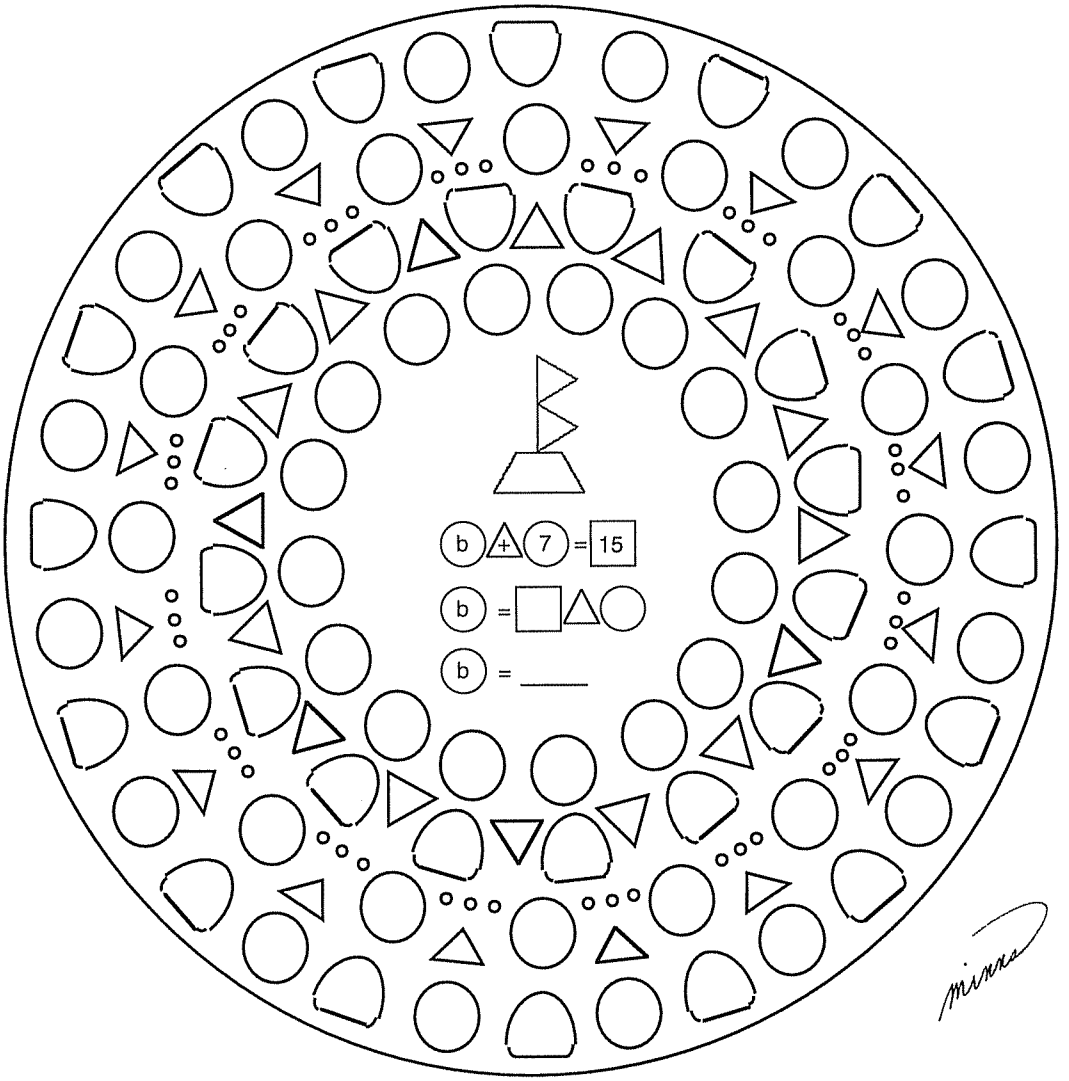
Patterns in Arithmetic

General Math - Booklet 4 PDF

Patterns and Algebraic Thinking

Parent/Teacher Guide



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Based on methods developed by Prof. Michael Butler at the
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General Math: Booklet 4 PDF - Patterns and Algebraic Thinking

Contents

This booklet is dedicated to Andrew J. Galambos. We are grateful for the positive influence he has had our lives.

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The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

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Parent/Teacher Guide

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Greater Than, Less Than, or Equal To

Purpose

The purpose of this lesson is to exercise mathematical reasoning using familiar symbols and ideas. This lesson is explicitly *not* for the sole purpose of teaching the use of these symbols. The lesson requires the student to write her answer in her Math Journal. To make that easier, the teacher and the student talk it through first, then settle on a way to explain it in words using mathematical vocabulary. Then the student writes it down, along with the problem. This exercise will not only develop her fluency with mathematical vocabulary but also build the kind of mathematical reasoning that underpins understanding.

Prerequisites

Previous work with greater than, less than, and equal to problems.

Materials

Greater Than, Less Than, or Equal To - Worksheets 1 - 4, pages 1 - 4
Red and green crayons or markers

Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **"How did you get that?" "Show me what you mean." "Build a model of that." "Tell me more so I can understand what you are saying."**

Warm Up

It is important to do this warm-up. Give the problems using the green and red colored spots to stand in for 'any number.' This is an important step to getting students to think generally, using principles rather than arithmetic to find a solution.

"Should this number sentence have a greater than, less than, or equal to, in order to make an always true sentence: R + G ? G + R?" "An equal sign because the Commutative Property of Addition says you can always switch the positions of the numbers when you are adding and still get the same answer."

Note

If she is not sure, have her make some trials with numbers less than 10, e.g., 2 + 3 = 3 + 2.

"What about R x G ? G x R?" "Same as before. They are equal because of the Commutative Property of Multiplication."

"What about this one: If neither G nor R is zero, one or two, and both are positive numbers, what sign would make this an always true number sentence: G + R __ G x R ?" "The sign should be less than, < , because multiplication is lots of additions of one of the numbers, so G x R would be more than G + R."

"What about G + R ? G - R? Assume G is a positive number." "It would be G + R > G - R because adding a number to another number will give a larger quantity

than subtracting that same number."

"Can you find any examples where G + R < G - R would not be true?" The answer will be unique. If she gives you a negative number, give her a huzza! for such great thinking.

Lesson

Greater Than, Less Than, or Equal To - Worksheet 1, page 1
The top nine problems review the use of the signs <, >, and =. The example at the bottom ties the top nine problems together with the red and green work done in the warm-up. The student is asked to explain in words why the answer she gave makes sense without simply using the arithmetic answer.

Practice Worksheets

Do Greater Than, Less Than, or Equal To - Worksheet 2, page 2, together.

Greater Than, Less Than, or Equal To - Worksheets 3 and 4, pages 3 and 4. You can do one worksheet per day to keep the work from getting tedious.

Test for Understanding

The student demonstrates understanding with the ability to clearly write the logic of her discussion on these pages. See the Answer Key.

Area: Nonstandard

Purpose The purpose of this lesson is to review estimation skills and for the student to review the construction of the general concept of area as covering a space. These spaces do not have to have a regular shape. The space sizes are quantified using differently sized beans. The relationships between the areas covered by each of the beans is also explored. This activity is more fully explored in General Math: Booklet 3.

Prerequisites None

Materials Area: Nonstandard - Worksheet, page 5
Glue, modeling clay, or Play-Doh
Pinto beans, split peas, and lima beans

Warm Up Place a raised border around the perimeter of the irregular shape on the worksheet. Glue works best but needs time to dry hard before you do the activity. Modeling clay works fine too, but it is a bit sticky and takes more time to put onto the paper. Students enjoy doing it, though.

Lesson Follow the instructions on the worksheet.

Note Some students will have difficulty manipulating the split peas. A tweezer may help. Some students need adults to help with this.

Practice Worksheets None

Test for Understanding Draw an irregular space about two-thirds the size of the space on the worksheet. Shade it in. Have the student estimate the number of each type of bean that will fit on that space. You are looking to see if he uses the data built up in the exercise above to give reasonable estimates. Have him then measure to check his estimations.

Area: Standard

Purpose

The purpose of this lesson is to use the construction of the procedure for calculating the area of a rectangle to strengthen mathematical reasoning and use of patterns to construct algebraic procedures. The formula for this procedure is so simple and obvious, students do not have to be told how to do this calculation but can discover it using what they already know about arrays in multiplication. This lesson also reinforces the idea of area as a covering of space and being able to measure the size of that cover.

Prerequisites

Area: Nonstandard

Materials

Area: Standard - Worksheets 1 - 5, pages 6 -10
Centimeter tiles and inch tiles
Centimeter ruler (There is a paper ruler to cut out on the bottom of Area: Standard - Worksheet 5, page 10.)

Note

Centimeter and inch tiles can be constructed from cutout grid paper if you do not have tiles. The Internet has printable grids you can print onto heavy paper and cut out. Do not use cubes as this suggests volume instead of a flat area.

Warm Up

Have the student play with the tiles and make designs and structures before you try to do the lesson. There are lots of great art activities that can be done with colored tiles.

Lesson Part 1

Have her study the new words box on Area: Standard - Worksheet 1. Have her trace the outside boundary edge–this is the perimeter. Then have her shade in the space within the boundary. This space is called area.

"When we estimate, we usually use a range. A range is the space between the highest number you think will fit and the lowest number it could possibly be to fill the space. Look at Figure 1. What is the highest number of centimeter tiles you think could fit within the boundary? The lowest number?"
Record her answers on the worksheet where it says "I guess between ____ and _____." A reasonable answer would be between ten and twenty.

Note

Have her make a range estimate without eyeballing the length of the sides and trying to count it out before she just guesses. Just reckon it. This develops estimation skills and confidence. Students who will not do this, who hang onto the idea that the estimate must be right on, especially need to work on reckoning and loosen up.

"What do you think the range is for this Figure 1?" Answer will be unique.

"Now measure the area by fitting the centimeter tiles exactly into the space and record that."

"Was your measured number within the range you reckoned?" If yes, the student circles Within Range. If the measured number is above the range estimated, the estimate was too low; if the measured number is below the range estimated, the estimate was too high. Circle the words that fit the estimate.
Continue this same pattern down to the bottom of the worksheet.

Lesson Part 2

Area: Standard - Worksheet 2, page 7, switches to one inch tiles.

Area: Standard - Worksheet 3, page 8 On this page, she will measure and record data and look for a pattern that will allow her to calculate how many centimeter tiles fit into a space.

Have her measure the length and width of Figure 1 and record the numbers on the chart. Then immediately have her fill the space with centimeter tiles and count up the total. Record.

After she has finished Figure 4, ask her if she sees any patterns. Draw her attention to looking at the numbers horizontally: length–width–total tiles.

Hopefully she will see the multiplication problems. If not, have her finish the page. If she does see the pattern, have her predict the number of tiles that will fit in Figures 5 and 6 based on the measurements of length and width.

The rule for calculation is 'To find the area of a rectangle, multiply the length times the width.' The algebraic rule is $A = lw$, or $A = L \times W$. Notice how many words are replaced by the mathematical formula.

Practice Worksheets

Area: Standard - Worksheets 4 and 5, pages 9 and 10
Extra practice on this topic can be found in many places. Try the Internet for interactive games and exercises. Try to find some that give the area and one side, where she has to figure out the length of the missing side.

Test for Understanding

Figures 6 and 7 on Area: Standard - Worksheet 5 are a great Test for Understanding.
Answers are in the Answer Key.

Number Lines

Purpose

The purpose of this lesson is to connect number sentence operations, 2 + 5 = 7, to movement on a number line. This exercise begins the groundwork for understanding operations with positive and negative numbers and later to vectors used in physical science.

