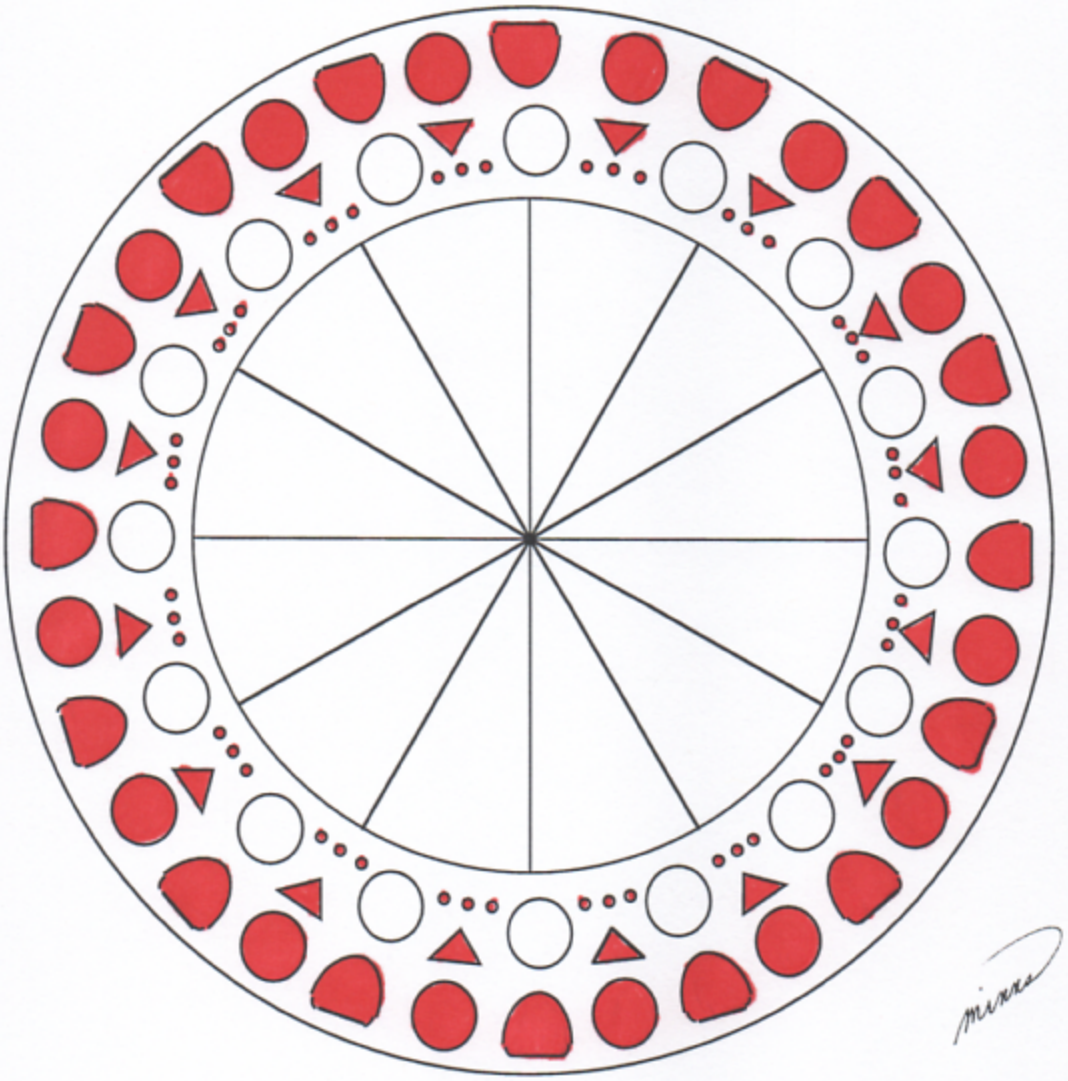
Patterns in Arithmetic

Fractions - Booklet 2

Developing Concepts and Beginning Operations

Sample Pages



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Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Fractions: Booklet 2 - Developing Concepts and Beginning Operations

Contents

The Post-Assessment is the same as Pre-Assessment: Part 2.

