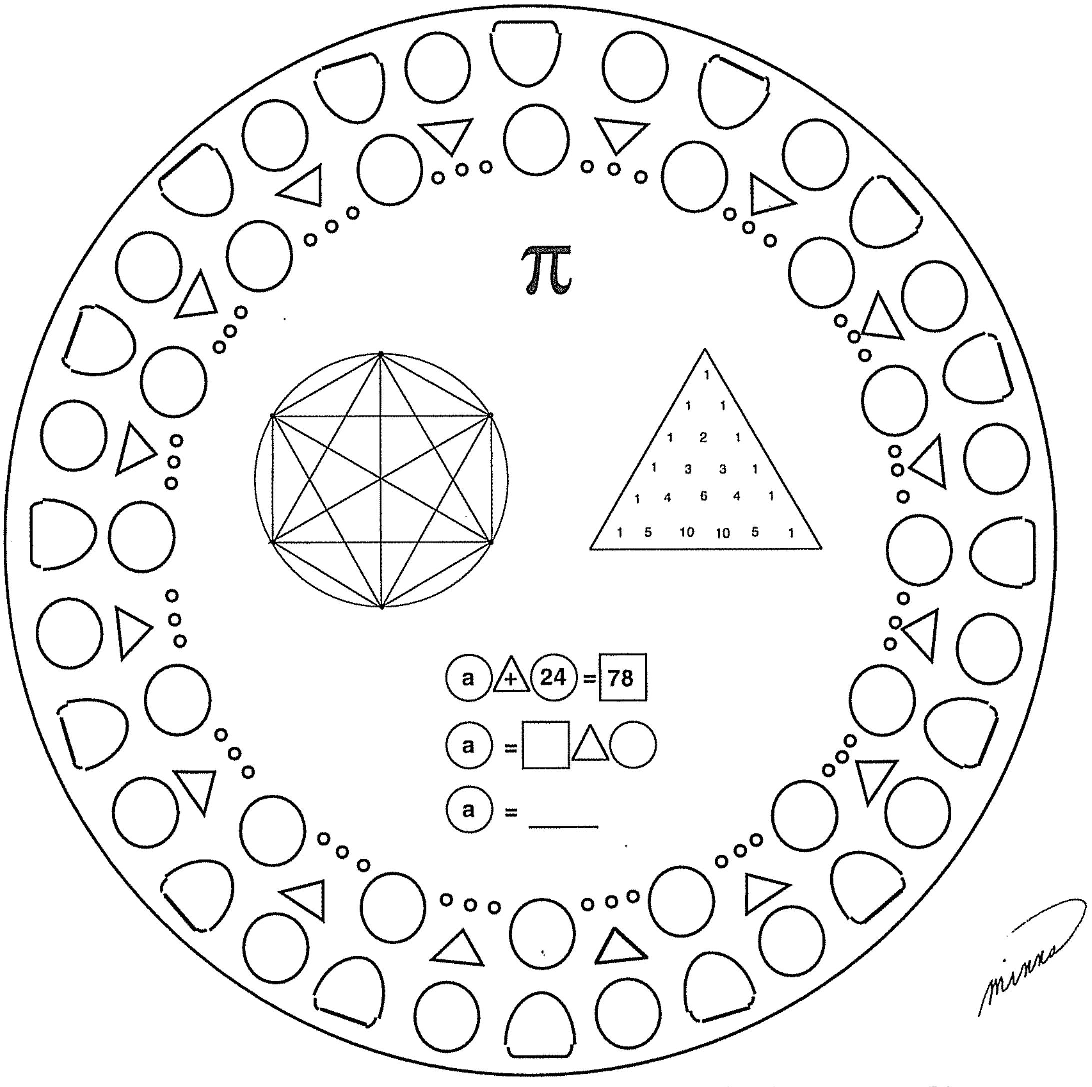
Patterns in Arithmetic

General Math - Booklet 6 PDF

Geometry of Circles, Algebraic Properties, and More Functions

Parent/Teacher Guide



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Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

General Math: Booklet 6 - PDF - Geometry of Circles, Algebraic Properties, and More Functions

Contents

This booklet is dedicated to pi. Completed on March 14, 3.14.

Acknowledgments

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For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

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The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

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Parent/Teacher Guide

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Pascal's Triangle

Purpose The purpose of this lesson is to get a little warm-up on finding patterns.

Prerequisites None

Materials Pascal's Triangle - Worksheets 1 - 4, pages 1 - 4
Crayons or colored pencils

Lesson Pascal's Triangle - Worksheet 1, page 1
Begin looking for patterns. Let the talk flow freely. There are many, many patterns in this triangle that will allow a student to fill in the correct numbers in the white triangles.

The main pattern is that the white triangle number is the sum of the two numbers above it, but do not tell him that. Fill in the numbers and check the Answer Key.

Pascal's Triangle - Worksheet 2, page 2
Use crayons to identify different patterns. For example: Use a red crayon to shade in the sequence of counting numbers, 1, 2, 3, 4, 5, etc. It moves down the second diagonal on both sides of the triangle.

Try circling odd numbers one color and even numbers another. Maybe there is a pattern, maybe not.
Make a Key at the bottom to tell what each color means.

Example: Red - shows the sequence of counting numbers.

Practice Worksheets Pascal's Triangle - Worksheets 3 and 4, pages 3 and 4

Look for patterns in multiples. Find the patterns and extend them.

Test for Understanding None

Circle Segments

Purpose

The purpose of this lesson is to reintroduce a geometric pattern and a T chart. This pattern concerns the number of chords that can be constructed in a circle with a given number of points on the circumference. A chord is a line segment connecting two points on the circumference of a circle. A chord that passes through the center of the circle is the diameter of the circle.

Prerequisites

Previous lessons on T charts presented in General Math: Booklet 4 - Graphing Number Patterns, General Math: Booklet 5 - Number Patterns: Functions, or other curricula such as *GEMS - Algebraic Reasoning*.

Materials

Circle Segments - Worksheets 1 and 2, pages 5 and 6
Ruler
Sharp pencil with an eraser
Colored pencils: purple, blue, green, yellow, orange, and red or your choice

Warm Up

Geometry Vocabulary: circumference, point, line segment, chord, diameter

Note

Using very sharp colored pencils makes the pattern more visible and creates a lovely graphic design. Using a ruler to make the lines straight creates an even lovelier pattern. The colors used can be changed to anything the student chooses.

Lesson Part 1

Circle Segments - Worksheet 1 Have the student follow the instructions Each circle has a given number of points placed on the circumference. Draw in all the possible chords that connect each point to every other point. Count up the number of chords that can be constructed for two, three, four and five points.

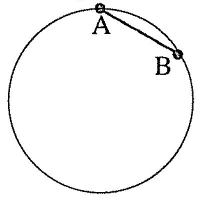
Then fill in the T chart and look for, and record the patterns he has found. Discuss the patterns he has found. Predict the number of chords for a ten point circle.

Lesson Part 2 Art Extension

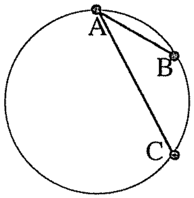
Circle Segments - Worksheet 2, page 6

On the first circle, use a purple pencil to color Point A and a blue pencil to color Point B. Then using the purple pencil, draw a line that makes a single purple chord between Points A and B.

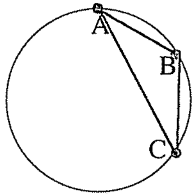
2 points



The second circle graphic has three points. Use the purple pencil to color point A. Color point B blue. Connect points AB and AC with a purple line with chords that extend from point A.

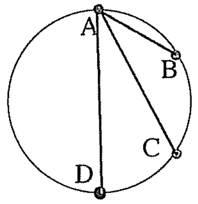


Color point C green. Use a blue pencil to draw one chord BC. Do not retrace the purple lines with blue.



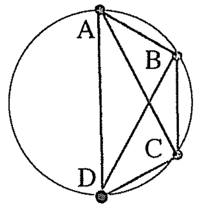
3 points

The third circle graphic has four points. Shade all four points. Shade point A purple, point B blue, point C green, point D yellow. Draw three chords that extend from the purple point to each of the other points. You will make three purple chords. Fill in the chart below the circles.

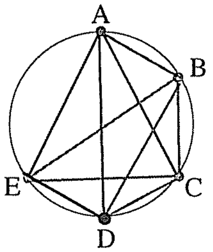


4 points

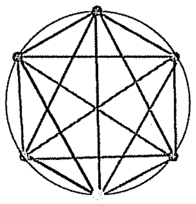
Draw blue chords between points BC and BD. There will be two blue chords. Finally, draw a green chord between points CD. There will be one green chord. Fill in the chart below the circle.



Color the five points with the same color pattern with the fifth point shaded orange. Beginning with point A, draw a purple chord to all the other four points. Move to the blue point and draw blue chords BC, BD, and BE. Next, move to the green point and draw chords CD and CE. Finally, make yellow chord DE. Fill in the chart below the circle.



5 points



6 points

For six points the sixth point is red. Draw in all the chords. Each point is connected to all the points clockwise around the circle. Fill in the chart below the circle. Check the T chart on Circle Segments - Worksheet 1.

Note

You may want to make larger circles on another piece of paper. Use a plate or a compass to trace circles. These take time, care, and precision to make and count. Make the line segments with a sharp colored pencil. Use a different color of your choice for the chords coming out of each point. Do not retrace chords that have already been made. You will see a pattern in the number of new line segments coming out of the points. The number will drop by one as you advance around the circle.

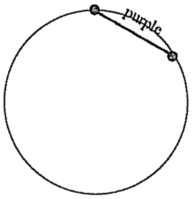
Extension

This pattern is an adding pattern that also shows up when counting points in a triangle. Go back to Pascal's Triangle - Worksheet 1, page 1. Count up the number of white triangles in each row. Add the total number of white triangles in row one to the total in row two. You will get a sum of three. Then add on the number in row four. The sum will be ten. This is the same pattern that shows up in the chords pattern in the circle chords.

This pattern when made with equally spaced points can be sewn on a piece of black velveteen with bright colored embroidery thread. It is stunning.

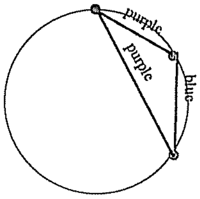
Circle Segments - Worksheet 2 answers

2 points



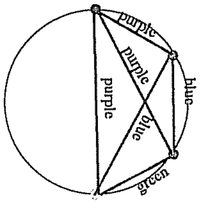
1 purple chord
1 total

3 points



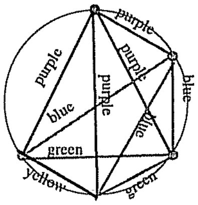
2 purple chords
+ 1 blue chord
3 total

4 points



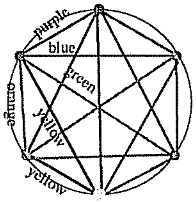
3 purple chords
2 blue chords
+ 1 green chord
6 total

5 points



4 purple chords
3 blue chords
2 green chords
+ 1 yellow chord
10 total

6 points



5 purple chords
4 blue chords
3 green chords
2 yellow chords
+ 1 orange chord
15 total

Only the new color chords were added.

Area and Perimeter

Purpose

The purpose of this lesson is to review the concepts of area and perimeter of rectangles and extend the review to a pattern-finding lesson that leads to a general relationship between area and perimeters of all rectangles. This lesson also introduces the concepts of maximization and minimization.

Prerequisites

Previous lessons on area and perimeter of rectangles, areas of triangles, and areas of parallelograms covered in *Patterns in Arithmetic:* General Math - Booklet 5

Materials

Area and Perimeter - Worksheets 1 - 3, pages 7 - 9
12 x 18 piece of construction paper
1 centimeter grid paper
Scissors
Tape or paste

Warm Up

Review the formula for calculation of area and perimeter of rectangles. Area = length times width, or $A = l\ w$. Perimeter is the sum of the measures of all four sides. The formula of perimeter is the sum of the two lengths and the two widths.
$P = 2\,l + 2\,w$
Area and Perimeter - Worksheet 1 Have the student write the procedures in words and with algebra. This can also be done in her Math Journal if it has not been completed in the past.

Lesson

Study in Structure: Area
"The areas are all going to be twenty-four square centimeters. Do you think the perimeters will be equal also?" Most students will not know or assume the answer is yes.

Have the student find all the rectangular shapes she can that have twenty-four squares in them. This is similar to the exercise done in Prime Factoring.

Answers: The possible rectangles are 1 x 24, 2 x 12, 3 x 8, and 4 x 6. A 1 x 24 has a perimeter of 50 cm., 2 x 12 has a perimeter of 28 cm., 3 x 8 has a perimeter of 22 cm., and the 4 x 6 has a perimeter of 20 cm.
After she has them all cut out, ask, **"Do you think all the perimeters are of equal length?"** "They are not equal."

Have the student paste the rectangles onto the construction paper in order from the longest perimeter to the shortest one.

The general rule is that for a given area, the greatest perimeter will be achieved by making a long, skinny rectangle. The least perimeter for a given area is to make a fat rectangle as close to a square as possible.

Area and Perimeter - Worksheet 2, A Study in Structures: Perimeter, page 8

Repeat this same exercise, but this time it is the perimeter that is held the same at twenty-four centimeters long.
"If several different shaped rectangles have the same perimeter of twenty-four centimeters, will the areas be the same too?" She may be warier this time.

Note

Some students will have difficulty with the idea of creating a rectangle with a specific perimeter. This line of questioning may help:
"What if the width of the rectangle is one centimeter? How long could the length be?" "Eleven." The two widths each take up one centimeter for a total of two centimeters. That leaves twenty-two left. The twenty-two must be split in half to make the top and the bottom lengths. So the rectangle will be 1 x 11.

Many students will say twenty-three, forgetting that the rectangle has four sides, not just two. (A rubber band is a useful manipulative here. Put a finger into each end of the loop and stretch it out to make a rectangle. She can see that there are two widths and two lengths. The total must be twenty-four.)

Have her draw this out and count the centimeters as she draws a line around the edge of the rectangle.

The perimeter will be 1 + 1 + 11 + 11 = 24. The area will be eleven square centimeters.

"What if the width is two centimeters? Then how long will the length of the rectangle be?" "2 + 2 is 4. So there are twenty centimeters for the two lengths,which means each length is ten centimeters."

Test for Understanding

"What will the area of this rectangle be?" "Twenty square centimeters, or 2 x 10."

Answers:	1 + 1 + 11 + 11 = 24	1 cm. x 11 cm. = 11 sq. cm. in area.
	2 + 2 + 10 + 10 = 24	2 cm. x 10 cm. = 20 sq. cm. in area.
	3 + 3 + 9 + 9 = 24	3 cm. x 9 cm. = 27 sq. cm. in area.
	4 + 4 + 8 + 8 = 24	4 cm. x 8 cm. = 32 sq. cm. in area
	5 + 5 + 7 + 7 = 24	5 cm. x 7 cm. = 35 sq. cm. in area
	6 + 6 + 6 + 6 = 24	6 cm. x 6 cm. = 36 sq. cm. in area.

Now complete the bottom section of Area and Perimeter - Worksheet 2 on Rules of Relationships. Check her answers to make sure they are correct.

Practice Worksheet

Area and Perimeter - Logic Puzzle, page 9 The lines are not to scale intentionally. Tell her not to eye ball the numbers. The total areas of figures B – K must add up to the total area of the large square A, which is 17 x 17, or 289 square units.

Note

Review calculation of area of triangles and parallelograms. She will have to use her logic to figure out the area of each shape.

Sample logic for rectangle C is as follows: The right hand length of rectangle B is six. (Shade this in with a color.) The little section below this in rectangle C must be equal to one because the length of Figure F is eleven and the combined length of figures E and K are twelve, so the difference is one. This means that the length of rectangle C is seven. The area then is 4 x 7 or 28.

Test for Understanding

Pose these questions.

1. If you had thirty-six feet of fence and you wanted to make a dog kennel with the greatest area for your dog to play in, what should be the length and width of the dog kennel? Length = 9 feet Width = 9 feet

2. If you are a general at a fort and you want the smallest amount of perimeter for your men to defend but the greatest area inside for them to live in, what shape should you make your fort? The shape should be the most square rectangle.

Geometry of Circles

Purpose

The purpose of this lesson is to develop fluency with the geometric vocabulary related to circles. This lesson is not an independent lesson. There are activities which must be teacher-guided.

Prerequisites

Circle Segments

Materials

Geometry of Circles - Radius and Diameter, page 10, Geometry of Circles - Circumference, page 11, and Geometry of Circles - Pi π, page 12
Meter stick
Several firm circular objects such as CDs, DVDs, plates, lids, Frisbees, etc.
Strip of paper that is at least 45 centimeters long
Calculator
Optional: Modeling clay

Warm Up

Review the vocabulary and labeling of points and line segments. Have him use a ruler to make a straight line and place a point on both ends of the line, then label each point with a capital letter; make at least three labeled line segments (label one line segment HT). A line segment is identified by calling out the letters on each end of the line segment. Call out the letters of each line segment and have him point to the correct line segment.
Example: Line segment HT. H•———•T

Activity

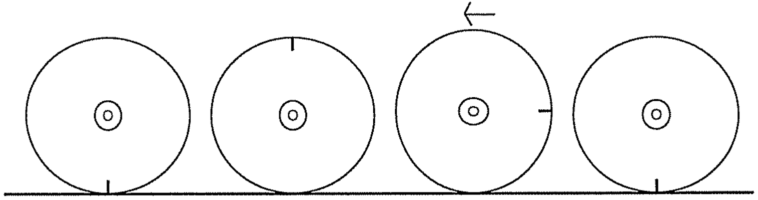
Measuring a Circumference

Before beginning the worksheets on Circumference, do this activity.

The length of a circumference is measured by rolling a circular object on its edge one full turn. Wheels are measured this way.

Get a forty-five centimeter strip of paper and a DVD. **"Guess how many centimeters the circumference is of this DVD."** The answers will vary.

Make a mark on the very edge of the DVD. Place the DVD up on its edge with the point you made at the bottom in contact with the paper. Make a mark on the paper where the marked edge of the DVD is touching the paper. *Carefully* roll the DVD along a straight line down the paper. Roll it all the way around until the point you made on the DVD is again in contact with the paper. Mark that point on the paper. Now measure the length of the line from the start point of your roll to the end point of your roll. This is how the length of a curved line is measured.



Use centimeters to measure; it is much more accurate. Record fractions of centimeters as decimals. For example, three centimeters and four millimeters would be written as 3.4 cm. This will make your calculations much more accurate. Repeat two more times to be sure you get an accurate measure. All three measures should agree very closely. Record your measurements on a piece of paper. Now measure the length of the diameter of the DVD. Record that with your data on a table like this:

Object	Circumference = C	Diameter = d	C ÷ d = ?
DVD	38 cm.	12 cm.	3.16
Yogurt lid			

When he has measured at least five different circle circumferences and diameters, and recorded them on the chart, ask:
"Which is longer, the circumference or the diameter?" "The circumference is always longer."

"How much longer? What is the relationship between a diameter and the circumference? How many diameters fit into a circumference?" "About three."

"Use division to see how many diameters equal the length of a circumference for each of the objects you rolled. You can use a calculator."

"What pattern do you see?" "The answer always comes out to be about three and a bit more."

This is a good place to end the session.

Lesson

Do all three of these worksheets with the student. These are not independent activities. Use the Answer Key to be sure you are filling out the worksheets correctly. Have him read the page aloud and fill in all the blanks.

Worksheets

Geometry of Circles - Radius and Diameter, page 10
There are always two radii in a diameter. This is expressed in algebraic terms as diameter is equal to radius times 2, or 2r: d = 2r. A radius is always half of a diameter, so r = ½ d, or d/2.

Radius and diameter are easily measured with a ruler. A circumference is a curved line and defies accurate measure with a ruler or string, so we roll circular objects to measure the length of the edge. There is a famous pattern called pi, or π. The circumference of any circle is always equal to the length of three diameters. Actually, it is not exactly three diameters, but three and a bit more. This relationship is constant for all circles. This allows us to measure the diameter and then use π to calculate the circumference. Very cool. Students can discover this critical pattern by measuring and filling in the chart above. Do not tell him how to calculate π.