Background Information

Most students at this point in their development think of 5 - 2 = 3 as a 'Take Away' where the sentence means 'Start with five and remove two, what is left?' problem or a 'difference between' problem, where the sentence means 'How much larger is five than two?' 5 - 2 can also be a movement on a number line. In this context, the sentence 5 - 2 = 3 would mean 'Start at +5 and move to the left or backwards two spaces. You land on + 3.' The -2 is not a quantity that is removed or compared, but a direction (backwards) and distance moved (two steps). Later in physics this idea translates to vectors, where scientists show speed and direction of movement of a physical object.

Prerequisites

Previous lessons in number lines are helpful.

Materials

Number Lines, page 11
Sidewalk chalk or masking tape

Warm Up

Make a walking number line (see instructions below). Tell the student that a + sign means move to the right by walking forward up the number line from zero; a minus sign means move to the left by walking backwards down the line back towards zero. He does not turn around – see the note below for the explanation of why.

Have him begin on zero. Give him a series of numbers such as: 0, + 4 (go four steps to the right/forward, stop on the number 4), + 5 (go five steps to the right/ forward, stop on the number 9), -4 (four steps backwards to the left, stop on the number 5), +3 (go three steps forward to the right, stop on the number 8), -8 (eight steps backwards to the left, stop on the number 0). He will end up back at zero.

Have him make up one for you or for another student in the group. Have him write it down first.

Now, walk out a number sentence on the number line and have him write the sentence to go with your movement. For example: Begin on 0, walk forward seven steps, walk backward three steps.
"What number sentence did I walk?" "(+) 7 - 3 = 4."

Note

(+) 7 - 3 = 4 The plus sign in the ( ) is not usually shown at this level. In 7 - 3 = 4 the + in front of the 7 is understood and thus is not written. Later, he will encounter number sentences that look like this: -11 + 5 = -6, where the sign of the first number must be shown. It is not a bad habit to put the + sign on the first movement when you are doing number lines.

Walking Number Line: Draw a ten to twenty foot line on a walkway using chalk, or on a carpet with masking tape. On the tip of the left end, place a six inch vertical line and label it 0. Have him stand on the 0 with his toes just touching the line. He takes a normal step forward with one foot remaining on the 0. Place a vertical line (again about six inches long) at the tip of his toes. Then take another step, marking this spot with a 2 and so on. The space between numbers on the number line is now determined by the length of your student's normal step. The number line should look like the number line in problem 7 of the Number Lines worksheet. You do not need to go all the way to twenty if space precludes it. Label each position with a number. If you are on a carpet, use Post-it Notes or index cards to number each step.

Lesson

Take out the Number Lines worksheet.
"Look at the kangaroo. What number sentence did he hop?" "+8 + 4 = 12."

"Write that number sentence on line 1 near the rabbit."
Continue this way down the worksheet. Check the answer key to be sure you are doing it correctly.

Note

Problem 3 is a multiplication number sentence of 200 x 3 as well as 200 + 200 + 200 = 600. Be sure he writes both number sentences.

Extension

Do some division number sentences. Start on 12, step backwards two numbers in one giant step. How many two giant steps do you need to do to get back to 0? 12 ÷ 2 = 6 can also be understood as 12 - 2 - 2 - 2 - 2 - 2 - 2 = 6. The six does not mean where you ended up but how many giant two steps you needed to make to get back to 0!

Practice Worksheets

There are no practice worksheets. But you are encouraged to do one of these types of problems a day for a while. Do all four operations.

Test for Understanding

Use problem 9 on the Number Lines worksheet to assess his understanding. At the least you want to see an addition number sentence with a drawing on the number line that matches it. The second problem below the line should be a different operation. He should be able to do this without assistance. If he needs help, keep working on this idea until he can do the problem independently.

Note

Why you do not turn around: A minus sign means move to the left by walking backwards down the line back towards 0. He does not turn around. The reason is that walking backwards is the first minus or negative sign, or reversing action.
5 - 2: Forward five steps, back two steps.

If you turn around, that is a second minus sign, which is a different action. So imagine you are at 7 and you turn around facing back towards the 0, and then take two steps back. You will end up not at 5, but at 9. This number sentence would be 7 - (-2). Turning around, the first negative, and then walking backwards, the second negative, has the same end point as adding two. Two negatives together are a posi-

tive. Neat, don't you think?

Extension

For an advanced student: Try extending the number line past 0 into the negative zone. The number line continues on to the left with the first number to the left of the 0 being -1, the second -2.

Greater Than, Less Than is defined by a position on this number line. The farther to the right a number is, the greater is the number value. So $-1 > -2$. Minus 1 is greater than minus two. Think of debt and this makes perfect sense.

Subtraction is just the addition of a negative number. Try this: $-5 + 6 = \_\_$. To walk this, you stand at 0 facing towards the right (going up) and step backwards five steps. Then you walk forward six steps and end up at +1.

How would you walk this one? -5 - +6? Walk backwards five steps, turn around and walk forward six steps and end up at minus eleven. Isn't that fun!

Key to Algebra: Booklet 1 has many problems like this using a football field as a model. An answer key is useful when you are playing with this concept to be sure you are getting it right.

Challenge for the Teacher

Using this model, figure out why -2 x -3 is +6!
Answer: -2 x -3 can be thought of as - (-2, -2, -2). The three is the number of backwards two steps you take, the minus in front of the three tells you to turn around before you do it. So stand on 0 facing up. Turn around and take three giant two steps backwards and you end up at +6.

Difference Between: Patterns

Purpose The purpose of this lesson is to teach the pattern-finding strategy of taking the difference between numbers in a series. This skill is used often in algebra to identify functions.

Prerequisites None

Materials
Difference Between: Patterns, page 12
Meter stick
Base Ten Cubes (the smallest Cuisenaire Rod will work too) or markers
Scratch paper

Warm Up Take out the meter stick and the cubes. Place a cube on the 5, on the 10, and on the 15.
"Where should the next cube go? How do you know?" "It should go on the 20. I know because it is going up by five every time."

"Good thinking. Mathematicians call the 'going up by five every time' difference between. The difference between each cube is five. Place a cube on the 7, 10, and 13. Continue the pattern. What is the difference between?" "Three."

"Sometimes the differences between can change and make a pattern. Place a cube at 1, 2, 4, 7, 11. Write the difference between each cube on a bit of paper and place it between the cubes next to the meter stick. This will help you find the pattern."
The pattern will be that the difference between 1 and 2 is 1, between 2 and 4 is 2, between 4 and 7 is 3, between 7 and 11 is 4.

"Where should the next two cubes go?" "16 (up by five) and 22 (up by six)."

"Place cubes on the meter stick using this pattern until you run out of room on the stick. How many more cubes do you think you will need? Try it."

Note On the Difference Between: Patterns worksheet, little arrow-like marks between each number in the pattern means 'the difference between the two numbers above' the little mark. On problem 1, the first two numbers in the pattern are given, 10 and 20, and two differences between. This is enough information to fill out the entire pattern. To determine a pattern, you need at least two pairs of numbers, or one pair and a difference between in another location. It may be helpful to use the cubes to help the student visualize and verify the patterns.

Practice Worksheets None, but add a problem like this once or twice per week in her practice.

Test for Understanding Have her make a pattern for you to continue. Let her decide how hard to make it. Can she make her own for you to solve that has the needed parts for you to figure out the pattern? If so, she understands.

Sums and Differences: Patterns

Purpose The purpose of this lesson is to develop fluency in mathematical reasoning, and patterns while introducing negative numbers on the number line.

Note Some students will have a high degree of difficulty with this exercise, especially at the end. Drop it if it is too hard. While we are building up mathematical muscles with this lesson, it is not a critical concept.

Prerequisites Number Lines and previous lessons

Materials Sums and Differences: Patterns - Worksheets 1 - 4, pages 13 - 16
Walk-on Number Line Extension that includes negative numbers is optional but helpful.

Warm Up Review adding and subtracting on the number line using the number line at the top of Sums and Differences: Patterns - Worksheet 1.
Solve 4 + 5 = ___ using the number line. Start at + 4 (by placing a pencil on that number), go to the right five spaces, end up at +9. Solve 10 - 6 = ___. Start at +10, go to the left six (or walk backwards on the number line), end up at +4. Solve 3 - 9 = ___. Start at +3, go to the left nine, end up at -6.

Lesson Using the number line to help, fill in all the answers to both sets of problems on Sums and Differences: Patterns - Worksheet 1, page 13. Look for and talk about the patterns the student sees. Help him use mathematical vocabulary to write about his patterns. For example, the sums of all the problems on the left are equal to twelve.

Note
Be sure to have this dialogue

Have him look across the two columns at the sums and differences of the same pair of numbers.
"What is the sum of nine and three?" "Twelve."

"What is the difference between nine and three?" "Six."

"Which pair of numbers has a sum of twelve and a difference of zero?" "Six and six."

"Which pair of numbers has a sum of twelve and a difference of four?" "Eight and four."

"Which pair of numbers has a sum of twelve and a difference of eight?" "Two and ten."

Worksheet Sums and Differences: Patterns - Worksheet 2, page 14, now uses this pattern to locate a specific pair of numbers with a given sum and difference. On the top problem, we are looking for a pair of numbers whose sum is twelve and whose difference is eight. This is shown at the top of the worksheet under the words:

Their Sum	Their Difference
12	8

The pattern used before to find all the pairs of numbers that have twelve as a sum will be used. The first is 12 + 0. That is the pattern shown under Their Sum. Then there is an arrow directing him to find the difference between those two numbers 12 - 0 = 12. This is not the pair we are looking for. We want a pair that has a sum of twelve and a difference of eight. As he works down the column, he will find the answer at 10 + 2 and 10 - 2. The sum is twelve and the difference is eight.

Have him write twelve and eight under the words Two Numbers written on the left margin.
Repeat this same process for the second problem. Find a pair of numbers whose sum is thirteen and whose difference is five. Begin in the addition column with 13 + 0 and 13 - 0.
Then go to 12 + 1 and 12 - 1, then to 11 + 2 and 11 - 2 and so on until you find the pair that has a sum of thirteen and a difference of five. See the Answer Key to be sure he is doing the work correctly.

"Notice the pattern in the differences." On Sums and Differences: Patterns Worksheet 1, all the differences were even numbers.

"What pattern do you see in the differences this time?" "All the differences are odd numbers."

"Why are all the differences on the bottom problem now odd numbers?" The answers will be unique.

Practice Worksheets

Sums and Differences: Patterns - Worksheets 3 and 4, pages 15 and 16
On Sums and Differences: Patterns - Worksheet 4, the top section gives the two numbers and the student must find the sums and differences. Problems 4 and 5 require him to use his logic to fill in the numbers needed. He needs to realize that the second number is larger than the first and all he has to do is add the six and the seven to get thirteen, which is the other number. This thinking will be hard for some students. Then the lower four problems require him to use another piece of paper to do the pattern search.

Test for Understanding

Watch to see what he does when he encounters problems 6 through 9 on the worksheet.

Does he get a piece of paper and write out the sequences to help him find the pair he needs? Does he proceed with confidence or not? How he works with the problems will tell you how he is doing. It is not important to push this exercise. If it is too hard, come back to it later.

Rainbow Patterns

Purpose The purpose of this lesson is to delight the mind with a lovely pattern and to practice pattern analysis.

Prerequisites Previous lessons

Materials Rainbow Patterns - Worksheets 1 - 4, pages 17 - 20
Colored pencils or crayons
Make multiple copies of Rainbow Patterns - Worksheet 4

Lesson Notice the graphic on the top, just to the left of the title of Rainbow Patterns - Worksheet 1, page 17. This shape is the pattern the student will draw. Each line is in a different color. The order of the colors is not important.

"Draw a high-arching colored line from the number 1 to the number 10. What is the sum of those two numbers?" "Eleven."

"Make the next arch from 2 up high over to the 9. What is the sum of those two numbers?" "Also eleven."

