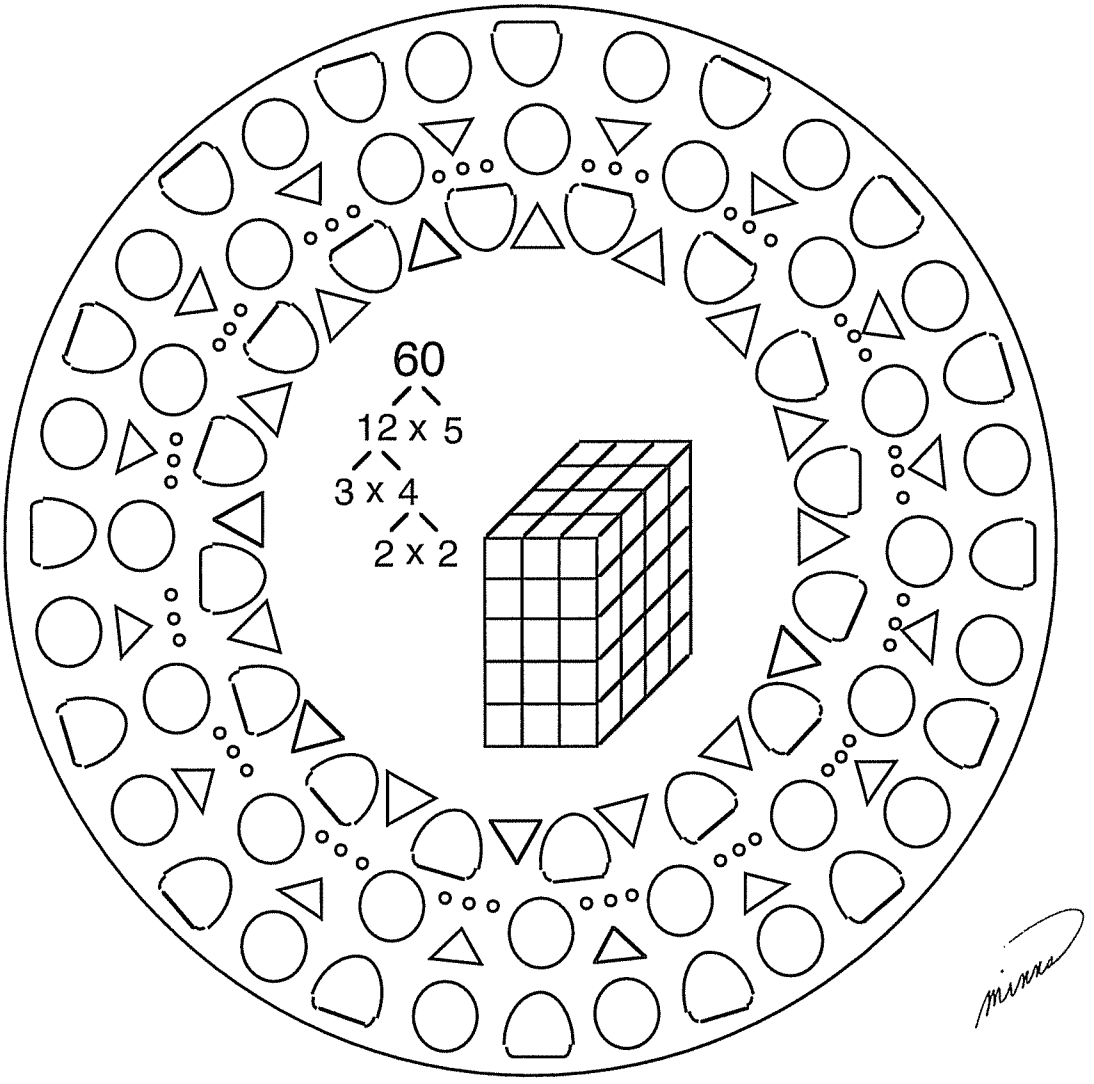
Patterns in Arithmetic

Multiplication - Booklet 3 PDF

Properties and Factoring

Parent/Teacher Guide



By Alysia Krafel, Suki Glenn, and Susan Carpenter

Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Multiplication: Booklet 3 - PDF - Properties and Factoring

Contents

I dedicate this booklet to my wonderful husband, Tom Glenn. Without his love and support I could not have finished this math series.
Suki Glenn

Acknowledgments

The knowledge, patience, and dedication of Professor Michael Butler made the UCI Farm Elementary School and this mathematics program possible. Special thanks go to Alysia Krafel and Susan Carpenter, who helped develop much of the math materials based on the teachings, ideas, and insights of Professor Butler.

For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

The books would never have been completed if the students at Chrysalis Charter School in Redding, California, under the guidance of Alysia and Paul Krafel, hadn't needed them. Thank you for your patience through all of the draft copies.

Susan Carpenter edited, added her wise words, useful suggestions, and helped make the Answer Keys a reality. Karán Founds-Benton contributed her meticulous editing skill and knowledge. Diligent and thorough copy editing was done by Jacqueline Logue.

The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

Patterns in Arithmetic: Multiplication - Booklet 3 PDF
Parent/Teacher Guide

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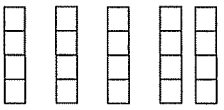
Pre-Assessment: Part 1 Date _____

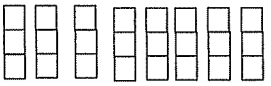
Put a question mark next to any problem you do not know how to do.

1. Look at the picture. Write the addition and multiplication number sentence.

a. 

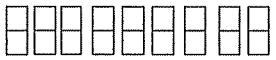
__ + __ + __ + __ + __ = _____ Addition

__ x __ = _____ Multiplication

b. 

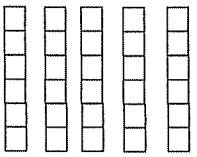
__ + = _____ Addition

__ x __ = _____ Multiplication

c. 

__ + = _____ Addition

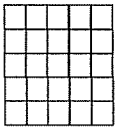
__ x __ = _____ Multiplication

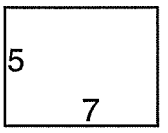
d. 

__ + = _____ Addition

__ x __ = _____ Multiplication

2. Here is a multiplication number sentence: 3 x 7 = 21.
   Write the meaning of each number in the multiplication number sentence.
   a. The 3 means the _____.
   b. The 7 means the _____.
   c. The 21 means the _____.

3. a. What is the area of this rectangle? _____ 

   b. What is the area of this rectangle? _____ 

Pre-Assessment: Part 2 - Worksheet 1 Date _____

Put a question mark next to any problem you do not know how to do.

1. Show all the ways of associating (grouping) these three numbers: 2, 5, 6

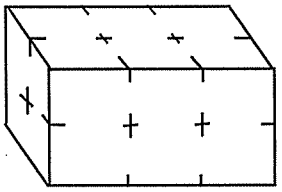
(__ x __) x ___ = ___ x ___ = ___

(__ x __) x ___ = ___ x ___ = ___

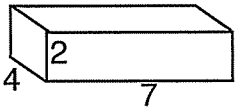
(__ x __) x ___ = ___ x ___ = ___

2. Find the volume of these rectangular solids.
Length (L) Width (W) Height (H)

a. Volume _____



b. Volume _____



c. Volume _____

L = 3; W = 6; H = 2

3. What is a Factor? Use numbers to help you explain.

_____

_____

_____

4. What is a Prime Number?

_____

_____

_____

5. List the first ten Prime Numbers starting with 2.

2 ___ ___ ___ ___ ___ ___ ___ ___ ___

Pre-Assessment: Part 2 - Worksheet 2 Date _____

6. Prime Factor 24, 50, and 37. Record the Prime Factors in the boxes.

a. ☐

24

/ \

__ x __

b. ☐

50

/ \

__ x __

c. ☐

37

/ \

__ x __

7. Factor 50 and 80 by tens.

By tens

a. 50

/ \

__ x __

By tens

b. 80

/ \

__ x __

8. a. Solve these problems.

6	60	600	6,000
x 8	x 8	x 8	x 8

b. What patterns do you notice in the answers to these problems?

_____

_____

_____

c. Use the pattern to find the answer to 600,000 x 8. _____

9. Use factoring by tens to show how to get the product of 400 x 6.

400 x 6

/ \

__ x __ x __

(__ x __) x __

__ x __

_____

Use the Answer Key to check for errors.

Assessment Guide

Purpose

The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Pre-Assessment: Part 1 is review material from *Patterns in Arithmetic:* Multiplication: Booklet 2. Pre-Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

The Post-Assessment is administered to determine if the student learned the material that was presented in this booklet. A comparison of the score on Pre-Assessment: Part 2 to the score on the Post-Assessment will give both you and your student a sense of growth. The Post-Assessment is the same as Pre-Assessment: Part 2.

Prerequisites

Patterns in Arithmetic: Multiplication - Booklet 2

Materials

Pre-Assessment: Part 1 and Part 2, pages 1- 3
Pre-Assessment: Part 1 and Part 2 Score Sheets, pages 8 and 9 in this booklet
Post-Assessment Score Sheet, page 10
Cuisenaire Rods

Instructions

Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

Note

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

Do the assessment in two parts. Give Pre-Assessment: Part 1 and check it for readiness for this booklet. The answers are in the Answer Key. If he is not ready for this booklet, there is no point in giving Pre-Assessment: Part 2. If he passes all the readiness items, then give Pre-Assessment: Part 2.

Assessment Guide

After scoring Pre-Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc. Score sheets that match the Assessment Guide for Pre-Assessment: Part 1, Pre-Assessment: Part 2, and the Post-Assessment follow. The answers for Pre-Assessment: Parts 1 and 2 are in the back of the Answer Key. The Post-Assessment is on pages 39 and 40.

Assessment Criteria for Pre-Assessment: Part 1

Can the student:

1. Concept (Multiplication: Booklets 1 and 2)

A. identify correctly the addition number sentence associated with the picture in three of the four problems?

B. identify correctly the multiplication number sentence associated with the picture in three of the four problems?

If he identifies the problem in item 1a as 5 x 4 instead of 4 x 5, it means he is not familiar with the convention used in these books. The *Patterns in Arithmetic* series always places the size of the group first, then the operations sign. The second number tells the number of groups. Reteach this convention but give him a Yes on the score sheet.

C. write the correct answer to three of the four problems?

2. Concept (Multiplication: Booklets 1 and 2)

The three may be identified as the number of groups or the number in each group. Whichever one is used for the three, the other one must be used for the seven.

A. identify the correct meaning of the three?

B. identify the correct meaning of the seven?

C. identify twenty-one as the total? The words 'the answer' are not sufficient to score a Yes on this item.

3. Standard Area (Multiplication: Booklets 1 and 2)

A. give the correct area on problem a, which gives a picture?

B. give the correct area on problem b, which gives only the numbers?

Note: If he gives the answer as 20 on 3a or 24 on 3b, he has area confused with perimeter. The answers should also be given in square units. If the numbers are correct, but he leaves off the square units, remediate this while you are teaching the lesson on area in Multiplication: Booklet 3.

Booklet Selection Guide based on results of Pre-Assessment: Part 1

If he gets a No on 1A, 1B, 2A, and 2B, he is not ready for this booklet. It is possible to know the multiplication table and not understand the concept or what the multiplication number sentence means. Begin with Booklet 1 to build his conceptual understanding of multiplication and the multiplication number sentences. If he knows his multiplication tables, you can focus on the first part of Multiplication: Booklet 1 and move onto Booklet 2, hitting the parts he does not know. Use the assessments in Multiplication: Booklet 2 to determine what he does and does not understand about single digit multiplication.

If he gets a Yes on 1A, 1B but gets a No on 2A, 2B, or 3A, he is not quite ready for this booklet; begin with Multiplication: Booklet 2. If Multiplication: Booklet 2 has been completed, remediate and retest. Then give Pre-Assessment: Part 2 and begin this booklet. Multiplication: Booklets 2 and 3 can be taught concurrently once the number sentence is understood.

If he gets a Yes on six or more of the items in Pre-Assessment: Part 1, give Pre-Assessment: Part 2.

Assessment Criteria for Pre-Assessment: Part 2

Can the student:

1. Associative Property (Multiplication: Booklet 3)

A. write the numbers into the parentheses on at least one problem?

B. fill in all three problems with the numbers in different orders?

C. fill in the correct numbers on the second line of two of the three problems?

D. give the answer sixty on any of the problems?

This group of items tests if he knows that the numbers can be placed in any order of combination and the product will be unchanged. The goal is for him to recognize what associating is and to be able to use the parentheses to show all the orders in which these three numbers can be multiplied.

2. Volume Puzzles (Multiplication: Booklet 3)

A. determine the correct volume of a rectangular solid with a picture showing the cubes (a)?

B. determine the correct volume of a rectangular solid with a picture with only numbers and no cubes shown (b)?

C. determine correctly the volume of a rectangular solid given only the length, width, and height (c)?

3. Discovering Prime Numbers (Multiplication: Booklet 3)

A. explain and/or show an example of a factor?

2 Points: A factor is a number that results in a product (answer) when multiplied by another factor. Example: $2 \times 3 = 6$. Two and three are factors of six.

1 Point: A factor is a number used to make an answer such as $2 \times 3 = 6$.

4. Discovering Prime Numbers (Multiplication: Booklet 3)

A. explain what a prime number is?

2 Points: A number that has only itself and one as factors.

1 Point: A number that can only be made one way (referring to the rectangular arrays drawn in the lesson).

5. Discovering Prime Numbers (Multiplication: Booklet 3)

A. list eight of the first ten prime numbers?

If he includes 9, 15, 25, or 27 in this list, it indicates he thinks that all odd numbers are prime.

6. Prime Factoring (Multiplication: Booklet 3)

A. prime factor correctly two of the three numbers?

B. place the prime factors from largest to smallest in the box?

C. realize thirty-seven can not be factored?

7. Factoring by Tens (Multiplication: Booklet 3)

A. factor both of the numbers correctly by factors of ten?

8. Expanded Tables (Multiplication: Booklet 3)

A. give the correct answers to three of the four problems?

B. explain that the number forty-eight will always occur as the first two numbers in the product but the number of zeros after the forty-eight will change? Or that the place value of the forty-eight will change?

C. extend that pattern to get forty-eight followed by five zeros? If he used addition to get the answer, or the standard multiplication format, give a No. To get a Yes on this item, just the answer should be written down. You might also see little dots above the zeros where he counted the number of them.

9. Using Factoring with Expanded Tables (Multiplication: Booklet 3)
    A. give the correct product at the bottom?
    B. show that the four hundred is factored into 4 x 100 in the first factoring line?
    C. show the reordering of the factors to combine the 4 x 6 in the second factoring line?

Booklet Selection Guide based on results of Pre-Assessment: Part 2
If his score is 17 or more Yes answers on the criterion score sheet, then he does not need this booklet. Move on to Booklet 4
If the score is 12 - 15 Yes answers on the criterion score sheet, use this booklet to remediate the weak areas and move on to Booklet 4.
If the score is 11 or below, this is the correct booklet for your student.

Whenever remediation is needed, rely upon the following process, which is used throughout the *Patterns in Arithmetic* series to develop understanding of a concept.

