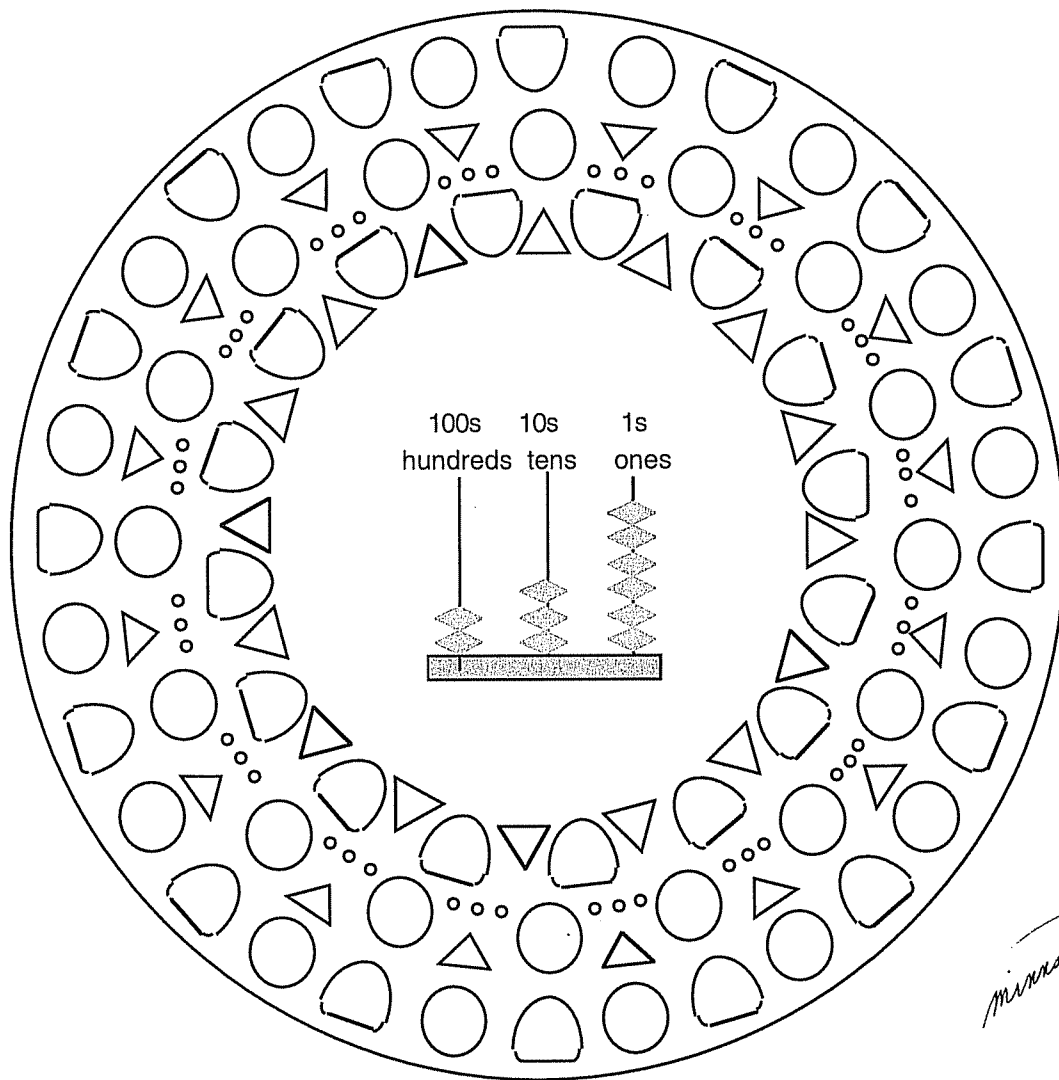


# Patterns in Arithmetic

## General Math - Booklet 3 PDF

### Place Value, Measurement, and Geometry

# Parent/Teacher Guide



By Alysia Krafel, Susan Carpenter, and Suki Glenn

Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the  
UCI Farm Elementary School  
University of California, Irvine

# General Math: Booklet 3 PDF - Place Value, Measurement, and Geometry

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We dedicate this booklet to Mary Baratta-Lorton. You have been such an inspiration to so many teachers and helped so many children get a firm foundation in mathematics.

## Acknowledgments

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For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

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Susan Carpenter edited, added her wise words, useful suggestions, and helped make the Answer Keys a reality. Karán Founds-Benton contributed her meticulous editing skill and knowledge. Diligent and thorough copy editing was done by Jacqueline Logue.

The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

Patterns in Arithmetic: General Math - Booklet 3 PDF

Parent/Teacher Guide

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## Patterns

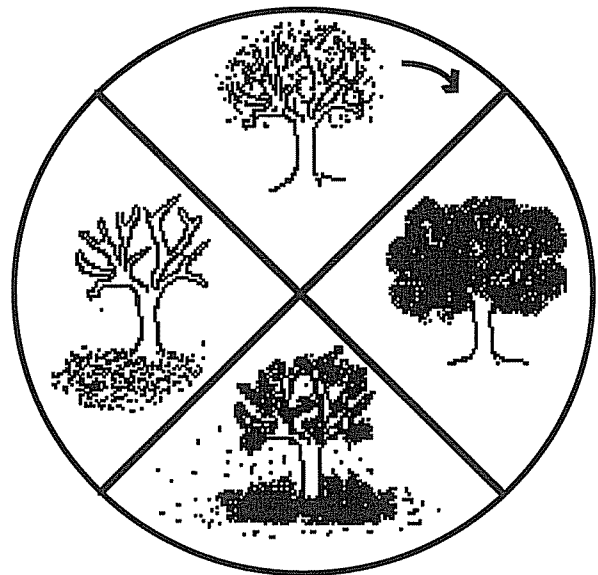
**Purpose** The purpose is to delight the student with the beautiful and playful side of mathematics. To develop awareness and skill in areas of mathematics other than arithmetic. To nourish the pattern lover in all of us.

**Concurrent** These activities can be done in any order you choose. They should be done concurrently with other lessons all year long. Students can work independently once they understand general instructions.

Students love mathematical play, which nourishes the fanciful as well as helps them develop a liking of math. Arithmetic calculations are important but can become a psychological grind unless the beautiful and the delightful in mathematics are enjoyed along the way.

## Cycles

**Activity** Cycles are patterns of regularly occurring events that repeat themselves. What series of things in the world go around and around and around—never stopping—always repeating, over and over again? Nature abounds with such cycles. Together, brainstorm about cycles you have experienced, such as cycles of a day, of a school year, of a tree during a year. Have the student think of a cycle and then draw it on a divided circle. The circle can have as many sections as needed. Draw an arrow to show in which direction the cycle flows. Here is an example of one cycle. It is the yearly cycle of an oak tree.



## Purpose **Number Patterns**

The purpose is to develop, describe, and extend number patterns involving all operations and to introduce formal number pattern analysis, for example, looking for a rule. Learning to see and enjoy number patterns as well as auditory, tactile, and visual patterns is a critical skill for developing problem-solving methods and algebra. Finding patterns is also a delight to the senses.

**Prerequisites** If the student is new to this series, have him begin with the patterns in *Patterns in Arithmetic: Books 1 and 2*. Review sound, visual, and movement patterns.

**Worksheets** Patterns: Geometry - page 1; Number Patterns - Worksheets 1 and 2, pages 3 and 4  
Number Patterns: Missing Numbers - page 30

## Geometry

**Purpose** The purpose is to introduce the student to geometric solids, lines, line segments, rays, and angles.

**Prerequisites** None

**Worksheets** Rectangle City, page 2, is a fun page to encourage creativity and playfulness.

Geometric Solids, page 5, does need a set of geometric solids. Otherwise the worksheet is self-explanatory.

Lines, Line Segments, Rays, and Angles - Worksheets 1 - 3, pages 6 - 8  
The worksheet is self-explanatory.

## What's My Rule?

**Purpose** The purpose is to learn to spot patterns in number series. When you are looking for a pattern that uncovers a hidden operation, you look at what number you started with and what number you ended with and try to figure out what happened to the numbers in between. This is one thing scientists and mathematicians do when they discover new 'formulas.' A game that exercises this skill is What's My Rule?

**Activity** To play, think of a simple operation such as  $+2$ . The student gives you a number and you add two to it and say the new number back. For example, if she says "one," you say "three." She says "five" and you say "seven." The number she gives you is called the independent variable, or the number in. The number you say back is called the dependent variable, or the number out.

Record the information on a chart like this:

IN	OUT
1	3
5	7
9	<u>?</u>

What's My Rule?  $+2$

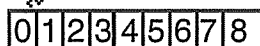
When a student thinks she knows the rule, let her predict the number out loud. Finally, when everyone can predict successfully, let someone formulate the rule of plus two. Students love to make these up for each other using easy addition, subtraction, multiplication, and division operations.

Play this game regularly. It's a good rainy day and in-the-car game.

One teacher called this activity Black Box and made a symbolic black box out of a milk carton decorated with gears and levers with a slide inside that flipped a card upside down. A card would be put in a slot in the top of the box and come out so the number written on the back of the card came out a bottom slot. The students then guessed the rule. When the box appeared, the students' minds focused to discover the relationship between the In and Out number of the day.

**Worksheet** What's My Rule? - Worksheets 1 and 2, pages 13 and 14

## Number Lines



### Purpose

The purpose is to connect a number line (representation) with the real world. To use a student's whole body (kinesthetic learning) to connect to a number line.

### Introduction

1) Record daily the days of a year. Use a roll of adding machine tape hooked to the wall at a comfortable writing level for a student. Start with zero. On day one have the student record a '1.' On each successive day write the next number.

2) Make a number line large enough to walk on using a roll of plastic, vinyl, chalk on a sidewalk, or butcher paper. Have the numbers fit the size of her steps. Before recording on a number line, have the student do the following for each operation. Also make up more problems and have the student make up her own.

Sample problems on a number line from one to twenty:

### Addition

"Start on zero. Count six steps. Add on seven. Where do you end up?" (13) The student will say 'one' as she steps from zero to one.

"What problem is this?"  $6 + 7 = 13$

"Start on zero. Count three steps and add on five steps. Where do you end up?" \_\_\_\_\_ "What problem is this?" \_\_\_\_\_  $3 + 5 = 8$

### Subtraction

"Start on zero. Count seventeen and stop, facing the twenty. Step backwards eight steps, counting as you go. Where do you end up?" (9)

"What problem is this?"  $17 - 8 = 9$

"Start on zero. Count fifteen and stop, facing the twenty. Step backwards seven steps, counting as you go. Where do you end up?" \_\_\_\_\_ (8)

"What problem is this?" \_\_\_\_\_  $15 - 7 = 8$

### Multiplication

"Start at zero, and hop two numbers, five times. Count each hop. Where do you end up?" (10) "What problem is this?" "Two times five." ( $2 \times 5 = 10$ ).

"Start at zero, and hop three numbers, four times. Count each hop. Where do you end up?" (12) "What problem is this?" "Three times four." ( $3 \times 4 = 12$ )

### Division

"Start at fifteen, and jump back by threes to zero. How many jumps?" (5) "How many jumps of three are in fifteen?" "Five." "What is the division number sentence?" "Fifteen divided by three." ( $15 \div 3 = 5$ ) "What is the subtraction number sentence?"  $15 - 3 - 3 - 3 - 3 - 3 = 0$

Compare the size of the jumps for all operations. "Which operations have equal sized jumps?" "Multiplication and division." "Which operations have different sized jumps?" "Addition and subtraction. Sometimes they are the same,  $2 + 2 = 4$ ."

Sketch out a number line together and draw the actions for each operation on the paper.

### Worksheets

Number Lines - page 15

## Difference Between

### Purpose

The purpose is to explore subtraction by the comparison of the sizes of numbers and to link the operation of subtraction and counting. For example, how much larger is nine than four?

### Materials

Any manipulative (Pennies are used in the example.)

### Lesson Session 1

**"Make a pile of nine pennies. Then make a pile of five pennies."**

**"Which set has the most?"** "The nine pile."

**"How do you know?"** "It looks bigger." Or, "I counted it."

**"How much larger?"** Or, **"How many more pennies in the larger pile? How can you figure this out?"** The answers will vary.

There are different ways to solve this. Encourage each student to solve this in her own way. Here are a few solutions students have come up with:

"I have five in this pile and counted pennies until I got to nine. Nine is four more than five." (Started with five and 'counted on'—six, seven, eight, nine.)

The counters are placed in two rows, one above the other. The student compares the two lines with one-to-one correspondence and counts the difference.

"I know that  $5 + 5$  is 10, and nine is one less than ten, so nine is four more than five."

"I subtracted. I have nine and I took away five and I have four left."

Nine pennies are laid out and each of the five pennies are placed on top of a penny that is laid out already. There are four pennies not covered up. "Five is four less than nine."

**"Count out thirteen counters and eight counters. Which number is larger? How much larger? What is the difference between thirteen and eight? How much smaller is eight than thirteen? How did you get the answer?"** Continue with several more examples. Encourage the student to ask the questions.

Finally, have the student draw a picture of the counters and write a number sentence or write a statement. If she counted on, as in the first example above, she may write the problem as an addition problem. Another student may write the problem as a subtraction problem. Compare solutions and encourage different ways to solve it.

### Session 2

Eventually we do want a student to see the problem as a subtraction problem. There are several ways to do this. Ask the student to find different ways of solving the problem. Or if more than one student is working on the problem, have the students compare the different ways in which they solved the problem. Have the students take turns being teacher and student and teach the other person a different way to solve the problem.

Another approach is to have the student build a model. One solution is to line up the counters.



"What numbers go with this problem?" "15, 8, and 7."

"Show what each number represents." Compare the difference.

"How much larger is fifteen than eight?" "Fifteen is seven larger than eight."

Then line the numbers up: 15 8 7.

"Can you make a number sentence with these numbers?"

Answer:  $15 - 8 = 7$  or  $15 = 8 + 7$

Or, try this way using the Families of Facts lesson. "**The answer is seven. What were the other numbers in this problem?**" "Fifteen and eight."

"What can you do with a fifteen and an eight to get seven as an answer?"

" $15 - 8 = 7$ ."

Have her prove each equation with counters. The student may need to play around with the manipulatives and the numbers to figure this out. If the student writes  $8 - 15 = 7$ , the use of counters will prove it is not correct. Let her self-correct by asking, "**Show me the eight, show me the fifteen, and show me the seven,**" instead of saying, "**That is not right.**" Allow the student time to reason this out. The relationship is not always obvious to students.

Once the student has figured out that a difference can be calculated by subtraction, have a discussion about the difference between comparison problems and 'Take Away,' or 'What's Left?' problems. Both are solved by the same operation, but they appear differently in real world situations.

How much change will I get when I buy this ball with my dollar? This is a 'Take Away' or 'What's Left?' problem.

How much more expensive is a candy bar than a package of gum? This is a comparison problem, solved by finding the difference.

Make up several examples of both together. Then have the student make up both types. Look for difference between problems in the real world. Car ads often boast, "We'll beat their price and pay you the difference."

**Worksheet** None.

## Sums and Differences

**Purpose** The purpose is to find the sum or difference in a pair of numbers and apply it to a number line.

**Prerequisites** Difference Between, Number Lines using addition and subtraction

**Materials** Sums and Differences, page 16  
Any manipulative, if needed

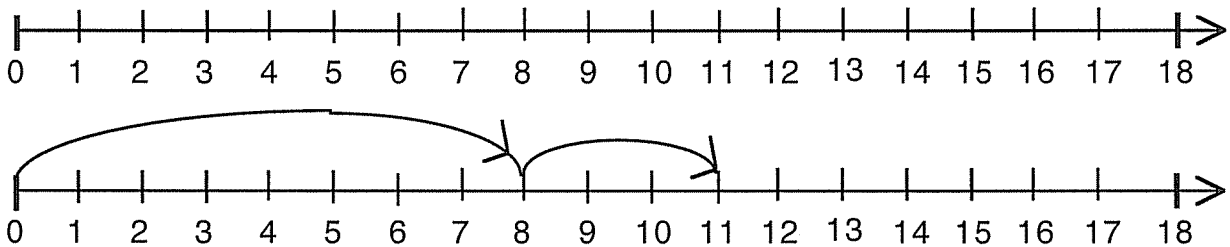
**Lesson** Give two numbers such as ten and six. **"How do you find a sum?"** "By addition."

**"How do you find a difference?"** "The difference is figured out by subtraction."

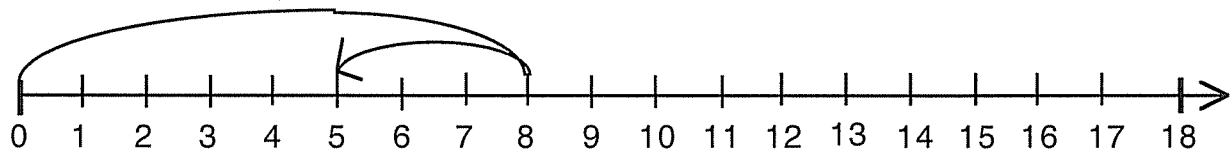
**"What is the sum of ten and six?"** "Sixteen."

**"How do you know?"** "I added them." **"What is the difference?"** "Four."

Draw a number line. Give two numbers, such as eight and three. Draw on the number line the sum of eight and three. **"What is the sum of eight and three?"** "Eleven."



Draw another number line. Draw on the number line the difference between eight and three. **"What is the difference between eight and three?"** "Five."



**Worksheet** Sums and Differences, page 16  
The student can finish the page independently if she has no difficulty.



## Place Value to One Hundred and One Thousand

**Purpose** The purpose is to review place value in the hundreds place and to extend the understanding of place value to numbers up to ten thousand.

