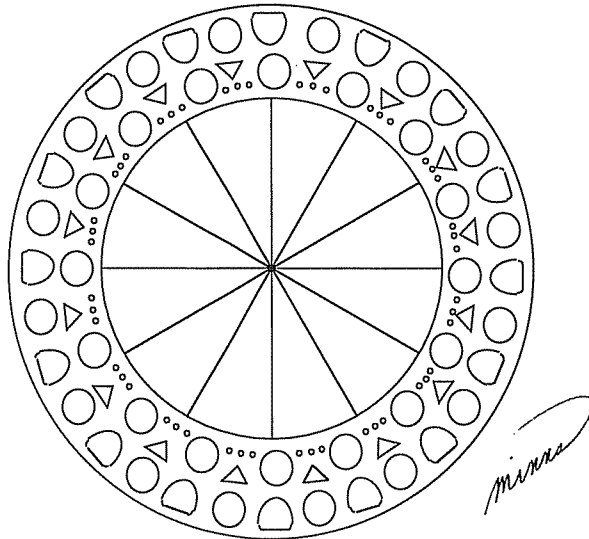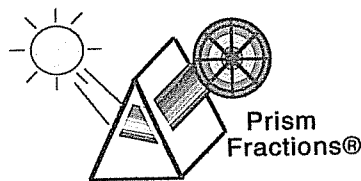# Patterns in Arithmetic
Fractions Placement PDF
Booklets 1 and 2
Parent/Teacher Guide

Booklet 1 - Basic Concepts
Booklet 2 - Developing Concepts and Beginning Operations

Prism
Fractions®

By Alysia Krafel, Susan Carpenter, and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Fractions Placement PDF - Booklets 1 and 2

Student Placement Assessment worksheets are located at the end of each Assessment Guide

Booklet 1 - Basic Concepts
This booklet is for seven to nine-year-old students (third grade).
Prerequisite: Free Explorations with fraction manipulative and pattern blocks
This booklet:
- works with concrete models to teach what a fraction is, how to read and write fraction notation, and begins equivalent fractions all using a student made booklet.

Booklet 2 - Developing Concepts and Beginning Operations
This booklet is designed for eight to ten-year-old students (fourth grade).
Prerequisite: Fractions: Booklet 1 or previous instruction in basic fractions
This booklet:
- works more deeply on the concept of a fraction and its relationship to the whole and that the definition of the whole can change.
- develops procedures for adding and subtracting like fractions and multiplication of a fraction by a whole number.

Answer Keys for Fractions: Booklet 1 and 2 Placement Assessments are at the end of the Assessment Guides.

Fractions: Booklet 1 Placement Assessment Guide

Purpose The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. The Assessment is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

Prerequisites None

Materials Assessment - Worksheets 1 - 4, pages 7 - 10
Score sheets, pages 5 and 6
Prism Fractions or fraction circles
Pattern blocks

Note If you do not have the pink or black pattern blocks in your set, make them with pattern blocks; directions are on page 4.

The student may use pattern blocks and fraction circles or Prism Fractions Circles, to complete the test. Give him at least fifteen minutes of free exploration time with the materials before the test.

Instructions Instruct the student to attempt all the problems. If the student does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

It is acceptable to read the items to a student. We are assessing math, not reading. But, do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

After scoring the assessment, use results with the Booklet Selection Guide to place your student. The answers are in the Answer Key.

Assessment Guide This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc.
Score sheets that match the Assessment Guide for the Assessment follow.
All criteria are from Fractions: Booklet 1

Assessment Criteria Can the student:
1. My Fractions Book: Beginning
   A. use a fraction manipulative to identify the one-third piece?

2. My Fractions Book: Beginning
      A. use the fraction manipulative to identify the one-fourth piece?
      B. indicate it takes four-fourths to make a whole?

3. My Fractions Book: Beginning
      A. identify the one-sixth piece?
      B. indicate it takes six-sixths to make a whole?

4. Sorting Fractions
      A. demonstrate that he understands that the two halves must be of equal size by picking out the three designs that are cut into two equal parts?

5. My Fractions Book: Beginning
      A. write the name of the fraction given in standard fraction notation?

6. Greater Than, Less Than
      A. use a greater than, less than symbol to show that one-fourth is less than three-fourths?
The test is to see if he can use the manipulative to determine the relative sizes of the fractional units given.

7. Numerators Greater Than One
      A. divide the rectangle into four roughly equal parts?
      B. shade in three of the four parts?

8. Greater Than, Less Than
      A. choose the larger fraction from a pair of two unlike fractions with ones in both numerators?

9. Greater Than, Less Than
      A. choose the larger fraction from a pair of two unlike fractions with different numbers in the numerator?
      B. Rubric for 'How do you know?'
           1 point: I measured and $\frac{4}{5}$ is bigger than $\frac{3}{6}$.
           2 points: Fifths are bigger than sixths and four is bigger than three so four-fifths must be larger than three-sixths.

10. Changing Wholes: Beginning
      A. choose the correct whole given the size of the one-sixth block?
Give him pattern blocks to solve this problem. This item tests his understanding of the relationship between parts and wholes in fractions. He must choose the larger block that takes six greens to cover it.

11. Changing Wholes: Beginning
      A. use pattern blocks to find the value of the blue block if yellow equals one?
      B. find the value of the blue block if the yellow equals one-half?

12. Changing Wholes: Beginning
      A. shade in half of each pizza?
      1 point - The wholes are not the same.
      2 points - The size of the half is made by the size of the whole, or if the whole is bigger, then the half will be bigger too.
This is to test if he understands that the whole can be defined as anything you like. Once that definition is made, all other fractional units are established by their relationship to the whole.

13. My Fractions Book: Equivalence
      A. use the manipulative to match several smaller pieces to the size of the $\frac{1}{2}$ piece?
      B. list at least two of the four equal fractions possible?
Have him use Fraction circles or Prism Fractions. This is to test if he understands that 'equal' means covering the same area. Can he match up a greater number of smaller pieces to cover the exact same area that the larger one-half piece covers?

14. My Fractions Book: Equivalence
      A. use the manipulative to match the nine-twelfths to the three-fourths?

15. My Fractions Book: Equivalence
      A. use the manipulative to match two-sixths to the one-third?

16. Equivalence: Recording
      A. use the manipulative to determine four halves would cover two wholes?
      B. extend the number pattern to fill in missing numbers?

17. My Fractions Book: Number Lines
      A. identify that the arrow points to one-third in the problem?
To interpret a number line, the student must first understand that the space between the zero and the one is the whole, and that the number of equal sections the space between zero and one is divided into determines the fractional part. Many students have difficulty with fractional number lines.

