# Patterns in Arithmetic
General Organization - Grade 3 PDF
Parent/Teacher Guide

Foreword by Professor Michael Butler
Introduction
Free Exploration, or How Not to Be a Nag
Farm School Math Notes
Teaching Techniques
Arithmetic Conventions
Take Delight
Math Journal
Grading and Dialogues

Sequence Guides
Lesson Planning
Materials
Resources
Base Ten Materials
Assessment Guide
Assessment Worksheets
Assessment Answer Key
Series Overview
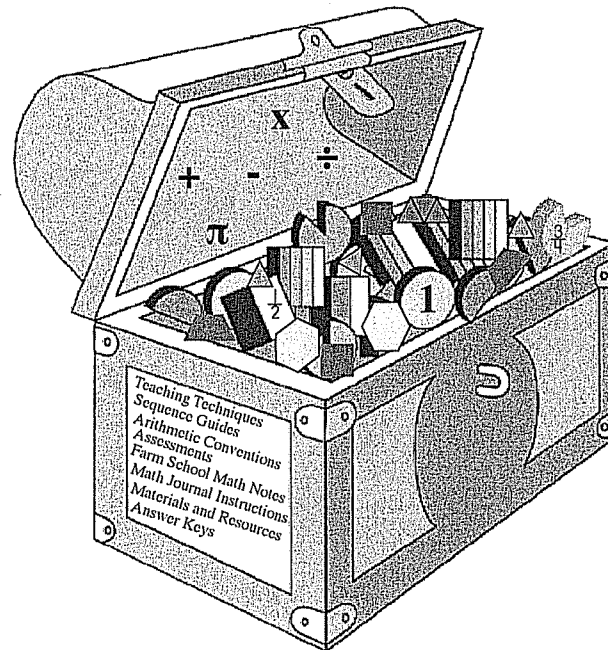
By Alysia Krafel, Susan Carpenter, and Suki Glenn
Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Contents

This booklet is dedicated to my father, Robert Lueck, an educator and school administrator for many years. He would be delighted to know I followed in his footsteps. He was a model of organization and resourcefulness.   Suki Glenn

Acknowledgments
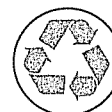
Foreword

To the reader,

I am delighted to write a foreword to this lovely book on mathematics. The work described here reflects a more general Farm School approach, but thinking about how people come to like and be good at mathematics played an important part in developing that approach.

Years ago I was a young professor at UC Irvine, and although I had long been fascinated by the act of learning, this was my first teaching job. Among other things I taught mathematics. The experience was immensely rewarding but unsettling. I thought of math as beautiful, richly ordered, and fun. Most of my students in those required courses appeared to think of it, at least at first, as arbitrary, impenetrable, incoherent, and dull; some of them found it scary.

A few students did not, cheerfully pushing and pulling at a formula, for instance, and asking: What would happen if this part of the denominator were in the numerator? What would happen if I reversed this and that part? What would happen if I made this piece very large or very small? What would be a simpler form or a more general form of the expression? They engaged in this systematic play for the fun of it, but their reinventing or recasting of the material of mathematics also helped them see why something was the way it was; it helped them understand. In fact, the students I started listening to each seemed to carry with them a kind of "understanding kit." They had an expectation that math would make sense; they knew when a particular expression or idea did not yet make sense to them, and when it did; and they had developed skills and stamina for getting from the first state to the second, and the habit of doing so. The math they came to know in this way, they owned.

These happy few were regarded by the others (and by most of my colleagues) as having a peculiar knack. There was no shame in not having it; that was just the luck of the genetic draw. Or did the attitude of the rest of the class toward math have to do with the way they had been educated? Their reports of their pre-college math study matched what I found when I started visiting schools, especially elementary schools, and reading texts of that era: My students had been spending most of their time memorizing calculation recipes and learning to run them more or, often, less well.

But that wasn't at all what the kind of people who had discovered the math did. Mathematicians look for and find patterns in formal objects, extend them, seek counter examples, figure out why the patterns work, and then, finally, publish an account of one way that they work. The last is the public part, but the rest is what they do. Almost none of my undergraduate students seemed to have had much experience with that. There was an odd disjunction between what practitioners did and what schools asked students to do, a disjunction that was deeper and odder the more you looked at it. It was as though we had plucked the fruit "mathematics" for use in schools, peeled it, and fed students the rind instead of the flesh.

Much the same thing seemed to be true in other areas. What working historians did, for example, or scientists was rarely much like what schoolchildren did, so it wasn't surprising that undergraduates found it hard to think that way when asked. Again, it was as though teachers had discovered what it was that delighted practitioners, that drew them to their discipline— and in fact kept it a discipline, a thing that people were willing to spend their lives in, over generations—and having found these sources of delight in practice, schools threw them away and taught the residue.

So we and the times and UCI being young, and there being some farmhouses available on the edge of the campus, some of us made a school to redress these wrongs. We wanted a school where children would learn to do what finders and makers do, not just master more or less badly and mechanically some scattered things they had worked out. The students would ideally acquire some of the skills and habits of mind of mathematicians and historians and writers and scientists and artists, and even learn to do what good thinkers do when they are thinking well, independent of a particular practice; and they would learn to find matters of interest in and around themselves, and to develop and sustain those interests, as creators of new art and knowledge must do. These were not the only aims of the school, called the Farm School, but they were central. In this sense we were elitist in our ambitions for children, but populist in our belief that most children could realize those ambitions.

We are all older now, but thanks to the dedicated work of people like Suki Glenn, Susan Carpenter, and Alysia Krafel over the years, the Farm School's approach has endured and evolved. If you also want your children to learn to do mathematics, this book will prove a subtle, wise, pleasure-giving and compassionate guide.

Professor Emeritus Michael Butler
Former Director, UCI Farm Elementary School
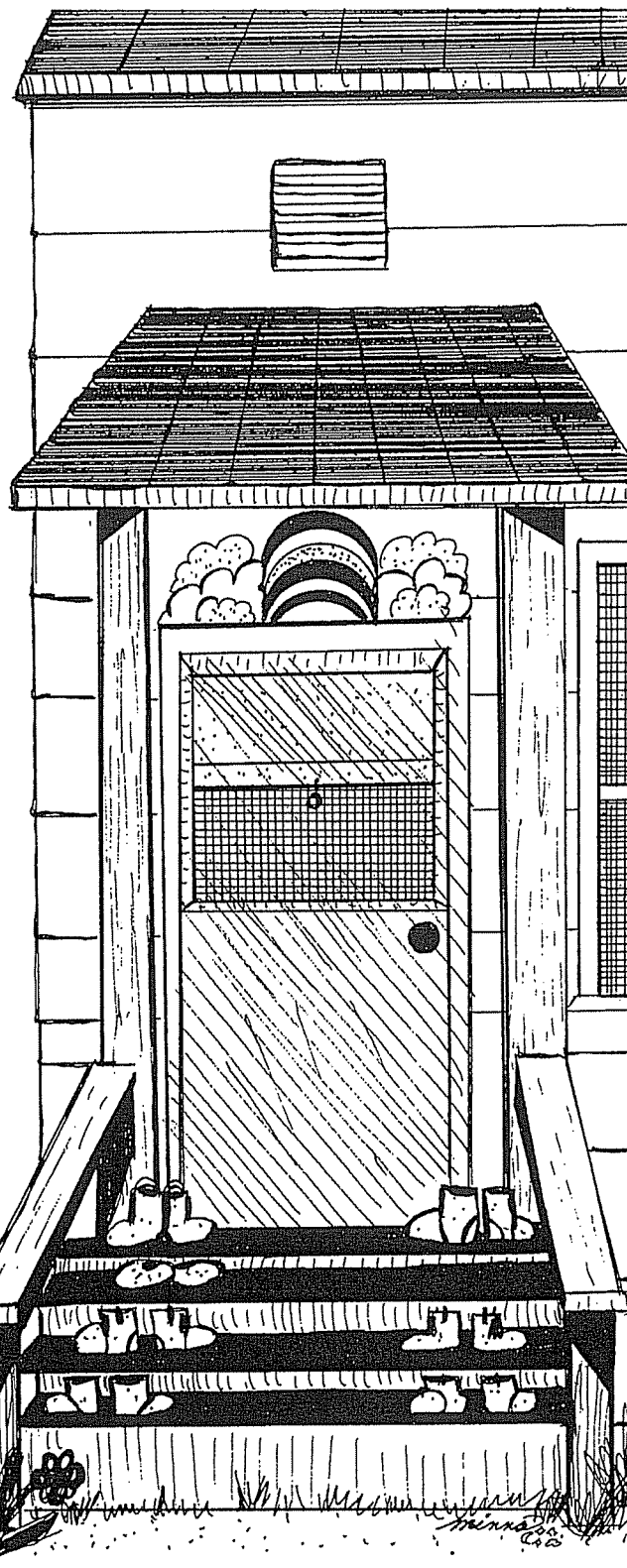
Introduction

University of California, Irvine, Professor Emeritus Michael Butler: "We wanted a school where children would learn to do what finders and makers do, not just master more or less badly and mechanically some scattered things they had worked out. The students would ideally acquire some of the skills and habits of mind of mathematicians and historians and writers and scientists and artists, and even learn to do what good thinkers do when they are thinking well, independent of a particular practice; and they would learn to find matters of interest in and around themselves, and to develop and sustain those interests, as creators of new art and knowledge must do. These were not the only aims of the school, called the Farm School, but they were central. In this sense we were elitist in our ambitions for children, but populist in our belief that most children could realize those ambitions."

Patterns in Arithmetic is the math program developed at the UCI Farm Elementary School for elementary aged students and now also published with adult learners in mind. At the heart of mathematics is the idea of patterns. Mathematicians are people who find and use patterns in numbers, shapes, and relationships, exploring through these patterns the workings of the world and universe. Mathematicians also explore patterns simply for their beauty and for the great pleasure discovering patterns can give.

This mathematics program aims to teach students and their teachers to think like mathematicians, not just memorize some of the things that mathematicians have found out. Students "do" mathematics from the beginning, so there is an emphasis on inquiry and discovery, on invention, and on learning what genuine understanding feels like and how to achieve it.

Students create their own methods for solving arithmetic problems. They use concrete materials to build and prove their answer. Then they are asked to solve it a different way and to verify that answer. The goal is understanding: knowing what it feels like when one understands (which often is accomplished through "Aha" flashes of insight); knowing what it feels like when one does not understand and what to do then; learning perseverance and how to push through feelings of uncertainty and frustration.

Learners create, routinely look for, and discover patterns in numbers, shapes, the sky, the world. They invent problems for themselves and others, estimate, predict, and look for answers that make sense; they develop heuristics (practices that help them solve problems, e.g., draw a picture), and look for relationships (the more the more).

Ideally they develop the meta-skills of a good mathematician: *the 'understanding' loop*: They know when they understand and when they don't, and they keep at it until they do; *the 'conjecture and test' loop:* They sense when something can be generalized and press to the limit of their ability to do so.

The Parent/Teacher Guide scripts the lesson to create this learning interaction. Ultimately, the learner will ask herself these thinking questions.

The following process is used throughout the *Patterns in Arithmetic* series to develop understanding of a concept:
1. Introduce the concept with a manipulative. Orally discuss it. Build it. Verify it. Practice it. Repeat the experience with a different manipulative (oral manipulative).
2. Use manipulatives to explore the concept again. This time record it with pictures (pictorial representation). Practice it. Use worksheets.
3. Record the problem with numbers (abstract, symbolic), which links the concrete to the pictorial to the abstract.
4. Practice to fluency.
5. Practice for speed.

About Free Exploration, or How Not to Be a Nag

by Alysia Krafel

I recall an incident years ago when my six-year-old nephew was learning to set the table. Dinner was almost ready and there he was popping his fists down onto fork tines to flip them into the air. The stage was set for an unpleasant confrontation. I see now that the problem is one of time frames. We needed him to get the job done; he needed to explore. When we press a child to use a tool seriously and with skill before we have provided fiddle time, we set him up to be seen as a behavior problem and ourselves to be seen as nags. If a child can do his necessary playing and exploring with materials when he is not under pressure or expectations, before he needs to use them as we wish him to use them, not only will he learn the new skill faster and with more understanding, he will learn it with more delight (or at least with less complaining). The first association will be of joy and competence rather than frustration and forced anxiety.

Learning to use math manipulatives is like learning to use other tools. An adult will look at a screwdriver and use it to drive screws. A child will roll it on the floor, listening intently as the grooves in the plastic handle thump. She will roll it over her tongue, poke it into cracks, bang it on the table, spin it, or balance it on her finger. When finally satisfied with the investigation, she will watch you screw in the screws and then try it herself. And so it is with all of our doodads.

Play, unhindered exploration, is very important to children. At the Farm School, we came to realize that this self-directed learning style was not an impediment but a boon to our aims. We found that children who had enough time to fiddle with the plexiglass tiles came to their first fractions lessons knowing that the smaller the pieces there were, the more of them they had; that you could cover the black ($^1/_2$) tile[1] with the yellow ($^1/_4$) tile or orange ($^1/_6$) pieces, but you couldn't with the clear ones ($^1/_3$). When they began to

work with the formal fractions lessons, they brought into action all that their senses already knew about these tiles. As soon as the labels "halves" and "fourths" were attached to the colored tiles, they knew that $^1/_2 = {}^2/_4$. They found the patterns very quickly and mastered the materials faster than the previous children who had not been allowed fiddle time.