1. Introduce the concept with a manipulative. Orally discuss it. Build it. Verify it. Practice it. Repeat the experience with a different manipulative (oral manipulative).
2. Use manipulatives to explore the concept again. This time record it with pictures (pictorial/representation). Practice it. Use worksheets.
3. Record the problem with numbers (abstract/symbolic), which links the pictorial with the abstract.
4. Practice fluency.
5. Practice for speed.

Begin each lesson with a warm-up and review. Always end a lesson with a success before the student is tired. It is best to end while the student is still enjoying the lesson.

Ask questions or make statements, such as: **"Are you sure?"** or **"Build it."** or **"What gave you the clue?"** or **"Show me how you got that."** or **"Prove it."** even when a student is correct. This is important to do often. Many students will ask an adult, "Am I right?" rather than answering definitively. Confidence in a student's response must come from within. A student needs to self-check and have confidence in his or her ability and knowledge. Asking the student if he or she is right, even when correct, will encourage self-confidence and the ability to self-check.

Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **"How did you get that?" "Show me what you mean." "Build a model of that." "Tell me more so I can understand what you are saying."**

Pre-Assessment: Part 1 Score Sheet

Name _____ Date _____

Can the student:

1. Concept (Multiplication: Booklets 1 and 2)
   - Yes No A. identify correctly the addition number sentence associated with the picture in three of the four problems?
   - Yes No B. identify correctly the multiplication number sentence associated with the picture in three of the four problems?
   - Yes No C. write the correct answer to three of the four problems?

2. Concept (Multiplication: Booklets 1 and 2)
   - Yes No A. identify the correct meaning of the three?
   - Yes No B. identify the correct meaning of the seven?
   - Yes No C. identify twenty-one as the total?

Note: The words 'the answer' are not sufficient to score a Yes on this last item.

3. Standard Area (Multiplication: Booklets 1 and 2)
   - Yes No A. give the correct area on problem a, which gives a picture?
   - Yes No B. give the correct area on problem b, which gives only the numbers?

Items Correct = _____ = _____%
Items Possible = 8

Pre-Assessment: Part 2 Score Sheet

Name _____ Date _____

Can the student:

1. Associative Property (Multiplication: Booklet 3)
   - Yes No A. write the numbers into the parentheses on at least one problem?
   - Yes No B. fill in all three problems with the numbers in different orders?
   - Yes No C. fill in the correct numbers on the second line of two of the three problems?
   - Yes No D. give the answer sixty on any of the problems?

2. Volume Puzzles (Multiplication: Booklet 3)
   - Yes No A. determine the correct volume of a rectangular solid with a picture showing the cubes (a)?
   - Yes No B. determine the correct volume of a rectangular solid with a picture with only numbers and no cubes shown (b)?
   - Yes No C. determine the correct volume of a rectangular solid given only the length, width, and height (c)?

3. Discovering Prime Numbers (Multiplication: Booklet 3)
   - 1Pt. 2 Pts. A. explain and/or show with an example what a factor is?

4. Discovering Prime Numbers (Multiplication: Booklet 3)
   - 1Pt. 2 Pts. A. explain what a prime number is?

5. Discovering Prime Numbers (Multiplication: Booklet 3)
   - Yes No A. list eight of the first ten prime numbers?

6. Prime Factoring (Multiplication: Booklet 3)
   - Yes No A. prime factor correctly two of the three numbers?
   - Yes No B. place the prime factors from largest to smallest in the box?
   - Yes No C. realize thirty-seven can not be factored?

7. Factoring by Tens (Multiplication: Booklet 3)
   - Yes No A. correctly factor both of the numbers by factors of ten?

8. Expanded Tables (Multiplication: Booklet 3)
   - Yes No A. give the correct answers to three of the four problems?
   - Yes No B. explain the pattern?
   - Yes No C. extend that pattern to get forty-eight followed by five zeros?

9. Using Factoring with Expanded Tables (Multiplication: Booklet 3)
   - Yes No A. give the correct product at the bottom?
   - Yes No B. show that the four hundred is factored into 4 x 100 in the first factoring line?
   - Yes No C. show the reordering of the factors to combine the 4 x 6 in the second factoring line?

Items Correct = _____ = _____% 17 Yes points to pass, or 80%

Items Possible = 22

Post-Assessment Score Sheet Name _____ Date _____

Can the student:

1. Associative Property (Multiplication: Booklet 3)
   - Yes No A. write the numbers into the parentheses on at least one problem?
   - Yes No B. fill in all three problems with the numbers in different orders?
   - Yes No C. fill in the correct numbers on the second line of two of the three problems?
   - Yes No D. give the answer sixty on any of the problems?

2. Volume Puzzles (Multiplication: Booklet 3)
   - Yes No A. determine the correct volume of a rectangular solid with a picture showing the cubes (a)?
   - Yes No B. determine the correct volume of a rectangular solid with a picture with only numbers and no cubes shown (b)?
   - Yes No C. determine the correct volume of a rectangular solid given only the length, width, and height (c)?

3. Discovering Prime Numbers (Multiplication: Booklet 3)
   - 1Pt. 2 Pts. A. explain and/or show with an example what a factor is?

4. Discovering Prime Numbers (Multiplication: Booklet 3)
   - 1Pt. 2 Pts. A. explain what a prime number is?

5. Discovering Prime Numbers (Multiplication: Booklet 3)
   - Yes No A. list eight of the first ten prime numbers?

6. Prime Factoring (Multiplication: Booklet 3)
   - Yes No A. prime factor correctly two of the three numbers?
   - Yes No B. place the prime factors from largest to smallest in the box?
   - Yes No C. realize thirty-seven can not be factored?

7. Factoring by Tens (Multiplication: Booklet 3)
   - Yes No A. correctly factor both of the numbers by factors of ten?

8. Expanded Tables (Multiplication: Booklet 3)
   - Yes No A. give the correct answers to three of the four problems?
   - Yes No B. explain the pattern?
   - Yes No C. extend that pattern to get forty-eight followed by five zeros?

9. Using Factoring with Expanded Tables (Multiplication: Booklet 3)
   - Yes No A. give the correct product at the bottom?
   - Yes No B. show that the four hundred is factored into 4 x 100 in the first factoring line?
   - Yes No C. show the reordering of the factors to combine the 4 x 6 in the second factoring line?

Items Correct = _____ = _____% 17 Yes points to pass, or 80%

Items Possible = 22

Introduction to Factoring

Factoring is a tool used with multiplication to break numbers into smaller units. Alysia Krafel always tells students that factoring is like a screwdriver. This tool will enable operations like simplification of fractions, division, and long multiplication.

The answer to a multiplication problem is called a *product*. A product is produced by multiplying two numbers together. These two numbers are the *factors* of that product.

In multiplication we have *factors* and *multiples*. Here is a set of statements that will clarify these two related ideas.

Four is a *factor* of twelve. This statement means four can be multiplied by another number to produce twelve.

Twelve is a *multiple* of four. This statement means that twelve can be produced by multiplying another number by four.

Three and four form a *factor pair* that will produce twelve. Twelve can be produced with several different *factor pairs*: 1 x 12, 2 x 6, and 3 x 4. Twelve is then a *multiple* of 1, 2, 3, 4, 6, and 12. All numbers have themselves and one as factors. 1, 2, 3, 4, 6, and 12 are factors of twelve.

There are three types of factors commonly used at this grade level. They are: Prime Factors, Composite Factors and Factors of Ten.

,
A *Prime Number* is any number that can be evenly divided by only itself and one. Example: Seven can not be divided evenly by any number other than one group of seven or seven groups of one.
The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29. Notice that 1 is not in the list. One is not defined as a prime number. Notice that all prime numbers other than two are odd numbers, but not all odd numbers are prime. Think nine (3 x 3) and fifteen (3 x 5), for example.
Prime Factors are the 'bones' of a multiple. Prime factors are a set of prime numbers that when multiplied together will produce a specific multiple. For example: 3 x 3 x 2 are the prime factors of eighteen. 7 x 5 x 2 are the prime factors of seventy.

Prime numbers and prime factors will be used by students at this grade level to simplify fractions. If a fraction contains a prime number, it can only be simplified if the other number is a multiple of the prime. For example: $\frac{7}{28}$ can be simplified because twenty-eight is a multiple of seven. $\frac{7}{18}$ cannot be simplified because seven is prime and eighteen is not a multiple of seven. There are no common factors to divide by. A nifty way to simplify a really nasty fraction is to prime factor both the numerator and the denominator and cancel out all the matching numbers. What will be left is the simplified fraction. This is how algebraic fractions are simplified.

Example: Take this difficult little fraction. The prime factors are listed after the fraction. The common factors are canceled out; what is left is multiplied, and you are left with a simplified fraction. Voila!

$$\frac{18}{96} \quad \frac{3}{3}\frac{3}{2}\frac{2}{2} \quad \frac{}{2}\frac{}{2}\frac{}{2} = \frac{3}{2}\frac{}{2}\frac{}{2}\frac{}{2} = \frac{3}{16}$$

A *Composite Number* is any number that is not prime. When writing 24 = 4 x 6, the numbers four and six are composite factors of twenty-four. Composite factors are included in a list of all possible factors of a number. These are important to recognize when simplifying fractions. If the numbers in the fraction are composites, there is a high probability that the fraction can be simplified by looking for common factors. The fraction $\frac{4}{6}$ is not simplified because both the numerator and the denominator are composite numbers that share two as a factor. Students must understand all of these words–prime, composite, and factoring–before they try to learn how to form equivalent fractions or simplify fractions.

Factors of Ten are numbers in the place value system. These include ones, tens, hundreds, thousands, tenths, hundredths, and so on. Factoring numbers by multiples of ten is a tool used to determine the product of numbers like 600 x 30 and how to set the decimal for 0.5 x 0.07. Factors of ten are also used when working with exponents.

In this booklet, the student will learn how factoring is done. She will experience the booklet as somewhat unconnected items. The connections will occur when she encounters the operations where factoring is used as a tool. These booklets will be Place Value: Booklet 3, Multiplication: Booklet 4, Division: Booklet 3, and Fractions: Booklets 4, 5, 6, and 7.

Concept

Purpose The purpose of this lesson is to review the basic concept of multiplication with blocks and addition and multiplication number sentences. If you are new to this series, read the note below for the multiplication convention used throughout these books.

Prerequisites Basic understanding of the concept of multiplication

Materials Concept, page 4
Unifix cubes or other interlocking cubes

Lesson Give Concept, page 4, and have the student look at the picture at the top of the page. Have her build the multiplication problem pictured, five stacks of four linked cubes.
"What number is being added?" "The fours."

"How many times is the four being added?" "Five times."

"What is the addition problem shown here?" "4 + 4 + 4 + 4 + 4 = 20."

"What multiplication problem can that be turned into?" "4 x 5 = 20."

Build the problem in the second picture.
"What number is being added?" "Six."

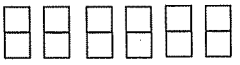
"How many times is the six being added?" "Three times."

"Write the addition problem in the box next to the picture."

"What is the multiplication problem you need to write in the blanks?"
"6 x 3 = 18."

Have her finish the rest of the page. Do not insist that she build the problems. Many students can solve the problem with the picture. Check the work immediately.

Test for Understanding The last two problems are Tests for Understanding. Can she draw a picture or build with blocks the addition problem shown and turn it into a multiplication number sentence? Can she make up her own problem and do all three steps? For example,

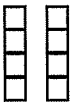
 = 12 2 x 6 = 12

Note There are two ways to interpret a multiplication number sentence.
4 x 2 = 8 can mean 2 + 2 + 2 + 2 = 8 or 4 + 4 = 8. Both ways are mathematically correct. There is a profound difference between 'four groups of two' and 'two groups of four.' Students will become familiar with this difference (and eventually discover that both concepts share the same answer) by building models to represent the multiplication problem.

In order to make models and to be able to visualize a multiplication problem, a

student needs to know which number in the multiplication problem represents the *size* of the groups to be made and which number represents the *number of times* that group will be made or added. The choice of which representation each number bears is arbitrary. But it is important for explicit thinking, model building, and proving answers that the numbers to the written problem have the same meaning every time. *The language used and the form of the models built must be consistent.* This series adopts the following convention.

4 x 2 = 8 as 4 + 4 = 8
Four, two times



The sentence, read from left to right, says, "Make a group of four, two times." The first number tells the *size* of the group to be made, and the second number tells the *number of times* the group is added. The reason for this choice is that when building models of number sentences it is easier for students if the first number of the sentence always tells them how many blocks to start with and the second number always tells them what to do next. The order of the words and the action of the hands must correspond. Build a group a certain size, then repeat that action a certain number of times.

The language that corresponds to this meaning of 4 x 2 is "Make a group of four, two times." Or shortened to "Build four, two times."

This may sound awkward if you are used to saying, "Make two groups of four." This language switches the positions of the numbers, which can be a source of confusion.

Number Lines

Purpose The purpose is to review multiplication as evenly spaced skips on a number line. This is a visual representation of skip counting.

Prerequisites Previous experience with number lines

Materials
Number Lines, page 5
Centimeter ruler (30 centimeters)
Cuisenaire Rods
Unlined paper

Warm Up Number lines are very abstract for most students. To make them concrete, use a ruler and Cuisenaire Rods. Place the ruler onto a sheet of unlined paper. Have the student place a light green rod (three cm) on the paper and push it up against the ruler. Be sure the first rod sits exactly at the zero mark, which may not be at the end of the ruler. The end of the rod should match up with the three on the ruler. Have him draw an arc from the zero to the three above the rod. Look at the arcs drawn on the worksheet to see the shape of the line that needs to be drawn.

Now, add another light green rod. It will line up with the six. Draw an arc from the end of the first rod to the end of the second. Continue this process until you run out of ruler.

Have him skip count by threes, beginning with zero, by tracing the arcs with his fingers and reading the number on the ruler that matches the end of each rod. Have him imagine that his finger is a kangaroo. You can add a 'boing' sound effect for each jump to make it more fun.

Have him slide the rods so they are above the arc lines. Then have him extend each arc line down to where the line touches the ruler. He has now made his own number line. Have him skip count again using the numbers on the ruler. The rods are still there to act as a physical reminder of what counting by threes looks like.

"How many jumps do you have to take to get to fifteen?" "Five."

"What multiplication problem is jumping three five times?" "Three times five."

"What number will you land on if you jump seven times?" He does this. "Twenty-one."

"Using your kangaroo finger, show three times four." He makes four jumps and lands on twelve.

"What multiplication problem did you just do?" "Three times four."
Keep the ruler, arcs, and rods on the table in front of him.

Lesson

Begin the worksheet, Number Lines. **"Which problem is the same as the number line you just made with the rods?"** "The first one." Have him trace with his finger the kangaroo hops while skip counting by three.

Test for Understanding

The first question asks, 'How many numbers are skipped in each jump?' The answer is three. Have him write that in. Next is, 'How many jumps?' This means how many jumps will the kangaroo make to get to the end. The answer is twelve. The number sentence is 3 x 12 = 36.