This booklet is dedicated to Agnes Mason Wilkerson, former Farm School Administrator. We are all grateful for her many years of outstanding service and excellent people skills at the Farm School.

Acknowledgments

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For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

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Susan Carpenter edited, added her wise words, useful suggestions, and helped make the Answer Keys a reality. Karán Founds-Benton contributed her meticulous editing skill and knowledge. Diligent and thorough copy editing was done by Zephyr Alfanash and Jacqueline Logue.

The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

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Parent/Teacher Guide

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Free Explorations and Pattern Block Games

Purpose

The purpose of this lesson is to encourage play with the materials and gently begin formal work with manipulatives.

Prerequisites

Pre-Assessment

Materials

Free Exploration - Worksheets 1 and 2, pages 7 and 8
Pattern blocks
Cuisenaire Rods
Prism Factions®

Warm Up

Free Exploration - Worksheet 1 Set out all the manipulatives (pattern blocks, Cuisenaire Rods, and Prism Factions) intended to be used for instruction on the table at once. Give the student plenty of time to play. Encourage the making of designs. Put on some music. Consider making a gallery of designs to put on the board to brighten the room. Have him trace and color one design into the workbook.

Worksheets

Free Exploration - Worksheet 2 Encourage the student to mix colors in each circle. This lets him know which colors can go together to form a complete whole. Have him trace and color to match the manipulative being used.

Games

Flower Power Game, page 9
This game can be more complex than it looks at first glance. If the players use only yellow blocks, the player to go first will always win. This encourages the second player to use a smaller block. There are multiple strategies having to do with even and odd numbers of blocks on the board as well as the ways the empty spaces can be filled.

Predator Game, page 10
Use yellow, red, green, and blue pattern blocks, a Predator Game board for each player, and one die. You can speed up the game with two dice. Take turns rolling the die. Take as many green blocks as rolled. Trade blue blocks and any other color block for green blocks whenever possible. When a student thinks he is done trading, he passes the die to signify the end of his turn. If he missed a trade, another player can point it out and then take the blocks that should have been traded, make the trade and place the blocks on his board. The first person to fill a Predator game board is the winner.

Variation

Use a fraction die with $^1/_2$, $^1/_3$, and $^1/_6$ on it.
Students enjoy these games. It is beneficial to play them many times.