The problem the teachers at the Farm School had encountered was that we wanted to teach x amount of material in a certain time frame. When the children needed three weeks of fiddle time with the fraction tiles, we experienced that as a delay, a delay that made us nervous. We just had to teach and have the children master fractions by the end of the school term. The solution was to introduce the materials long, long before (in many cases years before) we intended to use them as teaching tools. After ample free exploration time, not only did the children not resist the use of the materials in the specific way that the formal lessons demanded, they eagerly attended to the new way to interact with the now familiar materials. The new work was for them a continuation of the old play. The experience did not feel all that different to them. Since the struggle between teacher and child had been eased, the adversarial relationship became a co-worker relationship. The instruction then proceeded easily. The children and the teachers learned together to truly understand the world of numbers and enjoyed doing it.

My life suddenly changed when I had a baby. When giving her a bath one day, I had the idea to give her as a toy one thing she would later use as a tool, a cup. In the tub, she could pour, drink (or sputter), spill, bang, and splash to her heart's content without undoing whatever housework managed to get done. She would talk into the cup and put it on her head. Doing these things at the table with a cup filled with milk would have caused an uproar just as fiddling with colored tiles during a fractions lesson did. She had had the cup as a toy many months before we ever put milk into it for

[1] As used in *Fraction Tiles* by Lee Jenkins and Peggy McLean

her to drink at the table. Interestingly enough, when given milk in the cup at the table, she reacted in the same way as the children in the fractions lesson did. Since pouring liquid from the cup was old hat and drinking milk from it like Mommy and Daddy was new, she tried very hard to do it right because that's what she was ready to do, that was the new fun. She wouldn't pour milk out at the table and say "oh no" when she spilled it. Later, when she showed an interest, spoons and bowls were added to the toy shelf and to the bath set. While in the tub recently, she filled her bowl with water, bent down, and began to blow bubbles into it. She dumped out the water and put the bowl on her head and said, "Hat." I thought to myself, "I'm glad that was warm water on her naked body instead of hot tomato soup on her white sweater!"
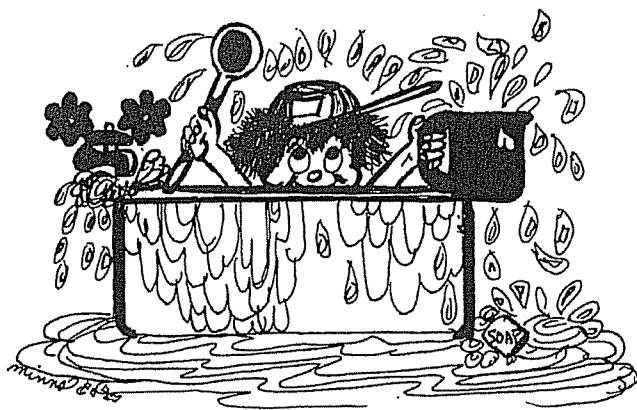
An unintended side effect of all of this was that she became competent with a cup at fourteen months and with a spoon and bowl by sixteen.

Having done all of that, I was surprised recently to find myself remaking that same mistake of not allowing exploration as a parent as I had as a beginning teacher. When my daughter put eggs and Cheerios in her cup of milk, I said, "No put food in cup! No put eggs in milk!" I was expecting her to be like me, to use tools as tools, food as food, instead of tools and food as toys. Resisting my constant scolding, she persisted in this behavior. Obviously, she needed to explore how things, like scrambled eggs and Cherrios, would interact with milk in her cup. My choice

was either to press her to "act properly at the table," triggering tears and tantrums, or to allow her to explore with the knowledge (or at least great hope) that when she was satisfied, she would stop. This is always a difficult decision for me to make. One doesn't want to be so permissive as to develop a totally undisciplined child. On the other hand, one doesn't want to be a repressive nag either. Since food couldn't be a toy anywhere else but at the table, I gave in on unconventional mixing, squashing food through fingers, and food painting but drew the line on throwing. So with a certain amount of plaintive sighing, I watched her put my lovingly prepared, warm eggs into her milk, got out the dish towels, and added them to her toy box. To my delight, she ate the cold, milky eggs out of her cup with the spoon without as much mess as one would expect! My husband realized that the current fascination was food and liquid together. Now we put the liquid in the bowl with the food to begin with, and everyone is happy. She certainly is not like me.

I wonder how many things children must learn, like setting the table, feeding the cats, making the beds and so on, would move more easily if we parents allowed the child to play with the new materials long before we or society demanded that they use them properly and with skill.

Farm School Math Notes by Michael Butler

A. Some things students and teacher should be doing routinely:

1. *Inventing problems*: For practice and self-teaching. For other people, including the teacher. As puzzles; to give and get pleasure. To show understanding of a principle. To link math to the world. To stretch creative power. To develop mathematical taste.

2. *Finding patterns:* In numbers, shapes, etc. Find the rule; 'black box.' Math or formal patterns in worldly stuff--motion, people, plants, cars, stars. 'Getting' a puzzle.

3. *Giving plausible answers and why:* Estimation/prediction/plausible guessing. Error bounds. Recognizing that some estimates are better than others. Iteration to improve estimates. Quickly inventing problems whose answers are 'about' such-and-such--e.g., between m and n. Believing that answers ought to make sense.

4. *Learning about and using heuristics:* Make a picture. Bookkeeping and, in general, 'cases.' Reduction to absurdity. If then. Plausible reasoning. Find a solution for simpler numbers, for a simpler case. Get *some* result; change it. What's a problem like this? What have I got so far?

5. *Learning about and using kinds of relations:* The more the more, the more the much more, etc. Plots; worldly cases. 'Variable' thinking. 'Control' -- 'other things being equal.' Refining explanations ['true for boys but not girls'] in terms of variables.

6. *Exploring the big ideas:* Scale. Symmetry. Congruence. Ratio. Equivalence. Measurement. Teacher alertness to these in anything.

B. Meta-skills for good mathematicians and students of mathematics:

--*the 'understanding' loop*: They know when they understand and when they don't, and they keep at it until they do.

--*the 'conjecture and test' loop:* They sense when something can be generalized and press to the limit of their ability to do so.

That children can do these things is one big test of the teacher's success.

C. The other is:
Children *like* working with numbers, 'thinking stuff' --as shown by their spontaneously choosing to do math and thinking stuff, visibly enjoying it, being absorbed in it, and not saying, "I don't want to, this is icky," etc.

They can do it.

They do do it.

Farm School Math Notes: Routine Questions, Challenges, and Responses

1. *Inventing problems:*   Make your own.
   Invent a problem like this for me, for your friend, and for a younger child.
   Find a story in your life that is like this problem.

2. *Finding patterns:*   What patterns do you see?
   What will happen next?
   What do you notice about_____? (plants, days, people, cars, playing, the sky)
   What is the same?
   What is different?

3. *Estimating:*   Guess the answer.
   What do you predict will happen?
   The answer will be about how big?
   What is an impossible answer to this?
   In what ball park will the answer be?

4. *Heuristics:*
   (Methods of discovering answers.)
      How can you show that?
   Draw a picture of it.
   How did you get that answer?
   What part of it can you do?
   What problems like this are easier (or harder)?
   Solve it another way.
   Build it.
   What gave you the clue?

5. *Relationships:*   Is there a relationship here? For example: The more the words, the more the time it takes to read the writing.
   The smaller the number, the fewer the combinations.
   Is the relationship true for everything?
   What part of this is in this?

6. *Big Ideas:*   How can we measure it? Is that design symmetrical?
   Can this pattern be repeated bigger?

   --the 'understanding' loop: Build that answer.
   Prove that answer.
   How do you know it is right?
   What gives you the clue?
   Challenge both correct and incorrect answers.
   Create more problems using your methods.

   --the 'conjecture and test' loop: What other problems work like this?
   When won't it work?

Teaching Techniques

Expect math to make sense. Look for sources of enjoyment.

Treat yourself and student as mathematicians. Mathematicians make up problems for themselves and others. Students who do this increase their control over the learning process, reinforce and show understanding, and strengthen their image of themselves as mathematicians.

Use two or more manipulatives for the concrete stages of concept development. It will increase the student's flexibility in thinking. Make many copies of the blank practice pages so you can gradually increase the difficulty of the problems and still have extra for periodical review. Cover pages with contact paper or put them in clear sheet protectors to make them reusable.

Begin each lesson with a warm-up and review. Always end the lesson with a success before the student is tired. It is best to end while the student is still enjoying the lesson.

Ask questions such as: **"Are you sure?"** or **"Build it."** or **"What gave you the clue?"** or **"Show me how you got that."** when a student is correct. This is important to do often. A student needs to rely on the self for knowledge ("Am I right?"). Confidence in knowing if a student is right or wrong, or at least close, must come from within the student, not from an external source. By asking the student if he or she is right even when right will make the student be confident and learn how to self-check.

Be sure the student is the one to solve the problem. Avoid giving too much help or direction. Time constraints during a lesson or impatience can lead a teacher to push too hard and show the student how to solve a problem the student could have solved without interference.

Explore other ways of solving a problem. Ask who has another idea over and over again. Mathematicians test each idea several ways to see if it will work or not. Encourage second and third thoughts.

Listen to each student explain how he or she solved a problem. It reinforces auditory learners and lets all students solidify concepts. Persist, talk, and think about what the student has done until you understand it.

Give some unsolvable problems (e.g., break up 4 six different ways). Allow a student to realize no solution is a possibility, a limit, and important information. It stretches the student's mind and teaches what failure is not.

Challenge a student who appears bored. The challenge could be working with higher numbers, recording answers for the group, or making up problems for all to solve.

Math Journal

It is interesting to keep a math journal of work and discoveries. Have students get in the habit of articulating and recording their thoughts, equations, and methods for solving problems. Instructions are on page 11.

Arithmetic Conventions

It is essential that learners build models of problems, draw pictures to prove their own answers, show understanding, and communicate their ideas to others. This program adopts the following conventions. The first number of an equation tells the amount or number of blocks to start with. The second number is seen as part of the operator, with the sign, and tells what to do next. The order of the words and the action of the hands correspond. Each operation—addition, subtraction, multiplication, and division—uses the same convention throughout the entire *Patterns in Arithmetic* series, Books 1 and 2 and Booklets.
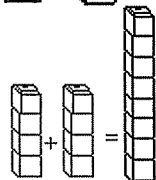
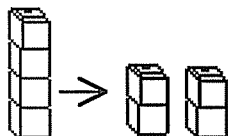$4 + 2 = ?$ means start with four and add two more.
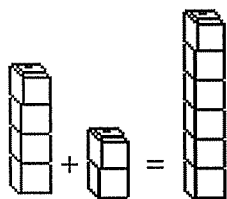
$4 - 2 = ?$ means start with four and take away two.
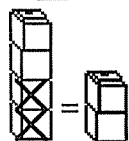
$4 \times 2 = ?$ means start with four and add it two times ($4 + 4 = 8$).

$4 \div 2 = ?$ means start with four and divide it into groups of two (or make two groups).

Make Your Own

Make your own problems should be done routinely. The student acting as a mathematician invents problems, which strengthens her control over the learning process. This reveals to the teacher her understanding of the concepts, how her mind works, and her current level of competency and confidence. It is a creative act for an empowered learner, and through its use powerful habits of mind will grow.

Time Span

Time span is a general guide for how long a lesson may take or the length of a practice phase. Since each individual will vary in her pace based upon age and experience, do not worry about externally suggested time frames or compare one student to another. The time spans given are for an average student at the third-grade level (about eight years old). Adults who are teaching themselves will pace themselves by their understanding, ease, and fluency with each concept.

At the Farm School students invented unique, successful ways to solve problems and discovered that although the standard regrouping algorithm has particular advantages in a base ten system, it is only one way among many. This invention laid a foundation for understanding *all* arithmetic algorithms (addition or subtraction or multiplication or division) because they are all built on the same model, a process with three parts: decomposing a large problem into many small problems, solving each small problem, and adding the results. When students understand this, each new arithmetic procedure has a familiar feeling, because it is seen to be part of the same family of algorithms.

The idea of breaking up difficult problems into smaller ones and then solving the little problems first can be applied to all of life. For example, a difficult piece of music is broken up into smaller parts to be practiced separately, then played all together later. And specifically to *Patterns in Arithmetic* a large third-, fourth-, and fifth-grade set of arithmetic lessons is broken up in a different way: by operations and by fractions and then recombined in booklet form for learners of all ages to do in their own chosen order.

The philosophy of the Farm School is based on the insights, ideas, and teachings of Professor Michael Butler. Over the years the work of many teachers has resulted in a mathematics program that aims to teach students to think like mathematicians.

"Anything children learn from adults can be learned more readily from another child." Welcome to this learning adventure.