The next problem asks him to draw 6 x 6 on the number line. Wait to see what he does.
If he needs assistance, use the dark green rods and the ruler again. Begin laying down six rods next to the ruler. Then have him draw the first arc on the paper above the rod like he did before. Then have him draw that same arc on the smaller number line on the worksheet. Wait. See if he can finish alone without the rods. If he cannot, go back to the rods, the ruler, and then back to the worksheet. Then skip the next two problems. Take out the purple rods and repeat the warm-up exercise with the purple rods and compare it to the fifth number line on the worksheet. Have him record the number sentence and the skip jumps, which are the same.

If he can do the problem alone, let him proceed down the worksheet alone, helping only if needed. The third and fourth number lines are again a Test for Understanding. If he can do these alone, he has demonstrated understanding of this concept.

If he has difficulty with the third and fourth problems, have him skip them and try the fifth and sixth problems. Some students will need to use the rods to figure out how to end on thirty in the sixth problem. Then have him go back and try the third and fourth problems.

On the last number line, tell him he can not use jumps of four. Let him figure out what other size jumps will work. Twos and eights will work.

Practice

If you feel your student needs more practice, make number line problems using blank paper and rulers with or without rods. Avoid using grid paper, as all the extra lines confuse most students.

Commutative Property

Purpose The purpose is to review the meaning of the numbers in a multiplication number sentence and the Commutative Property of Multiplication.

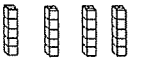
Prerequisites Investigation of the Commutative Property and previous introduction to the basic concepts of multiplication as taught in *Patterns in Arithmetic:* Multiplication - Booklet 1

Materials Commutative Property, page 6
Unifix cubes or other cubes

Warm Up Review the parts of a number sentence. On the board or a large piece of paper, draw three large rectangles shaped like the ones at the top of the Commutative Property worksheet. In the first rectangle write, 'The number of cubes in a stack, or the size of the group.' Have her build a stack of five cubes. In the second rectangle write, 'The number of stacks, or the number of groups.' Have her build three more stacks of five cubes. Write in the third box '= the number of cubes in all.' Have her count the number of cubes in all. Have her record this in her Math Journal.

"How many cubes are in each stack?" "Five."
"Record this. How many times did you build the stack of five cubes?"
"Four times."
"Write x 4 after the 5. How many cubes in all?" "Twenty."
"Write = 20. This is the multiplication number sentence."

The number in each group.	x The number of groups.	= The number of cubes in all.



5 x 4 = 20

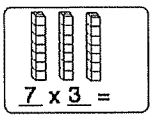
"What would happen if the numbers four and five changed places?" The answers will vary. **"Build it." "Prove it." "Record it."**

Lesson Have her study the first example on the worksheet and then count the number of cubes in each stack to see that there are seven. Draw her attention to the vertical way of writing the problem and the number sentence way of writing it. Go to the problem to the right of the example problem.
"How many cubes are in each stack?" "Three."
"Record this."
"How many stacks?" "Seven."
"Record this."
"How many cubes in all?" "Twenty-one."
"Record this."
"What is the name of this Property?" "Commutative."

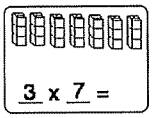
Record the meaning of each number in a multiplication number sentence.

The number in each group.	x The number of groups.	= The number of cubes in all.

7
x 3
7 x 3 =



3
x 7
3 x 7 =



If there are no problems, have the student finish this page independently.

Math Journal In her Math Journal, have her record a definition of the Commutative Property.

Test for Understanding The Make Your Own section commutes two problems in pictures and numbers.

Area Puzzle

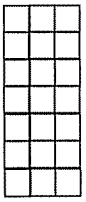
Purpose The purpose to review the connection between multiplication problems, arrays, and area of rectangles.

Note Do not teach perimeter in the same lesson as you teach area. Students at this age get these two concepts mixed up. Perimeter should be taught with string so students can see it is a measure of length. Area is taught with tiles as a measure of square space.

Prerequisites *Patterns in Arithmetic:* Multiplication - Booklet 2

Materials Area Puzzle, page 7
Flat, square tiles. Do not use cubes as it confuses area and volume. One inch square tiles cut from file folder material for the do-it-yourselfer.

Warm Up Using tiles instead of Unifix cubes, rebuild the problems on the Commutative Property done on the previous page. Instead of building three towers of seven cubes, make the sevens out of tiles. Then push the tiles together to make an array. Repeat with the other problems shown on Commutative Property, page 6.



Review the word 'array.' An array is an arrangement of tiles into a number of equal rows. The arrangement forms a rectangle. The rectangle can be named as a multiplication problem. Three rows of seven tiles is labeled as 7 x 3. If you still have Multiplication: Booklet 2, flip though the booklet to look at all the colored arrays that were used to learn how the Distributive Property of Multiplication was demonstrated.

Lesson **"What is an area?"** "It is a place or a space."

"When we measure the area of a rectangle, what are we measuring?" "How much space is inside the edges."

"If I build a perimeter fence around a space (draw a rectangle), then what part is the area?" "The space inside the lines."

"Can you shade it in?" "Yes."

"The area of rectangles are measured with arrays. In the array of 7 x 3, how many squares filled the space?" "Twenty-one."

"Twenty-one whats?" "Twenty-one squares."

"On Area Puzzle, page 7, what is the area of rectangle G?"

Note Pay attention to how he gets the answer. Does he count all the squares? Does he count one row and double it?

"What is the area of rectangle M?" If he begins to count, stop him and tell him to figure out what multiplication problem is shown by the array.

"The array is four tiles across. How many rows of four are there?" "Seven."

"What multiplication problem is that?" "Four times seven which is twenty-eight.",

"Twenty-eight whats?" "Twenty-eight squares."

Solve the riddle 'What do elves do after school?' by doing each problem.
Have him begin with the problems that show the squares in the array. If he wants to build them with the tiles, allow that.

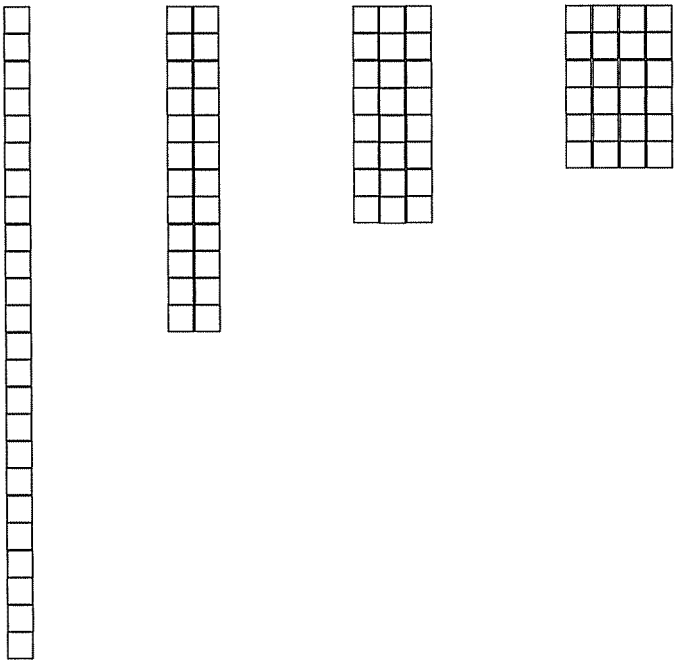
Watch to see what he does on K and E. Does he build? Does he draw in the lines and count, or does he use the multiplication array to find the answer?
Same thing on the two O problems. Some students will need to draw in the lines. Most will not by this time. A student who needs to draw or build should have more practice with the concept of area of rectangles.

Additional practice on this concept can be made with grid paper, made-up problems, or from problem sets on the Internet.
This free website has interactive practice on this concept: AAA Math - Geometry. Search the Internet for AAA math area, and you will get to this site. Scroll down for the practice sets: http://www.aaamath.com/geo78_x3.htm.

Test for Understanding

Build an array that has twenty-four tiles and six in each row. **"How many rows will there be?"** "Four."
"Build all the rectangles that have an area of twenty-four."

1 x 24 2 x 12 3 x 8 4 x 6



Using Parentheses

Purpose

The purpose is to introduce the parentheses as a tool for grouping numbers together to indicate order of combination and to name these parentheses 'Do me first.'

Materials

Paper and a pencil

Lesson

"Write on a piece of paper 1 + 2 x 3 = ? and solve it." Most will add the 1 + 2 to get 3 and then multiply the 3 x 3 to get 9.
Draw a pair of parentheses ( ) on the paper. **"These are called parentheses and when numbers are written inside them it means, 'Do me first.' "** Then give an example.
Write the same problem two ways: (1 + 2) x 3 = __ and 1 + (2 x 3) = __.
"Solve both problems. Will the answers be the same?"
Many students are surprised that the answers are different.
"Write three, one digit numbers on the paper. Put a + sign between two of the numbers and a x sign between the other two." Example: 5 + 6 x 2 = __. Have her put the parentheses in different places and find the answers. (5 + 6) x 2 = 22 and 5 + (6 x 2) = 17.

Have her try multiplication and subtraction signs, such as 4 x 5 - 3 = __.
(4 x 5) - 3 = 20 - 3 = 17 and 4 x (5 - 3) = 4 x 2 = 8.

Next have her try addition and subtraction together, such as (2 + 5) - 4 = __ and 2 + (5 - 4) = ___. **"What happens?"** "The answers are the same."

Try putting the - before the +. (7 - 3) + 2 = ___, and 7 - (3 + 2) = ___.
"What happens this time?" The answers are different."
"What about (+ and +) or (x and x)? " "The order you combine the numbers in doesn't matter."
"What patterns do you notice?" "The answer changes only when you mix multiplication with addition or subtraction or if you put (- and +) with the - first."
"When is it important to have parentheses in a problem." "When changing the order of combinations changes the answer."

This lesson should be playful and short. Most students will catch on to using the parentheses quickly. Some will have trouble seeing the overall patterns of when answers change and when they don't, but that's not a major concern at this point. Do not press for explanations of why the change in the order of combination changes some answers and not others.

Test for Understanding

Give a few problems with parentheses in them and see if the student does the operation inside the parentheses first. Give these problems: (5 x 8) - 2 and 5 x (8 - 2). For the purposes of this lesson, the student does not need to be able to predict in advance whether or not the answer will change when the order of operations changes. It's nice if she can, but not necessary.

Associative Property

Purpose The purpose is to demonstrate the Associative Property of Multiplication using volume of rectangular solids and various groupings of numbers using parentheses.

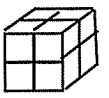
Background The Associative Property of Multiplication (and Addition) allows the combination of three or more numbers in any order. $2 \times 3 \times 4 = 4 \times 2 \times 3$. If all the signs are multiplication signs, changing the order will not change the answer. This property is used in taking volumes of containers, in factoring, and has many applications in fractions. A 3-D rectangle is called a rectangular solid or a rectangular prism.

Associating three numbers in multiplication means to multiply all three together. But in reality, you can multiply only two of the three together at once. You must multiply two numbers together and then multiply the product of those first two by the third number. The order of these pairings can change without changing the final product.

Prerequisites Basic multiplication tables, free exploration with linking cubes and Commutative Property

Materials Associative Property - Worksheets 1 and 2, pages 8 and 9
Associative Property: Volume Puzzles - Worksheets 1 and 2, page 10 and 11
Associative Property: Investigation - Worksheets 1 - 5, pages 12 - 16
Cubes – snap-together type are the best. LinkerCubes work the very best. Regular cubes can be used and taped together to hold the structure together.

Warm Up After free exploration time: **"Start with one cube. Add cubes to make a larger rectangular solid that has two blocks on each side. Guess how many cubes it will take."**
"How long is your rectangular solid"? "Two cubes."



"How wide is your rectangular solid"? "Two cubes."

"How tall is your rectangular solid"? "Two cubes."

"How many cubes does it take in all"? "Eight cubes."

Lesson Part 1 Begin with Associative Property - Worksheet 1, page 8. Follow the instructions. If you are using regular cubes, use tape to hold each layer together. Put the tape along the boundaries between cubes.
Have her build the first solid and count the total number of cubes used. To make this easier, there are no hidden cubes inside the solid.
Now have her use more cubes to build the second solid. There is a space on the worksheet for her to estimate and enter her best guess for how many cubes will be in this solid.

Note

Do not tell her the formula for volume at this time. As always, let her discover it for herself.

A few students will notice at this point that the solid is the same one every time but rotated. The solid always has a row of two, of three, and of four cubes. This would be a very insightful thought. She may want to just rotate the solid instead of building a new one. Allow this. In fact, it is a great mathematical solution that saves time.

If by the end of the page she does not notice that the solid is actually the same solid each time, have her take her first solid and rotate it and write down the length, width and height. Then have her rotate it again. Use the Answer Key to be sure she is doing the page correctly.

Now give Associative Property - Worksheet 2, page 9. Some students will want to skip building this one. If she tells you she already knows the total number of cubes in the solid, ask her to explain how she knows. She may count the cubes or squares she can see. This solid has cubes in the center that are not visible from the outside. Or, she may have realized that all she has to do is to multiply the length by the width, by the height. But, for a physical confirmation of her thoughts, do not skip building.

Do allow her to build it only once and then rotate it to see that the total does not change when the position of the numbers changes. 5 by 3 by 4 will have the same number of cubes as a 3 by 4 by 5 and the same as the 4 x 5 x 3, but the shape will look a bit different.

Worksheet

Associative Property: Volume Puzzle - Worksheet 1, page 10 **"Build the drawings of I, U, R, and L"** (the drawings on the top row.) **Count the number of cubes in each shape. The number of cubes in a solid is called the volume."**

Test for Understanding

See if she can figure out the volume of problem L, the drawing in the center of the page. Have her use her pencil to connect the lines so she can better visualize the cubes. Can she figure it out without building? Has she discovered a method for calculating it yet?

Part 2

Warm Up

Go to the Associative Property: Volume Puzzle - Worksheet 1 and do only problem E. Suggest that she try to figure out the total number of cubes before she builds it. See if she realizes all she has to do is multiply the three numbers together. Then have her build it to confirm.

Part 3

Calculating Volume

Warm Up

Associative Property: Volume Puzzle - Worksheet 1 Have her look at problem R in the top row. **"How could you calculate that there would be twelve cubes in the rectangular prism shown in problem R?"**

Several answers are possible. One conversation might go like this:
"I can see that there are six cubes on the top level and that there are two levels, so I could just multiply six times two. How could you find the six without counting the six cubes on the top level?" "I could see that there are two rows of three and multiply two times three."

"So what multiplication problem could you do that would give you the answer twelve that uses the numbers in the length, width, and height of the prism?" "I could just multiply all three numbers together."

"Let's see if that works with problem L on the top row. What multiplication problem do you see there?" "I see two wide, three long, and five high. That would be 2 x 3 x 5, or 30."

"Does that match what you got when you built it a few days ago?" "Yes."

"So what pattern can you use to figure out the volume of a rectangular prism?" "You just multiply the width times the length times the height in any order and it will tell you."

"Try your pattern with Problem I in the second row." "I see three wide, three deep, and three high, so that would be twenty-seven."

"Build it to see if you are right."

Problems S and T are two-step problems. Do problem S first.
"How can you use your multiplication pattern to find the total number of cubes in figure S?" "On the bottom you just see six long, two wide and two tall. So you multiply those to get twenty-four. Then you just add the four on top to get twenty-eight."