"Keep going with the pattern. Connect 3 to 8, 4 to 7, 5 to 6, etc. What patterns do you notice?" "I am always going one in from each side. And the sum is always eleven."

"Try it again with a different series of numbers. Go to Rainbow Patterns - Worksheet 2."
"Connect the first high arch from 21 to 30. What is the sum?" "Fifty-one."

"What do you connect next?" "The 22 and the 29."
"What do you predict the sum will be?" "Fifty-one, it is!"

"Finish the pattern."
Turn to Rainbow Patterns - Worksheet 3, page 19.
"This time we are going to multiply the numbers in each pair. Do you think the products will be the same like it was with the sum?" The answers will vary.

Have her fill in the products. The products are not the same, but they do form a pattern.
"What patterns do you see?" Answers will vary, but she may notice that as she gets closer to the center of the series, the product gets greater, they alternate odd and even, the one in the center is the greatest, and the one on the outside edge is the least.

"Take the differences between each product and record them in the column under 'Differences.' The first one is done for you."

"What pattern do you see in the differences?" "They are all two apart; the differences are a sequence of odd numbers."

"Try the same thing again with a different pattern." Use Rainbow Patterns - Worksheet 4 for this exploration.

Questions to Pose

"Does the pattern work if there are spaces between the numbers? Try this: 2, 4, 6, 8, 10, 12, 14, 16, or this: 3, 6, 9, 12, 15, 18, 21, 24." "Yes."

"How does the difference pattern change when you use these series of numbers?" "The differences for a pattern of adjacent numbers comes down to two apart; for the two times tables, they end up eight apart; and for the three times tables eighteen apart. If you looked at the four, five, and six times tables, you would probably find a pattern in the pattern of the differences. Patterns are everywhere!

"Does it work if the numbers are not evenly spaced 2, 3, 5, 6, 7, 9, 12?" "No."

Patterns to Notice

1. The series of numbers does not have to be with only adjacent numbers.
2. **"Which pair always produces the greatest product?"** "The pair in the center of the series."

Practice Worksheets

Multiple copies of Rainbow Patterns - Worksheet 4. Give a new series every day for arithmetic practice.

Test for Understanding

None

Coordinate Graphing

Purpose

The purpose of this lesson is to develop the concept of ordered pairs and their relationship to the x and y axis.

Teacher Background

Coordinate graphing is used to locate a specific point in a two dimensional area. The horizontal line is called the 'x' axis. The vertical line is called the 'y' axis. Where the two intersect locates a point. To locate a point, an ordered pair of numbers is used. These are often called coordinates. An ordered pair looks like this: (3, 1). This means beginning at the lower, left hand corner of the grid, go horizontally along the x axis three intervals, then go vertically upward along the y axis one interval. On Coordinate Graphing - Worksheet 1, the point (3, 1) is marked on the grid.

Prerequisites

Previous lessons on this topic are helpful.

Materials

Coordinate Graphing - Worksheets 1 - 5, pages 21 - 25
Ruler
Make a copy of Coordinate Graphing - Worksheet 2, page 22.

Warm Up

Label the points of the line that are already drawn on the grid. The lines look like greater than > and less than < signs. These will form the whiskers of a cat that will be drawn (but don't tell the student that). Have her put three dots on each whisker to mark where the whisker intersects the x and y lines. The whisker above the starting dot (3, 1) has three intersections, (3, 3), (4, 4), and (3, 5). Have her write the coordinates of this whisker on the list on the lower left hand side of the worksheet. Repeat with the second whisker. See the Answer Key to be sure the points are recorded correctly.

Lesson Part 1

"What is the coordinate of the printed dot on the grid?" "Over three and up one."

"This point is the first point on your drawing. Notice that (3, 1) is at the top of the list of coordinates on the left hand side of the worksheet." Point to the list.

"What are the coordinates of the second point on the list?" "(1, 3)." Have her place a heavy dot on this location. Starting at 0, go to the right one interval, and up three intervals and put a dot.

"Using a ruler, draw a line connecting point (3, 1) to point (1, 3)." See the Answer Key to be sure you are doing it correctly.

"Make a dot at coordinate (1, 6) and connect it to the last dot you made at (1, 3)."
Continue down the list, connecting the lines as you go until you get to point (7, 1). Do not connect point (7, 1) to the next point on the list, (5, 4). That short list of four points are connected only to each other. If she does this correctly, she will get

a picture.

Worksheet

Coordinate Graphing - Worksheet 2, page 22, is for the student to make her own design and record the coordinates. First, make a copy for the Test for Understanding below.

Lesson Part 2

Play Treasure Hunt on Coordinate Graphing - Worksheet 3, page 23. This game is similar to Bingo and Battleship. Just follow the instructions. There are extra grids to use for the game. Let her use Coordinate Graphing - Worksheet 3, and you use Coordinate Graphing - Worksheet 4 to play against her.

Practice Worksheet

Copy Coordinate Graphing - Worksheet 4 for playing more Treasure Hunt games.

There are more games on the Internet; try the Fun Brain site for some good games.

Test for Understanding

On the copy of Coordinate Graphing - Worksheet 2, give her a series of points that will draw a right triangle on the same grid. See if she can label and locate points independently. Any grid can be used if a copy wasn't made first of Coordinate Graphing - Worksheet 2.

Number Patterns

Purpose The purpose of this lesson is to further develop a sense of patterns.

Prerequisites Previous lessons

Materials Number Patterns: Adding Shapes, page 26
Number Patterns: Missing Numbers, page 27
Up and Down Sums page 28
Square Numbers, pages 29 and 30
Pattern blocks

Warm Up Explore with the pattern blocks. Trace the outlines of blocks placed adjacent to each other, as is shown on the Number Patterns: Adding Shapes worksheet, and trace the perimeters to refresh their vocabulary.

Lesson Number Patterns: Adding Shapes, page 26. Using the pattern blocks, build the sequence for triangles and record the perimeters on the chart at the bottom of the page. Then do squares, then hexagons. Assume the side always has a unit of one.
"What patterns do you see?"
"Triangles add one unit of perimeter each time a block is added. Squares add two units of perimeter each time a block is added. Hexagons add four units of perimeter each time a block is added."

"What do you think the pattern will be for pentagons?" A possible reasoning strategy: Each time a side is added one more unit of perimeter is added. Squares (four sided) added two units of perimeter each time you added a block. Hexagons (six sided) added four units of perimeter each time you added a block. Since pentagons are two more sides than triangles and one less side than hexagons, it adds three units each time.

Note To do the pentagons, you can draw a series of houses. The drawings do not have to be perfect pentagons for the pattern to be revealed.

Check the answers in the Answer Key to see the chart is filled out correctly.
"What did the pentagons add?" "Pentagons added three units of perimeter each time I added a block."

Many students may be confused by this pattern because it is counterintuitive. It seems like when the next square is added, for example, that the total perimeter should go up by three, not four. The reason it only goes up by two is because when the next square is added both squares lose a length of perimeter in the side that is touching. When the next square is added, the middle square loses two lengths of perimeter, one on each side. This is tricky for students to see. They must understand this pattern before they can do the heptagons because you can't easily draw these.

Practice Worksheets

Number Patterns: Missing Numbers, page 27, is for independent practice.

Square Numbers - Worksheets 1 and 2 (Before you do Square Numbers - Worksheet 1 look at Square Numbers - Worksheet 2 to see what the pattern looks like when you build 'the next largest square' with tiles.)
Up and Down Sums, page 28

Test for Understanding

Explain why the perimeter on the added squares only goes up by two instead of three.
The answer is given in the note above.

What's My Rule?

Purpose

The purpose is to learn to spot patterns in number series. When you are looking for a pattern that uncovers a hidden operation, you look at what number you started with and what number you ended with and try to figure out what happened to the numbers in between. This is one thing scientists and mathematicians do when they discover new "formulas." A game that exercises this skill is What's My Rule?

Activity

To play, think of a simple operation such as + 2. The student gives you a number and you add two to it and say the new number back. For example, if she says, "one," you say "three." She says, "five" and you say, "seven." The number she gives you is called the independent variable, or the number In. The number you say back is called the dependent variable, or the number Out.
Record the information on a chart like this:

IN	OUT
1	3
5	7
9	?

What's My Rule? +2

When a student thinks she knows the rule, let her predict the number out loud. Finally, when everyone can predict successfully, let someone formulate the rule of plus 2. Students love to make these up for each other using easy addition, subtraction, multiplication, and division operations.

Play this game regularly. It's a good rainy day and in-the-car game.

One teacher called this activity Black Box and made a symbolic black box out of a milk carton decorated with gears and levers with a slide inside that flipped a card upside down. A card would be put in the slot in the top of the box and come out so the number written on the back of the card came out a bottom slot. The students then guessed the rule. When the box appeared, the students' minds focused to discover the relationship between the In and Out number of the day.

Worksheet

What's My Rule? - Blank page to copy is on the following page.

Sample games:

IN	OUT
2	4
6	8
9	___
20	___

What's My Rule?_____

IN	OUT
7	4
3	0
10	7
8	___
20	___

What's My Rule?_____

*Answers: 9 11 22 20 17
5 8 20 17

Rule: Add 2 Rule: Subtract 3

What's My Rule? Date ______________

IN	OUT
___	___
___	___
___	___
___	___
___	___

What's My Rule?______________

IN	OUT
___	___
___	___
___	___
___	___
___	___

What's My Rule?______________

IN	OUT
___	___
___	___
___	___
___	___
___	___

What's My Rule?______________

IN	OUT
___	___
___	___
___	___
___	___
___	___

What's My Rule?______________

IN	OUT
___	___
___	___
___	___
___	___
___	___

What's My Rule?______________

IN	OUT
___	___
___	___
___	___
___	___
___	___

What's My Rule?______________

Graphing Number Patterns

Purpose

The purpose of this lesson is to connect number patterns to coordinate graphing. Showing a pattern on a graph makes a 'picture' of the pattern and allows us to easily find the rule or function being shown by the pattern.

Teacher Background

Study Huts and Flags - Worksheet 3. It is easy to see what the rule of this pattern is. The number of flags is equal to two times the number of houses. When a student is first introduced to this idea, he builds the patterns with blocks, then finds the answers by counting the blocks. The numbers are recorded on a chart called a T chart. When he notices a pattern or a rule, he can figure out how many flags would be needed for ten houses without having to build ten houses.

T charts are used to list the x and y pairs. The T shape is more obvious on Worksheet 4. It has an x and a y above a horizontal line and a vertical line between the x and the y to form a T. Using the number pairs on the T chart for x and y, we create an ordered pair that can be placed as a point on a graph. We then can put a picture of this pattern on a graph. What is fun is that the number pattern $y = 2x$ will create a geometric pattern on the graph. All the points will be in a straight line. This delights most students.

In algebra, the rule of a pattern is called the function. We choose the value of 'x.' Because we choose what 'x' is, this variable is called the independent variable. We choose the number of houses we are going to look at. The value of the 'y' is dependent on what the value of x is and by the function or the rule. The number of flags is dependent on the number of houses. The rule says take the number of houses and multiply it by two because every house has two flags. In algebra, we would write this pattern as a function like this: $y = 2x$. This means take the number of houses you have and multiply it by two to find out how many flags there will be. Knowing the function of a pattern allows us to calculate the nth value of the pattern. What is meant by the nth value is the 'y' value of any 'x' without having to count them. How many flags would you need for 101 houses? You could choose any number of houses and figure out how many flags you would need.

Mathematicians and scientists use this to figure out how the world works, using scientific formulas such as distance is equal to rate (how fast you are going) times how long you go. This is written as $d = rt$. Mastery of this concept of graphing functions and finding rules will take many years. It is one of our most important algebraic tools.