**Prerequisites** Place value activities in *Patterns in Arithmetic: Place Value - Book 2* or *Patterns in Arithmetic: Book 2 - General Math Section*

**Activities**

1. How long will a line of one hundred of each of these be: pennies; Unifix cubes; centimeter cubes or colored cubes; pinto or lima beans? Have the student estimate by placing a piece of masking tape or by stretching a length of string. After making the first estimation, make a length of ten objects. Let her modify the estimate. Continue adding objects until the count gets up to fifty. With this information have her estimate one more time. Compare the estimates.

2. Draw a square that ten standing students could fit into. How big would the square be if you wanted to stand one hundred? Have students notice seating capacity signs in theaters and restaurants. Can you seat one hundred students on a school bus? Are there one hundred seats per row in a theater?

3. What in the world weighs one hundred pounds? Make a list of things weighing one hundred pounds.

4. Have the student estimate how high in a jar one hundred beans will go. Or select a range of sizes of food jars. Which one will hold just one hundred beans?

5. How far is one hundred feet? How far is one hundred of your paces set heel to toe?

6. Close your eyes and raise your hand when ten seconds have passed; lower it when one hundred seconds have passed.

7. How long does it take to hop one hundred times? How long does one hundred turns of a jump rope take?

8. Number from zero to one hundred on place value strips.

9. What was going on in the world one hundred years ago?

10. What can you buy for one hundred dollars?

### How much is one thousand?

1. Continue the numbers on the place value strips into four digits and count as far as the student wants to go (see *Patterns in Arithmetic: General Math - Booklet 2*). This will take several days and can be done by several students. Estimate how long the strip will be when it is finished. Tape it up on the wall or ceiling when it's done. If an error is made in the counting, the student can cut out the wrong part and tape in a corrected version.

**Worksheet** Student Workbook - Place Value Strips: Four Digits, page 17

2. Using centimeter grid paper, cut out one square, then a row of ten squares, then a large square of one hundred cm squares. Write a dark one hundred on the one hundred cm square. Have the student estimate how much space will be taken up by one thousand small cm squares. Then proceed to cut out nine more squares of one hundred. Paste them next to each other on a large piece of paper. Label each one hundred as you go up—two hundred, three hundred, four hundred, etc. When you are finished, draw a dark, heavy line around the entire one thousand. Use this picture of one unit, one ten, one hundred, and one thousand to make a chart. "**How many units are in one thousand? How many tens in one thousand? How many one hundreds in one thousand? What pattern do you notice?**" "There are one thousand units, one hundred tens, and ten hundreds. They go down by factors of ten."

3. Solve addition problems with place value blocks that go into the thousands, e.g.,  $295 + 458 + 289 = 1,042$ .

4. Estimate where, on a one gallon jar, the top of one thousand pinto beans will be. A fast but not as accurate way to fill the jar is to count out one hundred beans and weigh them. Then multiply that weight by ten and weigh out that many beans.

5. Using the jar of one thousand beans as a standard, fill another jar to a level that is less than the level of the one thousand and have everyone in the family or class guess how many beans are in the jar. Later, refill the jar to more than one thousand and estimate again.

6. What in the world weighs one thousand pounds? This is a good research project. Have students make a list of animals and objects that might weigh around one thousand pounds. Then look them up in the encyclopedia or on the Internet. Draw pictures of the things that weigh around one thousand pounds and make a display of them.

7. What was going on in the world one thousand years ago? Have the student guess what kinds of machines people had. Were there dinosaurs then? Who lived where you live now? What was the population of the world then; what is it now?

8. There is a book about John F. Kennedy called *One Thousand Days*. How many years is that? Is that a full presidential term? What happened to Mr. Kennedy?

9. What can you buy with one thousand dollars?

10. How far is one thousand feet? Walk off one thousand paces. It is fun to pace it off on the beach if you have one available. How many feet make one mile?

## Note

These activities are important for developing concepts of numbers in the world. We all know that the national debt each year is in the trillions of dollars, but how many of us have even an inkling of how much that is in physical terms? (one trillion seconds is about thirty-two thousand years!) The physical grasp of number sizes is critical for us to have in order to really understand our mathematical world. The activities are fun for most students and help them to realize that numbers are alive; that they are a part of our everyday world and not just something we do in school. It gives the written numbers on the paper a physical meaning.

## Test for Understanding

1. This is a test of the abstract concept of numbers to one hundred and one thousand. "What does  $99 + 1 = ?$  What does  $1,000 - 1 = ?$  What does  $399 + 1 = ?$ " If these can not be done in the head, it would be safe to assume that the student's abstract understanding of place value is weak.
2. Build a number with the Base Ten Blocks. Have the student read the number.
3. Write a number; have the student build it with Base Ten Blocks.
4. Write a number and have the student draw a picture of the blocks she would use to build a model of the number.

## Testimonial by a Farm School graduate

"For the UCI administration, to whom it may concern:  
My name is Ross Venook. I am a freshman at Stanford University, a Woodbridge High School graduate, and a graduate of Farm School. Those are in reverse chronological order only. Most of all, I am a student of life. I owe this to Farm School...

... Circle math and counting in base 4--the way to a freer, better conceptualization of mathematics in kindergarten. The idea of 'place value' is the most difficult concept for young children to understand, mainly because it conflicts with their only base (no pun intended) in numbers, namely sequential counting. The base 4 counting taught me to regard ten as an arbitrary choice in the same position as other numbers in their own bases. By separating numbers into 10s, 100s, 1,000s, etc., as a method to arithmetic, I developed an early understanding of number concept. Beyond that, counting in base 4 didn't seem to have helped me much—until Pre-Calculus, when I began to hear the '1, 2, 3, BING' of my sixth year in my head when my teacher tried to explain to the class of high school juniors and seniors what different bases were. She was failing where my first grade Farm School teacher had succeeded. Why? Because the teachers at Farm School knew that the number 10 is not particularly special as a base, and that until we were taught otherwise (such as through rote methods taught in all public schools anywhere near Farm School), we could get past such limited thinking."

Excerpt by Ross Venook from a letter written in the 1990s in support of the Farm School.

## Trading Game: Recording

**How to Play** Colored blocks (start with four and eventually use ten colors), or any other counters  
**Materials** One die



Orange Blue Red Green

**Game Plan** Start with four colored blocks. It doesn't matter what color. For the purpose of explanation the colors used here are consistent.

Set four different colored blocks in a row. Arrange the blocks so the first unit is on the right, corresponding to place value getting larger when moving to the left.

The green block is the starting color. In this game three is the trading amount.

Three green blocks trade for one red block.

Three red blocks trade for one blue block.

Three blue blocks trade for one orange block.

The object of the game is for everyone to get at least one orange block. Students take turns rolling one die, getting the first color, and then trading when ready. After students get one orange block, they can accumulate more while others are getting their first orange block. Play for three orange blocks when students are more experienced.

**Recording** Play many trading games with different bases without recording.

**Materials** Trading Game Record Sheet, page 19  
 One die and colored blocks

**Lesson** "Record what happens on each turn using the recording sheet."

Turn	Number Rolled	Purple	Yellow	Orange	Blue	Red	Green
1	5					1	2
2	4				1	0	0
3	2				1	0	2
4	5				1	2	1

"When the trading number is three, what digits appear in your record?" "Zero, one, two."

"Change the trading number to four. Notice which numbers appear on the sheet. Predict what numbers will be on the sheet if the trading number is five. What if the trading number is six?"

"Explain the pattern you see."

"Only the numbers zero to three appear on the recording sheet when the trading number is four because whenever you get four of anything you trade up. Four green blocks will show on the record sheet as a one in the red column." "What else is like this?"

**Variation** Play the trading game up to one thousand with Base Ten Blocks. Pick up ten times the amount rolled on one die.

## Place Value: Three Ways to Record

### Purpose

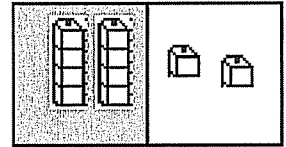
The purpose is to develop an understanding of the numeration system and place value concepts of base four, five, and ten. Also to record place value three ways: the picture of the manipulative, short notation, and the expanded form. Reminder: in this book a 'cuckoo' is a group of four blocks.

### Prerequisites

Previous experience counting in base four. See *Patterns in Arithmetic: Book 2*.

### Materials

Place Value: Three Ways to Record: Base Four, page 20  
Place Value Board: a shaded piece of paper on the left and a white piece of paper on the right.  
Unifix cubes



### Lesson

Have the student build the first problem on the worksheet with Unifix cubes.

**"Today you are going to be working with cuckoos (base four). Counting in cuckoos, how would you read the board?"** "Two cuckoos and two."

**"Record that amount on the worksheet."**

$$\begin{array}{r} 2 \text{ cuckoos} + 2 \text{ units} = 22 \\ ( ) + ( ) = \_\_\_\_ \end{array}$$

**"Record 2 cuckoos and 2 units in expanded form."**

$$\begin{array}{r} 2 \text{ cuckoos} + 2 \text{ units} = 22 \\ (20) + (2) = 22 \end{array}$$

Have the student finish the worksheet independently. If the student has difficulty or needs to review counting in different bases, see *Patterns in Arithmetic: Book 2 - General Math*.

### Worksheets

Give Place Value: Three Ways to Record: Base Five, page 21, for the student to complete independently.

Place Value: Challenge, page 22

## Place Value: Three Ways to Record - Base Ten

### Purpose

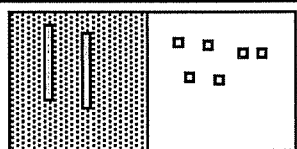
The purpose is to link place value and circle math and to test the student's understanding that the 'two' in twenty-six stands for 'twenty.'

### Materials

Place Value: Three Ways to Record - Base Ten - Worksheet 1, page 23  
Place Value Board  
Base Ten Blocks or Cuisenaire Rods

### Lesson

"Build twenty-six on the Place Value Board with Base Ten Blocks. Record on Place Value: Three Ways to Record - Base Ten - Worksheet 1."



\_\_\_\_\_ tens \_\_\_\_\_ ones = \_\_\_\_\_

○ + ○ = \_\_\_\_\_

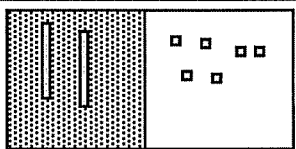
"How many tens do you have?" "Two."

"How many ones?" "Six."

"How many is that all together?"

"Twenty-six." "How do you write twenty-six?" "26."

Record that on this line:  
\_\_\_\_\_ tens and \_\_\_\_\_ ones = \_\_\_\_\_.



  2   tens   6   ones =  26 

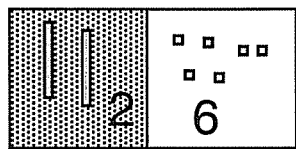
○ + ○ = \_\_\_\_\_

"Why do you write it 26 instead of 206?"

"Because the two really means twenty if it is in the tens place. 206 would be two hundred and six."

"Two tens equals how many blocks?" "Twenty."

"Record that number in the first circle. Record the ones in the second circle."



  2   tens   6   ones =  26 

○(20) + ○(6) =  26 

"Record the number of tens and ones on the workboard."

Build and record one more problem together if needed.

### Worksheets

Have the student finish the page independently. Place Value: Three Ways to Record - Base Ten - Worksheet 2, page 24, is for you to make additional problems if needed. Worksheet 3, page 25, has three digit problems. Continue practice with Place Value, pages 26 -29.

### Test for Understanding

Start with a number such as thirty-seven, break it up first into circles and then build it on the Place Value Board. Ask the value of each digit.

## Trading Game: Number Values

**Purpose** The purpose is to make a connection between language and math by posing a puzzle and having the student write, draw pictures, and write a number sentence of the solution.

**Prerequisites** Play the Trading Game many times in different bases and with different trading amounts. Be sure all students know how to make their own trades.

**Materials** Dice  
Colored blocks or other counters, six different colors  
Paper and pencil

**Lesson** Play one game of the trading game using six colors with three as the trading amount. Use two dice to help speed up the game.

The colors are: purple blue green yellow orange red



For a large group, play a mock game with only one player rolling the dice. Ask different students to explain how to make the trades.

The game can be stopped after a few students have traded up to green. Point to the red and ask, "**How much is it worth?**" "One." Record this for all to see. Have each student record independently.

**"If the red is one, how many red blocks is the orange worth?"**

"Three." Again record this.

**"The yellow is worth...?"** This may take some thinking and time to figure out. "Nine." Another way of phrasing it is, "**How many reds does it take to make a yellow?**" The students may build a model if they need to.

The number pattern so far is 1, 3, 9.

The rest of the lesson will be done independently or in small cooperative groups. **"Write the answer to the following questions and explain how you got the answer. Write as if you were going to tell someone else how you figured out the answer. Draw pictures and write a number sentence."**

**"How many red blocks do you need to get a green block? Continue the pattern. What is the next number in the series?"** "Twenty-seven."

1, 3, 9, \_\_\_\_

**Challenge** **"Continue the pattern. "1, 3, 9, 27, 81, 243."**

**"How many reds do you need to make a purple?"** "Two hundred forty-three red."

**"How many reds do you need to make a blue?"** "Eighty-one red."

**"How many greens do you need to make a purple?"** "Nine green."

## Rounding Off

### Rounding Off Tens

**Purpose** The purpose is to induce the rules for rounding off numbers to the nearest ten or hundreds place.

**Prerequisites** What's My Rule?

**Lesson** Play What's My Rule? with an operation rounding off to the nearest ten. **"What number would you like to put in? For this game limit the number to two digits."** "Seventy-six." **"If the 'in' number is seventy-six, the 'out' number is eighty. What other number would you like to try?"** "Seventy-two."

Record the numbers and the rounded off numbers on a piece of paper or the chalkboard.

**"If I put in eighty-nine, predict what the out number will be."** "Ninety."

After the students can correctly predict what the out number will be, ask, **"What's the rule?"**

IN	OUT
76	80
72	70
35	40
17	20
43	40
50	50
86	90

1) If the ones digit is less than five, the tens digit remains the same and the ones digit changes to zero.

2) If the ones digit is five or more, add one to the tens digit, and change the ones digit to zero.

**"Mathematicians call this 'Rounding off' a number to the nearest ten."**

**"When have you done this, seen it, or heard it?"**

"About three hundred kids were there."

"It costs around twenty dollars."

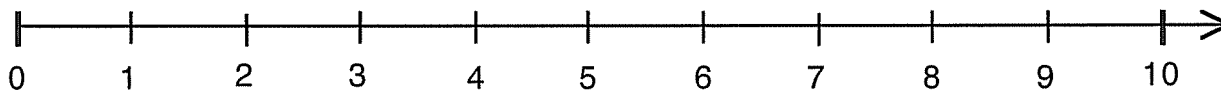
"I have to get up around seven in the morning."

**"Does this work with a three digit number? Try it."**

### Worksheets

Rounding Off: Tens - Worksheet 1, page 31

Draw a number line on a piece of paper from 0 to 10.



**"Put your finger on the three. Is your finger closer to zero or to ten?"** "The zero."

**"Put your finger on the eight. Is your finger closer to zero or to ten?"** "The ten."

**"Going to the closest number with a zero in it is called rounding off."**

Have the student finish the worksheet on her own.

Rounding Off: Tens - Worksheet 2, page 32, continues with rounding off using Base Ten Blocks.

Rounding Off: Tens - Worksheet 3, page 33, continues with rounding off using a meter stick for a number line.

Rounding Off: Tens - Worksheets 4 and 5, pages 34 and 35. Rounding off is used to estimate answers with addition. This helps students to tell whether an answer makes sense or not.



## Rounding Off Hundreds

Play What's My Rule? with an operation rounding off to the nearest hundred. **"Let's make the 'in' number over one hundred but less than one thousand."**

"Three hundred seventy-six." **"Four hundred."**  
 "Two hundred seventy-three." **"Three hundred."**

After the students can correctly predict what the out number will be, ask, **"What's the rule?"**

1) Find the digit in the hundreds place. If the digit to the right of it is less than five, do not change the hundreds digit but change every digit to the right of it to zero.

2) If the digit is five or greater, add one to the hundreds digit and change all the digits to the right of it to zero.

**"Mathematicians call this operation 'rounding off.' Now that we have rounded off to the nearest ten and the nearest hundred, would anyone like to try rounding off for a thousand or million?"** Encourage further generalizing of the rule.

IN	OUT
376	400
273	300
449	400
617	600
589	600
138	100
161	200

## Worksheets

Rounding Off: Hundreds - Worksheets 1 and 2, pages 37 and 38

## Extensions

1) Collect rounded-off numbers in a class collection or Math Journal.

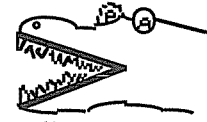
2) Play What's My Rule? with monetary amounts.

IN	OUT
\$14.50	\$15.00
\$29.99	\$30.00
\$ 6.39	\$ 6.00

3) Discuss when accuracy and precision is necessary and when it is not. How are 'in the right ball park,' the range will be between two hundred and three hundred, and rounding off similar? How are they different?

4) Discuss the range of numbers that could be rounded off to forty: 35, 36, 37, 38, 39, 40, 41, 42, 43, 44. Discuss the range of numbers that could be rounded off to four hundred: 350, 351, 352... 449. Discuss the range of numbers that could be rounded off to four thousand.