"What multiplication problem do you see in that little part on top?" "Two by two by one."

"Can you write the number sentence for these two parts that would give you the answer twenty-eight?" Most students will need help with this.

"First write the multiplication problem for the bottom part and put parentheses around it." Student writes (6 x 2 x 2).

"Now write the multiplication problem for the top part." Student writes (2 x 2 x 1).

"How do you put those two together to get the total?" "I add them."

"Where do you put the + sign?" "Between the two sets of parentheses." Student writes (6 x 2 x 2) + (2 x 2 x 1) = 28.

"Now try problem T."

Note

This might help. Figure T has two rectangular solids hooked together. Have her build the top part of problem T in one color of blocks and the base of it in another color of blocks.

"What multiplication problem do you have to do to find the total number of cubes in problem T?"
"I have two. The top one is a cube of two by two by two, and the bottom one is two by four by two."

"Write that out with numbers." T = (2 x 2 x 2) + (2 x 4 x 2) = 24 cubes.
Have her finish the page and get the message.

Practice Worksheets Lesson Part 4

Associative Property: Volume Puzzle - Worksheet 2, page 11

Associative Property: Investigation - Worksheets 1 - 5, pages 12 - 16, are written to the student. Have her work though the worksheets on her own. Do encourage a clear answer on the explanation questions. Make sure she writes answers in complete sentences in legible writing.

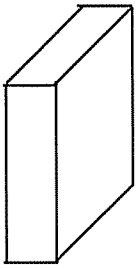
Associative Property: Investigation - Worksheet 3, page 14, is somewhat tedious, but important. Most students need help with this page. What does it mean physically to multiply three numbers together? (2 x 3) x 4 means to add two three times and then do that four times.

See the Answer Key to make sure the work is done correctly.
End the session after the completion of Associative Property: Investigation - Worksheet 3, page 14. Continue the next day with Associative Property: Investigation - Worksheets 4 and 5, pages 15 and 16.

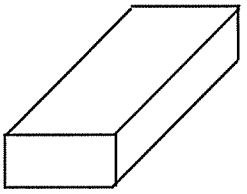
Test for Understanding

Find all the ways of associating these numbers: 3, 6, and 10. Then draw the buildings that these three numbers would create if the first number in the set is length, the second width, and the third height. (__ x __) x __
length width height

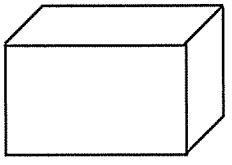
(3 x 6) x 10



(6 x 10) x 3



(10 x 3) x 6



Discovering Prime Numbers

Purpose

The purpose is to introduce prime and composite numbers using tiles to make rectangles. The making of rectangles is a physical way to factor a number. Some numbers can be arranged in two or more rectangular shapes. Eight tiles, for example, can be arranged in a single row of eight or a double row of four. Some numbers, such as seven, can only be arranged in a single row. In this lesson, the student will discover which numbers of tiles can be arranged in only a single row; these numbers are defined as prime numbers. A prime number is any number that has only itself and one as a factor. The numbers that have more than one arrangement are defined as composite numbers. Composite numbers have two or more pairs of factors.

Recognizing if a number is prime or composite is important to elementary students when they simplify fractions.

Prerequisites

Fluency with the multiplication tables and knowledge of odd and even numbers

Materials

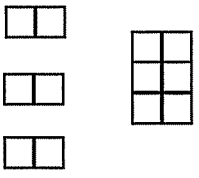
Discovering Prime Numbers: Worksheets 1 - 5, pages 17 - 21
Colored pencils
Twenty-five tiles Any kind of tile will work. One inch tiles can be obtained at home improvement centers for very little cost. Cut them loose from the netting and you have a great manipulative for all kinds of activities on factoring, area, and patterns. Tiles can also be made with heavy paper, but these are harder for students to manipulate.

Note

It is best to avoid cubes, as students can confuse factoring with volume when cubes are used for this activity.

A multiplication chart may be helpful for students who have not mastered the multiplication tables. Use the Speed Tests, pages 43 - 46, to practice the tables.

Warm Up



Give the student time to play with the tiles before beginning the lesson. Fifteen minutes may be adequate if he has had free exploration time previously.

"Use the tiles to build the multiplication problem 2 x 3." Most students will make three groups of two. Have him push them into a rectangular array with two rows of three. Tell him that this array shows the problem 2 x 3 or 3 x 2 depending on how you look at it.

Lesson Part 1

Do Discovering Prime Numbers - Worksheet 1 together.
When you get to the bottom of the first page, have him go back over the rectangles and label them with a heavy pencil.
The first square is 1 x 1, the second is 2 x 1, the third is 3 x 1, the fourth has two: 4 x 1 and 2 x 2, etc.

Then have him select a color for each number. Lightly color in all the rectangular arrays of a number the same color. For example, both the 4 x 1 and the 2 x 2 would be colored the same color. This becomes important on the next two pages where the rectangles will take up lots of space and can get mixed up.

Now begin Discovering Prime Numbers - Worksheet 2, page 18. Have him take out eight tiles. In the example it shows a drawing of two times four. It is labeled 2 x 4. Have him lightly shade in this rectangle with a color he has not used yet.
"Is there any other way to arrange eight tiles?" "Yes, I could make one row of eight."

"Draw one row of eight tiles to the right of the rectangle shown on your worksheet. How should this one be labeled?" "1 x 8."

"How should it be colored in?" "The same color as the 2 x 4."

Have him take one more tile to make nine tiles. Repeat this same process. Some students will think that because nine is an odd number, there can be only one rectangle. If this happens have him explore different ways to arrange the tiles. To confirm, he can look at a multiplication chart and find the nines. He will find the nine in the three times table. So nine can be made with one row of nine and three rows of three. Have him draw both rectangles, label them, and shade them in.

Eventually, you will run out of colors. It is fine to repeat them. The purpose of the shading is to keep the rectangles with the same total number of tiles sorted from their neighbors.

Have him finish Discovering Prime Numbers - Worksheets 2 and 3 the same way. This activity can be a bit tedious for some. Have him work for fifteen more minutes and stop. Continue the activity the next day. Most students take two to three days to complete both pages working in thirty minute sets.

Do not continue to Discovering Prime Numbers - Worksheet 4 until Worksheets 2 and 3 are completed. Check the work using the Answer Key to make sure that all the factors were found.

Lesson Part 2

Discovering Prime Numbers - Worksheet 4, page 20
The student will now go back though all the rectangles he drew and sort them into two groups: the numbers that had only one rectangle and the numbers that had more than one. Work on this page together.

There is a little investigation on the addition of odd and even numbers. Understanding will be assisted by using tiles and arranging them in pairs. An odd number will have one tile unpaired. When you put two odd numbers together, the 'odd man out' of each number get together and make an evenly paired group.

Practice Worksheets

Discovering Prime Numbers - Worksheet 5, page 21
Have him do this worksheet in pencil. Errors are likely. The goal is to end up with only prime numbers circled.

Note

Divisibility rules are useful here. The rule for multiples of three is that if the digits in the number add up to a number in the three times table, then the number is a multiple of three and therefore not prime. Look at the number eighty-seven. It looks so much like a prime number, but 8 + 7 = 15 and therefore eighty-seven is in the three times table and not prime.

Gus the Bus Driver, page 22, is an exercise in two-step multiplication word problems.

Test for Understanding

"Explain what a prime number is in writing. You may use a drawing to help you."

A good answer: A prime number is any number that has only itself and one as a factor.

An OK answer: A prime number is a number that can only be made one way in multiplication. Such as seven. ☐☐☐☐☐☐☐ Seven tiles made into a rectangle.

Math Journal

Write the definition of a prime number.

Extension

Look up Eratosthenes. He was a Greek mathematician living in ancient Egypt and a very interesting person.

Divisibility

Purpose The purpose of this lesson is to explore divisibility rules for 2, 5, and 3.

Prerequisites The ability to do simple division problems such as $39 \div 3$.

Materials Paper and pencil

Lesson Part 1 Give this set of problems: $12 \div 2$, $8 \div 2$, $14 \div 2$, $16 \div 2$, $10 \div 2$, $24 \div 2$, $32 \div 2$, $15 \div 2$, $28 \div 2$, $42 \div 2$, $23 \div 2$

"What do you notice about these problems?" "They are all divided by two."

"What else?" "The ones digits are all even except in $15 \div 2$ and $23 \div 2$."

"How are the answers different in these problems?" "They have a remainder."

"What else is different about these problems?" "The ones digits are odd."

"Write a rule of divisibility for problems divided by two." Give more examples if needed. Possible answers:

If a number ends with an even number it is evenly divisible by two. Divisible by two means when you divide by two there are no remainders.
A number is divisible by two if its ones digit is 0, 2, 4, 6, or 8.

"Can you find a problem that is divisible by two in which this rule doesn't work?" Student tries several problems on her own. "No."

Lesson Part 2 Give this set of problems: $20 \div 5$, $15 \div 5$, $35 \div 5$, $40 \div 5$, $5 \div 5$, $50 \div 5$, $25 \div 5$, $65 \div 5$

"What do you notice about these problems?" "They are all divided by five."
"What else?" "The ones digits are all zero or five."

"Write a rule of divisibility for problems divided by five." If a number ends in a 5 or a 0, it is divisible by 5.

Lesson Part 3 Give this set of problems: $12 \div 3$, $9 \div 3$, $27 \div 3$, $24 \div 3$, $36 \div 3$, $15 \div 3$, $18 \div 3$, $42 \div 3$, $33 \div 3$

"What do you notice about these problems?" "They are all divided by three."

"What else?" "There are no remainders."

"Can you see any patterns that will help you find a rule of divisibility? This rule is harder to figure out. If the student can't find any rule, give her a hint.

"Find the sum of each of the numbers being divided. Is it divisible by three?"

"Write a rule of divisibility for problems divided by three." A number is divisible by three if the sum of its digits is divisible by three.

Test for Understanding "Are these numbers divisible by three?" 51, 28, 45, 21, 25
yes no yes yes no

Prime Factoring

Purpose

The purpose is to teach the process of prime factoring. Prime factoring is a method of revealing the base factors of any number, the 'bones' if you will. Factoring is a tool, a screwdriver, which allows us to break up numbers using multiplication. It allows us to rearrange numbers to make problems easier, to simplify fractions, and find patterns.

Prerequisites

Previous lessons, Divisibility lesson, and the ability to do simple division problems such as $39 \div 3$. Students need to be able to test numbers such as thirty-nine or seventy-eight, which do not appear in the multiplication table but which are not prime. Both of these numbers are divisible by three.

Basic rules: A number is divisible by

2 If the number ends with an even number.
If you divide by two and there are no remainders.
If its ones digit is 0, 2, 4, 6, or 8.

3 If the sum of the digits in a number equals a number in the three times table.
If the sum of its digits is divisible by three.

4 If the last two digits of the number are divisible by four.

5 If the number ends in a five or a zero.

6 If a number follows the divisibility rules for both two and three.

10 If a number ends in zero.

Materials

Discovering Prime Numbers - Worksheets 4 and 5, pages 20 and 21
Prime Factoring - Worksheets 1 - 9, pages 23 - 31
Prime Factor Puzzles - Worksheets 1 and 2, pages 32 and 33
Prime Factoring Practice - Worksheets 1 and 2, pages 34 and 35
Show You Know, page 36

Warm Up

Review Discovering Prime Numbers.

"What is a prime number?" "Any number that can be divided only by itself and one. Or, any number that has only itself and one as factors."

"Write the numbers 2 though 30. Remember, 1 is not either prime or composite. Circle any prime numbers and cross out all composite numbers. You can use your list on Discovering Prime Numbers worksheet 4 if you need to."

When she gets to twenty-six, she will be beyond the numbers she investigated in the last lesson.

"Can any even number except two be prime?" "No."

"Why not?" "Because all even numbers can be divided by two."

"What about odd numbers?" "Some odd numbers are prime and some are not."

"How can you tell the difference?" "If an odd number is in the times tables other than the first row, then it is not prime."

Finish the task on the first thirty numbers. The only new prime number is twenty-nine.

Listing All Factors

Here is a handy tool for listing factors. Here is a way to find all the factor pairs for twelve. Begin with 1 x 12. Write them on opposite ends of your factor list like this:

1 12

Then go to 2. 2 x 6 = 12. Add these to the list, again putting the factors on opposite sides of the list moving to the inside like this:

1 2 6 12

Then go to 3. 3 x 4 = 12. Add these to the list again putting the factors on opposite sides of the list moving to the inside like this:

1 2 3 4 6 12

What numbers are between 3 and 4? None. So there are no more factors of twelve to list.

This is what the list would look like for 18. 1 x 18, 2 x 9, and 3 x 6.

1 2 3 6 9 18

Now all we have to do is look for any factors of 18 that are between 3 and 6. We can examine 4 and 5. Neither of these is a factor of 18. So we know there are no more factors of 18.

"Make a list of all the factors of twenty." "1, 2, 4, 5, 10, 20."

"What does the word factor mean? What is a factor?" "It is a number you multiply by to make another number."

"List two multiples of twenty." "Forty and eighty."

"How is a multiple different from a factor?" "A multiple is the answer or the product of multiplying by two factors."

"How many whole number factors does twenty have?" "Six."

"How many whole number multiples does twenty have?" "An infinite number."

"Which of the factors of twenty are prime numbers?" "Two and five."

"Which of the multiples of twenty are prime numbers?" "None because they all have twenty as a factor."

Follow the instructions on Prime Factoring - Worksheets 1 and 2, pages 23 and 24.

The prime factors of twelve listed at the bottom of Prime Factoring: Worksheet 2 will be the same as the ones listed on Prime Factoring - Worksheet 1.

No matter what factor pair you start with, the prime factors at the bottom will be the same.

Do Prime Factoring - Worksheet 3 together. Use the Answer Key to be sure you are doing the work correctly. The last one, sixteen, is a little puzzle.

On the practice sheets she will encounter numbers that are not prime, but do not show up on the multiplication chart. Examples: Seventy-eight and thirty-nine. She will need to use the divisibility rules to determine if the number is prime or a composite.

Practice Worksheets

Prime Factoring - Worksheets 4 - 9, pages 26 - 31

Spread these worksheets over several days while you work on other new topics.

Check each page as soon as it is completed.

Notes

Reteach if the student frequently lists composite numbers in the prime factor list. Watch for addition instead of multiplication in the factor trees. It is not uncommon to see ten factored into 5 x 5. She was thinking 5 + 5 is 10. Remind her she must use the multiplication number sentence for 5 + 5 = 10, which is 5 x 2, not 5 x 5.

Lesson Part 2

Prime Factor Puzzles - Worksheets 1 and 2, pages 32 and 33

Warm Up

List the prime factors of sixty. The factors are 2, 2, 3, and 5.

"What will the product be if you multiply 2 x 2 x 3 x 5 together?" Wait until she does the problem. "You get sixty back again."

"Does this always work? Try prime factoring another number. Then multiply the prime factors together and see." "It does work every time."

Lesson

Solve the first problem on Prime Factoring Puzzles - Worksheet 1 together.