Test for Understanding

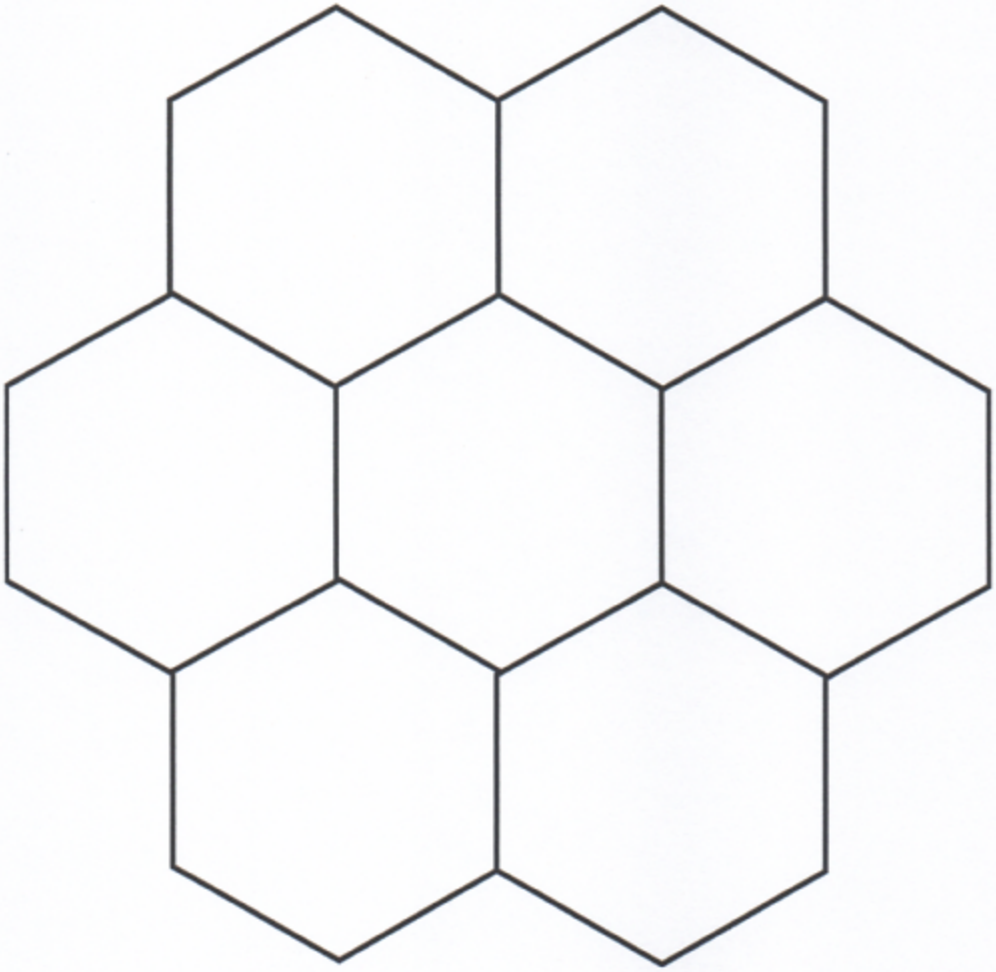
"Describe your strategies for playing this game."

Have the students describe difference strategies they have found playing the game over time. Look for understanding that smaller pieces allow more turns and that the smaller pieces can be fit together to cover the same area as the larger ones. Notice if he can correctly predict which piece will fit exactly into the space available for each move. If he selects the wrong piece to fit into a space several times, allow more time to play.

