Rainbow Rectangles and Area and Perimeter Logic Puzzle

Purpose The purpose is to delight a student with a lovely set of patterns and then use these patterns to review the area and perimeter of rectangles.

Prerequisites Basic multiplication tables

Materials Rainbow Rectangles - Worksheets 1 - 6, pages 8 - 13
Crayons or colored pencils
A blank piece of white paper

Warm Up Use a blank piece of white paper. Turn the paper sideways so the long side is on the bottom. On the piece of white paper write this sequence of numbers across the bottom of the page:

7 8 9 10 11 12 13 14

Choose a crayon, for example, blue. Put a blue circle around the seven, and a blue circle around the fourteen. Connect the two circles with a high arching blue line that will be the top of your rainbow. (See the figure on Rainbow Rectangles - Worksheet 1 on the right hand side of the page.) Now choose a green. Circle the eight and the thirteen in green. Draw a connecting arch between these two numbers. This arch will be under the blue one. Repeat with the nine and twelve, and the ten and eleven, choosing a different color for each pair. Now add the numbers in each pair.

"What do you find?" "The sums of each pair are always the same."

"Flip the page over and write another series of consecutive numbers, at least seven, and use an odd number of them this time."

What do you find?" "The sums of each pair are always the same."

"What about the odd number in the middle?" "If you double it, the sum is the same as the others."

"Isn't that cool? We call this a rainbow pattern. All kinds of interesting things happen with this pattern."

Note See a famous use of this pattern in the Extension at the end of the lesson.

Lesson Part 1 Begin with Rainbow Rectangles - Worksheet 1. Have the student trace the upside down rainbow with colored pencils. On the left is the pattern of the sums of the pairs. After he adds the pairs, he will again see the sums are equal.

"Do you think the products of the pairs will be equal also?" Most students will say they do not know.

"Try it." "The products are not equal."

"But what pattern do you see in the numbers as you go down the column?" "The numbers go up."

"Do the numbers go up in a regular pattern?" To answer this question, he must look at the differences between the numbers. There is a second column to the right of the column of products. Take the difference between the first two products twenty and twenty-seven. The difference is seven. Continue down the column taking the difference between the numbers.

Note

See the Answer Key to clarify how the pattern works.

"What pattern do you see?" "The sequence of odd numbers."

"Take the difference between each odd number now and write it to the right in the next column of spaces. What do you see?" "The differences are all two."

Continue on with the worksheet, answering the questions about the patterns. End the session.

Note

This part of the lesson uses the number series 2 - 10 and uses the pairs to form the length and width of rectangles. The lesson not only reviews the calculation of both area and perimeter, but also investigates the relationship between the area and the perimeter. In this lesson, the perimeter is held steady and the area changes in a predictable way. Most students will be surprised to find that the area changes when the perimeters are held steady. The conclusion we want him to see is that the closer the rectangle is to a square shape, where the length and width are nearly the same (the center pair of the rainbow pattern), the larger the area will be.

Lesson Part 2

On Rainbow Rectangles - Worksheet 2, follow the questions on the worksheet. To answer the short essay question at the bottom, draw his attention to the fact that the perimeter numbers match the series of numbers worked with on the top of Rainbow Rectangles - Worksheet 1.

"How is the perimeter of each rectangle related to the sum of the pairs on Rainbow Rectangles - Worksheet 1?" "The perimeter is twice the length of the sum of the pair because each pair is added twice to form the edges of the perimeter."

"At the bottom of the page, use a colored pencil to trace the pair in the rainbow that was used to make the edges of Rectangle C."

Practice Worksheets

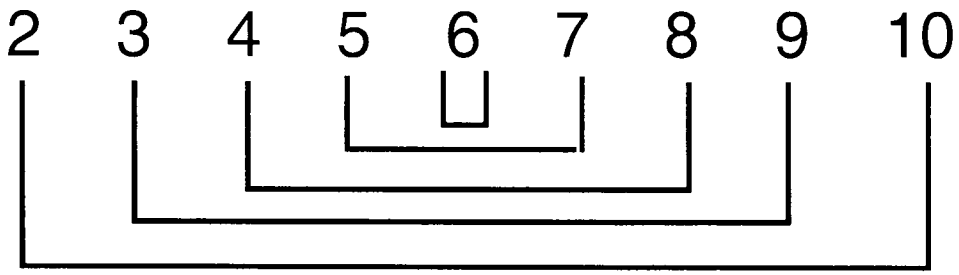
Rainbow Rectangles - Worksheets 3 - 5, pages 10 - 12

Test for Understanding

Rainbow Rectangles - Worksheet 6, page 13. See how much of this problem he can do alone. Help him if needed; the questions he asks will tell you what he does and does not understand.