Take Delight

The following are some words of wisdom from Alysia Krafel, who helped develop many of the math materials contained in the lessons:

>Take genuine delight in the learning of your student. This is your most powerful tool. There is much that is delightful in good thinking, a new solution, or an insightful comment. There is much danger for us adults when we come to think that we have seen it all, particularly when working on seemingly simple, "childish" problems. Often we are so intent on getting the student to see things our way that we miss the new perspective uttered by the student seeing the world for the first time. Take delight in those new perspectives–they are a gift to you. Often these perspectives will be simple and obvious yet profound in their truth. Be careful that you don't cast them aside with a "but of course."
>
>Be aware enough to separate a good thought path from the correctness of an answer. A student [in the process of] explaining will often find a good path but follow it poorly. The original idea may be a good one even if it does not produce the correct solution. Instead of saying "No, that is not right," honor the attempt. "That was a good idea; I understand what you think even though it doesn't work here. It was a good try." Or ask questions that will test the hypothesis in other ways. "Are there other cases where this will work? Can you think of problems where it won't work?"
>
>Be aware also that a student in a comfortable learning situation rarely says nonsensical things. Even an answer that may seem absurd to you makes sense to the student. Try to understand the model the student is using, the model that will make sense of what the student says. When you listen carefully to what the student is struggling to articulate, you show that you value his thoughts by your behavior. Your act of valuing and delighting in your student's thoughts will go many miles further than any reprimand you could give. You will, by your praise, cultivate a desire to search and test, and this will strengthen him against the frustration of error. The student will learn that thinking is fun.
>
>If delight is to be your greatest asset, then impatience will be your greatest liability. Impatient teaching is humiliating to the learner. One aspect of impatience is expecting students to give snap answers to questions. The better the question the more time a student will need to mull it over. You must allow the student time to think. Silent times in my teaching have been as long as three to five minutes. Give students time to think.

A question often asked by teachers is, "How do I know what to do next, and in what order do things need to be done?" Use the Assessment Guides and the Sequence Guides to determine the next step in the learning process. The guides are for sequencing. They are not a time schedule. The Tests for Understanding help teachers determine when students are ready for the next step. And ultimately let self-motivated students of all ages know what they understand and their next moves.

Math Journal

Materials A blank lined-paper journal–this can be a hardbound composition book, a colored re-port folder with lined paper in it, or a loose-leaf folder in which additional pages can be added as needed. Use wide-lined paper for this age group.

Post-it® Notes for Tabs–these are optional but very helpful to mark journal sections such as thoughts, discoveries, vocabulary glossary, algorithms, and formulas for easy reference.

Computer/Internet (optional, but wonderful)–amathsdictionaryforkids.com. This site has a complete, illustrated, interactive glossary of mathematical terms.

Set Up Set up the Math Journal. Number the first fifty pages. Set aside the first three pages for a table of contents. Label the opening page as the Table of Contents. The first entry will be Vocabulary Glossary pages 4 - 30.
On page 4 label the top of the page Vocabulary Glossary A for all of the vocabulary words starting with the letter A. Label subsequent pages Glossary B on page 5, Glos-sary C on page 6, and so on to Z.

Place a labeled tab on the first page of the Glossary.

Students record thoughts, methods of solving problems, definitions, vocabulary, and formulas they discover. Many lessons ask the student to record a short answer or an essay question in the journal. It is useful for the student to write a first draft of an an-swer in the workbook. Then discuss it and refine the explanation using the glossary to substitute mathematical vocabulary for vague wording. Then have her copy the edited version of her answer into the Math Journal.

Although pictures will be useful to illustrate many concepts, discourage doodling in the Math Journal.

An example of a journal entry:
Area: The space inside the boundaries of a flat figure.
Area of a rectangle: The formula is length x width or (l x w = A).

Grading, Dialogues, and Warning Note on Online Resources

In my practice as a teacher, I, Alysia, rarely had to give a letter grade to a student for work completed. The work was always examined and noted for student control of the topic. If a mistake was made, the student was asked to try to find where the mistake was located, and what caused the error and to correct it.

Construction of understanding and explaining one's thinking require that the individual take a personal psychological risk. Grading is by its nature judgmental and can create an unacceptable level of risk that will retard the ability of a student to succeed in construction and increase the chance that she will plead to be given the formula for a procedure she knows she does not understand.

Sometimes giving the formula to a student is appropriate. A few students will be cognitively unable to construct understanding. For them, accuracy and retention are important. Oftentimes, this choice can be made, and then later a second attempt at understanding can be made working backwards from knowing the answer to finding the sense of it.

If grading is required, use the Assessments for your personal measurement of the level of each student's understanding. If a student is asked to identify and correct errors, she is likely to spot her own carelessness or ask for a reteach so she can do the corrections. The grade can be based on both the effort made, to do this and the correct result. Sometimes the Tests for Understanding will reveal a developmental block indicating that the student is not ready to learn this concept. The student should be given an incomplete, not an F, for this situation and given more instruction if possible.

Dialogues Note

All teacher dialogues are in bold type and are in quotes. All student dialogues are in plain type and are in quotes. Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **"How did you get that?" "Show me what you mean." "Build a model of that." "Tell me more so I can understand what you are saying."**

Warning Note on Online Resources

There are thousands of sites online that provide worksheets, lessons, and games for kids to learn math. Take care when you use 'free' sites. Many of them are free because they are installing cookies in your computer to collect data on your child's computer use for marketing purposes. This data is used for pop-up ads and can be sold to third parties. If you do not want this collection to occur, avoid free game sites especially. If your child is using free applications on smartphones and the like, the GPS data on the physical location of the user is being collected by third parties. The sites that require subscription are much less likely to monetize your data in this way. Use online resources with care. There is a mountain of good stuff online, but you must be picky and read the privacy notices. If there are no privacy notices, that should act as a red flag.

Sequence Guides

The following sequence presents a model from which a teacher can get ideas for designing a plan. Use the Assessment in this booklet and in other booklets by subject to precisely plan a student's program. Plan a week at a time and revise as needed.

This is a proposed sequence, not a schedule. Each student will move at his or her own pace, taking a year, more or less. In addition, adults who want to understand arithmetic can try a strand and proceed independently and quickly.

It is important to take the time to explore new things and current interests of students instead of doing all of the planned lessons. Learning should be spontaneous, and time must be allowed for topics that are beyond the scope of *Patterns in Arithmetic*, which focuses primarily on arithmetic. These subjects are introduced but need further study: geometry, measurement, logic, and attribute studies. Lessons not covered in this book include decimals, time, problem solving, probability, and money. Recommended sources are **double starred in the sequence chart and are listed in Resources.

These booklets for Grade 3 assume the student has the skills generally learned in second grade at age seven. The Assessments place students into the math sequence by skill, not by age.

"Use your professional judgment and trust your intuition. If you feel you need to move more quickly, do so. If you feel you need to move more slowly, do that. *Always remember*: our focus is the children. Our goal is to meet those children's needs. The focus is *never* on "getting through" these lessons or keeping to the sequence in spite of children's needs."
Mary Baratta-Lorton, *Mathematics Their Way*

Lessons progress developmentally from left to right according to the order in which they should be taught. Most lessons depend upon concepts and skills learned in earlier lessons. The exception is the General Math booklets, where subjects can be interchanged.

Year-round schools, homeschoolers, and adults will expand or contract the schedule as needed.

Begin year

Grade	Months ——>		
Subjects ↓	Title and ——> location of lesson and/or		
	worksheet		

Abbreviations	AWV = Answers Will Vary	Add = Addition	Meas = Measurement
	BUWV = Break Ups Will Vary	Sub = Subtraction	Multi = Multiplication
	Div = Division	Fract = Fractions	PV = Place Value
	Geo = Geometry	Manip = Manipulative	TG = Trading Game

Grade 3	August	September	October
Place Value General Math	*Trading Game, TG: Review,* PV: Bklt 2 *Number Patterns* GM: Bklt 3 **Pattern Factory	*PV Strips, Counting Cuckoos, Manip Two Digits, TG: Logic Puzzle,* PV: Bklt 2 *Coordinate Graphing, What's My Rule?, Number Lines,* GM: Bklt 3	*TG: Record, PV: 3 Digits, 3 Digit Record,* PV: Bklt 2 §*Difference Between, Sums and Differences, PV to One Hundred and One Thousand, Trading Game,* GM: Bklt 3
Addition	*Assessment* - in this Booklet *Speed Test* Add: Bklt 3	**CM Add: Review, CM Add, CM Add: Missing Numbers, CM Add: 3 Digits, Gus the Bus Driver,* Add: Bklt 3	*Expanded Addition, CM Add: Four Digits, Families of Facts* Add: Bklt 3
Subtraction	§*Regrouping Intro and Philosophy, Pre-Assessment: Part 1 Speed Test* Sub: Bklt 3	**Circle Math: Review, Fixing Problems, Difference Between* Sub: Bklt 3	*Difference Between: Patterns, Gus the Bus Driver* Sub: Bklt 3
Multiplication	*Pre-Assessment: Part 1* *Multiplication Concept, Number Sentence: Review* Multi: Bklt 1	*Party Word Problems, Rainbow Times, Trains on Tracks,* Multi: Bklt 1	*Number Lines, Attribute Grids, Building Word Problems, Multiplication Facts Chart, Building Times Tables* Multi: Bklt 1
Division			
Fractions	*Pre-Assessment: Part 1 In the Kitchen* Fract: Bklt 1	*Free Exploration, MFB: Beginning,* Fraction Circles - make designs *What Fraction is This?,* Fract: Bklt 1	*Fractions Guessing Game, Numerators Greater Than One, Shrinking Circles, Whole Numbers as Fractions,* Fract: Bklt 1
Probability and Statistics	**Used Numbers, make frequency graphs on pets, favorite foods etc.	**Used Numbers, make frequency graphs on shoe sizes, height, number of people in the family, etc.	Continue **Used Numbers
Geometry	Free explore with Pattern Blocks and Stamps *Rectangle City, Patterns: Geometry* GM: Bklt 3	*Lines, Line Segments, Rays, and Angles, Geometric Solids,* GM: Bklt 3 *Names of Pattern Block shapes. Make shape books.	Art Projects using line types *Measurement: Nonstandard, Area: Nonstandard* GM: Bklt 3

*= review, Italic = lesson title, ** = outside resource, Bklt = Booklet, CM = Circle Math Addition, § = in Teacher Guide only, MFB = My Fractions Book, TG = Trading Game

Grade 3	November	December	January
Place Value General Math	*PV to 100 and 1,000, TG: Number Values*, PV: Bklt 2 *Trading Game, Place Value: Challenge, Number Patterns: Missing Numbers* GM: Bklt 3	*Abacus Sub Game, PV: Sub - Base 5*, PV: Bklt 2 *Rounding Off: Tens, Rounding Off: Hundreds, Greater Than, Less Than, or Equal To, Using Parentheses*, GM: Bklt 3	*PV: 6 Digits, What's My Rule?, Rounding: 10s*, PV: Bklt 2, *Three Ways to Record Base Ten*, More Number Lines, Negative numbers on number line
Addition	*Missing Numbers, Patterns in Sums* Add: Bklt 3	*CM: Equality* Add: Bklt 3	*Practice, Speed Tests* Add: Bklt 3
Subtraction	*Discovering Easier Ways, Gus the Bus Driver: Bus Stop Problems* Sub: Bklt 3	*Tens and Ones Manipulative and Practice* Sub: Bklt 3 or Base Ten Subtraction	*Nonstandard Regrouping, Nonstandard Regrouping and IOUs*, Sub: Bklt: 3 or Base Ten Subtraction - Regrouping
Multiplication	*Continue building times tables—one per week, Daily Skip Counts, Merry Mix Up Drill, Patterns in Tables*, Multi: Bklt 1	*Daily Skip Count, weekly Merry Mix Up Drill, Patterns in Tables* Multi: Bklt 1	*Daily Skip Count, weekly Merry Mix Up Drill, Patterns in Tables* Multi: Bklt 1
Division		*Pre-Assessment: Part 1 §Concept* Div: Bklt 1	*Dream Garden Drawing, Garden Word Problems, Number Sentence* Div: Bklt 1
Fractions	*Fraction Card Game, Greater Than and Less Than, Recording Fractions, Sorting Fractions, Spelling Words*, Fract: Bklt 1	*Pattern Block Games, MFB: Pattern Blocks, Pattern Block Fractions, Changing Wholes, Rulers as Number Lines*, Fract: Bklt 1	*Pattern Block Designs, MFB: Number Lines, Whole Numbers as Fractions: Number Lines, Numerators, Greater Than 1*, Fract: Bklt 1
Probability and Statistics	Tally charts, coin flips, colored cube draws with frequency graphs, etc. in **Used Numbers	Tally charts in **Used Numbers	Vocabulary – certainly, probably, likely, unlikely, improbable, impossible
Geometry	Symmetry – Art **Pattern Block Activities Book A Read clocks and calendars	*Measurement: Standard - Volume*, GM: Bklt 3 Capacity of cups, pints, quarts, gallons	*Area: Standard, Geoboard Area, Tile Floors*, GM: Bklt 3

*= review, Italic = lesson title, ** = outside resource, Bklt = Booklet, CM = Circle Math Addition, § = in Teacher Guide only, MFB = My Fractions Book