## Greater Than, Less Than, or Equal To



**Purpose** The purpose is to present signs and concepts of equality and inequality.

**Prerequisites** Times Tables are helpful but not mandatory.

**Materials** Greater Than, Less Than, or Equal To - Worksheets 1 - 5, pages 39 - 43  
Any counter

**Lesson Session 1** Start by showing pictures or drawings of common traffic and warning signs like red lights, green lights, stop signs, handicapped parking, no smoking symbols, etc. Ask the student what the signs mean. Discuss why people use symbols, like the circle with a diagonal line through it to give a warning, instead of words. They are quicker, take up less space, don't require a person to read, and transcend oral language differences.

**"What kinds of signs do you use when you write stories? What does this sign '?' mean? There are many others: the period . comma, ! " : ; etc."**

**"Write down any signs or symbols that are used in mathematics. There are several you know: + - x ÷ = \$. Explain what each sign means and write a number sentence or relationship that uses the signs."**

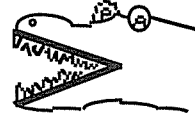
Write these on a piece of paper or the board.  $2 < 3$ ,  $0 < 4$ ,  $5 < 7$

**"Write a few more that would fit into this group."** The idea is for him to induce the meaning of the sign. Give him "Yes," "No" feedback on each relationship he writes. When he has written several correct ones, ask him to explain his pattern. He should say something such as, "The number in front of the sign is smaller than or less than the number after it."

Repeat the procedure with these examples.  $11 > 2$ ,  $4 > 0$ ,  $3 > 1$ .

This sign  $<$  means less than and this sign  $>$  means greater than. These symbols are easy to mix up. Making alligator mouths helps students remember. The alligator always eats the larger number, and the big end is always beside the larger number.

Greater Than:  $4 > 3$   
 $4 > 3$  Four is greater than 3.



"Yum! I want the bigger one!"

The big part  $>$  opens to the larger number.

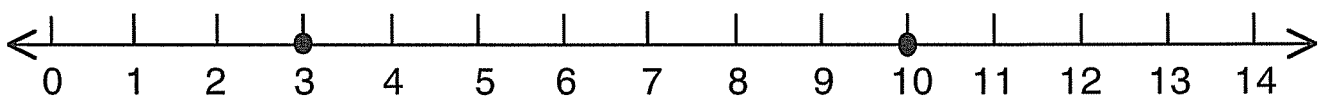
Less Than:  $5 < 7$



Alligator, alligator  
In a greedy fit.  
Always has to gobble up  
The greater digit!

$5 < 7$  Five is less than seven. The big opening still goes towards the larger number. Record these symbols in the student's Math Journal.

Draw a number line. Place a dot on two different numbers.



"Compare the two numbers, then put in the signs that make these number sentences true."

$$3 \bigcirc 10 \quad 10 \bigcirc 3$$
$$3 < 10 \quad 10 > 3$$

Have the student complete these sentences.

"If  $3 < \underline{\quad}$ , then three \_\_\_\_\_ on the number line."

"If  $3 < 10$ , then three is to the left of ten on the number line."

"If  $10 > \underline{\quad}$ , then ten \_\_\_\_\_ on the number line."

"If  $10 > 3$ , then ten is to the right of three on the number line."

## Worksheets

Greater Than, Less Than, or Equal To - Worksheets 1 and 2, pages 39 and 40

## Session 2

Go over Greater Than, Less Than, or Equal To - Worksheets 1 and 2 to make sure the student understands the use of the symbols. Then have the student write an easy addition problem such as  $4 + 3 = 7$ . Have him write a second one, for example,  $4 + 5 = 9$ .

"Is  $4 + 3$  equal to  $4 + 5$ ?" "No."

"If I write  $4 + 3$  \_\_\_\_\_  $4 + 5$ , what sign should I put in between them to make a true statement?" " $<$ ."

The sentence will look like this:  $4 + 3 < 4 + 5$ .

Another possible answer is this,  $4 + 3 \neq 4 + 5$ . This  $\neq$  means not equal to. If the student gives this response, accept it, and acknowledge his knowledge. Ask him to give another sign that would also make the sentence true.

Repeat with different numbers. Try different signs. Encourage creative answers. In a large group, have the students share their work. Here are some possible sentences.

$$2 \times 3 = 1 \times 6 \quad 2 + 3 + 4 < 20 - 10 \quad 20 \div 2 > 10 \div 2$$

## Test for Understanding

1. "What sign makes this a true sentence?" Can the student supply the correct sign and place appropriate numbers? Examples:  $4 + 9$  \_\_\_\_\_  $10 - 5$

2. "Fill in a number that will make this sentence true."

$$3 + \underline{\quad} < 2 + 9$$

$$3 + \underline{\quad} > 2 + 9$$

$$2 + \underline{\quad} < 14 - 1$$

## Worksheets

If the student does these problems with little difficulty, give Greater Than, Less Than, or Equal To: Worksheets 3 - 5, pages 41 - 43.

## Using Parentheses

**Purpose** The purpose is to introduce the parentheses as a tool for grouping numbers together to indicate order of combination. They are introduced with the language 'Do me first.'

**Prerequisites** Basic addition, subtraction, and multiplication

**Materials** Using Parentheses, page 44  
Paper and pencil

**Lesson** **"Write on a piece of paper  $1 + 2 \times 3 = ?$  and solve it."** Most will add the  $1 + 2$  to get three and then multiply the  $3 \times 3$  to get nine.  
Draw a pair of parentheses ( ) on the paper. **"These are called parentheses, and when numbers are written inside them, it means, 'Do me first.'"**

Then give an example.

Write the same problem two ways:  $(1 + 2) \times 3 = \underline{\quad}$  and  $1 + (2 \times 3) = \underline{\quad}$ .

**"Solve both problems. Will the answers be the same?"**

Many students are surprised that the answers are different.

**"Write three, one digit numbers on the paper. Put a + sign between two of the numbers and a x sign between the other two."** Example:  $5 + 6 \times 2 = \underline{\quad}$ . Have her put the parentheses in different places and find the answers.  $(5 + 6) \times 2 = 22$  and  $5 + (6 \times 2) = 17$ .

Then have her try it with multiplication and subtraction signs, such as  $4 \times 5 - 3 = \underline{\quad}$ .  
 $(4 \times 5) - 3 = 20 - 3 = 17$  and  $4 \times (5 - 3) = 4 \times 2 = 8$

Then have her try addition and subtraction together, such as  $(2 + 5) - 4 = \underline{\quad}$  and  $2 + (5 - 4) = \underline{\quad}$ . **"What happens? Do the answers change?"** "No."

Try putting the - before the +.  $(7 - 3) + 2 = \underline{\quad}$ , and  $7 - (3 + 2) = \underline{\quad}$

**"What happens this time?"** "The answers are different;  $(7 - 3) + 2 = 6$  and  $7 - (3 + 2) = 2$ ."

This lesson should be playful and short. Most students will catch on to using the parentheses quickly. After the student understands, give the worksheet.

### Worksheet

Using Parentheses, page 44

### Test for

**Understanding** Give a few problems with parentheses in them and see if the student does the operation inside the parentheses first. Give problems similar to the ones done in the lesson.

## Measurement: Nonstandard

### Capacity

**Prerequisites** Free exploration with rice or sand

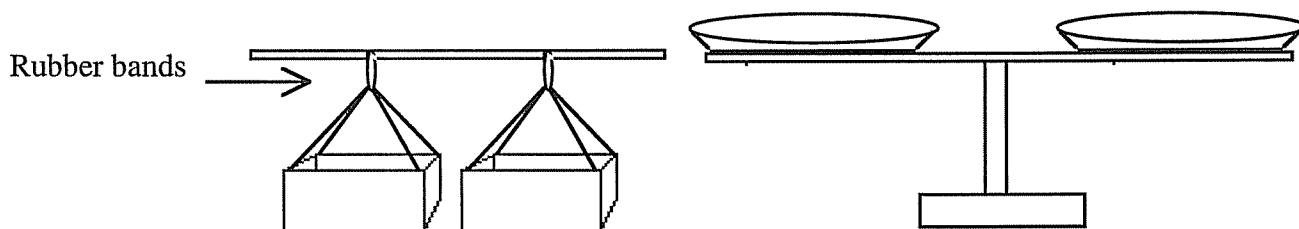
**Materials** A small cup or funnel with a small opening labeled 1  
Assortment of containers with different volumes labeled with letters starting with A.  
Label them out of order by volume.  
Rice in a sealed container set in a tub large enough to catch spills

Fill the funnel with rice, holding a finger over the end, and then let the rice flow into a container. Or use a cup and put the rice into containers. It is wise to have the student practice filling the funnel and letting the rice flow into containers before completing the following. Guess first, record the guess, and write the letter of the containers on a piece of paper.

- A) Find a container that holds two funnels (or cups) of rice.
  - B) Find a container that holds five funnels of rice.
  - C) Find a container that holds the same as one funnel of rice.
  - D) Find a container that holds less than one funnel of rice.
- Make your own problems.

### Weight

**Session 1** Have the student design and build a simple balance. Examples:



**Session 2** Have the student freely explore weighing a large assortment of small objects, such as pennies, balls, toys, stuffed animals, Unifix cubes, or pattern blocks.

**Materials** Student-made balance, weights such as pennies or paper clips, and a large assortment of small objects

Weigh and compare the different objects. Look for relationships of more, less, and the same (or equal). For example, seven Unifix cubes equal one stuffed animal. The stuffed kitty weighs more than the teddy bear. The giraffe weighs less than the ball. Have the student devise a way to record these relationships.

Have the student compare the weights of the objects using her hands,. Then have her order the objects from lightest to heaviest and verify the ordering with the balance. **"If you think that object one is heavier than object two, what would you expect to happen when you put them on the balance?"** "The side with object two on it will go up." Repeat this activity many times with several different objects.

### Session 3

Make nonstandard weights with paper clips, poker chips, large washers, and small Ziploc™ bags filled with rice. Make a chart of the weight relationships of the nonstandard weights. How many paper clips balance a poker chip? How many poker chips balance a block? How many blocks balance a small Ziploc bag of rice? How many bags of rice balance a little bottle of water?

Using these weights, have the student weigh several of the small objects used in the previous part. An empty salt shaker may weigh three blocks, a washer, and four paper clips. Have the student make the chart and record the weights of the objects on a chart.

Next, using different objects, have the student hold them in the hand and guess the weight in terms of blocks, washers, and paper clips. Record the estimates and then weigh the objects. Hopefully the estimates will get more accurate with practice.

### Length

#### Materials

Unifix cubes  
Paper clips  
Wooden building blocks  
Several objects to measure

#### Activities

A) Have the student use different length objects to measure things in the room. For example, the table is thirty Unifix cubes long and twenty Unifix cubes wide. The same table is nine blocks long and five blocks wide.

B) Mark how tall the student is on the wall with an erasable marker. **"How many paper clips tall do you think you are? Guess how many blocks and Unifix cubes also."**

C) Measure a room with your feet, one foot placed directly in front of the other. Compare the number of steps for all the students in the class or members in a family. Are the measurements all the same? Who has the correct amount? Discuss standard versus nonstandard measurement. For example, use the smallest and largest measurements in the class and calculate the cost of buying ten feet of red licorice.

#### Worksheets

Measurement: Nonstandard - Worksheets 1 and 2, pages 45 and 46

### Time

#### 1) Things I Do in a Day

#### Materials

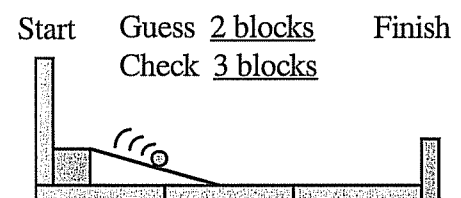
Sheets of paper cut into fourths  
Construction paper for covers cut into fourths  
Crayons or markers  
Stapler

#### Activities

A) Make a little book of eight things a student regularly does in a day: getting up, eating breakfast, going to school, etc. Order the things from first to last and staple the book together.

B) Expand the theme to Years of My Life.

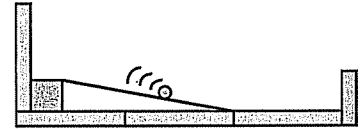
#### 2) Marble Races: Guess and check



**Materials** Marbles  
Blocks  
Stop watch or clock with a second hand

**Activities** A) Practice counting in seconds by saying alligator in between the numbers. "One, alligator, two, alligator." Watch the second hand on an analog clock, or a digital clock, or a stopwatch to see how close you can come to accurately counting by alligators.

B) Build a ramp that takes a marble one second to go from start to finish.  
Guess how many blocks long the ramp will need to be.  
Check after the ramp is tested.



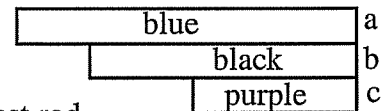
C) Build a ramp that takes a marble two seconds to roll from start to finish.  
Guess how many blocks long the ramp will need to be. Check after the ramp is tested.

**"What happens when a block for the ramp with a different angle is used?"**  
**"Will it take a longer or shorter time if the angle is less steep?"**

### Transitive Property

**Purpose** The purpose is to concretely introduce the Transitive Property, which states: If a relationship holds between A and B and between B and C, then it also exists between A and C. For example, if object A is larger than object B, and object B is larger than object C, then object A is larger than object C. In algebra it looks like this: if  $a > b$  and  $b > c$ , then  $a > c$ . Conversely, if  $b < a$  and  $c < b$ , then  $c < a$ .

**Activities** Build this model with Cuisenaire Rods.



Compare the relationship of the longest rod to the shortest rod.

**"If the blue rod is longer than the black rod, and the black rod is longer than the purple rod, then, the blue rod is \_\_\_\_\_ than the purple rod. Fill in the blank."** (longer)

Build another model. If the orange rod is longer than the brown rod, and the brown rod is longer than the yellow rod, then, the orange rod is \_\_\_\_\_ than the yellow rod. (longer) Have the student build several models and compare the relationships.

Reverse the process and compare the shortest rod to the longest rod. If the purple rod is shorter than the black rod, and the black rod is shorter than the blue rod, then, the purple rod is \_\_\_\_\_ than the blue rod. (shorter) Have the student build several models and compare the relationships.

Substitute number values for the rods after the student is confident. For example, after building the model (shown above) the statement would be, "If nine is greater than seven, and seven is greater than four, then nine is greater than four." Build several models and record several statements. For example: If  $9 > 7$  and  $7 > 4$ , then  $9 > 4$ .

Next use letters. If the blue rod is 'a,' the black rod is 'b,' and the purple rod is 'c.'

**"Fill in the blanks."**  $\_\_ > \_\_$   $\_\_ > \_\_$   $\_\_ > \_\_$

**"Fill in the blanks for the converse relationship."**  $\_\_ < \_\_$   $\_\_ < \_\_$   $\_\_ < \_\_$

Build several models and compare the relationships.

## Area: Nonstandard

**Purpose** The purpose is to develop models for the concept of area and to investigate the relationship between the size of a unit and the number of units needed in a particular measurement.

**Prerequisites** None

**Materials** Area: Nonstandard - Worksheets 1 and 2, pages 47 and 48  
Split peas, pinto beans, and lima beans  
Small containers for the beans, e.g., yogurt containers or paper cups  
(Substitutions for beans: tiny sticky dots for split peas and punched-out holes made from Post-it® Notes for lima beans)

**Lesson Session 1** Give the student a container full of pinto beans and Area: Nonstandard - Worksheet 1. Have him take a handful of beans to see how large the beans are. **"About how many pinto beans do you think it will take to cover Figure 1? You may guess a range, for example, between ten and fifteen beans. Record the estimate on the line labeled 'My guess between \_\_ and \_\_' under the figure. Place the beans on the figure, squeezing on as many as possible. There should be no beans outside the line. Count the beans and record the number on the line marked 'Actual number.' "** If the guess is within the range, he should circle 'within range' on the worksheet. If the guess was higher than the range, circle 'too high' on the worksheet. If the guess was lower than the range, he should circle 'too low.' Repeat this procedure for all the figures on Area: Nonstandard - Worksheets 1 and 2, pages 47 and 48. Watch to see if the estimates improve. Notice what size ranges a student picks.

**"Before doing Figures 7 and 8, guess which figure will take more beans to cover it. Why did you choose this figure?"** Many students will guess Figure 8 because it is taller and gives the impression of a larger size. Actually, Figure 7 is slightly larger. After the student tests the figures and sees that Figure 7 is larger, have a discussion about packages in stores. Many people think taller implies bigger. Bottles with long slender necks are a good example of a way that manufacturers try to get you to think that the bottle of X is bigger than brand Y's bottle. Have your student look for examples of this in stores and figure out how to check labels to find and compare the actual sizes.

**"What did you notice as you worked these problems? Did you get a closer estimation? How did your ranges change?"**

**Note** Some students equate being off on their estimates with being wrong. They think they must have an exact number or they are not doing well. They will either change the guess to match the actual number, pile the estimated number of beans into the figure, or accidentally on purpose miscount the beans.

One reason for this behavior is that the student's experience with numbers is usually one of precision. They are accustomed to one exact answer, not estimated answers that deal with a probable range of numbers. Learning to estimate probable ranges



for answers is a very important mathematical skill. It helps one to spot grossly incorrect or improbable answers and helps develop the idea of reasonableness of answers. Look for ranges in daily life, such as, "I'll be home between 5:00 and 5:30."

Encourage students to relax and to play with this activity. Do the worksheet beside them. Let them see by your response that being off is both OK and a useful tool for refining the estimates. Model how to use the 'within range, too high, and too low' to improve your guess on the next figure. Also let them know that almost no one, not even grown-ups, will get the estimate exactly right time and time again. The probability of being able to guess the actual number precisely more than once is almost zero. The larger the figure, the higher the difference will tend to be. Students can discover this by comparing the ranges for larger and smaller figures.