You are to figure out what numbers go in the blanks. Wait. See if she can figure out what to do. There are two ways to solve this. One is to start at the bottom and work backwards. On the last line are the factors 3 x 2. That means the number above must be six. The blank number above the 6 must be equal to 6 x 2 or 12. The number above the twelve must be 2 x 12 or 24. The other way is to multiply the prime factors in the box. 3 x 2 x 2 x 2 = 24.

The problem on the right works the same way, except the prime factors are not listed in the box. She needs to realize that the numbers listed in the factor tree are the prime factors. Again she can work up the tree, or multiply all the prime factors

together and then work down filling in the blanks as she goes.

Note

Do not help unless needed.

The next set of problems on the lower left require the student to multiply the primes together to find out the starting number.

Practice Worksheets

Check the work and assign practice.

Prime Factoring Puzzles - Worksheet 2, page 32

Prime Factoring Practice - Worksheets 1 and 2, pages 34 and 35

Test for Understanding

Show You Know, page 36 The answers are in the Answer Key.

Multiplying Factors

Purpose

The purpose of this lesson is to demonstrate to the student that when two numbers with a known product are factored and their factors reordered and multiplied together, the product will be unchanged. In previous lessons, the student has learned that no matter how you factor a number you will get the same prime factors at the end. He has also learned that multiplying the prime factors of a number together will produce the original number as a product. Now we will take two numbers that have known products, factor them, reorder the factors, and multiply. He will see that the final product is unaltered as when he multiplied the original numbers together.

This concept is important because when we multiply large numbers such as 400 x 60 together, they are factored to (4 x 100) x (6 x 10). Then the rule of the Associative Property of Multiplication is used to reorder the numbers. They are reordered to (4 x 6) x (100 x 10) to get 24 x 1,000 or 24,000. This is the source of the 'multiply the four times the six and add three zeros' trick. When multiplying large numbers like this, he needs to trust that the pattern works because it is difficult to determine the truth of 24,000 manipulatively. Thus, in this lesson the pattern is demonstrated with numbers the student already knows the answers to.

Prerequisites

Previous lessons

Materials

Multiplying Factors, page 37

Warm Up

Find the prime factors of eight and four. They are (2 x 2 x 2) x (2 x 2).

"What is the product of eight and four?" "Thirty-two."

"What product do you think you will get if you multiply all the prime factors of eight and four together?" Let him predict and then try it.

"What happened?" "I got thirty-two, the same as when I multiplied eight times four."

"Do you think that will always work?" "Maybe."

"How could you test it?" "I could try another one."

"What problem would you like to try?" "6 x 4."

"What is the product of six times four?" "Twenty-four."

List the prime factors of these numbers and then multiply them all together.
6 x 4 = (2 x 3) x (2 x 2) = 24

Lesson

Use Multiplying Factors worksheet. Have him factor the four and write the numbers on the second line. Repeat with the eight.

"Will the pattern still work if the factors you use are not prime?"
Listen to the response. If he says, "Of course it will work because no matter how you break it down, all the factors will get multiplied and the product will be the same." You will know he is understanding the concept. Most students will be much less sure at this point.

"Factor the eight into four times two, and put those on the second line under the eight. Now, reorganize the numbers. Put all the twos first and the four last. What happens?"
"You get the same answer as before. It does not matter how you factor it, you still get the same answer."

Note

Many students will at this point realize this process can be used to help you find the answer to a multiplication fact you can not remember. If he can not remember what six times eight is, he can factor the six into two times three. $6 \times 8 = (2 \times 3) \times 8$ He can then multiply three times eight to get twenty-four and then double it. This understanding creates mathematical power.

Complete the worksheet together. It is not necessary to break the numbers all the way down to prime factors. Use the Answer Key to be sure he is doing the problems correctly.

Test for Understanding

"Show how to use factoring to find the answer to 9 x 8 if you can't remember the multiplication factor."

Solutions are varied. Some examples: $9 \times 8 = 9 \times (2 \times 2 \times 2)$ = Double nine three times; 18, 36, 72
$9 \times 8 = 3 \times (3 \times 4) \times 2 = 3 \times 12 \times 2$ or $36 \times 2 = 72$

Factoring into Tens

Purpose The purpose of this lesson is to introduce the concept of factoring by tens. This tool is used in long multiplication and division.

Prerequisites Multiplying Factors and Expanded Tables in Multiplication: Booklet 2

Materials Factoring into Tens: Worksheets 1 and 2, pages 38 and 39, Factor by Tens and Prime Numbers, page 40, Using Factoring with Expanded Tables, page 41, Expanded Tables: Review, page 42

Warm Up Prime factor a number. Multiply numbers together by factoring them as was done in the last lesson.

Lesson Part 1 Do Factoring into Tens - Worksheet 1 with your student. Check the Answer Key to be sure you are doing it correctly.

Factoring into Tens - Worksheet 2 is harder than it looks.
Problem 1
First find all the prime factors of one hundred twenty. Suggest she begin with factoring one hundred twenty by tens. $10 \times 12 = (2 \times 5) \times (3 \times 2 \times 2)$ So the prime factors of one hundred twenty are 5, 3, 2, 2, 2.
Problem 2
To find all the composite factors of one hundred twenty (of which there are many) use the Associative Property to make all the possible groupings you can find of the prime factors.
Begin by writing 1 x 120 on opposite sides of the paper like this:
1 120

To make the search orderly, move from left to right. The next number is two.
1 2 120

To find what the other factor is, use the Associative Property and change the grouping of the prime numbers. We can put the two by itself and multiply the other prime factors together to get $2 \times (5 \times 3 \times 2 \times 2) = 2 \times 60 = 120$.

Now add the sixty to the list of composite factors of one hundred twenty. Now you might think, I could have found sixty without all this writing. True, but you will not find all the composite factors easily without this handy tool. Just wait.
1 2 60 120

The next number is three. Add the three to the list.
1 2 3 60 120

Use the prime factors to find the other number in the pair by putting the three alone and use the other four numbers together.

3 x (5 x 2 x 2 x 2) = 3 x 40. Add forty to the list.
1 2 3 40 60 120
The next number is four. Here is a possible questioning strategy.

"Is four a factor of one hundred twenty?" "Yes, you can make it with 2 x 2 from the prime number list."
(2 x 2) x (3 x 5 x 2) = 4 x 30 So now add thirty to the list.
1 2 3 4 30 40 60 120

"The next number is five. Is five a factor of one hundred twenty?" "Yes, it's in the prime factor list. Also, one hundred twenty ends in zero, so it is a multiple of five."

"Use the list of prime factors and put the five alone and then group the others."
5 x (3 x 2 x 2 x 2) = 5 x 24 Now that is one you may not have thought of!
So we can now add five and twenty-four to our list.
1 2 3 4 5 24 30 40 60 120

"What about six?" "Six is made with 2 x 3. Two and three are in our prime factor list."
6 x ? = 120 (2 x 3) x (5 x 2 x 2) = 6 x 20. Add them to the list.
1 2 3 4 5 6 20 24 30 40 60 120

Now we know the last factors of one hundred twenty must be between six and twenty. Seven? No, seven is a prime and is not on our prime factor list.

"What about eight? Eight does not look like a factor of one hundred twenty. Can eight be made from the prime factor list of 5 x 3 x 2 x 2 x 2 ?" "Yes, 2 x 2 x 2 is 8."
"So what is the other number?" "(2 x 2 x 2) x (3 x 5) = 8 x 15 = 120."

You definitely would not have easily found this one. Add these to your list.
1 2 3 4 5 6 8 15 20 24 30 40 60 120

"What about nine? What prime factors would we need to make a nine?" "3 x 3, so nine is not a factor of one hundred twenty."

"What about ten?" "We already know that 10 x 12 is equal to one hundred twenty."
1 2 3 4 5 6 8 10 12 15 20 24 30 40 60 120

"The only number left is eleven." "Eleven can't be a factor of one hundred twenty because eleven is prime and eleven is not in our prime factor list."

"So we have found all the composite factors of one hundred twenty. Put circles around all the factors on your list that are composite factors of one hundred twenty."

Finish this page and end the session.

Test for Understanding

Factor by Tens and Prime Numbers, page 40, is an assessment. At this point we want to see that she can prime factor a number on her own and peel zero off a number by factoring by tens.

Lesson Part 2

Use Factoring with Expanded Tables, page 41, and Expanded Tables: Review, page 42. Do the worksheets together and check with the Answer Key to be sure they are being done correctly.

Extension

Finding the composite factors of a number and associated problems is continued in the lesson Prime Numbers Meet the Associative Property and (affectionately called) Rat Cage Problems. which can be found in *Patterns in Arithmetic:* General Math - Booklet 5.

Note

A useful way to practice the times tables is to give Speed Tests on pages 43 - 45. The same tests are repeated but the problems are in different orders on each page. These can be given periodically throughout the year. Speed Test Graph, page 46, can be used throughout the year to graph and show the student's progress throughout the year.

Worksheets

Give the Post-Assessment - Worksheets 1 and 2, pages 48 and 49. The Pre-Assessment: Part 2 and the Post-Assessment are the same. The answers for both are on the last page of the Answer Key.

Proceed to *Patterns in Arithmetic:* Multiplication - Booklet 4.

Multiplication Facts Chart

Purpose

The purpose is to develop fluency with multiplication facts and to find patterns in the multiplication tables one through ten.

Prerequisites

Understanding basic multiplication

Materials

Multiplication Facts Chart, page 47

Lesson

The student fills out the Multiplication Facts Chart from memory.

She then looks for patterns in the completed chart. There are many patterns in this chart.

Can the student find the sequence of squares?

One activity is to make multiple copies of the filled-out chart and shade the multiples of each times table.

If there are several students, have them compare and discuss the different patterns found.

Test for Understanding

None

Post-Assessment - Worksheet 1 Date _____

Put a question mark next to any problem you do not know how to do.

1. Show all the ways of associating (grouping) these three numbers: 2, 5, 6

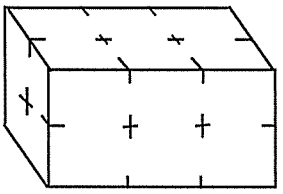
(__ x __) x ___ = ___ x ___ = ___

(__ x __) x ___ = ___ x ___ = ___

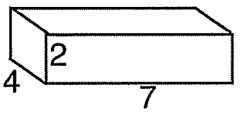
(__ x __) x ___ = ___ x ___ = ___

2. Find the volume of these rectangular solids.
Length (L) Width (W) Height (H)

a. Volume _____



b. Volume _____



c. Volume _____

L = 3; W = 6; H = 2

3. What is a Factor? Use numbers to help you explain.

_____

_____

_____

4. What is a Prime Number?

_____

_____

_____

5. List the first ten Prime Numbers starting with 2.

2 ___ ___ ___ ___ ___ ___ ___ ___ ___

Post-Assessment: Worksheet 2 Date _____

6. Prime Factor 24, 50, and 37. Record the Prime Factors in the boxes.

a. ☐ 24 / \ __ x __

b. ☐ 50 / \ __ x __

c. ☐ 37 / \ __ x __

7. Factor 50 and 80 by tens.

By tens

a. 50 / \ __ x __

By tens

b. 80 / \ __ x __

8. a. Solve these problems.

6	60	600	6,000
x 8	x 8	x 8	x 8

b. What patterns do you notice in the answers to these problems?

_____

_____

_____

c. Use the pattern to find the answer to 600,000 x 8. _____

9. Use factoring by tens to show how to get the product of 400 x 6.

400 x 6

/ \

__ x __ x __

(__ x __) x __

__ x __

_____

Use the Answer Key to check for errors.

Patterns in Arithmetic

Multiplication: Booklet 3

Properties and Factoring

Answer Key for the Student Workbook

By Suki Glenn and Susan Carpenter

Answer Key Legend

AWV = answer(s) will vary
BUWV = break up will vary
OWV = order will vary

Pattern Blocks
r = red trapezoid
g = green triangle
y = yellow hexagon
o = orange square
b = blue parallelogram
t = tan rhombus

Cuisenaire Rods
1 w = white
2 r = red
3 lg = light green
4 p = purple
5 y = yellow
6 dg = dark green
7 bk = black
8 bn = brown
9 bl = blue
10 o = orange

Note: Some items and pages are left out of the answer key.
1) Some pages in which the answers are open-ended or will vary.
2) Make your own problems. Since students create their own problems and solutions, these sections give valuable information about the level of confidence and competence. It can be a useful source of curriculum for other students.
3) Practice pages.
4) Workboards.
5) The answers are in the Parent/Teacher Guide.

Patterns in Arithmetic: Multiplication - Booklet 3
Answer Key for the Student Workbook

Printed on recycled paper.

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Fallbrook, CA 92088

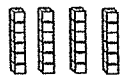
ISBN 978-1-935559-34-4

Multiplication - Booklet 3

Assessments are on the last page.

Concept

Look at the picture. Write the addition and multiplication number sentences.
Example:

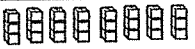


6 + 6 + 6 + 6 = 24 Addition

6 x 4 = 24 Multiplication

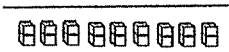
Addition 4+4+4+4+4=20

Multiplication 4 x 5 = 20



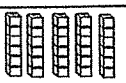
Addition 3+3+3+3+3+3+3+3=24

Multiplication 3 x 8 = 24



Addition 2+2+2+2+2+2+2+2+2=18

Multiplication 2 x 9 = 18

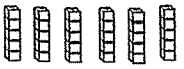


Addition 6+6+6+6+6=30

Multiplication 6 x 5 = 30

Addition 7+7+7=21

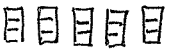
Multiplication 7 x 3 = 21



Addition 5+5+5+5+5+5=30

Multiplication 5 x 6 = 30

Draw the picture.



3 + 3 + 3 + 3 + 3 = 15

3 x 5 = 15

Make your own.

Addition

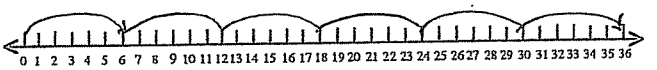
Multiplication ___ x ___ = ___

4

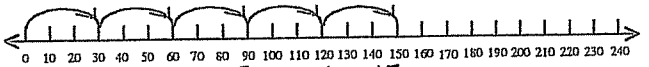
Number Lines

How many numbers are skipped in each jump? 3 How many jumps? 12

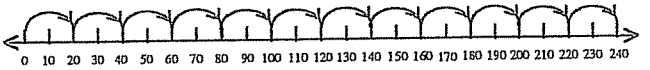
Record the number sentence: 3 x 12 = 36.



Record the skip jumps: 6 x 6 = 36.



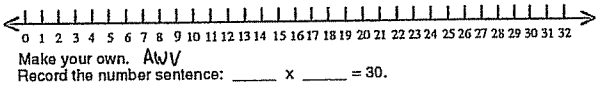
Record the number sentence: 30 x 5 = 150.



Record the skip jumps: 20 x 12 = 240.

Record the number sentence: 4 x 8 = 32.

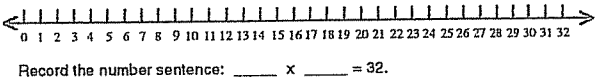
Record the skip jumps: 4 x 8 = 32.