Prerequisites

Sums and Differences - What's My Rule, and Coordinate Graphing

Materials

Graphing Number Patterns - Worksheets 1 - 8, pages 32 - 39
Ruler
Pattern blocks and tiles or cubes

Warm Up

Review ordered pairs or play Treasure Hunt.

Lesson

Begin with Graphing Number Patterns - Worksheet 1. Start the student on building a few houses and chimneys. When he has built three, ask, **"How many chimneys will ten huts have?"** "Ten."
"How do you know that? You only built three." "I can tell because there is only one chimney on each hut, so ten huts will need ten chimneys."

Huts	Chimneys
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10

Have him fill in the chart.
Turn to Graphing Number Patterns - Worksheet 2. Have him write 'Huts and Chimneys' on the line where it says 'Graph Title.' Above the X on the T chart on the right hand side of Graphing Number Patterns - Worksheet 2, write the word Huts.
"Number downward under the X, one through ten."

Above the Y on the T chart, write the word Chimneys.

Huts X	Chimneys Y
1	1
2	2
3	3
4	4
5	5

"How many chimneys does one hut have?" "One."

"Write 1 in the Chimney column just to the right of the 1 in the Huts column." Repeat with all the other number pairs for huts and chimneys. You are recopying the chart on Graphing Number Patterns - Worksheet 1 onto a more generic form.

Circle the first pair of numbers 1, 1 on the T chart.
"Do you remember what an ordered pair of (1, 1) tells you to do on a graph?" "Go over one and up one and put a point."

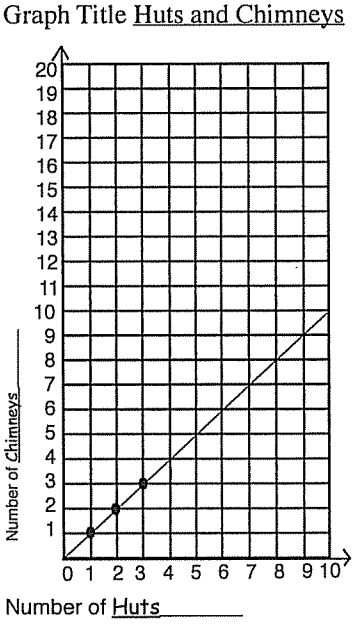
Huts X	Chimneys Y
1	1
2	2
3	3
4	4
5	5

"Put a point on the graph at (1, 1)." We have to label what each part of the ordered pair means.

"Where does the first one come from?"
"The number of huts."

On the x axis (the horizontal line at the bottom of the graph), have him write the words 'Number of Huts.'
"Where does the second one come from?" "The number of chimneys on the hut." On the vertical y axis, have him write, 'Number of Chimneys.' Have him turn the paper sideways to do this.

Repeat this process for two more points, (2, 2) and (3, 3). Remember (2, 2) means two houses have two chimneys.
"Do you see a pattern in the way the points go on the graph?" "They line up in a line." If he does not see the pattern, have him connect the points with a line using the ruler.



"Put your ruler on the page so that each dot just shows above the ruler. Connect the dots. Extend the line down to (0, 0) right at the corner of the graph."

Note This location (0, 0) is called the origin.

"This is how you draw a 'picture' of a pattern on a graph."
Check the Answer Key to be sure you have done the work correctly.
End the lesson here.
On Graphing Number Patterns - Worksheet 3, page 34, Huts and Flags, and Worksheet 5, page 36, Stars and Points, the pattern of instruction is repeated.

Worksheets Graphing Number Patterns - Worksheets 3 - 8, pages 34 - 39

Test for Understanding What we want to see at this point in his work is that he can look at a pattern and extend it. Can he identify the rule? Can he put the x, y points on a graph and connect the points?

Use Graphing Number Patterns - Worksheets 7 and 8 as a Test for Understanding. Observe him working. Do not assist him. Watch to see what he can do and what he can not do.

You can give him more practice on patterns you make up, or you can let this one rest until next year. This topic will come up every year in his study of mathematics all the way though Algebra II and beyond. This is just the beginning.

Algebraic Thinking

Purpose The purpose of this lesson is to introduce algebraic notation used to build number sentences.

Prerequisites Previous lessons

Materials Algebraic Thinking - Worksheets 1 - 9, pages 40 - 48
Number tiles or cutouts of the tiles included with this lesson

Warm Up Review the math vocabulary of sum, product, difference and quotient, greater than, less than, and equal to.

Lesson Part 1 Follow the instructions on Algebraic Thinking - Worksheet 1, page 40. Have the student move the three tiles around inside the number sentence to find the correct solution.

"What does the ( ) around the (_ x _) mean?" "It means 'do me first,' so I need to multiply two numbers and then add the third number to that product."

"Study the Puzzle. What pattern do you notice in the first three problems?" "They are all (__ x ___) + _____."

"(2 x 4) + 6 is read, 'Quantity 2 x 4 plus 6, or the product of 2 and 4 added to 6, or the sum of 6 and the product of 2 and 4.' "

"Move the numbers 2, 4, and 6 around in the boxes. Try different arrangements. When you find a solution, record it on your worksheet. Do the same with the bottom three problems."

Most students will solve these very quickly.

Move onto Algebraic Thinking - Worksheet 2, page 41. Make more number tiles with the operators +, - , x , ÷ and equality and inequality signs <, >, =.

This time she must leave the numbers given in exactly the same location as they are and place the operation and equality signs in to make a true sentence. Have her move the tiles around to search for solutions and then record. If she does not do this, she will erase a lot and put holes in the pages. Check her solutions in the Answer Key.

Practice Worksheets Algebraic Thinking - Worksheets 3 - 5, pages, 42 - 44

Lesson Part 2 On Algebraic Thinking - Worksheet 6, page 45, we want her to connect the notations of the number sentences to properties of arithmetic.

"How do you know three has to go into the box on the first problem?" "Be-

cause both sides are multiplication and both sides have a four, so both sides have to have a three also."

"What property of multiplication is being used here?" "Commutative."

"What does that mean?" "It means that the order that you combine the numbers does not change the answer."

"In math talk what do you call the answer?" "The product."

Encourage her to use correct mathematical vocabulary in her explanations. If there is not enough room for her to write on the worksheet, have her write it in her math journal.

Practice Worksheet

Algebraic Thinking - Worksheet 7, page 46

Lesson Part 3

On Algebraic Thinking - Worksheet 8, page 47, a box is substituted for the (2 + P). See if she can figure out how to do these top four problems on her own. Listen to her thinking and it will reveal her level of understanding. Many fourth grade students will not be able to think this out without help.

The help is in the middle of the page. If she can not decode the top four problems, skip down to $30 = 6 \times (4 + B)$. The reasoning for how to solve this problem is given just below the problem. Do this together, then go back and try the top problems again.

Test for Understanding

She should be able to do Algebraic Thinking - Worksheet 9, page 48, on her own as a Test for Understanding. She can use a multiplication chart to help if she does not know her tables well enough.

Families of Facts: Missing Numbers

Purpose The purpose of this lesson is to connect what the student already knows about Families of Facts to algebraic notation and convey the idea that algebra can be used to express the general rule about the relationships between families of numbers.

Prerequisites Previous lessons and patterns coloring on Families of Facts: Missing Numbers - Worksheet 1, page 49. See pages 27 and 28 (in this booklet) for instructions on how to introduce the color patterns in Families of Facts.

Materials
Families of Facts: Missing Numbers - Worksheets 1 - 4, pages 49 - 52
Colored acetates or colored chips
Colored pencils or crayons
Math Journal

Warm Up With Families of Facts: Missing Numbers - Worksheet 1 and the instructions on pages 27 and 28, use acetates or translucent colored chips to review the addition and subtraction number families.

Lesson Families of Facts: Missing Numbers - Worksheet 2, page 50 Color in the circles and boxes the colors indicated.

"What pattern do you see?" "The color pattern stays the same even when the numbers change."

Finish the top four problems on Families of Facts: Missing Numbers - Worksheet 2.
"What pattern do you see in the colors?" "The pattern is always the same."

Then at the bottom of the page ask, **"Does the pattern change if the numbers change?"** "No."

Have him write the letter 'a' in the circle with the five, the letter 'b' in the circle with the six and the letter 'c' with the eleven, which he will write in. Have him color the circles and the box.

"Since the pattern does not change when the numbers change, can you drop the numbers and just use the variables (the letters) to stand for any number?" Please note the a, b, and c colors: a is yellow, b is blue, and c is green. Have him try to do this just using the letters and the colors.

Note This is the algebraic notation for the general pattern of fact families for addition and subtraction.

Yellow + Blue = Green	is the same as	a + b = c	
Blue + Yellow = Green		b + a = c	ADDITION
Green - Yellow = Blue		c - a = b	INVERSE OF ADDITION
Green - Blue = Yellow		c - b = a	

This pattern will hold no matter what the numbers are. This pattern is used to solve equations with missing numbers.

On Families of Facts: Missing Numbers - Worksheet 3, have him color in the seven yellow, the nine blue, and the sixteen green. Now have him place the operator, the +, into the triangle. Now using the same color pattern as before, fill in the numbers and the operators. See the Answer Key to be sure you are doing it correctly.

"We can not use this pattern to find out the value of a missing number in a number sentence. These number sentences are also called equations. We 'solve for' the missing number which is shown as a letter."

"On Families of Facts: Missing Numbers - Worksheet 3 in the middle of the page you see a + 9 = 16. To figure out what 'a' is, look at our patterns to see which equations has '= a.'"

"What color is 'a' in our patterns?" "Yellow."

"Which equation number on the top of Families of Facts: Missing Numbers - Worksheet 2 has '= yellow' as a solution?" "Number four."

You are looking for c - b = a, or green - blue = yellow. (It could be number 3 depending on which order he wrote them in.)

"Can you solve the next one? It is j + 11 = 21." "It is 21 - 11 = j and j = 10." Have him finish the rest of the page alone if possible.

Worksheet

Families of Facts: Missing Numbers - Worksheet 4, page 52

Test for Understanding

On number 10 on Families of Facts: Missing Numbers - Worksheet 4, can he solve the problem if you write in K + F = M and tell him to solve for F?

Answer: K + F = M F = M - K, and the last part has no answer because you are not using numbers.
If so, he understands this concept. Watch to see if he writes it from memory or if he goes back to refer to his previous worksheets. If he does not look back, he is solid. If he does, he is getting there. If he can not process the question but was able to do the worksheets, stay with numbers a little longer.

Cube City on page 53 is a just for fun art page.

Families of Facts: Missing Numbers
Instructions for color patterns

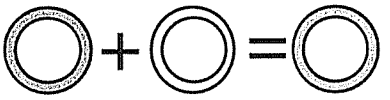
Materials

Family of Facts: Missing Numbers - Worksheet 1, page 49
Transparent plastic chips or blue and yellow acetate sheets or cellophane
*Prisma colored pencils or crayons or watercolor markers
Overhead projector (classroom)

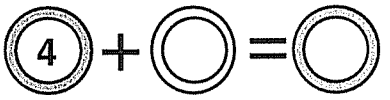
Lesson

Place the transparent materials against a lighted window. Use an overhead projector for a classroom. Start with the blue chip and place the yellow chip on top.
"What color is it now?" "Green."
Have the student state what happened, e. g., blue plus yellow equals green. Next, start with the yellow chip and place the blue chip on top.
"What color is it now?" "Green." Have the student state what happened, e.g., yellow plus blue equals green. The student can repeat with colored transparent chips.

On problem 1, using the crayons or colored pencils, have the student trace the circumference (edge) of the first circle blue, the second circle yellow, and the third circle green. If using light colored markers or colored pencils the whole circle may be shaded.



Choose a number and write it in the blue circle.