### **Challenge**

Area: Nonstandard - Worksheet 3, page 49, is an optional Challenge Problem. It requires tries or drafts of success.

### **Session 2**

#### **Materials**

Area: Nonstandard - Worksheets 4 - 6, pages 50 - 52  
Pinto beans, lima beans, and split peas

#### **Note**

The concept for Area: Nonstandard - Worksheets 4 and 5 is that it takes many small units to equal a few large units. The larger the unit we measure by, the fewer of them we will need. For example, five hundred pennies is equal to five dollars and twelve inches is equal to one foot. Experienced students (and adults) have a sense of this in the real world, but this knowledge does not automatically transfer to formal measurements. Developing the connection between units and numbers will help students in place value, fractions, decimals, and real world math problems. This activity uses different sized beans to measure area. The student will experience how many more smaller beans than larger beans are needed to cover a figure.

On Area: Nonstandard - Worksheet 6, page 52, the student is asked to make a chart of the number of beans it took to cover each figure with the different sizes of beans. The patterns to notice are that the number of split peas used is always more than the number of pinto or lima beans used. The number of lima beans is always less than the number of pinto beans or split peas. Students might notice that the number of pinto beans used on a given figure is roughly twice that of lima beans and that the number of split peas used is roughly three times that of the pinto beans.

The student then makes a line graph of the results on the chart in Area: Nonstandard - Worksheet 6. The student can use this chart or make up one of his own.

## Reflections, Categories, and Continuums

**Purpose** The purpose is to ascertain what a student remembers about a subject, to pool group knowledge and to provide an idea and word bank for sorting and classifying.

**Materials** Paper and pencil  
Reflections, Categories, and Continuums, pages 53 and 54 (Give after Session 2.)

**Session 1** Pick a subject, for example, time, the sea, measurement tools, or symbols. **"Write down everything you can think of related to this subject in three minutes."**

**"Who would like to share a list? Who else would like to add something?"**

**"Now, please group your words into categories. What might be the name of a category?"** After the students sort their words, they might share it with a neighbor.

**Session 2** **"What is the opposite of hot?"** "Cold." "Big?" "Little." "Young?" "Old."

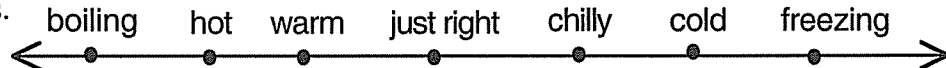
**"Let us call hot and cold two points on a line."**



**"Hot and cold describe what?"** "Temperature." **"What is another word describing temperature?"** "Warm."

**"We'll fill in more points on the line using words describing temperature. Is warm closer to hot or cold?"** "Hot."

**"Should we place it to the right or to the left of hot? Why?"** Continue placing all ideas.



**"We call this a temperature continuum. Each part continues into the next."**

To make a continuum, draw a line with an arrow at each end to show the line continues. Decide what to arrange in order, such as units of time, length, or weights. Label the continuum subject and place as many units on it as you can in the order of the smallest to the largest, hottest to coldest, or whatever works for your subject.

Compare your continuum to others and see if you can think of more things to add to it.

**Worksheets** Reflections, Categories, and Continuums, pages 53 and 54

**Extension** Use the categories from the reflections for continuum ideas. This is a very useful assessment tool when you want to see what a student remembers about a subject. Place it in the Math Journal.

## Measurement: Standard

<b>Purpose</b>	The purpose is to measure using standard tools and units and to derive the relationship among standard units and chart them to improve estimates.
<b>Prerequisites</b>	Nonstandard measurement and free exploration with the tools
<b>Materials</b>	Measurement: Volume, page 55 Containers that hold a cup, a pint, a quart, a half gallon, and a gallon Water, sand, or rice
<b>Note</b>	Measurement should always be taught manipulatively with real tools. As a general rule, students should estimate the weight, length, or capacity of the things they are measuring. It makes the measuring more fun and will develop their ability to compute measures in their head fairly accurately. Graphs, charts, or pictures showing the relationships between units are meaningless to students at this level unless students make their own charts to record the results of their own measurements. The chart should include a column for guesses as well as actual measurements. Making their own charts now will help students understand other people's charts later. Measuring with rulers, weighing and pouring are activities students love to do.

## Capacity or Volume

<b>Activity</b>	Using water or sand, have the student investigate the size relationships of the containers. How many cups are in a pint? How many pints are in a quart? How many quarts are in a half gallon? How many half gallons are in one gallon? Record the results on the worksheet Measurement: Standard - Volume.
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Then, have the student use logic to figure out other relationships. For example, if you know there are two cups in a pint and two pints in a quart, then how many cups would be in a quart? Have the student guess first, then try to test it out logically. Some students will need to draw pictures. Some students, especially students under seven, may not be able to answer at all and will need to use the containers to get an answer. Have these students pour the one cup measure into the pint measure, then empty the pint into the quart. **"How many more cups do you think will be needed to fill the quart?"** You will need to keep the containers around and ask relationship questions regularly before they remember them. All students should check their answers with the containers. (Chart the relationships.)

One fun way to make students more aware of capacity measures is to ask the sizes that common products come in. **"What things in the store are sold in gallon containers? In quarts? Pints? Why is whipping cream sold in pints but not in gallons? Why are some things sold by container size and other things by weight? Why don't you buy a quart of Cheerios?"**

Introduce the liter container also, since many drinks are sold in liter containers.

## Weight

<b>Materials</b>	A student-made or commercial balance, standard weights, and a large assortment of small objects
<b>Activity</b>	Students love to weigh things. Standard weights of pounds and ounces can be obtained at the supermarket. Dry beans are sold in one-pound units and make good weights. (The packages are not very sturdy. You can rebag them in Ziploc bags to prevent a mess.) Water can be used to make ounce measures. 1 cup = 8 oz. 1/8 cup of water = 1 oz. Pour 1/8 cup of water into a container on one pan of your balance.

Place a plastic bag on the other. Put rice or beans into the bag until it balances with the water, then seal the bag. Paper clips strung together can also be used to make approximate ounce weights.

Finally, repeat Session 3 in Nonstandard Measurement with standard weights. Use metric and imperial measures if possible. Do not have the student calculate conversions at this point.

## Length

**Materials** Rulers (imperial and metric)  
Tape measures  
Paper and pencils

**Activity** Simply have the student estimate, measure, and chart everything in her environment.

Have her use a yard stick to give measurements ,in inches, feet, and yards.  
Have her use a meter stick to give measurements in centimeters and meters.

A fun activity for parents to do in the car is to have the student estimate a mile. The parent watches the odometer and tells the student when to start reckoning. The student says, "Now!" when she thinks the car has traveled one mile. The parent states how far she actually drove and computes how close she estimated.

## Activities Time

- 1) Teach how to tell time.
- 2) Have the student make charts of the times she regularly does things in her life: getting up, eating breakfast, doing school things, etc.
- 3) Estimate units of time. Have the student close her eyes and raise her hand when she thinks ten seconds have passed. Vary the units of time up to one minute.
- 4) Keep records of the number of things that can be done in one minute, like distance run or walked, jumps in a minute, etc. **"Can you make your bed, or get dressed in one minute?"**
- 5) Relative time activity for large groups. **"Is one second a long or short time?"** Most will say it is very short. Have the students sit in a circle. A leader starts the game by clapping his or her hands once. Then the person sitting to the right of the leader claps. Then the next person claps and so on around the circle. How many seconds does it take to go around the circle? Do it several times trying to reduce the time spent. Then divide the total number of seconds taken by the group on their fastest time by the number of people in the group. You will find that each person took only a fraction of a second to respond. In this game, a second is a long time. **"How many people can respond and clap in one second?"**

**Resources** *This Book Is About Time* by Marilyn Burns

## Area: Standard

**Purpose** The purpose is to develop models for the concept of standard area and to investigate the relationship between the size of a unit and the number of units needed in a particular measurement.

**Prerequisites** Area: Nonstandard

**Materials** Area: Standard - Worksheets 1 and 2, pages 56 and 57  
Centimeter tiles or graph paper

**Note** In this lesson the term 'area' is introduced. A region is a closed, defined space. Some mathematics textbooks define 'area' as the amount of space contained within that region. Many books do not define 'area' at all but simply tell the student how to calculate it. Please do not teach your student the formula for calculating area at this time. There are three reasons for this. The first is that a student needs to develop a concrete sense of what area is and to develop ways to express a measure of it. Square inches is only one of many ways to measure an area. The second reason is that it might be implied from the formula that area is a concept applying only to rectangular surfaces. Third, the most powerful reason for not showing it at this time is that with a little more experience in concepts of area and multiplication, a student will discover the formula for himself. Discovery feels good and gives power to the student's sense of himself as a thinker and mathematician. The discovery of the formula ensures his understanding of it.

### Lesson

#### Session 1

In this session, the student guesses and measures how many tiles will fit in the figures. Area: Standard - Worksheet 1 is in centimeters and Worksheet 2 is in inches. The student may figure out his own way to compute the area.

#### Session 2

In this session, the student will find units to measure the area of various surfaces in the room. **"How many pinto beans do you think it would take to measure the area of the tabletop you are working on?"** A groan will be a common response. **"Why would using pinto beans not be a great way to do it?"** "They are too small. It would take forever." **"What unit of measure other than a pinto bean would make finding the area of the tabletop easier?"** Common solutions are building blocks, hands, books, notebook papers, or newspaper sheets, etc. (One group of students measured the entire floor area of the classroom with bath towels.) Have the students estimate a range of how many units of whatever the measuring tool is that it will take to cover the table. Finally, measure and record it. Then have them make a list of ten or so items in the room that have an area that could be measured. Common items are hallways, doors, floors, tops of tables, desks, counters, and shelves. If there are several students working in a group, have each student measure the area of these ten items using different units of measure.

Make a chart of the results and discuss them in terms of size of the units and the number of them it took to measure the same areas. Note that a ruler or a linear measurement tool is not used to cover a two-dimensional surface. Notice any of the

more, the more type relationships. **"What do you think a good unit of measure would be if you wanted to measure the area of a soccer or football field, or even a city?"** "Yards or miles or blocks in a city."

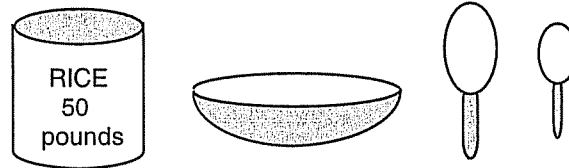
## Test for Understanding

1. Draw a small figure, no more than two square inches in area, and ask the student to estimate the number of pinto beans that would fit inside. The student's estimate should be fairly close. A gross error in estimation indicates that the student is having difficulty relating the size of the bean to the size of the figures or that he was unable to generalize from previous experience. More practice estimating the size of things in the world in general would be helpful.

2. Ask the student to explain what the word "area" means. Many will explain with hands as much as with words. He will often place his hands on the table and slide them over the surface, trying to find words for what his hands know. Another way to explain is to rub the tips of his fingers around the inside of a figure. If he has difficulty explaining in words, ask, **"Can you show me which part of this figure is the area? What is not in the area?"**

3. This question is on volume. As in the larger the bean, the fewer the beans are needed to measure the area, this tests the student's understanding of the concept of the larger the unit of measure, the fewer of them needed.

Show this picture:



If you want to move some rice from the can to fill the bowl as quickly as possible, which scoop would you use? Why?

Answer. The larger scoop would require less time because it transfers more rice per scoop.

Challenge! Why might a small child be able to do the work faster with the small scoop?

Answer. The large scoop when full will be heavier than the smaller one. A small student might do the work faster with the small scoop because there would be less effort, strain and struggle with the weight. For a larger person, the difference in weight would not be a consideration.

Ask the student how the measurement of area relates to the student's world.

## Practice Worksheets

Give these worksheets as independent activities.

Geoboard Area, page 58

Tile Floors: Worksheets 1 - 6, pages 59 - 64

## Expansions

1. Draw a floor plan of your room. Use the whole (8 1/2 x 11) paper. Draw in the furniture, the door, and the closet. Imagine you are building a doll house or a fort

with your blocks and you want to build it as big as you can. You would use the whole room if you could, but your mom says that you must leave a walkway at least one foot wide from the door to the closet, dresser, and your bed. Draw the outside walls of the house or fort on your floor plan. Shade the area inside. Design the interior of a fort or doll house. Guess how many  $8\frac{1}{2} \times 11$  inch pieces of paper would cover the area of your building.

2. Do you like to sweep or vacuum floors? If you could choose a room in your house to sweep or vacuum, which one would you choose? Why would you choose it? The idea here is to get the student to connect the amount of work or time it takes to sweep a room with the amount of floor space, area, the room has.

3. The following questions relate to the size of measuring units and the number of units used, the concept worked on in Session 2.

A. Eating: Get out a bowl of your favorite dry cereal and an extra bowl. You could eat it with a teaspoon or a tablespoon. **"Which spoon would you prefer and why? Which spoon will give you the most bites? Which spoon should you use if you are in a hurry? How many more teaspoon bites will there be than tablespoon bites? Guess and measure. Measure the number of spoonfuls of cereal by dipping into the full bowl and dumping into the empty bowl, counting as you go."**

B. Walking: A man walks out to the street to pick up the newspaper. His small son follows him. **"Will they take the same number of steps? Who will take more? How many more do you think?"** Pace off a certain distance counting your steps. Guess how many steps an adult will take pacing off the same distance. Then have an adult pace it off. **"Why does the adult take fewer steps?"**

4. Play "Mother May I?" with giant steps, baby steps, scissor steps, sideways steps, hopping, tippy-toes, and heel-toe steps.

5. Collect many other examples of "the bigger, the less" or "the more, the less." For example: the faster you run, the less time it takes to get there. Role play or mime a life situation where the audience guesses: The more, the more: the less, the less, etc., relationships.

6. Maps and floor plans. Have the students draw maps of imaginary places and real places. (The maps do not need to be to scale. That is too difficult a task for most students at this level.) This activity can be integrated with geography lessons on various land forms such as islands, peninsulas, bays, deltas, and so on. Some possible maps:

A) Draw an imaginary kingdom that has seven different land forms in it.

B) Hide a "treasure" and draw a map to it. Have the students make their own treasure maps.

C) Draw a floor plan of an imaginary palace, dungeon, maze, or spaceship.

This activity plants ideas that lead to the understanding of the concepts of orientation, scale, proportion, and dimension. Maps of imaginary places can also spark or add to the writing of stories. The famous epic fantasies *The Hobbit* and *The Lord of the Rings* by Tolkien, and many others, are illustrated with lovely maps. The story *Watership Down* has good maps of an entirely different scale.

## Perimeter of Rectangles

**Purpose** The purpose is to help students understand that perimeter is a measurable attribute around the *RIM* of a shape and is measured in linear units such as inches or centimeters. Do not teach this lesson at the same time as area is taught. Otherwise, students tend to get the two concepts confused.

**Prerequisites** Free exploration with Geoboards

**Materials** Geoboards and rubber bands  
Permanent markers  
Cotton kite string To prepare, tie a string longer than the perimeter of a Geoboard onto the upper left-hand nail of each Geoboard.

This activity may seem like a lot of unneeded work, since students can find perimeter by counting the spaces between the nails of a rectangle outlined by the rubber band. But this activity is important because it sets the visual image of a linear measurement tool (in this case, the marked string) used to measure the outside edge of a shape. This image will help prevent confusion between calculating area and perimeter later. Perimeter can be measured with a string or ruler having one dimension of length, whereas area must be measured with tiles having two dimensions of length and width.

**Lesson** **"We are going to make a Geoboard ruler for finding the perimeter of shapes on the Geoboard. Work with a partner and mark your string in units the same length apart as the pegs on your Geoboard."** One way is to have one student stretch the string around the outside edge nails on the Geoboard. Have the other student use a marker to place a dark mark on the string where each nail is located. This will create a nonstandard ruler of Geoboard units. **"What is the distance around the entire Geoboard? Pull the ruler out to the side to count it."**

Now make a 2 x 4 rubber band rectangle that has a corner where your string is tied. Use your string ruler to follow the rubber band and find the distance around (or the perimeter of) the rectangle. Pinch the string at the point it touches the place you began and pull it off the rectangle to stretch it out straight. Count off the length measurement and record it. Make a different size rectangle. Measure the perimeter of it.

	Perimeter
Geoboard	_____
2 x 4 rectangle	_____
__ x __ rectangle	_____

Again, this may seem too simple. But, the visual image of the stretched-out string will pay big dividends later. Students will discover that the easiest way to figure out perimeter is to count the sections along the edge of the shape. Discovery fuels further investigations. Rejoice with them.



## Averages

<b>Purpose</b>	The purpose is to manipulatively teach the concept of averaging.
<b>Prerequisites</b>	None
<b>Materials</b>	Averages, Worksheets 1 - 3, pages 65 - 67 Unifix cubes
<b>Warm Up</b>	Have the student freely explore with the Unifix cubes for several minutes or so before beginning the lesson.
<b>Lesson</b>	<p>Have her pick out of the Unifix cube box seven black cubes, six white cubes, three yellow cubes, two red cubes, and two blue cubes. Separate them into separate groups by color.</p> <p>Make up a story about the cubes that goes something like this.</p> <p>Once upon a time there was a village of people who kept their food in colored boxes. Each neighborhood had food boxes of a different color. The neighborhood with black boxes had seven boxes. The one with white boxes had six boxes, the one with yellow had only three, and the ones with red or blue had only two each. In each neighborhood the same number of people lived. Some people had lots of food and some were starving. The leaders of the town decided this was not fair. They decided that everyone should come to the town square with their boxes. The leaders told the neighborhoods that every group should have the same number of boxes. There would still be five neighborhoods, but each one would have an equal number of food boxes.</p> <p>So the people came to the town square—some happy, some grumbling—and stacked their boxes in piles.</p> <p>Have her snap together all the black cubes into a stack. Do this with all the other colors as well.</p> <p>Now, fix the stacks so that each stack is the same, no matter what colors are in each stack. The people who had lots of boxes thought this process was unfair, but they did as they were asked to do.</p> <p><b>“How many food boxes did each neighborhood end up with?” “Five.”</b></p>
<b>Worksheets</b>	Averages - Worksheets 1 - 3, pages 65 - 67

## Patterns in a Counting Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

### Purpose

To identify and describe patterns and to introduce the sum of digits.