Grade 3	February	March	April/May
Place Value General Math	*Rounding Off Large Numbers*, PV: Bklt 2 *Place Value*, GM: Bklt 3 Build a model of 10,000 **Hands On Attribute Blocks	*Estimating Addition, Speed Tests, CM: Review, Practice*, PV: Bklt 2 *Reflections, Categories and Continuums* GM: Bklt 3	*Patterns in Sums: Column Add, Gus the Bus Driver, Assessment*, PV: Bklt 2 *Averages, §Patterns in a Counting Chart* GM: Bklt 3
Addition	*Practice* Add: Bklt 3	*Picture My Learning* Add: Bklt 3	Make up mixed addition and subtraction word problems, Use *Practice - Worksheet 2* as a Post-Assessment in Add: Bklt 3
Subtraction	*Gus the Bus Driver: Differences, Standard Regrouping: Manipulative*, Sub: Bklt 3, or Base Ten Subtraction	*Standard Regrouping: CM, Standard Regrouping: Short Way, Gus the Bus, Families of Facts*, Sub: Bklt 3 or Base Ten Subtraction	*100s, 10s and 1s: Manipulative, 3 Digit Regrouping: Manipulative, 3 Digit Regrouping: Recording Practice, Riddle*, Sub: Bklt 3 or Base Ten Sub
Multiplication	*Daily Skip Counts, Merry Mix Up Drill, Patterns in Tables 6-9s, Properties of Mult., §Tiles, §Commutative Property, Associative Property*, Multi: Bklt 1	*Uneven Stacks, Factoring: Manipulative and Recording, Daily Skip Count, weekly Merry Mix Up Drill*, Multi: Bklt 1 Continue word problems	*Multi by Addition, Picture My Learning, Post-Assessment*, Multi: Bklt 1, Practice multiplication and division word problems
Division	*Monkeys on the Roof, Word Problems: Two Steps, Recording*, Div: Bklt 1	*Missing Factors, Remainders, Families of Facts: Patterns and Practice*, Div: Bklt 1	*Word Problems: Two Formats, Families of Facts: Connections, Post-Assessment*, Div: Bklt 1
Fractions	*MFB: Equivalence, MFB: Equivalence - Number Lines, Equivalence: Manipulative*, Fract: Bklt 1	*Equivalence: Recording Word Problems, §Fractional Parts of Sets* Fract: Bklt 1	*Post-Assessment* Fract: Bklt 1
Probability and Statistics	**GEMS - Frog Math **GEMS - In All Probability **Used Numbers	Spinner Games–Single Outcome Probability in **GEMS - In All Probability	**GEMS–Math on the Menu
Geometry	Time: Things I do in a day, a week, a month, a year – make an illustrated little book. When is a second a long time?	*§Perimeters of Rectangles*, Measure tables and books with string using centimeter and inch rulers.	*§Measurement: Standard* GM: Bklt 3

*= review, Italic = lesson title, ** = <u>outside resource</u>, Bklt = Booklet, CM Add = Circle Math Addition, § = in Teacher Guide only, MFB = My Fractions Book, CM = Circle Math

Lesson Planning

The Sequence Guides provide a math skills model to assist in planning lessons. Adapt and supplement it for your own situation. Since the student is the focus, speed up or slow down according to her pace. Schedule a weekly lesson planning in a quiet place for half an hour or more. Make a copy of the following page for each week.

Choose topics from each subject in the Sequence Guides chart (e.g., General Math, Addition, Subtraction, etc.). The lessons progress developmentally from left to right from August to May. Specifically, use the student's results from the Assessment to indicate where to begin instruction. Finally, check for prerequisites in the lessons to make sure the student has the skills and/or previous experience to easily succeed in a particular lesson. Then use the student's "Make your own" problems as a guide for what to do next, e.g., reteach the lesson, review with manipulatives, give harder problems, give easier problems, go to the next lesson, and/or practice fluency. Use blank practice pages to create exactly what the student needs.

If a student is bored, it often means she is not being challenged. Remedy this by giving problems with larger numbers, more intricate patterns, missing numbers, and by having her make up her own challenging problems including a hard problem she is not yet able to do.

The weekly lesson plan is based on the model that every concept is taught in a consistent pattern: initially learning from the concrete (e.g., body and blocks), through the pictorial (e.g., a drawing), and on to the abstract (e.g., number sentences), and finally practicing fluency and speed.

Weekly Lesson Plan (a sample for the beginning of the year)
Discuss and practice how to care for the manipulatives and establish a cleanup routine.

Exploration (concrete/manipulative)
1. Pattern blocks
2. Cuisenaire Rods

Guided Lessons (oral/manipulative)
1. GM - What's My Rule? P/TG
   Number Lines P/TG
2. Add - Circle Math: Review P/TG SW
3. Multi - Multiplication Concepts, Number Sentence P/TG
4. GM - Patterns PT/G, SW

Supervised Practice (manipulative to representational)
1. GM - What's My Rule? SW
2. Add - Circle Math: Review SW
3. Multi - Multiplication Concept SW
4. GM - Review Trading Game P/TG (play - don't record)

Independent Practice (abstract/symbolic)
1. Sub - Practice subtraction facts - appropriate size problems
2. GM - Number Patterns SW
3. Add - Practice addition facts - appropriate size problems
4. GM - Play Trading Game
5. Take out and return manipulatives properly

Daily list (for student) Pages are in SW.
1. Play with blocks
   GM - Patterns: Geometry
   GM - What's My Rule?
2. Free exploration with blocks
   Multi - Number Sentence
   Sub - Speed Test
   Add - Circle Math practice (copy blank pages and make appropriate size problems) P/TG
3. GM - Build patterns with blocks
   GM - Rectangle City
   Add - Practice basic addition facts
   _____
4. GM - Play Trading Game
   Multi - Number Sentence
   GM - Make a number line from 0 - 25
   _____
5. Play with different manipulatives
   Sub - Practice basic subtraction facts
   GM - Lines, Line Segments, Rays, and Angles
   _____

Lessons are from Student Workbooks and
Parent/Teachers Guides
General Math: Booklet 3
Multiplication: Booklet 1
Addition: Booklet 3
Subtraction: Booklet 3

Abbreviations
SW = Student Workbook   P/TG = Parent/Teacher Guide
Add = Addition   Sub = Subtraction
GM = General Math   Multi = Multiplication

Weekly Lesson Plan (for teacher)

Exploration (concrete/manipulative)

1. _____

2. _____

3. _____

Guided Lessons (oral /manipulative)

1. GM _____

2. Add _____

3. Sub _____

4. Multi _____

5. Div _____

6. Fract _____

Supervised Practice (manipulative to representational)

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

Independent Practice (abstract/symbolic)

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

Daily list (for student)

1. _____

_____

_____

_____

2. _____

_____

_____

_____

3. _____

_____

_____

_____

4. _____

_____

_____

5. _____

_____

Materials

You will need the following tools to complete this program. This supply list is for one to two students. Classes will need to upscale appropriately.

General Supplies
- No. 2 pencils (Ticonderogas are the best), pink erasers, colored pencils, crayons, sidewalk chalk, masking tape, cellophane tape, scissors, 30 centimeter ruler, 100 centimeter ruler, a standard ruler, and a yardstick. A twenty-five-foot tape measure is helpful.
- Colored acetates or cellophane in green, blue, and yellow (Can use report covers, which are sold in most office supply stores. These are nice and rigid.) Cut into 1 cm by 10 cm strips.
- Grid paper can be downloaded from the Internet or ordered from a distributor.

Optional
- Small write-on/wipe-off white boards. Use masking tape or permanent markers to make the workboard circles and squares. Wipe-off markers, board cleaner, old cotton or wool socks to erase the boards. You can also use laminated workboards with the overhead markers.
  Note: The markers used on white boards and acetates are different.
- Cafeteria trays or large cookie sheets are useful to contain manipulatives.

Manipulatives
- 20 one-inch tiles: These can be ordered or made from stiff poster board and an X-Acto knife.
- Dice: a set of 4 green and 4 red standard dice
- Lima beans and pinto beans are good, inexpensive, all-purpose counters.
- Split peas (for General Math: Booklets 3 and 4)
- Base Ten Blocks: 5 one thousands cubes, 20 one hundreds flats, 30 ten rods, 100 unit cubes, 30 centimeter chips (These can be made with stiff poster board.)
- Unifix Cubes: 1 set of 100
- Cuisenaire Rods: 2 to 3 trays You need ten rods of each color.
- Pattern blocks: 1 bucket
- Pattern blocks: pink double hexagon, black chevron set
  Pattern block directions for pink or black blocks:
  Pink: Glue two yellow hexagons together to make this shape. Paint pink if you wish.
  Black: Glue two blue rhombi together to make this shape. Paint black if you wish.
  Or make them from sturdy paper.
- Prism Fractions circles: 1 set (available from Pattern Press)
  Prism Fractions square and circle fraction manipulative paper sets are supplied with the *Patterns in Arithmetic* fraction series booklets (available from Pattern Press).
- A lined composition book for each student or a three-ring binder with 50 sheets of lined paper for the Math Journal
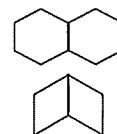
Resources

Mathematics Their Way – Addison Wesley Used for kindergarten and first grade. In first grade, *Mathematics Their Way* is combined with *Patterns in Arithmetic*.

Pattern Factory for all ages–Uses manipulatives to generate patterns that can be figured out, graphed, and turned into functions.

<u>Gems</u> Units: Available from Lawrence Hall of Science in Berkeley, California, and many other distributors. Most cost $15 to $23. They also have great science units.
1. *Frog Math*: Grades 1 and 2 Introduces probability and attributes

2. *Secret Formulas*: Grades 2 to 4–Teaches measurement of various ingredients to make a salad dressing.

3. *Math on the Menu*: Grades 3 to 5–Teaches strategies for determining all the possible combinations of a set of items, in this case, tacos This strategy is important for the understanding of probability.

4. *In All Probability*: Grades 3 to 5–Develops concepts of probability.

5. *Algebraic Reasoning*: Grades 3 to 5–Introduces Linear Graphing and Algebraic Functions

6. *Quadice*–Great for a family with students at different math levels for grades 4 - 8. It is a wonderful exploration of the probabilities of various combinations of four dice.

Family Math–Great resource for all ages for family math games.

Hands On Attribute Blocks

Key to Fractions: Book 1, grade 3, after completing *Patterns in Arithmetic*: Fractions - Booklet 1.

Key to Algebra: Book 1, grade 6, teaches operations with positive and negative numbers.

Key to Decimals: Books 1 to 4 and answer keys grades 4 and 5.

Pattern Block Activities, Books A and B

Problem Solver Series–Teaches eight problem-solving strategies with word problems of all kinds. Easy to use.

Measurement–the old *Measure Matters* by Creative Publications is wonderful. If you can find one of these sets, get them. Other hands-on materials for teaching measurement are widely available online or from multiple catalogs. One of the best ways to teach customary weights is in the grocery store and in the pharmacy for metric weights such as milligrams.

Used Numbers or other resource for teaching data and statistics
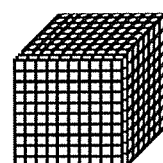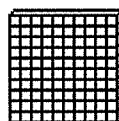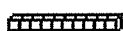
Base Ten Materials

Base Ten Counters

	ones	tens	hundreds	thousands	
Base Ten Blocks					wood, plastic, or paper
Cuisenaire Rods					
Bean Sticks					
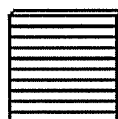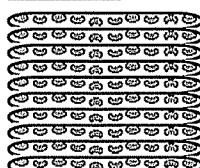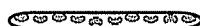
Cuisenaire Rods

Use the white (one) and the orange (ten) rods.

For one hundred use ten orange rods rubber banded or glued together. DAP™ Fun-Tak Reusable Adhesive can be used to temporarily hold rafts together.

Inexpensive paper cubes representing a thousand can be purchased from ETA Cuisenaire. ETA-Cuisenaire (800) 445-5985 www.etacuisenaire.com
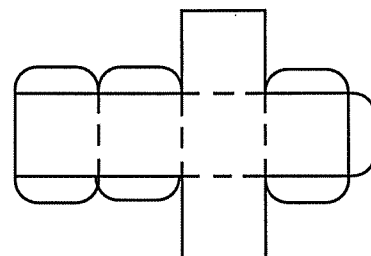
Instructions for making thousands cubes
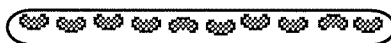### Materials
Poster board
Centimeter grid paper
Paper glue or paste

Cut out precisely six 10 x 10 centimeter squares from the grid paper. These should be the same size as the one hundred flat Base Ten Block. Arrange the six cut out squares on the poster board and paste them down so the grid paper will be on the outside. Leave room for tabs. Draw in tabs like the illustration. Cut out the figure including the tabs. Score along the dotted lines with a ruler and pen. Flip it over, and fold it into a cube, and glue the tabs to hold it together.
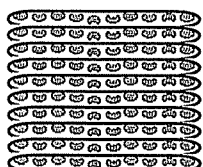
Base Ten Bean Sticks

An inexpensive alternative is to make Base Ten Bean Sticks.

Materials: small beans, Popsicle sticks, glue

Activity: Glue ten beans to a Popsicle stick. Make about twenty of these.

Rafts can be made to represent 100 with ten bean sticks and rubber bands or glue ten sticks to a cardboard square.

Assessment Guide

For students entering Grade 3

Work manipulatively with each student to observe his level of confidence and emotional relationship to math, as well as his skill level. Choose a problem in each operation and ask him to build the problem, solve it, and prove the answer. Have him make and solve an easy, medium, and very hard problem in addition and subtraction. Then repeat this process in multiplication and division if he is familiar with these concepts. **"You don't have to solve the hard ones if you can't; just make them up."**

Push to the limits of the student's ability. Each mistake provides valuable information. Ask the student what he is doing or thinking. Listen.