18. My Fractions Book: Number Lines
      A. identify that the arrow points to three-fifths?

19. Numerators Greater Than One: Number Lines
      A. identify that the arrow points to three-fourths?
If he gave the answer three-eighths, he counted the spaces on the whole number line, not just the spaces between the zero and the one. This is a very common error.

20. My Fractions Book: Number Lines
      A. use the number line to identify equal fractions by drawing a line from one number line to a second number line?
Make sure he does not use the fraction manipulative to answer this question. He must draw in the line.

Booklet Selection Guide based on results of the Assessment
21 points or more: Move to Fractions: Booklet 2. Remediate on number lines and equal fractions if either of those areas are marked with a No.
A score of 11 - 20 indicates beginner level understanding of basic fractions concepts.
For a student of age nine or less, do the entire Fractions: Booklet 1.

A student who is age ten or more should do a modified Fractions: Booklet 1. You can skip items that he got correct on the Assessment. Focus on the My Fractions Book especially on the sections on equivalent fractions and number lines. You can skip the sections on Changing Wholes and Greater Than, Less Than, as these are strongly covered in Fractions: Booklet 2. Reassess and begin instruction in Fractions: Booklet 2.

A score of less than ten points indicates this booklet is the correct place to begin for a student of any age.

Whenever remediation is needed, repeat the following process, which is used throughout the *Patterns in Arithmetic* series to develop understanding of a concept.
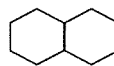
1. Introduce the concept with a manipulative. Orally discuss it. Build it. Verify it. Practice it. Repeat the experience with a different manipulative (oral manipulative).
2. Use manipulatives to explore the concept again. This time record it with pictures (pictorial/representation). Practice it. Use worksheets.
3. Record the problem with numbers (abstract/symbolic). This links the pictorial with the abstract.
4. Practice fluency.
5. Practice for speed.

Ask questions or make statements, such as: **"Are you sure?"** or **"Build it."** or **"What gave you the clue?"** or **"Show me how you got that."** or **"Prove it."** even when a student is correct. This is important to do often. Many students will ask an adult, "Am I right?" rather than answering definitively. Confidence in a student's response must come from within. A student needs to self-check and have confidence in his or her ability and knowledge. Asking the student if he or she is right, even when correct, will encourage self-confidence and the ability to self-check.

Pattern block directions for pink or black blocks:
Pink: Glue two yellow hexagons together to make this shape.
Paint pink if you wish.

Black: Glue two blue rhombi together to make this shape.
Paint black if you wish.

Assessment Score Sheet Name _____ Date _____

Can the student:

1. My Fractions Book: Beginning
    Yes No   A. identify the one-third piece?

2. My Fractions Book: Beginning
    Yes No   A. identify the one-fourth piece?
    Yes No   B. indicate that it takes four-fourths to make a whole?

3. My Fractions Book: Beginning
    Yes No   A. identify the one-sixth piece?
    Yes No   B. indicate that it takes six-sixths to make a whole?

4. Sorting Fractions
    Yes No   A. choose the three correct pictures of a figure divided into halves?

5. My Fractions Book: Beginning
    Yes No   A. write the name of the fraction, one-third?

6. Greater Than, Less Than
    Yes No   A. use a greater than, less than symbol to show that $\frac{1}{4} < \frac{3}{4}$?

7. Numerators Greater Than One
    Yes No   A. divide the rectangle into four roughly equal parts?
    Yes No   B. shade in three of the four parts?

8. Greater Than, Less Than
    Yes No   A. choose the larger fraction from a pair of two unlike fractions with ones in both numerators?

9. Greater Than, Less Than
    Yes No   A. choose the larger fraction from a pair of two unlike fractions with different numbers in the numerators?
    1 pt. 2 pts.   B. 'How do you know?'

10. Changing Wholes: Beginning
    Yes No   A. choose the correct whole given the size of the one-sixth block?

11. Changing Wholes: Beginning
    Yes No   A. find the value of the blue block if yellow equals one?
    Yes No   B. find the value of the blue block if the yellow equals one-half?

12. Changing Wholes: Beginning
    Yes No   A. shade in half of each pizza?
    1 pt. 2 pts.   B. Explain–use the rubric

13. My Fractions Book: Equivalence
   Yes No A. match several smaller pieces to the size of the one-half piece?
   Yes No B. list at least two of the four equal fractions possible?

14. My Fractions Book: Equivalence
   Yes No A. match nine-twelfths to the three-fourths?

15. My Fractions Book: Equivalence
   Yes No A. match two-sixths to the one-third?

16. Equivalence Recording
   Yes No A. determine four halves cover two wholes?
   Yes No B. extend the number pattern to fill in missing numbers?

17. My Fractions Book: Number Lines
   Yes No A. identify that the arrow points to one-third?

18. My Fractions Book: Number Lines
   Yes No A. identify that the arrow points to three-fifths?

19. Numerators Greater Than One: Number Lines
   Yes No A. identify that the arrow points to three-fourths?

20. My Fractions Book: Number Lines
   Yes No A. use the number line to identify equivalent fractions?

1 point per Yes

Items Correct = _____ = _____%
Items Possible =   30

For placement information see Booklet Selection Guide, page 4

Assessment - Worksheet 1

Assesses basic concept in:

A. Fractional units using a manipulative
B. Changing sizes of the whole
C. Fractional units on a number line
D. Equal fractions with a manipulative and number lines

A familiar fraction manipulative such as Fraction Circles or Prism Fractions and pattern blocks must be available to the student. The examiner must observe the student throughout the entire assessment. Give assistance only in reading. Note if the student uses the manipulative or not on each item.
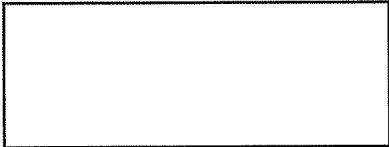
Put a question mark next to anything you do not know yet.