Geometry of Circles - Circumference, page 11
"Can you measure the circumference of a circle with a ruler?" "Yes, but it is really hard and not accurate because the edge is curved."

"How did you measure the circumference of the DVD?" "I rolled the DVD on its circumference one full rotation."

"Take out the table you made when you rolled things to measure the circumference. How could you use the pattern you found to figure out what the circumference of a circle is without having to measure it?" "I could measure the diameter of the circle and then multiply it by three and a bit more to get the length of the circumference."

"This three and a bit more is called pi and written like this, π."

Complete page 11 and check the answers in the Answer Key.

Optional Confirmation Activity

Use modeling clay and roll out thin logs and lay them on the circumference of the circles on the paper. Then peel the clay from the paper and measure the length of the clay log with a ruler. You can also then fold the circumference log into thirds and see if a third of the length of the circumference is about the same length as the diameter. Students enjoy this activity, and it confirms the pattern that there are three and a little more diameters in a circumference.

Practice Worksheet

Geometry of Circles - Pi π, page 12

Test for Understanding

Give this problem. Have him do the problem in his head first. **"The radius of a circle is three units. About how long would the circumference be?"**
"If the radius is three units, then the diameter is six units. The circumference would be about three times longer than the diameter. So the circumference would be a little more than eighteen."

Area of Circles

Purpose

To derive the formula to calculate the area of a circle from the formula for the area of a rectangle. The student cuts up a circle into small triangles and rearranges these parts to form a rectangle. The radius of the circle corresponds to the width of the rectangle. The circumference of the circle ends up on the top and bottom of the long side of the rectangle. Only half of the circumference equals the length of the rectangle. Through a carefully crafted series of steps, the student transforms the formula for a rectangle into the formula for a circle. This is powerful mathematics and illustrates the connection between geometry and algebra. It is more enlightening for the student to construct this formula the way Archimedes did it originally than to simply be told Area = πr^2.

This process also introduces the idea of the concept of limits, a critical concept used in calculus. In this case the cut-up circle is not really rectangular, but if you make those wedges really, really, really small when you reassemble them, they will be a rectangle because the curve of the circumference will disappear and for all intents and purposes be a straight line.

Preparation Note

It is very important for the teacher to do this activity herself before attempting it with a student. Follow the instructions and read the teacher guide as you go. This is new material for most adults.

Prerequisites

Area and Perimeter, Geometry of Circles

Materials

Area of Circles - Worksheets 1 - 4, pages 13 - 16, Circle City, page 17
Crayons
Good scissors with a sharp tip
A piece of colored paper cut in half

Warm Up

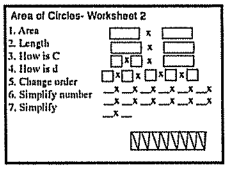
Give the student some area of rectangle problems, area of triangle problems, and some area of parallelogram problems.
"How is the formula for calculating the area of triangles and parallelograms based on rearranging the parts of a rectangle and using the formula A = lw?"
"All triangles are half of a rectangle. You can draw the rectangle around any triangle. The width of the rectangle becomes the height of the triangle. The length of the rectangle becomes the base of the triangle, $A = \frac{1}{2}bh$. The area of a parallelogram is just the triangle on one end cut off and pasted on the other end to form the rectangle. The width of the rectangle is the height of the parallelogram. The base is the same as the length, $A = bh$."

Pre-Assessment of Readiness for this lesson

Area of Circles - Worksheet 1, page 13, the top half
Have him fill in the blanks on the top half of this worksheet. Go over any weak areas before going on.

Be sure Steps 1 - 3 are done before doing the activity of cutting out the circle.

Lesson



Area of Circles- Worksheet 2
1. Area
2. Length
3. How is C
4. How is d
5. Change order
6. Simplify number
7. Simplify

Area of Circles - Worksheet 1, bottom half

"Cut along the dotted line to remove the circle. Then follow the instructions." Attach the colored paper with the cut up circle to the bottom right side of Area of Circles - Worksheet 2, page 14.

Note

For Mathematical Perfectionists: The figure looks like a parallelogram because we did not, and can't easily, cut the circle into small enough wedges to overcome the curve of the circle. The smaller the wedges are the more like a rectangle the figure will become. Try it. Cut the wedges as 32 or 64 and paste that together. Most students will not enjoy that level of perfection, but some will. You might. It is important not to use the formula for a parallelogram for this activity because the height of the triangle is not the radius. To derive the formula for a circle, you must imagine that this figure really is a rectangle, that the wedges are soooooo small that the height of the triangle and the radius are one. **"Is the area of this new figure, which we will call a rectangle, equal to the area of the circle you started with?"** "Yes, because I just cut it up and rearranged the pieces."

"Where is the circumference of your circle now?" "It is at the top and the bottom of the rectangle."

"What fraction of the circumference is on the bottom edge of the rectangle?" "Half of it."

"Draw a rectangle. What is the bottom edge of a rectangle called?" "The length."

"Under the bottom edge of your figure, put a capital L for length and write what it equals in the circle parts." "L = $\frac{1}{2}$ C."

If the student can not answer: **"Now, this bottom edge we are calling the length is what fraction of the circumference of your original circle?"** "Half."

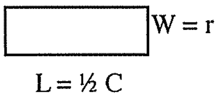
"How do you know this?" "The bright colored line used to be the circumference of the circle. When I cut it up and rearranged the pieces, half of the circumference is on the top of the rectangle and half is on the bottom."

"Would you agree then that the length of the rectangle, the bottom line, is equal to half of the circumference of your original circle?" "Yes."

"Write that as an equation." "L = $\frac{1}{2}$ C."

"After the capital L below the rectangle write = ½ C. This means the length of the bottom line of this rectangle is half of the original circumference. So, we have found an equivalency between a part of the circle and a part of the rectangle. We can use that to help calculate the area of this new figure."

"What part of your circle is on the short side of the rectangle?" "The radius."



"How do you know this?" "Because I put squiggly lines on the radii in my original circle. **"What do we call the short side of a rectangle?"** "The width."

"Label that on your figure and write what it is equal to in the circle measurement. The short side is the width, and it is the radius of the circle." "So I write W = r on one side of the rectangle."

Given the work you have done to this point, take some time to think about and play around with the formula for the area of a circle."

After you and your student have had a chance to exercise your mind, go to the top of Area of Circles - Worksheet 2. The parts of the circle, now shaped like a rectangle, are going to be used to figure out how to get the area, just like we did with triangles and parallelograms.

Teacher Prep Note

Use the Answer Key to help you move through this next section.
Start with line 1. **"When the area of a rectangle is calculated, how is it done?"** "Multiply the length times the width."

Important

"Do we use both the top and the bottom length?" "No, we use only one of them." Have him write L in the first box and W in the second. $= \boxed{L}$ x $\boxed{W}$

"What circle parts did you say was the same as the length of the rectangle?" "Half of the circumference."

"In line 2, under the L, write ½ C. We just changed the rectangle part into a circle part."
"What circle part did you say was the same as the width of the rectangle?" "The radius."

"In line 2, under the W, write r for radius in the second box. We again changed the rectangle part into a circle part."

"In line 3, we are going to split the number ½ away from the C. You will see why in a minute. Under the ½ C you see two arrows pointing to two smaller boxes. Put the ½ in the first box and the C in the second one. In the third box under the r, write the r again. We are not going to change that."

"Why is it OK to split these into two boxes and maybe rearrange the parts later? What property allows us to do this?" "The Associative Property of Multiplication because ½ C means ½ times C. Since all the signs are multiplication signs, they can be broken up and rearranged if we wish to." You may have to tell him this answer. He may not make the connection. In a class, usually someone will know.
"Why is the length of a rectangle called ½ C again?"
So now we have ½ x C x r written in line 3.

"Why would it be hard to measure the circumference of a circle with a ruler?" "Because it is curved."

"What part of the circle is easier to measure accurately?" "The diameter because there are two points on the circle and the center point. If there is a clear center point, you could also measure the radius."

"Which one of these easy-to-measure circle parts shows up in the rectangle at the top of the page?" "The radius."

"So in Line 4, we are going to change the hard-to-measure C into the easy-to-measure r, which shows up in the figure above."

"Go back to your formula for C. How can you calculate C using the radius?" "Pi times the diameter is equal to C. $C = \pi D$."

"How many radii are in a diameter?" "Two."

"How can you calculate C using the radius?" "Pi times radius times two."

"In line 4, you can see where we are going to turn that C which is hard to measure into radii. First, drop down the ½ from the box above. Where did that ½ come from again?" "Half of the circumference."

"Under the C in line 3, you see three arrows. That shows we are going to break up the C into its parts using the radius. We are going to separate all the pieces of the C into their own boxes."

"How is the circumference calculated using the radius?" "Pi times radius times two."

"What property of multiplication is used to do this?" "The Associative Property."

"In the boxes under the three arrows that point from the C, write each part that you use to calculate circumference with a radius. Put only one part in each box." Order does not matter. Many students will write pi in, then r, then 2 because that is how they think of it.

"Does the order matter?" "No, because we are using multiplication."

"In line 5, we are going to reorder the parts and put things that are alike next to each other. We always put numbers first. What are the numbers?" "½ and 2."

Have him write ½ in the first space and 2 in the second space. ½ x 2
"Pi is a number, so put that next." Have him do this. Now you have ½ x 2 x π.

"What is left?" "The two rs." Have him write these now. So you should have now on line 5: $\frac{1}{2} \times 2 \times \pi \times r \times r$.

"Now simplify. Can we get rid of any boxes?" Wait.
You are listening for $\frac{1}{2} \times 2$ is equal to one. Some students will see the r squared also.
"So, on line 6 put a 1 in the first blank and drop everything else down. What do you have now?" "$1 \times \pi \times r \times r$,"

"Can we drop anymore boxes out?" Wait. "What is r times r?"

Note

It is common for students to want to say 2 r. That would be r + r not r times r. Refer back to how exponents are used to show the prime factors of numbers.

"Now on line 7, we have only circle parts with all the extra stuff taken out or simplified $A = \pi r^2$, which is the formula to calculate the area of the circle from the formula to calculate area of a rectangle."

Cool, don't you think? Now that is a thing of beauty! Mathematics rocks! Thank you, Archimedes.
Look up Archimedes. He is a very important and interesting figure in history of both mathematics and science. Read how he died and why. That is fascinating too.

Practice Worksheets

Area of Circles - Worksheets 3 and 4, pages 15 and 16

Test for Understanding

Require him to write out clear answers to these questions. Open book.

1. Why did we cut up the circle and rearrange the parts?
   To make the circle parts into a rectangle.
2. Why did we do that?
   To make it easy to calculate the area. The area of a rectangle is easy to do.
3. What circle part did you use to be the same as the rectangle part called length?
   Half of the circumference of the circle.
4. What circle part did you use to be the same as the rectangle part called width?
   The radius.
5. In line 6, you ended up with two radii in your formula. One came from a rectangle part - the squiggly line. Where did the second one come from?
   It came from changing the C, circumference, from π d to π 2 r. We used the radius to calculate the circumference.

Challenge Test for Understanding

Explain how the formula for a circle comes from the parts of a rectangle. Use what is on Area of Circles - Worksheet 2 and explain every step in words. Explain where each part comes from and how and why you changed the pieces or moved them around. Use the line numbers to help you explain what is going on.
The answer would be a written version of the conversation in this lesson.

Worksheets

Circle City, page 17 - a page for fun and creativity
Variables, page 18 There are no instructions as this worksheet is self-explanatory.

Playing with Parentheses

Purpose

The purpose of this lesson is to review and solidify operations with Parentheses and the identification of the properties of multiplication. These concepts are foundations for working with equations in algebra. The lesson also works with finding patterns, making rules for them, and then trying to think out why the pattern works the way it does.

Prerequisites

Experience with the properties of arithmetic–Commutative, Associative, and Distributive–is helpful.

Materials

Playing with Parentheses - Worksheets 1 - 5, pages 19 - 23
Order of Operations, page 24 (There are no instructions; the page is self-explanatory.)

Warm Up

Do a speed drill on addition, subtraction, multiplication, and division facts.
Review the terms *Commutative*, *Associative*, and *Distributive* and give arithmetic problems that show each property.
Examples:
For Addition and Multiplication
Commutative: 3 + 4 = 4 + 3 3 x 4 = 4 x 3
Associative: (3 + 4) + 5 = (3 + 5) + 4 = (4 + 5) + 3
(3 x 4) x 5 = (3 x 5) x 4 = (4 x 5) x 3
For Multiplication and Division
Distributive: 96 x 3 = (90 x 3) + (6 x 3)
96 ÷ 3 = (90 ÷ 3) + (6 ÷ 3)

Lesson

"What do these marks, ( ), tell you to do?" "Do what is inside them first."

"They are also used to group numbers. Parentheses are often used in two-step word problems. For example, how would you write out the number sentence for this problem?"
You buy five ribbons for two dollars each and nine balloons for three dollars each. How much money are you spending? *Require* him to write out the number sentence.
Answer: (5 x $2) + (9 x $3) = $10 + $27 = $37

"What answer would you get if you did not have the Parentheses and just combined the numbers from left to right?" "5 x 2 + 9 x 3 = 57."

"Which answer is correct?" "The first one."

"Prove it." The answers will vary. He may draw a picture.

"Why did the second, incorrect answer come out to be a much larger answer than the first one?" Most students will not be able to easily answer this question.

Answer: Multiplication is more powerful than addition. Adding the nine to the (5 x 2) makes the number being multiplied nineteen instead of just nine, so the answer goes up by twenty because you multiplied the ten (from 2 x 5) by three instead of just adding it onto the twenty-seven. This concept is explored on Playing with Parentheses - Worksheet 2.

Now begin Playing with Parentheses - Worksheet 1, page 19

Assessment

Ask him to answer the first question by thinking about the properties of addition rather than working with the numbers right away. This will let you know how well he is using the principles of arithmetic to formulate an answer. This is developing in students at this age.

Have him confirm his answer with the numbers. If he cannot answer the question theoretically, have him try the numbers and see what happens and write a rule. Require him to make up another example to see that this pattern works no matter what the numbers are.

Answer: In addition, it does not change the answer if you change the grouping or the order in which you combine the numbers. This is the Associative Property of Addition. In simple terms: In addition, order of combination does not matter.

Most students will not know what to say about subtraction or division having never really encountered a sequence of subtractions.

Note

Playing with Parentheses: Worksheets 2 - 4 give number sentences with mixed signs. Can the Commutative and Associative properties work if all the signs in the number sentence are not the same? The answer is *no*. Mixed signs create another set of situations. Misuse of grouping symbols is a large source of errors in algebra.

Playing with Parentheses - Worksheet 2 explores the power of multiplication and provides practice at writing rules.

Insist he use proper mathematical language when writing rules.
Example: Adding to create the largest sum possible before multiplying will always produce a larger product.

Playing with Parentheses - Worksheet 3 explores the power of the divisor in division and provides practice at writing rules. The larger the divisor is, the less the quotient will be. Divisors really cut the size of numbers down. This is very powerful. This is explored in the first set.

In the second set, the Parentheses work on increasing the dividend versus increasing the divisor. Since divisors are so powerful at reducing numbers, the more numbers exposed to a divisor, the lower the answer will be. So, by dividing first and then adding, the second number, the one being added, is not exposed to the divisor.

Note that the rules for multiplication and division are the inverse of each other. In multiplication, you want to add first and then multiply to get the largest answer. In division, to get the largest answer you divide first and then add.

Practice Worksheets

Playing with Parentheses - Worksheets 4 and 5, pages 22 and 23
Order of Operations, page 24

Test for Understanding

Put the parentheses in these number sentences so that the correct answers are obtained.

Problems	Answers
1. 3 x 3 + 5 = 24	3 x (3 + 5) = 24
2. 9 + 6 x 3 = 27	9 + (6 x 3) = 27
3. 14 ÷ 2 + 12 x 3 = 3	14 ÷ (2 +12) x 3 = 3
4. 14 ÷ 2 + 12 x 3 = 43	(14 ÷ 2) + (12 x 3) = 43

Families of Facts: Missing Numbers

Purpose The purpose of this lesson is to extend the now well-developed understanding of Families of Facts to algebraic representations of this pattern. The pattern will then be used to solve single variable equations.

Prerequisites Previous experience with Families of Facts is helpful. The concept is reviewed in this lesson.

Materials Families of Facts: Missing Numbers - Worksheets 1 - 6, pages 25 - 30
Blue, yellow, and green crayons or colored pencils
Clear yellow and blue chips or cellophane is helpful if the student is not familiar with this concept.

Warm Up Using the yellow and blue chips, experiment with this pattern. Yellow + blue is green. Place the blue chip over the yellow chip so the color she now sees is green. (Holding them up to the light will make it clearer.) Repeat beginning with the blue chip. Blue + yellow is also green. Green take away yellow is blue. Green take away blue is yellow.

On Families of Facts: Missing Numbers - Worksheet 1, work with 7 + 5 = 12 first. Color in the circle with the seven in it yellow, with the five blue. Fill in the twelve and color it green. Note that the sum is in a box.

Yellow + blue is green. Blue + yellow is also green. The circles show that the order is reversed. Fill in the twelve again and color the circles and box. Keep the five as blue, the seven yellow and the twelve as green.

"What comes next?" "Green take away yellow is blue." Shade in the circles and box and fill in the numbers 12 – 7 = 5.

"What comes next?" "Green take away blue is yellow." Have her shade in the circles and box and fill in the numbers 12 – 5 = 7.

"Does this same pattern hold for multiplication and division?" "Yes." Have her shade in the circles and boxes and fill in the numbers.

Have her complete the rest of the page alone. This should be easy and nearly automatic at this point.

Lesson Part 1 Families of Facts: Missing Numbers - Worksheet 2
"Choose two numbers under twenty and use them to fill in the circles and the boxes." Give her time to do this. Require her to color in the circles and squares.

"Does the color pattern change when you change the numbers?" "No."

"When a pattern stays the same even when the numbers are changed, we can use letters to stand for any number and show the general pattern with algebra. Algebra is the language of mathematical patterns."

Have her read the paragraph at the bottom, then fill in the numbers.
"Can you write all four sentences using only a + b = c?" If she has difficulty have her shade the a as yellow, the b as blue and the c as green. Then she can copy the color pattern and put in the letters.

"Go back to Families of Facts: Missing Numbers - Worksheet 1, and write a, b, and c over the colors and numbers. Does the pattern stay the same?" "Yes."

Repeat this procedure for the multiplication and division sentences.

Test for Understanding

1. Give this number sentence: J + Q = W See if she can write the other three sentences with the same letters. Allow her to use the colored pencils if needed. Answer: J + Q = W, Q + J = W, W – J = Q, W – Q = J

2. Give this sentence: T – W = P. See if she can write the other three sentences. Answer: W + P = T, P + W = T, T – P = W

Lesson Part 2

Families of Facts: Missing Numbers - Worksheet 3
Tell her that the sign +, -, x, or ÷ goes into the triangle. These are called operators.

"What sign goes into the triangle on sentence number 1?" "Plus, because seven plus nine is sixteen."

"Can you complete sentence 2?" "Yes, I know it is 9 + 7 = 16 because the box is at the end of the sentence."

"What would it mean if the box were at the beginning of the sentence?" "That it was the subtraction number sentence."

Note

This next part is critical. We are going to link what she knows about Families of Facts patterns and use them to solve equations. The procedure you memorized back in school to subtract the number from both sides is based on this pattern.

Let her think about the question in the center of the page. The question is: If you have the equation a + 9 = 16, what equation do you use to find the value of a?

We do this nearly automatically in our thinking, but slow down and think about it. You either think what plus nine is sixteen and count up or you subtract nine from sixteen. The latter is what is done in algebra or when there are large numbers.

"What are the other three sentences that go in the family with a + 9 = 16?" Have her write them out. "9 + a = 16, 16 – a = 9, and 16 – 9 = a.

"Which one says = a?" "16 – 9."

"Write that on the line. That is how we find out the value of a, by subtracting nine from sixteen."