Give the written assessment to find out what the student knows and doesn't know and to identify strengths, areas of weakness, and confusion of the concepts. **"Do every problem on the worksheet that is possible for you to do now. Put a ? next to any problem you can't solve."**

Clarify directions but do not assist the student with any math problem. Do not teach or correct during the evaluation. Make notes about what the student does know and what needs to be learned in future lessons. Use this information to decide where to begin instruction. At the end of the assessment is the record sheet listing basic math knowledge required to succeed in *Patterns in Arithmetic* Booklets for Grade 3. Individual lessons list prerequisites.

Materials

Assessment Worksheets, pages 31 - 39
Assessment record sheet, page 40
Pencil
Base Ten Blocks
Colored blocks
Counters

Summary of Assessment Tools

General Concepts
- Place Value
- Associative Property of Addition
- Number Sentences
- Families of Facts
- Difference Between
- Expansion of Numbers
- Concept of Multiplication
- Fraction Notation
- Conservation of Number

Arithmetic Skills
- Addition
- Subtraction
- Multiplication
- Division

Speed Tests

Patterns

Assessment Tools

There are nine worksheets in five sections. The first section is on general concepts. The second section evaluates arithmetic skills. The third section reviews Circle Math. The fourth section is a speed test on basic addition and subtraction facts. The fifth section is on patterning. It is best not to give all five sections at once. One each day is suggested.

Make sure the student receives no assistance in completing any part of the worksheet other than clarification of directions. Tell the student to do every problem on the worksheet that is possible for him to do. Tell the student to put a '?' next to any problem that he can't solve. Do not give any lessons at this time.

Along with the written evaluation test, it is useful to work manipulatively with each student to observe his level of confidence and emotional relationship to math, as well as his skill level.

General Concepts

1. Place Value
This problem set tests the understanding of numeration. Can the student write the numerals that represent a given number? Pictures of ones, tens, and hundreds are shown. The student is asked to write the numerals that go with the pictures. Pictures a and b are warm-ups. Pictures c and e test the identification of the ones and the tens place. Pictures d, f, g, and i test the concept of place holders, using zeros to show no ones or no tens. Picture j tests the concept of grouping by tens. There are twelve ones shown next to two tens. There is an additional ten in the ones group and the student records 32, not 312 or 22. The "Write the number that comes after" (Problem k) tests place value symbolically: what number comes after a number that has a 9 in the ones place? The concept of place value is abstract and very important. Any uncertainty in this area can be remedied using blocks and recording. (See the Place Value sections in *Patterns in Arithmetic: Books 1 and 2.*)

2. Associative Property of Addition
Does changing the order of the numbers in an addition problem change the answer? If a student knows that it does not, then he would select the two addition problems that have the same numbers as the given problem without having to check his equality by actually adding them. If the student adds to find the answer, assume the concept is weak. The two problems that equal the given problem are 5 + 6 + 3 (the first box in the top row) and 3 + 5 + 6 (the second box in the bottom row).

3. Number Sentences
This section tests the ability to formulate a number sentence in response to a word problem. If a calculation error occurs, give the student another problem of the same type. If the student simply puts down the final answer, ask him to tell you how he got the answer to each problem. Then have him write down the problem (number sentence) that he did to find the answer. If he can't do that, review the lesson on number sentences for that operation. Addition and Subtraction Number Sentences are in *Patterns in Arithmetic: Book 1* and the Multiplication Number Sentence is in *Patterns in Arithmetic: Book 2.* If he doesn't know how to approach the problems at all, then focus on the word problems in the number sentences lessons.

Answers:

a. $3 + 5 + 4 = 12$ Nan has 12 balls.
b. $17 - 6 = 11$ Henry has $11 left.
c. $3 \times 7 = 21$ Toby collects 21 eggs each week.

4. Families of Facts

This problem tests for understanding of Families of Facts. Can the student produce the three related number sentences? If the student has trouble thinking of the subtraction problems, then you can conclude that the concept of reversal of operations, the main idea in the Families of Facts, needs to be reviewed. If he can't think of the other addition problem, review the Commutative Property of Addition. The three other number sentences are: $3 + 2 = 5$, $5 - 2 = 3$, and $5 - 3 = 2$.

5. Difference Between

This problem tests the understanding of the concept of difference between. Can the student say how much larger one number is than another, or how many numbers are in between two given numbers? Note whether the student counts or uses subtraction. Counting indicates a lower level of understanding. Give harder problems, with a greater distance between the numbers, to press the student to come up with a more efficient method of figuring. Answer: 24 is 9 larger than 15.

6. Expansion of Numbers

This problem set tests whether the student recognizes that there are many ways to build or expand (break up) a number. The student should not have any difficulty doing this problem set without a manipulative, other than fingers, for calculating. The 23 is broken up in three entirely different ways. Examples: $23 = 8 + 5 + 9 + 1$, or $23 = 10 + 10 + 3$, or $23 = 7 + 9 + 7$, and so on.

7. Concept of Multiplication

This problem set tests the student's ability to recognize a given multiplication problem from a pictorial representation of a physical model and to create a picture for a given problem. In the first picture there are six groups of two squares. This represents $2 \times 6 = 12$; the 2 is repeated 6 times. A student who is new to this math series may write 6×2. That is OK. Switching to the format used in this series is not difficult. Review Multiplication Concepts and Conventions in the Multiplication Section.

If the student is unable to do the problem at all, you will need to go back to the Concept of Multiplication and review it as well as the Multiplication Number Sentence. The second picture shows three groups of four, or $4 \times 3 = 12$. The picture of $3 \times 5 = 15$ should look similar to this:
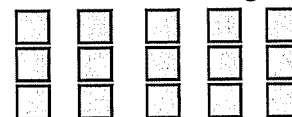
8. Fraction Notation

This problem set tests the concept of fractional notation (how one writes a fraction).
The 1/2 is shown by box 3 and by box 8. The 1/3 is shown by box 6. The 3/4 is shown by box 7. Does the student know that the bottom number (denominator) tells the number of pieces the whole is divided into? Choosing box 10 for 1/3 (because it has three pieces shaded) or box 4 for 1/2 indicates confusion on this point. Does the student know that each piece must be of equal size? Choosing box 2 for 1/2 or box 5 for 1/3 indicates confusion on this point. Does the student know the top number (numerator) tells how many pieces of a certain size should be shaded in? Choosing boxes 8 or 9 for 3/4 indicates confusion on this point. Can the student identify the 1/3 on the number line? In Fractions: Booklet 1 review Pattern Block Fractions, Recording Fractions: Prism Set, and My Fractions Booklet: Number Lines.

9. Conservation of Number (in the Circle Math section)

Does the student know that the way a problem is expanded (broken up) does not change the answer? As long as there are no errors in breaking up the numbers 26 and 19, the answers to the little subtraction problems (in the circles) will sum up to the same answer. This is because the 26 and the 19 do not change their values when they are broken up. As long as you are subtracting 19 from 26, the answer will remain 7, regardless of the method of calculation.

If the student is new to this program and does not know how to do Circle Math, give this test instead. Have the student count seven blocks into your hand. Close your hands over all the blocks, mix them up. Leave some blocks in one hand and some in the other one. Open your hands palms up and ask, **"How many now?"** Sometimes you have to ask, **"How many are in both hands?"** If the student:

A) Says the correct amount, mix the blocks up and put a different combination in each hand but use the same amount of blocks. Again ask how many blocks. If the student says the correct amount, add another block and repeat with a couple of different combinations. If the student says the correct amount, make it a higher number (but raise the amount only by two or three blocks). Mix the blocks up again and ask how many. Have him look and tell you how many blocks there are in total with an increasing amount of blocks. Record the highest number he does correctly or stop at about twenty blocks. The student understands conservation.

B) Stops and counts each block or guesses the wrong amount, have him count the blocks one by one. Mix them up and have him count again. Ask, **"How many blocks altogether?"** If he counts by touching each block and does not automatically say the total amount of blocks, it means he doesn't understand conservation. Try a lower number of blocks. Record the highest number he can do correctly. To remediate go back to *Patterns in Arithmetic: Book 2* and start with Breaking Up Numbers: Three Circles. The student is not ready for Grade 3 Booklets if he doesn't conserve numbers.

Arithmetic Skills

Unobtrusively observe the student doing the problems. Note his speed and whether or not he uses counting to find answers. If he does use counting, do not prohibit it. Simply note it and plan drill work on the basic facts to improve fluency. If he seems to be struggling or shows signs of frustration, remind him to skip problems that he feels are too difficult.

The problems are organized into seven sets with three problems in each set. The first problem in each set is an expanded problem. The second and third problems are to be done the short notation way. Look at problems D and F for example. In problem F, the student adds the 4 and the 8 to make 12. He writes a 2 in the ones place and carries the 1 to the second column and writes it above the 6. Then 1 is added to the 6 and the 2 to get 9. The 9 will be written next to the 2 to get the final answer of 92. Many students do not understand that the 1 that he carries to the second column is actually a ten. How the 1+6+2 turns into a 90 is somewhat of a mystery. He has practiced adding by rote without thinking about the true value of the numbers he is working with. Problem D will reveal this weakness if it is there. In problem D, the student adds the 9 and the 5 to get 14. The 4 is written in the ones place and a 10 is carried and written above the 20 in the second column. The 10, 20, and 30 are added to get 60. The 60 is added to the 4 to get 64. See the example below. Problem P, an expanded borrowing problem, is also shown.

$$
\begin{array}{ll}
\text{D.} & \overset{10}{29} = \underline{20} + \underline{9} \\
& + 35 = \underline{30} + \underline{5} \\
\hline
& \underline{64} = \underline{60} \ \underline{+4}
\end{array}
\qquad
\begin{array}{ll}
\text{P.} & 53 = \overset{40}{\cancel{50}} + \overset{10}{\underline{3}} \\
& -27 = \underline{-20} + \underline{7} \\
\hline
& \underline{26} = \underline{20} + \underline{6}
\end{array}
$$

If on problems D and P, you see the student do something similar to this,

D.
$$29 = \underline{20} + \underline{9}$$
$$+ \ 35 = \underline{30} + \underline{5}$$
$$\overline{\underline{55} = \underline{51} + \underline{4}}$$

P.
$$53 = \underline{\cancel{50}} \overset{49}{} + \underline{{}^1 3}$$
$$- \ 27 = \underline{-20} + \underline{-7}$$
$$\overline{\underline{35} = \underline{29} + \underline{6}}$$

then you know that there is confusion on the values of the numbers. If this happens or if the student does not know how to do an expanded problem, see the Circle Math Addition and Regrouping sections in *Patterns in Arithmetic: Book 2* and the subtraction section in *Subtraction: Booklet 3*.

Problems A, B, and C test 2 digit addition without carrying.
Problems D, E, and F test 2 digit addition with carrying into the tens place.
Problems G, H, and I test 3 digit addition with carrying into the hundreds place. No carrying to tens
Problems J, K, and L test 3 digit addition with carrying to both the tens and the hundreds
Problems M, N, and O test 2 digit subtraction without regrouping (borrowing).
Problems P, Q, and R test 2 digit subtraction with regrouping. Some students will use Circle Math to solve these and may use nonstandard regrouping. If so, begin work on standard regrouping.
Problems S, T, and U test regrouping in two places. This is not covered in *Patterns in Arithmetic: Book 2*, but a few students will be able to do it based on what they already know.
Problems V, W, and X test calculation of basic multiplication facts.
Problems Y and Z test short, two digit multiplication. Most students will do this with addition at this point.

Word Problems
This page tests if a student can formulate an addition, a subtraction, and a multiplication word problem. If he can't, then he doesn't understand the structure of word problems and should review them.

Notes for Arithmetic Skills
Items that are missed should be examined to determine if the error is conceptual or if it is a miscalculation difficulty. Conceptual errors will tend to be repeated from problem to problem. If miscalculations on a single operation are frequent, work on basic facts is in order. It is also possible that the student has not gained fluency with a certain type of problem. Widespread mistakes usually indicate the student worked too fast or he is feeling poorly about his ability to do math.

Two Common Conceptual Errors

$$37$$
$$+46$$
$$\overline{713}$$

1. This answer indicates that the student does not understand place value and has not mastered carrying. The fact that the 7 is pushed into the hundreds place shows lack of understanding of place value. The student also does not have a sense of a reasonable answer (that 37 + 46 can't possibly be 713). After the test is finished, have the student make a model of 713 with the blocks. Then do the problem with blocks. He will be able to see that the total is much less than 713. Some students will then correct their own error. Lots of practice on this type of problem should then follow. A student who can't correct the error should review the lessons on regrouping in *Patterns in Arithmetic: Book 2.*

2. This error indicates a lack of understanding of the meaning of the positions of the numbers in a subtraction problem. The student, when confronted with 2 - 9, changes it to

$$32$$
$$-19$$
$$\overline{17}$$

9 - 2, not realizing that this completely alters the problem and that the bottom number is what is taken away from the top number. Review basic subtraction in Circle Math Review in the Subtraction section of this book. Then give the student a chance to redo the problem knowing that the situation is 2 - 9. He may or may not know how to proceed from that point. It may be necessary to review the subtraction section in *Patterns in Arithmetic: Book 2.*

Circle Math

Do not give this section to a student who is new to this series.