1. What is the color of the 1/3 piece in your set? _____

2. Cover the whole with fourths. How many fourths does it take to cover a whole?
_____

3. Cover the whole with sixths. How many sixths does it take to cover a whole?
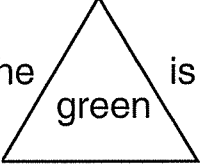_____

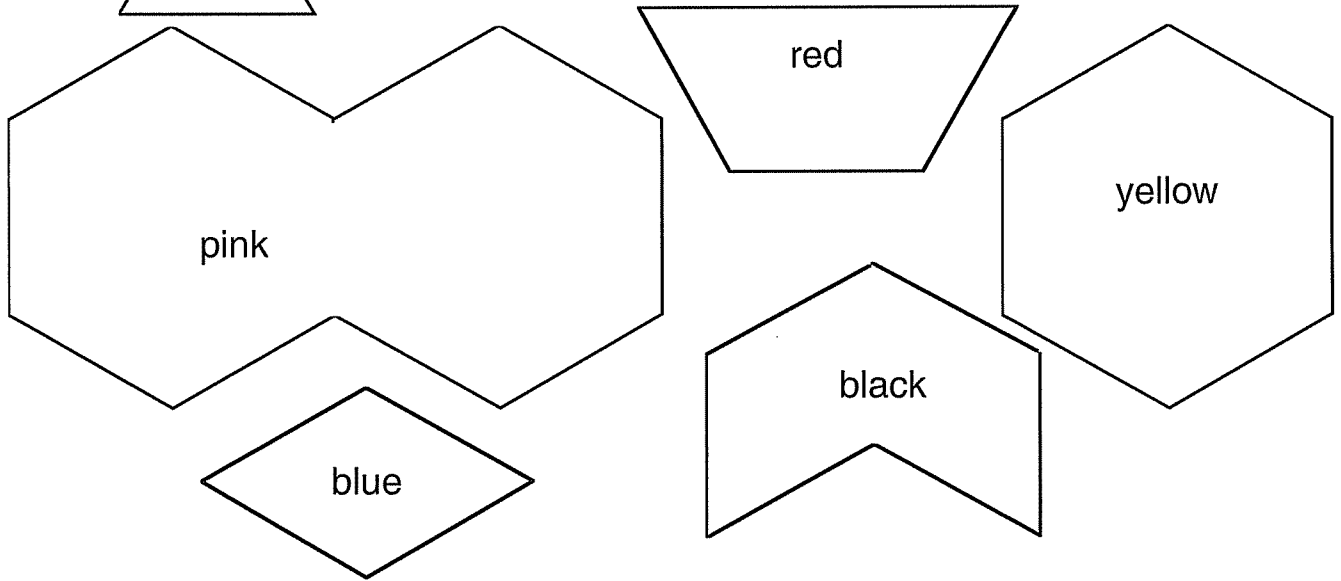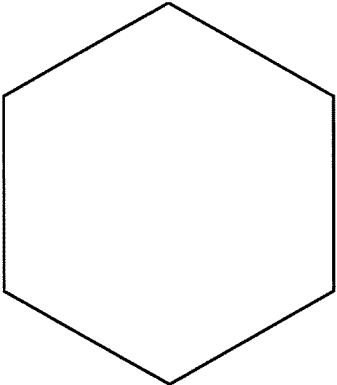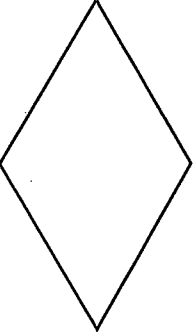4. Circle all of the designs below that are cut in half.
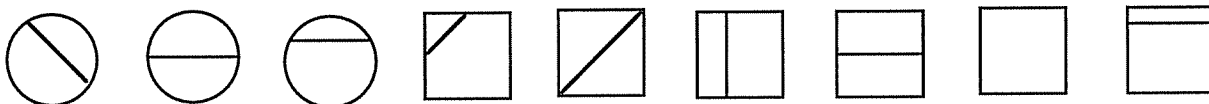
5. Write the name of this fraction in words. $\frac{1}{3}$ _____

6. Write in the correct < or > sign. $\frac{1}{4}$ $\quad$ $\frac{3}{4}$

7. Shade in $\frac{3}{4}$ of this box.

8. Circle the fraction that shows a larger amount. $\frac{1}{4}$ $\quad$ $\frac{1}{3}$

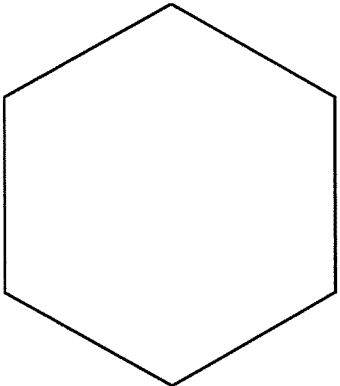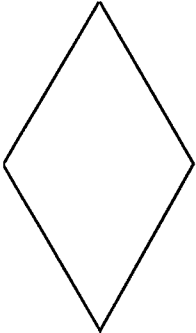9. Which fraction shows a larger amount $\frac{4}{5}$ or $\frac{3}{6}$? _____

How do you know? _____
_____

Date_____

Put a question mark next to anything you do not know yet.

10. If the (green) is equal to $\frac{1}{6}$, which of these figures is equal to one whole? _____

red

pink

yellow

black

blue

11. If (hexagon) equals 1, then what does (rhombus) equal? _____

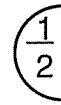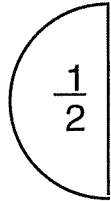If (hexagon) equals $\frac{1}{2}$, then what does (rhombus) equal? _____

Put a question mark next to anything you do not know yet.

12. Here are two pizzas.   Shade in half of each pizza.

A   B

This is half of pizza A.   This is half of pizza B.

$\frac{1}{2}$ $\frac{1}{2}$

The pieces are both halves, but they are not the same size.

How can this be? _____

13. List all the fractions you can make with your set of fraction pieces that are

equal to $\frac{1}{2}$. _____

14. Trade three-fourths for twelfths with your fraction pieces. How many twelfths cover the same area as three-fourths? _____

15. How many sixths are needed to cover one-third? _____

16. Fill in the missing numbers in this pattern.

$$2 = \frac{}{2} = \frac{}{3} = \frac{}{4} = \frac{}{6} = \frac{}{8}$$

17. The arrow points to which fraction on the number line? _____

0   1
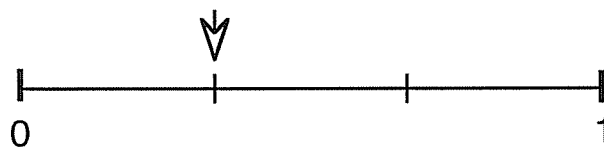
Assessment - Worksheet 4

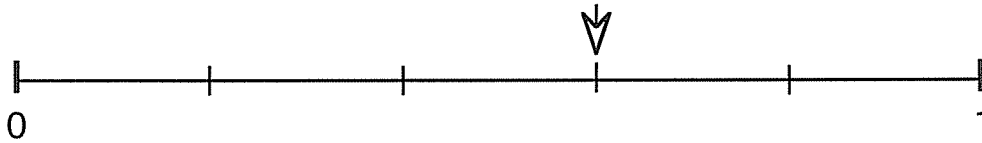Put a question mark next to anything you do not know yet.