### Materials

Patterns in a Counting Chart, pages 68 and 69  
Markers or crayons

### Lesson

Have the students make a chart numbered one to one hundred in sequence.

**"Look for number patterns in the rows, columns, and diagonals."** Share them out loud. Hearing patterns will speed up finding more.

Some possible patterns are:

1) In the first column, all the numbers end with one; in the second, they all end in two, etc.

2) The first numerals of the numbers in all the columns starting in the second row are 1,2,3,4,5,6,7,8,9.

3) Moving diagonally from one to one hundred, the numbers go 1, 12 (1, 2), 23 (2, 3), 34 (3, 4), 45 (4, 5), 56 (5, 6) etc. The last digit of the previous number is the first digit of the next number.

4) All the double numbers 11, 22, 33, etc., are on a diagonal.

Introduce the idea of sums of digits. For the number thirty-four, for example, the sum of the digits is seven or  $3 + 4 = 7$ . For the number seventy-nine,  $7 + 9 = 16$ . Since sixteen is still a two digit number,  $1 + 6 = 7$ . The answer to a sum of digits is always added until only one digit remains. Now look for sums of digits patterns.

**"Find all the numbers whose digits add up to seven. Circle those numbers with a yellow crayon. What pattern do you see? Will the pattern be the same for the other sums?"**

Find all the numbers whose digits add up to six. Circle them in green. Repeat this for numbers whose digits add up to two (circle in black), three (circle in brown), four (circle in purple), five (circle in blue), eight (circle in orange), nine (circle in red), and one (circle in pink). What do you notice?

**"Add the digits of the numbers on the diagonal from one to one hundred."** Ask, **"What pattern do you see?"** "The sequence of odd numbers." Do the same for the numbers two to ninety on the diagonal. **"What pattern do you see?"** "The sequence of even numbers." Repeat from three to eighty and four to seventy. **"What general pattern appears?"** "If you start with an even number, the sequence of even numbers appears. If you start with an odd number, the sequence of odd numbers appears."

### Challenge

What patterns appear if you add the digits of numbers down rows or columns? Explain why the patterns are the way they are. How would the pattern change if the chart started at zero?

### Extensions

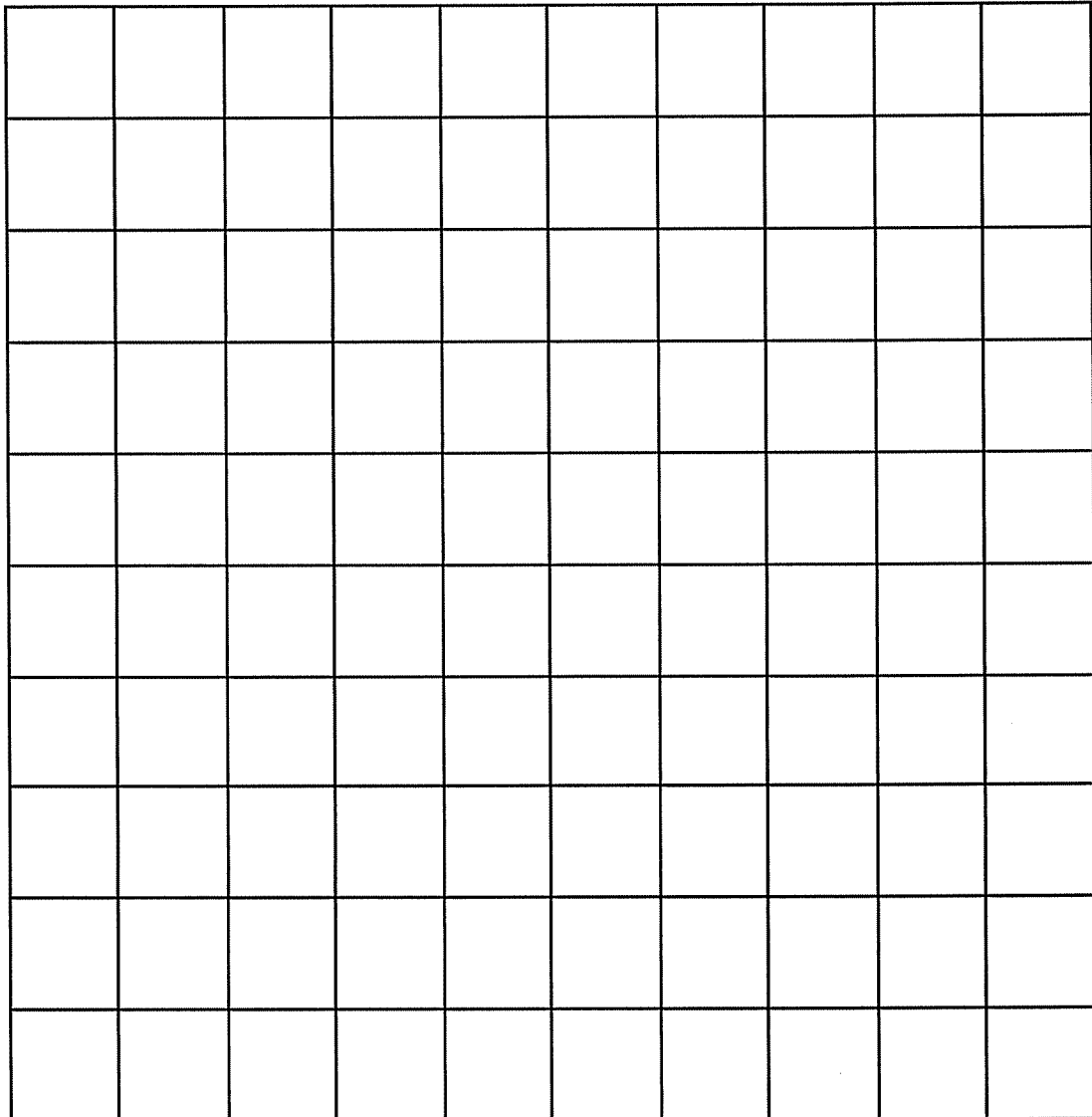
*Color It on the Hundred Chart* - Books 1 and 2 by Marcy Cook  
*Those Amazing Tables* by Joe Lieberman

# Patterns in a Counting Chart

Date \_\_\_\_\_

Make multiple copies.

10 x 10 Grid

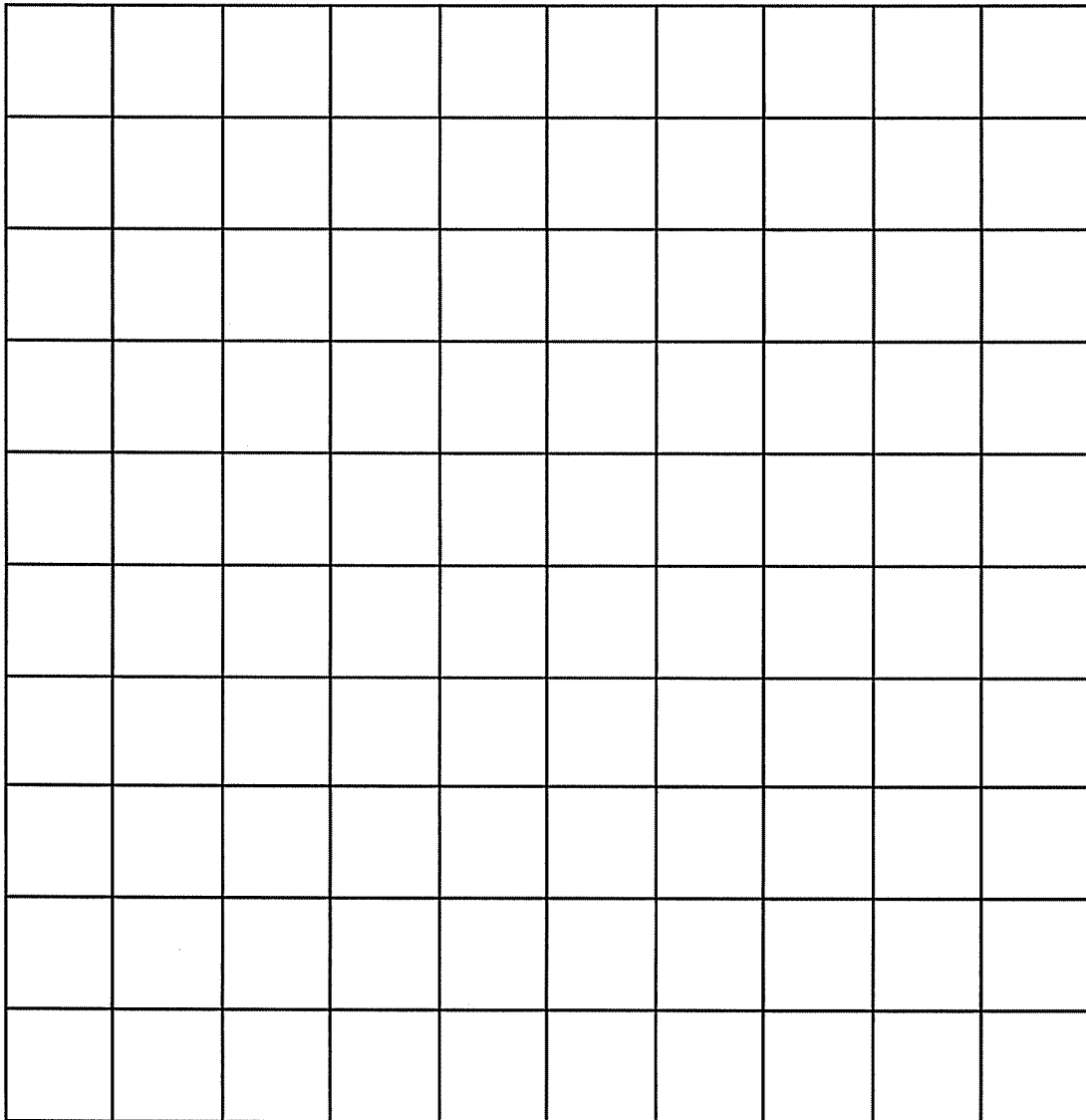


# Patterns in a Counting Chart

Date \_\_\_\_\_

Make multiple copies.

10 x 10 Grid



**Patterns in Arithmetic**  
**General Math: Booklet 3**  
**Place Value, Measurement, and Geometry**

**Answer Key**  
**for the**  
**Student Workbook**

**By Suki Glenn and Susan Carpenter**

# Answer Key Legend

AWV = answer(s) will vary

BUWV = break up will vary

OWV = order will vary

## Pattern Blocks

r = red trapezoid

g = green triangle

y = yellow hexagon

o = orange square

b = blue parallelogram

t = tan rhombus

## Cuisenaire Rods

1 w = white

2 r = red

3 lg = light green

4 p = purple

5 y = yellow

6 dg = dark green

7 bk = black

8 bn = brown

9 bl = blue

10 o = orange

Note: Some items and pages are left out of the answer key.

- 1) Some pages in which the answers are open-ended or will vary.
- 2) Make your own problems. Since students create their own problems and solutions, these sections give valuable information about the level of confidence and competence. It can be a useful source of curriculum for other students.
- 3) Practice pages.
- 4) Workboards.
- 5) The answers are in the Parent/Teacher Guide.

Patterns in Arithmetic: General Math - Book 3  
Student Workbook Answer Key  
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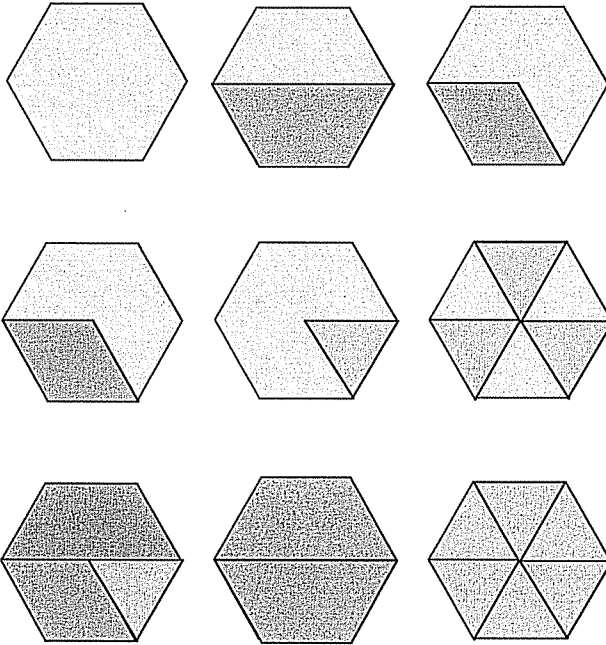


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# General Math - Booklet 3

## Patterns: Geometry

Build each of these hexagon shapes differently. \_\_\_\_\_



The most blocks 6.  
 The fewest blocks 1.  
 The smaller the blocks, the more it takes.

1

## Number Patterns - Worksheet 1

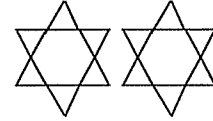
### Stars

Build with pattern blocks.

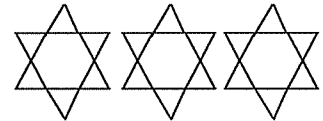
One star has 6 points.



Two stars have 12 points.



Three stars have 18 points.



How many points will 10 stars have? 60

Guess first \_\_\_\_\_

How do you know?

What's the rule?

Six times the  
number of stars  
equals the  
number of points.  
Star x 6 = points

Stars	Points
1	<u>6</u>
2	<u>12</u>
3	<u>18</u>
4	<u>24</u>
5	<u>30</u>
6	<u>36</u>
7	<u>42</u>
8	<u>48</u>
9	<u>54</u>
10	<u>60</u>

Challenge: How many points will 20 stars have? 120

3

## Number Patterns - Worksheet 2

### Windows

Build with pattern blocks.

A window one pane wide takes one square.



A window two panes wide takes four squares.



A window three panes wide takes 9 squares.



How many panes in the 10th window? 100

Guess first \_\_\_\_\_

How do you know?

What's the rule?

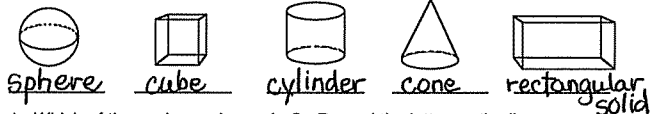
Width x width  
equals the squares  
Multiply each  
number by itself.  
Add the width as  
many times as  
many times as the  
width.  
w x w = squares

Width	Squares
1	<u>1</u>
2	<u>4</u>
3	<u>9</u>
4	<u>16</u>
5	<u>25</u>
6	<u>36</u>
7	<u>49</u>
8	<u>64</u>
9	<u>81</u>
10	<u>100</u>

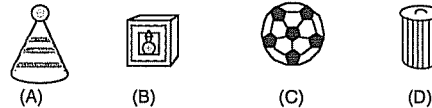
4

## Geometric Solids

With a set of geometric solids, find the sphere, rectangular solid, cube, cone, and cylinder. Label these shapes.

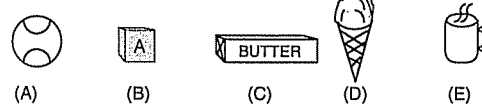


1. Which of these shapes is a cube? Record the letter on the line.



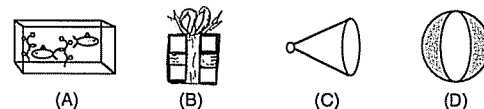
B

2. Which of these shapes is a cylinder?



E

3. Which of these shapes is a cone?



C

Find each of these shapes, write its name on the line, and draw it.

sphere baseball, soccer ball, beach ball

cube block, jack in the box, present

cone party hat, ice cream cone, megaphone

cylinder trash can, cup

rectangular solid fish tank, butter

5

## Lines, Line Segments, Rays, and Angles Worksheet 1

This is a picture of a line. A line never stops and the arrows at the ends mean that the line goes on forever

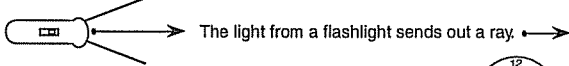


Part of this line is marked off by the end points B and C. The part of the line between B and C is called a line segment

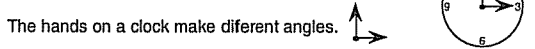


- |   |   |
|---|---|
| 1. The name of this figure is a <u>line</u> | 2. The name of this figure is a <u>line segment</u> . |
|   |   |
| 3. Draw and label line segment CD.          | 4. Draw a line through point F                        |

A ray starts at a point and goes out from the point.



An angle is made up of two rays starting from the same point.