1. In this problem, the student breaks up the 14 into the three circles, breaks up the 21, then solves the little addition problems in the circles, and records the totals in the three circles under the line. A final sum is obtained by adding up the numbers in the bottom circles. If the student has forgotten how to solve Circle Math problems, see Circle Math Addition: Review. Example:

$$14 = (5) + (4) + (5)$$
$$+21 = (9) + (8) + (4)$$
$$\overline{35 = (14) + (12) + (9)}$$

2. This problem is done the same way as Problem 1, but with different numbers.

3. This problem is done the same way as the previous two except that the broken up 19 is subtracted from the broken up 26. The final answer is obtained by adding the partial differences in the bottom row of circles. Example:

$$26 = (9) + (8) + (9)$$
$$-19 = (-7) + (-6) + (-6)$$
$$\overline{7 = (2) + (2) + (3)}$$

4. In this problem, the numbers have already been broken up. But you will notice that in the last circles, the number on the top is smaller than the number on the bottom. The student is instructed to 'fix' it so that the subtraction can be done. In this case the numbers in the bottom row can't be rearranged without re-creating the same difficulty. The easiest way to fix this situation is to 'trade' some numbers from the 17 to make the 5 larger. There are many ways to solve this. One way is in the example. If the student insists on completely scrubbing the given numbers and starting over, that's fine. There will be practice with this situation in the lesson Discovering Easier Ways.
Example:

$$26 = (\cancel{10}^{14}) + (4) + (\cancel{8}^{8})$$
$$-19 = (-7) + (-4) + (-8)$$
$$\overline{7 = (7) + (0) + (0)}$$

5. This is new material for most students. An advanced student may be able to solve the problem. It tests the limit of knowledge in regard to standard regrouping. The problem asks the student to solve the problem using tens and ones. The student may or may not borrow a ten. The standard solution to this type of problem is taught in the subtraction section in this book.

Conservation of Number, Concept 9, is discussed in the General Concepts Section.

Speed Test

The speed test measures fluency in basic addition and subtraction facts. Give the speed tests several days after the other Assessment Worksheets. This allows the student a few days of brushing up before having to work at top speed. The test is in four parts with six problems in each part. Each section is timed separately.

Part A tests basic sums to 9. Record results on Assessment Sheet, Addition 1.
Part B tests intermediate sums from 10 to 18. Record results on Assessment Sheet, Addition 2.
Part C tests basic subtractions from numbers 3 through 9. Record results on Assessment Sheet, Subtraction 1.
Part D tests intermediate subtractions from numbers 10 through 18. Record results on Assessment Sheet, Subtraction 2.
For most seven- and eight-year-old students, parts B and D are difficult to do quickly.

For a Large Group
Allow exactly 30 seconds for each section. This may seem like a very short time (especially to the students) but actually allows 5 seconds per problem. If a student thinks that he can not possibly work that fast, have him count out 5 seconds to see how long a time 5 seconds really is. This rate does not require an automatic response to the problem, which is what the goal is. For eight years old and up, the goal is an accurate response within 3 seconds by the end of the school year.

"You will have 30 seconds to do each part. Work as fast as you can but don't panic if you can't finish before the time is called. If you come to a problem you don't know the answer to, skip it and come back to it if you have time. You must stop at the end of each part. The word STOP is written on the test to show you where to stop."

Have the students help correct their work. To the side of each problem set is a 30 with a line under it followed by an = sign. To find the time spent per problem, count up the number of correct answers and write that number under the line beneath the 30. Divide the 30 by the number of correctly done problems to obtain the time spent per problem. Since these students probably don't know how to divide, put a chart on the board that gives the scores:
6 problems correct = 5 seconds or less per problem,
5 problems correct = 6 seconds per problem,
4 problems correct = 7 1/2 seconds per problem and so on. A score of 6 seconds per problem or more indicates lack of fluency. Make a game of trying to get faster, where the amount of improvement is rewarded rather than the speed itself. See Expansion at the end of this section.

For Individuals or Small Groups
Tell the student that you are going to time him and that he should work as quickly as he can. Time each section separately. Instead of giving 30 seconds per section, time how long it takes the student to complete the section. To obtain the time spent per problem, count up the correct answers and divide that number into the number of seconds it took the student to complete that section.
If the student is being home educated and is not required to take timed achievement tests, you may want to skip the speed tests. However, there are other reasons for knowing the basic facts quickly. First, it makes doing longer problems easier. Second, practicing thinking quickly is a mental calisthenic. Many students enjoy the challenge of trying to improve their speed, especially if there is no grade pressure.

Patterns

The student continues representational and numeric patterns. The blank line in front is for extending the pattern in the other direction. If patterns is new to a student, review Patterns in *Patterns in Arithmetic: Book 2.*

Organizing the Results

Use the Assessment: Record Sheet to record the student's aptitude on each skill and concept tested. If his current level is adequate, put a √ in the first box, labeled OK, next to that skill or concept. If you want to work further with the student on a particular thing, put a check in the second box, labeled NW for needs work.

In the Arithmetic Skills section, you will notice that on some items there are two sets of boxes. One set is labeled EX for expanded problems, and one is labeled SN for short notation. Record the results for the expanded problems separately from the short notation problems. It is common for a student to be able to do an expanded problem but not a short notation problem, or vice versa.

You will also notice that Circle Math problems are included in the list of skills to be checked off. Ignore these if the student has not done *Patterns in Arithmetic: Book 2.*

If your student is new to this series and is competent in addition and subtraction, you may want to skip Circle Math addition and subtraction or do it quickly later in the year after having done multiplication and division. If your student needs work on regrouping, it would make his understanding much stronger if you do the Circle Math lessons before you teach the standard methods of regrouping.

When you have finished recording all the results, look over your list of Needs Work items, if there are any. Note whether the items are individually developed ideas, like Families of Facts or the Associative Property, that are taught as a single idea or if the items are linked, such as multiplication number sentences, the concept of multiplication, and the multiplication facts. Did the student have difficulty on all the multiplication questions or only one section of them? Could the student do subtraction without regrouping but not the problems with regrouping? The answers to these kinds of questions will help you decide where to start.

Using the Results

On another piece of paper, write a list of the things you want to work on with that student. Note whether you want to review a skill separately or if it is the starting point for a whole series of lessons. For example, a student who had difficulty with the multiplication word problem may need just a quick brush-up on word problems, or he may need to work on the entire concept of multiplication itself which spans several lessons. (The latter situation would be diagnosed by difficulty on all or most of the multiplication questions.)

Put your list into an order that seems reasonable to you and use it to begin your lessons.

Needing further work on some items in *Patterns in Arithmetic: Book 1* or *Book 2* does not mean that the student can't begin working on some of the things in Grade 3 Booklets. Work on addition or subtraction can proceed separately from work on multiplication and division. Work on fractions is related to division, but they are not directly linked to each other at this point. Items needing further work would hold up the beginning of new material in Grade 3 Booklets only if mastery of the material that needs to be worked on is a prerequisite for the new lesson.

In planning your approach, you might consider this point. If it is the beginning of a new school year or session or if you are integrating a new student, *do not start with trouble areas*. Start on something brand-new and fresh or not related to the weak material. A student who is struggling with arithmetic, or one who says that he hates math, might start on geometry or patterns and cycles. Psychologically, this makes a world of difference to the student. It avoids the "Oh no, not this again" syndrome and helps him feel he is making real progress. His good feelings about the work is one of your greatest allies. You can circle around to the trouble areas after he has gained momentum.

You will find yourself working on several fronts at once. That's the way learning goes. We humans hardly ever learn things in a linear fashion. One way not to be overwhelmed by this is to use the Sequence Guides to chart what has been accomplished. (These sheets can also serve home schoolers who need to show progress to local school districts.)

Students love charting their own progress. Invite them to participate in directing their own learning. Try to follow current interests, and change the order of lessons to suit the student's needs. Our ordering is a possible path, not the only path a learner might take as he adds to his skills and knowledge. A student who is eager to learn to divide, for example, need not do all the multiplication lessons first. Since multiplication and division are closely linked, doing division first would assist the learning of multiplication later. The conventional route is to learn multiplication first, but that would not take precedence over a strong desire to learn division.

Expansion
One way to increase speed on basic addition and multiplication facts is to use Picture My Learning (at the end of the addition and multiplication sections in this book and in the booklets Addition: Booklet 3 and Multiplication: Booklet 1). The teacher or another student gives the problem, pointing with a pencil to the appropriate square if possible; the student writes in the answer as fast as he can. If the correct response is given in 3 seconds or less, the student then colors that problem square green. A 4 to 6 second response square is colored orange. A 7 to 10 second response square is colored yellow. An incorrect response or no response after 10 seconds square is colored red.

One way to speed the process up is to write the letter "g" for green, "o" for orange, "y" for yellow and "r" for red in the squares and color them in after all the problems have been given.

If you are working with a large group, give the problem, then after 3 seconds say," Green." Any student who answered within that time span writes a little "g" in the square. After 6 seconds say, "Orange," and any student who answered in that time span writes an "o" in his square. At 10 seconds say yellow, at 11 seconds say red and give the correct answer. Any student who missed the problem automatically codes the square for red. A student who writes an answer in 4 seconds, for example, realizes the answer is wrong, and rewrites it by 9 seconds would code the square yellow.

The value of this type of chart is that it shows in a very graphic way the student's improvement as the distribution of colors shifts from red and yellow to green and orange. It also shows the student which problems need work (generally fewer than the student would have guessed), allowing him to focus on the few that are hard and not be concerned about the others. This method results in very rapid improvement for most students. Most enjoy coloring in the charts. Twice per week seems to be an adequate number of times to do this activity.

Assessment: General Concepts - Worksheet 1   Date _____
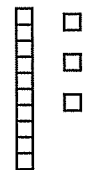
1. Place Value

This little square □ stands for 1.

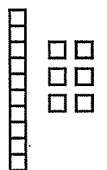This tower ▯ stands for 10.

This big square ▦ stands for 100.

Write the number that each drawing stands for.
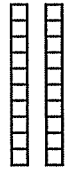
Example:
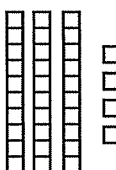
▯ □ □ □ = 13

a. □□ □□ □□ □□ =
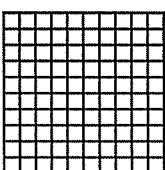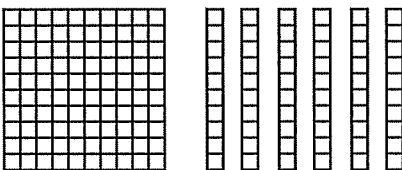
b. □□ □□ □ =
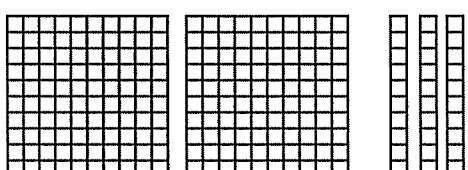
c. ▯ □□ □□ □□ =
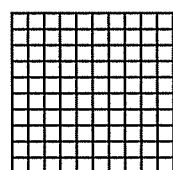
d. ▯▯ =
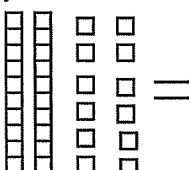
e. ▯▯▯▯ □ □ □ □ =

f. ▦ =

g. ▦ ▯▯▯▯▯ =

h. ▦▦ ▯▯▯ □ □ =

i. ▦ □ =

j. ▯▯ □ □ □ □ □ □ □ □ =

k. Write the number that comes <u>after</u> each of these numbers.

39,____   99,____

109,____   111,____

2. Associative Property

   Don't add these! Just look at the problems. Circle the two that will have
   the same answer as this addition problem, 6 + 3 + 5 = _____.

5 + 6 + 3 = ___.	2 + 4 + 5 = ___.	5 + 6 + 2 = ___.
4 + 5 + 3 = ___.	3 + 5 + 6 = ___.	4 + 6 + 5 = ___.

3. Number Sentences

   Write the number sentence (the problem you do) that goes with each word
   problem. Then find the answer.

   a. Nan has 3 blue balls, 5 green balls, and 4 red balls. How many balls does
   Nan have?
   Number Sentence _____

   b. Henry had $17. Somehow he lost $6. How much does he have left?

   Number Sentence _____

   c. Every day Toby collects 3 eggs from the chicken coop. How many eggs
   does Toby collect each week?
   Number Sentence _____

4. Families of Facts

   Use only these three numbers to make four different number sentences.

   $$2, 3, 5$$
   DO NOT use any other numbers.
   The first one is done for you.

   $$2 + 3 = 5$$

5. Difference Between

   Which number is larger: 24 or 15? _____

   How much larger? _____

6. Expansion of Numbers

   Break up 23 three different ways.

   $$23 = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$$
   $$23 = \underline{\quad} + \underline{\quad} + \underline{\quad}$$
   $$23 = \underline{\quad} + \underline{\quad}$$

7. Concept of Multiplication

What multiplication problem is shown in this picture?

_____ ✕ _____ = _____
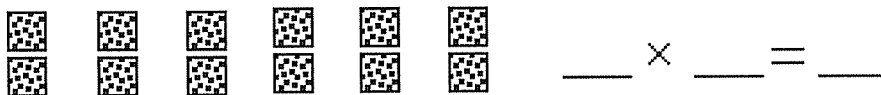
What multiplication problem is shown in this picture?
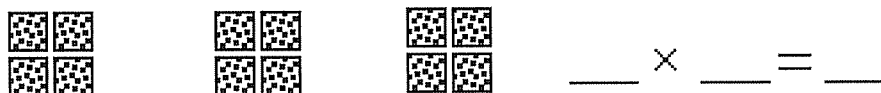
_____ ✕ _____ = _____

Draw a picture of 3 x 5 = 15.