18. The arrow points to which fraction on the number line? _____

```
                            ↓
        ⊢———————+———————+———————+———————+———————⊣
        0                                       1
```

19. What fraction is the arrow pointing to? _____

```
              ↓
        ⊢——+——+——+——+——+——+——+——⊣
        0        ___     1              2
```

20. How many twelfths are in two-thirds? _____

   Draw a line on the number line to prove your answer.

```
        ⊢————————+————————+————————⊣
        0                          1
```

```
        ⊢—+—+—+—+—+—+—+—+—+—+—+—⊣
        0                        1
```
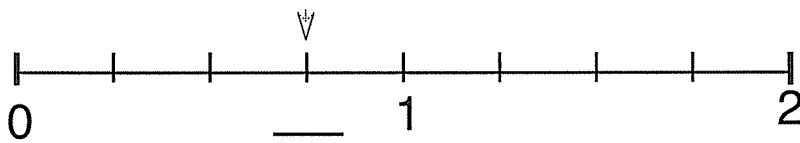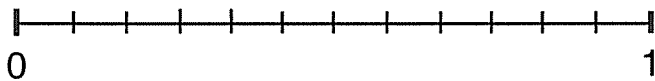
Fractions: Booklet 2 Placement Assessment Guide

Purpose
The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet.

Assessment: Part 1 is review material from the last booklet and is used to determine student readiness for this booklet. Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

Prerequisites
Patterns in Arithmetic: Fractions - Booklet 1 or previous instruction in fractions commonly taught in third grade textbooks

Materials
Assessment: Part 1 - Worksheets, pages 10 - 13; Assessment: Part 2 - Worksheets, pages 12 and 13
Score sheets, pages 7 - 9
Prism Fractions or fraction circles
Pattern blocks

Note
If you do not have the pink or black blocks in your pattern block set, make them with pattern blocks; directions are on page 6.

Allow the student to use pattern blocks and fraction circles or Prism Fractions to complete the test. Give him at least fifteen minutes of free exploration time with the materials before the test.

Instructions
Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

Do the assessment in two parts. Give Assessment: Part 1 and check it for readiness for this booklet. If he is not ready for this booklet, there is no point in giving Assessment: Part 2. If he passes all the readiness items, then give Assessment: Part 2. After scoring the assessment, use results with the Booklet Selection Guide to place your student. The answers are in the Answer Key.

Assessment Guide
This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc. Score sheets that match the Assessment Guide for both Assessment: Part 1 and Part 2 follow.

All criteria in Assessment: Part 1 are from Fractions: Booklet 1
Can the student:
1. My Fractions Book: Beginning
    A. identify the one-third piece from a fraction manipulative set he is using?

2. My Fractions Book: Beginning
    A. identify the one-fourth and the one-sixth pieces in the fraction manipulative set he is using?
    B. indicate that it takes four-fourths and six-sixths to make a whole?
These items test if he understands how to use the fraction manipulative.

3. Sorting Fractions
    A. demonstrate that he understands that the two halves must be of equal size by picking out the three designs that are divided into two equal parts?

4. My Fractions Book: Beginning
    A. write the correct name of the fraction given in standard fraction notation?

5. Greater Than, Less Than, or Equal To
    A. use a greater than, less than sign to correctly show that $\frac{1}{4} < \frac{3}{4}$?
    B. choose the larger fraction from a pair of two unlike fractions with ones in the numerator?
    C. choose the larger fraction from a pair of two unlike fractions with different numbers in the numerator? This is to test if he can use the manipulative to determine the relative sizes of the fractional units given.

6. Changing Wholes: Beginning
    A. choose the correct whole given the size of the one-sixth block?
Give him pattern blocks to solve this problem. This item tests his understanding of the relationship between parts and wholes in fractions. He must choose the larger block that takes six greens to cover it.

7. Numerators Greater Than One
    A. divide the rectangle in problem a into three roughly equal parts?
    B. shade in two of the three parts in problem a?
    C. identify the fraction one-sixth shown by the graphic in problem b?
    D. use the language 'three out of four' when referring to fractions?
This is to test if he can create and interpret graphic representations of fractional units.

8. Shrinking Circles
    A. shade in half of each pizza?
    Rubric points   1 point - The wholes are not the same.
                     2 points - The size of the half is made by the size of the whole, or if the whole is bigger, then the half will be bigger too.
This is to test if he understands that the whole can be defined as anything you like. Once that definition is made, all other fractional units are established by their relationship to the whole.
9. My Fractions Book: Equivalence
    A. use the manipulative to match several smaller pieces to the size of the one-half piece?
    B. list at least two of the four equivalent fractions possible?
Have him use Fraction Circles or Prism Fractions. This is to test if he understands that 'equal' means

covering the same area and can match up a greater number of smaller pieces to cover the exact same area the larger one-half piece covers.

10. My Fractions Book: Equivalence
    A. use the manipulative to match nine-twelfths to the three-fourths?
    B. use the manipulative to match two-sixths to the one-third?

11. Equivalence Recording
    A. use the manipulative to determine how many halves would cover two wholes?
    B. extend the number pattern to fill in missing numbers?

12. My Fractions Book: Number Lines
    A. identify that the arrow points to one-third in problem a?
    B. identify that the arrow points to three-fourths in problem b?
If he gave the answer three-eighths, he counted the spaces on the whole number line, not just the spaces between the zero and the one. This is a very common error.

To interpret a number line, the student must first understand that the space between the zero and the one is the whole. Then he must know that the denominator is determined by the number of sections that the space between zero and one is divided into. Many students have difficulty with fractional number lines.

13. My Fractions Book: Number Lines
    A. use the number line to identify equivalent fractions by drawing a line from one number line to a second number line?
Make sure he does not use the fraction manipulative to answer this question. He must draw in the line.

Booklet Selection Guide based on results of Assessment: Part 1
Critical concepts: If items 1, 2A, 2B, 7A, 7B, and 7C are marked No, he should begin with Fractions: Booklet 1. Do not give Assessment: Part 2.

A score of 13 or less indicates a beginner level understanding of basic fractions concepts.
For a student of age nine or less, do the entire Fractions: Booklet 1 start to finish. Do not give Assessment: Part 2 of this assessment.
A student who is age ten to adult should do a modified Fractions: Booklet 1. You can skip items that he got correct on Assessment: Part 1. Focus on the My Fractions Book sections on equivalent fractions and number lines and shorten tracing and labeling sections in the first part of that section. You can skip the sections on the Changing Wholes and Greater Than, Less Than, or Equal To, as these are strongly covered in Fractions: Booklet 2. Reassess on items you remediated. Then give Assessment: Part 2 and begin instruction in Fractions: Booklet 2.