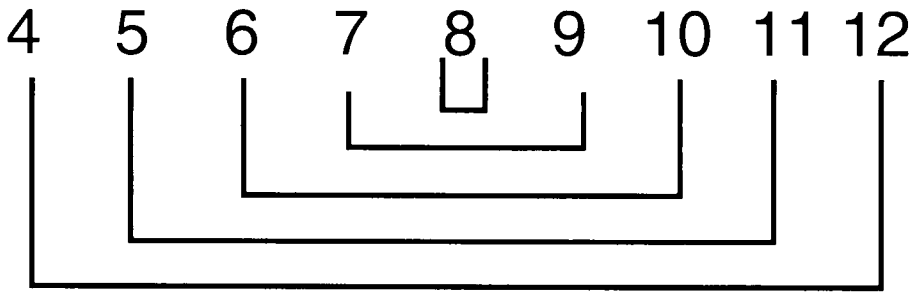
Rainbow Rectangles - Worksheet 1 Date _____

Here is a series of numbers. Multiply the numbers that are connected by the dark lines. Record the results below.



2 + 10 = _____	2 x 10 = _____
3 + 9 = _____	3 x 9 = _____
4 + 8 = _____	4 x 8 = _____
5 + 7 = _____	5 x 7 = _____
6 + 6 = _____	6 x 6 = _____

When these pairs are added, the sums are _____.
When the pairs are multiplied, the products are not equal.
Circle the pair that has the largest product.
Try it again on another sequence. See if the same pattern appears.



What is the sum of each pair? _____ Now multiply.

4 x 12 = ____ 5 x 11 = ____ 6 x 10 = ____ 7 x 9 = ____ 8 x 8 = ____

Which pair had the largest answer this time? _____
Make a rule for choosing the multiplication problem with the largest answer. Try a few more sequences to be sure.

My rule is _____

_____

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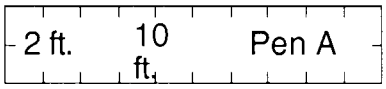
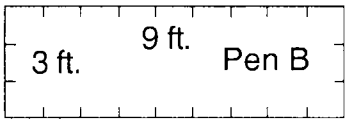
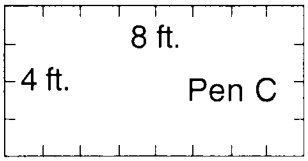
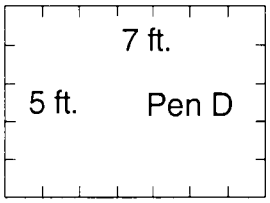
Rainbow Rectangles - Worksheet 2 Date _____

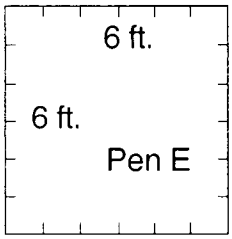
These rainbow patterns help us solve problems dealing with area and perimeter of rectangles. Here are 5 animal pens. Circle the one that looks the roomiest.











Use the numbers or the lines inside each drawing to calculate the area and perimeter of each pen.

Area of Pen A = _____sq. ft. Perimeter of Pen A = _____ feet.

Area of Pen B = _____sq. ft. Perimeter of Pen B = _____ feet.

Area of Pen C = _____sq. ft. Perimeter of Pen C = _____ feet.

Area of Pen D = _____sq. ft. Perimeter of Pen D = _____ feet.

Area of Pen E = _____sq. ft. Perimeter of Pen E = _____ feet.