8. Fraction Notation

There are ten boxes below.

Circle the one that has $\frac{1}{2}$ shaded in.

Put an X on the one that has $\frac{1}{3}$ shaded in.
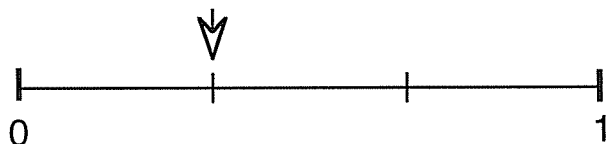
Underline the one that has $\frac{3}{4}$ shaded in.

1   2   3   4   5

6   7   8   9   10
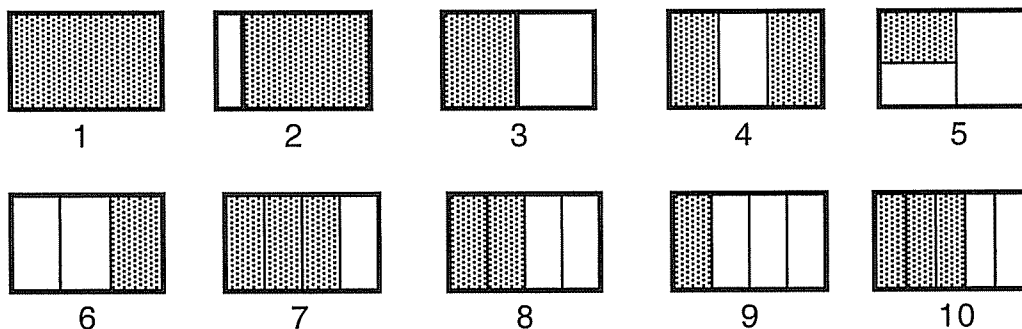
In a fraction what is the top number called? _____

In a fraction what is the bottom number called? _____

What information does the bottom number give? _____

The arrow points to which fraction on the number line? _____

0   1

Here are two ways to solve arithmetic problems.

Expanded way   Short way

$$34 = 30 + 4$$
$$\underline{+25 = 20 + 5}$$
$$59 = 50 + 9$$

$$34$$
$$\underline{+\ 25}$$
$$59$$

Solve these problems the Expanded way.   Solve these problems the Short way.

A.   10s   1s

$$52 = \underline{50} + \underline{2}$$
$$\underline{+\ 37 = \underline{\quad} + \underline{\quad}}$$
$$\underline{\quad} = \underline{\quad} + \underline{\quad}$$

B.   C.

$$23$$ $$51$$
$$\underline{+\ 12}$$ $$\underline{+\ 47}$$

D.

$$29 = \underline{\quad} + \underline{\quad}$$
$$\underline{+\ 35 = \underline{\quad} + \underline{\quad}}$$
$$\underline{\quad} = \underline{\quad} + \underline{\quad}$$

E.   F.

$$52$$ $$64$$
$$\underline{+\ 49}$$ $$\underline{+\ 28}$$

G.

$$147 = \underline{\quad} + \underline{\quad} + \underline{\quad}$$
$$\underline{+\ 271 = \underline{\quad} + \underline{\quad} + \underline{\quad}}$$
$$\underline{\quad} = \underline{\quad} + \underline{\quad} + \underline{\quad}$$

H.   I.

$$284$$ $$472$$
$$\underline{+\ 393}$$ $$\underline{+\ 454}$$

J.

$$364 = \underline{\quad} + \underline{\quad} + \underline{\quad}$$
$$\underline{+\ 177 = \underline{\quad} + \underline{\quad} + \underline{\quad}}$$
$$\underline{\quad} = \underline{\quad} + \underline{\quad} + \underline{\quad}$$

K.   L.

$$385$$ $$201$$
$$\underline{+\ 345}$$ $$\underline{+\ 399}$$

Assessment: Arithmetic Skills - Worksheet 5   Date _____

Start at the top and work across these subtraction problems.

Solve these problems the expanded way.   Solve these problems the short way.

M.

$$46 = \underline{\quad} + \underline{\quad}$$
$$- 23 = \underline{\quad} + \underline{\quad}$$
$$\underline{\quad} = \underline{\quad} + \underline{\quad}$$

N.
$$38$$
$$- 27$$

O.
$$59$$
$$- 36$$

P.

$$53 = \underline{\quad} + \underline{\quad}$$
$$- 27 = \underline{\quad} + \underline{\quad}$$
$$\underline{\quad} = \underline{\quad} + \underline{\quad}$$

Q.
$$32$$
$$- 14$$

R.
$$76$$
$$- 38$$

These next three problems are difficult. If you don't know how to do them, skip them and go on to the next part.

S.

$$452 = \underline{\quad} + \underline{\quad} + \underline{\quad}$$
$$- 274 = \underline{\quad} + \underline{\quad} + \underline{\quad}$$
$$\underline{\quad} = \underline{\quad} + \underline{\quad} + \underline{\quad}$$

T.
$$347$$
$$- 158$$

U.
$$404$$
$$- 255$$

These are multiplication problems.

V.
$$3$$
$$\times 3$$

W.
$$5$$
$$\times 4$$

X.
$$7$$
$$\times 3$$

Y.
$$14$$
$$\times 5$$

Z.
$$25$$
$$\times 4$$

Patterns in Arithmetic: General Organization   Assessment
Parent /Teacher Guide   35

Word Problems

1. Write a word problem that uses this problem: 4 + 7 = 11.

2. Write a word problem that uses this problem: 14 - 8 = 6.

3. Write a word problem that uses this problem: 4 x 3 = 12.

1. Solve this addition problem.
   Use all the circles.

$$14 = \bigcirc + \bigcirc + \bigcirc$$
$$+21 = \bigcirc + \bigcirc + \bigcirc$$
$$\underline{\quad} = \bigcirc + \bigcirc + \bigcirc$$

2. Solve it again a different way.

$$14 = \bigcirc + \bigcirc + \bigcirc$$
$$+21 = \bigcirc + \bigcirc + \bigcirc$$
$$\underline{\quad} = \bigcirc + \bigcirc + \bigcirc$$

3. Solve this subtraction problem.
   Use all the circles.

$$26 = \bigcirc + \bigcirc + \bigcirc$$
$$-19 = \bigcirc\!- + \bigcirc\!- + \bigcirc\!-$$
$$\underline{\quad} = \bigcirc + \bigcirc + \bigcirc$$

4. Fix this problem.

$$26 = (17) + (4) + (5)$$
$$-19 = (-7) + (-4) + (-8)$$
$$\underline{\quad} = \bigcirc + \bigcirc + \bigcirc$$

5. Break up this problem into tens and ones.

$$26 = \bigcirc + \bigcirc$$
$$-19 = \bigcirc\!- + \bigcirc\!-$$
$$\underline{\quad} = \bigcirc + \bigcirc$$

Concept #9 Conservation of Number

Problems 3, 4, and 5 are broken up

different ways. Does that change the

answer?

Explain_____

_____

_____

_____

Section 1: Addition

A

8	4	3	5	2	5
+ 1	+ 2	+ 6	+ 4	+ 3	+ 3

30 =
___

STOP

B

7	6	9	9	6	8
+ 3	+ 8	+ 9	+ 4	+ 5	+ 7

30 =
___

STOP

Section 2: Subtraction

C

7	6	5	9	6	8
- 4	- 1	- 2	- 5	- 6	- 3

30 =
___

STOP

D

11	12	15	10	11	17
- 3	- 6	- 7	- 4	- 5	- 9

30 =
___

STOP

Assessment: Patterns - Worksheet 9   Date _____

Continue each pattern as far as you can.

____ ▽△▽△▽△▽△

____ ▭▥ ▭▥ ▭▥ ▭▥

____ □ ▭ ▭ ▭ ____ ____

____ 60   50 ____   30 ____ ____ ____

____ 1   10   100 ____ ____ ____

____ BA   BAA   BAAA ____ ____ ____

____ 20   18   16   14 ____ ____ ____

____ □ ▭ ⊞ ▦ ____ ____

____ Y Z X Y Z X Y Z X Y_____

____ 1   2   4   8 ____ ____ ____

Make up two more patterns.

Assessment: Record Sheet

Name _____

Date _____

General Concepts

	OK	NW
1. Place Value to the hundreds	☐	☐
2. Commutative and Associative Properties	☐	☐
3. Number sentences - addition	☐	☐
- subtraction	☐	☐
- multiplication	☐	☐
4. Families of Facts	☐	☐
5. Difference Between	☐	☐
6. Concept of Multiplication	☐	☐
7. Fraction Notation	☐	☐
8. Expansion of Numbers (Circle Math)	☐	☐
9. Conservation of Number (Circle Math)	☐	☐

Arithmetic Skills

Addition

	OK	NW		
1. Calculation of Basic Facts	☐	☐		
2. Calculation of Intermediate Facts	☐	☐		
3. Nonstandard Addition (Circle Math)	☐	☐		
4. Two digits - No regrouping EX	☐	☐	SN ☐	☐
5. Two digits - Regrouping EX	☐	☐	SN ☐	☐
6. Three digits - Regrouping to hundreds place only EX	☐	☐	SN ☐	☐
7. Three digits - Regrouping to tens and hundreds place. EX	☐	☐	SN ☐	☐
8. Word problems	☐	☐		

Subtraction

	OK	NW		
1. Calculation of Basic Facts	☐	☐	Speed_____sec/prob
2. Calculation of Intermediate Facts	☐	☐	Speed_____sec/prob
3. Nonstandard Subtraction (Circle Math)	☐	☐		
4. Two digits - No regrouping EX	☐	☐	SN ☐	☐
5. Nonstandard Regrouping (Circle Math)	☐	☐		
6. Two digits - Regrouping EX	☐	☐	SN ☐	☐
7. Three digits - Regrouping to tens and hundreds place EX	☐	☐	SN ☐	☐
8. Word problems	☐	☐		

Multiplication

	OK	NW
1. Calculation of Basic Facts from 2 - 5 Times Tables	☐	☐
2. Two digit x one digit using addition	☐	☐
3. Word problems	☐	☐

	OK	NW
Patterns	☐	☐

OK = adequate
NW = needs work
EX = expanded
SN = short notation

Where to go from here: record on another sheet of paper.

Answer Key

Assessment: General Concepts - Worksheet 1

1. Place Value

This little square ☐ stands for 1.

This tower ⧮ stands for 10.

This big square ▦ stands for 100.

Write the number that each drawing stands for.

Example: ⧮☐☐☐ = 13

a. = 8

b. = 5

c. = 16

d. = 20

e. = 34

f. = 100

g. = 160

h. = 232

i. = 101

j. = 32

k. Write the number that comes <u>after</u> each of these numbers.

39, 40   99, 100
109, 110   111, 112

31

Assessment: General Concepts - Worksheet 2

2. Associative Property

Don't add these! Just look at the problems. Circle the two that will have the same answer as this addition problem, 6 + 3 + 5 = 14.

| (5 + 6 + 3 = ___) | 2 + 4 + 5 = ___ | 5 + 6 + 2 = ___ |
| 4 + 5 + 3 = ___ | (3 + 5 + 6 = ___) | 4 + 6 + 5 = ___ |

3. Number Sentences

Write the number sentence (the problem you do) that goes with each word problem. Then find the answer.

a. Nan has 3 blue balls, 5 green balls, and 4 red balls. How many balls does Nan have?
Number Sentence $3 + 5 + 4 = 12$

b. Henry has $17. Somehow he lost $6. How much does he have left?
Number Sentence $\$17 - 6 = \$11$

c. Every day Toby collects 3 eggs from the chicken coop. How many eggs does Toby collect each week?
Number Sentence $3 \times 7 = 21$ or $3+3+3+3+3+3+3 = 21$

4. Families of Facts

Use only these three numbers to make four different number sentences.
(2, 3, 5) DO NOT use any other numbers.
The first one is done for you.

$2 + 3 = 5$ $5 - 2 = 3$
$3 + 2 = 5$ $5 - 3 = 2$

5. Difference Between

Which number is larger: 24 or 15? 24

How much larger? 9

6. Expansion of Numbers

Break up 23 three different ways. AWV Examples:

$23 = 7 + 12 + 4 + 0$
$23 = 3 + 10 + 10$
$23 = 20 + 3$

32

Assessment: General Concepts - Worksheet 3

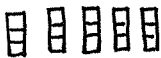**7. Concept of Multiplication**

What multiplication problem is shown in this picture?

$2 \times 6 = 12$

What multiplication problem is shown in this picture?

$4 \times 3 = 12$
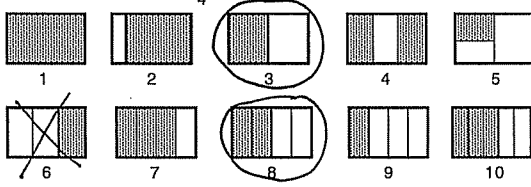
Draw a picture of 3 x 5 = 15.

8. Fraction Notation

There are ten boxes below.

Circle the one that has $\frac{1}{2}$ shaded in.