- |                                  |                                |
|----------------------------------|--------------------------------|
| 5. Name this figure <u>angle</u> | 6. Name this figure <u>ray</u> |
|                                  |                                |

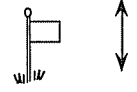
- |                |                   |                                    |
|----------------|-------------------|------------------------------------|
| 7. Draw a ray. | 8. Draw an angle. | 9. Which angle is larger (A) or B? |
|                |                   |                                    |

10. Write the name of each figure.

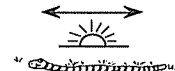
- |                  |                 |                         |                |
|------------------|-----------------|-------------------------|----------------|
| <br><u>angle</u> | <br><u>line</u> | <br><u>line segment</u> | <br><u>ray</u> |
|------------------|-----------------|-------------------------|----------------|

## Lines, Line Segments, Rays, and Angles Worksheet 2

A flag pole is an example of a vertical line. It goes up and down.



The sun going down over the horizon and a king snake lying in the grass are examples of horizontal lines.



A surfboard on a wave is an example of a diagonal line.



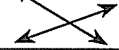
- |  |  |
|--|--|
| 1. This line is in what position?<br><u>diagonal</u>   | 2. This line is in what position?<br><u>vertical</u> |
|  |  |
| 3. This line is in what position?<br><u>horizontal</u> | 4. This line is in what position?<br><u>diagonal</u> |
|  |  |

- |  |                             |                               |
|--|-----------------------------|-------------------------------|
| 5. Draw a horizontal line.   | 6. Draw two vertical lines. | 7. Draw three diagonal lines. |
|  |                             |                               |
| 8. Find an example of each kind of line around you. Write the name and draw a picture of each. |                             |                               |

- |                 |                     |               |
|-----------------|---------------------|---------------|
| horizontal line | vertical line       | diagonal line |
|                 | <u>Example:</u><br> |               |

## Lines, Line Segments, Rays, and Angles Worksheet 3

Lines that meet at a point are intersecting lines.



Lines that never meet are called parallel lines.



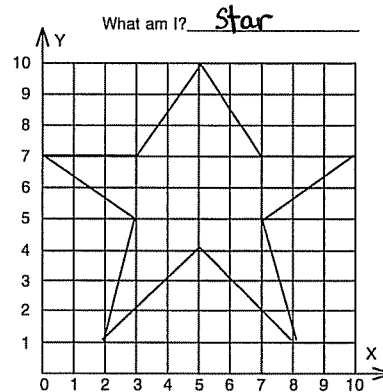
- |   |   |
|---|---|
| 1. What kind of lines? <u>C</u>                     | 2. What kind of lines? <u>B</u>                     |
|   |   |
| A. vertical B. intersecting<br>C. parallel D. angle | A. vertical B. intersecting<br>C. parallel D. angle |

- |                                     |   |
|-------------------------------------|---|
| 3. Draw a line parallel to line AB. | 4. Draw a line that intersects line CD. |
|                                     |   |

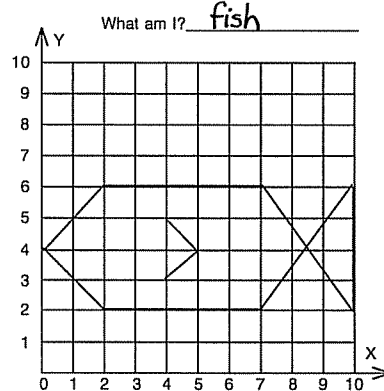
- |  |   |
|--|---|
| 5. Draw two lines parallel to line YZ. | 6. Draw two lines intersecting line EF. |
|  |   |

- |  |   |
|--|---|
| 7. Find and draw two examples parallel lines. <u>AWV</u> | 8. Find and draw two examples of intersecting lines. <u>AWV</u> |
|--|---|

## Coordinate Graphing - Worksheet 1 - What am I?



## Coordinate Graphing - Worksheet 2 - What am I?





# What My Rule? - Worksheet 1

IN	OUT
2	4
6	8
9	11
20	22

What's My Rule?  $+2$   
In plus 2 = out

IN	OUT
10	5
8	4
6	3
4	2
100	50
12	6

What's My Rule?  $\div 2$   
Or  $\frac{1}{2}$  of in equals out  
In  $\div 2 =$  out

IN	OUT
2	12
3	13
10	20
7	17
40	50

What's My Rule?  $+10$   
In  $+10 =$  out

IN	OUT
7	4
3	0
10	7
8	5
20	17

What's My Rule?  $-3$   
In  $-3 =$  out

IN	OUT
0	0
1	2
2	4
3	6
8	16

What's My Rule?  $\times 2$   
In  $\times 2 =$  out

Make your own.

IN	OUT
—	—
—	—
—	—
—	—
—	—

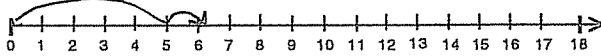
What's My Rule?  $\text{AWV}$

13

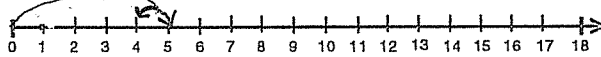
# Sums and Differences

Two Numbers	Their Sum	Their Difference
10, 6	16	4
9, 3	12	6
1, 7	8	6
4, 9	13	5

Draw the number line showing the sum of 5 and 1.

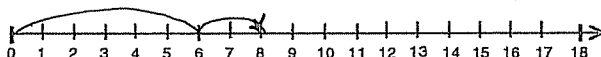


Draw the number line showing the difference between 5 and 1.

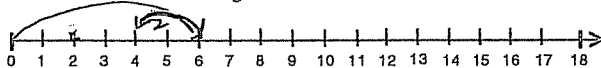


5, 1	6	4
4, 11	15	7
7, 7	14	0

Draw the number line showing the sum of 6 and 2.



Draw the number line showing the difference between 6 and 2.



6, 2

8                      4

How did you figure it out? I added the two numbers to get the sum and subtracted to get their difference.

16

# Number Lines

- Write the number sentence for number line A.  $8+4=12$
- Write the number sentence for number line B.  $4+3+7=14$
- Write the number sentence for number line C.  $20+20+20=60$  or  $20 \times 3=60$
- On number line D draw  $12+7=19$
- On number line E draw  $40+30=70$
- Write the number sentence for number line F.  $12-5=7$
- Write the number sentence for number line G.  $17-6=11$
- On number line H draw  $16-9=7$
- Draw your own number line. Above the number line, make up a subtraction problem. Draw it and write the number sentence.  $\text{AWV}$

15

# Place Value: Three Ways to Record - Worksheet 15

<p>2 cuckoos 2 units = 22 <math>(20) + (2) = 22</math></p>	<p>1 cuckoos 2 units = 12 <math>(10) + (2) = 12</math></p>
<p>2 cuckoos 3 units = 23 <math>(20) + (3) = 23</math></p>	<p>2 cuckoos 1 units = 21 <math>(20) + (1) = 21</math></p>
<p>3 cuckoos 0 units = 30 <math>(30) + (0) = 30</math></p>	<p>1 cuckoos 3 units = 13 <math>(10) + (3) = 13</math></p>
<p>0 cuckoos 3 units = 3 <math>(0) + (3) = 3</math></p>	<p>3 cuckoos 2 units = 32 <math>(30) + (2) = 32</math></p>

20

Place Value: Three Ways to Record - Worksheet 2  
Base Five

 $\begin{array}{r} 3 \quad \quad 2 \text{ units} = 32 \\ \hline (30) + (2) = 32 \end{array}$	 $\begin{array}{r} 2 \quad 4 \quad \quad \text{units} = 24 \\ \hline (20) + (4) = 24 \end{array}$
 $\begin{array}{r} 1 \quad \quad 3 \text{ units} = 13 \\ \hline (10) + (3) = 13 \end{array}$	 $\begin{array}{r} 2 \quad \quad 0 \text{ units} = 20 \\ \hline (20) + (0) = 20 \end{array}$
 $\begin{array}{r} 4 \quad \quad 3 \text{ units} = 43 \\ \hline (40) + (3) = 43 \end{array}$	 $\begin{array}{r} 3 \quad \quad 1 \text{ units} = 31 \\ \hline (30) + (1) = 31 \end{array}$
 $\begin{array}{r} 0 \quad \quad 4 \text{ units} = 4 \\ \hline (0) + (4) = 4 \end{array}$	 $\begin{array}{r} 2 \quad \quad 3 \text{ units} = 23 \\ \hline (20) + (3) = 23 \end{array}$

21

Place Value: Challenge

In a strange land, far, far, away...

Predict what the rest of this chart would look like.

0	0	0
0	0	1
0	0	2
1	1	0
1	1	1
1	1	2
2	2	0
2	2	1
2	2	2
0	1	0
1	0	1
1	0	2

WHAT HAPPENS?

In strange land away...

Try to figure this out!

Predict what the rest of this chart would look like.

0	0	0	X
0	0	1	U
0	0	2	*
1	1	0	X
1	1	1	X
1	1	2	U
2	2	0	*
2	2	1	X
2	2	2	X
0	1	0	U
1	0	1	*
1	0	2	X
0	1	0	U
1	0	1	*
1	0	2	X

Place Value: Three Ways to Record - Worksheet 1  
Base Ten

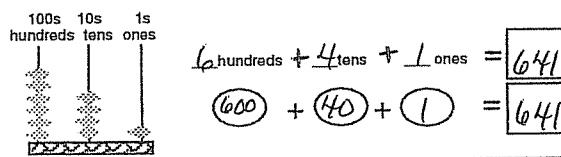
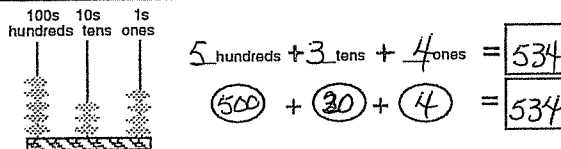
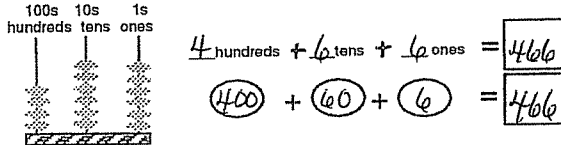
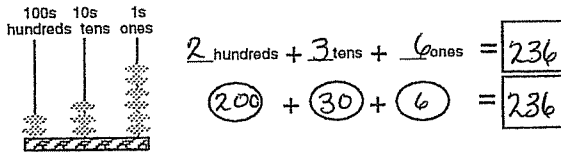
 $\begin{array}{r} 2 \text{ tens } \quad 3 \text{ ones} = 23 \\ \hline (20) + (3) = 23 \end{array}$	 $\begin{array}{r} 3 \text{ tens } \quad 4 \text{ ones} = 34 \\ \hline (30) + (4) = 34 \end{array}$
 $\begin{array}{r} 2 \text{ tens } \quad 7 \text{ ones} = 27 \\ \hline (20) + (7) = 27 \end{array}$	 $\begin{array}{r} 5 \text{ tens } \quad 8 \text{ ones} = 58 \\ \hline (50) + (8) = 58 \end{array}$
 $\begin{array}{r} 3 \text{ tens } \quad 9 \text{ ones} = 39 \\ \hline (30) + (9) = 39 \end{array}$	 $\begin{array}{r} 4 \text{ tens } \quad 6 \text{ ones} = 46 \\ \hline (40) + (6) = 46 \end{array}$
 $\begin{array}{r} 6 \text{ tens } \quad 5 \text{ ones} = 65 \\ \hline (60) + (5) = 65 \end{array}$	 $\begin{array}{r} 5 \text{ tens } \quad 5 \text{ ones} = 55 \\ \hline (50) + (5) = 55 \end{array}$

23

Place Value: Three Ways to Record - Worksheet 3  
Base Ten

 $\begin{array}{r} 1 \text{ hundreds } \quad 3 \text{ tens } \quad 5 \text{ ones} = 135 \\ \hline (100) + (30) + (5) = 135 \end{array}$	 $\begin{array}{r} 2 \text{ hundreds } \quad 4 \text{ tens } \quad 7 \text{ ones} = 247 \\ \hline (200) + (40) + (7) = 247 \end{array}$
 $\begin{array}{r} 3 \text{ hundreds } \quad 4 \text{ tens } \quad 6 \text{ ones} = 346 \\ \hline (300) + (40) + (6) = 346 \end{array}$	 $\begin{array}{r} 2 \text{ hundreds } \quad 0 \text{ tens } \quad 8 \text{ ones} = 208 \\ \hline (200) + (0) + (8) = 208 \end{array}$
 $\begin{array}{r} 3 \text{ hundreds } \quad 5 \text{ tens } \quad 7 \text{ ones} = 357 \\ \hline (300) + (50) + (7) = 357 \end{array}$	 $\begin{array}{r} 3 \text{ hundreds } \quad 5 \text{ tens } \quad 5 \text{ ones} = 355 \\ \hline (300) + (50) + (5) = 355 \end{array}$

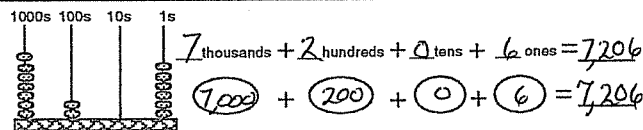
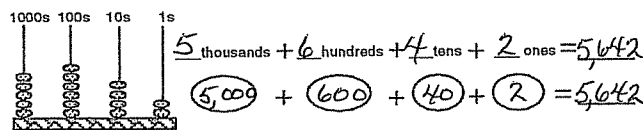
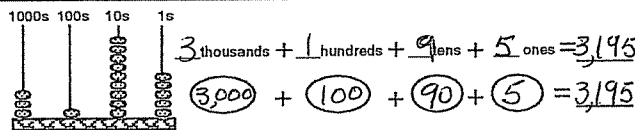
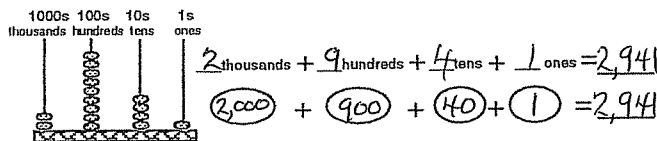
### Place Value - Worksheet 1



$600 + 10 + 2 = 612$
$600 + 30 + 7 = 637$
$500 + 20 + 0 = 520$
$800 + 0 + 2 = 802$

26

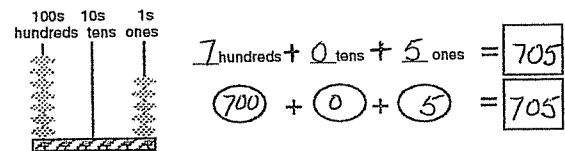
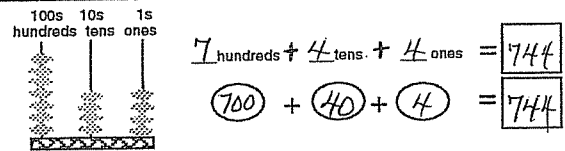
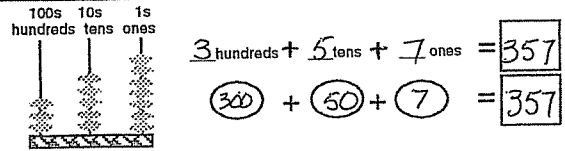
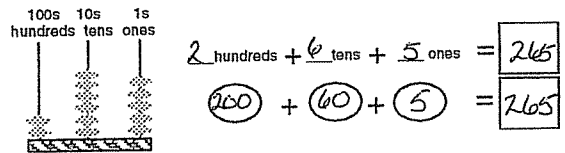
### Place Value - Worksheet 3



$3,000 + 100 + 50 + 4 = 3,154$
$5,000 + 200 + 40 + 9 = 5,249$
$7,000 + 200 + 80 + 1 = 7,281$
$9,000 + 700 + 30 + 8 = 9,738$

28

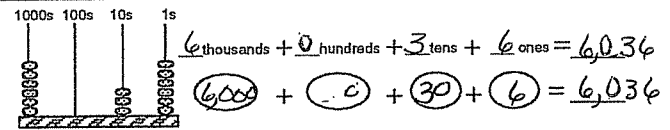
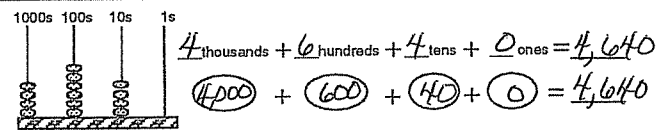
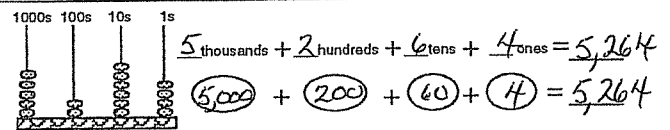
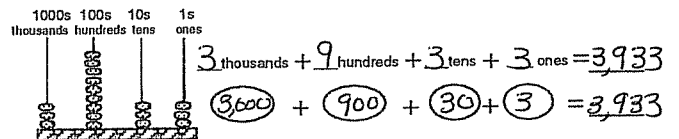
### Place Value - Worksheet 2



$100 + 50 + 4 = 154$
$400 + 30 + 8 = 438$
$700 + 90 + 2 = 792$
$400 + 0 + 0 = 400$

27

### Place Value - Worksheet 4



$4,000 + 300 + 90 + 0 = 4,390$
$2,000 + 600 + 0 + 2 = 2,602$
$5,000 + 800 + 20 + 7 = 5,827$
$4,000 + 0 + 10 + 2 = 4,012$

29

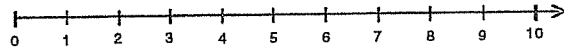
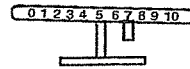
## Number Patterns: Missing Numbers

Write the missing numbers to complete the pattern.