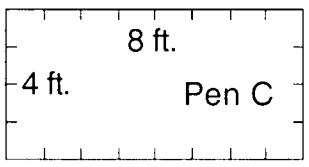
What do you notice about the perimeters? _____

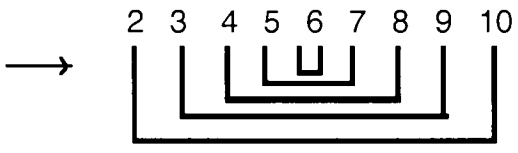
What do you notice about the areas? _____

When you compare the length, width, and area of each pen to the number series and multiplication problems that you first worked with, what connection do you see?_____

_____

_____





Pen C = 32 sq. ft.

Number Patterns: Functions

Purpose

The purpose is to connect manipulative, physical structures to T charts used to record the total number of blocks in the structures, using ordered pairs of numbers used to graph the pattern. The numbers are graphed and then the pattern is extended. This is a major tool used by scientists to find formulas to describe relationships in nature. For example: Isaac Newton used data from astronomers concerning the movements of planets to find the relationship between distance between planets, their size, and how they affect each other's movements to find the function for the force of gravity. This skill forms the base for a significant part of what is studied in Algebra I.

Prerequisites

The ability to generate a mathematical rule given a pair of numbers that follows the rule and to place a point on a graph given its coordinates. Coordinate graphing, What's My Rule?, Number Patterns: Functions, and T charts presented in *Patterns in Arithmetic:* General Math - Booklets 3 and 4. Or *GEMS–Algebraic Reasoning*, play the Function Machine Game (look this up online). This site has a good one: www.mathplayground.com/functionmachine.html

Note

This lesson is not a beginners lesson in functions.
There are several meanings to the symbol 'x': as multiplication or times sign in arithmetic; to identify the 'x' or horizontal axis on a graph, as 'y' is the vertical axis; to express the unknown in algebra, e.g., x - 3 = 5, x = 8). In function lessons it is used to identify the independent variable and is graphed on the x axis. Therefore, multiplication problems will now take the form of 3x, e.g., if x = 2, then 3x = 6.

Materials

Number Patterns: Functions - Worksheets 1 - 9, pages 50 - 58
Cubes
A colored pencil
A ruler

Warm Up

Play What's My Rule?

Lesson Part 1

Begin with Number Patterns: Functions - Worksheet 1, Thrones. Read the story with the student. Have fun role playing the different parts. Ham it up by using different voices for the characters in the story.

Work together on Number Patterns: Functions - Worksheet 2, Thrones. Fill in the chart for Throne 4.

"What did the boy do each time he built a throne?" "First he built a rectangular shape. Then he took away one cube to make a seat."

"Predict how many cubes Throne 5 will have." "It will have nine."

The student fills in the chart, on page 51, up to Throne Number 6 and answers this question about the function, **"How do you calculate the total number of cubes**

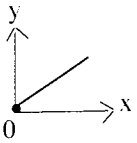
used in a throne?" "Multiply the throne number (x) by two and then take away one." **"How do you write that in an algebraic form?"** "2x - 1."

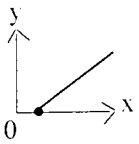
"Predict what the graph will look like and sketch it on the bottom of the page."

Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **"How did you get that?" "Show me what you mean." "Build a model of that." "Tell me more so I can understand what you are saying."**

Note

The Test for Understanding is embedded. If his sketch passes through the origin, he is not yet understanding T charts well enough to graph clearly. The line starts at one, not zero.





Mathematicians have figured out a way to solve this kind of problem. Let's keep exploring.

Now take out Number Patterns: Functions - Worksheet 3.
"Look at the top of Number Patterns: Functions - Worksheet 2 that you just completed. Graph the first pair of numbers." Throne 1 uses one block. So the ordered pair is (1, 1). Go over to the one on the x axis, and up just one little box on the y axis and place a point. Check the Answer Key.

"Put all the points on the graph. What do you notice?" "They make a straight line." **"Connect them."**

"What if the graph had more numbers?" "It would probably be straight forever."