Put an X on the one that has $\frac{1}{3}$ shaded in.

<u>Underline</u> the one that has $\frac{3}{4}$ shaded in.
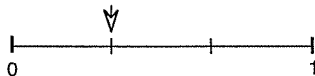
1 2 3 4 5
6 7 8 9 10

In a fraction what is the top number called? numerator

In a fraction what is the bottom number called? denominator

What information does the bottom number give? number of pieces in the whole

The arrow points to which fraction on the number line. $\frac{1}{3}$

0   1

33

Assessment: Arithmetic Skills - Worksheet 4

Here are two ways to solve arithmetic problems.

Expanded way

$34 = 30 + 4$
$+25 = 20 + 5$
$59 = 50 + 9$

Short way

34
$+ 25$
59

Solve these problems the Expanded way.

A.   10s   1s
$52 = 50 + 2$
$+ 37 = 30 + 7$
$89 = 80 + 9$

D.
$29 = 20 + 9$
$+ 35 = 30 + 5$
$64 = 50 + 14$

G.
$147 = 100 + 40 + 7$
$+ 271 = 200 + 70 + 1$
$418 = 300 + 110 + 8$

J.
$364 = 300 + 60 + 4$
$+ 177 = 100 + 70 + 7$
$541 = 400 + 130 + 11$

Solve these problems the Short way.

B.
23
$+ 12$
35

C.
51
$+ 47$
98

E.
52
$+ 49$
101

F.
64
$+ 28$
92

H.
284
$+ 393$
677

I.
472
$+ 454$
926

K.
385
$+ 345$
730

L.
201
$+ 399$
600

34

Assessment: Arithmetic Skills - Worksheet 5

Start at the top and work across these subtraction problems.

Solve these problems the expanded way.

M.   BUWV Example:

$$46 = \underline{40} + \underline{6}$$
$$-\ 23 = -\underline{20} + -\underline{3}$$
$$\overline{23 = 20 + 3}$$

Solve these problems the short way.

N.
$$38$$
$$-\ 27$$
$$\overline{11}$$

O.
$$59$$
$$-\ 36$$
$$\overline{23}$$

P.
$$\overset{40}{53} = \underset{\cancel{50}}{} + \overset{+10}{3}$$
$$-\ 27 = -\underline{20} + -\underline{7}$$
$$\overline{26 = 20 + 6}$$

Q.
$$\overset{2}{\cancel{3}}2$$
$$-\ 14$$
$$\overline{18}$$

R.
$$\overset{6}{\cancel{7}}6$$
$$-\ 38$$
$$\overline{38}$$

These next three problems are difficult. If you don't know how to do them, skip them and go on to the next part.

S.
$$452 = \overset{300}{\cancel{400}} + \overset{40}{\cancel{50}} + \overset{10+}{2}$$
$$-\ 274 = -\underline{200} + -\underline{70} + \underline{4}$$
$$\overline{178 = 100 + 70 + 8}$$

T.
$$\overset{2}{\cancel{3}}\overset{3}{4}7$$
$$-\ 158$$
$$\overline{189}$$

U.
$$\overset{3}{\cancel{4}}\overset{9}{0}4$$
$$-\ 255$$
$$\overline{149}$$

These are multiplication problems.

V.
$$3$$
$$\times 3$$
$$\overline{9}$$

W.
$$5$$
$$\times 4$$
$$\overline{20}$$

X.
$$7$$
$$\times 3$$
$$\overline{21}$$

Y.
$$14$$
$$\times 5$$
$$\overline{70}$$

Z.
$$25$$
$$\times 4$$
$$\overline{100}$$

Page 34 - Answers will vary.

Assessment: Arithmetic Skills - Worksheet 7

1. Solve this addition problem. BUWV Use all the circles.   **Example:**

$$14 = (10) + (3) + (1)$$
$$+21 = (7) + (7) + (7)$$
$$\overline{35} = (17) + (10) + (8)$$

2. Solve it again a different way.

$$14 = () + () + ()$$
$$+21 = () + () + ()$$
$$\overline{35} = () + () + ()$$

AWV Example:

3. Solve this subtraction problem. Use all the circles.

$$26 = (10) + (10) + (6)$$
$$-19 = (-10) + (-3) + (-6)$$
$$\overline{7} = (0) + (7) + (0)$$

4. Fix this problem.

$$26 = \overset{4}{(\cancel{1}7)} + (4) + \overset{+3=8}{(5)}$$
$$-19 = (-7) + (-4) + (-8)$$
$$\overline{7} = (7) + (0) + (0)$$

5. Break up this problem into tens and ones.

$$26 = \overset{10}{(\cancel{2}0)} + \overset{+10}{(6)}$$
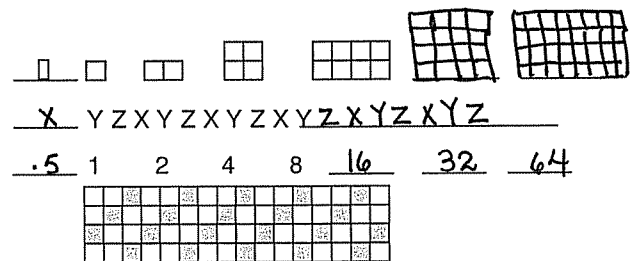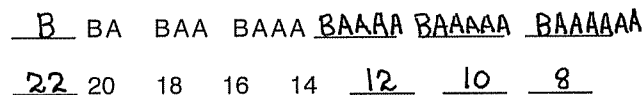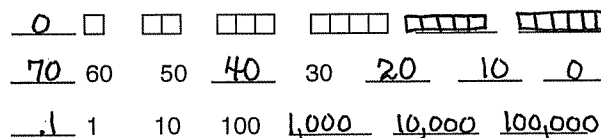$$-19 = (-10) + (-9)$$
$$\overline{7} = (0) + (7)$$

Concept #9 Conservation of Number

Problems 3, 4, and 5 are broken up different ways. Does that change the answer? Explain **Answers stay the same no matter how the numbers are broken up.**

Assessment: Speed Test - Worksheet 8

Section 1: Addition

A.

8	4	3	5	2	5	
+1	+2	+6	+4	+3	+3	30 =
9	6	9	9	5	8 STOP	—

B.

7	6	9	9	6	8	
+3	+8	+9	+4	+5	+7	30 =
10	14	18	13	11	15 STOP	—

Section 2: Subtraction

C.

7	6	5	9	6	8	
-4	-1	-2	-5	-6	-3	30 =
3	5	3	4	0	5 STOP	—

D.

11	12	15	10	11	17	
-3	-6	-7	-4	-5	-9	30 =
8	6	8	6	6	8 STOP	—

Assessment: Patterns - Worksheet 9

Continue each pattern as far as you can.

(triangle patterns)

$$\underline{0}\ \ \square\ \ \square\square\ \ \square\square\square\ \ \square\square\square\square\ \ \square\square\square\square\square$$

$$\underline{70}\ 60\ \ 50\ \ \underline{40}\ \ 30\ \ \underline{20}\ \ \underline{10}\ \ \underline{0}$$

$$\underline{.1}\ 1\ \ 10\ \ 100\ \ \underline{1,000}\ \ \underline{10,000}\ \ \underline{100,000}$$

$$\underline{B}\ BA\ \ BAA\ \ BAAA\ \ \underline{BAAAA}\ \ \underline{BAAAAA}\ \ \underline{BAAAAAA}$$

$$\underline{22}\ 20\ \ 18\ \ 16\ \ 14\ \ \underline{12}\ \ \underline{10}\ \ \underline{8}$$

(square grid patterns)

$$\underline{X}\ YZXYZXYZXYZ\underline{XYZ}$$

$$\underline{.5}\ 1\ \ 2\ \ 4\ \ 8\ \ \underline{16}\ \ \underline{32}\ \ \underline{64}$$

Patterns in Arithmetic: Series Overview

Books

Contains:
Addition
Subtraction
Multiplication
Division
General Math
Place Value
Beginning Fractions

Grade 1	Grade 2
Parent/Teacher Guide **Book 1**	Parent/Teacher Guide **Book 2**
Student Workbook **Book 1**	Student Workbook **Book 2**

Booklets by Subject

Parent/Teacher Guide
Student Workbook

General Organization
Includes:
Philosophy
Teaching Techniques
Arithmetic Convention, ,
Sequence Guides
Lesson Planning
Materials
Resources
Math Journal
Series Overview

General Math
Includes:
Patterns
Geometry
Measurement
Place Value
Pre-algebra
Graphing
Functions
and more

	Grade 3	Grade 4	Grade 5	Grade 6
General Organization	Grade 3	Grades 4 and 5	Grades 4 and 5	
Addition	Booklet 3			
Subtraction	Booklet 3 Base Ten	Booklet 4		
Multiplication	Booklet 1	Booklet 2 Expanded Tables Flip Bk *Booklet 3	Booklet 4	
Division	Booklet 1	Booklet 2 Base Ten	Booklet 3	
General Math	Booklet 3	Booklet 4	Booklet 5 Booklet 6	
Place Value		*Booklet 2	Booklet 3	
Fractions / Fractions Tool Chest	Booklet 1	Booklet 2 *Booklet 3	Booklet 4 Booklet 5 Booklet 6 Booklet 7	Booklet 8
Decimals		*Key to Decimals* Book 1	*Key to Decimals* Book 2	*Key to Decimals* Book 3

*These booklets are used in more than one grade.

Patterns in Arithmetic Series Overview

Books - Contains all subjects listed below
Book 1 - Grade 1
Book 2 - Grade 2

General Organization and Philosophy
General Organization - Grade 3
General Organization - Grades 4 and 5
Fractions Tool Chest - Understanding Fractions

Booklets by Subject

Addition
Booklet 3 - Regrouping (carrying) into Four Digits - Grade 3

Subtraction
Booklet 3 - General Principles of Regrouping (borrowing) - Grade 3
Base Ten Subtraction - Chrysalis Charter School - Grade 3
Booklet 4 - Large Numbers and Practice - Grade 4

General Math
Booklet 3 - Place Value, Measurement, and Geometry - Grade 3
Booklet 4 - Patterns and Algebraic Thinking - Grade 4
Booklet 5 - Geometric Formulas, Linear Functions, and Division Relationships - Grade 5
Booklet 6 - Geometry of Circles, Algebraic Properties, and More Functions - Grades 5 and 6

Multiplication
Booklet 1 - Basic Concepts - Grade 3
Booklet 2 - Beginning Long Multiplication and Basics of Distribution - Grade 4
Expanded Tables Flip Book - Grades 3, 4, and 5
Booklet 3 - Properties and Factoring - Grades 4 and 5
Booklet 4 - Working with Large Numbers and Decimals - Grade 5

Division
Booklet 1- Basic Concepts - Grades 3 and 4
Booklet 2 - Breaking Up Division - Grade 4
Base Ten Division - Chrysalis Charter School - Grade 4
Booklet 3 - Working with Double Digit Division - Grade 5

Place Value
Booklet 2 - Rounding Off and Estimating - Grades 3 and 4
Booklet 3 - Into the Millions, Exponential Notation, and Operations Review - Grades 4 and 5

Fractions
Booklet 1 - Basic Concepts - Grade 3
Booklet 2 - Developing Concepts and Beginning Operations - Grade 4
Booklet 3 - Mixed Numbers and Improper Fractions - Grades 4 and 5
Booklet 4 - Equivalent Fractions - Grades 4 and 5
Booklet 5 - Simplifying Fractions - Grades 4 and 5
Booklet 6 - Multiplication - Grades 4 and 5
Booklet 7 - Addition and Subtraction of Unlike Fractions - Grade 5
Booklet 8 - Understanding Division - Grades 5 and 6

Patterns in Arithmetic: Fractions Series Overview

Books

Contains: Division
Addition   General Math
Subtraction   Place Value
Multiplication Beginning Fractions

Grade 1	Grade 2
Parent/Teacher Guide Book 1	Parent/Teacher Guide Book 2
Student Workbook Book 1	Student Workbook Book 2

Booklets

Fractions Tool Chest - Understanding Fractions
Includes: Philosophy, Farm School Math Notes, Teaching Techniques, Arithmetic Conventions, Fraction Series Overview, Sequence Guide, Prism Fractions Instructions, Materials and Resources, Series Overview for Patterns in Arithmetic

Student Workbook and Parent Teacher/Guide come as a set

Booklet 1 - Basic Concepts

Booklet 2 - Developing Concepts and Beginning Operations

Booklet 3 - Mixed Numbers and Improper Fractions

Booklet 4 - Equivalent Fractions

Booklet 5 - Simplifying Fractions

Booklet 6 - Multiplication

Booklet 7 - Addition and Subtraction of Unlike Fractions

Booklet 8 - Understanding Division

Set of Fraction Booklets 1 - 8, Free shipping for a complete program.

Placement Assessment: in every booklet and available for free on the website: www.patternpress.com
An Answer Key to the Student Workbook is in each Parent/Teacher Guide

Prism Fractions® manipulatives
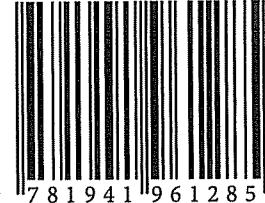Prism Fractions Paper Set - squares and circles
Colored acetate set for Booklet 6 - Multiplication
Plastic Prism Fractions Circles Set - 51 pieces