"Find j + 11 = 21; you see below this another number sentence with symbols. Look up at the top of the page to see the other number sentences with those same symbols. What number should go in the box here?" " Twenty-one."

"What operator is needed?" "Subtraction, so I put a minus sign in the triangle and the eleven in the circle."

"So, what number sentence tells you what j is equal to?" "21 – 11."

Watch as she finishes the page. Check the work as soon as it is completed. Now repeat the same thought pattern to complete Families of Facts: Missing Numbers - Worksheet 4. Check the work as soon as it is completed.
Try one harder problem from Families of Facts: Missing Numbers - Worksheet 6 together before you assign the practice pages.

Practice Worksheets

Families of Facts: Missing Numbers - Worksheets 5 and 6

Test for Understanding

Give a problem similar to the ones on Families of Facts: Missing Numbers - Worksheet 6, but change the order and put the = value at the beginning and see if she can work her way through it.

Example: 45 = 52 – x What is the equation that will tell you the value of x?
52 - 45 = 7

Families of Facts: Vertical Format

Purpose The purpose of this lesson is to connect the lessons learned in Families of Facts: Missing Numbers to the Vertical Format used when checking addition and subtraction problems and solving basic equations using larger numbers.

Prerequisites Previous lessons

Materials Families of Facts: Vertical Format - Worksheets 1 and 2, pages 31 and 32

Warm Up Families of Facts: Vertical Format - Worksheet 1
In the second row, there appears a single problem in all its Families of Facts formats. She is to find the missing number in each problem. The answer is the same in all of them. Then she determines which of the four gives the most direct answer. The best way of course is to use the format of the last problem.
At the bottom of the page she is asked to see that if the subtracted number and the answer do not sum up to be the starting number, then the answer is incorrect.

Lesson Families of Facts: Vertical Format - Worksheet 2
The top two rows of the page have problems in pairs. Do the first two problems with her. Then move on and have her finish on her own.

First comes a subtraction problem with a box around the answer. It is followed by an addition problem with a box for the missing number, which will be the same number as the answer in the preceding subtraction problem. She should copy the answer to the subtraction problem into the box in the addition problem. Then she adds the two numbers to see if the sum matches the Start With number. If it does not, she should redo the subtraction to check for an error.

In the bottom section, do the first two problems on the second half of the page with her. There is a problem with a variable 'q' instead of a box for the missing number. Where is says q =, she should write 943-278 = _____. Using the margins of the page to do the calculations, she should do the subtraction to find q, and then add the value of q to 278. She is asked a question on the right side of the problem, 'Does q + 278 equal 943?' If it does not, then the answer she came up with for q is incorrect and she should redo the problem.

Practice Worksheet Complete Families of Facts: Vertical Format - Worksheet 2 independently. Check the work immediately.

Test for Understanding If she can do the problems at the bottom of the page independently and get the correct answer, she is demonstrating that she understands how to use the Families of Facts pattern.
In the "Make your own' section, give her this: $K - r = P$, what is the number sentence that will give you r? Answer: $r = K - P$

Families of Facts: Multiplication and Division

Purpose The purpose of this lesson is to connect the Families of Facts pattern to multiplication and division and then use that pattern to check the accuracy of calculations and to solve simple equations. This lesson will feel the same as the last lesson except the student will be using multiplication and division instead of addition and subtraction.

Prerequisites Previous lessons and proficiency in single digit division

Materials Families of Facts: Multiplication and Division - Worksheets 1 - 6, pages 33 - 38

Warm Up Families of Facts: Multiplication and Division - Worksheets 1, 2, and 3 review the basic relationships of fact families in multiplication and division and using those relationships to check answers to division problems. Do these pages with him. Check the work immediately using the Answer Key.

Lesson Families of Facts: Multiplication and Division - Worksheet 4
Do the top row with him. He will need some scratch paper to do the calculations. He already knows the basics of this process. Solve 228 ÷ 4, placing the quotient into the circle. Then fill in the number sentence below. Have him check his division by multiplying 57 x 4 to check that it does indeed equal 228.

Then skip down to 432 ÷ q = 9.
"Write out the four number sentences that would be in this fact family." If he has difficulty, use Families of Facts: Multiplication and Division - Worksheet 2 as an example. "432 ÷ q = 9 is the first one. Then there would be q x 9 = 432, 9 x q = 432, and 432 ÷ 9 = q.

"Which problem will give you the value of q?" "432 ÷ 9 = q."

"What did you get for the quotient?" "Forty-eight."

"Does q x 9, or 48 x 9, equal 432?" "Yes."

"What would it mean if it didn't?" "That I made a mistake somewhere."

Practice Worksheet Families of Facts: Multiplication and Division - Worksheets 5 and 6

Test for Understanding Give the problem A ÷ B = C. **"What does A equal?"** "B x C."

Relationships

Purpose The purpose of this lesson is to use algebraic thinking to express the inverse relationships between numbers in multiplication factors used in number sentences.

Prerequisites Previous lessons on Relationships in General Math: Booklet 5

Materials Relationships, page 39

Warm Up Review factors: Have the student write all the factors of 20: 1 x 20, 2 x 10, 4 x 5.

Lesson **"Fill in the missing numbers in the boxes. The numbers that go into the boxes are the factors of the number in the 'c' column."**

Note The numbers in the 'a' column in the first problem are ascending while the ones in the second problem are descending.

Have her fill in the answers to the three questions on the lower left. The first two relate to the work she has already done.

Assessment The third question requires her to use what she knows about multiplication. Have her think it out. Then use the 'Make your own example' column to test her response. Use the Answer Key to be sure the worksheet is filled out correctly.

Test for Understanding Change the number sentences to ÷ signs and ask the same three questions that are asked on the worksheet on the lower left. This is tricky because she is accustomed to seeing the c as the dividend. You are going to change it to a without mentioning it. This will cause her to rethink the relationships.
$a \div b = c$ If c remains the same and a gets larger, then b will ______________.

Answer: If the quotient remains the same and a is increased, then b must increase also. The relationship between a and b remains constant as it does in equivalent fractions.
Examples: $12 \div b = 4$ $(b = 3)$ $16 \div b = 4$ $(b = 4)$ $20 \div b = 4$ $(b = 5)$

$a \div b = c$ If c remains the same and a gets smaller, then b will ______________.
Answer: If the quotient remains the same and a is decreased, then b must be decreased also. The relationship between a and b remains constant as it does in equivalent fractions.
Examples: $12 \div b = 4$ $(b = 3)$ $8 \div b = 4$ $(b = 2)$ $4 \div b = 4$ $(b = 1)$

$a \div b = c$ If a remains the same and b gets larger, then c will ______________.
Answer: If the dividend remains the same and b, the divisor, is increased, then c, the quotient must decrease. The relationship between b and c is inverse.
Examples: $12 \div 2 = c$ $(c = 6)$ $12 \div 3 = c$ $(c = 4)$ $12 \div 4 = c$ $(c = 3)$

Properties: Review

Purpose The purpose of this lesson is connect algebraic statements of important number properties to those number properties shown with numerals.

Prerequisites Previous lessons

Materials Properties: Review - Worksheet, page 40

Warm Up **"In the Family of Facts, you wrote sentences like this: 4 x 5 = 20 and 5 x 4 = 20. Is it always true that you can switch the order of the numbers you are multiplying and still get the same answer?"** "Yes."

"Does this work for addition too?" "Yes."
"Can you remember the name of the property that these are examples of?" "Commutative."

"Is subtraction commutative?" "No."

"Show me an example." "9 - 7 is not equal to 7 - 9."

"What about division? Is 15 ÷ 5 = 5 ÷ 15?" "No."

Note There is a relationship between the answers, though. 9 - 7 = 2 and 7 - 9 = - 2. Is this pattern always true? Try it. In the division problem above the answers are not equal but they are related: 15 ÷ 5 is 3 and 5 ÷15 is one-third. The answers are reciprocals to each other.

"Find the prime factors of seventy-five. Then change the order of the factors and multiply them together. Does the order you multiply them in change the answer?" "No."
"What property of multiplication is this? Do you remember?" "The Associative Property."

Lesson Study the algebraic sentences that describe the patterns you just reviewed. Remind her that algebra is mainly a way of writing down patterns that do not change if you change the numbers.

Give the problem 32 x 5.
"How do you do a problem like this?" "You break up the thirty-two into thirty plus two and then you multiply the five times each of those numbers."

"Would you do the same thing if I gave you different numbers? What if I gave you 76 x 4?" "The numbers would be different, but I would do the same procedure."

"In a problem like this, you take the four to the seventy and then to the six. Do you remember the word for this process?" "Distribution."
Study the algebra for Distribution.
Work together to label each number sentence given in the middle of the page with one of the properties listed above. Check your work before you go on.
At the bottom of the page, make up examples of your own following the patterns above.

Practice Worksheet None

Test for Understanding None

Difference Between

Purpose

The purpose of this lesson is to solve a problem using a pattern.

Prerequisites

Basic understanding of subtraction

Materials

Difference Between - Worksheets 1 and 2, pages 41 and 42
Difference Between: Patterns - Worksheets 1 - 3, pages 43 - 45

Warm Up

Review the vocabulary of sums and differences.
"What is a sum?" "The answer to an addition problem."

"What is a difference?" "The answer to a subtraction problem."

Lesson

Have him do Difference Between - Worksheet 1 alone and look for patterns. Have him use correct mathematical vocabulary to discuss the patterns he sees.

Example: The first column in each set is a sequence of consecutive descending numbers. The second column is a sequence of the same consecutive numbers in reverse order.

The sums of each pair are always equal to twelve.
The differences show a sequence of even numbers. The lower half of the differences are negative numbers.

Worksheet

Difference Between - Worksheet 2
This time the sum and difference columns have a connecting arrow to show a pairing. For example 11 + 1 = 12 is paired with 11 – 1 = 10. In this example the sum of the two numbers is twelve and the difference is ten.

"Which two numbers have a sum of twelve and a difference of eight?" Have the student solve the equations.
"Write those two numbers on the left hand side of the paper under 'Two Numbers.' " 10 and 2

"What two numbers have a sum of sixteen and a difference of four?" 10 and 6
The worksheet provides the pattern blanks for him to fill in.

Practice Problems

Find a pair of numbers that have a sum of 30 and a difference of 6. 12 and 18
Find a pair of numbers that have a sum of 25 and a difference of 9. 17 and 8
Find a pair of numbers that have a sum of 100 and a difference of 93. Not solvable with whole numbers. All the differences are even numbers.

Practice Worksheets

Difference Between: Patterns - Worksheets 1 - 3, pages 43 - 45

Test for Understanding

None other than solving the practice problems.

Pattern Language Puzzles (Algebra)

Purpose The purpose of this lesson is to connect what students know about the Distributive Property of Multiplication to algebra and then use that connection to solve equations logically.

Prerequisites Previous lessons

Materials Pattern Language Puzzles (Algebra) - Worksheets 1 - 4, pages 46 - 49

Warm Up Pattern Language Puzzles (Algebra) - Worksheet 1 gives a little review of the distribution number sentence students have learned–the booklets on multiplication. The way the pattern works is that the number to be multiplied, in this case the twenty-three, is broken up into smaller numbers, in this case, 20 + 3. This is done so that the multiplication tables can be used. Each of these smaller numbers are then multiplied by the multiplier, which in this case is six. In the second line, the two smaller multiplication problems are shown. These are 20 x 6 and 3 x 6. On the third line, the products of these smaller problems are written and summed to find the final product.

This is how all long multiplication problems work. Choose another set of numbers like 97 x 6. The multiplier, in this case the 6, must be only a single digit number. The pattern is slightly different if you have a double digit multiplier, and the algebra is more complex.

Note The number being multiplied does not have to be broken up into tens and ones, it can be broken up many different ways in several parts. But what never changes is that each part of the broken-up number must be multiplied by the multiplier. Example: 12 x 7 can equal (8 + 4) x 7. The seven would be multiplied by the eight to equal fifty-six, and by the four to equal twenty-eight. 56 + 28 = 84.

The explanation the student needs to write is similar in wording to what is written above in the warm-up paragraph. Check the Answer Key to see how the final answer should appear.

Lesson Pattern Language Puzzles (Algebra) - Worksheet 2 To solve these problems the student will have to use the work-backwards strategy using what she knows about Families of Facts and missing numbers.

"How can you tell what *a* is equal to in the first problem?" "It must be equal to four because the nine is broken into a + 5. So the *a* must be equal to four."

"So what is (a x 6) equal to?" "Twenty-four."

She will write that number in the third line and check to see if the two partial products add up to fifty-four, a = 4.

Have her complete the page independently. Make sure she completes the problem where she makes up her own problem. Check the work as soon as she is finished.

Move on to Pattern Language Puzzles (Algebra) - Worksheet 3.
These problems offer a new complication. There are now two variables, Q and a. **"What numbers can you tell are in the second row of the distribution number sentence?"** "(a x 5) and (8 x 5)."

"You know what (8 x 5) is, so put that on the line in the third row." "It's forty."

"How can you figure out what (a x 5) is?" "I can subtract forty from the sixty. That will tell me that (a x 5) is twenty. So I can write that on the other line."

"What is the value of a then?" "It must be four because five times four is twenty."

"Now how do you find the value of Q?" "If a is equal to four, then on the top line (a + 8) x 5, the (a + 8) must be equal to twelve. So Q is twelve."

"All the problems on this page follow the same pattern. So finish the problems on your own."

Practice Worksheet

Pattern Language Puzzles (Algebra) - Worksheet 4

Test for Understanding

Using the 'Make your own' format, see if she can fill in the blanks correctly. Watch her do it and see what order she does it in. If she fills in line three first and works backwards, she is ready for the challenge problem. If she works forwards and fills in the final product last, give her another problem and ask her to start with the third line and work backwards. If she can not do that, her understanding is not quite there.

Challenge problem:

____ x ____ = (___ + ___) x _____
= (___ x ___) + (___ x ___)
____ = 45 + 54

To do this problem, she must think about what forty-five and fifty-four have in common. They have to have a common factor.

____ x ____ = (___ + ___) x _____
= (___ x 9) + (___ x 9)
99 = 45 + 54

The common factor is nine. So that is the multiplier.

11 x 9 = (5 + 6) x 9
= (5 x 9) + (6 x 9)
99 = 45 + 54

Now the other two numbers can be filled in.

Number Patterns: Functions

Purpose

The purpose of this lesson is to further develop the concept of an algebraic function by connecting a geometric pattern to a T chart and to a graph. The student builds a sequence of geometric structures that increase in size with a regular pattern. The structure number and total number of blocks in that structure become the x and y of the T chart and the coordinate points on the graph.

Prerequisites

General Math: Booklet 5 - Number Patterns lesson, or *GEMS - Algebraic Reasoning* (commercially available curriculum), What's My Rule?, see pages 33 and 34 in this booklet.

Materials

Number Patterns: Functions - Worksheets 1 - 9, pages 50 - 58, What's My Rule?, page 59
Rainbow cubes or Unifix cubes (Any kind of colored blocks will work. Dyed sugar cubes will work in a pinch.)

Background Information

If this written explanation does not make sense, do not be concerned. You will understand it by doing it. Some people like to see the map first.

Look at the Answer Key for Number Patterns: Functions - Worksheets 1 and 2 as you read this. You can see on the worksheet there are drawings of Structures A and B. Notice on the charts that the first column is for the Building Number and that each successive building follows a pattern. In the first building there is a single block. The next building adds a new block to the top and next to both sides. This same pattern is repeated in Building 3. When a series of block buildings is built, the number of the building, or a its position in the pattern, is labeled the independent variable and is given the letter x in algebra. On a graph–see Number Patterns: Functions - Worksheet 2–the Building Number is shown across the bottom on the horizontal line. This bottom horizontal line is called the x axis. This axis is labeled as 'The Building Number.'

The total number of blocks in the building is dependent on what Building Number it is. The more blocks being added according to the pattern, the more blocks there will be in each successive structure. Notice that the second column is labeled 'Number of Blocks Needed.' This second column is also called the dependent variable and is given the letter y in algebra. On a graph, the y is shown on the vertical axis.

The scale of numbers, or the numbering pattern, on the x and y axes do not need to match. Notice on Graph 1, on the x axis there are five little squares between number one and number two. On the y axis, there is only one little square between one and two. This is to stretch the graph out to make it easier to see the relationship between the points.

When placing the points on the graph, read across the chart. For Building 2 on Number Patterns: Functions - Worksheet 1, for example, see the number two and to the right in the next column, see the number four for the total number of blocks in Building Number 2. This is called an ordered pair (2, 4). An ordered pair is always graphed so that the first number, the 2, means go over two. The second number, the 4, means go up four. So (2, 4) means go to the right two (along the x axis) and up four (along the y axis) and put in a point.

Notice when all the points for the chart are placed on the graph, a straight line is formed. That is because there was a pattern that added three blocks to each new structure. When graphing a pattern and it makes a straight line, it is called a *linear pattern* or a *linear function*. For easy patterns, you will be able to figure out the relationship between the x and the y. These are called *functions*. A function is a formula that tells you what to do to the x to get y. $y = 2x + 4$ means to pick a value for x (which is why it is called the *independent variable*), multiply it by two and add four, and that is what y will be. The value of y depends on what number you choose for x. That is why the y is called the *dependent variable*.

Warm Up

Play 'What's My Rule?' or the 'Function Machine Game.' Instructions follow this lesson on pages 33 and 34. A recording sheet is also in the Student Workbook on page 59.

Lesson

Read the instructions on Number Patterns: Functions - Worksheet 1, page 50. Study the T chart. Have the student build Building 1 with a red block. To build Building 2, have him add on blue blocks to the red block. This will help him see the geometric pattern. Build Building 3 by adding on green blocks.

"What is the pattern?" "Each building adds on three blocks."

"How are you going to find out how many blocks are in the fiftieth building?" "I could keep adding threes, but that would take a while."

"Can you figure out what the rule is for each pair? For example: What can you do to a two to make four that is the same as what you would do to a three to make seven, or a four to make ten?"

Hint 1: The pattern is easier to see with the larger numbers: (4, 10), (5, 13), (6, 16).
Hint 2: How many 'arms' are in each building? Three.
Hint 3: Look at Building 3. What multiplication problem is formed by the blue and green blocks? 2 x 3. What about Building 4? Three times three.
Answer: The function is $B = 3A - 2$, or in common language, multiply the Building Number by three and then subtract two to get the total number of blocks.
"Where does the x 3 come from?" "From the three arms of the building."

"Where does the -2 come from?" This one is really tricky. To see the minus two, look at Building 2. How many blocks do you need to add to make the model be 2 x 3? Add the blocks in. You can see that 2 x 3 makes six blocks, which is two more than the building has. Those two blocks must be removed to make the building in the picture.
Look at Building 3. To get the total, the function says to multiply 3 x 3 and then

take away two. Move your cubes around and then add what you need to make a model of 3 x 3, three columns of three. How many blocks did you have to add to the structure to make 3 x 3? Two more. That is why the function requires a minus two at the end.

Cool, don't you think? Finish the T chart, check the Answer Key, and then graph the first six points on the graph on Number Patterns: Functions - Worksheet 2, page 51. Label the x axis 'Building Number' and the y axis 'Number of Blocks Needed.' Remember, the first number in each pair is graphed on the x axis, and the second on the y axis. Check the Answer Key to be sure the graph is completed properly.

Repeat this same process with Structure B. This time there are five 'arms' on each building. The function this time is B = 5A - 4. Graph the function on Number Patterns: Functions - Worksheet 3, page 52.

Practice Worksheets

Number Patterns: Functions - Worksheets 4 - 8, pages 53 - 57

Note

Number Patterns: Functions - Worksheet 4, Table 1–page 53, graph it on page 54. When a T chart has a pair, (0, 8), it means that the function has a + 8 in it. You can tell because if x is zero, anything multiplied by it is also going to be zero, so the eight in the y column comes from an addition of eight. To find the function, subtract eight from the number in the y column. The resulting number will be the product of the x number times another number. The other number will be the same number for every pair in the table. For example: The chart says that if x is two, then y is 14. 14 – 8 = 6. So two times what number is six? Three. Try multiplying by three and adding eight onto the next pair on the chart. The next pair is x = 3, y = 17. So, take three times three and add eight. Yes, you get seventeen. So the function is y = 3x + 8.