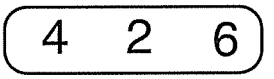


Choose a different number and write it in the yellow circle.

Add the two numbers together to see what the number in the green circle will be.

Record the sum (answer) in the green circle.

Record the three numbers, 4, 2, and 6 in the box.



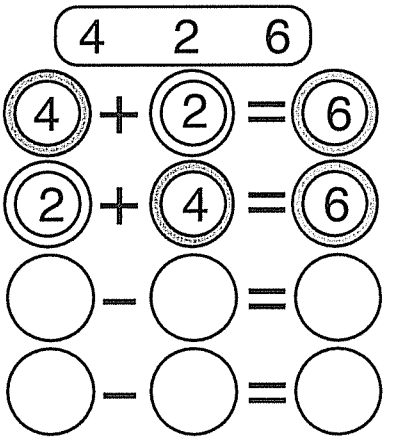
"Using only these three numbers, 4, 2, and 6, make a different addition number sentence that is true."

In the next set of circles, color the first circle yellow, the second circle blue, and the last circle green. Record the numbers.

Record the sum in the green circle.

Note that each color of circle always has the same value. The blue circle equals four, the yellow circle equals two, and the green circle equals six. Allow students to discover this. The color pattern helps students understand the Families of Facts. Some students will rely on the colors for making their own Fact Families, while others will not.

The worksheet will now look like: →

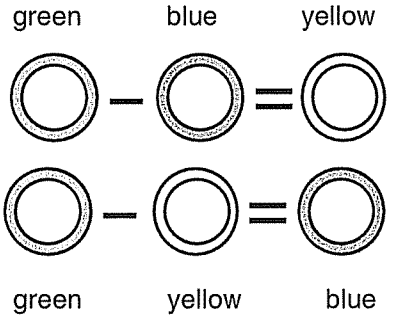


Use the overhead projector or, if at home, place the transparent materials against a lighted window. Start with the yellow and the blue chip together. Ask, **"What color is it?"** "Green." **"What color will be left if the yellow chip is taken away?"** Check the prediction. Take away the yellow chip. **"What color is left?"** "Blue."

Again start with the yellow and the blue chip together. **"What color will be left if the blue chip is taken away?"** Take the blue chip away. **"What color is left?"** "Yellow." The student can repeat with colored transparent chips.

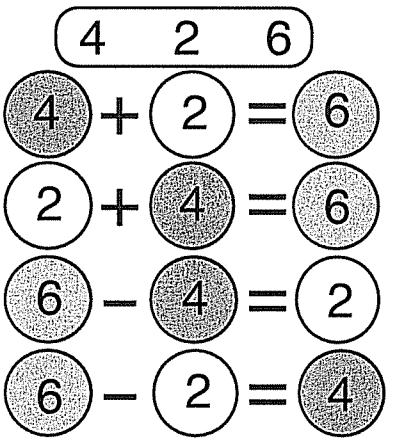
On the worksheet, color the next row of circles. Color the first circle green, the second circle blue, and the third circle yellow.

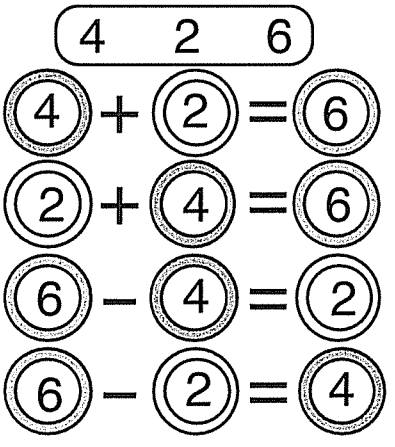
In the last row of circles, color the first circle green, the second circle yellow, and the third circle blue.



"Using only these three numbers 4, 2, and 6, make two more number sentences that are true but different." 6 - 4 = 2 and 6 - 2 = 4.

Have the student use manipulatives to prove the sentences and then record the numbers on the worksheet.





Have the student make up another set of numbers in problem 2.

*Prisma colored pencils will blend the colors together so blue + yellow changes to green.

Patterns in Arithmetic

General Math: Booklet 4

Patterns and Algebraic Thinking

Answer Key for the Student Workbook

By Suki Glenn and Susan Carpenter

Answer Key Legend

AWV = answer(s) will vary
BUWV = break up will vary
OWV = order will vary

Pattern Blocks
r = red trapezoid
g = green triangle
y = yellow hexagon
o = orange square
b = blue parallelogram
t = tan rhombus

Cuisenaire Rods
1 w = white
2 r = red
3 lg = light green
4 p = purple
5 y = yellow
6 dg = dark green
7 bk = black
8 bn = brown
9 bl = blue
10 o = orange

Note: Some items and pages are left out of the answer key.
1) Some pages in which the answers are open-ended or will vary.
2) Make your own problems. Since students create their own problems and solutions, these sections give valuable information about the level of confidence and competence. It can be a useful source of curriculum for other students.
3) Blank practice pages
4) Workboards
5) Games
6) Self correcting pages
7) Instructions only pages

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Student Workbook Answer Key

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General Math - Booklet 4

Greater Than, Less Than, or Equal To Worksheet 1

Which is more 3 + 4 or 3 + 3? 3+4

Only one of these number sentences is true. Circle it.

3+4=3+3 | 3+4<3+3 | 3+4>3+3

Put in the signs that make these number sentences true. You don't have to calculate them all. A few of them can be done just by looking at them.

3+4 < 3x3 | 12÷4 < 8 | 4x2 < 4+5

11 - 3 > □ +6 | 100 = 99+1 | 7+9 = 9+7

AWV

< > = Logic Problems

1. Look at the pair of problems. Reason out which problem should have the larger answer.
2. Record your guess in the "Guess" face.
3. Write down your logic.
4. Check by calculation on the paper or with a calculator.
5. Write in the < , > , or = sign between the problems and evaluate your guess in the "How did you do?" target. Draw an arrow on the target. The closer to the correct answer, the closer the arrow is drawn to the bull's-eye.

Example:

Which is more 35 + 12 or 17 + 35?

Use this space to do your calculations.

My logic is: both sides had a 35 in it. 17 is more than 12. So 35+17 has to be more than 35+12.

35 +17 = 52

35 +12 = 47

35+12 < 17+35

Greater Than, Less Than, or Equal To - Worksheet 2

Use your logic first. Enjoy.

Put in the signs that make these number sentences true.

Use this space to calculate.

1. Which is more 3 x 2 or 4 x 2? Guess.

My logic is: 3 is less than 4, so if you are multiplying both 3 and 4 by 2, the large factor (4) will have a larger answer.

3x2 < 4x2

Check. Draw an arrow.

2. Which is more 57 or 100 x 0? Guess.

My logic is: Any number times zero equals zero, so 57 has to be greater.

57 > 100 x 0

Check. Draw an arrow.

3. Which is more 1 x 2 x 3 or 2 x 3 x 4? Guess.

My logic is: 2x3x4 numbers a greater than 1,2,3.

1x2x3 < 2x3x4

Check. Draw an arrow.

Challenge! AWV

Fill in the blanks to make this a true sentence, ___ - ___ < ___ + ___

Greater Than, Less Than, or Equal To - Worksheet 3

4. Which is more 345 - 65 or 345+ 65?

My logic is: The addition problem will be more because the first and second numbers are the same.

345 - 65 < 345+ 65

5. Which is more 18 + 21 + 25 or 21 + 19 + 25?

My logic is: 19 > 18 and the other addends are the same in both equations.

18 + 21 + 25 < 21 + 19 + 25

6 Which is more 98 - 12 or 25 + 29?

My logic is: 98 is much larger and only 12 is being subtracted.

98 - 12 > 25 + 29

Challenge! AWV

Fill in the blanks to make this a true sentence, ___ + ___ > ___ + ___

Greater Than, Less Than, or Equal To - Worksheet 4

7. Which is more 68 x 2 or 68 + 2?

My logic is: Any multiplication problem will be greater if the only difference is a + or x.

68x2 > 68+2

This one is tricky!

8. Which is more 8 x 8 or 16 x 4?

My logic is: They are equal. All multiples of 4.

8x8 = 16x4?

9 Which is more 0 + 1 or 0 x 1?

My logic is: Any number time 0 is 0.

0+1 > 0x1

Challenge! AWV

Fill in the blanks to make this a true sentence.

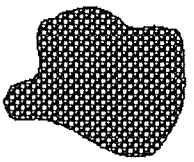
___ x ___ = ___ + ___ > ___ - ___ < ___ + ___

Area: Nonstandard

Estimate how many pinto beans it will take to cover the shaded area below. Record your estimation below. Then measure and record it. Next cover this figure with the pinto beans and record the actual amount. Repeat with split peas and lima beans.

To help the beans stay in place, make a three dimensional border around the edge of each figure with glue or modeling clay.

	Estimate	Actual
How many pinto beans?	_____	16
How many split peas?	_____	42
How many lima beans?	_____	7



Trace one lima bean.
How many pinto beans fit in that space? 4
How many split peas fit in that space? 9

Trace one pinto bean.
How many split peas fit in that space? 2

Use your logic to answer these questions.

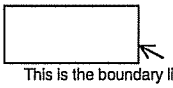
1. One lima bean covers about the same area as 4 pinto beans.
2. One pinto bean covers about the same area as 2 split peas.
3. One lima bean covers about the same area as 9 split peas.
4. A figure that takes 30 pinto beans to cover it will take 7½ lima beans.
5. A figure that takes 100 pinto beans to cover it will take 200 split peas.
6. A figure that takes 100 lima beans to cover it will take 900 split peas.
7. Write your own.

Area: Standard -Worksheet 1

Use centimeter tiles or graph paper.

New Word **Area** (âr'e•ə) pronounced air-ee-uh

Area is the flat space inside a boundary line.



This is the boundary line or perimeter.

This shaded part is the area.

Guess and measure how many tiles will fit on the figure.

Figure 1

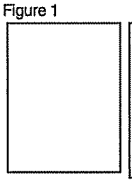


Figure 1 - centimeter tiles
I guess between ___ and ___
Actual number 12
Within range too high too low

Figure 2

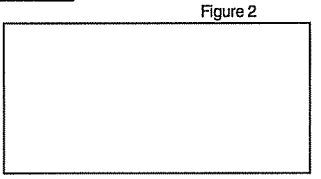


Figure 2 - centimeter tiles
I guess between ___ and ___
Actual number 32
Within range too high too low

Figure 3

Figure 3 - centimeter tiles
I guess between ___ and ___
Actual number 48
Within range too high too low

Area: Standard -Worksheet 2

Use inch tiles or graph paper.

Guess a range no wider than five. Measure how many tiles will fit on the figure.

Figure 1

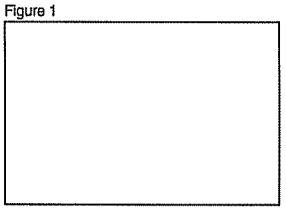


Figure 1 - inch tiles
I guess between ___ and ___
Actual number 6
Within range too high too low

Figure 2

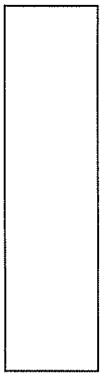


Figure 2 -inch tiles
I guess between ___ and ___
Actual number 4
Within range too high too low

Figure 3

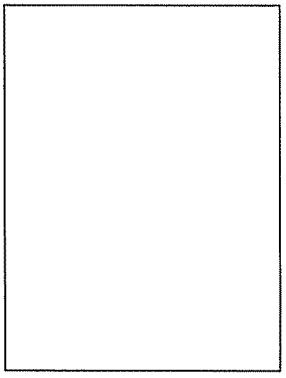


Figure 3 - inch tiles
I guess between ___ and ___
Actual number 12
Within range too high too low

Area: Standard -Worksheet 3

Use a centimeter ruler. Measure the length and width of each rectangle. Cover each rectangle with centimeter tiles. Enter the data on the chart.