14 - 18 points will allow a pass into Fractions: Booklet 2 with remediation on the items missed.

If Items 5 and 6 were missed, do not remediate these, as these topics are strongly covered in Fractions: Booklet 2.

If your student is new to this program, the My Fractions Book section of Fractions: Booklet 1 is very

useful. The activities of tracing and labeling can be shortened. Focus on the drawings of equivalent fractions and number lines, which are the areas most likely to need remediation. These two concepts are not covered in Fractions: Booklet 2. Do the needed sections of Fractions: Booklet 1, retest using Assessment: Part 1, then give Assessment: Part 2, and proceed to Fractions: Booklet 2.

19 points are needed to pass into Fractions: Booklet 2 with no remediation. Give Assessment: Part 2 and proceed to Fractions: Booklet 2.

Assessment Criteria for Assessment: Part 2
All criteria in the Assessment: Part 2 are from Fractions: Booklet 2
Can the student:
1. Greater Than, Less Than, or Equal To
      A. put the fractions in order from least to greatest with a manipulative?
      B. put the fractions in order without the use of the manipulative?
This item tests to see if the student recognizes that a large number in the denominator indicates a smaller fractional piece. If he can put the first three in order, he is demonstrating he understands this. The item then tests to see if the student realizes that three-fifths is little more than a half, and four-fifths would be the largest fraction. If he uses the manipulative, he is not quite sure, but he can figure it out.

2. Changing Wholes
      A. give one-sixth as the value for the green piece in problem a where the yellow block is defined as one? A No on this item indicates lack of experience with pattern blocks, or an undeveloped sense of the basic concept of fractional values.
      B. give one as the value of the blue block when the value of the yellow block is three?
A Yes on Item B indicates good understanding of this concept because the value of a single block, the yellow, is given as a number greater than one. This is a new kind of problem. He must calibrate the relationship between the blue and the yellow to realize the blue is actually the whole, or one.

3. Changing Wholes
      A. give one twenty-fourth as the value of the green piece in problem a?
      B. give one-ninth as the value of the blue piece in problem b?
      C. give the correct answers on problems a and b without the use of a manipulative? A Yes on this item indicates strong understanding of this concept.

4. Addition of Like Fractions
      A. give the correct answer on two of three problems?
Fractions do not need to be simplified to be correct. Does he understand that only the numerators are added and not the denominators? The denominators are not added because they tell only which fraction piece to use, not how many.

5. Subtraction of Like Fractions
      A. give the correct answer on two of three problems?

6. Fractions as Ratios
      A. give the denominator as fourteen, which is the total number of letters in the box?
      B. give the correct numerator as six, which is the number of capital letters in the box?
This item tests to see if he understands that the whole is determined by the total number of things in a

set. The numerator is the number of a particular part of that set.

7. Fractions as Parts of Wholes - using a picture

Parts of Wholes problems require the student to understand that the whole can be defined as a set of objects or as a whole number greater than one. All fractional values are then determined in relationship to whatever the whole is.

      A. circle half of the triangles in a group of four?

      B. color in two of the six boxes to show one-third of a group of six?

These items test if a student can use a graphic to select a fraction of a group.

8. Fractions as Parts of Wholes - using a picture

      A. circle nine of the twelve letters in the box?

This item is testing to see if the student knows that he must count all the letters in the square, divide that number into four equal parts and circle three of those parts. If you see a single circle enclosing nine of the letters, you know he did this calculation in his head. You may also see evidence of four smaller divisions with three selected in a darker circle, which means he needed to visually identify the fourths and then select three of them. This indicates the graphic most likely is still needed.

9. Fractions as Parts of Wholes

This item tests to see if he can calculate the part with only one in the numerator. This requires that he knows to divide the whole number into the number of parts indicated by the denominator of the fraction, ten divided by two in the case of problem a.

      A. give the correct answer in two of the three problems?

      B. give the correct answers without evidence of a drawing or use of a manipulative?

10. Fractions as Parts of Wholes

This items tests the above concept, plus the second step needed of adding more than one group. In the first item, he must divide twenty-five by five to get one-fifth of twenty-five and then multiply that quantity by three because the numerator says there are three of the five groups needed.

      A. give the correct answer in two of the three problems?

      B. give the correct answers without evidence of a drawing or use of a manipulative?

11. Fractions as Parts of Wholes

This item tests if he can use the procedure to calculate a fraction times a whole number problem with a two-step process and a multiplication (x) sign.

      A. give the correct answer in two of the three problems?

Booklet Selection Guide based on results of Assessment: Part 2

14 or more Yes, move to Fractions: Booklet 3 - Improper and Mixed Fractions.

11 - 13 Yes, remediate weak areas, retest, and move to Fractions: Booklet 3.

10 or under, with a pass on Part 1, means that this is the perfect booklet for your student.

Whenever remediation is needed, rely upon the following process, which is used throughout the *Patterns in Arithmetic* series to develop understanding of a concept.

1. Introduce the concept with a manipulative. Orally discuss it. Build it. Verify it. Practice it. Repeat the experience with a different manipulative (oral manipulative).

2. Use manipulatives to explore the concept again. This time record it with pictures (pictorial/representation). Practice it. Use worksheets.

3. Record the problem with numbers (abstract/symbolic), which links the pictorial with the abstract.
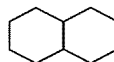4. Practice fluency.
5. Practice for speed.

Ask questions or make statements, such as: **"Are you sure?"** or **"Build it."** or **"What gave you the clue?"** or **"Show me how you got that."** or **"Prove it."** even when a student is correct. This is important to do often. Many students will ask an adult, "Am I right?" rather than answering definitively. Confidence in a student's response must come from within. A student needs to self-check and have confidence in his or her ability and knowledge. Asking the student if he or she is right, even when correct, will encourage self-confidence and the ability to self-check.

Pattern block directions for pink or black blocks:
Pink: Glue two yellow hexagons together to make this shape.
Paint pink if you wish.

Black: Glue two blue rhombi together to make this shape.
Paint black if you wish.

Please note that the dialogues in most lessons are idealized, with a student giving all the correct answers. Teacher dialogues are in bold type and student dialogues are in regular type. The dialogue you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as: **"How did you get that?" "Show me what you mean." "Build a model of that." "Tell me more so I can understand what you are saying."**

Can the student:
1. My Fractions Book: Beginning
        Yes No   A. identify the one-third piece?

2. My Fractions Book: Beginning
        Yes No   A. identify the one-fourth and the one-sixth pieces from the fraction manipulative set he is using?
        Yes No   B. indicate that it takes four-fourths and six-sixths to make a whole?