Number Patterns: Functions - Worksheet 4, Table 2, page 53–graph it on page 55. Now when you graph this function, notice that the line crosses the y axis at + 8. You can see the x 3 because the line goes up three in between each point.

Number Patterns: Functions - Worksheet 4, Table 3, page 53, is kind of a stinker. Graph it on page 56. Graph it before figuring out the function. The location where the line crosses the y axis will tell you if anything is being added. You will notice that the line crosses the y axis at negative eight. This means that there is a subtraction of eight in the function. The pattern goes up by eights, which tells you there is a multiplication of eight going on. So the function is y = 8x – 8.
Number Patterns: Functions - Worksheet 4, Table 4, page 53–graph it on page 57.

Test for Understanding

Use Number Patterns: Functions - Worksheet 9, page 58, as an assessment. See if he can figure out the functions on his own from the tables or from the graphs.

Note

If he can not do these problems on his own, do not push. This topic will come up over and over in pre-Algebra and Algebra I.

What's My Rule?

Purpose

This lesson's purpose is learn to spot patterns in number series. When you are looking for a pattern that uncovers a hidden operation, you look at what number you started with and what number you ended with and try to figure out what happened to the numbers in between. This is one thing scientists and mathematicians do when they discover new 'formulas.' A game that exercises this skill is What's My Rule?

Activity

To play, think of a simple operation such as + 2. The student gives you a number and you add two to it and say the new number back. For example, if she says "one," you say "three." She says "five" and you say "seven." The number she gives you is called the independent variable, or the number in. The number you say back is called the dependent variable, or the number out.
Record the information on a chart like this:

IN	OUT
1	3
5	7
9	?

What's My Rule? +2

When a student thinks she knows the rule, let her predict the number out loud. Finally, when everyone can predict successfully, let someone formulate the rule of plus 2. Students love to make these up for each other using easy addition, subtraction, multiplication, and division operations.

Play this game regularly. It's a good rainy day and in-the-car game.

One teacher called this activity Black Box and made a symbolic black box out of a milk carton decorated with gears and levers with a slide inside that flipped a card upside down. A card would be put in the slot in the top of the box and come out so the number written on the back of the card came out a bottom slot. The students then guessed the rule. When the box appeared, the students' minds focused to discover the relationship between the In and Out number of the day.

Worksheet

What's My Rule? - Blank page to copy is on the following page.

Sample games:

IN	OUT
2	4
6	8
9	___
20	___

What's My Rule?___________

IN	OUT
7	4
3	0
10	7
8	___
20	___

What's My Rule?___________

*Answers: 9 11 8 5
20 22 20 17
Rule: Add 2 Rule: Subtract 3

What's My Rule?

Date ______________

IN	OUT
___	___
___	___
___	___
___	___
___	___

What's My Rule?______________

IN	OUT
___	___
___	___
___	___
___	___
___	___

What's My Rule?______________

IN	OUT
___	___
___	___
___	___
___	___
___	___

What's My Rule?______________

IN	OUT
___	___
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___	___
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___	___

What's My Rule?______________

IN	OUT
___	___
___	___
___	___
___	___
___	___

What's My Rule?______________

IN	OUT
___	___
___	___
___	___
___	___
___	___

What's My Rule?______________

Geometric Patterns

Purpose

The purpose of this lesson is to further advance the student's ability to extend a geometric pattern and use a graphic and a T chart to determine the function (the formula) for calculating the total number of dots in any figure of a certain shape.

Prerequisites

Previous lessons

Materials

Geometric Patterns - Worksheets 1 - 3, pages 60 - 62

Lesson Part 1

Break this lesson up and do one pattern each day.

Geometric Patterns - Worksheet 1
Sequence of Squares:
Have the student fill in the number of dots for the first four squares. Remind him that he is looking for a single arithmetic procedure that will give the correct answer for every square.

"What can you do to a two to get four?" "I could add two, or multiply by two."

"What can you do to a three to get nine?" "I could add six, or multiply by three."

"What can you do to a four to get sixteen?" "I could add twelve, or multiply by four."

"What is the same in all three cases?" "I multiply the Square Number (or the length of one side of the square) by itself. Or take the area of the square. Or 'square' each x number to get y."

Have him finish the chart. The nth number refers to any number you would like to put in.
"What is the general pattern, and how do you write it with algebra?" "$y = x^2$ or $nth = n^2$."

"Graph the first six terms of this function on the graph on Geometric Patterns - Worksheet 2."

Lesson Part 2

Sequence of Oblongs: The first thing to notice is that each oblong is an array that shows a multiplication problem.
The first oblong is: 1 x 2 = 2 dots.
The second is: 2 x 3 = 6 dots.
The third is: 3 x 4 = 12 dots.
The fourth is: 4 x 5 = 20 dots.

"What patterns do you see in the numbers of the multiplication problems (not the answers)?" "The first pattern is that the first number of the multiplication problem is the same as the Oblong Number. The second pattern is that the second number in the multiplication problem is equal to the Oblong Number plus one."

"So, what would be the number of dots in the fifth Oblong?" *Wait.* Give him time to think it out. "Thirty because the fifth oblong will be five dots high and (5 + 1) dots long."

"What would be the number of dots in the sixth Oblong?" "Forty-two because the next oblong will be six dots high and (6 + 1) dots long."

"So, how could you use this pattern to figure out how many dots would be in the 50th oblong?" "It would be fifty dots high and fifty-one dots long so it would have 2,550 dots."

It can be written in algebra by saying that the number of dots in an oblong in this pattern is N (oblong number) times (N + 1), the oblong number plus one. This is written as a formula by saying that the Number of Dots in each oblong, which can be called D, is equal to N x (N+ 1). If this is multiplied using the Distributive Property of Multiplication (multiply N by both numbers in the parentheses) we get this: N x (N+ 1) = (N x N) + (N x 1) = $N^2 + N$.

Go back and use this formula on the oblongs shown in the graphic. Do it for the fourth oblong. The N number is four. According to the pattern, to get the total number of dots in this oblong, multiply (4 x 4) and add four. That would give the answer of twenty dots, which is correct.

"Does this pattern work for the second and third oblongs also?" "Yes."

That means the function to find the total number of dots in the nth (or any oblong one wishes) is to take the N number, square it, and then add on another N. Cool, don't you think?

"Put the first six terms of this sequence on the graph on Geometric Patterns - Worksheet 2, Oblong Numbers. Label it. Compare it to the shape of the line for the sequence of squares."

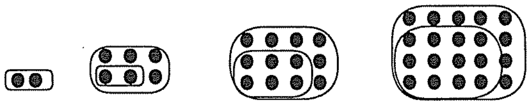
Note

Notice that the graph of the sequence of oblongs has a similar shape as the sequence of squares, but it is always above by the n number.
"If they are on same graph, will the two ever cross?" (The student may want to graph this on Geometric Patterns graph in a different color.) "No, they will get farther and farther apart as the n number increases."

Extension to the oblong pattern

The sequence of total number of dots in each oblong is 2, 6, 12, 20, 30, 42.... Have him look at the difference between the patterns, 6 - 2 = 4, 12 - 6 = 6, 20 - 12 = 8, 30 - 20 = 10, 42 - 30 = 12. The sequence of differences is 4, 6, 8, 10, 12. It goes up by two each time.

To see why, examine this:



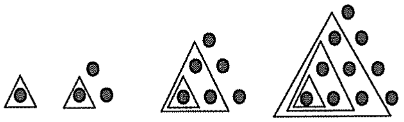
Have him draw this in on his picture.
"What patterns do you see?" "The sequence of differences is 4, 6, 8, 10, 12. It goes up by two each time."

"Compare the number of dots added on from one oblong to the next and the Oblong Number. What do you see?" "The number of dots added to Oblong 2 to make Oblong 3 is six. That is equal to two times the new Oblong Number."

Notice that in the fourth Oblong, there are eight new dots added; this is equal to 2 x 4. So if the Oblong Number is doubled, you can find out the number of dots that are added. This is another way to extend the pattern, but the algebra is much more difficult.

Lesson Part 3

Sequence of Triangles:
To analyze this pattern, look at Triangle 2. It has three dots in it. Now look at Triangle 3. Using a pencil, put a thin line around the three dots in the lower left hand corner of Triangle 3. You are sketching in Triangle 2. This helps to show what is added in the new triangle.



"What pattern do you see?" "Triangle 3 has three more dots than Triangle 2 does. Triangle 4 has four more dots than Triangle 3 does."

"How can this pattern help you figure out how many dots are going to be in Triangle 5?" "I take the total number of dots in Triangle 4, which is ten, and add five. There will be fifteen dots in Triangle 5."

Continue the pattern until the eighth triangle.
"Graph this pattern on Geometric Patterns - Worksheet 2, Geometric Patterns and label it. Compare it to the other two graphs. How is it the same? How is it different?" "This line is curved also, but not as steeply as the two before it because it uses only addition and not multiplication like the first two did."

This addition pattern will work up to Triangle 6 or even 10, but to do it all the way to 50 would be very tedious. What is needed now is the function. But the algebra is very difficult and is beyond students at this age.

Practice Worksheet

Geometric Patterns - Worksheet 3

Test for Understanding

None

Investigating Cubes

Purpose

The purpose of this lesson is to review the sequence of squares and extend that study to another related geometric pattern that is common in the world, the sequence of cubed numbers. The paper models show the power of exponents and develop a visual image of 'squaring' and 'cubing' a number.

Prerequisites

Previous lessons

Materials

Investigating Cubes - Worksheets 1 - 4, pages 63 - 66; 67 and 68 are grid paper
Orange pattern blocks or any other square, tile-like block
Linking cubes or centimeter cubes from the Base Ten Blocks set, sugar cubes can work in a pinch.
15 sheets of centimeter grid paper copied onto heavy white paper, page 41
Crayons
Glue stick
Cellophane tape
Pointed scissors
Optional: old newspaper

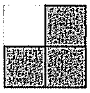
Note

This lesson takes several sessions to complete. The visual images are powerful.

Warm Up

Investigating Cubes - Worksheet 1 This is a review of the sequence of squares. Have the student build with the blocks and record only the numbers in the first two columns of the chart. These first two columns are 'Number of blocks on one side' and 'Total number of blocks.' She will build a model of this and record the numbers in the third column.

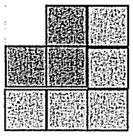
Visual Project: Using the centimeter grid paper, pages 67 and 68 in the Student Workbook, have her cut out a single square. Then have her draw the next square, a 2 x 2 that will have four centimeters inside. Have her color it red, then cut it out.



Glue the white square you cut first on top of the upper left hand centimeter of the red square. Three small centimeter red squares will be sticking out. Those three red squares are the squares that were added to the total when she went from one squared, to two squared. This 3 should be entered into the third column, the 'Number of blocks added.'

So the third row of the chart now reads:

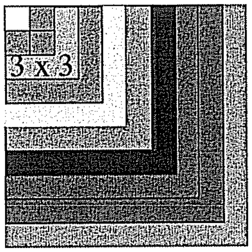
Number of blocks on one side	Total number of blocks	Number of blocks added	Number squared
2	4	3	2



Draw a square that is 3 cm. x 3 cm. Color it Spring Green (chartreuse, the color of the three centimeter Cuisenaire Rod). Glue the red 2 x 2 square to the upper left hand corner of this green square. Four of the small centimeter green squares are being covered. The five green centimeter squares sticking out represent the squares that were added. Record this in the third column.

The fourth row of the chart reads:

Number of blocks on one side	Total number of blocks	Number of blocks added	Number squared
3	9	5	3



Continue in this same way until the final 10 x 10 square is made. Keep gluing the last square to the upper left hand corner of the new square.
The sequence of colors is: 1 x 1 - white, 2 x 2 - red, 3 x 3 - chartreuse, 4 x 4 - purple, 5 x 5 - yellow, 6 x 6 - dark green, 7 x 7 - black, 8 x 8 - brown, 9 x 9 - dark blue, 10 x 10 - orange.

In the column 'Number of blocks added,' the sequence of odd numbers appears. The difference between each number in the sequence of odd numbers is two. Have her show the difference between each number in this column. This number is important when working on the sequence of cubes. See the Answer Key for an example.

Investigating Cubes - Worksheet 2 Follow the instructions on the page. Have her build as many of the cubes as she has patience for, but at least to 4 cubed. The rest can be calculated. Stop the session here.

Lesson Part 1

Investigating Cubes - Worksheet 3 - Making Cubes
She will now make a paper model of one through ten cubed. This model is time consuming to make but will give her a strong visual image of the power of an exponent. She will be able to see the progression of 13, 23, 33, 43, 53, etc.

Lesson Part 2

Have her cut the cubes out using the pattern shown on the worksheet. The net shows the six sides of the cube as they would look opened up and flattened out. The nets for the first two cubes are shown. The center of each net is the square of the number. For example: Examine the second graphic. Notice that the 2 centimeter long x 2 centimeter wide square in the center of the net is the 2^2 , the square number worked on in the warm-up. By adding the third dimension of height, we create 2^3. This cube will be two centimeters long, two centimeters wide, and two centimeters tall. We also call this 3-D. Have her notice how much larger the cube is than the square it is based on. That is the power of an exponent.

The color sequence of the cubes will be the same as the color sequence of the squares she did in the warm-up. The first cube will be small and somewhat difficult to tape. You can substitute a white Cuisenaire Rod cube if you do not want to try to make this 1 x 1 x 1 cube.

Note

If you have her add a little tab to one side of each flap in the net, it will make it much easier to tape the cubes together. You can also stuff the cubes with newspaper to make them less floppy.

After all the cubes are made, have her place the cubes in a tower using the same pattern she used with the squares. Start with the orange 10 x 10 x 10 cube. Place the dark blue cube on top of the orange cube. Place the lower left hand corner of the blue cube on top of the upper left hand corner of the orange cube. Tape it in place. Continue lining up the rear corners of the cubes and make a tower.

Have her notice the increase in size, dimension, and number of the cube tower, with the paper square pattern made in the warm-up. That difference in size and space is the difference between squaring and cubing a number.

Exponents are also called 'powers.' 5^2 is called five to the second power. 5^3 is called five to the third power. Have her imagine the size 5^{10}. 5^{10} built with centimeter cubes would be five cubes that hold a billion centimeter cubes each. Each cube would be ten meters long, ten meters wide, and ten meters tall! Now that is power!

Lesson Part 3

Follow the instructions on Investigating Cubes - Worksheet 4.
Answer to the question about the patterns: The sequence of squares series of differences between is all twos after two rows of differences between. The sequence of cubes is all sixes, and you have to go to three rows.

Optional Extended Pattern

On another piece of paper, write out the sequence of numbers to the fourth power. Do the first eight numbers. Use a calculator. Begin taking the difference between the numbers in each row, just like it was done for the sequence of squares and the sequence of cubes. Rows of differences for 4th power are four. The repeating difference is 24, or 1 x 2 x 3 x 4.
The meta pattern is the factorial pattern 1 x 2 = 2 1 x 2 x 3 = 6 1 x 2 x 3 x 4 = 24

1^4 2^4 3^4 4^4 5^4 6^4 7^4 8^4
1 16 81 256 625 1296 2401 4096
15 65 175 369 671 1105 1695
50 110 194 302 434 590
60 84 108 132 156
24 24 24 24

Think on this:	Squared Numbers	Cubed Numbers	Fourth Power
Rows of differences	2	3	4
Repeating Difference	2	6	24

Questions:
"Based on this chart, how many rows of differences would there be for numbers to the 5th power?" "Five."
"What pattern do you see in the repeating difference numbers?" "The same number as the power number."
"What would the final repeating difference in the numbers to the fifth power be?" "Five rows." The repeating difference would be 1 x 2 x 3 x 4 x 5, or 24 x 5, which is 120.
The repeating difference pattern, not easy to see, is what is called a factorial pattern.

Squared Numbers x 2 = 2 or (2 x 1)
Cubed Numbers x 3 = 6 or (3 x 2 x 1)
Fourth Power x 4 = 2 or (4 x 3 x 2 x 1)
Fifth Power x 5 = 2 or (5 x 4 x 3 x 2 x 1) = 120

Test for Understanding

None

Investigating Cubes - Grid Paper - make multiple copies on heavy paper

Averages Up and Down

Purpose The purpose of this lesson is to deepen understanding of how averages work. The student is asked to alter a set of numbers to change the average of those numbers by a specific amount.

Prerequisites The ability to calculate an average of a set of numbers

Materials Averages Up and Down - Worksheets 1 - 5, pages 69 - 73
Cube counters

Warm Up Physically review what an average is by finding the average of a set of numbers with blocks. Use the first set of numbers on Averages Up and Down - Worksheet 1. Have the student make a tower of six cubes for text messages on Monday, another tower of five cubes for text messages on Tuesday, another tower of six cubes for text messages on Wednesday, and tower of three cubes for text messages on Thursday.

A mean average is when the total number of towers is kept the same, but the blocks are moved around so that each of the four towers is of equal height. Have him make high places low and low places high. When this is done, he will have four towers with five blocks in each tower. This number, five, is the average number of text messages Laurel received each day that week.

Build the average for the second set of numbers on Averages Up and Down - Worksheet 1. This one will have six towers. When all six towers are equalized, each will contain seven blocks.
"What is the procedure for calculating an average?" "Add up all the numbers in the set and then divide by the number of numbers in the set."

"When building the average with the blocks, when does the division happen?" "It happens when each tower is made the same height."

"Why does it make sense that to calculate the average, you have to add before you divide?" "You need to know how many blocks you have in all. Then you divide the total number of blocks evenly between each tower."

"Can you change the number of towers?" "No."

"Why not?" "Because that is the numbers in the set of towers."

Lesson Have him complete all the questions on Averages Up and Down - Worksheet 1 except the last one. The last question begins this new addition to averaging. The new idea is, how do higher and lower numbers affect the average and how can one figure out how to change the numbers in a set to increase or decrease the average by a specific amount?

"How many messages would Judi have to receive on Saturday to change her average to eight messages?" *Wait.* Let him think about this.

The current average is seven for a set of six numbers. One of the first things many students will try in raising an average by one is to add one to the Saturday number and make it ten.

Have him build the set with cubes to see why that does not work. Building will help him realize that to increase the average by one, he must add one block to each stack. So six blocks need to be added to the total so that when he divides by six the answer is eight, not seven. He needs a total of forty-eight blocks. The problem requires him to add all six new blocks to the Saturday number only. This will make the Saturday number fifteen.

To raise the average by one point, he needs to add six to the set. Do not tell him this. Let him figure it out. He can figure it out with the blocks.

Solutions: Some students will work backwards and build six towers of eight blocks each. Then with that total they will rebuild the original set of numbers and put all the extra blocks on the last tower.

Some students will experiment with only the Saturday number by raising it until the total divided by six equals eight.

Move on to Averages Up and Down - Worksheet 2. The worksheet leads him through a series of similar problems. Have him use blocks to help if he needs it. Give him time to think through each problem.

Practice Worksheets

Averages Up and Down - Worksheets 3 and 4 Many students will be unable to do these problems independently.

Note

When there is a zero in the set of numbers, this space must be counted as one of the numbers in the set when the division is done. The zero pulls the average down, as all the other numbers have to be lowered to share the total with that zero.

The Michael referred to in the problems is Prof. Emeritus Michael Butler who is the progenitor of this math program. It was when he gave problems like this to his college students that he realized that his students did not know how to play with numbers and could not solve novel problems that did not fit with the standard procedures they had memorized. He realized this was a problem created by weak mathematics instruction in their formative years.

Answers will vary for the new sets. In problem 2 on Averages Up and Down - Worksheet 3, the student must take the total of the original set–7, 11, 19, 28, 40– which is 105, and divide it by 5 to get an average of 21. He must realize that to raise the average to 25, he needs to add 4 to every number to increase the total to 5 x 25, which is 125. Once he realizes this pattern, these problems become easy.

Each number does not have to be raised by four; simply add twenty to the total. He could add this twenty to one number and still raise the average by the required amount.

