

Patterns in Arithmetic

Fractions Placement PDF

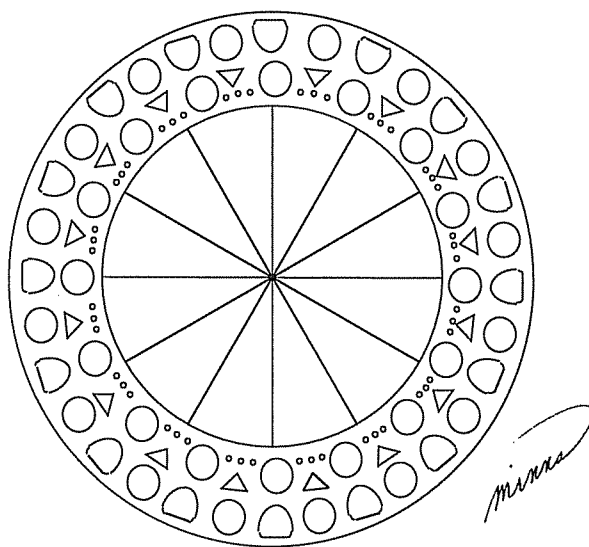
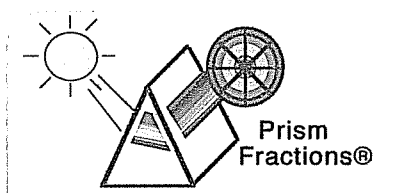
Booklets 6, 7, and 8

Parent/Teacher Guide

Booklet 6 - Multiplication

Booklet 7 - Addition and Subtraction of Unlike Fractions

Booklet 8 - Understanding Division



By Alysia Krafel, Susan Carpenter, and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Fractions Placement PDF - Booklets 6, 7, and 8

Student Placement Assessment worksheets are located at the end of each Assessment Guide.

Booklet 6 - Multiplication

Designed for nine to eleven-year-old students (fifth grade).

Prerequisite: Booklets 3, 4, and 5

This booklet:

- works with concrete models to teach what it means to multiply a fraction by a fraction.
- uses manipulatives and drawings to help students understand this seemingly easy procedure. Multiplying fractions is easy to do but not that easy to understand. For example: why does the answer to a multiplication problem in fractions have an answer that is less than either fraction you started with?

Booklet 7 - Addition and Subtraction of Unlike Fractions

Designed for ten to twelve-year-old students (fifth and sixth grade).

Prerequisite: Booklets 3, 4, 5, and 6

This booklet:

- uses the concept of equivalent fractions to develop a common denominator that allows the addition and subtraction of unlike fractions first with manipulatives, and then with calculation.

Booklet 8 - Understanding Division

Designed for ten to twelve-year-old students (fifth and sixth grade).

Prerequisite: Booklets 3, 4, 5, and 6

This booklet:

- uses manipulatives and drawings to develop the concept of division with fractions.
- works on the concept of what the quotient means.
- teaches how remainders are handled.
- teaches how to use patterns to calculate the answers without having to use a drawing.

Answer Keys for Fractions: Booklet 6, 7, and 8 Placement Assessments are at the end of the Assessment Guides.

Patterns in Arithmetic: Fractions Placement PDF
Booklets 6, 7, and 8
Parent/Teacher Guide
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Fractions: Booklet 6 Placement Assessment Guide

Purpose The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Assessment: Part 1 is review material from the last booklets and is used to determine student readiness for this booklet. Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

Prerequisites *Patterns in Arithmetic: Fractions - Booklets 1 - 5*

Materials Assessment: Part 1, pages 8 and 9 and Assessment: Part 2, page 10
Score sheets, pages 5 - 7

Note *Do not* provide manipulatives of any kind.

Instructions Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

Do the assessment in two parts. Give Assessment: Part 1 and check it for readiness for this booklet. The answers are in the Answer Key. If he is not ready for this booklet, there is no point in giving Assessment: Part 2. If he passes all the readiness items, then give Assessment: Part 2.

After scoring Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

Assessment Guide This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc. Score sheets that match the Assessment Guide for both Assessment: Part 1 and Part 2 follow.

Assessment Criteria for Assessment: Part 1

Can the student:

1. Addition of Like Fractions (Fractions: Booklet 2) and Simplification (Fractions: Booklet 5)
 - A. add the numerators and not the denominators in two of the three problems?
 - B. simplify two of the three sums?

2. Subtraction of Like Fractions (Fractions: Booklet 2)
- A. subtract the numerators and not the denominators in problem a?
 - B. visualize the whole as three-thirds and subtract two-thirds in problem b?
 - C. Bonus Point: Extend what he knows to get the correct answer to problem c, an unfamiliar but accessible problem? This item tests to see if he can visualize the $2\frac{1}{2}$ subtract one whole and then one-fourth, which the student knows is half of the half, leaving one and one-fourth. A correct answer here indicates strong growth in conceptual understanding, which is formally taught in Fractions: Booklet 7.

3. Greater Than, Less Than, or Equal To (Fractions: Booklet 2)
- A. identify $\frac{1}{2}$ as being the least?
 - B. identify $\frac{5}{6}$ as being the greatest?
 - C. order correctly $\frac{6}{8}$, $\frac{2}{3}$, and $\frac{6}{10}$?
 - D. Bonus Point: simplify $\frac{6}{8}$ or $\frac{6}{10}$?

This item tests conceptual understanding of fractional values. It identifies students who can use reason or drawings to solve a problem without the use of a manipulative.

4. Fractions as Parts of Wholes (Fractions: Booklet 2)
- A. divide the eight items into four equal groups of two?
 - B. give the correct answer to all three problems?
5. Mixed Numbers to Improper Fractions (Fractions: Booklet 3)
- A. give the correct answer to two of three problems?
 - B. calculate rather than draw a graphic to get the answers?
6. Improper Fractions to Mixed Numbers (Fractions: Booklet 3)
- A. give the correct answer on two of three problems?
 - B. simplify the fraction in problem b?
 - C. calculate rather than draw a graphic to get the answers?

7. Equivalence (Fractions: Booklet 1), The Mighty One (Fractions: Booklet 4)
- A. fill in the correct numbers to show the values of one?

8. Equivalence: Calculating (Fractions: Booklet 4)
- A. fill in the correct numerator in two of three problems?
 - B. calculate rather than draw a graphic to get the answers?
 - C. use the short notation on both problems?

9. Simplification: Calculating (Fractions: Booklet 5)
- A. write in the multiplier in items a and c?
 - B. give the correct answer in items b and d?
 - C. give the correct answer in item e as a mixed number or an improper fraction?
 - D. identify the multiplier in item a as 1 on item f?

10. Associative and Commutative Properties (Multiplication: Booklet 2)
- A. fill in the numbers without calculation except for the last number?
 - B. identify item a as the Commutative Property?
 - C. identify item b as the Associative Property?

11. Parts of Wholes as Multiplication of Fractions (Fractions: Booklet 2)

A. add the numerators correctly and not the denominators?

B. identify the multiplication problem as $\frac{2}{5} \times 5 = \frac{10}{5}$?

If he scores a Yes on B, it shows ability to generalize to a new situation. This topic was briefly introduced in Fractions: Booklet 2.

12. Equivalence: Calculating (Fractions: Booklet 4)

A. pick out two of the three equal fractions?

13. Fractions as Parts of Wholes (Fractions: Booklet 2)

A. cut the group into three sections and shade two of them to get the answer in item a?

B. correctly fill in the number sentence in item b?

14. Parts of Wholes as Multiplication of Fractions (Fractions: Booklet 2)

A. show the multiplication problem as an addition problem?

Booklet Selection Guide based on results of Assessment: Part 1

If the student scores less than 20 points, excluding items 13 and 14, on Assessment: Part 1, remediate weak areas and retest. Fractions are complicated and difficult for many students. Be sure you are not moving too quickly. Give more practice. Do not give Assessment: Part 2 until this work is completed.

Readiness concepts: Items 1A, 5A, 6A, 6B, 9A - D, 10B, and 10C. Any of these items with a No marked on the score sheet need immediate reteaching. Refer back to the appropriate booklets. These skills are needed to complete this booklet. Reteach and retest before giving Assessment: Part 2.

Important concepts to review: 2A, 3A, 3B, 8A, and 12A. Any of these items marked No should be retaught concurrently with this booklet. These skills will be needed for Fractions: Booklet 7.

All Parts of Wholes topics are reviewed in the beginning of this booklet. If Items 4A, 4B, 11A, 11B, 13A, 13B and 14A are marked with No on the score sheet, complete the review section at the beginning of Fractions: Booklet 6, then retest these items to make sure they are mastered. Not having mastered these concepts will not stop you at this point. Give Assessment: Part 2 and begin the booklet if these are the only weak areas.