3. Sorting Fractions
        Yes No   A. pick out the three pictures that correctly show a half?

4. My Fractions Book: Beginning
        Yes No   A. write the name of the fraction in words? Spelling does not count.

5. Greater Than, Less Than, or Equal To
        Yes No   A. use a greater than, less than sign to show that one-fourth is less than three-fourths?
        Yes No   B. choose the larger fraction from a pair of two unlike fractions with ones in the numerator?
        Yes No   C. choose the larger fraction from a pair of two unlike fractions with different numbers in the numerator?

6. Changing Whole: Beginning
        Yes No   A. choose the correct whole given the size of the one-sixth block?

7. Numerators Greater Than One
        Yes No   A. divide the rectangle in problem a into three roughly equal parts?
        Yes No   B. shade in two of the three parts in problem a?
        Yes No   C. identify the name of the fraction shown by the graphic in problem b?
        Yes No   D. use the language 'three out of four' when referring to fractions?

8. Shrinking Circles
        Yes No   A. shade in half of each pizza?
        1 point   The wholes are not the same.
        2 points   The size of the half is made by the size of the whole, or if the whole is bigger, then the half will be bigger too.

9. My Fractions Book: Equivalence
        Yes No   A. use the manipulative to match several smaller pieces to the size of the half piece?
        Yes No   B. list at least two of the four equivalent fractions possible?

10. My Fractions Book: Equivalence
      Yes No   A. use the manipulative to match nine-twelfths to the three-fourths?
      Yes No   B. use the manipulative to match two-sixths to the one-third?

11. Equivalence: Recording
      Yes No   A. use the manipulative to determine four halves cover two wholes?
      Yes No   B. extend the number pattern to fill in missing numbers?

12. My Fractions Book: Number Lines
      Yes No   A. identify that the arrow points to one-third in problem a?
      Yes No   B. identify that the arrow points to three-fourths in problem b?

13. My Fractions Book: Number Lines
      Yes No   A. draw a line from two-thirds on one number line to the eight-twelfths on the bottom number line?

Items Correct = _____ = _____%   72% needed to begin Fractions: Booklet 2.
Items Possible = 25   This is 18 or more Yes items.
                                  Less than 72% (age 10 or under), review items in Fractions:
                                  Booklet 1, do additional practice in the areas missed,
                                  and retest with a pass on Assessment: Part 1. This is the
                                  perfect booklet for your student.

Assessment: Part 2 Score Sheet Name _____ Date _____

Can the student:

1. Greater Than, Less Than, or Equal To
    Yes No   A. put the fractions in order from least to greatest with a manipulative?

2. Changing Wholes
    Yes No   A. give one-sixth as the value for the green block in problem a?
    Yes No   B. give one as the value of the blue block in problem b?

3. Changing Wholes
    Yes No   A. give one twenty-fourth as the value of the green piece in problem a?
    Yes No   B. give one-ninth as the value of the blue piece in problem b?
    Yes No   C. give the correct answers on problems a and b without the use of a manipulative?

4. Addition of Like Fractions
    Yes No   A. give the correct answer on two of three problems?
                        Fractions do not need to be simplified to be correct.

5. Subtraction of Like Fractions
    Yes No   A. give the correct answer on two of three problems?

6. Fractions as Ratios
    Yes No   A. write fourteen as the denominator?
    Yes No   B. write six as the numerator?

7. Fractions as Parts of Wholes - using a picture
    Yes No   A. circle two of the triangles in a group of four?
    Yes No   B. color in two of the six boxes to show one-third of a group of six?

8. Fractions as Parts of Wholes - using a picture
    Yes No   A. circle nine of the twelve letters in the box?

9. Fractions as Parts of Wholes - calculating with only one in the numerator
    Yes No   A. give the correct answer in two of the three problems?
    Yes No   B. give the correct answers without a drawing or use of a manipulative?

10. Fractions as Parts of Wholes - two step process
    Yes No   A. give the correct answer in two of the three problems?
    Yes No   B. give the correct answers without a drawing or use of a manipulative?

11. Fractions as Parts of Wholes - calculating with a two step process and a multiplication (x) sign
    Yes No   A. give the correct answer in two of the three problems without a drawing
                        or a manipulative?

Items Correct = _____ = _____%
Items Possible =   18

Assessment: Part 1 - Worksheet 1

The student may use a manipulative for the test.

Put a question mark next to anything you do not know how to do.

1. What is the color of the 1/3 piece in your set? _____

2. a. Cover the whole with fourths. How many fourths does it take to cover a whole? _____

   b. Cover the whole with sixths. How many sixths does it take to cover a whole? _____
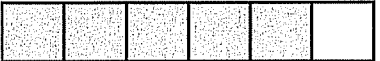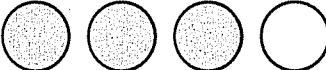
3. Circle all of the designs below that are cut in half.
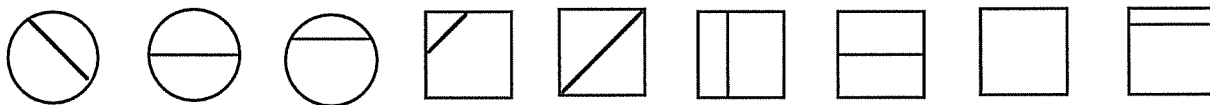
4. Write the name of this fraction in words. $\frac{2}{3}$ _____

5. a. Write in the correct < or > sign. $\frac{1}{4}$ $\frac{3}{4}$

   b. Circle the fraction that shows a larger amount. $\frac{1}{4}$ $\frac{1}{3}$

   c. Which fraction shows a larger amount $\frac{4}{5}$ or $\frac{3}{6}$? _____

   d. How do you know? _____

_____

6. If the is equal to $\frac{1}{6}$, which of these figures is equal to one whole?
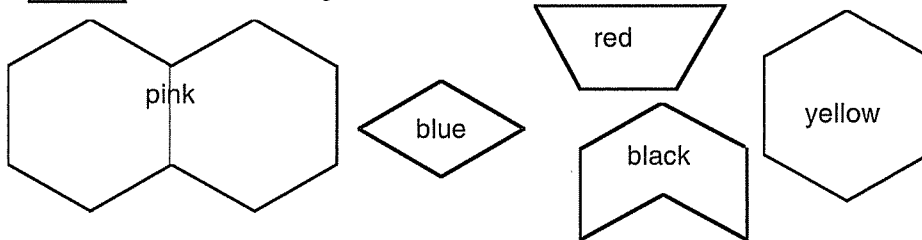
7. a. Shade in $\frac{2}{3}$ of this rectangle.
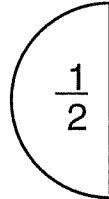
   b. What fraction of this rectangle is not shaded? _____

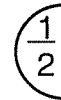   c. ____ out of ____ of the circles are shaded.