To lower it to fifteen, he must have the total be 5 x 15, or seventy-five. This means he must lower the total from one hundred five to seventy-five, a drop of thirty.

Test for Understanding

The question at the bottom of Averages Up and Down - Worksheet 4 is a test. If he can explain the procedure clearly, he understands what he is doing.

Possible Answer: To raise or lower the average of a set, first find the average of the original set. Write down the sum of the numbers in the original set. Then multiply the number of items in the set by the average you are trying to end up with. This will tell you what the new sum must be. Take the difference between the original sum and the new sum. This will tell you the number that must be added or subtracted to the original set to raise or lower the average of the new set. It does not matter how you increase the values of the numbers in the set, just that the sum is raised by the needed amount.

Averages Up and Down - Worksheet 5 is a challenge for any student who can answer the question above. This problem asks the average to be changed for the lowest four scores to raise the average of the group to a certain number.

General Math - Booklet 6

Pascal's Triangle - Worksheet 1

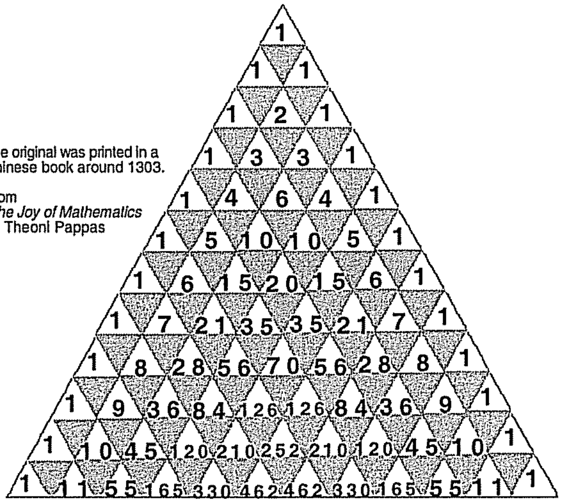
Blaise Pascal (1623-1662) was a famous French mathematician and scientist. Pascal's Triangle was named after him because he discovered so many patterns in this triangle of numbers.

Find the pattern and continue filling in the numbers in the white triangles. This pattern continues without end.

*After you have figured out the pattern, guess what the numbers will be in the last row at the bottom.

The original was printed in a Chinese book around 1303.

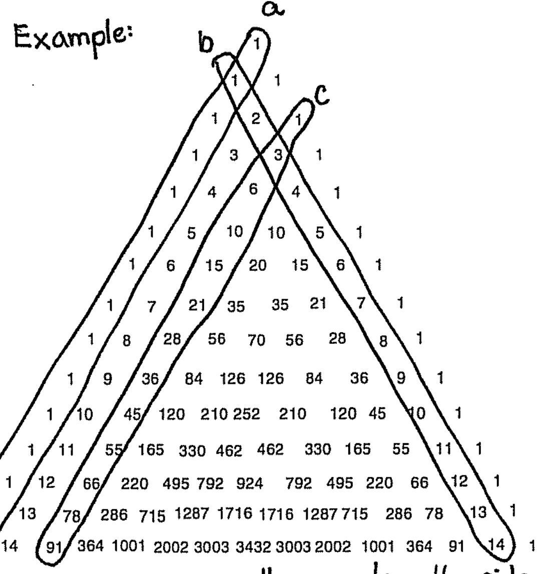
From *The Joy of Mathematics* by Theoni Pappas



*Write your guesses here.

Pascal's Triangle - Worksheet 2

Circle and color all of the patterns you can find.
Make a key that explains the pattern with words or numbers.

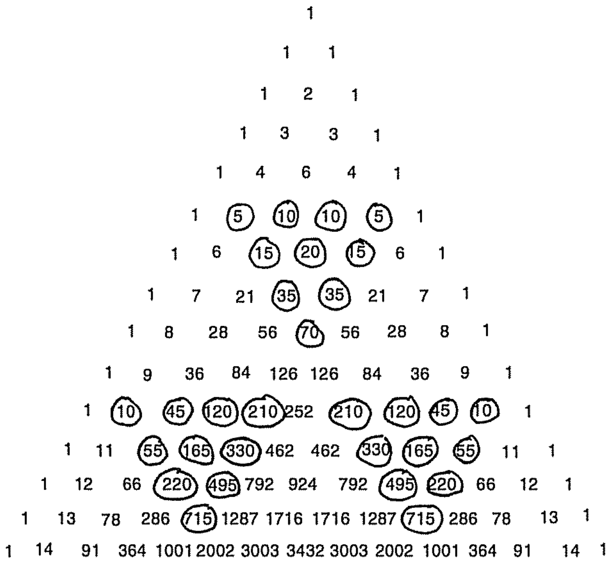


Key

a. all ones along the side
b. 1,2,3....14
c. sequence of triangular numbers

Pascal's Triangle - Worksheet 3

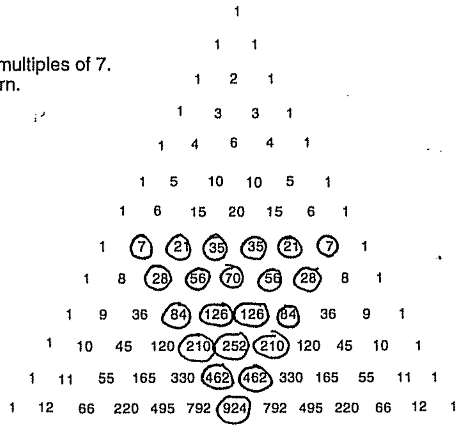
Circle all of the multiples of 5.
Look for a pattern.



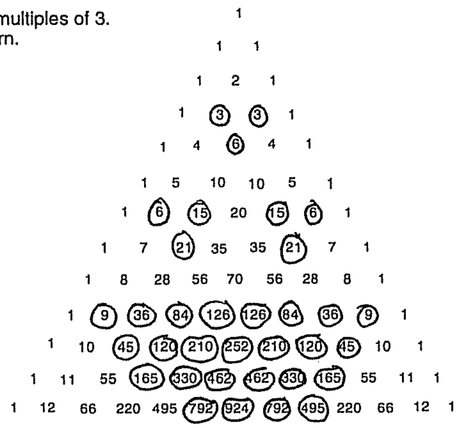
Pascal's Triangle - Worksheet 4

Pattern

Circle all of the multiples of 7.
Look for a pattern.



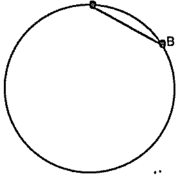
Circle all of the multiples of 3.
Look for a pattern.



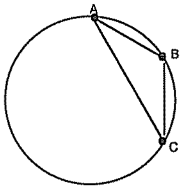
Circle Segments - Worksheet 1

Each circle below has a given number of points placed on the circumference. Draw in all the possible chords that connect every point to every other point. Count the number of chords that can be constructed for two, three, four, and five points. Fill in the T chart using the patterns you found.

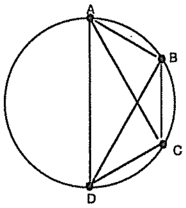
A circle with two points has one chord.



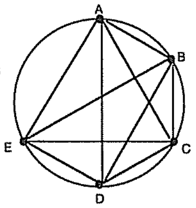
A circle with three points has three chords.



A circle with four points has 6 chords.



A circle with five points has 10 chords.



Number of points	Number of chords
1	0
2	1
3	3
4	6
5	10
6	15
7	21
8	28

What patterns do you notice?

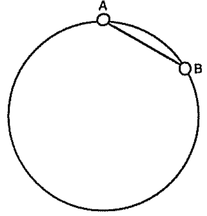
Differences

1 >2
3 >3
6 >4
10 >5
15 >6
21 >7
28

Circle Segments - Worksheet 2

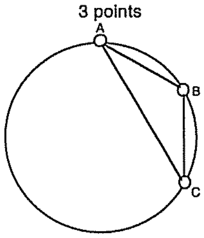
Color the point at the top of each circle purple. Going clockwise color the points these colors; blue, green, yellow, orange, and red. Draw purple chords starting from the top point. Then draw chords the color of the points the color from which they originate.

Connect 2 points



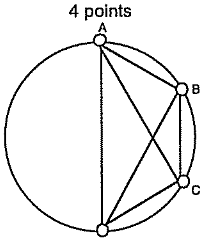
1 purple chord(s)
___ blue chord(s)
___ green chord(s)
___ yellow chord(s)
+ ___ orange chord(s)
1 Total chords

3 points



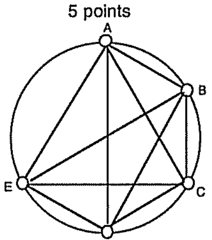
2 purple chord(s)
1 blue chord(s)
___ green chord(s)
___ yellow chord(s)
+ ___ orange chord(s)
3 Total chords

4 points



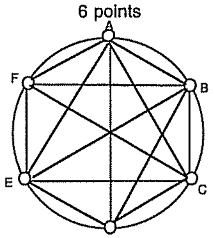
3 purple chord(s)
2 blue chord(s)
1 green chord(s)
___ yellow chord(s)
+ ___ orange chord(s)
6 Total chords

5 points



4 purple chord(s)
3 blue chord(s)
2 green chord(s)
1 yellow chord(s)
+ ___ orange chord(s)
10 Total chords

6 points



5 purple chord(s)
4 blue chord(s)
3 green chord(s)
2 yellow chord(s)
+ 1 orange chord(s)
15 Total chords

7 points
(6+5+4+3+2+1)
21 total chords

8 points
(7+6+5+4+3+2+1)
28 Chords

The number will drop by one as the points advance around the circle. This is the pattern in the number of new line segments coming out of points.

Area and Perimeter - Worksheet 1

A Study in Structures: **Area**

What is an "area"? The amount of space inside a shape.

1. How do you find the area of a square or rectangle? Describe the procedure in words. Measure the length of one side and the width of another adjacent side, the multiply the amounts together.
2. Now write it in pattern language (algebra). L x W = A

Do a study.

You will need a large piece of paper (12 x 18 inches) and graph paper

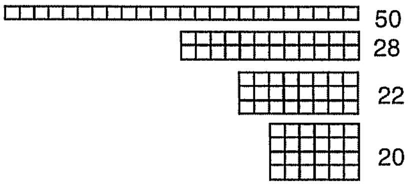
Use the graph paper to make rectangles. Keep the areas of the rectangles at 24 square centimeters.

Find all the different shapes of rectangles you can make. Cut them out.

Lay them (do not paste) on the large paper.

Figure out the perimeter of each rectangle and label them.

Organize the rectangles in order from the largest to smallest perimeter.



Area = 24 on all.
Perimeter changes.

Area and Perimeter - Worksheet 2

A Study in Structures: **Perimeter**

What is a "perimeter"? The distance around the outside of a shape.

1. How do you find the perimeter of a square or rectangle? Describe the procedure in words. Measure the length of all sides and add them together. Or measure two adjacent sides (length and width), multiply each by 2, then add the products.
2. Now write it in pattern language (algebra). L+L+W+W or (L x 2) + (W x 2) = P

Do a study. Use the graph paper to make rectangles with the perimeters of 24 sq. cm.

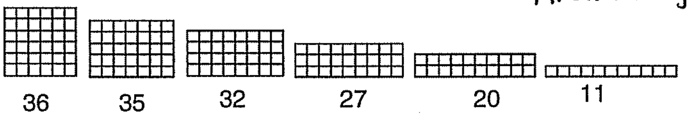
Find all the different shapes of rectangles you can make. Cut them out.

Lay them (do not paste) on the large paper.

Figure out the area of each rectangle and label them.

Organize the rectangles in order from the largest to smallest area.

Perimeter = 24 in all.
Area changes.



Rules of Relationships

If you make several shapes of rectangles with equal areas, will the perimeters be equal too? no

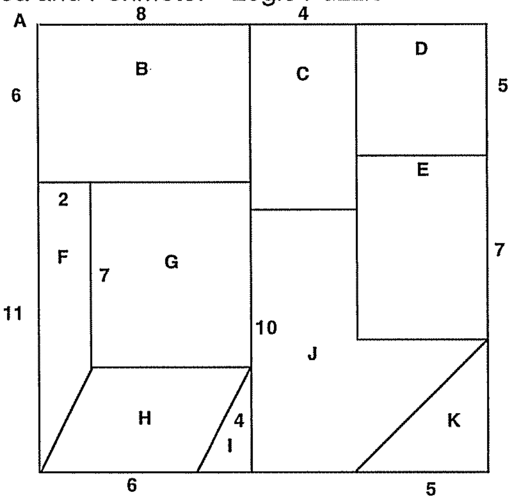
If you make a rectangle long and skinny, the less the area will be.

The more the length, the less the area.

If you want to maximize the area of a rectangle with a given perimeter, you should shape the rectangle to be as close to a square shape as possible.

If you want to maximize the perimeter with a given area, you should shape the rectangle to be as close to a long rectangular shape as possible.

Area and Perimeter - Logic Puzzle



Shape		Dimensions	Area
Large square	A	17 x 17	= 289
Rectangle	B	8 x 6	= 48
Rectangle	C	7 x 4	= 28
Square	D	5 x 5	= 25
Rectangle	E	7 x 5	= 35
Combination	F	(7 x 2)+(14 x 2)÷2	= 18
Rectangle	G	7 x 6	= 42
Parallelogram	H	6 x 4	= 24
Triangle	I	(2 x 4)÷2	= 4
Combination	J	(5 x 5)÷2+(10 x 4)÷2	= $52\frac{1}{2}$
Triangle	K	(5 x 5)÷2	= $12\frac{1}{2}$

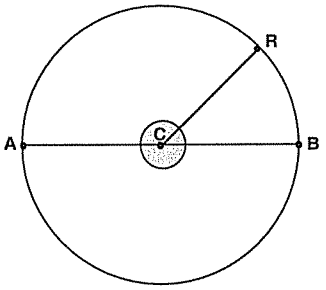
Geometry of Circles - Radius and Diameter

The rim of a DVD has the shape a circle.

Point C is the center of the circle.

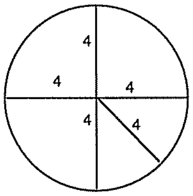
Line segment CR is a radius of the circle. A ***radius*** connects the center with a point on the circle.

Line segment AB is a ***diameter*** of the circle. A diameter connects 2 points on the circle and goes through the center.



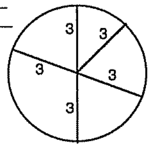
Label the radius and the diameter.

The outside edge is called a ***circumference***.



The length of each radius is 4 centimeters.
The length of each diameter is 8 centimeters.
r = 4cm
d = 8 cm

The length of each radius is 3 centimeters.
The length of each diameter is 6 centimeters.
r = 3cm
d = 6 cm



Find two circular objects and measure them.

1. r = ______
   d = ______
2. r = ______
   d = ______

Measure the radius of the DVD pictured above. 4cm

Measure the diameter of the DVD. 8cm

The radius is Shorter than the diameter .

What is the relationship in length between the diameter and the radius?

Radius is half the length of the diameter.

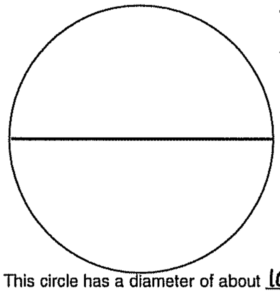
Write an equation in pattern language (algebra) that states the relationship.

$d \div 2 = r$ or $d = 2r$ or $r = \frac{1}{2}d$

Geometry of Circles - Circumference

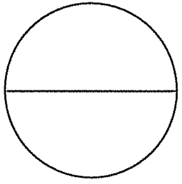
The circumference of a circle is the distance around the circle. Do the activity with your teacher . What patterns did you find? ______

Use the pattern you found in the table you made to figure out the length of the circumference of the circles below

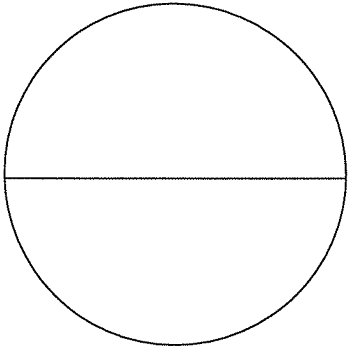


This circle has a diameter of about 8 centimeters.
There are about 3 diameter lengths in a circumference, so the circumference should be around 24 centimeters.

This circle has a diameter of about 5 centimeters.
There are about 3 diameter lengths in a circumference, so the circumference should be around 15 centimeters.



This circle has a diameter of about 10 centimeters.
There are about 3 diameter lengths in a circumference, so the circumference should be around 30 centimeters.



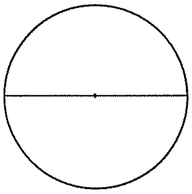
Geometry of Circles - Pi π

In all the circles the circumference divided by the diameter is about 3.
In pattern language C ÷ d ≈ 3. (≈ means about)

It happens that this is true for all circles. In fact the circumference divided by the diameter is a little larger than 3. It is 3.1415926.... Most people use the number 3.14. It is called pi, or π. Therefore, C ÷ d = π.

Archimedes (287 -212 B.C.), a Greek, was one of the people who discovered this, and that is why the Greek letter π is used for pi.

If you know the diameter of a circle, how can you find the
You already know that C ÷ d = π. Hint: Use a Families of Facts to help.
C = π x d.



diameter of this circle.

Diameter = 6 centimeters

Circumference = 18.84 cm

Radius = 3 cm

Area = 28.26 sq. cm.

Give the definitions

D = Diameter The distance across a circle through the center.

R = Radius The distance from the center of a circle to any point on the circumference.

C = Circumference The distance around the rim of a circle.

pi = 3.14...

Write an equation for calculating each of these.

C = d x pi or d x 3.14 D = 2r A = pi x r^2

Area of Circles - Worksheet 1

On the circle below:

1. Trace a diameter of the circle with a heavy line.
2. Trace each radius with a small squiggly line. ∿∿
3. Use a bright colored crayon to trace the circumference of the circle. Make this line fairly thick.
4. Pi is the number of diameters in the circumference of a circle.
5. The symbol for Pi is π. The value of Pi as a decimal number is 3.14 .
6. The formula for calculating the circumference using the diameter is C = πd.
7. How many radii are in a diameter? 2 d = 2 r r = $\frac{1}{2}$d
8. The formula for calculating the circumference using the radius is C = π x 2 r
9. The formula for calculating the area of a rectangle is A = L x W .

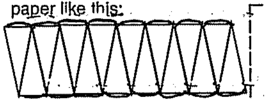
Activity

First: Cut out the circle. Make sure the bright colored traced circumference line shows.

Second: Carefully cut along each dotted squiggly radii line.

Third: Count to be sure you have 16 wedges.

Fourth: Use double sided tape or aste to attach the wedges to the colored paper like this:



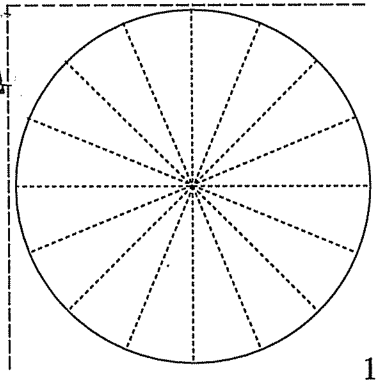
Vocabulary

C = circumference

d = diameter

r = radius

A = area



Area of Circles - Worksheet 2

1. Area of a rectangle = $\boxed{L}$ x $\boxed{W}$
2. Length of this rectangle Width of this rectangle (from the cut-up circle parts) = $\boxed{\frac{1}{2}C}$ x $\boxed{r}$
3. How is C calculated? = $\boxed{\frac{1}{2}}$ x $\boxed{\pi d}$ x $\boxed{r}$
4. How is d calculated? (In terms of r) = $\boxed{\frac{1}{2}}$ x $\boxed{\pi}$ x $\boxed{2}$ x $\boxed{r}$ x $\boxed{r}$
5. Change the order. = $\frac{1}{2}$ x 2 x π x r x r
6. Simplify the numbers. = 1 x π x r x r
7. Simplify again. = π x r^2 Area of a circle

Attach the colored paper here.