Make your own. AWV

Record the number sentence: ___ x ___ = 30.

Record the skip jumps: ___ x ___ = 30.



Record the number sentence: ___ x ___ = 32.

Record the skip jumps: ___ x ___ = 32.

In multiplication all the skip jumps are the same size.

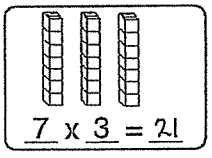
5

Commutative Property

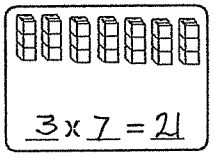
Record the meaning of each number in a multiplication number sentence.

number of blocks in each group	x	number of times each group is made	=	the total number of blocks

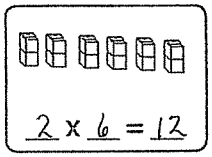
7 x 3 = 21



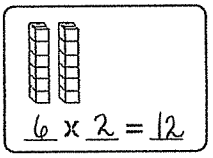
3 x 7 = 21



2 x 6 = 12



6 x 2 = 12



Are your answers the same or different? same

Why? The amount of blocks stay the same. The number sentence is written two ways.

Continue this pattern. Draw the pictures.

3 x 4 = ___

___ x ___ = ___

6

Area Puzzle

What do elves do after school?

24	18	35	28	32	25	27	36	20
G	N	O	M	E	W	O	R	K

Figure out the area of these rectangles. Place the letter beside the rectangle in the box below its area in square units to solve this riddle.

O = 27

K = 20

E = 32

N = 18

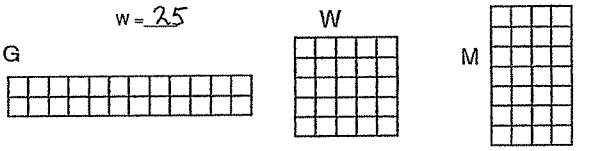
W = 25

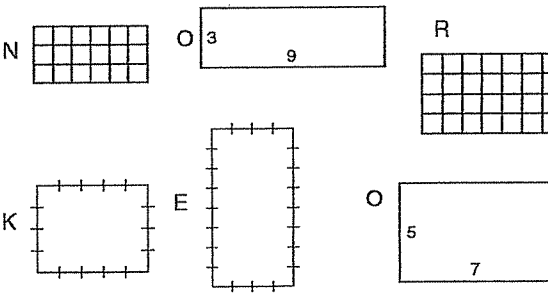
G = 24

R = 36

M = 28

O = 35



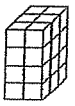


7

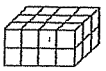
Associative Property - Worksheet 1

Take out a set of linking blocks.
Build a figure that is two blocks long, three blocks wide and four blocks high. It will look something like this:

This figure is called a rectangular solid.
It's a three dimensional or 3D rectangle.
In these lessons the size of the sides are always listed in this order: length, width, height.
This solid would be called 2 by 3 by 4.



How many blocks did you use to build this rectangular solid? 24



Now build this solid.

This one is a 4 by 3 by 2.

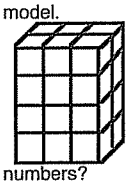
How many blocks will this solid take?

guess ____ actual 24

Compare the number of blocks you used for the first solid to the number of blocks you used in the second solid.
What did you find out?

Arrange the solid another way. Draw a picture of your model.

It could be 3 by 2 by 4.



What changes when you change the arrangement of the numbers?
The shape of the solid.
What does not change? The total number of blocks.

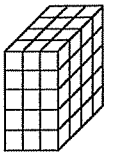
Associative Property - Worksheet 2

Build another rectanglular solid like this: 3 by 4 by 5

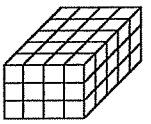
length	width	height
3	4	5

How many blocks will it take to build this new solid?

guess ____ actual 60



Now build this solid.



This one is a 4 by 5 by 3.

How many blocks will this solid take?

guess ____ actual 60

length	width	height
4	5	3

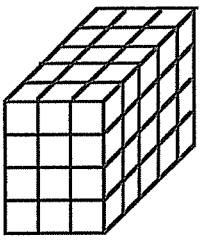
Arrange the solid another way. Draw a picture of your model.

It would be 3 by 5 by 4.

length	width	height
3	5	4

How many blocks will this one take?

guess ____ actual 60



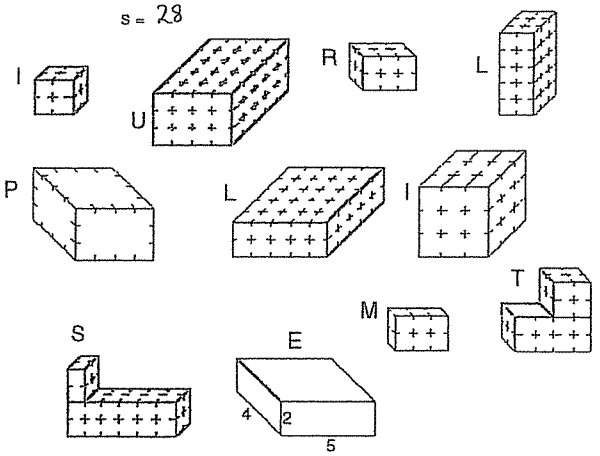
Associative Property: Volume Puzzle - Worksheet 1

What kind of pliers do you use in arithmetic?

6	60	30	24	8	48	50	27	40	12	28
M	U	L	T	I	P	L	I	E	R	S

Figure out the volume of these figures. Each block stands for 1 cube. Place the letter beside a figure in the box below its volume in cubic units to solve this riddle.

T = 24
I = 8
P = 48
E = 40
L = 30
S = 28
I = 27
L = 50
M = 6
U = 60
R = 12



Associative Property: Volume Puzzle - Worksheet 2

Why did the amoeba fail the math test?

The answer to this question is written in code. Solve these volume problems. Then write the letter next to a problem above it's answer to solve this riddle.

Length (L) Width (W) Height (H)

B E C A U S E I T M U L T I P L I E D
147-56-30-64-126-75-1 72-2 70-54-144-27-48-140-36-96-42-

B Y D I V I D I N G.
60-8 90-120-125-84-210-40-105-13

6	D	L = 1; W = 2; H = 3	210	D	L = 3; W = 7; H = 10
60	B	L = 3; W = 4; H = 5	30	C	L = 5; W = 1; H = 6
16	E	L = 2; W = 4; H = 2	90	D	L = 6; W = 5; H = 3
8	Y	L = 2; W = 2; H = 2	54	U	L = 9; W = 3; H = 2
64	A	L = 4; W = 4; H = 4	72	I	L = 2; W = 9; H = 4
125	V	L = 5; W = 5; H = 5	140	P	L = 4; W = 5; H = 7
147	B	L = 7; W = 7; H = 3	96	I	L = 6; W = 8; H = 2
42	E	L = 7; W = 2; H = 3	75	S	L = 3; W = 5; H = 5
27	T	L = 3; W = 3; H = 3	48	T	L = 4; W = 1; H = 6
48	I	L = 8; W = 2; H = 3	40	I	L = 2; W = 4; H = 5
132	G	L = 6; W = 2; H = 1	105	N	L = 7; W = 5; H = 3
36	L	L = 3; W = 6; H = 2	126	U	L = 6; W = 3; H = 7
70	M	L = 2; W = 5; H = 7	144	L	L = 8; W = 9; H = 2
84	I	L = 7; W = 6; H = 2	120	I	L = 5; W = 3; H = 8
56	E	L = 4; W = 2; H = 7			

Associative Property: Investigation - Worksheet 1

In science, we learn that there are things called Laws of Nature. That the solar system is held together by the force of gravity is one such law. There are in mathematics too. These laws are often called properties. You will explore one such property today. It is called the **Associative Property of Multiplication** This property is about the order in which we combine numbers.

Solve this multiplication problem. 2 x 3 x 4 = 24
You are being asked here to multiply three numbers together.
You can only multiply two numbers together at a time.
Which two did you multiply together first? 2 x 3 = 6

Then what did you do? 6 x 4 = 24

What would happen if you multiplied the numbers together in a different order?
The answer should stay the same.

Solve this problem. 3 x 4 x 2 = 24

Explain how you did it this time.
3 x 4 = 12 12 x 2 = 24

Did changing the order of the numbers change the final answer? no

Solve this problem. 4 x 2 x 3 = 24

Did changing the order of the numbers change the final answer? no

Associative Property: Investigation - Worksheet 2

You did a math experiment. You multiplied 2 x 3 x 4. Then you multiplied 3 x 4 x 2. Then you multiplied 4 x 2 x 3. What did you find out?

Every time the answer was the same.

Did the order in which you paired the numbers change the result? no
This is what the Associative Property of Multiplication says. It says that the order in which you multiply a series of numbers will not change the final answer. This is a law of multiplication.

You will use this property later to help you solve large whole number and fraction problems more easily

We use a symbol like this ( ) to group numbers together. These are called parenthesis (pair- n - the - seez). Parenthesis tell you to do what is inside first.

(2 x 3) x 4 = means multiply 2 x 3 first to get 6.
6 x 4 = now multiply the 6 times the 4 to get 24.

Do these. (3 x 4) x 2 = 12 x 2 = 24 (4 x 2) x 3 = 8 x 3 = 24

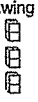
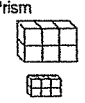
Find all three ways of associating these three numbers. 3, 4 , 5
The first one is done for you.

(3 x 4) x 5 = 12 x 5 = 60 (4 x 5) x 3 = 20 x 3 = 60

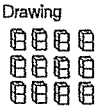
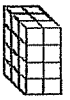
(3 x 5) x 4 = 15 x 4 = 60

Associative Property: Investigation - Worksheet 3

Here are four ways to record the problem 2 x 3.

Multiplication	Addition	Drawing	Prism
2 x 3 = 6	2 2 +2 6		

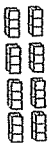
Here are four ways to record the problem (2 x 3) x 4.

Multiplication	Addition	Drawing	Prism
(2 x 3) x 4 =	2 2 2 2 2 2 2 2 +2 +2 +2 +2 6 6 6 6		

Show the other two ways to associate the numbers 2, 3, and 4. Record the multiplication problem with parenthesis, the addition problem, and the drawing for each way.

(3 x 4) x 2 =

3 3
3 3
3 3
+3 +3
12 12



(4 x 2) x 3 =

4 4
+4 +4
8 8
+8

Build a model of 2 x 3 x 4 with linking cubes or Unifix cubes.

Associative Property: Investigation - Worksheet 4

Find all the ways of associating these three numbers. 2, 5 , 6

(2 x 5) x 6 = 10 x 6 = 60 (2 x 6) x 5 = 12 x 5 = 60

(5 x 6) x 2 = 30 x 2 = 60

Math Experiment
You found that there are three ways to associate 3 numbers in multiplication.
How many ways are there to do it with 4 numbers? 3 without repeating
Make a guess. _____

Use these four numbers 2, 3, 5, 6 You may use a calculator if you want.
The first one is done for you.

(2 x 3) x (5 x 6) = 6 x 30 = 180

(5 x 2) x (3 x 6) = 10 x 18 = 180

(5 x 3) x (2 x 6) = 15 x 12 = 180

Associative Property: Investigation - Worksheet 5

Find all the ways of associating these three numbers with multiplication.
You may use a calculator if needed. 3, 5, 7

(3 x 5) x 7 =
15 x 7 = 105

(7 x 3) x 5
21 x 5 = 105

(__ x __) x __
__ x __ =

Is the product the same every time? yes

The product is the answer to a multiplication problem.

Try it again. This time use addition instead of multiplication.
Use 3, 5, 7 again. Do you think all the sums will be the same? AWV

(3 + 5) + 7 =
8 + 7 = 15

(7 + 5) + 3
12 + 3 = 15

(3 + 7) + 5
10 + 5 = 15

What did you find out? All the answers are the same.

Try it one more time. This time the signs will be mixed and both x and + will be used. Will the answers be the same every time this time? AWV
You may use a calculator . Use 3, 5, and 7 again.

(3 x 7) + 5 =
21 + 5 = 26

(5 x 7) + 3
35 + 3 = 38

(3 x 5) + 7 =
15 + 7 = 22

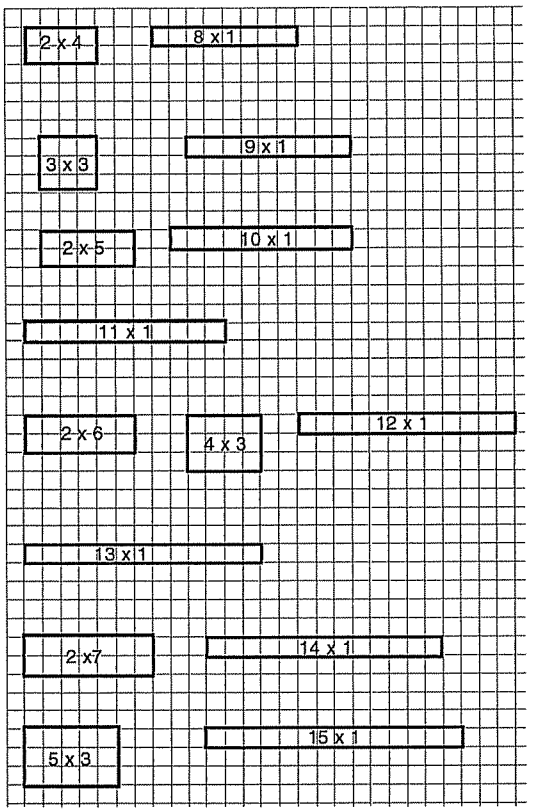
(7 + 5) x 3
12 x 3 = 36

(3 + 7) x 5 =
10 x 5 = 50

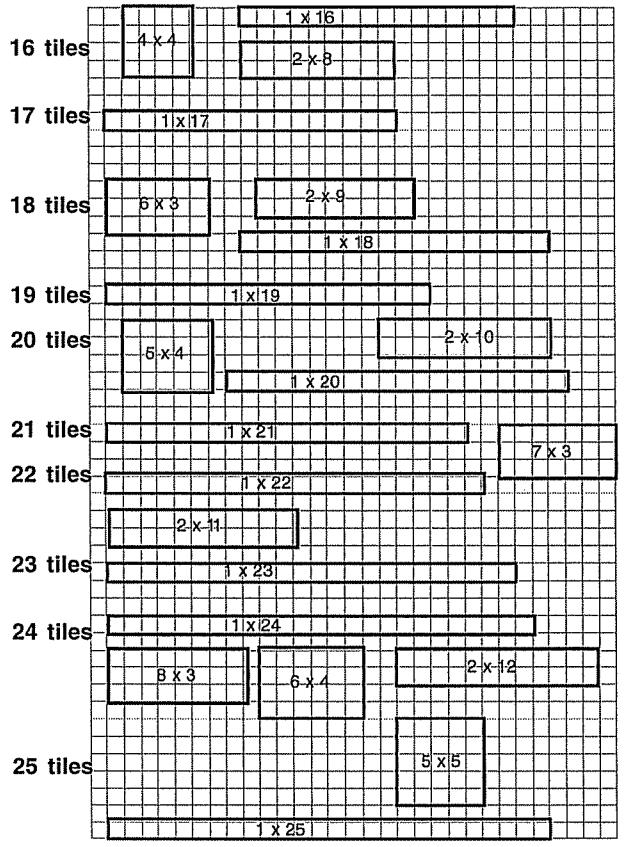
(5 + 3) x 7
8 x 7 = 56

What happens when you mix addition and multiplication
All the answers are different.