If Items 4A, 4B, 11B, 13A, 13B, and 14A are marked with Yes on the score sheet, you might consider skimming over some of the review parts at the beginning of this booklet.

Assessment Criteria for Assessment: Part 2

All criteria in Pre-Assessment: Part 2 are taught in Fractions: Booklet 6.

Can the student:

1. Multiplication

A. identify the multiplication of fractions problem shown in the drawing?

2. Multiplication - ones in the numerator

A. give the correct answer?

B. prove the answer with a drawing?

It is common to find that a student can answer A but not B. This is an indication of a memorized procedure without understanding. This is also true of Item 3.

3. Multiplication - numbers other than one in the numerator

- A. give the correct answer?
- B. prove the answer with a drawing?

4. Cancelling

- A. get the correct answer on both problems?
- B. show the cancelling work?
- C. use the standard short notation for this procedure?

The student may still be using the rewrite and reorder notation. This is acceptable but indicates the student needs more work to master the shorter, faster notation.

5. Commutative Property Meets Multiplication of Fractions

- A. explain why cancelling works?
0 point - Did not answer or the answer made no sense.
1 point - It works because you simplify the fractions before you multiply.
2 points - It works because you change the order of the numerators to create fractions that can be simplified before you multiply.
- B. give at least one of the properties, Associative or Commutative?

6. Multiplication

- A. give the correct answer in one of two problems? May do it the long way and simplify at the end.
- B. use cancelling? May use the rewrite, reorder strategy.
- C. use the short notation for cancelling?

7. Multiplying Mixed Numbers

- A. change both mixed numbers to the correct improper fraction?
- B. use cancelling?
- C. get the correct answer?

8. Multiplying Mixed Numbers

- A. get the correct answer on problem a?
- B. get the correct answer on problem b?

These last two problems don't have any shortcuts and have lots of arithmetic. We are testing to see if the student can navigate all the steps and get the correct answer in the end.

Booklet Selection Guide based on results of Assessment: Part 2

The student needs to score 15 points or more to pass. Each Yes counts for one point. If the student scores 15 points or more, move on to Fractions: Booklet 7 - Addition and Subtraction of Uncommon Denominators. If the student scores 13 points or less, Fractions: Booklet 6 is the correct booklet.

Can the student:

1. Addition of Like Fractions (Fractions: Booklet 2)

Simplification (Fractions: Booklet 5)

- Yes No A. add correctly on two of the three problems?
 Yes No B. simplify two of the three sums?

2. Subtraction of Like Fractions (Fractions: Booklet 2)

- Yes No A. subtract correctly in problem a?
 Yes No B. subtract correctly in problem b?
 Yes No C. Bonus Point: give correct answer to problem c?

3. Greater Than, Less Than, or Equal To (Fractions: Booklet 2)

- Yes No A. identify $\frac{1}{2}$ as being the least?
 Yes No B. identify $\frac{5}{8}$ as being the greatest?
 Yes No C. correctly order $\frac{6}{8}$, $\frac{2}{3}$, and $\frac{6}{10}$?
 Yes No D. Bonus Point: simplify $\frac{6}{8}$ or $\frac{6}{10}$?

4. Fractions as Parts of Wholes (Fractions: Booklet 2)

- Yes No A. divide the eight items into four equal groups of two?
 Yes No B. give the correct answer to all three problems?

5. Mixed Numbers to Improper Fractions (Fractions: Booklet 3)

- Yes No A. give the correct answer to two of three problems?
 Yes No B. calculate rather than draw a graphic to get the answers?

6. Improper Fractions to Mixed Numbers (Fractions: Booklet 3)

- Yes No A. give the correct answer on two of three problems?
 Yes No B. simplify the fraction in problem b?
 Yes No C. calculate rather than draw a graphic to get the answers?

7. Equivalence (Fractions: Booklet 1), The Mighty One (Fractions: Booklet 4)

- Yes No A. fill in the correct numbers to show the values of one?

8. Equivalence: Calculating (Fractions: Booklet 4)

- Yes No A. fill in the correct numerator in two of three problems?
 Yes No B. calculate rather than draw a graphic to get the answers?
 Yes No C. use the short notation on both problems?

9. Simplification: Calculating (Fractions: Booklet 5)

- Yes No A. write in the multiplier in problems a and c?
 Yes No B. give the correct answer in problems b and d?
 Yes No C. give the correct answer in problem e as a mixed number or an improper fraction?
 Yes No D. identify the multiplier in problem a as 1 on item f?

10. Associative and Commutative Properties (Multiplication: Booklet 2)

- Yes No A. fill in the correct numbers?
Yes No B. identify item a as the Commutative Property?
Yes No C. identify item b as the Associative Property?

11. Parts of Wholes as Multiplication of Fractions (Fractions: Booklet 2)

- Yes No A. add correctly in problem a?
Yes No B. identify the multiplication problem as $\frac{2}{5} \times 5 = \frac{10}{5}$?

12. Equivalence: Calculating (Fractions: Booklet 4)

- Yes No A. pick out two of the three equal fractions?

13. Fractions as Parts of Wholes (Fractions: Booklet 2)

- Yes No A. shade in six of the nine circles?
Yes No B. write $(9 \div 3) \times 2$ in problem b?

14. Parts of Wholes as Multiplication of Fractions (Fractions: Booklet 2)

- Yes No A. show the multiplication problem as an addition problem?

Items Correct = _____

Items Possible = 33

Notes:

Can the student:

1. Multiplication

Yes No A. identify the multiplication of fractions problem as $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$?

2. Multiplication - ones in the numerator

Yes No A. give the correct answer of $\frac{1}{6}$?

Yes No B. prove the answer with a drawing?

3. Multiplication - numbers other than one in the numerator

Yes No A. give the correct answer of $\frac{8}{15}$?

Yes No B. prove the answer with a drawing?

4. Cancelling

Yes No A. get the correct answer on both problems?

Yes No B. show the cancelling work?

Yes No C. use the standard short notation for this procedure?

5. Commutative Property Meets Multiplication of Fractions

A. explain why cancelling works?

0 point - Did not answer or the answer made no sense.

1 point - It works because you simplify the fractions before you multiply.

2 points - It works because you change the order of the numerators to create fractions that can be simplified before you multiply.

Yes No B. give at least one of the properties, Associative or Commutative?

6. Multiplication

Yes No A. give the correct answer in one of two problems? May do it the long way and simplify at the end.

Yes No B. use cancelling? May use the rewrite, reorder strategy.

Yes No C. use the short notation for cancelling?

7. Multiplying Mixed Numbers

Yes No A. change both mixed numbers to the correct improper fraction?

Yes No B. use cancelling?

Yes No C. get the correct answer?

8. Multiplying Mixed Numbers

Yes No A. get the correct answer on problem a?

Yes No B. get the correct answer on problem b?

Items Correct = _____

Items Possible = 19

78% needed to begin Fractions: Booklet 7

This is 15 or more Yes items.

11 - 13 Yes: review weak areas, retest, and move to Fractions: Booklet 7 when skills are improved.

Put a question mark next to anything you do not know how to do..

1. Solve. Simplify all answers.

a. $\frac{2}{3} + \frac{2}{3} =$

b. $1\frac{1}{4} + \frac{1}{4} =$

c. $1\frac{5}{8} + \frac{1}{8} =$

2. Solve.

Challenge.

a. $\frac{5}{6} - \frac{2}{6} =$

b. $1 - \frac{2}{3} =$

c. $2\frac{1}{2} - 1\frac{1}{4} =$

3. Put these fractions in order from the least to the greatest.

$\frac{2}{3}$

$\frac{6}{8}$

$\frac{6}{10}$


$\frac{5}{6}$

$\frac{1}{2}$

4. Solve and show:

 $\frac{1}{4}$ of 8 = _____

 $\frac{2}{4}$ of 8 = _____

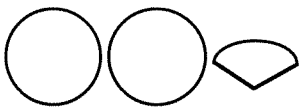
 $\frac{3}{4}$ of 8 = _____

5. Change to improper fractions.

a. $2\frac{1}{3} = \frac{\quad}{3}$

b. $3\frac{1}{2} =$

c. $4\frac{3}{5} =$



6. Change to mixed numbers.

a. $\frac{8}{3} =$

b. $\frac{22}{10} =$

c. $\frac{47}{6} =$



7. Fill in the blanks. $1 = \frac{\quad}{5} = \frac{\quad}{10} = \frac{\quad}{100} = \frac{200}{\quad}$

8. Solve.

a. $\frac{2}{3} = \frac{\quad}{12}$

b. $\frac{\quad}{4} = \frac{9}{12}$

c. $\frac{4}{7} = \frac{\quad}{28}$

9. Simplify. a. $\frac{14}{21} \div \frac{2}{3} = \frac{\quad}{\quad}$ b. $\frac{10}{100} \div \frac{3}{10} = \frac{\quad}{\quad}$ c. $1\frac{3}{15} = \frac{\quad}{\quad}$

f. What is the value of this number? _____

10. Solve. a. $8 \times 5 = 5 \times \square$ b. $2 \times 4 \times 3 = 3 \times \square \times 2 = \underline{\quad}$

Fill in the blanks. What property is shown by problem a? _____

What property is shown by problem b? _____


Properties: Associative Commutative Distributive Mighty One

11. Solve. a. $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \underline{\quad}$

b. Write it as a multiplication problem. _____ x _____ = _____

12. Circle all the fractions below that are equal to $\frac{3}{5}$.

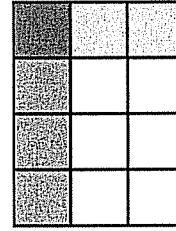
$\frac{6}{10}$ $\frac{8}{10}$ $\frac{9}{15}$ $\frac{3}{10}$ $\frac{9}{25}$ $\frac{15}{25}$