Area of Circles - Worksheet 3

The formula for finding the area of a circle is:

A = π x r^2 or A = 3.14 x r x r

Find A if r = 5

A = 3.14 x 5 x 5

A = 78.5 square units

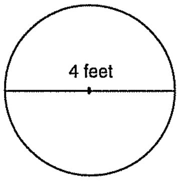
Find A if d = 12

The formula for finding radius is: r = d ÷ 2

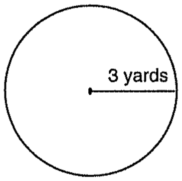
A = 3.14 x 6 x 6

A = 113.04 square units

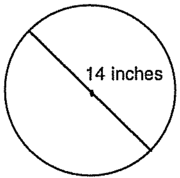
Find the area of each circle. Use 3.14 for π.



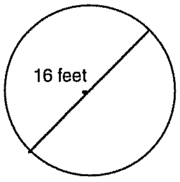
12.56 sq. feet



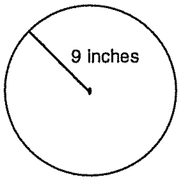
28.26 sq. yards



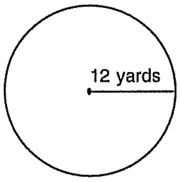
153.86 sq. inches



200.96 sq. feet



254.34 sq. inches

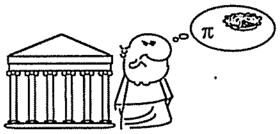


452.16 sq. yards

Area of Circles - Worksheet 4

s

Practice

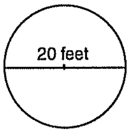


Who invented the general method of finding the area of a circle?____________

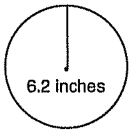
After solving a problem write the letter under it in the blank above the correct answer to name the mathematian.

A	R	C	H	I	M	E	D	E	S
22	1926.5	25	120.7	200.96	78.5	15.89	16	379.9	314

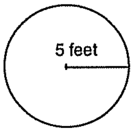
Find the area of each circle. Use 3.14 for π.



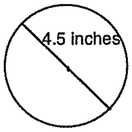
314 sq. ft.
S



120.7 sq. in.
H



15.89 sq. ft.
M



15.89 sq. in.
E

Fill in this chart.

radius	diameter	area
11 in.	22 in. A	379.9 sq. in. E
25 ft. C	50 ft.	1926.5 sq. ft. R
8 yd.	16 yd. D	200.96 sq.yd. I

Variables

Some equations use letters instead of numbers. The letters are called variables and stand for numbers.
Example:
6 + n   x - 3   5y
↖ ↑   ↗
variable

Variables can be replaced by a number. This process is called substitution.
Example:
If n = 10, 5 + n = 5 + 10 or 15

If x = 4, 6 - n = 6 - 2 or 4

If y = 6, 3y = 3 x 6 or 18

Solve. Let a = 2, b = 3, and c = 6.

1. a + 6 = 8
2. a + c = 8
3. (6 • b) - 5 = 13
4. c - b = 3
5. 15 - c = 9
6. a + b + c = 11
7. 3a + 3c = 24
8. 19 - 2b = 13
9. 5c - 3a = 24
10. b^2 = 9
11. 3(a + c) = 24
12. 2c - (b - a) = 11

Make and solve 4 problems of your own. AWV
Let __ = ___, ___ = ___, and ___ = ___

Playing with Parentheses - Worksheet 1

If all the signs in a number sentence are addition signs, what will happen if you move the parenthesis ( ). The answer will not change.

You make up one: AWV

(12 + 4) + 3 = 19   (___ + ___)+ ___ = ____
12 + (4 + 3) = 19   ___ + (___ + ___) = ____

What about subtraction? The answer will be different.

(12 - 4) - 3 = 5   (___ - ___) - ___ = ____
12 - (4 - 3) = 11   ___ - (___ - ___) = ____

(12 - 8) - 2 = 2
12 - (8 - 2) = 6

Which equations are equal?   You make up some:
Multiplication

(12 x 4) x 3 = 144   (___ x ___) x ___ = ____
12 x (4 x 3) = 144   ___ x (___ x ___) = ____

Division

(18 ÷ 6) ÷ 3 = 1   (___ ÷ ___) ÷ ___ = ____
18 ÷ (6 ÷ 3) = 9   ___ ÷ (___ ÷ ___) = ____

Which two operations are not changed by the order of combinations.
addition
multiplication

This is the Associative Principle.

It applies to only to addition and multiplication

Playing with Parentheses - Worksheet 2
Multiplication and Addition

Add first   Multiply then add
(7 + 8) x 5 = 75   7 + (8 x 5) = 47

(9 + 4) x 8 = 104   9 + (4 x 8) = 41

(3 + 6) x 2 = 18   3 + (6 x 2) = 15

Which way always produces the larger answer? Add first, then x.
Why? The equation with addition in the parenthesis will be the larger amount. Adding to create the largest sum before multiplying always produces a larger product.

Based on this discovery predict how you think the rule would change if you changed the + sign to a - sign. Try some problems.

Examples:

(8 - 2) x 3 = 18   18 - (2 x 3) = 12

(12 - 3) x 2 = 18   12 - (3 x 2) = 6

(18 - 4) x 2 = 28   18 - (4 x 2) = 10

Write a rule for getting the largest answer for a pair of equations that use - and **x**.

My rule is: Subtract first, then multiply.

Playing with Parentheses - Worksheet 3
Division and Addition

Try it with division and addition. Which order of operation (÷ then + or + then ÷) do you think will produce the larger answer?________

Add first   Divide then add

12 ÷ (4 + 2) = 2   (12 ÷ 4) + 2 = 5

18 ÷ (3 + 6) = 2   (18 ÷ 3) + 6 = 12

24 ÷ (6 + 2) = 3   (24 ÷ 6) + 2 = 6

Do you think that putting the ÷ sign after the + sign will change the pattern of how to get the largest answer?____Try it.

(20 + 10) ÷ 2 = 15   20 + (10 ÷ 2) = 25

(4 + 8) ÷ 2 = 6   4 + (8 ÷ 2) = 8

(9 + 6) ÷ 3 = 5   9 + (6 ÷ 3) = 11

Try some on your own. AWV

(___+___ ÷ ___ = ____   (___+___)÷ ___ = ____

(___+___ ÷ ___ = ____   (___+___)÷ ___ = ____

Write a rule for getting the largest answer for a pair of equations that use ÷ and +.

My rule is: Divid first, then add.

Playing with Parentheses - Worksheet 4
Division and Subtraction

Try it with division and subtraction. Which order of operation(÷ then - or - then ÷) do you think will produce the larger answer?______________

Subtract first
12 ÷ (4 - 2) = 6
18 ÷ (6 - 3) = 6
24 ÷ (6 - 4) = 12

Divide then subtract
(12÷ 4) - 2 = 1
(18÷ 6) - 3 = 0
(24÷ 6) - 4 = 0

Do you think that putting the ÷ sign after the - sign will change the pattern of how to get the largest answer?_____Try it.

Subtract first
(20 - 10) ÷ 2 = 5
(8 - 4) ÷ 2 = 2
(9 - 6) ÷ 3 = 1

Divide then subtract
20 - (10 ÷ 2) = 15
8 - (4 ÷ 2) = 6
9 - (6 ÷ 3) = 7

What happened? The order makes adifference.

Now try this set of problems:
20 ÷ (10 - 2) = 2.5   (20 ÷ 10) - 2 = 0

Make up problems using the same numbers with all four possibilities:
(___ - ___) ÷ ___ = ____   ___ - (___ ÷ ___) = ____

___ ÷ (___ - ___) = ____   (___ ÷ ___) - ___ = ____

Which will give the largest answer?
Write a rule for getting the largest answer for a pair of equations that use ÷ and -.
My rule is: Divide first, then subtract to get the larger answer with each equation having the division sign after the subtraction sign.

Playing with Parentheses - Worksheet 5
A New Property

Look at the equation (3 + 5) x 4

The procedure you would do is: Add 3 + 5, then multiply by 4.
You get 32.

What will happen if you multiply each part of the addition problem (the 3 by 4) and (the 5 by 4) then add the two answers
The answer is the same.

It is written like this: (3 + 5) x 4
(3 x 4) + (5 x 4)
12 + 20 = 32

Put signs between the two equations (=, <, >) to show their relationship.
=, <, >

(3 + 5) x 4 = (3 x 4) + (5 x 4)
Have you seen this before? yes
Doing what kind of problems? Two digit multiplication.

This is the Distributive Property of Multiplication. This is what is behind your procedure for long multiplication.

Here is an example:
125
x 4

125 x 4 = (100 + 20 + 5) x 4
= (100 x 4)+(20 x 4)+(5 x 4)
= 400 + 80 + 20
= 500

Order of Operations

When solving a problem, always perform the operation within the parenthesis first.
Example
(3 x 4) + 6
12 + 6 = 18

2(7) the multiplication problem is 2 x 7

3•8 the multiplication problem is 3 x 8

After all parenthesis are renamed, the operations are solved in this order.

1. multiply
2. divide
3. add or subtract from left to right

Example
56 - 20 ÷ 4
56 - 5 = 51

What would the answer be if the subtraction problem (56 - 20) was done first? _______

1. (5 + 4) x 6 = 54
2. (20 + 4) ÷ 2 = 12
3. (7 - 2) • (15 - 10) = 25
4. 7 + (4 + 9) = 20
5. (18 ÷ 9) + (4 • 6) = 26
6. (15 x 4) ÷ 6 = 10
7. (7 - 2) • (15 - 10) = 25
8. (9 x 9) ÷ 3 = 27
9. 13 + (15 ÷ 3) = 18
10. (12 + 6) x (12 ÷ 3) ÷ 3 = 24
11. 3(8 -2) + 6 = 24
12. (12 ÷ 4) + 6 = 9
13. 13 + (15 ÷ 3) = 18
14. (15 + 6) x (10 - 9) ÷ 3 = 7
15. 6 + (32 ÷ 4) - 12 = 2
16. (24 • 2) + (12 ÷ 2) - 6 = 48

Families of Facts: Missing Numbers - Worksheet 1

Solve these problems using the Families of Facts color pattern.

7 = yellow 5 = blue box = green

⑦ x ⑤ = $\boxed{35}$ ⑦ + ⑤ = $\boxed{12}$
⑤ x ⑦ = $\boxed{35}$ ⑤ + ⑦ = $\boxed{12}$
$\boxed{35}$ ÷ ⑦ = ⑤ $\boxed{12}$ - ⑦ = ⑤
$\boxed{35}$ ÷ ⑤ = ⑦ $\boxed{12}$ - ⑤ = ⑦

4 = yellow 9 = blue box = green

④ x ⑨ = $\boxed{36}$ ④ + ⑨ = $\boxed{13}$
⑨ x ④ = $\boxed{36}$ ⑨ + ④ = $\boxed{13}$
$\boxed{36}$ ÷ ④ = ⑨ $\boxed{13}$ - ④ = ⑨
$\boxed{36}$ ÷ ⑨ = ④ $\boxed{13}$ - ⑨ = ④

8 = yellow 6 = blue box = green

⑧ x ⑥ = $\boxed{48}$ ⑧ + ⑥ = $\boxed{14}$
⑥ x ⑧ = $\boxed{48}$ ⑥ + ⑧ = $\boxed{14}$
$\boxed{48}$ ÷ ⑧ = ⑥ $\boxed{14}$ - ⑧ = ⑥
$\boxed{48}$ ÷ ⑥ = ⑧ $\boxed{14}$ - ⑥ = ⑧

Families of Facts: Missing Numbers - Worksheet 2

Make your own set of Families of Facts problems. Outline each shape with the colors from worksheet 1. Fill in the numbers.

$\bigcirc \times \bigcirc = \square$ $\quad \bigcirc + \bigcirc = \square$

$\bigcirc \times \bigcirc = \square$ AWV $\quad \bigcirc + \bigcirc = \square$

$\square \div \bigcirc = \bigcirc$ $\quad \square - \bigcirc = \bigcirc$

$\square \div \bigcirc = \bigcirc$ $\quad \square - \bigcirc = \bigcirc$

Does the pattern change if the numbers change?________________

Use pattern language (algebra) to show the relationship.
Fill in the numbers below.

a	b	c	a	b	c

$\textcircled{7} \times \textcircled{8} = \boxed{56}$ $\quad \textcircled{7} + \textcircled{8} = \boxed{15}$

$\textcircled{8} \times \textcircled{7} = \boxed{56}$ $\quad \textcircled{8} + \textcircled{7} = \boxed{15}$

$\boxed{56} \div \textcircled{7} = \textcircled{8}$ $\quad \boxed{15} - \textcircled{7} = \textcircled{8}$

$\boxed{56} \div \textcircled{8} = \textcircled{7}$ $\quad \boxed{15} - \textcircled{8} = \textcircled{7}$

The language of mathematicss says "Let a, be equal to 7" and "Let b, be equal to 8." Therefore, a x b would be equal to 56 which is now labeled as c. Since the pattern of what you do doesn't change when you change the numbers, you can write the pattern using letters that stand for any number.

Families of Facts: Missing Numbers - Worksheet 3

Use the Families of Facts patterns to solve these equations.

Put signs in the $\triangle$ .Outline each shape with the colors from worksheet 1.

1 $\textcircled{7} \overset{+}{\triangle} \textcircled{9} = \boxed{16}$

2 $\textcircled{9} \overset{+}{\triangle} \textcircled{7} = \boxed{16}$

3 $\boxed{16} \overset{-}{\triangle} \textcircled{7} = \textcircled{9}$

4 $\boxed{16} \overset{-}{\triangle} \textcircled{9} = \textcircled{7}$

This pattern is used to find values of missing numbers.

If $\textcircled{a} + \textcircled{9} = \boxed{16}$, which equation above tells what= a? 16-9=7 *

Use Families of Facts to solve these equations.

$\textcircled{j} + \textcircled{11} = \boxed{21}$

$\textcircled{j} = \boxed{21} \overset{-}{\triangle} \textcircled{11}$

$\textcircled{j} =$ 10

$\textcircled{t} + \textcircled{6} = \boxed{24}$

$\textcircled{t} = \boxed{24} \overset{-}{\triangle} \textcircled{6}$

$\textcircled{t} =$ 18

$\textcircled{f} + \textcircled{17} = \boxed{26}$

$\textcircled{f} = \boxed{26} \overset{-}{\triangle} \textcircled{17}$

$\textcircled{f} =$ 9

Make your own. AWV

$\bigcirc + \bigcirc = \square$

$\bigcirc = \square \triangle \bigcirc$

$\bigcirc =$ ______

*Answer
16 - 9 = a

Families of Facts: Missing Numbers -Worksheet 4
Inverse Operations

Use the Families of Facts patterns to solve these equations.

Put signs in the $\triangle$ .Outline each shape with the colors from worksheet 1.

1 $\textcircled{6} \overset{\times}{\triangle} \textcircled{9} = \boxed{54}$

2 $\textcircled{9} \overset{\times}{\triangle} \textcircled{6} = \boxed{54}$

3 $\boxed{54} \overset{\div}{\triangle} \textcircled{6} = \textcircled{9}$

4 $\boxed{54} \overset{\div}{\triangle} \textcircled{9} = \textcircled{6}$

This pattern is used to find values of missing numbers.

If $\textcircled{a} \times \textcircled{9} = \boxed{54}$,which equation above tells what= a? division *

Use Families of Facts to solve these equations.

$\textcircled{d} \times \textcircled{8} = \boxed{24}$

$\textcircled{d} = \boxed{24} \overset{\div}{\triangle} \textcircled{8}$

$\textcircled{d} =$ 3

$\textcircled{k} \times \textcircled{7} = \boxed{42}$

$\textcircled{k} = \boxed{42} \overset{\div}{\triangle} \textcircled{7}$

$\textcircled{k} =$ 6

$\textcircled{p} \times \textcircled{6} = \boxed{48}$

$\textcircled{p} = \boxed{48} \overset{\div}{\triangle} \textcircled{6}$

$\textcircled{p} =$ 8

Make your own. AWV

$\bigcirc \times \bigcirc = \square$

$\bigcirc = \square \triangle \bigcirc$

$\bigcirc =$ ______

*Answer
54 ÷ 9 = a

Families of Facts: Missing Numbers - Worksheet 5
Pattern Language (Algebra)

1. $\textcircled{c} \overset{+}{\triangle} \textcircled{8} = \boxed{17}$

   $\textcircled{c} = \boxed{17} \overset{-}{\triangle} \textcircled{8} = 9$

   $\textcircled{c} =$ 9

2. $\textcircled{13} \overset{+}{\triangle} \textcircled{g} = \boxed{24}$

   $\textcircled{g} = \boxed{24} \overset{-}{\triangle} \textcircled{13}$

   $\textcircled{g} =$ 11

3. $\textcircled{r} \overset{+}{\triangle} \textcircled{16} = \boxed{24}$

   $\textcircled{r} = \boxed{24} \overset{-}{\triangle} \textcircled{16}$

   $\textcircled{r} =$ 8

4. $\textcircled{b} \overset{+}{\triangle} \textcircled{7} = \boxed{42}$

   $\textcircled{b} = \boxed{42} \overset{-}{\triangle} \textcircled{7}$

   $\textcircled{b} =$ 35

5. $\textcircled{a} \overset{+}{\triangle} \textcircled{24} = \boxed{78}$

   $\textcircled{a} = \boxed{78} \overset{-}{\triangle} \textcircled{24}$

   $\textcircled{a} =$ 54

6. $\textcircled{m} \overset{+}{\triangle} \textcircled{35} = \boxed{81}$

   $\textcircled{m} = \boxed{81} \overset{-}{\triangle} \textcircled{35}$

   $\textcircled{m} =$ 46

Put in the circles, squares, and triangles.

7. $\textcircled{17} \overset{+}{\triangle} \textcircled{t} = \boxed{53}$

   t = $\boxed{53} \overset{-}{\triangle} \textcircled{17}$

   t = 36

8. $\textcircled{s} \overset{+}{\triangle} \textcircled{65} = \boxed{81}$

   $\textcircled{s} = \boxed{81} \overset{-}{\triangle} \boxed{65}$

   $\textcircled{s} =$ 16

9. $\textcircled{w} \overset{+}{\triangle} \textcircled{37} = \boxed{65}$

   w = $\boxed{65} \overset{-}{\triangle} \textcircled{37}$

   w = 28

Make your own. AWV

10. $\bigcirc \triangle \bigcirc = \square$

    $\bigcirc = \square \triangle \bigcirc$

    $\bigcirc =$ ______

Families of Facts: Missing Numbers - Worksheet 6
Pattern Language (Algebra)

1. $\boxed{17}$ △ (n) = (8)
(n) = $\boxed{17}$ △ (8)
(n) = 9

2. $\boxed{245}$ △ (g) = (137)
(g) = $\boxed{245}$ △ (137)
(g) = 108

3. $\boxed{152}$ △ (r) = (57)
(r) = $\boxed{152}$ △ (57)
(r) = 95

4. $\boxed{53}$ △ (t) = (17)
(t) = $\boxed{53}$ △ (17)
(t) = 36

5. $\boxed{465}$ △ (s) = (343)
(s) = $\boxed{465}$ △ (343)
(s) = 122

6. $\boxed{165}$ △ (d) = (123)
(d) = $\boxed{165}$ △ (123)
(d) = 42

7. $\boxed{344}$ △ (p) = (55)
(p) = $\boxed{}$ △ ○
(p) = ____

8. $\boxed{234}$ △ (m) = (81)
(m) = $\boxed{234}$ △ (81)
(m) = 153

9. $\boxed{52}$ △ (y) = (45)
(y) = $\boxed{52}$ △ (45)
(y) = 7

Make your own.

10. $\boxed{}$ △ ○ = ○
○ = $\boxed{}$ △ ○
○ = ____

Families of Facts: Vertical Format - Worksheet 1

7	8	15	15
+ 8	+ 7	- 7	- 8
15	15	8	7

What happens when you add these two numbers?*

* You get the top number.

Use what you know about families of facts to fill in the blanks.

93	$\boxed{94}$	187	187
+ $\boxed{94}$	+ 93	- $\boxed{94}$	- 93
187	187	93	$\boxed{94}$

Which problem is most useful in finding the missing numbers?** 187-93

You can also use this to check your subtraction.