Discovering Prime Numbers - Worksheet 2



Discovering Prime Numbers - Worksheet 3



Discovering Prime Numbers - Worksheet 4

Go back through your investigation and find all the numbers that had only one rectangle.
List the numbers here:
1, 2, 3, 5, 7, 11, 13, 17, 19, 23

These numbers are called PRIME NUMBERS.
A prime number is a number that has only itself and one as a factor.
The only way to make 17 in multiplication is 17 x 1. This means that 17 is a prime number. The number 1 is not considered to be a prime number.

Go back through your investigation and find all the numbers that had two or more rectangles. These are numbers that can be made with several different factors.
List those numbers here:
4, 6, 8, 9, 10, 12, 14, 15, 16, 25
18, 20, 21, 22, 24, 25

These numbers are called COMPOSITE (kahm -pahz-it) NUMBERS.
A composite number is any number that has not only itself and one as factors but other numbers too. An example of this is the number 9. Nine can be made in multiplication with 9 x 1 and also 3 x 3.

Look up the word composite in the dictionary.
Copy the first definition. Mixture of things.

How many prime numbers are even? 1
How many prime numbers are odd? 8
Are composite numbers even or odd? both
Give examples 4, 9, 16, 21

Guess the answer to these equations.
Check with numbers and see if you were correct.

Even + Even = even
Check 2 + 4 = 6
Check 4 + 10 = 14

Even + Odd = odd
4 + 1 = odd 5
6 + 3 = odd 9

Odd + Odd = even
Check 3 + 5 = 8
Check 9 + 3 = 12

Odd + Even = odd
3 + 10 = 13
5 + 12 = 17

Use the SIEVE OF ERASTOSTHENES (next page) to discover all the prime numbers between 2 and 100.
Learn prime numbers. They will help you when you learn fractions.

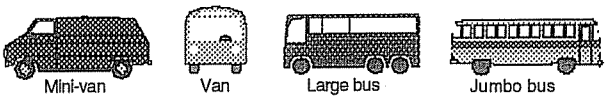
Discovering Prime Numbers - Worksheet 5

The Greek mathematician Erastosthenes (275-194 B.C.) invented this technique of a numerical sieve to discover the prime numbers smaller than some given number

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	33	~~34~~	~~35~~	~~36~~	37	~~38~~	39	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	49	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	69	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

1) Cross out 1. It is not considered a prime.
2) Circle 2, the smallest positive even prime. Cross out all of the multiples of two.
3) Circle the next prime, it is 3, Cross out all of the multiples of 3. Every 3rd number will be crossed out.
4) Circle the next open number. It is 5. Cross out every 5th number.
5) Continue the process until all the numbers up through 100 are either circled or crossed out.

Gus the Bus Driver



Gus, the bus driver, now has his own bus company. He has a Mini-van, a Van, a Large bus, and a Jumbo bus. He has also hired a mechanic, Madge, to help keep the buses in good shape.

1. Gus decided to get new windshield wipers for three buses. Each wiper cost $4.00 each and each bus needs two. How much will he spend on one bus?. $8.00 How much will he spend on three buses? $24.00

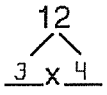
2. When Gus went shopping, Madge, the mechanic, slipped him a mail-in rebate. "One dollar off each wiper blade when you buy four or more." How much money can he get back? $6.00
How much will he spend on the wipers for three buses? $18.00

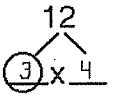
3. The field trip to see the lilacs at Evring Park, (75 miles), picnic a the Living History Farm, (17 miles), back to school (66 miles), was planned for two different classes. How many miles would Gus drive over the two days? 316

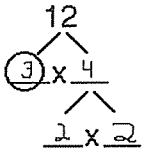
4. Gus' favorite gas station sells candy and sodas. He decides to surprise the kids on the last week of school with a tootsie roll pop as they get of f the bus. The pops are 10¢ each. His 60 seat bus usually carries 44 kids home. How much money will Gus need? $4.40 How much change will he get from a $10.00 bill? $5.60.

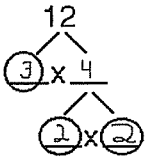
Prime Factoring - Worksheet 1

This is how you discover the prime factors of a composite number.

Step 1.  Factor the top number.

Step 2.  Circle any prime factor.

Step 3. 

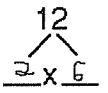
Step 4. 

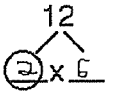
Step 5. These circled numbers are the prime factors of the number you started with, in this case 12. Prime factors are usually listed in a box after the number being factored in order from largest to smallest. Separate them with a comma.

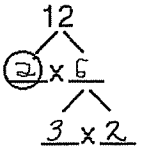
12 | 3, 2, 2 |

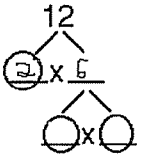
Prime Factoring - Worksheet 2

We will factor 12 to prime numbers again. This time we will use 2 x 6 to start instead of 3 x 4. Do you think the prime factors of 12 at the end will be the same or different? Same

Step 1  Factor the top number.

Step 2  Circle any prime factor.

Step 3 

Step 4 

Step 5 List the prime factors in order of size. Are they the same as before? ____

12 | 3, 2, 2 |

Prime Factoring - Worksheet 3

OWV

4 = 2 x 2 → 2, 2

6 = 3 x 2 → 3, 2

8 = 2 x 4 = 2 x 2 x 2 → 2, 2, 2

9 = 3 x 3 → 3, 3

10 = 5 x 2 → 5, 2

14 = 2 x 7 → 7, 2

15 = 5 x 3 → 5, 3

16 = 2 x 8 = 2 x 4 x 2 x 2 → 2, 2, 2, 2 or 4 x 4 = 2 x 2 x 2 x 2

Prime Factoring - Worksheet 4

18 = 3 x 6 = 2 x 3 x 3 → 3, 3, 2

18 = 2 x 9 = 3 x 3 x 2 → 3, 3, 2

21 = 3 x 7 → 3, 7

22 = 2 x 11 → 11, 2

24 = 8 x 3 = 2 x 4 x 3 = 2 x 2 x 2 x 3 → 3, 2, 2, 2

25 = 5 x 5 → 5, 5

Prime Factoring - Worksheet 5

26 = 13 x 2 → 13, 2

56 = 2 x 28 = 2 x 14 x 2 x 7 → 7, 2, 2, 2

44 = 4 x 11 = 2 x 2 x 11 → 11, 2, 2

32 = 8 x 4 = 4 x 2 x 2 x 2 x 2 → 2, 2, 2, 2, 2

50 = 5 x 10 = 5 x 2 x 5 → 5, 5, 2

42 = 7 x 6 = 7 x 2 x 3 → 7, 3, 2

58 = 2 x 29 → 29, 2

98 = 2 x 49 = 2 x 7 x 7 → 7, 7, 2

Prime Factoring - Worksheet 6

27 = 9 x 3 = 3 x 3 x 3 → 3, 3, 3

54 = 27 x 2 = 9 x 3 x 2 = 3 x 3 x 3 x 2 → 3, 3, 3, 2

42 = 6 x 7 = 2 x 3 x 7 → 7, 3, 2

47 = x → prime

36 = 6 x 6 = 2 x 3 x 2 x 3 → 3, 3, 2, 2

64 = 32 x 2 = 8 x 4 x 2 = 4 x 2 x 2 x 2 x 2 = 2 x 2 x 2 x 2 x 2 x 2

72 = 8 x 9 = 4 x 2 x 3 x 3 = 2 x 2 x 2 x 3 x 3 → 3, 3, 2, 2, 2

30 = 2 x 15 = 2 x 3 x 5 → 5, 3, 2

Prime Factoring - Worksheet 7

45 → 9 × 5 → 5, 3, 3 (9 → 3 × 3)

51 → 17 × 3 → 17, 3

76 → 19 × 4 → 19, 2, 2 (4 → 2 × 2)

22 → 11 × 2 → 11, 2

48 → 6 × 8 → 3, 2, 2, 2, 2 (6 → 2 × 3; 8 → 2 × 4; 4 → 2 × 2)

85 → 17 × 5 → 17, 5

35 → 7 × 5 → 7, 5

75 → 15 × 5 → 5, 5, 3 (15 → 3 × 5)

Prime Factoring - Worksheet 8

30 → 3 × 10 → 5, 3, 2 (10 → 2 × 5)

81 → 9 × 9 → 3, 3, 3, 3 (9 → 3 × 3; 9 → 3 × 3)

43 → 43 × 1 → prime

90 → 3 × 30 → 5, 3, 3, 2 (30 → 10 × 3; 10 → 5 × 2)

99 → 3 × 33 → 11, 3, 3 (33 → 3 × 11)

74 → 37 × 2 → 37, 2

62 → 31 × 2 → 31, 2

88 → 2 × 44 → 11, 2, 2, 2 (44 → 2 × 22; 22 → 2 × 11)

Prime Factoring - Worksheet 9

60 → 2 × 30 → 5, 3, 2, 2 (30 → 2 × 15; 15 → 3 × 5)

35 → 7 × 5 → 7, 5

40 → 2 × 20 → 5, 2, 2, 2 (20 → 2 × 10; 10 → 2 × 5)

66 → 3 × 22 → 11, 3, 2 (22 → 2 × 11)

80 → 2 × 40 → 5, 2, 2, 2, 2 (40 → 4 × 10; 4 → 2 × 2; 10 → 2 × 5)

95 → 19 × 5 → 19, 5

68 → 34 × 2 → 17, 2, 2 (34 → 17 × 2)

78 → 39 × 2 → 13, 3, 2 (39 → 13 × 3)

Prime Factoring Puzzles - Worksheet 1

Fill in the missing numbers.

24 → 3 2 2 2 (24 → 2 × 12; 12 → 2 × 6; 6 → 3 × 2)

60 → 5 3 2 2 (60 → 3 × 20; 20 → 2 × 10; 10 → 2 × 5)

108 → 9, 3, 2, 2

54 → 3, 3, 3, 2 (54 → 3 × 18; 18 → 3 × 6; 6 → 2 × 3)

84 → 7, 3, 2, 2

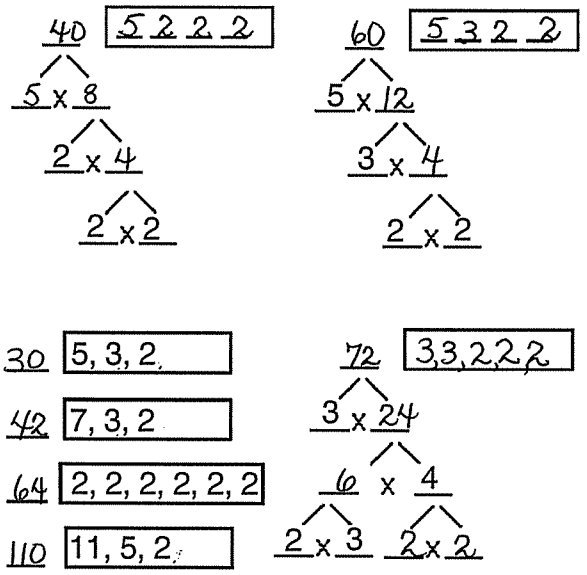
81 → 3, 3, 3, 3

90 → 9, 5, 2,

What are the prime factors of 1000? 5, 5, 5, 2, 2, 2 (1000 → 10 × 100; 10 → 5 × 2; 100 → 10 × 10; 10 → 5 × 2; 10 → 5 × 2)

Prime Factoring Puzzles - Worksheet 2

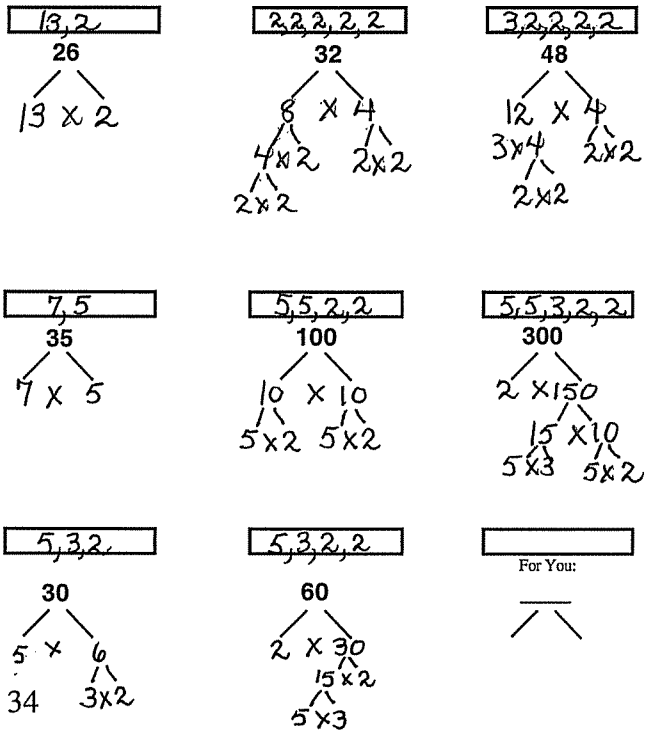
Fill in the missing numbers.



What are the prime factors of 100?

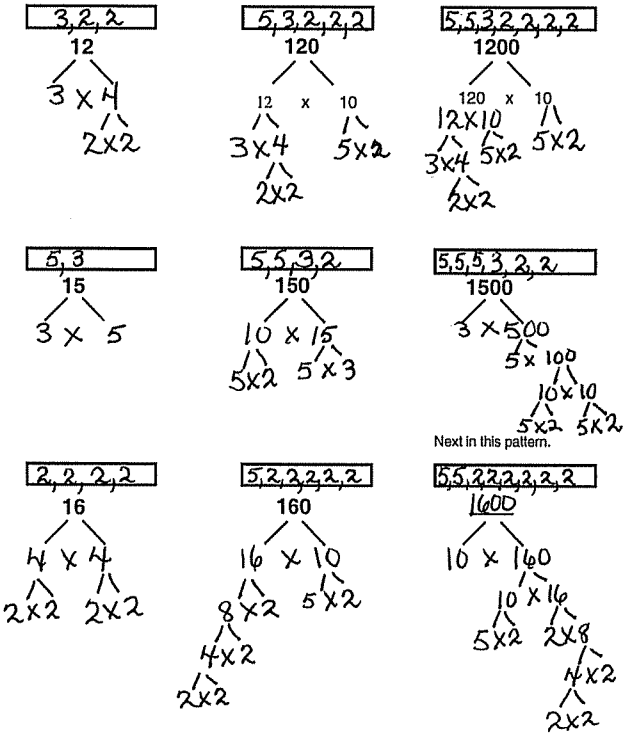
Prime Factoring Practice - Worksheet 1

Factor these to prime numbers. List the prime factors in the box. Remember to put them in order from largest to smallest. Put commas between the prime factors.