13. $\frac{2}{3}$ of 9 a. Use the picture to show $\frac{2}{3}$ of 

b. Write the number sentence. (_____ ÷ _____) x _____ = _____

14. $\frac{2}{3} \times 9 = \underline{\quad}$ Find the answer with iterative addition.

1. What multiplication problem does this show? _____



2. a. What is $\frac{1}{3}$ of $\frac{1}{2}$? _____ b. Draw a picture to prove the answer is correct.

3. Solve. a. $\frac{4}{5} \times \frac{2}{3} =$ _____ b. Draw a picture to prove the answer is correct.

4. Use cancelling to solve. a. $\frac{4}{7} \times \frac{7}{8} =$ _____ b. $\frac{15}{4} \times \frac{8}{9} =$ _____

5. a. Explain why cancelling works. _____

b. What property of mathematics is used? _____

6. Solve. a. $\frac{2}{3} \times \frac{15}{22} =$ _____ b. $\frac{8}{3} \times \frac{9}{4} =$ _____

7. Solve. $2\frac{2}{3} \times 2\frac{1}{4} =$ _____

8. Solve. a. $3\frac{2}{3} \times \frac{8}{9} =$ _____ b. $2\frac{1}{5} \times 3\frac{1}{5} =$ _____

Fractions: Booklet 7 Placement Assessment Guide

Purpose The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Assessment: Part 1 is review material from the last booklet and is used to determine student readiness for this booklet. Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

Prerequisites *Patterns in Arithmetic: Fractions - Booklet 4 (Equivalent Fractions) and Fractions - Booklet 5 (Simplifying Fractions)*

Materials Assessment: Part 1 - Worksheets 1 and 2, pages 8 and 9, and Assessment: Part 2, page 10
Score sheets, pages 6 and 7

Instructions Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

Do the assessment in two parts. Give Assessment: Part 1 and check it for readiness for this booklet. The answers are in the Answer Key. If he is not ready for this booklet, there is no point in giving Assessment: Part 2. If he passes all the readiness items, then give Assessment: Part 2.

After scoring Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

Assessment Guide This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc. Score sheets that match the Assessment Guide for both Assessment: Part 1 and Part 2 follow.

Assessment Criteria for Assessment: Part 1

The main skills needed to add and subtract fractions with unlike denominators are the abilities to find equivalent fractions, convert improper fractions to mixed fractions, and to simplify a fraction. If these skills are weak, they must be remediated before attempting this booklet. Use this Assessment to uncover areas that need reteaching.

Can the student:

1. Addition of Like Fractions (Fractions: Booklet 2)

- A. give the correct answer on two of the three problems?

A student who adds the denominators at this point is demonstrating lack of understanding of the function of a denominator.

2. Addition of Like Fractions (Fractions: Booklet 2) (Fractions: Booklet 3 - Improper to Mixed Fractions)

- A. give the correct improper fraction in two of the three problems?
B. give the correct mixed number in two of the three problems?

3. Addition of Mixed Numbers with Like Denominators (Fractions: Booklet 3)

- A. add the whole numbers and the fractions in two of the three problems?

Point out the example to the student. Do not explain. We are testing to see if the student can perform a series of operations to get the correct answer. First, the whole numbers and the fractions need to be added. This creates a whole number and an improper fraction. Then the student must simplify to create a mixed number with a whole number and a proper fraction.

- B. give the correct mixed number sum on two of the three problems?

4. Equivalence: Representational (Fractions: Booklet 4)

- A. draw the two-thirds and the five-sixths correctly?

The box should be divided into three or six roughly equal sections. Then two of the three and five of the six should be lightly shaded in.

- B. draw the change from two-thirds to twelfths and the change from five-sixths to twelfths?

In Item B, each third should be broken down into four smaller sections. Each sixth should be broken down into two sections.

- C. identify the multiplier as $\frac{4}{3}$?

5. Equivalence: Calculating (Fractions: Booklet 4)

- A. fill in the correct multiplier on two of the three problems?
B. fill in the correct missing numerators on problems a and b?
C. fill in the correct missing denominator on problem c?

6. Equivalence: Calculating (Fractions: Booklet 4)

- A. fill in the correct missing number on two of the three problems?

7. Equivalence: Calculating (Fractions: Booklet 4)

- A. explain how the multiplier is determined?

1 point - You divide by four.

2 points - You find the relationship between the denominators by dividing the three into the twelve. You then use that number as the multiplier. Or, you find the relationship between the denominators by dividing the three into the twelve and multiplying the four by both the three and the two.

- B. explain why the multiplier must be equal to one?

1 point - Because that is the only way to make the two fractions equal.

2 points - Because one times any number is the same number. To keep the two fractions equal, the only number you can multiply by is one.

8. Simplification: Calculating (Fractions: Booklet 5)

- A. give the correct answer in two of the three problems?
- B. show the Mighty One in two of the three problems?

Some students will do this part mentally. If the numbers in the Mighty One are left blank, ask him to fill them in. If he can not, he does not understand how this process works.

9. Simplification: Calculating (Fractions: Booklet 5)

- A. explain where the number that makes up the Mighty One when simplifying come from?
- B. state that the value of the number being divided into the numerator and denominator is one?

10. Simplification: Calculating (Fractions: Booklet 5)

- A. list the four common factors of 24 and 32 in problem a?
- B. circle the greatest of the common factors, eight, in the list in problem a?
- C. explain that the resulting fraction would not be in lowest terms or simplified all the way?
- D. explain that the greatest common factor will simplify the fraction to its lowest terms or to the fewest possible pieces?

Booklet Selection Guide based on results of Assessment: Part 1

The student should score 80% or better on the Assessment: Part 1 before moving into this booklet without any review. Reteach any items on which the student received a No on the score sheet. The Assessment Guide indicates where each concept is taught.

If the student is new to this program and gets No scores on the explanation sections 7A, 7B, 8B, 9A, and 9B but Yes on the items that require only calculation, you can proceed with this booklet. The student will be able to do the problems. It would be wise to go back and redo Fractions: Booklets 4 and 5 if the student does not understand how the multiplier and the greatest common factors used are equal to one. This is a very, very important concept. The Mighty One is a major tool for many arithmetic and algebraic procedures.

Assessment Criteria for Assessment: Part 2

Items in this part will be taught in Fractions: Booklet 7 unless otherwise indicated.

Can the student:

1. Addition of Unlike Fractions: Manipulative

- A. change the unit to fruits or any other common unit?

The general principle in combining any quantity is that the units must match. You must add inches to inches, feet to feet, and so on. This is the reason fractions must have common denominators in order to be combined. This is a very basic principle.

2. Addition of Like Fractions (Fractions: Booklet 2)

- A. add two fractions with like denominators?

3. Addition of Unlike Denominators - easy one

- A. find the correct common denominator?
- B. find the correct equivalent fraction to $\frac{1}{4}$?
- C. give the correct sum?
- D. simplify the sum?