"Where would the seventh throne's total number of blocks show up?" Have him get a colored pencil and put it on the seven on the x axis. Trace that line upwards all the way to the top of the page. Now, take your regular pencil out, and using a ruler, extend the line made of points until it crosses the colored line for the seventh throne and draw a point. Have him place his finger on the point and trace the horizontal line moving left until he reaches the y axis. The number on the scale of the y axis should match the number he would have gotten on the chart for the seventh throne. This is a way to use the graph to tell what the total number of cubes is needed for any throne. See the Answer Key to be sure he is doing the work correctly.

Warm Up

Play What's My Rule? or the Function Machine Game - see Prerequisites.

Lesson Part 2

Number Patterns: Functions - Worksheet 4 - Graph It, page 53. **"Now you are going to figure out a pattern by studying the numbers and then place that pattern**

on the graph on Number Patterns: Functions - Worksheet 5, page 54. Where do you record the x and y columns?" "The x column will be graphed on the horizontal line, the y column on the vertical line."

"So the first point would be placed where?" "Over zero and up eight, so it's placed at (0, 8)."

"Where is the second point placed?" "Over one and up eleven, so it's placed at (1, 11). How to think about what the function might be: When any number is multiplied by zero the resulting product will be zero. On this pattern, when you put in a zero you get back an eight. That tells you there is a + 8 on the end of this function. The pattern is going up by three each time. That tells you there is a multiplication by three going on. So if the x is one, multiply one times three and add eight. That gives the answer of eleven. Try it again with the two; $2 \times 3 + 8 = 14$. So $y = 3x + 8$ is the function for this pattern.

When graphing this function, only two to three points have to be graphed to tell where the line is. All other numbers in the table can be found from the graph like we did on the seventh throne.

Extension

Extension for advanced students (few fifth-graders will get this). You can also see the function in the graph. Notice that the line crosses the y axis at eight. That tells you there is a + 8 in the function. Then notice that as the points go up the graph they always go over one up three. That tells you that there is a multiplication by three in the function. So the function is $y = 3x + 8$.

Lesson Part 3

Number Patterns: Functions - Worksheet 6, Graph It, Table 2, page 55
Use the numbers from Table 2 on Number Patterns: Functions - Worksheet 4, Graph It, page 53.

Possible conversation to figure out the function:
"On the first line the x is a zero and the y is a two. What does that tell you about the function?" "That there is a + 2 on the end."

"So is the function $y = x + 2$?" "No because when I go to one, I can see the y value is not three but seven. That means something else is happening too."

"How can you tell what the something else is?" "The pattern is going up by a skip count of five. That means there is a multiplication by five first."

"So what is the function?" "$y = 5x + 2$."

"Graph it. What pattern do you notice?" "The line crosses the y axis at + 2. The points go over one and up five."

Number Patterns: Functions - Worksheet 7, Graph It, Table 3, page 56
The table is on Number Patterns: Functions - Worksheet 4, Graph It, page 53.

The function for this one is really tricky.

"What pattern do you see in the y side of the chart?" "The eight times table."

"But what is wrong with it?" "It should have the eight across from the one, not the two."

"So what do you have to do to the two to make it act like a one?" "Take away one."

"If you take away one before you multiply by eight, what happens?" "You get the numbers in the table."

"So what is the function?" "You have to take away one from the x and then multiply it by eight. Or $y = (x - 1)$ times eight, which is written $y = 8 (x - 1)$."

Continuing on Number Patterns: Functions - Worksheet 4, Table 4
This time, there are no y values given at all. The y values must be obtained from the graph on Number Patterns: Functions - Worksheet 8, Graph It, Table 4, page 57. **"What do you notice when you connect the points?"** "It is not a straight line but a very steep curve." **"That tells you there is an exponent attached to the x."**

Practice Worksheets

Number Patterns: Functions - Worksheet 9, page 58
The student completes the tables without graphs. Supply graph paper if the student needs to continue graphing. Copy Grid Paper, page 34

Test for Understanding

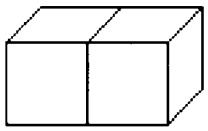
Have him predict where the graph of Table 3 on Number Patterns: Functions - Worksheet 9 crosses the y axis, or what the zero value of the table will be.

Make a graph for the function $y = x \div 2$. Use odd numbers also so the points will fall on fractional units. Use large square graph paper. Have him figure out the function from the graph.