Figure 1

Figure 2

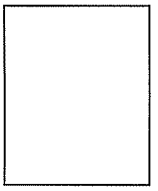


Figure 3

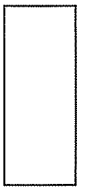


Figure 4

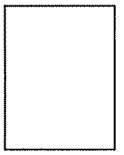
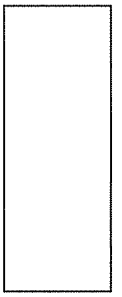
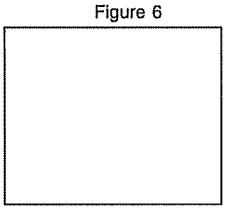


Figure 5



	Length	Width	Number of tiles to cover the figure
Fig. 1	2	2	4
Fig. 2	5	4	20
Fig. 3	5	2	10
Fig. 4	4	3	12
Fig. 5	8	3	24
Fig. 6	5	6	30

Figure 6



What patterns do you notice? lenght x width equals number of tiles

Write the rule for calculating area. l x w

Area: Standard -Worksheet 4

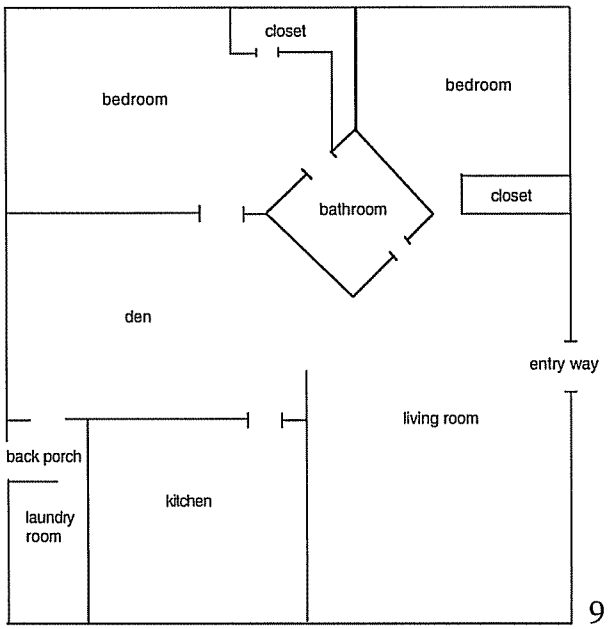
Here is a floor plan of a house. This house has no rugs. All the floors are wood.
It is your job to sweep the floors.
Use the drawing below and the centimeter tiles to help you answer these questions.

Which room would take the most time to sweep? living room

Which room would take the least time to sweep? laundry room

Which room would take the more time to sweep, the kitchen or the den? den

Which job would take more time, sweeping the kitchen and the den, or both bedrooms? kitchen and den



Area: Standard -Worksheet 5

How do you calculate area? Multiply the length times the width of a rectangle.

Write the rule in alegebraic terms. l x w

Measure the length and width of each rectangle. Use a centimeter ruler
There is a cut-out ruler at the bottom of the page.
Record the length, width, and area for each figure.

Figure 1

Length = 6 cm
Width = 3 cm
Area = 18 sq. cm

Figure 2

Length = 8 cm
Width = 2 cm
Area = 16 sq. cm

Figure 3

L = 4 cm
W = 2 cm
A = 8 sq. cm

Figure 4

Figure 5

Area = 4 sq. cm

Figure 6

L = 5 cm
W = 2 cm
A = 10 sq. cm

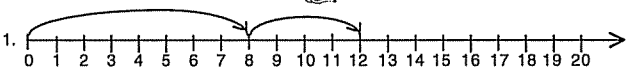
Area of the rectangle is 12 sq. cm

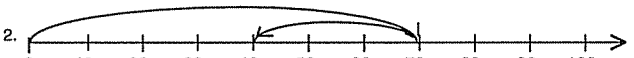
What is the area of the triangle?
6 sq. cm.

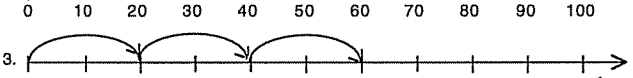
Challenge!
Figure 7

What is the area of the picture
5 sq. cm.

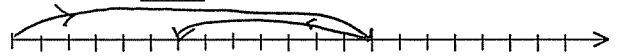
Number Lines

1. 

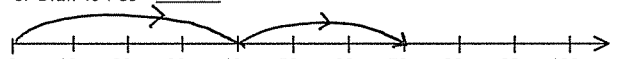
2. 

3. 

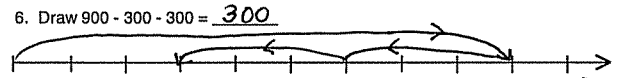
1. Write the number sentence for number line 1. 8+4=12
2. Write the number sentence for number line 2. 70-30=40
3. Write the number sentence for number line 3. 200+200+200=600
4. Draw 13 - 7 = 6



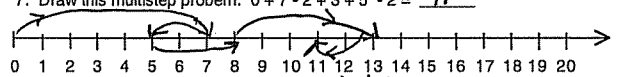
5. Draw 40 + 30 = 70



6. Draw 900 - 300 - 300 = 300



7. Draw this multistep problem: 0 + 7 - 2 + 3 + 5 - 2 = 11



8. Make up a multistep problem for a friend. AWV = ___

9. Draw your own number line. Above the number line, make up a problem. Draw it and write the number sentence. Below the number line, create a problem using a different operation. Draw it and write the number sentence.

Difference Between: Patterns

Complete the sequence and solve for the difference between each number.

1. 10, 20, 30, 40, 50, 60, 70
   The difference between: 10 10 10 10 10 10
   How did you get it? Subtraction

2. 0, 15, 30, 45, 60, 75, 90, 105
   The difference between: 15 15 15 15 15 15 15

3. 48, 42, 36, 30, 24, 18, 12, 6, 0
   The difference between: 6 6 6 6 6 6 6

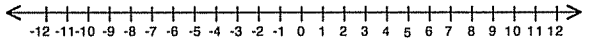
4. 0, 2, 6, 12, 20, 30, 42, 56
   The difference between: 2 4 6 8 10 12 14

5. Challenge! The famous Fibonacci Sequence
   1, 1, 2, 3, 5, 8, 13, 21, 34
   The difference between: 0 1 1 2 3 5 8 13

6. AWV
   ___, ___, ___, ___, ___, ___
   The difference between: 4 4 4 4 4

7. Make your own.
   The difference between:

Sums and Differences: Patterns - Worksheet 1



Solve all the ways of adding two numbers to equal the sum of twelve.

12 + 0 = 12
11 + 1 = 12
10 + 2 = 12
9 + 3 = 12
8 + 4 = 12
7 + 5 = 12
6 + 6 = 12
5 + 7 = 12
4 + 8 = 12
3 + 9 = 12
2 + 10 = 12
1 + 11 = 12
0 + 12 = 12

Now solve the diferences of each pair.

12 - 0 = 12
11 - 1 = 10
10 - 2 = 8
9 - 3 = 6
8 - 4 = 4
7 - 5 = 2
6 - 6 = 0
5 - 7 = -2
4 - 8 = -4
3 - 9 = -6
2 - 10 = -8
1 - 11 = -10
0 - 12 = -12

There are several patterns here. What do you see?

In addition all the answers equal twelve. One column gets smaller by one, the other larger by one. In subtraction the answers get smaller by two. The pattern continues in negative numbers.

Sums and Differences: Patterns - Worksheet 2

One Way to Analyze a Pattern

Two Numbers	Their Sum	Their Difference
10, 2	12	8

12 + 0 = 12 → 12 - 0 = 12
11 + 1 = 12 → 11 - 1 = 10
10 + 2 = 12 → 10 - 2 = 8

Continue the pattern. Continue the pattern.

9 + 3 = 12 → 9 - 3 = 6
8 + 4 = 12 → 8 - 4 = 4
7 + 5 = 12 → 7 - 5 = 2
6 + 6 = 12 → 6 - 6 = 0

Stop Stop

Try again with different numbers.

Two Numbers	Their Sum	Their Difference
9, 4	13	5

13 + 0 = 13 | 13 - 0 = 13
12 + 1 = 13 | 12 - 1 = 11
11 + 2 = 13 | 11 - 2 = 9
10 + 3 = 13 | 10 - 3 = 7
9 + 4 = 13 | 9 - 4 = 5
8 + 5 = 13 | 8 - 5 = 3
7 + 6 = 13 | 7 - 6 = 1

Sums and Differences: Patterns - Worksheet 3

Two Numbers	Their Sum	Their Difference
11, 4	15	7

15 + 0 = 15 | 15 - 0 = 15
14 + 1 = 15 | 14 - 1 = 13
13 + 2 = 15 | 13 - 2 = 11
12 + 3 = 15 | 12 - 3 = 9
11 + 4 = 15 | 11 - 4 = 7
10 + 5 = 15 | 10 - 5 = 5
9 + 6 = 15 | 9 - 6 = 3

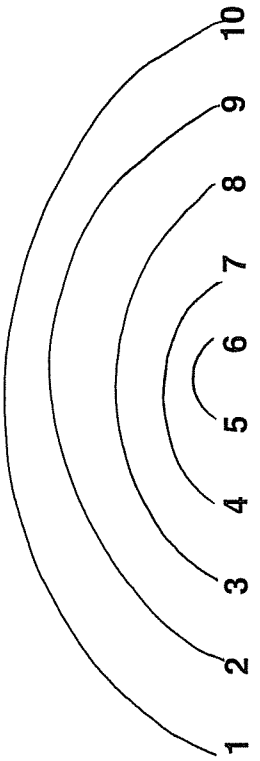
Two Numbers	Their Sum	Their Difference
7, 6	13	1

13 + 0 = 13 | 13 - 0 = 13
12 + 1 = 13 | 12 - 1 = 11
11 + 2 = 13 | 11 - 2 = 9
10 + 3 = 13 | 10 - 3 = 7
9 + 4 = 13 | 9 - 4 = 5
8 + 5 = 13 | 8 - 5 = 3
7 + 6 = 13 | 7 - 6 = 1

Sums and Differences: Patterns - Worksheet 4

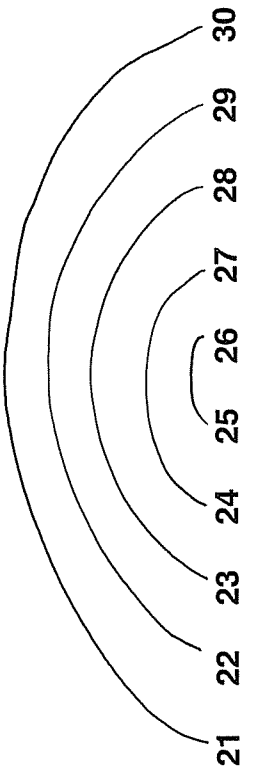
Two Numbers	Their Sum	Their Difference
1. 6, 16	22	10
2. 13, 4	17	9
3. 7, 12	19	5
4. 6, 13	19	7
5. 8, 3	11	5
6. 5, 3	8	2
7. 7, 8	15	1
8. 6, 9	15	3
9. 10, 5	15	5

Rainbow Patterns - Worksheet 1



1+10 = 11
2+9 = 11
3+8 = 11
4+7 = 11
5+6 = 11

Rainbow Patterns - Worksheet 2



21+30 = 51
22+29 = 51
23+28 = 51
24+27 = 51
25+26 = 51

Rainbow Patterns - Worksheet 3

Here is a series of nine numbers. Multiply the numbers that are connected by the dark lines. Record your results below.