8. Here are two pizzas.   Ⓐ Ⓑ   a. Shade in half of each pizza.
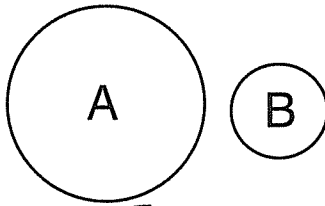
This is half of pizza A. $\frac{1}{2}$ This is half of pizza B. $\frac{1}{2}$

   b. The pieces are both halves, but they are not the same size.
       How can this be? _____

9. List all the fractions you can make with your set of fraction pieces that are
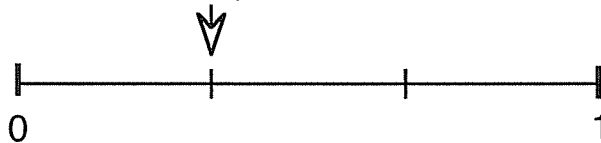   equal to $\frac{1}{2}$. _____

10. a. Trade three-fourths for twelfths with your fraction pieces. How many
        twelfths are equal to three-fourths? _____

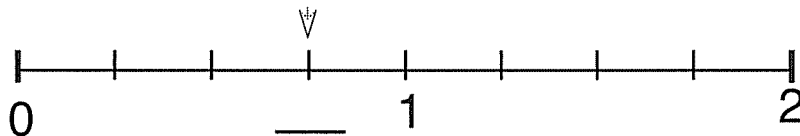   b. How many sixths are needed to cover one-third? _____

11. Fill in the missing numbers $2 = \dfrac{}{2} = \dfrac{}{3} = \dfrac{}{4} = \dfrac{}{6} = \dfrac{}{8}$
     in this pattern.

12. a. The arrow points to which fraction on the number line? _____

       ├────┼────┼────┤
       0   1

     b. What fraction is the arrow pointing to? _____

       ├──┼──┼──┼──┼──┼──┼──┼──┤
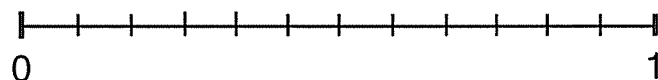   0   __   1   2
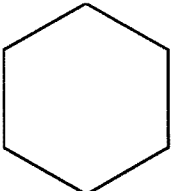
13. How many twelfths are in two-thirds? _____
     Draw a line on the number line to prove your answer.

       ├────────┼────────┤
       0   1

       ├──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┤
       0   1
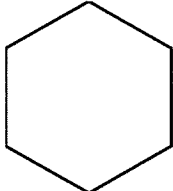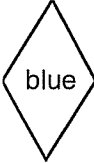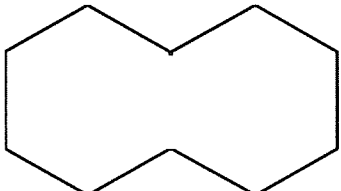
Put a question mark next to anything you do not know how to do.
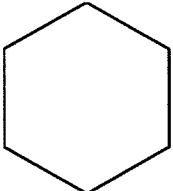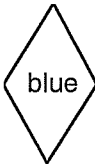
1. Put these fractions in order from the least to the greatest.

$$\frac{4}{5} \qquad \frac{1}{2} \qquad \frac{1}{6} \qquad \frac{3}{5} \qquad \frac{1}{4}$$

_____   _____   _____   _____   _____

2. a.   If [hexagon] equals 1, then what does [green triangle] equal? _____

   b. If [hexagon] equals 3, then what does [blue rhombus] equal? _____

3. a. If [two hexagons] equals $\frac{1}{2}$, then what does [green triangle] equal? _____

   b. If [hexagon] equals $\frac{1}{3}$, then what does [blue rhombus] equal? _____

4. Solve.   a. $\frac{1}{2}+\frac{1}{2}=$ b. $\frac{3}{5}+\frac{1}{5}=$ c. $\frac{6}{10}+\frac{2}{10}=$

5. Solve.   a. $\frac{7}{8}-\frac{1}{8}=$ b. $\frac{2}{3}-\frac{1}{3}=$ c. $\frac{5}{5}-\frac{1}{5}=$

6. What fractional part of the letters are capital letters? _____

```
a   B   c
            D      e
f   G   h  i      J

k   L   m      N
```
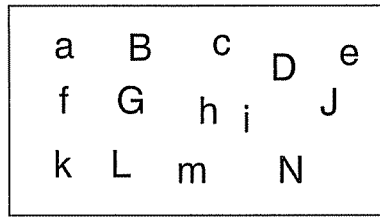
7. a. Circle $\frac{1}{2}$ of the group.   △ △ △ △

   b. Circle $\frac{1}{3}$ of the group.   ☐☐☐☐☐☐

8. Circle $\frac{3}{4}$ of the set.

```
   f    P      B  E  y

         L   Z    c
   t               X
       A     W
```

9. a. $\frac{1}{2}$ of 10 =   b. $\frac{1}{3}$ of 15 =   c. $\frac{1}{5}$ of 20 =

10. a. $\frac{3}{5}$ of 25 =   b. $\frac{2}{7}$ of 21 =   c. $\frac{4}{9}$ of 36 =

11. a. $\frac{3}{4}$ x 16 =   b. $\frac{4}{5}$ x 30 =   c. $\frac{6}{7}$ x 42 =

Fractions - Booklet 1
Assessment - Worksheet 1

Assesses basic concept in:
A. Fractional units using a manipulative
B. Changing sizes of the whole
C. Fractional units on a number line
D. Equal fractions with a manipulative and number lines,

A familiar fraction manipulative, such as, circle fractions or Prism Fractions and Pattern Blocks must be available to the student. The examiner must observe the student throughout the entire assessment. Give assistance only in reading. Note if the student uses the manipulative or not on each item.

1. What is the color of the 1/3 piece in your set? _orange_

2. Cover the whole with fourths. How many fourths does it take to cover a whole? _4_

3. Cover the whole with sixths. How many sixths does it take to cover a whole? _6_

4. Circle all of the designs below which are cut in half.

5. Write the name of this fraction in words. $\frac{1}{3}$ _one-third_

6. Write in the correct < > sign. $\frac{1}{4}$ < $\frac{3}{4}$

7. Shade in $\frac{3}{4}$ of this box.

8. Circle the fraction that shows a larger amount. $\frac{1}{4}$ ⓐ$\frac{1}{3}$

9. Which fraction shows a larger amount ⓐ$\frac{4}{5}$ or $\frac{3}{6}$? $\frac{4}{5}$
How do you know? _Fifths are larger than sixths._

$\frac{3}{6} = \frac{1}{2}$ $\frac{4}{5}$ is larger than $\frac{1}{2}$.