If, 15 - 7 = 8
then, 7 + 8 must = 15

If you did this problem 26 - 9 = 13

and added 9 + 13, what would you get? 22

If 26 - 9 = 13 does not make a true sentence when rearranged to be 9 + 13 = 26, then the original answer is wrong.

** 187 - 93.

Families of Facts: Vertical Format - Worksheet 2

937	158	834	347	723	275
- 158	+ $\boxed{779}$	- 347	+ $\boxed{487}$	- 275	+ $\boxed{448}$
$\boxed{779}$	937	$\boxed{487}$	834	$\boxed{448}$	723

762	485	941	398	763	576
- 485	+ $\boxed{277}$	- 398	+ $\boxed{543}$	- 576	+ $\boxed{187}$
$\boxed{277}$	762	$\boxed{543}$	941	$\boxed{187}$	763

943 - q = 278
q = 665

Does q + 278 = 943? yes

What does it mean if it doesn't?
Subtraction was done incorrectly.

834 - w = 641
w = 193

860 - m = 204
m = 656

642 - c = 360
c = 282

424 - y = 327
y = 97

428 - y = 273
y = 155

Make your own. AWV
____ - r = ____
r =

Families of Facts: Multiplication and Division - Worksheet 1

③ x ④ = $\boxed{12}$

$\begin{array}{r} ③ \\ \times ④ \\ \hline \boxed{12} \end{array}$

④ x ③ = $\boxed{12}$

$\begin{array}{r} ④ \\ \times ③ \\ \hline \boxed{12} \end{array}$

$\boxed{12}$ ÷ ③ = ④

$\begin{array}{r} ④ \\ ③\overline{)\boxed{12}} \end{array}$

$\boxed{12}$ ÷ ④ = ③

$\begin{array}{r} ③ \\ ④\overline{)\boxed{12}} \end{array}$

Families of Facts: Multiplication and Division - Worksheet 2

Study these examples and then make your own set.
You may want to build a model to work with.

④ x ⑤ = $\boxed{20}$

$\boxed{20}$ ÷ ④ = ⑤

$\begin{array}{r} ④ \\ \times ⑤ \\ \hline \boxed{20} \end{array}$

$\begin{array}{r} ⑤ \\ ④\overline{)\boxed{20}} \end{array}$

⑤ x ④ = $\boxed{20}$

$\boxed{20}$ ÷ ⑤ = ④

$\begin{array}{r} ⑤ \\ \times ④ \\ \hline \boxed{20} \end{array}$

$\begin{array}{r} ④ \\ ⑤\overline{)\boxed{20}} \end{array}$

Families of Facts: Multiplication and Division Worksheet 3

What happens when you multiply these two numbers?*

④ × ⑤ = $\boxed{20}$

⑤ × ④ = $\boxed{20}$

⑤ over ④)$\boxed{20}$

④ over ⑤)$\boxed{20}$

Use what you know about families of facts to fill in the blanks.

⑧ × ㉗ = $\boxed{216}$

㉗ × ⑧ = $\boxed{216}$

⑧ over ㉗)$\boxed{216}$

㉗ over ⑧)$\boxed{216}$

Which problem is most useful in finding the missing numbers?** 27 over 8)216

You can also use this to check division.

If 216 ÷ 8 = 27
then 27 x 8 must = 216

If you did this problem: 43 over 8)368

And then multiplied 43 x 8

Your answer would be 334. ***
Since 344 does not match the original dividend of 368, your first answer of 43 would be wrong.

* You get the product of the multiplication problem and the dividend of the division problem.

** 8)216

*** 344 instead of 368.

Families of Facts: Multiplication and Division Worksheet 4

$\boxed{228}$ ÷ ④ = (54)

④ × (54) = $\boxed{228}$

$\boxed{364}$ ÷ ⑦ = (52)

⑦ × (52) = $\boxed{364}$

$\boxed{770}$ ÷ ⑦ = (110)

⑦ × (110) = $\boxed{770}$

You put in the numbers.

(8 376 47)

$\boxed{376}$ ÷ ⑧ = ㊼

⑧ × ㊼ = $\boxed{376}$

(5 445 89)

$\boxed{445}$ ÷ ⑤ = (89)

⑤ × (89) = $\boxed{445}$

(203 7 29)

$\boxed{203}$ ÷ ⑦ = ㉙

⑦ × ㉙ = $\boxed{203}$

432 ÷ q = 9
q = 48

Does q x 9 = 432? yes
What does it mean if it doesn't? The division was done incorrectly.

768 ÷ w = 6
w = 128

368 ÷ m = 8
m = 46

448 ÷ c = 8
c = 56

324 ÷ y = 6
y = 54

Families of Facts: Multiplication and Division Worksheet 5

⑥ × (54) = $\boxed{324}$

(54) over ⑥)$\boxed{324}$

⑦ × (56) = $\boxed{392}$

(56) over ⑦)$\boxed{392}$

④ × (83) = $\boxed{332}$

(83) over ④)$\boxed{332}$

⑥ × (92) = $\boxed{552}$

(92) over ⑥)$\boxed{552}$

⑨ × (54) = $\boxed{486}$

(54) over ⑨)$\boxed{486}$

⑤ × (94) = $\boxed{470}$

(94) over ⑤)$\boxed{470}$

Families of Facts: Multiplication and Division Worksheet 6

1. w x 8 = 16
w = 16 ÷ 8 = 2
w = 2

2. k x 3 = 15
k = 15 ÷ 3 = 5
k = 5

3. 5 x g = 60
g = 60 ÷ 5 = 12
g = 12

4. m x 6 = 84
m = 84 ÷ 6 = 14
m = 14

5. 162 = s x 9
s = 162 ÷ 9 = 18
s = 18

6. r x 6 = 96
r = 96 ÷ 6 = 16
r = 16

7. a x 3 = 72
a = 72 ÷ 3 = 24
a = 24

8. 5 x p = 220
p = 220 ÷ 5 = 44
p = 44

9. 9 x t = 153
t = 153 ÷ 9 = 17
t = 17

10. v x 7 = 294
v = 294 ÷ 7 = 42
v = 42

Relationships

Fill in the blanks

a	x	b	=c
1	X	12	= 12
2	X	6	= 12
3	X	4	= 12
4	X	3	= 12
6	X	2	= 12
12	X	1	= 12

a	x	b	=c
12	X	1	= 24
8	X	3	= 24
6	X	4	= 24
4	X	6	= 24
3	X	8	= 24
2	X	12	= 24
1	X	24	= 24

If c remains the same and a gets larger, then b gets smaller.

If c remains the same and a gets smaller, then b" gets larger.

If a remains the same and b gets larger, then c gets larger.

Make your own example here. Example:

AWV

a	x	b	=c
2	X	4	= 8
2	X	5	= 10
2	X	6	= 12
2	X	7	= 14
2	X	8	= 16
2	X	9	= 18

Properties: Review

1. Commutative Property of Addition **A + B = B +A**
2. Commutative Property of Multiplication **A x B = B x A**
3. Associative Property of Addition **(A+ B) + C = A + (B +**
4. Associative Property of Multiplication**(A x B) x C = A x (B x C)**
5. Distributive Property **A (B + C) = (A x B) + (A x C)**

Label these problems with the name of the property each one is an example of.

10 x 9 = 9 x 10 Commutative Property of Multiplication

3 + (4 + 7) = (3 + 4) + 7 Associative Prop. of Addition

8 + 6 = 6 + 8 Commutative Prop. of Addition

7 (3 + 6) (7 x 3) + (7 x 6) Distributive Property

(4 x 10) x 5 = 4 x (10 x 5) Associative Prop. of Multiplication

Sneaky one: 3 x (5 x 6) = 3 x (6 x 5) Commutative Prop. of Multiplication

Make up one example of each AWV

1. Commutative Property of Addition __________
2. Commutative Property of Multiplication__________
3. Associative Property of Addition __________
4. Associative Property of Multiplication__________
5. Distributive Property __________

Difference Between - Worksheet 1

Solve all the ways of adding two numbers to equal the sum of twelve.	Now solve the differences between each of these pairs.
12 + 0 = 12	12 - 0 = 12
11 + 1 = 12	11 - 1 = 10
10 + 2 = 12	10 - 2 = 8
9 + 3 = 12	9 - 3 = 6
8 + 4 = 12	8 - 4 = 4
7 + 5 = 12	7 - 5 = 2
6 + 6 = 12	6 - 6 = 0
5 + 7 = 12	5 - 7 = -2
4 + 8 = 12	4 - 8 = -4
3 + 9 = 12	3 - 9 = -6
2 + 10 = 12	2 - 10 = -8
1 + 11 = 12	1 - 11 = -10
0 + 12 = 12	0 - 12 = -12

There are several patterns here. What do you see?

In addition the sums all equal twelve. One column gets smaller by one, the other gets larger by one. This also occurs in the subtraction set. In subtraction the answers get smaller by two and they are all even numbers. The pattern continues in the negative numbers.

Difference Between - Worksheet 2

How to analyze a pattern to find the difference.

Two Numbers	**Their Sum**	**Their Difference** Pattern in the differences.
10 , 2	12	8 ↓

12 + 0 = 12 → 12 - 0 = 12
11 + 1 = 12 → 11 - 1 = 10
10 + 2 = 12 → 10 - 2 = 8

Continue the pattern.

9 + 3 = 12 → 9 - 3 = 6
8 + 4 = 12 → 8 - 4 = 4
7 + 5 = 12 → 7 - 5 = 2
6 + 6 = 12 → 6 - 6 = 0

Two Numbers	**Their Sum**	**Their Difference**
10 , 6	16	4

16 + 0 = 16 → 16 - 0 = 16
15 + 1 = 16 → 15 - 1 = 14
14 + 2 = 16 → 14 - 2 = 12
13 + 3 = 16 → 13 - 3 = 10
12 + 4 = 16 → 12 - 4 = 8
11 + 5 = 16 → 11 - 5 = 6
10 + 6 = 16 → 10 - 6 = 4
9 + 7 = 16 → 9 - 7 = 2
8 + 8 = 16 → 8 - 8 = 0

Difference Between: Patterns - Worksheet 1

Here is a new pattern to analyze. Fill in the blanks.

Two Numbers	Their Sum	Their Difference
1, 11	12	-10
	6 + 6 = 12	6 - 6 = 0
	5 + 7 = 12	5 - 7 = -2
	4 + 8 = 12	4 - 8 = -4
	3 + 9 = 12	3 - 9 = -6
	2 + 10 = 12	2 - 10 = -8
	1 + 11 = 12	1 - 11 = -10
	0 + 12 = 12	0 - 12 = -12

Try this one on your own:

Two Numbers	Their Sum	Their Difference
1, -17	18	-16
	9 + 9 = 18	9 - 9 = 0
	8 + 10 = 18	8 - 10 = -2
	7 + 11 = 18	7 - 11 = -4
	6 + 12 = 18	6 - 12 = -6
	5 + 13 = 18	5 - 13 = -8
	4 + 14 = 18	4 - 14 = -10
	3 + 15 = 18	3 - 15 = -12
	2 + 16 = 18	2 - 16 = -14
	1 + 17 = 18	1 - 17 = -16
	0 + 18 = 18	0 - 18 = -18

Difference Between: Patterns - Worksheet 2

Two Numbers	Their Sum	Their Difference
10, 6	16	4
	16 + 0 = 16	16 - 0 = 16
	15 + 1 = 16	15 - 1 = 14
	14 + 2 = 16	14 - 2 = 12
	13 + 3 = 16	13 - 3 = 10
	12 + 4 = 16	12 - 4 = 8
	11 + 5 = 16	11 - 5 = 6
	10 + 6 = 16	10 - 6 = 4
	9 + 7 = 16	9 - 7 = 2
	8 + 8 = 16	8 - 8 = 0
	7 + 9 = 16	7 - 9 = -2
	6 + 10 = 16	6 - 10 = -4
	5 + 11 = 16	5 - 11 = -6
	4 + 12 = 16	4 - 12 = -8
	3 + 13 = 16	3 - 13 = -10
	2 + 14 = 16	2 - 14 = -12
	1 + 15 = 16	1 - 15 = -14
	0 + 16 = 16	0 - 16 = -16

Difference Between: Patterns - Worksheet 3

Find the difference.

The answer to a multiplication problem is a product.

The answer to a division problem is a quotient.

quotient
divisor ⟌ dividend

For these problems the dividend is larger than the divisor.

Factors	Their Product	Dividend ÷ Divisor = Quotient	
5, 10	50	10 ÷ 5	2
6, 2	12	6 ÷ 2	3
20, 4	80	20 ÷ 4	5
20, 5	100	20 ÷ 5	4
1, 2	2	2 ÷ 1	2
2, 4	8	4 ÷ 2	2
50, 10	500	50 ÷ 10	5

Pattern Language Puzzles (Algebra) - Worksheet 1

Solve this problem:

23 x 6 = (20 + 3) x 6
= (20 x 6) + (3 x 6)
138 = 120 + 18

Put in your own numbers. Use the same pattern. AWV

___ x ___ = (___ + ___) x ___
= (__ x __) + (__ x __)
___ = ___ + ___

If you change the numbers, does the pattern change? No

Explain how you do this problem. AWV

In Pattern Language (Algebra) it looks like this:

= (a + b) x c
= (a x c) + (b x c)
ac+bc = ac + bc

The pattern is always the same. You multiply both numbers in the ( ), parentheses, by the number after the X sign: (a + b) x c = a x c + b x c. In algebra we don't use the X sign because it looks like an x or a missing number. Therefore, in the equation (a + b) x c = ac + bc, ac means a x c.

Pattern Language Puzzles (Algebra) - Worksheet 2

Use the pattern to figure out what numbers the letter stands for
Work backwards from 54.

9 x 6 = (a + 5) x 6
= (a x 6) + (5 x 6)
24 + 30 = 54
a = 4

54
-30
24

9x6=54
24÷6=4

Your turn to try some on your own.

10 x 6 = (b + 2) x 6
= (b x 6) + (2 x 6)
48 + 12 = 60
b = 8

12 x 7 = (d + 4) x 7
= (d x 7) + (4 x 7)
56 + 28 = 84
d = 8

23 x 5 = (k + 9) x 5
= (k x 5) + (9 x 5)
70 + 45 =115
k = 14

Pattern Language Puzzles (Algebra) - Worksheet 3

Here is a different problem with two unknown numbers.

Q x 5 = (a + 8) x 5
= (a x 5) + (8 x 5)
20 + 40 = 60
a = 4
Q = 12

a x 5 = 20
a = 4
Q x 5 = 60
$5\overline{)60}$ 12

F x 8 = (d + 4) x 8
= (d x 8) + (4 x 8)
10 x + 32 =112
d = 10
F = 14

S x 5 = (k + 4) x 5
= (k x 5) + (4 x 5)
70 + 20 = 90
k = 14
S = 18

Pattern Language Puzzles (Algebra - Worksheet 4

D x 8 = (n + 4) x 8
= (n x 8) + (4 x 8)
72 + 32 = 104
n = 9
D = 13

T x 7 = (m + 6) x 7
= (m x 7) + (6 x 7)
56 + 42 = 98
m = 8
T = 14

P x 6 = (q + 5) x 6
= (q x 6) + (5 x 6)
72 + 30 = 102
q = 12
P= 17

Z x 12 = (y + 4) x 12
= (y x 12) + (4 x 12)
48 + 48 = 96
y = 4
Z = 8

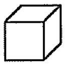
Number Patterns: Functions - Worksheet 1
Buildings

Complete the table to discover the pattern. Predict the number of blocks needed for the 50th building and then figure out the function. Build with cubes if needed. Graph both on the following pages.

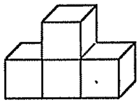
Building Number x	Number of Blocks Needed y
1	1
2	4
3	7
4	10
5	13
6	16
50	148
function	y=3x-2

Structure A.

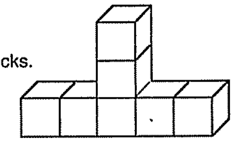
Building 1 needs 1 block.



Building 2 needs 4 blocks.

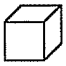


Building 3 needs 7 blocks.

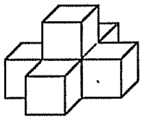


Structure B.

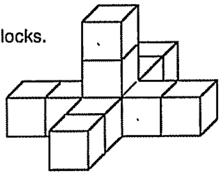
Building 1 needs 1 block.



Building 2 needs 6 blocks.

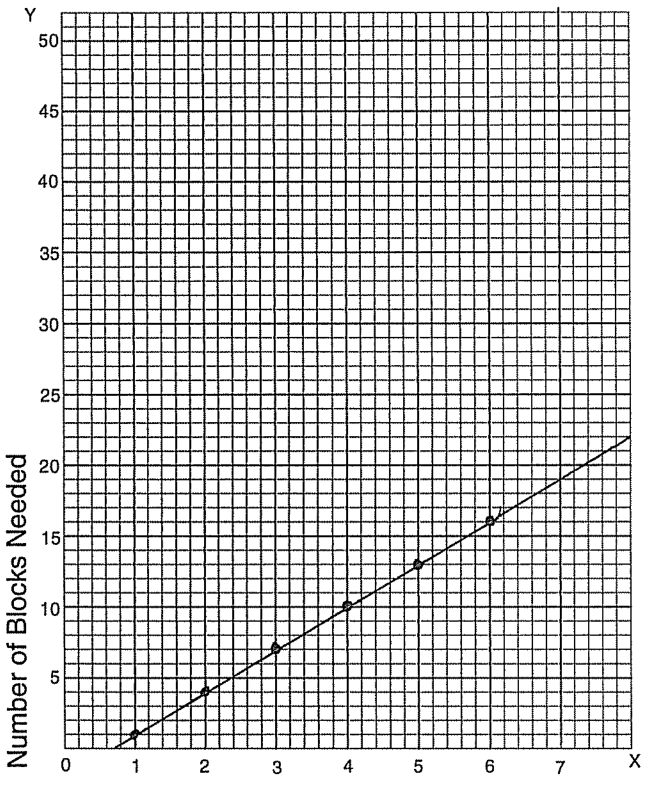


Building 3 needs 11 blocks.

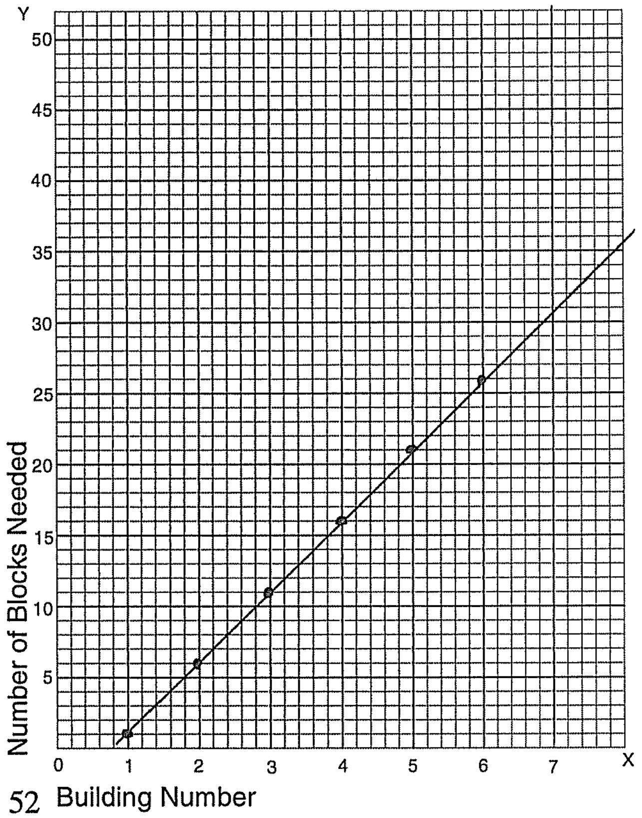


Building Number x	Number of Blocks Needed y
1	1
2	6
3	11
4	16
5	21
6	26
50	248
function	y=5x-4

Number Patterns: Functions - Worksheet 2
Graph 1 - Structure A



Number Patterns: Functions - Worksheet 3
Graph 2 - Structure B



Number Patterns: Functions - Worksheet 4
Graph it

Use (x, y) points to make a line on a graph. Find solutions to fill in the table using this method. Graph paper is on the following pages.