Flower Power Game

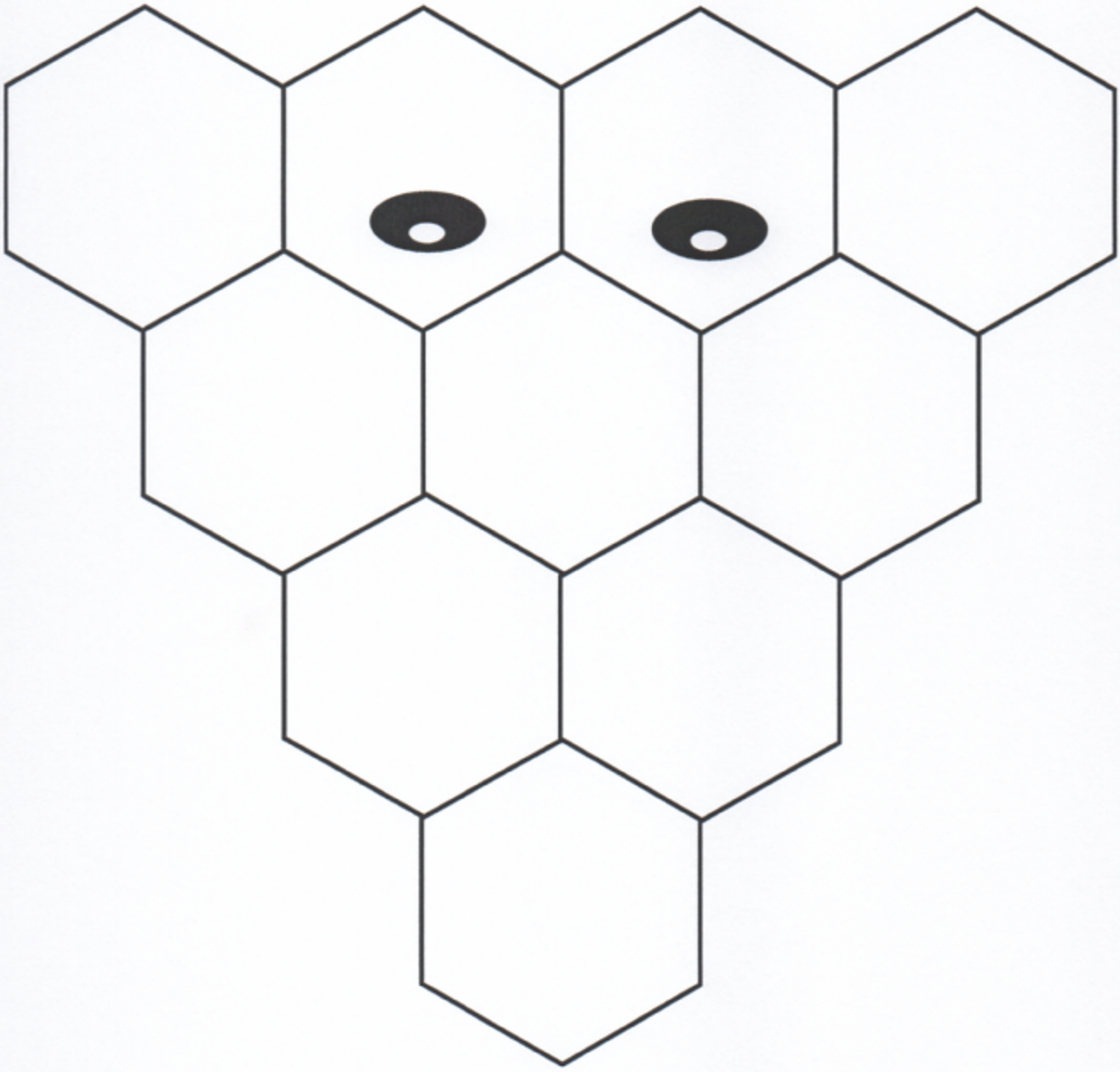
Date_____

2 players
Green, red, blue, and yellow pattern blocks, Flower Power game board.
Each player in turn chooses one pattern block and places it on the game board. No piece can cover a line.
The last player to place a block is the winner.



Predator Game Date_____

2 - 4 players
Use yellow, red, green, and blue pattern blocks, a Predator Game board for each player, and one die.
Take turns rolling the die and take as many green blocks as rolled. Trade blue blocks (or any other color block) for green blocks whenever possible. If you do not trade immediately, your opponent can take your two greens. Cover the board with yellow blocks. The first player to cover the board is the winner. Remember, if you do not trade ASAP, your opponent can steal them on his or her turn.



Changing Wholes

Purpose

The purpose of this lesson is to lay the foundation for the understanding of multiplication and division of fractions. All fractional pieces are defined in terms of the whole. The quantity that is defined as the whole can be changed. For example, with pattern blocks: If the yellow hexagon is defined as the whole, then the red trapezoid piece is one-half, the green triangle is one-sixth. But if the definition of the whole is changed to two hexagon blocks, then the one-half is one hexagon, the red trapezoid becomes one-fourth, and the green triangle is one-twelfth.

Note

This lesson introduces the very abstract idea that any quantity can be defined as a fraction of a set and that the size of the thing called the whole can change. What does not change is the relationship between the fractional unit and the whole. This lesson asks students to constantly change what is defined as the whole and then figure out the fractional value of all the parts of that whole.

The changing whole is a critical concept for understanding fractions. It is the understanding that a fraction is a relationship to a whole. This relationship is called a ratio. A half of a big pizza is not the same amount of pizza as a half of a small pizza. The fractions are equal, one-half, but the size of that half depends on the size of the whole. The unit of the fraction one-half a pizza or one-half a mile is also determined by what the whole is defined as. What makes this even trickier is that the whole may not be equal to one. I can define the whole group as six, like you would on a basketball team; one-half the team is three players. The value of a coin, or a pattern block, depends on what the whole is. The definition of the whole is changed over and over again. The student must then find the fraction of that whole by dividing it up into the correct number of equal pieces. This concept is the underpinning of ratio and proportion, which in turn underlies calculations of percentage, rates of change, and all division procedures.