2 3 4 5 6 7 8 9 10

		Differences	
2 + 10 = 12	2 x 10 = 20		
3 + 9 = 12	3 x 9 = 27	7	2
4 + 8 = 12	4 x 8 = 32	5	2
5 + 7 = 12	5 x 7 = 35	3	2
6 + 6 = 12	6 x 6 = 36	1	

When you add these pairs the sums are the same or equal.
When you multiply the pairs the products are not equal.
Circle the pair which gave you the largest product.
Try it again on another sequence. See if the same pattern appears.

4 5 6 7 8 9 10 11 12

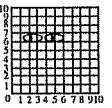
What is the sum of each pair? 16

Multiply		Differences	
4 x 12 =	48	7	2
5 x 11 =	55	5	2
6 x 10 =	60	3	2
7 x 9 =	63	1	
(8 x 8) =	64		

Circle the pair which had the largest answer this time and write it here. 8x8
Try a few more sequences on another piece of paper.

Make a rule for chosing the multiplication problem with the largest answer.
My rule is multiply the middle number of the set of odd numbers together.

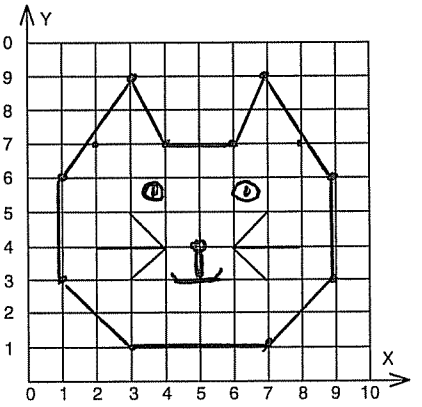
Coordinate Graphing Worksheet 1
What am I?



Always measure from (0, 0). What am I? Cat

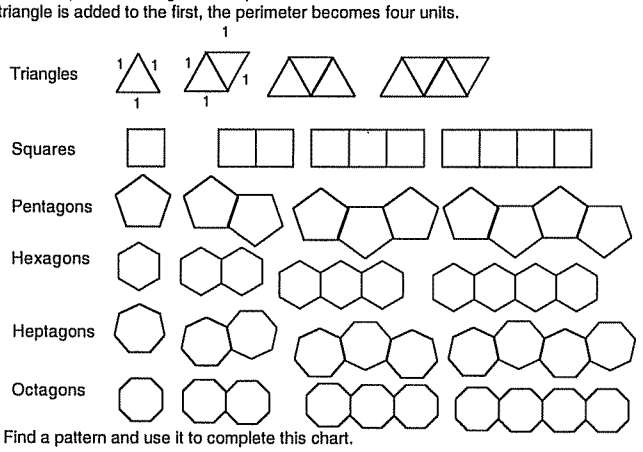
Over	Up	
(3,	1)	Make a dot.
(1,	3)	Make a dot, connect the dots.
(1,	6)	Make a dot, connect the dots.
(2,	7)	Make a dot. Continue connecting the dots.
(3,	9)	Make a dot.
(4,	7)	Make a dot.
(6,	7)	Make a dot.
(7,	9)	Make a dot.
(8,	7)	Make a dot.
(9,	6)	Make a dot.
(9,	3)	Make a dot.
(7,	1)	Draw a line between the dots. Start again.
(5,	4)	
(5,	3)	Connect dots.
(6,	3)	Connect dots.
(4,	3)	

Write the coordinates for the lines already drawn.
(3, 3)
(4, 4)
(5, 3)
(6, 4)
(7, 3)
(7, 5)



Number Patterns: Adding Shapes

Draw additional hinged shapes and record the perimeters in the space below
For example, one triangle has of perimeter of three units. When an identical triangle is added to the first, the perimeter becomes four units.



Find a pattern and use it to complete this chart.

Number of polygons	Perimeter of Triangles	Perimeter of Squares	Perimeter of Pentagons	Perimeter of Hexagons	Perimeter of Heptagon	Perimeter of Octogons
1	3	4	5	6	7	8
2	4	6	8	10	12	14
3	5	8	11	14	17	20
4	6	10	14	18	22	26
5	7	12	17	22	27	32
6	8	14	20	26	32	38
10	18	24	32	42	52	62

Number Patterns: Missing Numbers

Write the missing numbers to complete the pattern.

85, 95, 105, 115, 125, 135, 145, 155, 165

500, 600, 700, 800, 900, 1,000, 1,100, 1,200, 1,300

510, 515, 520, 525, 530, 535, 540, 545, 550, 555

600, 604, 608, 612, 616, 620, 624, 628, 632, 636

800, 900, 1,000, 1,100, 1,200, 1,300, 1,400, 1,500

542, 545, 548, 551, 554, 557, 560, 563, 567, 570

1,500, 1,504, 1,508, 1,512, 1,516, 1,520, 1,524, 1,528

6,000, 6,005, 6,010, 6,015, 6,020, 6,025, 6,030, 6,035

7,097, 7,098, 7,099, 7,100, 7,101, 7,102, 7,103, 7,104

1,997, 1,998, 1,999, 2,000, 2,001, 2,002, 2,003, 2,004

Make your own.

Up and Down Sums

This series of numbers goes up and down. Find the sum of each series.
Look for a pattern.

(1) = 1

1 + (2) + 1 = 4

1 + 2 + (3) + 2 + 1 = 9

1 + 2 + 3 + (4) + 3 + 2 + 1 = 16

1 + 2 + 3 + 4 + (5) + 4 + 3 + 2 + 1 = 25

1 + 2 + 3 + 4 + 5 + (6) + 5 + 4 + 3 + 2 + 1 = 36

1 + 2 + 3 + 4 + 5 + 6 + (7) + 6 + 5 + 4 + 3 + 2 + 1 = 49

1 + 2 + 3 + 4 + 5 + 6 + 7 + (8) + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 64

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + (9) + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 81

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + (10) + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 100

If the highest number is 20, what is the sum of this series? 400

What is the sum of the series if the highest number is n? n^2

Square Numbers - Worksheet 1

Build with orange pattern blocks or square tiles.

1st square  How many blocks? 1

Build the next size square. How many blocks? 4

Keep building the next size larger square. Write the total number of blocks below.

Can you find a pattern so you don't have to build all of the squares?

How many blocks will the 10th square have? 100 Guess.

Squares	Number of blocks on one side	Total number of blocks	Number of blocks added	Number Squared
0	0	0	0	0
1st	1	1	1	1
2nd	2	4	3	2
3rd	3	9	5	3
4th	4	16	7	4
5th	5	25	9	5
6th	6	36	11	6
7th	7	49	13	7
8th	8	64	15	8
9th	9	81	17	9
10th	10	100	19	10

What's My Rule

IN	OUT
2	4
6	8
9	11
20	22

What's My Rule? +2
in plus 2 equals out

IN	OUT
7	4
3	0
10	7
8	5
20	17

What's My Rule? −3
in − 3 = out

IN	OUT
10	5
8	4
6	3
4	2
100	50
12	6

What's My Rule? ÷2
or $\frac{1}{2}$ of in equals out

IN	OUT
0	0
1	2
2	4
3	6
8	16

What's My Rule? X 2

Make your own.

IN	OUT
2	12
3	13
10	20
7	17
40	50

What's My Rule? +10

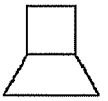
IN	OUT

What's My Rule? AWV

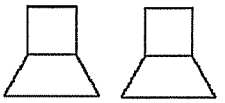
Graphing Number Patterns -Worksheet 1
Huts and Chimneys

Build with pattern blocks.

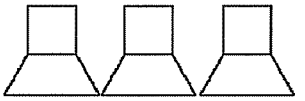
One hut has one chimney.



Two huts have two chimneys.



Three huts have 3 chimneys.



How many chimneys will 10 huts have? 10

Guess first _____

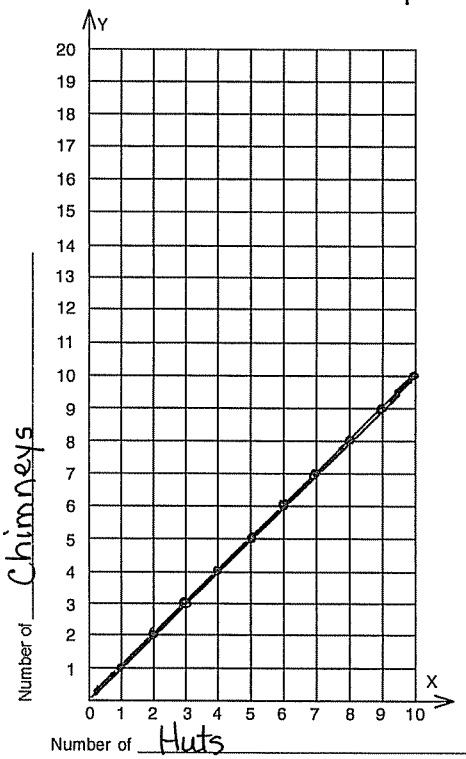
How do you know?
What's the rule?

huts = chimneys
or C = H
There is one chimney on each hut.

Huts	Chimneys
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10

Graphing Number Patterns -Worksheet 2

Graph title Huts and Chimneys

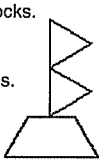


X	Y
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10

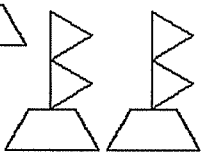
Graphing Number Patterns -Worksheet 3
Huts and Flags

Build with pattern blocks.

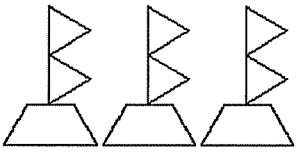
One hut has two flags.



Two huts have four flags.



Three huts have 6 flags.



How many flags will 10 huts have? 20

Guess first _____

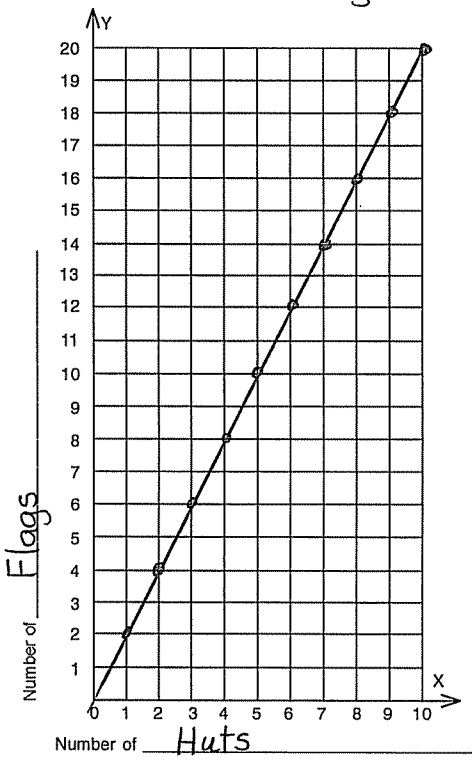
How do you know?
What's the rule?

huts x 2 = flags
or
F = 2H

Huts	Flags
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18
10	20

Graphing Number Patterns -Worksheet 4

Graph title Huts and Flags

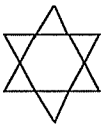


X	Y
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18
10	20

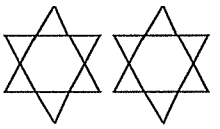
Graphing Number Patterns -Worksheet 5
Stars and Points

Build with pattern blocks.

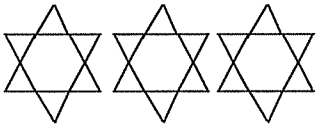
One star has 6 points.



Two stars have 12 points.



Three stars have 18 points.



How many points will 10 stars have? 60

Guess first _____

How do you know?