4. Addition of Unlike Fractions - more difficult
- find a correct common denominator on both problems?
 - find the correct equivalent fraction in three of the four fractions?
 - convert the improper fraction sum into a mixed number in problem a?
 - find the correct answers on both problems?
 - give the sum in the lowest terms on problem b?
 - Bonus Point - simplify the $\frac{3}{8}$ to $\frac{1}{2}$ before adding?
5. Subtraction of Unlike Fractions - easy one
- find the least common denominator?
 - find the correct equivalent fraction in both numbers?
 - find the correct difference?
 - simplify the answer?
6. Subtraction of Unlike Fractions - more difficult
- find common denominators for both problems?
 - find the lowest common denominators for both problems?
 - find the correct equivalent fraction for all four fractions?
 - give the correct answer to one of the two problems?
7. Least Common Multiples
- explain why the denominators must match?
 Rubric: 1 point You can't add them if they don't match.
 2 points In order to add or subtract, the unit the numbers are in must match because you can not add unlike units.
8. Least Common Multiples
- list four common multiples of 2 and 3?
9. Least Common Multiples
- list three common multiples for 4 and 6?
 - give the least common denominator?
 - explain why it matters?
 Rubric: 1 point Because it makes it easier
 2 points It makes the arithmetic easier, and the answer is in a simpler form at the end.
10. Working with Mixed Numbers
- give the correct answer for problem a?
 - in problems b and c, regroup a whole number to increase the fraction to an improper fraction to make the top fraction larger?
 - get the correct answer on problem b?
 - get the correct answer on problem c?

This problem is quite difficult. A common denominator must be found first and then the top number regrouped.

Booklet Selection Guide based on results of Assessment: Part 2

If your student scores 75% or better on Assessment: Part 2, use the error analysis chart on page 5 and reteach any weak areas. You can then move on to Fractions: Booklet 8 (Division of Fractions) or switch your focus to decimals and complete the *Key to Decimals* - Booklets 2, 3, and 4.

If understanding of the Identity Property of Multiplication (Mighty One) is weak, now would be a good time to do Fractions: Booklet 4 and Fractions: Booklet 5.

If the score is 74% or less, this booklet is a good match for your student.

Whenever remediation is needed, rely upon the following process, which is used throughout the *Patterns in Arithmetic* series to develop understanding of a concept.

1. Introduce the concept with a manipulative. Orally discuss it. Build it. Verify it. Practice it. Repeat the experience with a different manipulative (oral manipulative).
2. Use manipulatives to explore the concept again. This time record it with pictures (pictorial/representation). Practice it. Use worksheets.
3. Record the problem with numbers (abstract/symbolic). This links the pictorial with the abstract.
4. Practice fluency.
5. Practice for speed.

Error Analysis for Assessment: Part 2

Circle items scored No on the score sheet on the chart below. It may show patterns that will tell you where to focus on reteaching if needed.

Common Denominators	Equivalence	Answer Accuracy	Simplification
12A 13A	13B	13C	13D
14A	14B	14D	14E
15A	15B	15C	15D
16A	16C	16D	
		20A, B, C, D	

Can the student:

1. Addition of Like Fractions

Yes No A. give the correct answer on two of the three problems?

2. Addition of Like Fractions

Yes No A. give the correct improper fraction in two of the three problems?

Yes No B. give the correct mixed number in two of the three problems?

3. Addition of Mixed Numbers with Like Denominators (Fractions: Booklet 3)

Yes No A. add the whole numbers and the fractions in two of the three problems?

Yes No B. give the correct mixed number sum in two of the three problems?

4. Equivalence: Representational (Fractions: Booklet 4)

Yes No A. draw the two-thirds and the five-sixths correctly?

Yes No B. draw the change of both fractions to twelfths?

Yes No C. identify the multiplier as $\frac{4}{3}$?

5. Equivalence: Calculating (Fractions: Booklet 4)

Yes No A. fill in the correct multiplier on two of the three problems?

Yes No B. fill in the correct missing numerators on problems a and b?

Yes No C. fill in the correct missing denominator on problem c?

6. Equivalence: Calculating (Fractions: Booklet 4)

Yes No A. fill in the correct missing number on two of the three problems?

7. Equivalence: Calculating (Fractions: Booklet 4)

0 1 2 A. explain how the multiplier is determined?

0 1 2 B. explain why the multiplier must be equal to one?

8. Simplification: Calculating (Fractions: Booklet 5)

Yes No A. give the correct answer in two of the three problems?

Yes No B. show the Mighty One in two of the three problems?

9. Simplification: Calculating (Fractions: Booklet 5)

Yes No A. explain the Mighty One when simplifying is in the form of a common factor?

Yes No B. state that the value of this number is one?

10. Simplification: Calculating (Fractions: Booklet 5)

Yes No A. list the four common factors as 1, 2, 4, and 8?

Yes No B. circle 8, the greatest of the common factors in the list?

Yes No C. explain that the resulting fraction would not be in lowest terms or simplified all the way?

Yes No D. explain that the greatest common factor will simplify the fraction to its lowest terms or to the fewest possible pieces?

Items Correct = ____ = _____% 19 points or more to pass

Items Possible = 24

Can the student:

1. Addition of Unlike Fractions: Manipulative
 Yes No A. change the unit to fruits or any other common unit?

2. Addition of Like Fractions (Fractions: Booklet 2)
 Yes No A. add the numerators and not the denominators?

3. Addition of Unlike Denominators - easy one
 Yes No A. find the correct common denominator of twelfths?
 Yes No B. find the correct equivalent fraction of $\frac{3}{2}$?
 Yes No C. give the correct sum of $\frac{4}{12}$?
 Yes No D. simplify the sum to $\frac{1}{3}$?

4. Addition of Unlike Fractions - more difficult
 Yes No A. find a correct common denominator on both problems?
 Yes No B. find the correct equivalent fraction in three of the four fractions?
 Yes No C. convert the improper fraction sum into a mixed number in problem a?
 Yes No D. find the correct answers on both problems?
 Yes No E. give the sum in the lowest terms on problem b?
 Yes No F. Bonus Point - simplify the $\frac{3}{8}$ to $\frac{1}{2}$ before adding?

5. Subtraction of Unlike Fractions - easy one
 Yes No A. find the least common denominator?
 Yes No B. find the correct equivalent fraction in both numbers?
 Yes No C. find the correct difference?
 Yes No D. simplify the answer?

6. Subtraction of Unlike Fractions - more difficult
 Yes No A. find common denominators for both problems?
 Yes No B. find the lowest common denominators for both problems?
 Yes No C. find the correct equivalent fraction for all four fractions?
 Yes No D. give the correct answer to one of the two problems?

7. Least Common Multiples
 0 1 2 A. explain why the denominators must match?

8. Least Common Multiples
 Yes No A. list four common multiples of 2 and 3?

9. Least Common Multiples
 Yes No A. list three common multiples for 4 and 6?
 Yes No B. give the least common denominator?
 0 1 2 C. explain why it matters?

10. Working with Mixed Numbers
 Yes No A. give the correct answer for problem a?
 Yes No B. regroup a whole number correctly in problems b and c?
 Yes No C. get the correct answer on problem b?
 Yes No D. get the correct answer on problem c?

Items Correct = = %
 Items Possible = 30

Assessment - Part 1 - Worksheet 1 Name _____ Date _____

Put a question mark next to any problem you do not know how to do.

1. Solve. a. $\frac{1}{3} + \frac{1}{3} =$ b. $\frac{2}{5} + \frac{1}{5} =$ c. $\frac{3}{11} + \frac{6}{11} =$

2. Solve and change any improper fractions to mixed numbers.

a. $\frac{4}{5} + \frac{3}{5} =$ b. $\frac{6}{7} + \frac{5}{7} =$ c. $\frac{4}{9} + \frac{7}{9} =$

3. Solve and change all improper fractions to mixed numbers.

Example:
$$\begin{array}{r} 3\frac{3}{4} \\ + 1\frac{3}{4} \\ \hline 4\frac{6}{4} = 5\frac{1}{2} \end{array}$$

a.
$$\begin{array}{r} 2\frac{3}{5} \\ + 1\frac{4}{5} \\ \hline \end{array}$$

b.
$$\begin{array}{r} 6\frac{2}{9} \\ + 5\frac{8}{9} \\ \hline \end{array}$$

c.
$$\begin{array}{r} 2\frac{3}{7} \\ + 5\frac{6}{7} \\ \hline \end{array}$$

4. Draw the change of:

a. $\frac{2}{3}$ to $\frac{1}{12}$

b. $\frac{5}{6}$ to $\frac{1}{12}$

c. What is the multiplier in problem a? _____

5. Fill in the blanks.

a. $\frac{1}{3} \times \frac{\quad}{\quad} = \frac{1}{27}$ b. $\frac{5}{7} \times \frac{\quad}{\quad} = \frac{1}{42}$ c. $\frac{8}{9} \times \frac{\quad}{\quad} = \frac{32}{\quad}$

6. Fill in the missing numbers.

a. $\frac{7}{9} = \frac{\quad}{63}$ b. $\frac{\quad}{5} = \frac{18}{30}$ a. $\frac{5}{6} = \frac{15}{\quad}$

7. a. How do you know what this number is? _____

$$\frac{2}{3} \times \frac{\quad}{x} = \frac{8}{12}$$

b. Why does this number have to be equal to one? _____

$$\frac{1}{6} \times \frac{3}{x} = \frac{3}{18}$$

8. Simplify these fractions. Show the Mighty One.

a. $\frac{15}{18} \div \frac{1}{1} = \text{---}$

b. $\frac{7}{21} \div \frac{1}{1} = \text{---}$

c. $\frac{35}{56} \div \frac{1}{1} = \text{---}$

9. a. Where does this number come from? _____

$$\frac{12}{16} \div \frac{4}{4} = \text{---}$$

b. What is the value of this number? _____

10. List the common factors of 24 and 32.

a. _____

b. Circle the common factor you would use to simplify $\frac{24}{32}$.

c. What would happen if you used four to simplify $\frac{24}{32}$?

$$\frac{24}{32} \div \frac{4}{4} = \text{---}$$

d. Why is the greatest common factor used to simplify a fraction instead of any factor? _____

Assessment - Part 2

Name _____ Date _____

Put a question mark next to any problem you do not know how to do.