Assessment - Worksheet 2

10. If the △ green is equal to $\frac{1}{6}$, which of these figures is equal to one whole? _yellow hexagon_
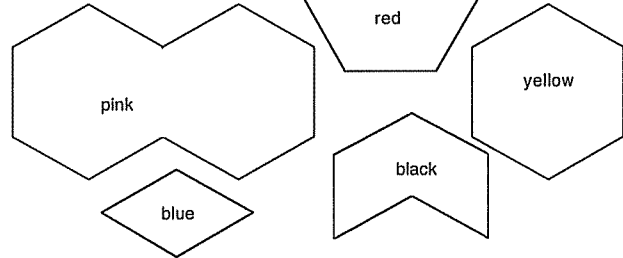
11. If ⬡ equals 1, then what does ◇ equal? $\frac{1}{3}$

If ⬡ equals $\frac{1}{2}$, then what does ◇ equal? $\frac{1}{6}$

Assessment - Worksheet 3

12. Here are two pizzas. Ⓐ Ⓑ Shade in half of each pizza.

This is half of pizza A. $\frac{1}{2}$ This is half of pizza $\frac{1}{2}$

The pieces are both halves, but they are not the same size.
How can this be? _If the whole is bigger, then the half is bigger._

13. List all the fractions you can make with your set of fraction pieces that are equal to $\frac{1}{2}$. _4ths, 6ths, 8ths, 10ths, 12ths_
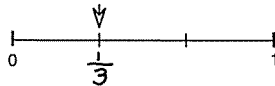
14. Trade three-fourths for twelfths with your fraction pieces. How many twelfths cover the same area as three-fourths? _9_

15. How many sixths are needed to cover one-third? _2_

16. Fill in the missing numbers in this pattern.

$2 = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = \frac{12}{6} \frac{16}{8}$

17. The arrow points to which fraction on the number line. $\frac{1}{3}$

Assessment - Worksheet 4

18. The arrow points to which fraction on the number line? $\frac{3}{5}$

19. What fraction is the arrow pointing to? $\frac{3}{4}$

20. How many twelfths are in two-thirds? _8_

Draw a line on the number line to prove your answer.

Answer Key: Fractions Placement

Fractions - Booklet 2

Assessment: Part 1 - Worksheet 1

1. What is the color of the 1/3 piece in your set? **Orange**

2. a. Cover the whole with fourths. How many fourths does it take to cover a whole? **4**

   b. Cover the whole with sixths. How many sixths does it take to cover a whole? **6**

3. Circle all of the designs below which are cut in half.

4. Write the name of this fraction in words. $\frac{2}{3}$ **two-thirds**

5. a. Write in the correct < or > sign. $\frac{1}{4}$ **<** $\frac{3}{4}$

   b. Circle the fraction which shows a larger amount. $\frac{1}{4}$ **($\frac{1}{3}$)**

   c. Which fraction shows a larger amount **($\frac{4}{5}$)** or $\frac{3}{6}$?

   d. How do you know? **Fifths are bigger than sixths and there are more of them.**

6. If the △ green is equal to $\frac{1}{6}$, which of these figures is equal to one whole?

7. a. Shade in $\frac{2}{3}$ of this rectangle.

   b. What fraction of this rectangle is not shaded? **$\frac{1}{6}$**

   c. **3** out of **4** of the circles are **shaded**. ○○○○

Assessment: Part 1 - Worksheet 2

8. Here are two pizzas. (A) (B) a. Shade in half of each pizza.

   This is half of pizza A. $\frac{1}{2}$ This is half of pizza B. $\frac{1}{2}$

   b. The pieces are both halves, but they are not the same size. How can this be? **Each is $\frac{1}{2}$ of a different size whole.**

9. List all the fractions you can make with your set of fraction pieces that equal to $\frac{1}{2}$. **4ths, 6ths, 8ths, 10ths, 12ths**

10. a. Trade three fourths for twelfths with your fraction pieces. How many twelfths are equal to three-fourths? **9**

    b. How many sixths are needed to cover one-third? **2**

11. Fill in the missing numbers in this pattern. $2 = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = \frac{12}{6} = \frac{16}{8}$

12. a. The arrow points to which fraction on the number line. **$\frac{1}{3}$**

    b. What fraction is the arrow pointing to? **$\frac{3}{4}$**

13. How many twelfths are in two-thirds? **8**
    Draw a line on the number line to prove your answer.

Assessment: Part 2 - Worksheet 1

1. Put these fractions in order from the least to the greatest.

   $\frac{4}{5}$ $\frac{1}{2}$ $\frac{1}{6}$ $\frac{3}{5}$ $\frac{1}{4}$

   $\frac{1}{6}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{5}$ $\frac{4}{5}$

2. a. If ⬡ equals 1, then what does △ green equal? **$\frac{1}{6}$**

   b. If ⬡ equals 3, then what does ◇ blue equal? **1**

3. a. If ⬢ equals $\frac{1}{2}$, then what does △ green equal? **$\frac{1}{24}$**

   b. If ⬡ equals $\frac{1}{3}$, then what does ◇ blue equal? **$\frac{1}{9}$**

4. Solve. a. $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$ or **1**   b. $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$ c. $\frac{6}{10} + \frac{2}{10} = \frac{8}{10}$ or $\frac{4}{5}$

5. Solve. a. $\frac{7}{8} - \frac{1}{8} = \frac{6}{8}$ b. $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ c. $\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$

Assessment: Part 2 - Worksheet 2

6. What fractional part of the letters are capital letters? **$\frac{6}{14}$**

7. a. Circle $\frac{1}{2}$ of the group. △ △ (△ △)

   b. Circle $\frac{1}{3}$ of the group.

8. Circle $\frac{3}{4}$ of the set.

9. a. $\frac{1}{2}$ of 10 = **5**   b. $\frac{1}{3}$ of 15 = **5**   c. $\frac{1}{5}$ of 20 = **4**

10. a. $\frac{3}{5}$ of 25 = **15**   b. $\frac{2}{7}$ of 21 = **6**   c. $\frac{4}{9}$ of 36 = **16**

11. a. $\frac{3}{4}$ x 16 = **12**   b. $\frac{4}{5}$ x 30 = **24**   c. $\frac{6}{7}$ x 42 = **36**

Patterns in Arithmetic: Fractions Placement PDF
Booklets 1 and 2

Parent/Teacher Guide