This concept is also critical to understanding fractional operations in multiplication and division. For example, let's say you baked a pan of brownies to sell at a bake sale. You sell a fourth of the pan to your first customer. Then your second customer comes up and offers to buy two-thirds of what you have left. So the second customer wants to buy two-thirds of the three-fourths. To cut this up, you have to think of the three-fourths of a pan you still have as the whole in order to cut it up into thirds. Then, in order to price the brownies the second customer bought, you have to think of the original tray of brownies as the whole and the fraction of the original whole brownie pan that the two-thirds of the three-fourths was. This is the problem $^{2}/_{3} \times {}^{3}/_{4} = \_\_\_$. This idea is a bit of a mind bender.

Fractions are, by their nature, ratios. This means that fractions have meaning only in relationship to whatever the whole is. This is a very, very important concept in mathematics, do not skimp on this lesson.

Prerequisites

All previous lessons in the booklet

Materials

Changing Wholes - Worksheets 1 - 8, pages 30 - 38
Parts 1 and 3: Pattern blocks and crayons

Part 2: Cuisenaire Rods, a centimeter ruler, and crayons

Part 1

Changing Wholes (with pattern blocks)

Warm Up

Allow the student to choose doing a design or playing a game with the pattern blocks. Then tell him he may not make designs during the lesson unless the worksheet asks him to.

Lesson

Use Changing Wholes - Worksheet 1, page 30. It will help clarify dividing sets of things into halves.

"If you want to divide something in half, how many parts do you need to divide the whole into?" "Two."

Put down four blocks on the table. **"What would half of this group of blocks be?"** "Two blocks."

Challenge Problem: **"What if I tell you that I have half of a group in my hand. I show you three blocks. How many blocks would be in the whole group?"** "Six."

"Why?" "Because if you have a half, that means you only have one out of the two groups. Since you showed me three blocks for one group, and I know I need two groups, I need to just *double* that number to get the whole."

Double or 'two times as much' are the words you are looking for.

A second line of questioning goes as follows:

"What is half of four?" "Two."

"How can the whole number two be called a half?" "Because it is half of four. It takes two twos to make four."

"Is the number two a fraction?" "No, it is a whole number."

"Then how can two be called a half when one-half is a fraction and two is a whole number?" "Two can be called a half if you call four a whole. It is the relationship between the two and the four that makes the one-half."

A clarifying question might be:

"So if all the money you have in the world is four dollars, and you gave me two dollars, then you would be giving me a fraction, one-half, of your money?" "Yes."

Do Worksheet 1 together. This worksheet asks you to think of whatever is in the left hand column as half of something in the right hand column. The example shows the number 2 in the left hand column connected to the 4 in the right hand column. This is because two is half of four.

"The next one shows the red pattern block. If the red pattern block is called a half, then what would be the whole?" "The yellow pattern block."

"Why?" "Because it takes two red blocks to make the yellow."

"What is the trapezoid half of?" "The hexagon." Have him draw the line connecting those two shapes.

"What would you connect a 20 to?" "The 40, because 20 is half of 40."

"Finish the rest of the page by yourself and then use the Answer Key to be sure you did them all correctly."

Worksheet

Changing Wholes - Worksheet 2, page 31

There are three groups of hexagons on this page. The top one defines the whole as one yellow. The student is asked to write down the fractional value of each color block when the whole is one yellow. Have him record those on the worksheet at the top. This should be easy, as these relationships have been covered before.

The middle problem defines the whole as two yellows. This means that the fractional values of the same blocks used in the first example will change. In the first problem, the red block is one-half. In the second problem, that same red block is now one-fourth.

"Color the double hexagon pink. We have decided that two yellows make a pink. What if we now say that the pink is equal to one? Then the pink is the whole. Now which block is the one-half?" "The yellow because it takes two yellows to make the pink."