1. If you add 2 oranges + 3 apples, you will have _____

2. Solve.

$$\begin{array}{r} \frac{2}{5} \\ + \frac{1}{5} \\ \hline \end{array}$$

3. Solve.

$$\begin{array}{r} \frac{1}{4} \\ + \frac{1}{12} \\ \hline \end{array}$$

4. Solve.

$$\begin{array}{r} \text{a. } \frac{3}{4} \\ + \frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{r} \text{b. } \frac{3}{5} \\ + \frac{3}{6} \\ \hline \end{array}$$

5. Solve.

$$\begin{array}{r} \frac{1}{3} \\ - \frac{1}{12} \\ \hline \end{array}$$

6. Solve.

$$\begin{array}{r} \text{a. } \frac{5}{6} \\ - \frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} \text{b. } \frac{5}{8} \\ - \frac{5}{12} \\ \hline \end{array}$$

7. Why must you find 'common denominators' to add fractions?

8. List four common multiples of 3 and 2. _____

9. a. When solving this fraction problem, $\frac{3}{4} + \frac{5}{6}$, list three common denominators that are possible. _____

b. What is the least common denominator? _____

c. Why does it matter? _____

10. Solve. a. 3

$$\begin{array}{r} - \frac{3}{4} \\ \hline \end{array}$$

b. $3 \frac{1}{4}$

$$\begin{array}{r} - 1 \frac{3}{4} \\ \hline \end{array}$$

c. $7 \frac{1}{3}$

$$\begin{array}{r} - 2 \frac{5}{8} \\ \hline \end{array}$$

Fractions: Booklet 8 Placement Assessment Guide

Purpose The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Assessment: Part 1 is review material from the past fraction booklets and is used to determine student readiness for this booklet. Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

Prerequisites *Patterns in Arithmetic*: Fractions - Booklets 3 - 7, or proficiency with proper and improper fractions, equivalence and simplification of fractions, common denominators, and multiplication of fractions

Materials Assessment: Part 1, page 7 and Assessment: Part 2, pages 8 and 9
Score sheets, pages 5 and 6
Colored pencils
Patterns blocks
Fraction manipulative

Instructions Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

Do the assessment in two parts. Give Assessment: Part 1 and check it for readiness for this booklet. The answers are in the Answer Key. If he is not ready for this booklet, there is no point in giving Assessment: Part 2. If he passes all the readiness items, then give Assessment: Part 2.

After scoring Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

Assessment Guide This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc. Score sheets that match the Assessment Guide for both Assessment: Part 1 and Part 2 follow.

Assessment Criteria for Assessment: Part 1

Can the student:

1. Changing Wholes (Fractions: Booklet 2)

- A. identify that the red block will be equal to the whole if the blue block is designated as $\frac{2}{3}$?
2. Improper Fractions to Mixed Numbers (Fractions: Booklet 3)
A. give the correct answer two of three times? Answers do not have to be simplified to be correct. We want to see if he can perform this operation.
3. Mixed Numbers to Improper Fractions (Fractions: Booklet 3)
A. give the correct answer two of three times? Answers do not have to be simplified to be correct. We want to see if he can perform this operation.
4. Equivalent Fractions (Fractions: Booklet 4)
A. supply the correct missing number in two of the three items?
5. Simplification of Fractions (Fractions: Booklet 5)
A. give the correct answer to three of the four problems?
6. Multiplication of Fractions (Fractions: Booklet 6)
A. give the correct unsimplified answer for two of the three items?
B. give the correct simplified answer for two of the three items?
7. Cancelling (Fractions: Booklet 6)
A. supply the correct simplified answer in two of the three problems?
B. use cancelling to simplify the numbers before he multiplies?
8. Multiplying Mixed Numbers (Fractions: Booklet 6)
A. supply the correct simplified answer in one of the two problems?
B. use cancelling to simplify the numbers before he multiplies?
9. Word Problem (Fractions: Booklet 6)
A. supply the correct answer?
10. Addition of Fractions with Unlike Denominators (Fractions: Booklet 7)
A. give the correct unsimplified answer two of three times?
B. give the correct simplified answer two of three times?
C. use forty-five as the common denominator on item c? If he used ninety, have him practice finding the least common factor of two numbers. This will be important when drawing division of fractions problems.

Booklet Selection Guide based on results of Assessment: Part 1

All of the skills in this assessment are needed in Division of Fractions. There must be a Yes on all items before proceeding. Any No items must be remediated before beginning this booklet. If the score is 75% or greater, you can probably begin the Division of Fractions Booklet and remediate needed items concurrently. This is not the case if he receives a No on multiplication of fractions, Items 6, 7, and 8. These items must be remediated before the booklet can be attempted. Begin with Fractions: Booklet 6.

Assessment Criteria for Assessment: Part 2

All items in this section are taught in Fractions: Booklet 8

Can the student:

1. Understand the question asked by a division problem?
 - A. give the answer, 'How many groups of 3 are in 12'?
 - B. give the answer, 'How many groups of $\frac{2}{3}$ are in 6'?
 - C. give the answer, 'What part of 4 fits into 3'?
 - D. circle $3 \div 4$?
2. Solving Problems with Pictures

Solve a division of fractions problem with a picture where the dividend is larger than the divisor?

 - A. give the correct answer on two of three problems using any method?
 - B. make a drawing to prove each answer that clearly shows both the dividend and the divisor?
 - C. show some kind of marking in the drawing that clearly indicates the counting of the number of divisor units in the dividend?
 - D. use a common denominator to clearly show each fraction?
 - E. explain that the remainder of $\frac{1}{7}$ is $\frac{1}{8}$ of the group of $\frac{2}{8}$ with a clear drawing to support the answer?
3. Solving Problems with Pictures

Solve a division of fractions problem with a picture where the divisor is larger than the dividend?

 - A. state that the quotient to $\frac{1}{4} \div \frac{1}{2}$ will be less than one?
 - B. give the answer that it will be less than one because $\frac{1}{2}$ is larger than $\frac{1}{4}$ and thus only part of the $\frac{1}{2}$ will fit into the $\frac{1}{4}$?
 - C. give the correct answer to two of three problems in items c, d, and e?
 - D. make a drawing to prove each answer that clearly shows both the dividend and the divisor?
 - E. clearly show the answer in the drawing?
4. Introducing Reciprocals
 - A. write a fraction and its reciprocal?
5. Using Multiplication to Solve Division Problems
 - A. explain why these two problems give equal answers? 2 Points: These two give equal answers because when you solve $12 \div \frac{1}{2}$, you are asking how many halves fit in twelve. Since there are two halves in each whole, it makes sense that the total number of halves would be gotten by multiplying 2×12 . 1 Point: Because they give the same answer.
6. Using Multiplication to Solve Division Problems
 - A. give the answer 'yes'?
 - B. explain or draw a clear picture to show that $\frac{3}{4} \div \frac{1}{2}$ is $1 \frac{1}{2}$, which is equal to $\frac{3}{4} \times 2$?
7. Standard Procedure
 - A. give the correct answer in two of the three problems?
8. Division with Mixed Numbers
 - A. give the correct answer in two of the three problems?

Booklet Selection Guide based on results of Assessment: Part 2

If the student can give correct answers when he calculates Items 2A, 3C, 4A, and 8A but is not able to explain or prove understanding with a clear drawing, then you have some choices:

Move on because he can do division of fractions and leave it at that.

Know that he does not understand the nature of division with fractions.

Or you can have him complete this booklet to investigate why the procedure works the way it does and develop understanding of the physical process that is occurring when fractions are divided. One teacher gave a student a choice of understanding division of fractions and working through this booklet or just being given the recipe for the standard procedure. This enlightened student chose to do this booklet. Completing this booklet may take three to four weeks for most students.

Whenever remediation is needed, rely upon the following process, which is used throughout the *Patterns in Arithmetic* series to develop understanding of a concept.