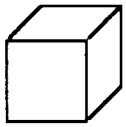
Number Patterns: Functions - Worksheet 1 Date_____

Thrones

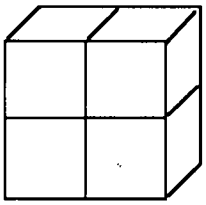
Once upon a time, a proud king strode into a humble village to address his docile subjects. "I need a throne!" he bellowed.



A terrified boy quickly brought him two cube boxes to sit on.

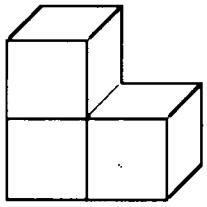


"That is too fat; take one cube away," shouted the silly king. So the competent boy took away one cube and the king sat down on it. "Throne 1 is too small and there isn't a back to lean against."



So, the flustered boy brought more cubes and stacked them like this:

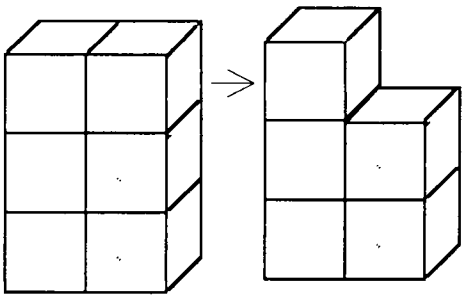
"That is better but I need a seat and backrest. Remove one cube," barked the unhappy king.



So the frustrated boy removed one cube to make Throne 2 look like this:

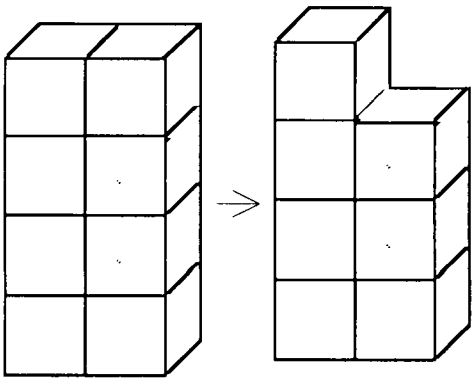
"I want a taller throne! Your heads are above mine!" cried the disgruntled king.

So the intrepid boy made Throne 3 by first making a stack then removing one cube to make the seat.



"I want an even taller throne," shrieked the pompous king.

So the patient boy built Throne 4 by first making a stack, then removing one cube to make the seat.



Finally, the happy king declared, "Throne 4 is great!"

The people of the village gave a great shout of relief. "Huzza! Huzza! Huzza!"

And the tired boy wiped his brow, "Whew."

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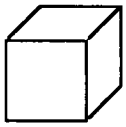
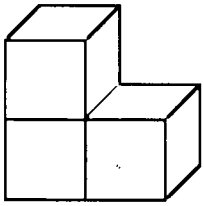
Number Patterns: Functions - Worksheet 2 Date_____

Thrones

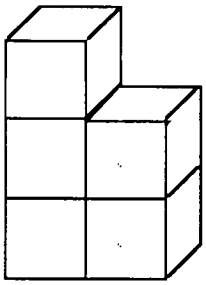
Throne 1 needs 1 cube.



Throne 2 needs 3 cubes.



Throne 3 needs 5 cubes.



X Throne	Y Cube Boxes
1	1
2	3
3	5
4	
5	
6	
Function	Y =

Record the number of cubes Throne 4 needs on the chart.
Write what the boy did as a mathematical function.

First, he _____

Then, he _____

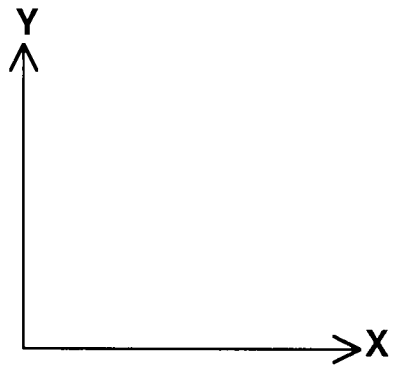
X is the throne number. Y is the total number of Cube Boxes used.