"How could you figure out what fraction of the whole the red block is ?" "I could put reds on to cover the pink and count them up. Or I could just see them in my mind. I already know that two reds cover a yellow, so it would take four reds to cover two yellows."

Note

Pay attention to which strategy he uses. If he gives you the second answer, it means the concept is becoming abstractly understood. This is what we are working towards.

Have him finish the page alone if possible.

Test for Understanding

Watch to see what he does on the last problem. This last problem defines the smallest block as the whole. He has not seen this before. If he does it alone, that's great. He gets it. If he needs help, have him put greens on top of the other blocks. You can even write the number 1 on the block and have him count. He already knows it takes three greens to cover a red, but the concept of the whole is what is shifting.

Note

If he calls the green one-third when he covers the red, it means he is confused about what the whole is. We have defined the green as being one whole. This means the red is equal to three. Tell him this. You can even write the number 1 on each block to help. Show him how the green is only one-third if the red is declared the one. If this confuses him, do more of this kind of problem.

Encourage him to build each one to make a clear picture in his mind. Watch him do the other two blocks.

Date_____

Changing Wholes - Worksheet 2

Use pattern blocks.

If 1 yellow = 1,
then 1 red = _____
1 blue = _____
1 green = _____

If 2 yellows = 1,
then 1 red = _____
1 blue = _____
1 green = _____
1 yellow = _____

If 3 yellows = 1,
then 1 red = _____
1 blue = _____
1 green = _____
1 yellow = _____

If 1 green = 1,
then 1 red = _____
1 blue = _____
1 yellow = _____

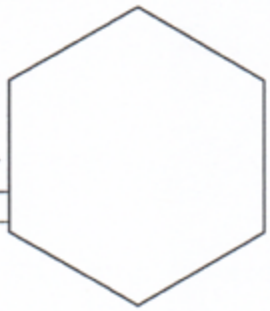
Make your own on another piece of paper.

Fractions - Booklet 2 Sample Pages Answers

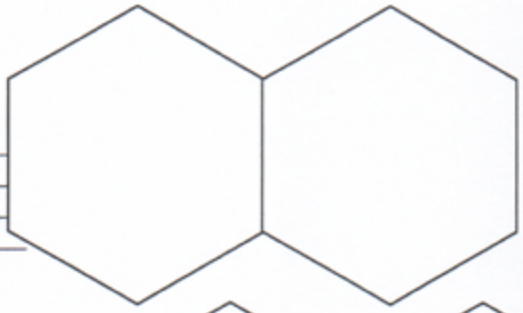
Changing Wholes - Worksheet 2

Use pattern blocks.

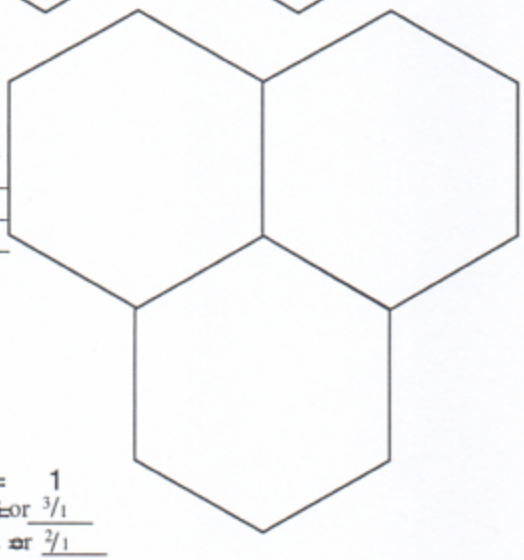
If 1 yellow, = 1
then 1 red = 1/2
1 blue = 1/3
1 green = 1/6



If 2 yellows, = 1
then 1 red = 1/4
1 blue = 1/6
1 green = 1/12
1 yellow = 1/2



If 3 yellows, = 1
then 1 red = 1/6
1 blue = 1/9
1 green = 1/18
1 yellow = 1/3



If 1 green, = 1
then 1 red = 3 or 3/1
1 blue = 2 or 2/1
1 yellow = 6 or 6/1