1. Introduce the concept with a manipulative. Orally discuss it. Build it. Verify it. Practice it. Repeat the experience with a different manipulative (oral manipulative).
2. Use manipulatives to explore the concept again. This time record it with pictures (pictorial/representation). Practice it. Use worksheets.
3. Record the problem with numbers (abstract/symbolic), which links the pictorial with the abstract.
4. Practice fluency.
5. Practice for speed.

Begin each lesson with a warm up-and review. Always end a lesson with a success before the student is tired. It is best to end while the student is still enjoying the lesson.

Ask questions or make statements, such as: “**Are you sure?**” or “**Build it.**” or “**What gave you the clue?**” or “**Show me how you got that.**” or “**Prove it.**” even when a student is correct. This is important to do often. Many students will ask an adult, “Am I right?” rather than answering definitively. Confidence in a student’s response must come from within. A student needs to self-check and have confidence in his or her ability and knowledge. Asking the student if he or she is right, even when correct, will encourage self-confidence and the ability to self-check.

Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. The dialogue you have with your student will be unique. What’s most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: “**How did you get that?**” “**Show me what you mean.**” “**Build a model of that.**” “**Tell me more so I can understand what you are saying.**”

Assessment: Part 1 Score Sheet

Name _____ Date _____

Can the student:

1. Changing Wholes (Fractions: Booklet 2)
Yes No A. identify that the red block will be equal to the whole?
2. Improper Fractions to Mixed Numbers (Fractions: Booklet 3)
Yes No A. give the correct answer two of three times?
3. Mixed Numbers to Improper Fractions (Fractions: Booklet 3)
Yes No A. give the correct answer two of three times?
4. Equivalent Fractions (Fractions: Booklet 4)
Yes No A. supply the correct missing number in two of the three items?
5. Simplification of Fractions (Fractions: Booklet 5)
Yes No A. give the correct answer to three of the four problems?
6. Multiplication of Fractions (Fractions: Booklet 6)
Yes No A. give the correct unsimplified answer for two of the three items?
Yes No B. give the correct simplified answer for two of the three items?
7. Cancelling (Fractions: Booklet 6)
Yes No A. supply the correct simplified answer in two of the three problems?
Yes No B. use cancelling to simplify the numbers before he multiplies?
8. Multiplying Mixed Numbers (Fractions: Booklet 6)
Yes No A. give the correct simplified answer in one of the two problems?
Yes No B. use cancelling to simplify the numbers before he multiplies?
9. Word Problem (Fractions: Booklet 6)
Yes No A. supply the correct answer?
10. Addition of Fractions with Unlike Denominators (Fractions: Booklet 7)
Yes No A. give the correct unsimplified answer two of three times?
Yes No B. give the correct simplified answer two of three times?
Yes No C. use forty-five as the common denominator on item c?

Items Correct = _____ = _____%

Items Possible = 15

Can the student:

1. Understand the question asked by a division problem?
 - Yes No A. give the answer, 'How many groups of 3 are in 12'?
 - Yes No B. give the answer, 'How many groups of $\frac{2}{3}$ are in 6'?
 - Yes No C. give the answer, 'What part of 4 fits into 3'?
 - Yes No D. circle $3 \div 4$?

2. Solving Problems with Pictures

Solve a division of fractions problem with a picture where the dividend is larger than the divisor?

 - Yes No A. give the correct answer on two of three problems using any method?
 - Yes No B. make a drawing to prove each answer that clearly shows both the dividend and the divisor?
 - Yes No C. show some kind of marking in the drawing that clearly indicates the counting of the number of divisor units in the dividend?
 - Yes No D. use a common denominator to clearly show each fraction?
 - Yes No E. explain that the remainder of $\frac{1}{2}$ is $\frac{1}{8}$ of the group of $\frac{2}{3}$ with a clear drawing to support the answer?

3. Solving Problems with Pictures

Solve a division of fractions problem with a picture where the divisor is larger than the dividend?

 - Yes No A. state that the quotient to $\frac{1}{4} \div \frac{1}{2}$ will be less than one?
 - Yes No B. explain why?
 - Yes No C. give the correct answer to two of three problems in items c, d, and e?
 - Yes No D. make a drawing to prove each answer that clearly shows both the dividend and the divisor?
 - Yes No E. clearly show the answer in the drawing?

4. Introducing Reciprocals
 - Yes No A. write a fraction and its reciprocal?

5. Using Multiplication to Solve Division Problems (for points see Assessment Criteria on page 5)
 - 2 1 0 pt. A. explain why these two problems give equal answers?

6. Using Multiplication to Solve Division Problems
 - Yes No A. give the answer yes?
 - Yes No B. explain or draw a clear picture to prove his answer?

7. Standard Procedure
 - Yes No A. give the correct answer in two of the three problems?

8. Division with Mixed Numbers
 - Yes No A. give the correct answer in two of the three problems?


Was he able to score Yes on items 2A, 3C, 4A, and 8A but No on many other items? See choices on page 4 in Booklet Selection Guide based on results of Assessment: Part 2

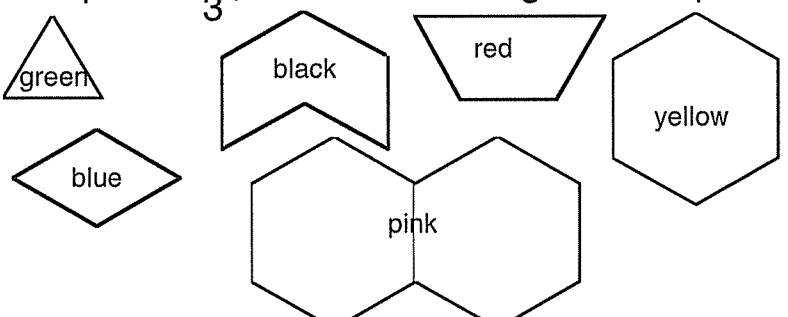
Items Correct = _____ = _____%

Items Possible = 21

Assessment - Part 1

Put a question mark next to any problem you do not know how to do.

1. If the  is equal to $\frac{2}{3}$, which of these figures is equal to one whole? _____



2. Change to an improper fraction. a. $3\frac{3}{4} = \underline{\hspace{2cm}}$ b. $6\frac{7}{8} = \underline{\hspace{2cm}}$ c. $7\frac{4}{9} = \underline{\hspace{2cm}}$

3. Change to a mixed number. a. $\frac{17}{5} = \underline{\hspace{2cm}}$ b. $\frac{29}{4} = \underline{\hspace{2cm}}$ c. $\frac{58}{8} = \underline{\hspace{2cm}}$

4. Fill in the missing numbers. a. $\frac{4}{5} = \frac{\hspace{1cm}}{15}$ b. $\frac{\hspace{1cm}}{4} = \frac{15}{20}$ c. $\frac{2}{3} = \frac{18}{\hspace{1cm}}$

5. Simplify. a. $\frac{32}{36} = \underline{\hspace{2cm}}$ b. $\frac{17}{34} = \underline{\hspace{2cm}}$ c. $\frac{19}{23} = \underline{\hspace{2cm}}$ d. $\frac{42}{54} = \underline{\hspace{2cm}}$

6. Multiply. a. $\frac{1}{3} \times \frac{1}{2} = \underline{\hspace{2cm}}$ b. $6 \times \frac{3}{4} = \underline{\hspace{2cm}}$ c. $\frac{4}{5} \times \frac{2}{3} = \underline{\hspace{2cm}}$

7. Use cancelling to solve. a. $\frac{4}{7} \times \frac{7}{8} = \underline{\hspace{2cm}}$ b. $\frac{2}{3} \times \frac{15}{22} = \underline{\hspace{2cm}}$ c. $\frac{8}{3} \times \frac{9}{4} = \underline{\hspace{2cm}}$

8. Multiply. a. $2\frac{2}{3} \times 2\frac{1}{4} = \underline{\hspace{2cm}}$ b. $7\frac{1}{2} \times 3\frac{2}{3} = \underline{\hspace{2cm}}$

9. If 25¢ is one-seventh the cost of a piece of ribbon, how much does the ribbon cost? _____

10. Solve. a.
$$\begin{array}{r} \frac{3}{4} \\ + \frac{2}{3} \\ \hline \end{array}$$

b.
$$\begin{array}{r} \frac{4}{5} \\ + \frac{1}{3} \\ \hline \end{array}$$

c.
$$\begin{array}{r} \frac{4}{9} \\ + \frac{3}{5} \\ \hline \end{array}$$

Assessment - Part 2 - Worksheet 1

Put a question mark next to any problem you do not know how to do.

1. What question does each problem ask?

a. $12 \div 3$ _____

b. $6 \div \frac{2}{3}$ _____

c. $3 \div 4$ _____

d. Circle the problem whose answer will be less than one.