Table 1 Use page 54.

x	y
0	8
1	11
2	14
3	17
4	20
5	23
6	26
function	3x+8
50	158

Table 2 Use page 55.

x	y
0	2
1	7
2	12
3	17
4	22
5	27
6	32
function	5x+2
50	247

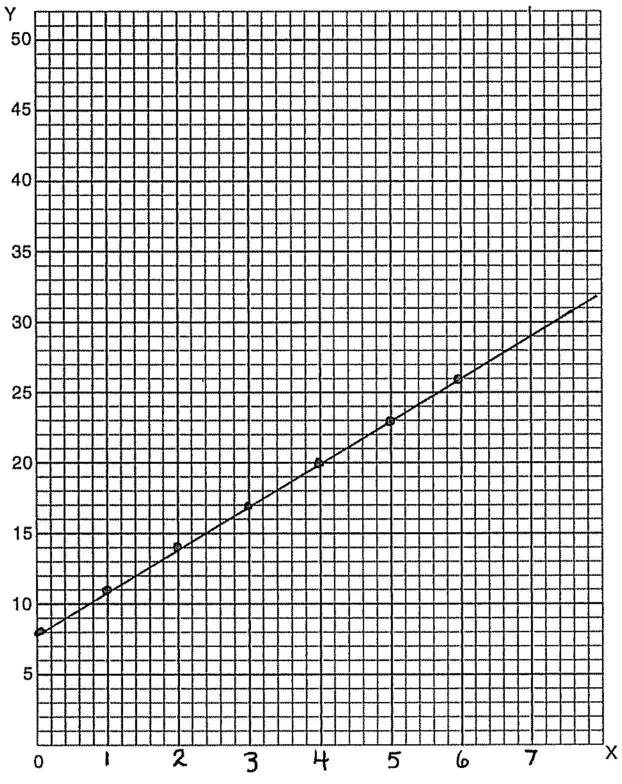
Table 3 Use page 56.

x	y
1	0
2	8
3	16
4	24
5	32
6	40
7	48
function	8x-8
50	392

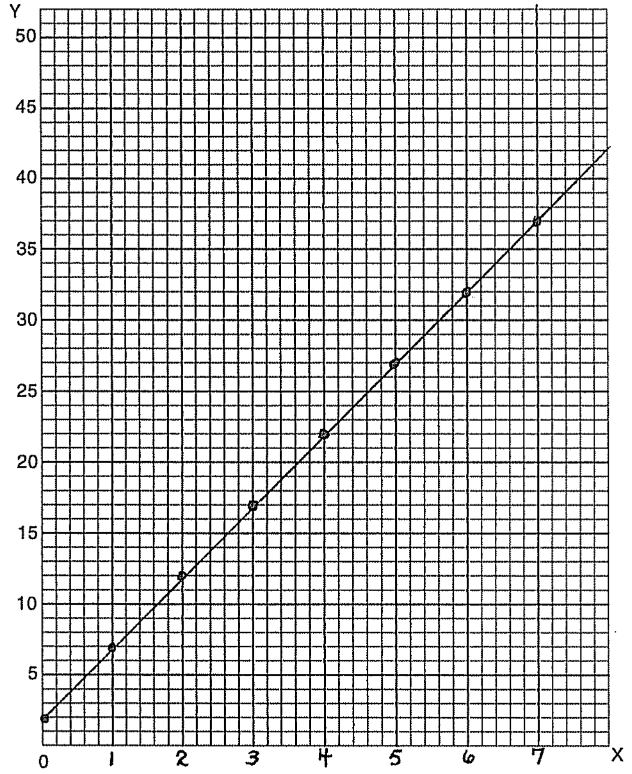
Table 4 Usevpage 57.
Do it backwards,
Get the points off of Table 4.

x	y
0	0
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
function	x^2

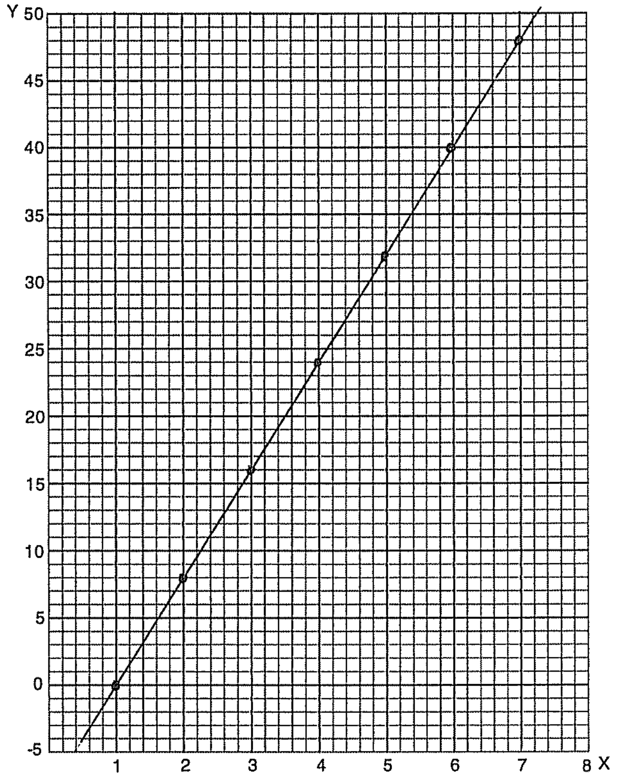
Number Patterns: Functions - Worksheet 5
Graph it - Table 1



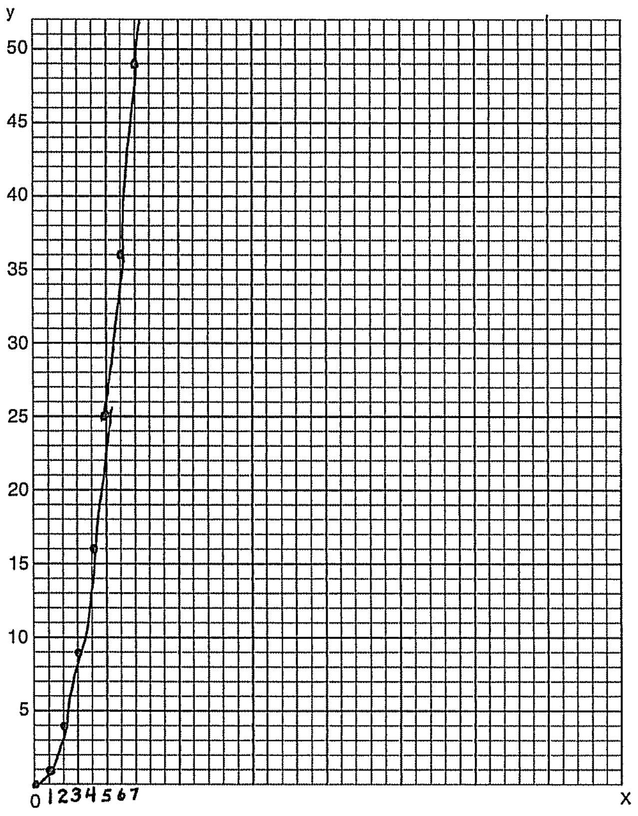
Number Patterns: Functions - Worksheet 6
Graph it - Table 2



Number Patterns: Functions - Worksheet 7
Graph it - Table 3



Number Patterns: Functions - Worksheet 8
Graph it - Table 4



Number Patterns: Functions - Worksheet 9
Tables

Table 1

x	y
1	3
2	4
3	5
4	6
5	7
6	8
7	9
50	52
function	X+2

Table 2

x	y
1	3
2	6
3	9
4	12
5	15
6	18
7	21
50	150
function	3x

Table 3

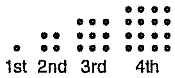
x	y
1	3
2	5
3	7
4	9
5	11
6	13
7	15
50	101
function	2x+1

Table 4

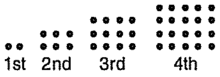
x	y
1	5
2	9
3	13
4	17
5	21
6	25
7	29
function	4x+1
50	201

Geometric Patterns - Worksheet 1

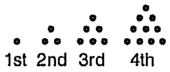
Numbers can be represented by dots in an array. The first four numbers of the square, oblong, and triangular numbers are pictured above the tables. Look for patterns and fill in the tables. After completing the table find the number of dots in the nth number for Square and Oblong Numbers. Graph the first six terms on the folowing page.



Square Number	Number of Dots
1st	1
2nd	4
3rd	9
4th	16
5th	25
6th	36
50th	2,500
nth	n^2

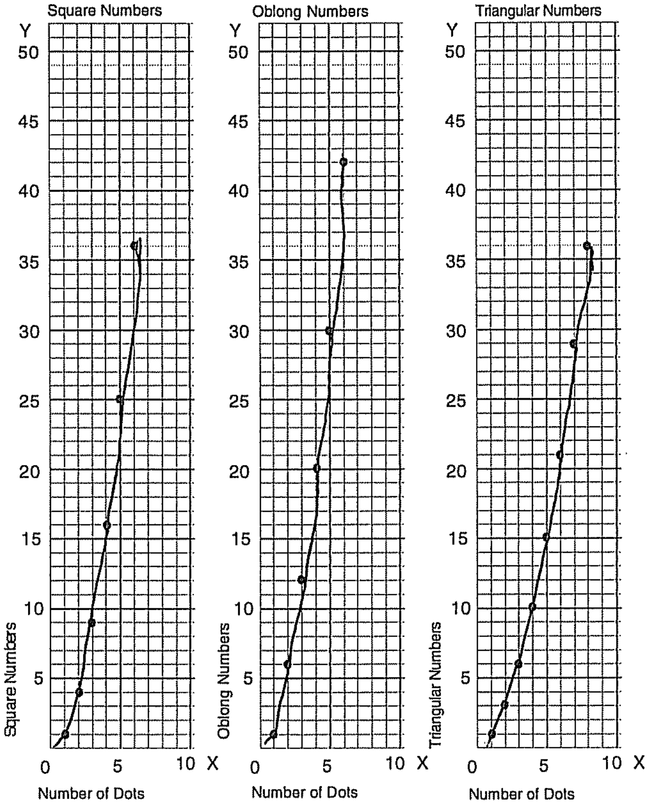


Oblong Number	Number of Dots
1st	
2nd	6
3rd	12
4th	20
5th	30
6th	42
50th	2,550
nth	n^2+1



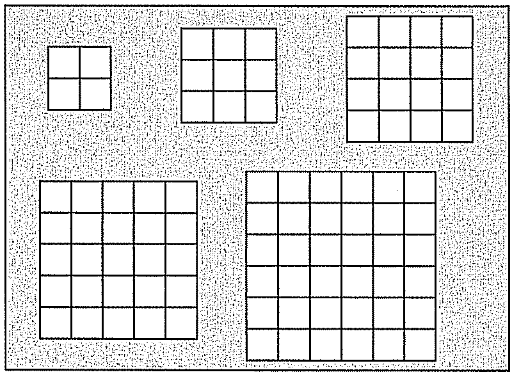
Triangular Number	Number of Dots
1st	1
2nd	3
3rd	6
4th	10
5th	15
6th	21
7th	28
8th	36

Geometric Patterns - Worksheet 2



Geometric Patterns - Worksheet 3

Each of these squares are made up of unit squares. Some of these units touch the shaded area on one side, some on two sides, and some on no sides. Fill in the tables using this picture.



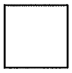
Length of One Side	Area of Squares	Number of Unit Squares Touching On:		
		2 sides	1 Side	No Sides
2	4	4	0	0
3	9	4	4	1
4	16	4	8	4
5	25	4	12	9
6	36	4	16	16
7	49	4	20	25
10	100	4	32	64
n	n^2	4	$4(n-2)$	$(n-2)^2$

What patterns do you notice? Area is the sequence of squares. 2 sides touching is always 4 because these are corners. One side touching is the 4 times table; there are 4 sides. No sides touching is the sequence of squares but 2 squares back. Each side of a square with one side touching the length of a side - 2X4.

Investigating Cubes - Worksheet 1, Square Numbers

Build with orange pattern blocks or square tiles.

1st square



How many blocks? 1

Build the next size square. How many blocks? 4

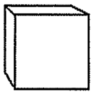
Keep building the next size larger square. Write the total number of blocks below.

Can you find a pattern so you don't have to build all of the squares?

How many blocks will the 10th square have? _______ Guess.

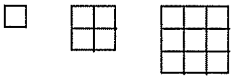
Squares	Number of blocks on one side	Total number of blocks	Number of blocks added	Number Squared
0	0	0	0	0
1st	1	1	1	1
2nd	2	4	3	2
3rd	3	9	5	3
4th	4	16	7	4
5th	5	25	9	5
6th	6	36	11	6
7th	7	49	13	7
8th	8	64	15	8
9th	9	81	17	9
10th	10	100	19	10

Investigating Cubes - Worksheet 2, Square Numbers



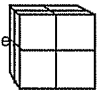
What is a cube?

Look back over the work you did on the sequence of squares.



Remember one squared is 1, two squared is 4, three squared 9.
There is a sequence of cubes also.

Use linking or centimeter cubes.



The first cube in the sequence is a single cube block.
Now, build a larger cube that is 2 cubes wide on each side.
How many blocks are needed to build this cube? ___

Use your cube models to fill in this chart.

			length		width		height		
1st cube has 1 block in it.	It is		1	x	1	x	1		
2nd cube has 8 blocks in it.	It is		2	x	2	x	2	=	8
3rd cube has 27 blocks in it.	It is		3	x	3	x	3	=	27
4th cube has 64 blocks in it.	It is		4	x	4	x	4	=	64
5th cube has 125 blocks in it.	It is		5	x	5	x	5	=	125
6th cube has 216 blocks in it.	It is		6	x	6	x	6	=	216
7th cube has 373 blocks in it.	It is		7	x	7	x	7	=	343
8th cube has 512 blocks in it.	It is		8	x	8	x	8	=	512
9th cube has 729 blocks in it.	It is		9	x	9	x	9	=	729
10th cube has 1,000 blocks in it.	It is		10	x	10	x	10	=	1,000

Investigating Cubes - Worksheet 4, Square Numbers

Patterns in the Sequence of Cubes

On Investigating Cubes - Worksheet 1 the Number of Blocks Added for each square was recorded. You made a model of this with colored squares.

Below is the written sequence that is used to find the pattern in the Number of Blocks Added.

The first row shows the sequences of squared numbers. The second row shows the Number of Blocks Added, or the difference between the total number of blocks and from one squared number to the next squared number. You noticed before that the second row is the sequence of odd numbers. The third row shows the difference between each of the numbers in the second row. Show that you can find the pattern by filling in the blanks.

1 4 9 16 25 36 49 64

3 5 7 9 11 13 15

2 2 2 2 2 2

What patterns occurred in the third row? The differences are all twos.

Now do the same thing with the sequence of cubes. In the top row is the sequence of cubes. The second row is the number of centimeter cubes added when going from one cubed number to the next cubed number. Use your model to help you get this number. Or you could take the difference from each number. The third row takes the difference between the differences between. The forth row takes the difference between the numbers in the third row.

Fill in the blanks. You may use a calculator. What patterns do you see?
The differences in row 3 are all sixes.

1 8 27 64 125 216 543 512

7 19 37 61 91 127 169

12 18 24 30 36 42.

6 6 6 6 6

Averages: Up and Down - Worksheet 1

Here are the number of text messages Laurel received each day.

6 Monday
5 Tuesday
6 Wednesday
3 Thursday
20

What is her average? 5

Will the average go up or down if she received 7 messages instead of 3 on Thursday?
up.

Her new messages would then be: 6, 5, 6, 7. The new average is 6.

Here are the number of text messages Judi received each day.

7 Monday
8 Tuesday
7 Wednesday
3 Thursday
8 Friday
9 Saturday
42

What is her average? 7

Will the average go up or down if she had 3 messages instead of 9 on Saturday? down.

Her messages would then be: 7, 8, 7, 3, 8, 3. The new average is 6.

How many messages would she have to receive on Saturday to change her average to 8 messages? 15

Averages: Up and Down - Worksheet 2

Here are the number of text messages Tom received each day.

7 Monday
7 Tuesday
6 Wednesday
8 Thursday
28

What is his average? 7

Will the average go up or down if he received 4 messages on Thursday instead of 8?
down

Predict by how much.______ His new amount of messages are: 7, 7, 6, 4.
New average. 6

How many messages would he have to receive on Thursday to change his average to 8 messages? 12

Here are the number of text messages Peter received each day.

9 Monday
8 Tuesday
7 Wednesday
8 Thursday
8 Friday
40

What is his average? 8

Will the average go up or down if he had 13 messages instead of 8 on Friday? up.

His messages would then be: 9, 8, 7, 8,13. The new average is 9.

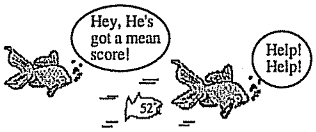
How many messages would he have to receive on Friday to change his average to 7 messages? 3

On a piece of paper write an average problem in which the average goes up.

Write another average problem in which the average goes down.

Averages: Up and Down - Worksheet 3

Save all your calculations.



Another word for average is the mean

The procedure for taking an average is: 1. Add the set of numbers.

2. Divide the total by the amount of numbers in the set.

Now, here are some problems which Michael gives to his college math

1. Study this set of numbers [7,11, 19, 28, 40].
   What do you think the mean is? Guess_______
   Calculate. The mean of this set is 21.

2. Now, using the same set of numbers [7,11, 19, 28, 40], change each number in the set so that the mean raises to 25.
   Example:
   New set: [9, 17, 23, 32, 44]

Can you do the same thing to each number? Yes, add four to each number.

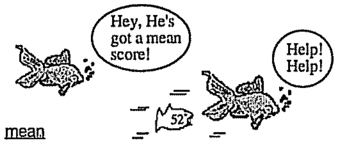
3. Do the same again, same set, [7,11, 19, 28, 40], and lower the mean to 15.
   Example:
   New set: [1, 5, 13, 22, 34] Take away six from each number.

4. Again, same set, [7,11, 19, 28, 40], and raise the mean to 36.
   Example:
   New set: [22, 26, 34, 43, 55]

What is the rule for lowering and raising the mean of a set?
Find the difference between the original mean and the new mean. Add or subtract that number to every number in the original set of numbers.

Averages: Up and Down - Worksheet 4

Save all your calculations.



Another word for average is the mean

1. Study this set of numbers [5, 9, 8, 14, 6, 0].
   What do you think the mean is? Guess_______
   Calculate. The mean of this set is 7.

2. Now, using the same set of numbers [5, 9, 8, 14, 6, 0], raise the mean to 8.
   Example:
   New set: [5, 9, 8, 14, 6, 6] $= 48 \div 6 = 8$

3. Do the same again, same set, [5, 9, 8, 14, 6, 0], and lower the mean to 5.
   Example:
   New set: [3, 7, 6, 10, 4, 0] $\begin{array}{r}42\\-12\\\hline 30\end{array}$ $30 \div 6 = 5$

4. Again, same set, [5, 9, 8, 14, 6, 0], and raise the mean to 9.
   New set: [5, 9, 8, 14, 6, 12] $\begin{array}{r}42\\+12\\\hline 54\end{array}$ $54 \div 6 = 9$

What is the rule for lowering and raising the mean of a set?
Find the average of the original set. Multiply the number of items in the set by the average you are trying to end up with. Find the difference between the original sum and the new sum. This number is added or subtracted to the original set to raise or lower the mean of the new set.

Averages: Up and Down - Worksheet 5
Challenge!

Save all your calculations.

1. Here are the teams scores [14, 18, 12, 19, 18, 7, 18, 6].
   What do you think the mean is? Guess_______
   Calculate. The mean of this set is 14.

2. Raise the mean to 18. Add 32

   New set: [AWV]

3. How much do your 4 lowest scoring athletes have to increase their scores to raise the team's mean to 18? Add eight to each of the four lowest scores.
   Show the improved scores.

   [22, 18, 20, 19, 18, 15, 18, 14]

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