Prime Factoring Practice - Worksheet 2

Factor these to prime numbers.

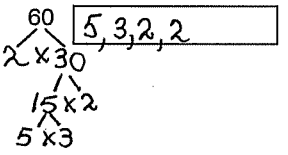


Show You Know

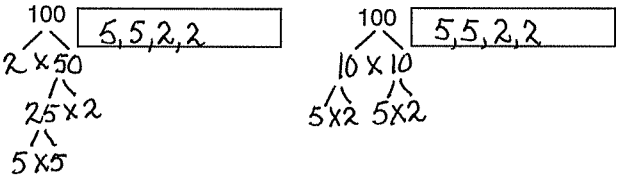
1. If you need to multipliy three numbers together, in what order should you do it? Any order.

2. Explain what a prime number is. A whole number that can only be divided evenly by itself and one.

3. Find the prime factors of 60. 60 — 5, 3, 2, 2



4. Prime factor 100 two different ways.



What pattern do you see in the prime factors in the box? They are the same numbers. Multiplied together they equal one hundred.

Multiplying Factors

4 x 8 = 6 x 8 =

2 x 2 4 x 2 = 32 2 x 3 2 x 4 = 48

2 x 2 x 4 x 2 = 32 2 x 3 x 2 x 4 = 48

What happens when you multiply the factors of a number together?

You get the original number.

7 x 6 = 6 x 4 =

1 x 7 3 x 2 = 42 2 x 3 2 x 2 = 24

1 x 7 x 3 x 2 = 42 2 x 3 x 2 x 2 = 24

20 x 8 = 30 x 6 =

10 x 2 1 x 8 = 3 x 10 2 x 3 =

10 x 2 x 1 x 8 = 160 3 x 10 x 2 x 3 = 180

Change the order of the numbers.

1 x 2 x 10 x 8 = 160 10 x 2 x 3 x 3 = 180

Factoring into Tens - Worksheet 1

Sometimes factoring a number into multiples of ten is more useful than factoring it with prime numbers.

Two examples: 20 = 2 x 10; 300 = 3 x 100 (or 3 x 10 x 10)

Factor these numbers by tens.

40 = 10 x 4

90 = 10 x 9

100 = 10 x 10

600 = 6 x 100 or 6 x 10 x 10

800 = 8 x 100 or 8 x 10 x 10

700 = 7 x 100 or 7 x 10 x 10

Factoring into Tens - Worksheet 2

What are the prime factors of 120? 5, 3, 2, 2, 2

What are all the composite factors of 120? 60, 40, 24, 15, 12, 8

Factor 120 by tens. 120 = 12 x 10 = 2 x 6 x 2 x 5 = 2 x 3 x 2 x 5

120 = 2 x 60 = 6 x 10 = 2 x 3 x 2 x 5

Prime factors of mystery number = 5, 2, 3, 2, 2, 2

mystery number = 240

Prime factors of mystery number = 7, 3, 3, 2

mystery number = 126

Factor to primes. Draw a circle around the prime numbers.

17 = 17, 1 (17 x 1)

81 = 3, 3, 3, 3 (9 x 9; 3 x 3, 3 x 3)

243 = 3, 3, 3, 3, 3 (3 x 81; 9 x 9; 3 x 3, 3 x 3)

Factor By Tens and Prime Numbers

Factor each number into prime numbers and then by tens.

50 = 5 x 10 = 2 x 5

Prime numbers: 5, 2, 5

By tens: 50 = 5 x 10

80 = 40 x 2 = 20 x 2 x 2 = 10 x 2 x 2 x 2 = 5 x 2 x 2 x 2

Prime numbers: 5, 2, 2, 2, 2

By tens: 80 = 8 x 10

150 = 3 x 50 = 25 x 2 = 5 x 5 x 2

Prime numbers: 5, 5, 3, 2

By tens: 150 = 15 x 10

700 = 7 x 100 = 50 x 2 = 25 x 2 x 2 = 5 x 5 x 2 x 2

Prime numbers: 7, 5, 5, 2, 2

By tens: 700 = 70 x 10

Using Factoring with Expanded Tables

8,000 x 4 — Factor 8,000 into tens.

8 x 1,000 x 4 — Rearrange and group.

(8 x 4) x 1,000 — Multiply what's inside the ( ).

32 x 1,000 — Find the product.

32,000

You try it.

700 x 3 = 7 x 100 x 3 = (7 x 3) x 100 = 21 x 100 = 2,100

500 x 7 = 5 x 100 x 7 = (5 x 7) x 100 = 35 x 100 = 3,500

400 x 6 = 4 x 100 x 6 = (4 x 6) x 100 = 24 x 100 = 2,400

2,000 x 7 = 2 x 1,000 x 7 = (2 x 7) x 1,000 = 14 x 1,000 = 14,000

What patterns do you notice when:

multiplying by 100 add 2 zeros.

multiplying by 1,000 add 3 zeros.

Expanded Tables: Review

3	30	300	3,000
x 3	x 3	x 3	x 3
9	90	900	9,000
7	**70**	**700**	**7,000**
x 4	x 4	x 4	x 4
28	280	2,800	28,000
8	**80**	**800**	**8,000**
x 7	x 7	x 7	x 7
56	560	5,600	56,000
6	**60**	**600**	**6,000**
x 8	x 8	x 8	x 8
48	480	4,800	48,000

Speed Tests - Worksheet 1

2 x 2 = 4	3 x 8 = 24	4 x 6 = 24	5 x 2 = 10
2 x 0 = 0	3 x 3 = 9	4 x 0 = 0	5 x 5 = 25
2 x 6 = 12	3 x 7 = 21	4 x 3 = 12	5 x 4 = 20
2 x 12 = 24	3 x 2 = 6	4 x 12 = 48	5 x 7 = 35
2 x 3 = 6	3 x 6 = 18	4 x 8 = 32	5 x 12 = 60
2 x 7 = 14	3 x 4 = 12	4 x 5 = 20	5 x 3 = 15
2 x 5 = 10	3 x 12 = 36	4 x 7 = 28	5 x 8 = 40
2 x 8 = 16	3 x 5 = 15	4 x 2 = 8	5 x 9 = 45
2 x 4 = 8	3 x 0 = 0	4 x 4 = 16	5 x 6 = 30
2 x 9 = 18	3 x 9 = 27	4 x 9 = 36	5 x 0 = 0
Total = ___	Total = ___	Total = ___	Total = ___
Sec/Problem = ___	Sec/Problem = ___	Sec/Problem = ___	Sec/Problem = ___
6 x 6 = 36	7 x 9 = 63	8 x 8 = 64	9 x 4 = 36
6 x 3 = 18	7 x 6 = 42	8 x 5 = 40	9 x 7 = 63
6 x 8 = 48	7 x 7 = 49	8 x 7 = 56	9 x 6 = 54
6 x 5 = 30	7 x 8 = 56	8 x 0 = 0	9 x 9 = 81
6 x 7 = 42	7 x 3 = 21	8 x 12 = 96	9 x 2 = 18
6 x 2 = 12	7 x 2 = 14	8 x 6 = 48	9 x 5 = 45
6 x 0 = 0	7 x 12 = 84	8 x 4 = 32	9 x 3 = 27
6 x 9 = 54	7 x 5 = 35	8 x 2 = 16	9 x 12 = 108
6 x 12 = 72	7 x 0 = 0	8 x 9 = 72	9 x 8 = 18
6 x 4 = 24	7 x 4 = 28	8 x 3 = 24	9 x 0 = 0
Total = ___	Total = ___	Total = ___	Total = ___
Sec/Problem = ___	Sec/Problem = ___	Sec/Problem = ___	Sec/Problem = ___

Speed Tests - Worksheet 2

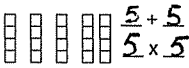
2 x 4 = 8	3 x 6 = 18	4 x 8 = 32	5 x 8 = 40
2 x 7 = 14	3 x 0 = 0	4 x 3 = 12	5 x 5 = 25
2 x 6 = 12	3 x 3 = 9	4 x 7 = 28	5 x 7 = 35
2 x 9 = 18	3 x 12 = 36	4 x 2 = 8	5 x 0 = 0
2 x 2 = 4	3 x 8 = 24	4 x 6 = 24	5 x 12 = 60
2 x 5 = 10	3 x 5 = 15	4 x 4 = 16	5 x 6 = 30
2 x 3 = 6	3 x 7 = 21	4 x 12 = 48	5 x 4 = 20
2 x 12 = 24	3 x 2 = 6	4 x 5 = 20	5 x 2 = 10
2 x 8 = 16	3 x 4 = 12	4 x 0 = 0	5 x 9 = 45
2 x 0 = 0	3 x 9 = 27	4 x 9 = 36	5 x 3 = 15
Total = ___	Total = ___	Total = ___	Total = ___
Sec/Problem = ___	Sec/Problem = ___	Sec/Problem = ___	Sec/Problem = ___
6 x 9 = 54	7 x 6 = 42	8 x 2 = 16	9 x 3 = 27
6 x 6 = 36	7 x 3 = 21	8 x 5 = 40	9 x 5 = 45
6 x 7 = 42	7 x 8 = 56	8 x 4 = 32	9 x 6 = 54
6 x 8 = 48	7 x 5 = 35	8 x 7 = 56	9 x 7 = 63
6 x 3 = 18	7 x 7 = 49	8 x 12 = 96	9 x 12 = 108
6 x 2 = 12	7 x 2 = 14	8 x 3 = 24	9 x 8 = 72
6 x 12 = 72	7 x 0 = 0	8 x 8 = 64	9 x 5 = 45
6 x 5 = 30	7 x 9 = 63	8 x 9 = 72	9 x 9 = 81
6 x 0 = 0	7 x 12 = 84	8 x 6 = 48	9 x 4 = 36
6 x 4 = 24	7 x 4 = 28	8 x 0 = 0	9 x 2 = 18
Total = ___	Total = ___	Total = ___	Total = ___
Sec/Problem = ___	Sec/Problem = ___	Sec/Problem = ___	Sec/Problem = ___

Speed Tests - Worksheet 3

2 x 6 = 12	3 x 4 = 12	4 x 3 = 12	5 x 6 = 30
2 x 3 = 6	3 x 7 = 21	4 x 5 = 20	5 x 0 = 0
2 x 8 = 16	3 x 6 = 18	4 x 6 = 24	5 x 3 = 15
2 x 5 = 10	3 x 9 = 27	4 x 7 = 28	5 x 12 = 60
2 x 7 = 14	3 x 2 = 6	4 x 12 = 48	5 x 8 = 40
2 x 2 = 4	3 x 5 = 15	4 x 8 = 32	5 x 5 = 25
2 x 0 = 0	3 x 3 = 9	4 x 5 = 20	5 x 7 = 35
2 x 9 = 18	3 x 12 = 36	4 x 9 = 36	5 x 2 = 10
2 x 12 = 24	3 x 8 = 24	4 x 4 = 16	5 x 4 = 20
2 x 4 = 8	3 x 0 = 0	4 x 2 = 8	5 x 9 = 45
Total = ___	Total = ___	Total = ___	Total = ___
Sec/Problem = ___	Sec/Problem = ___	Sec/Problem = ___	Sec/Problem = ___
6 x 8 = 48	7 x 2 = 14	8 x 8 = 64	9 x 9 = 81
6 x 3 = 18	7 x 0 = 0	8 x 5 = 40	9 x 6 = 54
6 x 7 = 42	7 x 6 = 42	8 x 7 = 56	9 x 7 = 63
6 x 2 = 12	7 x 12 = 84	8 x 0 = 0	9 x 8 = 72
6 x 6 = 36	7 x 3 = 21	8 x 12 = 96	9 x 3 = 27
6 x 4 = 24	7 x 7 = 49	8 x 6 = 48	9 x 2 = 18
6 x 12 = 72	7 x 5 = 35	8 x 4 = 24	9 x 12 = 108
6 x 5 = 30	7 x 8 = 56	8 x 2 = 16	9 x 5 = 45
6 x 0 = 0	7 x 4 = 28	8 x 9 = 72	9 x 0 = 0
6 x 9 = 54	7 x 9 = 63	8 x 3 = 24	9 x 4 = 36
Total = ___	Total = ___	Total = ___	Total = ___
Sec/Problem = ___	Sec/Problem = ___	Sec/Problem = ___	Sec/Problem = ___

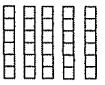
Pre-Assessment: Part 1

1. Look at the picture. Write the addition and multiplication number sentence.

a. 
5 + 5 + 5 + 5 + 5 = 25 Addition
5 x 5 = 25 Multiplication

b. 
3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24 Addition
3 x 8 = 24 Multiplication

c. 
2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 18 Addition
2 x 9 = 18 Multiplication

d. 
6 + 6 + 6 + 6 + 6 = 30 Addition
6 x 5 = 30 Multiplication

2. Here is a multiplication number sentence: 3 x 7 = 21.
Write the meaning of each number in the multiplication number sentence.
a. The 3 means the number of blocks in each group.
b. The 7 means the number of times each group is made.
c. The 21 means the total number of blocks.

3. a. What is the area of this rectangle? 25 sq. units 

b. What is the area of this rectangle? 35 sq. units 

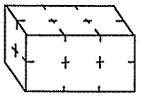
Pre-Assessment: Part 2 - Worksheet 1

1. Show all the ways of associating (grouping) these three numbers: 2, 5, 6

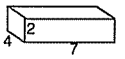
(2 x 5) x 6 = 10 x 6 = 60 (6 x 2) x 5 = 12 x 5 = 60 (6 x 5) x 2 = 30 x 2 = 60

2. Find the volume of these rectangular solids.
Length (L) Width (W) Height (H)

a. Volume 12 cu units 3 x 2 x 2



b. Volume 56 cu units 7 x 4 x 2



c. Volume 36 cu. units 3 x 6 x 2

L = 3; W = 6; H = 2

3. What is a Factor? Use numbers to help you explain.
A factor is a number that results in a product (answer) when multiplied by another factor.
2 x 3 = 6 – two and three are factors.

4. What is a Prime Number?
A number that has only itself and one as factors.

5. List the first ten Prime Numbers starting with 2.

2 3 5 7 11 13 17 19 23 29

Pre-Assessment: Part 2 - Worksheet 2

6. Prime Factor 24, 50, and 37. Record the Prime Factors in the boxes.

a. 3, 2, 2, 2
24 → 2 x 12 → 3 x 4 → 2 x 2

b. 5, 5, 2
50 → 5 x 10 → 2 x 5

c. 37, 1
37 → 1 x 37

7. Factor 50 and 80 by tens.

By tens
a. 50 → 5 x 10

By tens
b. 80 → 8 x 10

8. a. Solve these problems.

6	60	600	6,000
x 8	x 8	x 8	x 8
48	480	4,800	48,000

b. What patterns do you notice in the answers to these problems?
Forty-eight will always be the first two numbers in the product. The number of zeros after the forty-eight will change. The place value of forty-eight changes.

c. Use the pattern to find the answer to 600,000 x 8. 4,800,000

9. Use factoring by tens to show how to get the product of 400 x 6.

400 x 6
4 x 100 x 6
(4 x 6) x 100
24 x 100
2,400

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