2. Solve each problem. Make a drawing that clearly proves the answer.

a. $\frac{1}{2} \div \frac{1}{4} =$ _____

b. $6 \div \frac{2}{3} =$ _____

c. $\frac{3}{4} \div \frac{1}{3} =$ _____

d. $\frac{3}{4} \div \frac{2}{3} = 1 \frac{1}{8}$. Explain where the $\frac{1}{8}$ comes from. Why is it $\frac{1}{8}$?

3. Solve each problem. Make a drawing that clearly proves the answer.

a. Will the answer to $\frac{1}{4} \div \frac{1}{2}$ be greater than or less than one? _____

b. How do you know? _____

c. $\frac{1}{3} \div \frac{1}{2} =$ _____

Assessment - Part 2 - Worksheet 2

d. $\frac{1}{2} \div \frac{3}{4} =$ _____

e. $\frac{1}{4} \div \frac{3}{8} =$ _____

4. Write a fraction and its reciprocal. a. _____ b. _____

5. Explain why $12 \div \frac{1}{2} = 12 \times 2$. _____
_____6. Does the pattern shown in problem 5 work for this problem $\frac{3}{4} \div \frac{1}{2}$? _____

a. Prove it with a drawing.

b. Explain or draw why the 'invert and multiply' formula works in this division of fractions problem. _____

7. Solve these problems.

a. $\frac{3}{4} \div \frac{2}{3} =$ _____

b. $\frac{5}{6} \div \frac{4}{5} =$ _____

c. $\frac{3}{7} \div \frac{7}{12} =$ _____

8. Solve these problems.

a. $5 \div 2 \frac{1}{2} =$ _____

b. $2 \frac{2}{3} \div 1 \frac{1}{4} =$ _____

c. $6 \frac{4}{5} \div 9 \frac{1}{2} =$ _____


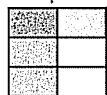
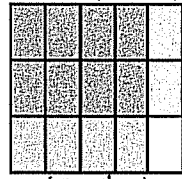
Fractions - Booklet 6

Assessment: Part 1 - Worksheet 1

- Solve. Simplify all answers.
 - a. $\frac{2}{3} + \frac{2}{3} = \frac{4}{3}$
 - b. $1\frac{1}{4} + \frac{1}{4} = 1\frac{2}{4} = 1\frac{1}{2}$
 - c. $1\frac{5}{8} + \frac{1}{8} = \frac{6}{8} = \frac{3}{4}$
- a. $\frac{5}{6} - \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$
 - b. $1 - \frac{2}{3} = \frac{1}{3}$
 - c. $2\frac{1}{2} - 1\frac{1}{4} = 1\frac{2}{4} = 1\frac{1}{2}$
- Put these fractions in order from the least to the greatest.

$\frac{2}{3}$	$\frac{6}{8}$	$\frac{6}{10}$	$\frac{5}{6}$	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{6}{8}$	$\frac{5}{6}$
- Solve and show:
 - $\frac{1}{4}$ of 8 = 2
 - $\frac{2}{4}$ of 8 = 4
 - $\frac{3}{4}$ of 8 = 6
- Change to improper fractions.
 - a. $2\frac{1}{3} = \frac{7}{3}$
 - b. $3\frac{1}{2} = \frac{7}{2}$
 - c. $4\frac{3}{5} = \frac{23}{5}$
- Change to mixed numbers.
 - a. $\frac{8}{3} = 2\frac{2}{3}$
 - b. $\frac{22}{10} = 2\frac{2}{10} = 2\frac{1}{5}$
 - c. $\frac{47}{6} = 7\frac{5}{6}$
- Fill in the blanks. $1 = \frac{5}{5} = \frac{10}{10} = \frac{100}{100} = \frac{200}{200}$
- Solve.
 - a. $\frac{2}{3} = \frac{8}{12}$
 - b. $\frac{3}{4} = \frac{9}{12}$
 - c. $\frac{4}{7} = \frac{16}{28}$

Assessment: Part 2

- What multiplication problem does this show? $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

- a. What is $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{6}$ b. Draw a picture to prove the answer is correct.
 
- Solve. a. $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$ b. Draw a picture to prove the answer is correct.
 
- Use cancelling to solve. a. $\frac{4}{1} \times \frac{2}{2} = \frac{1}{2}$ b. $\frac{5}{4} \times \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$
- a. Explain why cancelling works.

The order of the numerators can be changed to create fractions that can be simplified before multiplying.
- b. What property of mathematics is used?

Commutative property of multiplication
- Solve. a. $\frac{4}{1} \times \frac{2}{2} = \frac{1}{2}$ b. $\frac{2}{1} \times \frac{3}{1} = \frac{6}{1} = 6$
- Solve. $2\frac{2}{3} \times 2\frac{1}{4} = \frac{8}{3} \times \frac{9}{4} = \frac{72}{12} = 6$
- Solve. a. $3\frac{2}{3} \times \frac{8}{9} = 3\frac{7}{27}$ b. $2\frac{1}{5} \times 3\frac{1}{5} = \frac{7}{5}$
- $\frac{11}{3} \times \frac{8}{9} = \frac{88}{27}$ $\frac{11}{5} \times \frac{16}{5} = \frac{176}{25}$

Answer Key: Fractions Placement

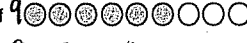
Assessment: Part 1 - Worksheet 2

- Simplify.
 - a. $\frac{14}{21} \div \frac{7}{7} = \frac{2}{3}$
 - b. $\frac{10}{100} \div \frac{10}{10} = \frac{1}{10}$
 - c. $1\frac{3}{15} = 1\frac{1}{5}$
 - f. What is the value of this number? 1
- Solve. a. $8 \times 5 = 5 \times 8$ b. $2 \times 4 \times 3 = 3 \times 4 \times 2 = 24$

Fill in the blanks. What property is shown by problem a? commutative

What property is shown by problem b? associative

Properties: Associative Commutative Distributive Mighty One
- Solve. a. $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{10}{5} = 2$
 - b. Write it as a multiplication problem. $\frac{2}{5} \times 5 = \frac{10}{5} = 2$
- Circle all the fractions below which are equal to $\frac{3}{5}$.





$\frac{6}{10}$ $\frac{8}{10}$ $\frac{9}{15}$ $\frac{3}{10}$ $\frac{9}{25}$ $\frac{15}{25}$
- $\frac{2}{3}$ of 9 a. Use the picture to show $\frac{2}{3}$ of 9.
 
 - b. Write the number sentence. $(9 \div 3) \times 2 = 6$
- $\frac{2}{3} \times 9 = 6$ Find the answer with iterative addition.
 $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{18}{3} = 6$

Fractions - Booklet 7

Assessment - Part 1 - Worksheet 1

- Solve. a. $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ b. $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$ c. $\frac{3}{11} + \frac{6}{11} = \frac{9}{11}$
- Solve and change any improper fractions to mixed numbers.
 - a. $\frac{4}{5} + \frac{3}{5} = \frac{7}{5} = 1\frac{2}{5}$
 - b. $\frac{6}{7} + \frac{5}{7} = \frac{11}{7} = 1\frac{4}{7}$
 - c. $\frac{4}{9} + \frac{7}{9} = \frac{11}{9} = 1\frac{2}{9}$
- Solve and change all improper fractions to mixed numbers.

Example: $3\frac{3}{4} + 1\frac{3}{4} = 4\frac{6}{4} = 5\frac{2}{4} = 5\frac{1}{2}$


 - a. $2\frac{3}{5} + 1\frac{4}{5} = 3\frac{7}{5} = 4\frac{2}{5}$
 - b. $6\frac{2}{9} + 5\frac{8}{9} = 11\frac{10}{9} = 12\frac{1}{9}$
 - c. $2\frac{3}{7} + 5\frac{6}{7} = 7\frac{9}{7} = 8\frac{2}{7}$
- Draw the change of:
 - a. $\frac{2}{3}$  to $\frac{4}{6}$ 
 - b. $\frac{5}{6}$  to $\frac{10}{12}$ 
 - c. What is the multiplier in problem a? $\frac{4}{3}$
- Fill in the blanks.
 - a. $\frac{1}{3} \times \frac{9}{9} = \frac{9}{27}$
 - b. $\frac{5}{7} \times \frac{6}{6} = \frac{30}{42}$
 - c. $\frac{8}{9} \times \frac{4}{4} = \frac{32}{36}$
- Fill in the missing numbers.
 - a. $\frac{7}{9} = \frac{49}{63}$
 - b. $\frac{3}{5} = \frac{18}{30}$
 - a. $\frac{5}{6} = \frac{15}{18}$

Assessment - Part 1 - Worksheet 2

7. a. How do you know what this number is? $12 \div 3 = 4$ so $4 \times \frac{2}{3} = \frac{8}{3}$ or $\frac{2}{3} \times \frac{4}{1} = \frac{8}{3}$
 $3 \times 4 = 12$ and the multiplier has to equal one, also $2 \times 4 = 8$.
- b. Why does this number have to be equal to one? Because of the Identity Property of Multiplication. One times any number is the same number.
8. Simplify these fractions. Show the Mighty One.
- a. $\frac{15}{18} = \frac{5}{6}$ b. $\frac{7}{21} = \frac{1}{3}$ c. $\frac{35}{56} = \frac{5}{8}$
9. a. Where does this number come from? It has to equal and 4 is the common factor of 12 and 16.
 $\frac{12}{16} \div \frac{4}{4} = \frac{3}{4}$
- b. What is the value of this number? The value of the number is one.
10. List the common factors of 24 and 32.
- a. 1 2 4 (8)
- b. Circle the common factor you would use to simplify $\frac{24}{32}$.
- c. What would happen if you used four to simplify $\frac{24}{32}$?
 $\frac{24}{32} \div \frac{4}{4} = \frac{6}{8}$ The answer would not be in the lowest terms or simplified all the way.
- d. Why is the Greatest Common Factor used to simplify a fraction instead of any factor? The GCF will simplify the fraction to its lowest term or to the fewest pieces.