10	15	20	25	30	35	40	45	50
50	60	70	80	90	100	110	120	130
120	125	130	135	140	145	150	155	160
60	64	68	72	76	80	84	88	92
800	900	1,000	1,100	1,200	1,300	1,400	1,500	1,600
69	66	63	60	57	54	51	48	45
500	504	508	512	516	520	524	528	532
600	605	610	615	620	625	630	635	640
197	198	199	200	201	202	203	204	205
997	998	999	1,000	1,001	1,002	1,003	1,004	1,005

30

## Rounding Off: Tens - Worksheet 1



Use the number line above. Put your finger on the 4. Is your finger closer to 0 or to 10? 0  
Put your finger on the 7. Is your finger closer to 0 or to 10? 10  
Going to the closest number with a zero in it is called rounding off.

These numbers are correctly rounded off to the nearest ten.

6 → 10      12 → 10      5 → 10  
15 → 20      42 → 40      38 → 40

These numbers are incorrectly rounded off to the nearest ten. ↗ means does not

32 ↗ 20      46 ↗ 40      75 ↗ 70  
62 ↗ 50      48 ↗ 45      36 ↗ 45

Which of these are correctly rounded off to the nearest ten? Circle the correct ones. Write the correct answer to problems that are not correct.

34 → 30      69 → 70      25 → 20      30  
65 → 70      17 → 10      20      48 → 50  
16 → 20      86 → 80      90      99 → 100

Round off these numbers.

42 → 40      67 → 70      AWW      — → 60  
85 → 90      78 → 80      — → —

31

## Rounding Off: Tens - Worksheet 2

Base Ten blocks can be used to help round off to the nearest ten.

Build 27.

Build the two groups of ten between which 27 falls. Draw the two groups here.

27 comes between the two nearest tens 20 and 30.  
Compare. 27 is closer to 30. 27 rounds off to 30.

- Is closer to or ? 33 rounds off to 30.
- Is closer to or ? 14 rounds off to 10.
- Is closer to or ? 48 rounds off to 50.
- Is closer to or ? 35 rounds off to 40.

46 comes between 40 and 50.

The halfway number is 45.

46 is (greater than, less than, or equal to) greater than 45

46 rounds to 50.

Round each number to the nearest ten. By convention halfway numbers round up.

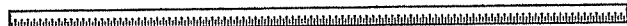
Number	Comes Between	Halfway number	Rounds to
32	30 and 40	35	30
65	60 and 70	65	70
17	10 and 20	15	20
51	50 and 60	55	50
86	80 and 90	85	90
74	70 and 80	75	70

Halfway numbers round up (up, down, or stay the same).

32

Answer Key: General Math - Booklet 3

## Rounding Off: Tens - Worksheet 3



Use a meter stick to help solve these problems.

Round them off to the nearest ten.

24 rounds off to 20      79 rounds off to 80  
58 rounds off to 60      25 rounds off to 30  
84 rounds off to 80      42 rounds off to 40  
32 rounds off to 30      67 rounds off to 70  
46 rounds off to 50      61 rounds off to 60  
75 rounds off to 80      53 rounds off to 50  
34 rounds off to 30      Make your own. AWW  
46 rounds off to 50      — rounds off to —  
99 rounds off to 100      — rounds off to —

List all the numbers which would round off to 30.

25 26 27 28 29 30 31 32 33 34

List all the numbers which would round off to 70.

65 66 67 68 69 70 71 72 73 74

33

### Rounding Off: Tens - Worksheet 4

Rounding is used to estimate and to see if answers make sense.

#### Rounding Sums

$$\begin{array}{r} 43 \rightarrow 40 \\ + 68 \rightarrow 70 \\ \hline 111 \quad \boxed{110} \\ \text{actual} \quad \text{estimate} \\ \text{answer} \end{array}$$

$$\begin{array}{r} 38 \rightarrow 40 \\ + 45 \rightarrow 50 \\ \hline 83 \quad \boxed{90} \\ \text{actual} \quad \text{estimate} \end{array}$$

$$\begin{array}{r} 73 \rightarrow 70 \\ + 29 \rightarrow 30 \\ \hline 102 \quad \boxed{100} \\ \text{actual} \quad \text{estimate} \end{array}$$

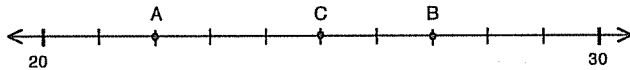
$$\begin{array}{r} 46 \rightarrow 50 \\ + 57 \rightarrow 60 \\ \hline 103 \quad \boxed{110} \\ \text{actual} \quad \text{estimate} \end{array}$$

$$\begin{array}{r} 54 \rightarrow 50 \\ + 37 \rightarrow 40 \\ \hline 91 \quad \boxed{90} \\ \text{actual} \quad \text{estimate} \end{array}$$

$$\begin{array}{r} 24 \rightarrow 20 \\ + 46 \rightarrow 50 \\ \hline 70 \quad \boxed{70} \\ \text{actual} \quad \text{estimate} \end{array}$$

34

### Rounding Off: Tens - Worksheet 6



- Point A = 22.  
Point A is closer to 20.  
Point A rounds off to 20.
- Point B = 27.  
Point B is closer to 30.  
Point B rounds off to 30.
- Point C = 25.  
Point C is closer to equal distance.  
Point C rounds off to 30.

Round each number to the nearest ten.

- 34 comes between 30 and 40.  
Halfway number 35.  
34 rounds off to 30.
- 51 comes between 50 and 60.  
Halfway number 55.  
51 rounds off to 50.
- 147 comes between 140 and 150.  
Halfway number 145.  
147 rounds off to 150.
- 363 comes between 360 and 370.  
Halfway number 365.  
363 rounds off to 360.
- 109 comes between 100 and 110.  
Halfway number 105.  
109 rounds off to 110.
- 249 comes between 240 and 250.  
Halfway number 245.  
249 rounds off to 250.
- 319 comes between 310 and 320.  
Halfway number 315.  
319 rounds off to 320.
- 173 comes between 170 and 180.  
Halfway number 175.  
173 rounds off to 170.

Make your own. Have one number round up and the other round down. *AWV*

- \_\_\_\_\_ comes between \_\_\_\_\_ and \_\_\_\_\_.  
Halfway number \_\_\_\_\_.  
\_\_\_\_\_ rounds off to \_\_\_\_\_.
- \_\_\_\_\_ comes between \_\_\_\_\_ and \_\_\_\_\_.  
Halfway number \_\_\_\_\_.  
\_\_\_\_\_ rounds off to \_\_\_\_\_.

What is the rule for rounding off to the nearest ten?

Go to the tens place and look to the numbers to the right. If that digit is 5 or greater, round the tens digit to the next higher number. If the digit is less than 5, leave the tens digit as is, and make the ones digit 0.

36

### Rounding Off: Tens - Worksheet 5

#### Rounding Differences

$$\begin{array}{r} 72 \rightarrow 70 \\ - 21 \rightarrow 20 \\ \hline 51 \quad \boxed{50} \\ \text{actual} \quad \text{estimate} \end{array}$$

$$\begin{array}{r} 89 \rightarrow 90 \\ - 43 \rightarrow 40 \\ \hline 46 \quad \boxed{50} \\ \text{actual} \quad \text{estimate} \end{array}$$

$$\begin{array}{r} 57 \rightarrow 60 \\ - 42 \rightarrow 40 \\ \hline 15 \quad \boxed{20} \\ \text{actual} \quad \text{estimate} \end{array}$$

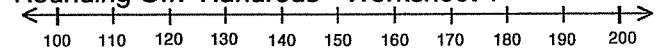
$$\begin{array}{r} 64 \rightarrow 60 \\ - 23 \rightarrow 20 \\ \hline 41 \quad \boxed{40} \\ \text{actual} \quad \text{estimate} \end{array}$$

$$\begin{array}{r} 69 \rightarrow 70 \\ - 57 \rightarrow 60 \\ \hline 12 \quad \boxed{10} \\ \text{actual} \quad \text{estimate} \end{array}$$

$$\begin{array}{r} 76 \rightarrow 80 \\ - 45 \rightarrow 50 \\ \hline 31 \quad \boxed{30} \\ \text{actual} \quad \text{estimate} \end{array}$$

35

### Rounding Off: Hundreds - Worksheet 1



Use the number line above. Put your finger on the 130. Is your finger closer to 100 or to 200? 100 Put your finger on the 180. Is your finger closer to 100 or to 200? 200

These numbers are correctly rounded off to the nearest hundred.

$$\begin{array}{lll} 180 \rightarrow 200 & 140 \rightarrow 100 & 150 \rightarrow 200 \\ 615 \rightarrow 600 & 342 \rightarrow 300 & 265 \rightarrow 300 \end{array}$$

These numbers are incorrectly rounded off to the nearest hundred:  $\rightarrow$  means does not

$$425 \rightarrow 500 \quad 612 \rightarrow 610 \quad 458 \rightarrow 400$$

$$782 \rightarrow 600 \quad 225 \rightarrow 100 \quad 175 \rightarrow 300$$

Which of these are correctly rounded off to the nearest hundred? Circle the correct ones. Write the correct answer to problems that are not correct.

$$\boxed{264 \rightarrow 300} \quad \boxed{125 \rightarrow 100} \quad \boxed{483 \rightarrow 500}$$

$$362 \rightarrow 360 \quad \boxed{865 \rightarrow 900} \quad 728 \rightarrow 800$$

Round off these numbers to the nearest hundred.

$$952 \rightarrow 1,000 \quad 678 \rightarrow 700 \quad \text{AWV} \rightarrow 700$$

$$854 \rightarrow 900 \quad 245 \rightarrow 200 \quad \rightarrow 500$$

$$827 \rightarrow 800 \quad 1,262 \rightarrow 1,300 \quad \rightarrow$$

What is the rule for rounding off to the nearest hundred?

Find the hundreds place. Is the digit to its right less than 5? Round to the amount of the hundreds place and place zeros in the tens and ones places. If it is five or greater, round up to the next highest hundred. Place zeros in the tens and ones places.

37

## Rounding Off: Hundreds - Worksheet 2

Round off each number to the nearest hundred.

$272 \rightarrow 300$

$705 \rightarrow 700$

$345 \rightarrow 300$

$284 \rightarrow 300$

$846 \rightarrow 800$

$416 \rightarrow 400$

$623 \rightarrow 600$

$8,360 \rightarrow 8,400$

$\text{AWV} \rightarrow 300$

$\rightarrow$

$\rightarrow$

Round off. Solve the rounded off problems.

Ball park estimate

$$\begin{array}{r} 885 \rightarrow 900 \\ - 414 \rightarrow 400 \\ \hline 500 \end{array}$$

$$\begin{array}{r} 265 \rightarrow 300 \\ - 193 \rightarrow 200 \\ \hline 100 \end{array}$$

$$\begin{array}{r} 615 \rightarrow 600 \\ - 272 \rightarrow 300 \\ \hline 300 \end{array}$$

$$\begin{array}{r} 192 \rightarrow 200 \\ - 129 \rightarrow 100 \\ \hline 100 \end{array}$$

$$\begin{array}{r} 805 \rightarrow 800 \\ - 423 \rightarrow 400 \\ \hline 400 \end{array}$$

$$\begin{array}{r} 527 \rightarrow 500 \\ - 384 \rightarrow 400 \\ \hline 100 \end{array}$$

38

## Greater, Than, Less Than, or Equal To Worksheet 1

$=, <, >, \neq$

$5 > 4 \quad 10 > 5 \quad 4 < 5 \quad 5 < 11$

$5 > 3 \quad 300 > 200 \quad 3 < 5 \quad 200 < 300 \quad 1 + 2 \neq 5$

$5 > 0 \quad 100 > 8 \quad 0 < 5 \quad 3 < 99 \quad 2 + 7 \neq 10$

$6 + 4 > 9 \quad 3 < 2 + 2 \quad 12 \neq 43$

Look at the examples above. What do you think '>' means?

Greater than or bigger than

Look at the examples above. What do you think '<' means?

Less than or smaller than

Look at the examples above. What do you think '≠' means?

We read it as 'not equal to.'

Is not the same as or not equal to

Put in the correct signs. Use only three ≠ signs.

$7 > 3 \quad 4 + 2 = 6 \quad 7,000,000 > 70,000$

$7 < 12 \quad 1,000 > 999 \quad 12 = 12$

$0 < 6 \quad 33 > 23 \quad 99 > 98$


$3 + 5 > 3 \quad 16 < 61 \quad 9 - 6 > 9 - 7$

$100 < 300 \quad 9 > 3 + 5 \quad 13 - 3 > 13 - 6$


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## Greater, Than, Less Than, or Equal To - Worksheet 2

The big part > opens towards the larger number. The alligator always eats the larger number.

6  3 Six is greater than three.

The big part < still opens towards the larger number.

5  7 Five is less than seven.

Put numbers into the blanks to make the number sentences true. AWV  
 $8 > 5 \quad 10 > \underline{\quad} \quad 26 > \underline{\quad} \quad \underline{\quad} > 13$

Do the same with these number sentences. AWV  
 $12 < 13 \quad 11 < \underline{\quad} \quad \underline{\quad} < 27 \quad 43 < \underline{\quad}$

Put the sign in the circles to make these number sentences true.

$15 \textcircled{=} 15 \quad 18 \textcircled{<} 29 \quad 76 \textcircled{>} 57 \quad 42 \textcircled{=} 42$

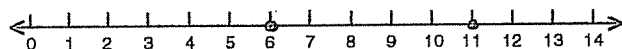
$25 \textcircled{>} 15 \quad 0 \textcircled{=} 0 \quad 35 \textcircled{<} 53 \quad 63 \textcircled{>} 54$

You put in the numbers this time. AWV

$\underline{\quad} = \underline{\quad} \quad \underline{\quad} < \underline{\quad} \quad \underline{\quad} > \underline{\quad} \quad \textcircled{\quad}$

Put in a sign tool

Place a dot on 11 and 6 on the number line. Compare.



Make these number sentences true.  $6 \textcircled{<} 11 \quad 11 \textcircled{>} 6$

If  $6 < 11$ , then 6 is on the left on the number line.

If  $11 > 6$ , then 11 is further to the right on the number line.

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Answer Key: General Math - Booklet 3

## Greater, Than, Less Than, or Equal To - Worksheet 3

Put in the symbols to make these number sentences true. Cut and paste the symbols at the bottom of the page or write in the correct symbols.

Use this space to calculate.

$1. \quad 3 \times 2 \textcircled{<} 4 \times 2$

$2. \quad 57 \textcircled{>} 100 \times 0$

$3. \quad 4 \times 5 \times 3 \textcircled{>} 2 \times 3 \times 5$

$4. \quad 19 + 37 \textcircled{=} 27 + 29$

$5. \quad 345 - 65 \textcircled{<} 345 + 65$

$6. \quad 4 \times 9 \textcircled{=} 6 \times 6$

$7. \quad 98 - 12 \textcircled{<} 25 + 62$

$= = = < < < < > > > >$

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## Greater, Than, Less Than, or Equal To - Worksheet 4

Write in the signs to make these number sentences true.



$400 + 20 + 3 \quad (\lt) \quad 432$

$264 \quad (\lt) \quad 426$

$1,000 + 90 \quad (\lt) \quad 1,900$

$2,400 \quad (\lt) \quad 2,000 + 400 + 4$

$1,322 \quad (=) \quad 1,000 + 300 + 20 + 2$

$300 + 60 + 5 \quad (=) \quad 365$

$2,000 + 800 + 60 + 1 \quad (\gt) \quad 2,816$

$20,030 \quad (\gt) \quad 2,030$

$695 \quad (\gt) \quad 659$

$2,000 + 3 \quad (\lt) \quad 2,030$

$3,552 \quad (\gt) \quad 3,000 + 500 + 50$

$42. \quad 25,603 \quad (\gt) \quad 25,503$

### Using Parentheses

We use a mathematical symbol like this ( ) to group numbers together. These are called parentheses (pair- n - the - seez). Parentheses tell you to do what is inside first.

$(2 \times 3) \times 4 =$  means multiply  $2 \times 3$  first to get 6.  
 $6 \times 4 =$  then multiply the 6 times the 4 to get 24.

Solve these.  $(3 \times 4) \times 2 =$   $(4 \times 2) \times 4 =$   
 $\underline{12} \times 2 = 24$   $\underline{8} \times 4 = 32$

$(5 \times 2) \times 6 =$   $(3 \times 1) \times 3 =$   
 $\underline{10} \times 6 = 60$   $\underline{3} \times 3 = 9$

Solve these. Be careful, these are a little different.

$(8 + 1) \times 6 =$   $(5 + 3) \times 2 =$   
 $\underline{9} \times 6 = 54$   $\underline{8} \times 2 = 16$

$(3 + 4) \times 5 =$   $(4 + 5) \times 7 =$   
 $\underline{7} \times 5 = 35$   $\underline{9} \times 7 = 63$

$(7 - 2) \times 4 =$   $(5 - 3) \times 8 =$   
 $\underline{5} \times 4 = 20$   $\underline{2} \times 8 = 16$

$(12 - 4) \times 3 =$   $(9 - 3) \times 8 =$   
 $\underline{8} \times 3 = 24$   $\underline{6} \times 8 = 48$

Make your own.

## Greater, Than, Less Than, or Equal To - Worksheet 5

= < >

Write the name of each sign and an equation using each sign.

Name	Equation	AWV	Name	Equation
>	Greater than	>	<	Less than
=	equal	=	≠	does not equal

Use money or money stamps to make the equation true.