Continue the pattern to Throne 6.

How do you calculate the total number of Cube Boxes used in a throne?

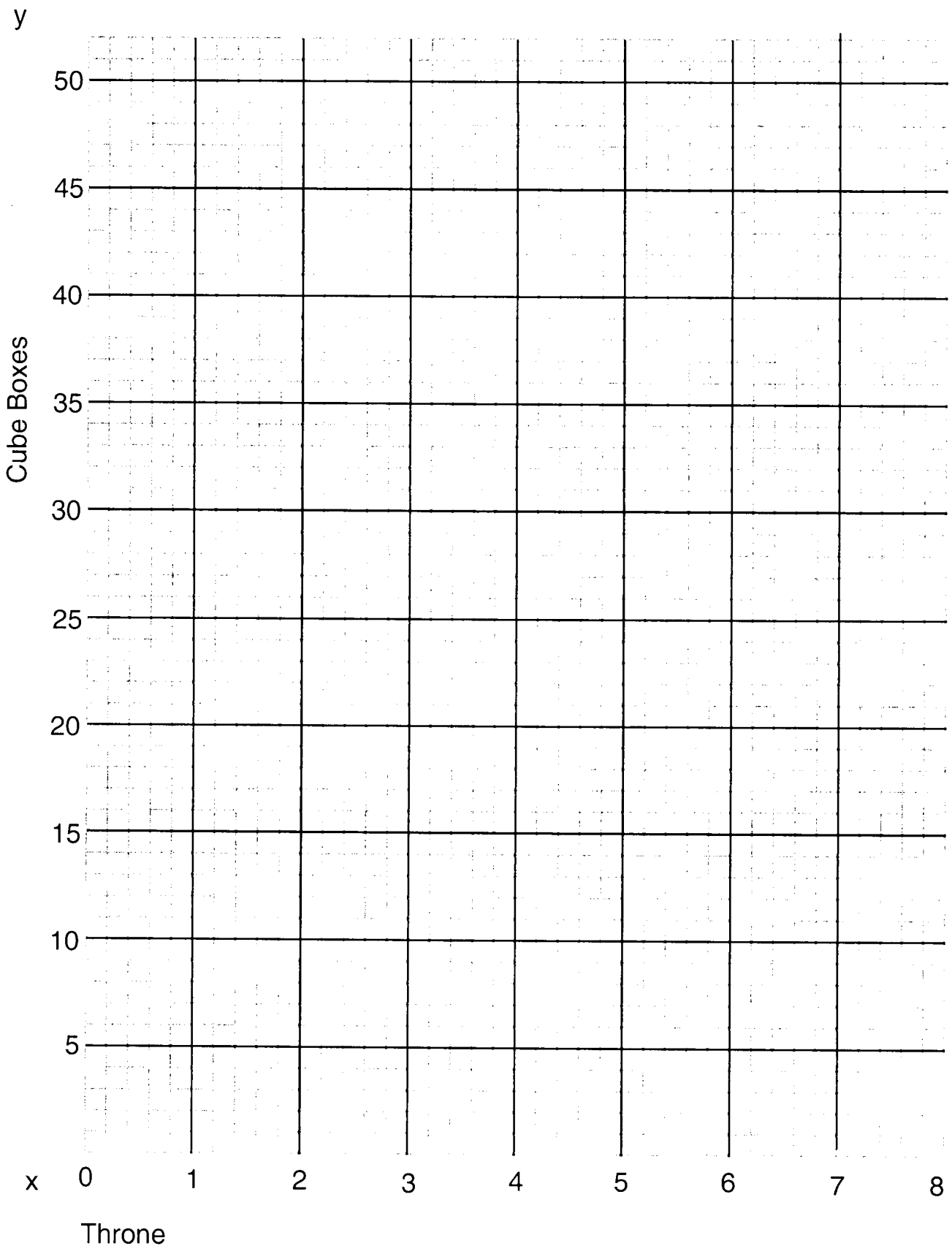
_____

What do you think the graph will look like? Sketch it. Do not put in the numbers. Then graph it on the following page.



Number Patterns: Functions - Worksheet 3 Date_____

Thrones - Graph 1



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