Fractions - Booklet 8

Assessment - Part 1

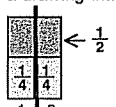
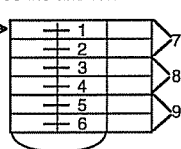
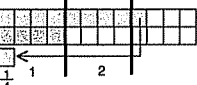
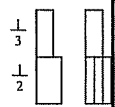
1. If the  is equal to $\frac{2}{3}$, which of these figures is equal to one whole? red
2. Change to an improper fraction. a. $3\frac{3}{4} = \frac{15}{4}$ b. $6\frac{7}{8} = \frac{55}{8}$ c. $7\frac{4}{9} = \frac{67}{9}$
3. Change to a mixed number. a. $\frac{17}{5} = 3\frac{2}{5}$ b. $\frac{29}{4} = 7\frac{1}{4}$ c. $\frac{58}{8} = 7\frac{1}{4}$
4. Fill in the missing numbers. a. $\frac{4}{5} = \frac{12}{15}$ b. $\frac{3}{4} = \frac{15}{20}$ c. $\frac{2}{3} = \frac{18}{27}$
5. Simplify. a. $\frac{32}{36} = \frac{8}{9}$ b. $\frac{17}{34} = \frac{1}{2}$ c. $\frac{19}{23} = \frac{19}{23}$ d. $\frac{42}{54} = \frac{7}{9}$
6. Multiply. a. $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ b. $6 \times \frac{3}{4} = 4\frac{1}{2}$ c. $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$
7. Use cancelling to solve. a. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$ b. $\frac{1}{2} \times \frac{5}{11} = \frac{5}{11}$ c. $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$
8. Multiply. a. $2\frac{2}{3} \times 2\frac{1}{4} = 6$ b. $7\frac{1}{2} \times 3\frac{2}{3} = 27\frac{1}{2}$
9. If 25¢ is one seventh cost of a piece of ribbon, how much does the ribbon cost? _____
10. Solve. a. $\frac{3}{4} + \frac{2}{3} = \frac{17}{12} = 1\frac{5}{12}$ b. $\frac{4}{5} + \frac{1}{3} = \frac{17}{15} = 1\frac{2}{15}$ c. $\frac{4}{9} + \frac{3}{5} = \frac{47}{45} = 1\frac{2}{45}$

Assessment - Part 2

Put a question mark next to any problem you do not know how to do.

1. If you add 2 oranges + 3 apples, you will have 5 fruits
2. Solve. $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$
3. Solve. $\frac{1}{4} + \frac{1}{12} = \frac{4}{12} + \frac{1}{12} = \frac{5}{12}$
4. Solve. a. $\frac{3}{4} = \frac{9}{12}$ b. $\frac{3}{5} = \frac{18}{30}$
 $\frac{2}{3} = \frac{8}{12}$ $\frac{3}{6} = \frac{15}{30}$
 $\frac{17}{12} = \frac{15}{12}$ $\frac{33}{30} = 1\frac{3}{30} = 1\frac{1}{10}$
5. Solve. $\frac{1}{3} = \frac{4}{12}$ 6. Solve. a. $\frac{5}{6} = \frac{10}{12}$ b. $\frac{5}{8} = \frac{15}{24}$
 $\frac{1}{12} = \frac{1}{12}$ $\frac{3}{4} = \frac{9}{12}$ $\frac{5}{12} = \frac{10}{24}$
 $\frac{3}{12} = \frac{1}{4}$ $\frac{1}{12}$ $\frac{5}{24}$
7. Why must you find 'common denominators' to add fractions?
The unit numbers must match because you can't add or subtract unlike units.
8. List four common multiples of 3 and 2. 6, 12, 18, 24
9. a. When solving this fraction problem, $\frac{3}{4} + \frac{5}{6}$, list three common denominators that are possible? 12, 24, 36
- b. What is the Least Common Denominator? 12
- c. Why does it matter? The arithmetic is easier and the answer is in the simplest form.
10. Solve. a. $\frac{2}{3} - \frac{3}{4} = \frac{8}{12} - \frac{9}{12} = -\frac{1}{12}$
- b. $\frac{7}{8} - \frac{1}{4} = \frac{7}{8} - \frac{2}{8} = \frac{5}{8}$
- c. $\frac{6}{7} - \frac{1}{3} = \frac{18}{21} - \frac{7}{21} = \frac{11}{21}$

Assessment - Part 2 - Worksheet 1

1. What question does each problem ask?
 a. $12 \div 3$ How many groups of 3 are in 12?
 b. $6 \div \frac{2}{3}$ How many groups of $\frac{2}{3}$ are in 6?
 c. $3 \div 4$ What part of 4 fits into 3?
 d. Circle the problem whose answer will be less than one.
2. Solve each problem. Make a drawing that clearly proves the answer.
- a. $\frac{1}{2} \div \frac{1}{4} = 2$ 
- b. $6 \div \frac{2}{3} = 9$ 
- c. $\frac{3}{4} \div \frac{1}{3} = 2\frac{1}{4}$ 
- d. $\frac{3}{4} \div \frac{2}{3} = 1\frac{1}{8}$. Explain where the $\frac{1}{8}$ comes from. Why is it $\frac{1}{8}$?
The remainder of $\frac{1}{4}$ is $\frac{1}{8}$ of the group of $\frac{2}{3}$.
 The new whole = $\frac{8}{12} = \frac{2}{3}$.
3. Solve each problem. Make a drawing that clearly proves the answer
- a. Will the answer $\frac{1}{4} \div \frac{1}{2}$ be greater than or less than one? less
- b. How do you know? Because $\frac{1}{2}$ is greater than $\frac{1}{4}$, so only part of the $\frac{1}{2}$ will fit in the $\frac{1}{4}$.
- c. $\frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$ 

Assessment - Part 2 - Worksheet 2

d. $\frac{1}{2} \div \frac{3}{4} = \frac{4}{6} = \frac{2}{3}$



e. $\frac{1}{4} \div \frac{3}{8} = \frac{8}{12} = \frac{2}{3}$

4. Write a fraction and its reciprocal. a. $\frac{2}{3}$ b. $\frac{3}{2}$

5. Explain why $12 \div \frac{1}{2} = 12 \times 2$. When solving $12 \div \frac{1}{2}$ you are asking how many halves fit in 12. Since there are 2 halves in each whole the total number of halves is solved by 12×2 .

6. Does the pattern shown in problem 5 work for this problem $\frac{3}{4} \div \frac{1}{2}$? yes

a. Prove it with a drawing.



b. Explain or draw why the 'invert and multiply' formula works in this division of fractions problem. when dividing $\frac{3}{4}$ by $\frac{1}{2}$, it is equal to $1\frac{1}{2}$, which is the same as $\frac{3}{4} \times 2$.

7. Solve these problems.

a. $\frac{3}{4} \div \frac{2}{3} = 1\frac{1}{2}$ b. $\frac{5}{6} \div \frac{4}{5} = 1\frac{1}{24}$ c. $\frac{3}{7} \div \frac{7}{12} = \frac{36}{49}$

8. Solve these problems.

a. $5 \div 2\frac{1}{2} = 2$ b. $2\frac{2}{3} \div 1\frac{1}{4} = 2\frac{2}{15}$ c. $6\frac{4}{5} \div 9\frac{1}{2} = \frac{68}{95}$

Patterns in Arithmetic: Fractions Placement PDF
Booklets 6, 7, and 8

Parent/Teacher Guide

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