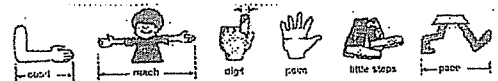
\_\_\_\_\_ > \_\_\_\_\_      \_\_\_\_\_ < \_\_\_\_\_

\_\_\_\_\_ > \_\_\_\_\_      \_\_\_\_\_ < \_\_\_\_\_

\_\_\_\_\_ = \_\_\_\_\_      \_\_\_\_\_ ≠ \_\_\_\_\_

\_\_\_\_\_ = \_\_\_\_\_      \_\_\_\_\_ ≠ \_\_\_\_\_

## Measurement: Nonstandard - Worksheet 1

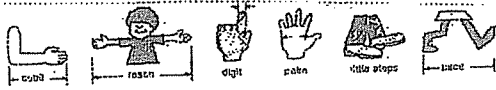


- Use these "body measures" to measure things around the room.
- First estimate (guess) what the measurement will be and record it. Then measure and record the actual number (measurement).
- Record what actual body measure you use, the unit, (example: 5 cubits).
- Compare your measurements with a friend. Are they the same? Why? Whose measurements are the easiest?

Object	Body Measurement	
	Estimate	Actual
length of table		
width of door		
height of this paper		
your height		
Make your own.		

## Measurement: Nonstandard - Worksheet 2 AWW

Use body measures again to measure the objects listed below. This time use three different measures for each object you measure.



1. Length of a table. **Body Measurement**

	1	2	3
Estimate			
Actual			

2. Your height. **Body Measurement**

	1	2	3
Estimate			
Actual			

3. Length of one orange Cuisenaire Rod. **Body Measurement**

	1	2	3
Estimate			
Actual			

Are some units better for certain jobs? Why?

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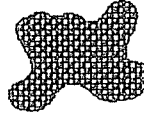
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## Area: Nonstandard - Worksheet 1

How many pinto beans will cover each of these figures? First, guess a range the answer will be in between, for example, between 15 and 20 beans. Record the actual number. If your guess was within the range, circle "within range." If your guess was higher than the range, circle the "too high." If your guess was lower, circle the "too low." To help the beans stay in place, make a three dimensional border around the edge of each figure with glue or modeling clay.

Try to improve your guesses as you go.

Figure 1



I guess between \_\_\_ and \_\_\_

Actual number 7

Within range  too high  too low

Figure 2

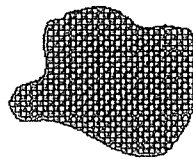


I guess between \_\_\_ and \_\_\_

Actual number 12

Within range  too high  too low

Figure 3

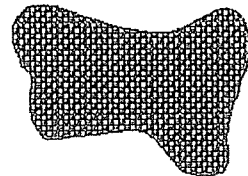


I guess between \_\_\_ and \_\_\_

Actual number 16

Within range  too high  too low

Figure 4



I guess between \_\_\_ and \_\_\_

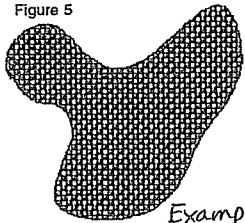
Actual number 22

Within range  too high  too low

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## Area: Nonstandard - Worksheet 2

Figure 5

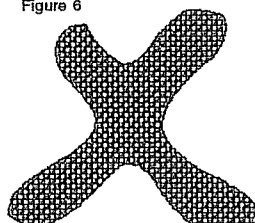


I guess between 25 and 27

Actual number 24

Within range  too high  too low

Figure 6



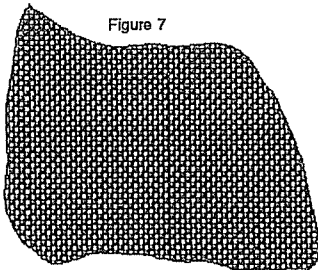
I guess between 20 and 25

Actual number 20

Within range  too high  too low

Guess first!  
Which will need more beans to cover it  
Figure 7 or Figure 8? \_\_\_\_\_

Figure 7

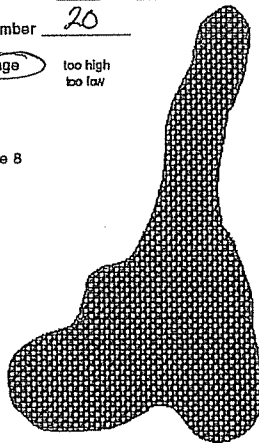


I guess between \_\_\_ and \_\_\_

Actual number 49

Within range  too high  too low

Figure 8



I guess between \_\_\_ and \_\_\_

Actual number 43

Within range  too high  too low

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## Area: Nonstandard - Worksheet 4

Take out Worksheet 1 and a container of lima beans. You are going to cover each of the four figures on Worksheet 1 with the lima beans. You will guess a range first, count, and then, figure out if your guess was within range, and higher or lower than the range.

Which bean is larger, the pinto bean or the lima bean? \_\_\_\_\_

PREDICT! Will it take more or less lima beans than pinto beans to cover a figure? \_\_\_\_\_

Explain your thinking.

CHECK IT OUT.

Figure 1 - lima beans
I guess between ___ and ___
Actual number <u>4</u>
Within range <input type="checkbox"/> too high <input type="checkbox"/> too low

Was your prediction right? \_\_\_\_\_ It took more / less lima beans to cover it than pinto beans. Will that be true for the other figures too? Try it.

Figure 2 - lima beans
I guess between ___ and ___
Actual number <u>6</u>
Within range <input type="checkbox"/> too high <input type="checkbox"/> too low

Figure 3 - lima beans
I guess between ___ and ___
Actual number <u>7</u>
Within range <input type="checkbox"/> too high <input type="checkbox"/> too low

Figure 4 - lima beans
I guess between ___ and ___
Actual number <u>9</u>
Within range <input type="checkbox"/> too high <input type="checkbox"/> too low

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### Area: Nonstandard - Worksheet 5

Take out Worksheet 1 and a container of split peas. You are going to cover each of the four figures on Worksheet 1 with the split peas. You will guess a range first, count, and then, figure out if your guess was within range, and higher or lower than the range.

Which bean is larger, the pinto bean or the split pea? pinto  
Use split peas. PREDICT!

Will it take more or less split peas to cover the figure than pinto beans? more  
Why? They are smaller.

Figure 1 - split peas  
I guess between \_\_\_ and \_\_\_  
Actual number 14  
Within range    too high  
                    too low

Figure 2 - split peas  
I guess between \_\_\_ and \_\_\_  
Actual number 34  
Within range    too high  
                    too low

Figure 3 - split peas  
I guess between \_\_\_ and \_\_\_  
Actual number 42  
Within range    too high  
                    too low

Figure 4 - split peas  
I guess between \_\_\_ and \_\_\_  
Actual number 64  
Within range    too high  
                    too low

Was your prediction right? \_\_\_\_\_

### Area: Nonstandard - Worksheet 6

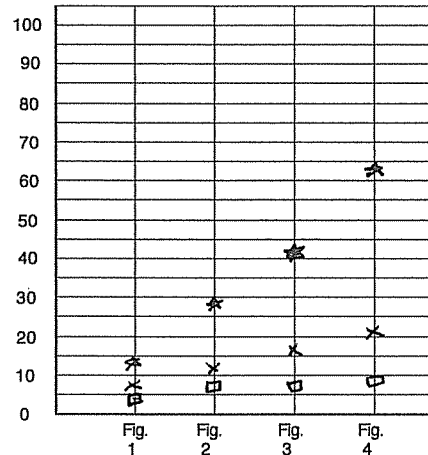
Fill in the chart. Next to each figure record how many split peas covered it, how many pinto beans covered it, and how many lima beans covered it.

	SPLIT PEAS	PINTO BEANS	LIMA BEANS
FIGURE 1	<u>14</u>	<u>7</u>	<u>4</u>
FIGURE 2	<u>34</u>	<u>12</u>	<u>6</u>
FIGURE 3	<u>42</u>	<u>16</u>	<u>7</u>
FIGURE 4	<u>64</u>	<u>22</u>	<u>9</u>

What patterns do you see in the numbers on the chart?

Now graph the results.

- Put a green dot or a ☆ for the amount of split peas.
- Put a brown dot or an X for the amount of pinto beans.
- Put an orange dot or a □ for the amount of lima beans.



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52 The larger the figure, the \_\_\_\_\_ beans it takes to cover it.  
The larger the bean, the \_\_\_\_\_ beans it takes to cover it.

### Measurement: Standard - Volume

Volume is how much will fit into a container. One way to measure volume is with ounces, cups, pints, quarts, and gallons.

Use sand, water or rice to figure out:

How many ounces (oz.) in one cup? 8

How many cups in pint? 2

How many cups in quart? 4

How many quarts in gallon? 4



Fill out the chart below.

2 cups = 1 pint

2 pints = 1 quart

4 quarts = 1 gallon

2 quarts = 1/2 gallon

1 quarts = 1/4 gallon

4 cups = 2 pint(s)

4 cups = 1 quart(s)

16 ounces = 1 pint(s)

32 ounces = 1/4 gallon(s)

4 cups = 1 quart

32 quarts = 8 gallon(s)

16 pints = 2 gallons

16 pints = 8 quarts

8 quarts = 2 gallons

16 quarts = 4 gallon(s)

Make your own.

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### Area: Standard - Worksheet 1

Use centimeter tiles or graph paper.

**New Words - Area** (ā'ē-ə) pronounced air-ee-uh

Area is the flat space inside a boundary line.

This shaded part is the area.

This is the boundary line or perimeter.

Guess and measure how many tiles will fit on the figure.

Figure 1

Figure 1 - centimeter tiles  
I guess between \_\_\_ and \_\_\_  
Actual number 12  
Within range    too high  
                    too low

Figure 2

Figure 2 - centimeter tiles  
I guess between \_\_\_ and \_\_\_  
Actual number 32  
Within range    too high  
                    too low

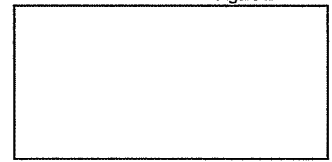


Figure 3



Figure 3 - centimeter tiles  
I guess between \_\_\_ and \_\_\_  
Actual number 48  
Within range    too high  
                    too low

56

## Area: Standard -Worksheet 2

Use inch tiles or graph paper.

Guess a range no wider than five. Measure how many tiles will fit on the figure.

Figure 1

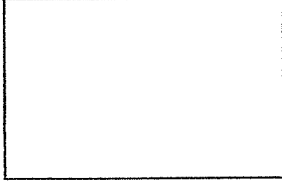


Figure 1 - inch tiles  
I guess between \_\_\_ and \_\_\_  
Actual number 6  
Within range      too high  
                                 too low

Figure 2 - inch tiles  
I guess between \_\_\_ and \_\_\_  
Actual number 4  
Within range      too high  
                                 too low

Figure 2



Figure 3

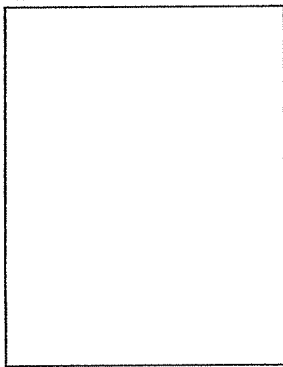
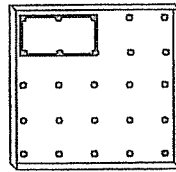


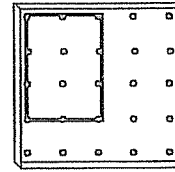
Figure 3 - inch tiles  
I guess between \_\_\_ and \_\_\_  
Actual number 12  
Within range      too high  
                                 too low

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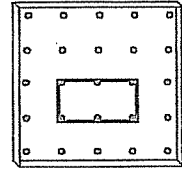
## Geoboard Area



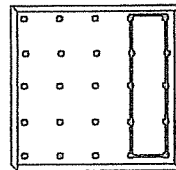
area 2



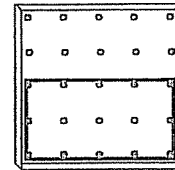
area 6



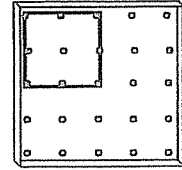
area 2



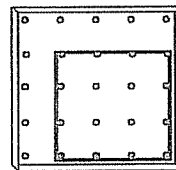
area 4



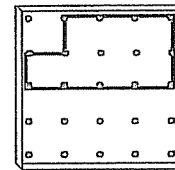
area 8



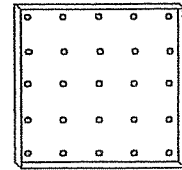
area 4



area 9



area 7



area \_\_\_

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## Tile Floors - Worksheet 1

Pick up one tile.

Find Floor 1.

Put the tile in the little square on Floor 1.

Now guess!

How many tiles do you think it will take to cover Floor 1? \_\_\_

Now cover the floor with your tiles.

How many tiles did it take? 4

Watch for patterns!

Now find Floor 2.

Guess!

How many tiles do you think it will take to cover Floor 2? \_\_\_

Now cover the floor with your tiles.

How many tiles did it take? 6

Now look at Floor 3 and Floor 4.

Guess!

Which one has more space or area inside? I think Floor \_\_\_ is bigger.

I guess that Floor 3 will hold \_\_\_ tiles.

I guess that Floor 4 will hold \_\_\_ tiles.

Now measure it. Floor 3 holds 9 tiles.

Floor 4 holds 8 tiles.

Look at Floor 5 and Floor 6.

Guess!

Which one has more area inside? I think Floor \_\_\_ is bigger.

Now measure.

Floor 5 holds 12 tiles. Floor 6 holds 12 tiles.

Are you surprised? \_\_\_\_\_

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## Tile Floors - Worksheet 2

Floor 7

Guess!

How many tiles will it take to cover Floor 7? \_\_\_

Measure.

How many tiles did it take? 5

Look at Floors 8 and 9.

Guess! Which one has more area? Floor \_\_\_

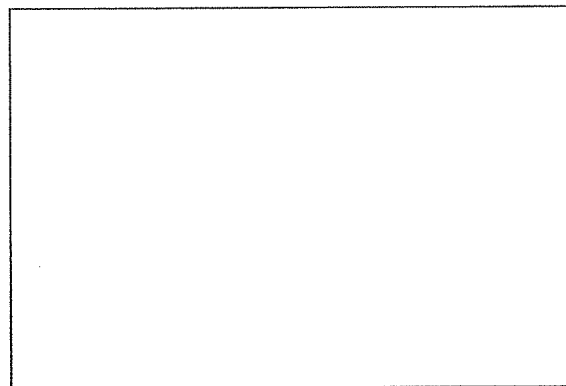
I think Floor 8 will hold \_\_\_ tiles and Floor 9 will hold \_\_\_.

Measure.

Floor 8 holds 18 tiles. Floor 9 holds 16 tiles.

Draw a rectangular floor that will hold 24 tiles.

Floor 10 Example:



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### Tile Floors - Worksheet 3

Use Tile Floors - Worksheets 1, 2, 4, 5, and 6 to fill out this chart. Number the sides of each floor as Floor 1 is numbered.

Floor 10	Floor 9	Floor 8	Floor 7	Floor 6	Floor 5	Floor 4	Floor 3	Floor 2	Floor 1	Area in square tiles	Length of sides			
											side 1	side 2	side 3	side 4
24	16	18	5	12	12	8	9	6	4		2	2	2	2
AWV	4	6	5	2	3	2	3	3	2		4	4	4	4

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### Averages - Worksheet 2



#### Problem 1

Let's say you are shooting 20 arrows per day at a target. These are your results for this week. Use blocks to figure out the number of arrows that hit the target per day.

Monday	5 arrows on target	Make a stack of 5 blocks.
Tuesday	2 arrows on target	Make a stack of 2 blocks.
Wednesday	4 arrows on target	Make a stack of 4 blocks.
Thursday	3 arrows on target	Make a stack of 3 blocks.
Friday	11 arrows on target	Make a stack of 11 blocks.

Now keep the same number of stacks (days) but move the blocks around until all the stacks have the same number in them.

What is your average number of arrows on target? 5

#### Problem 2

Notice that you had a really off day on Tuesday and a super lucky day on Friday. Sometimes to get a more accurate average we don't count the highest or lowest score. We only average the scores in the middle. Do that. Use only these days.

Monday	5 arrows on target	Make a stack of 5 blocks.
Wednesday	4 arrows on target	Make a stack of 4 blocks.
Thursday	3 arrows on target	Make a stack of 3 blocks.

This time what was the average number of arrows on target? 4

#### Problem 3

Do an experiment now. Get a pile of 10 pennies or pebbles and a paper cup or other container. Sit 3 feet away from the cup. Try to pitch 10 pennies into the cup. Count the number of times the pennies landed in the cup. Do this again 5 times. Record the data on the chart on the next page. Use blocks to find your average.

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### Averages - Worksheet 1



Here is a common problem in the world. Let's say you are learning to shoot baskets. Each day you shoot 15 baskets. You keep track of how many baskets you make. Then someone asks you how you are doing. Do you tell them what you got today? Do you tell them what you got yesterday or two days ago? Most people would ask, "What is your average?"

What they mean when they ask that is: if each day you made the same number of baskets, what would that number be? It means you took all your scores and evened them out so that they were all the same, that number is what an average is.

You will need some blocks. Unifix cubes are really good for this.

Let's say you kept track of how many baskets you made each day for a week. Your record looks like this:

Monday	3 baskets made	Make a stack of 3 blocks.
Tuesday	2 baskets made	Make a stack of 2 blocks.
Wednesday	6 baskets made	Make a stack of 6 blocks.
Thursday	5 baskets made	Make a stack of 5 blocks.
Friday	4 baskets made	Make a stack of 4 blocks.

Your stacks should look like this:



Now keep the number of stacks the same. Five stacks for five days. But move the blocks around until all the stacks are the same size.

How many are in each stack now? 4

There should be four in each stack. So the average number of baskets you made this week is four. The average is the number in a stack after the stacks are all the same size. Do more problems on the next page.

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