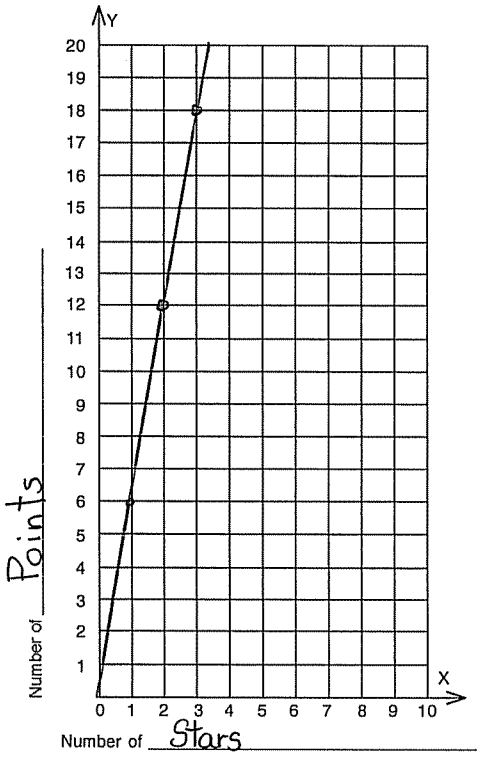
What's the rule?

Star x 6 = points
or
P = 6S

Stars	Points
1	6
2	12
3	18
4	24
5	30
6	36
7	42
8	48
9	54
10	60

Graphing Number Patterns -Worksheet 6

Graph title Stars and Points



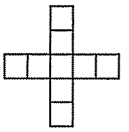
X	Y
1	6
2	12
3	18
4	24
5	30
6	36
7	42
8	48
9	54
10	60

Graphing Number Patterns -Worksheet 7
Crosses

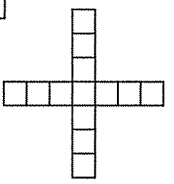
Build with pattern blocks or centimeter tiles.
All crosses have four arms.

This cross has one tile in each arm.
There are five tiles all together.

This cross has two tiles in each arm.
There are nine tiles all together.



The next cross has three tiles in each arm.
How many tiles all together? 13



How many tiles will it take to build a cross with 10 tiles in each arm? 41

Guess first _____

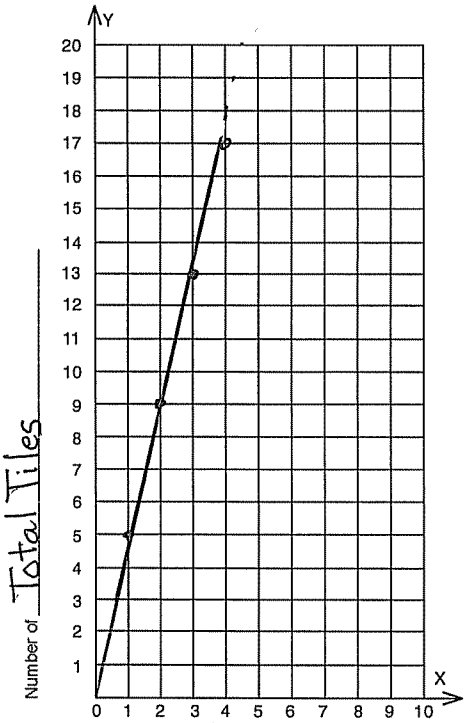
How do you know? What's the rule?

(Cross # times 4) + 1
or
T = 4a + 1

Tiles in arm	Total Tiles
1	5
2	9
3	13
4	17
5	21
6	25
7	29
8	33
9	37
10	41

Graphing Number Patterns -Worksheet 8

Graph title Crosses



X	Y
1	5
2	9
3	13
4	17

Algebraic Thinking - Worksheet 1

Using only the numbers 2, 4, 6 find all the answers.
Make tiles like the ones shown at the side.
Move the tiles around for each problem until
you find the answer you are looking for
Then record the numbers in the boxes.

(2 x 4) + 6 = 14

(6 x 2) + 4 = 16

(6 x 4) + 2 = 26

(4 x 6) - 2 = 22

(4 x 2) - 6 = 2

(6 x 2) - 4 = 8

Algebraic Thinking - Worksheet 2
Math Sentences

< = > + ÷ - x

Put signs in to make these number sentences true.
There are several different solutions. Examples: 8 < 4 + 7

1 + 4 = 5

2 + 3 = 5

6 = 2 x 3 or 6 > 2 + 3

6 + 2 > 3

Solve it a different way.

6 ÷ 2 = 3

8 + 2 = 12 − 2

9 = 4 + 5

9 − 4 = 5

9 < 4 + 6

11 > 4 + 5

7 + 2 = 5 + 4

Algebraic Thinking - Worksheet 3
Math Sentences

Cut out the tiles and paste in the sign that makes the number sentence true.
Use only the signs in the tiles. Each sign can only be used once.

Set 1

(3 + 4) = 7

(3 + 4) < 8

(3 x 4) = 12

(4 − 3) = 1

(4 − 3) > 0

>	<
+	x
-	+
=	=
-	=

Set 2

8 = (4 x 2)

(6 − 4) = 2

6 = (4 + 2)

(6 + 4) > 8

0 < (4 − 2)

+	=
-	x
=	>
=	+
<	-

Algebraic Thinking - Worksheet 4
Math Sentences

Cut out the tiles and paste in the sign that makes the number sentence true.
Use only the signs in the tiles. Each sign can only be used once.

Set 1

6 + 3 = 9

6 − 3 = 3

7 − 2 = 5

7 − 6 < 2

5 X 1 = 5

=	<
+	X
-	-
=	=
-	=

Set 2

8 + 3 = 11

11 − 5 = 6

7 X 2 = 14

11 > 7 − 5

2 − 0 = 2

-	>
X	-
+	=
=	=
-	=

Algebraic Thinking - Worksheet 5
Math Sentences

Use only the numbers in the box to make true sentences.

Order may vary with addition and multiplication

5, 3, 8 (5 + 3) = 8

5, 3, 8 (8 - 5) = 3 or 8-3=5

5, 5, 3, 3 (5 + 3) = (3 + 5

10, 2, 4, 8 (10 - 8) + 2 = 4

1, 2, 5, 3 (5 x 1) - 2 = 3

5, 5, 0, 3 5 + (3 x 0) = 5

4, 6, 6, 3 not possible ___ - (___ x ___) = ___

(4×3)-6=6 is possible

Algebraic Thinking - Worksheet 6
Math Sentences

Write the correct number in each box to make a true sentence. Think about the solution without doing the calculations.

Explain your thinking.

4 x 3 = [3] x 4 Commutative

2 x 5 x 2 = 5 x [1] x 4

[8] + 4 + 5 = 5 + 8 + 4 Associative

4 x 7 x 3 = [3] x 4 x 7 Associative

Now instead of a box, use a letter.

9 x 8 = J x 9
J = 8

3 x 6 x 4 = 6 x Q x 3
Q = 4

K x 4 = 4 x 5
K = 5

M + 4 + 6 = 4 + 6 + M
M = 0

Algebraic Thinking - Worksheet 7
Math Sentences

Solve these equations.

(5 + 4) x 2 = ?
9 x 2 = 18

3 x (7 + 2) = ?
3 x 9 = 27

? = 2 x (5 + 6)
22 = 2 x 11

(4 + 3) x 4 = ?
7 x 4 = 28

Now let's do one upside down and work backwards.
What number goes in the box to make the sentence true? 4

[4] x 3 = 12

What if I take that 4 and break it into two numbers? What number goes in the box now? 3

(1 + [3]) x 3 = 12

Solve these equations.

[3] x 5 = 15
(1 + [2]) x 5 = 15

Now let's put a letter where the box goes.

[3] x 5 = 15
(1 + J) x 5 = 15
J = 2

Algebraic Thinking - Worksheet 8
Math Sentences

[4] x 6 = 24
(2 + P) x 6 = 24
P = 2

[6] x 5 = 30
(4 + K) x 5 = 30
K = 2

45 = [9] x 5
45 = (4 + H) x 5
H = 5

Challenge!
40 = 5 x (A + 5)
What does A = 3 ?
Explain how you know
5X8=40 A+5=8 So A must be 3.

30 = 6 x (4 + B) What does B equal to? 1

Reasoning:
Well, 6 x what number equals 30? 5
So, the sum of the numbers in the ( ) must be 5. 5 x 6 equals 30.
If the number in the ( ) is 5, then what is B? 1
Four plus what is five? 1
So, B must be 1.

Now you solve some problems. Show your work here.

24 = 3 x (3 + J)

J = 5

18 = 2 x (6 + R)

R = 3

20 = 4 x (5 + M)

M = 0

Algebraic Thinking - Worksheet 9

Using only the numbers 3, 5, 9 find all the answers.
Make tiles like the ones shown at the side.
Move the tiles around for each problem until
you find the answer you are looking for
Then record the numbers in the boxes.

3
5
9

([3] x [5]) + [9] = 24

([3] x [5]) x [9] = 135

([9] x [5]) - [3] = 42

([3] + [5]) x [9] = 72

([5] x [9]) ÷ [3] = 15

([9] + [5]) - [3] = 11

([3] x [9]) + [5] = 32

Families of Facts: Missing Numbers - Worksheet 2

Solve these problems using the Families of Facts color pattern.

7 = yellow 8 = blue 15 box = green 4 = yellow 9 = blue 13 box = green

(7) + (8) = [15]
(8) + (7) = [15]
[15] - (7) = (8)
[15] - (8) = (7)

(4) + (9) = [13]
(9) + (4) = [13]
[13] - (4) = (9)
[13] - (9) = (4)

6 = yellow 8 = blue 14 box = green

(6) + (8) = [14]
(8) + (6) = [14]
[14] - (6) = (8)
[14] - (8) = (6)

Make your own. Outline each shape with the same color pattern. AWV

( ) + ( ) = [ ]
( ) + ( ) = [ ]
[ ] - ( ) = ( )
[ ] - ( ) = ( )

Does the pattern change if the numbers change? no

Use pattern language (algebra) to show the relationship.
Fill in the numbers below.

a	b	c
5	6	11
6	5	11
11	5	6
11	6	5

a	b	c
a	b	c
b	a	c
c	a	b
c	b	a

Families of Facts: Missing Numbers - Worksheet 3

Use the Families of Facts patterns to solve these equations.

Put signs in the △ . Outline each shape with the colors from worksheet 1.

1 (7) △ (9) = [16]
2 (9) △ (7) = [16]
3 [16] △ (7) = (9)
4 [16] △ (9) = (7)

This pattern is used to find values of missing numbers.

If (a) + (9) = [16], which equation above tells what a? 16 - 9 = a.

Use Families of Facts to solve these equations.

(j) + (11) = [21]
(j) = [21] △ (11)
(j) = 10

(t) + (6) = [13]
(t) = [13] △ (6)
(t) = 7

(f) + (7) = [12]
(f) = [12] △ (7)
(f) = 5

Make your own. AWV

( ) + ( ) = [ ]
( ) = [ ] △ ( )
( ) = _____

*Answer
a = 6 - 9

Families of Facts: Missing Numbers - Worksheet 4
Pattern Language (Algebra)

1. c + 4 = 9

   c = 9 − 4 = 5

   c = 5

2. 7 + g = 10

   g = 10 − 7

   g = 3

3. r + 6 = 14

   r = 14 − 6

   r = 8

4. 7 + t = 12

   t = 12 − 7

   t = 5

5. c + 5 = 17

   c = 17 − 5

   c = 12

6. b + 7 = 15

   b = 15 − 7

   b = 8

7. a + 10 = 18

   a = 18 − 10

   a = 8

8. m + 5 = 21

   m = 21 − 5

   m = 16

9. w + 12 = 15

   w = 15 − 12

   w = 3

Make your own. AWV

10. ○ △ ○ = □

    ○ = □ △ ○

    ○ = ____

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