

Evaluation

The Evaluation is an assessment tool to find out what the student knows and doesn't know. For example, before a student can write the symbol for a number (numeral), and understand the concept of number. If a student does not have one-to-one correspondence in counting, the student is not ready to start addition.

Do not teach or correct during any test. Record the information to use in planning an appropriate program. Make notes about what the student does and what needs to be learned in future lessons. The information will tell you where to begin instruction. At the end of the test is Basic Math Knowledge required to succeed in *Patterns in Arithmetic: Addition - Booklet 1*. In addition to individual lessons list prerequisites.

Summary of Evaluation Tools

Counting - Count out loud as high as you can.

Concept of Number/Cardinality - Teacher hides different amounts of a group of blocks.

One-to-one Correspondence - Count blocks one by one.

Conservation - Teacher asks how many blocks are in both hands, mixed up several times.

Symbolic Concept of Number/Written Number - Write the numeral 3 next to three blocks.

Instant Recognition/Subitizing - At a glance, how many blocks are there in all?

Count Backwards - Count out loud backwards.

Numeral Form - Write numerals from 0 - 9.

Numeral Recognition - Read numerals the teacher has written randomly.

Pattern - Teacher starts a pattern and the student continues it; student creates own pattern.

Order - Order a set of circles by size.

Which Is More? - Guess which pile of blocks is more.

Estimating Large Numbers - Without counting, guess how many objects are in jars.

Sorting - Sort attribute blocks into different sets.

Shapes - During other tests the student recognizes the shapes of blocks.

Addition - Teacher writes an addition problem, student proves understanding by building a model.

Materials for Evaluation Tools

Copy and use one Evaluation record sheet for each student

Small blocks or beans

Bag with five circles of various diameters

Scratch paper

Pencils

Blocks of several colors

Three small jars - sealed

Jar A - 35 small blocks (1 cm cube size)

Jar B - 100 lima beans

Jar C - 250 pinto beans

A bag of attribute blocks - about twenty blocks of different shapes, sizes, thicknesses, and colors.

Or prepare your own bags of sorting materials such as buttons, keys, or other small household items.

For more information see the Internet source: *Interactive STEM, Mathematics in Early Grades: Counting and Cardinality*

Evaluation Tools

Counting - To recite numbers, one by one, in sequence. It is a memorized process. Student counts out loud to 100 if possible.

Ask the student to count as high as she can. Once counting stops, reassure the student if she says she can't count any higher. Tell the student whatever number she counted up to is fine. If the student:

- A) Counts to x and stops. Write down the number. If the student counts to 100 and stops, ask if she knows the next number. She doesn't need to continue counting higher after she has said 101.
- B) Counts to x and mixes up numbers or starts counting numbers out of sequence. Write down the last number she said before she got mixed up. Do not correct the student, just allow her to count as high as she can. See Lessons.
- C) Misses a number out of sequence but stops herself and makes a correction. Note that she can self correct. If she misses many, she may need more practice with the counting sequence.
- D) Skips the same number in the counting sequence through each series of ten. Make a note about which number is missed and teach to this in a later session.

Lessons: See *Mathematics Their Way*, Chapter 4. Play counting games to one number larger than her mistake. Counting is a memorized sequence. If a student can count to 100, it does not necessarily mean she has an understanding of how much 100 is.

Concept of Number/Cardinality - The quantity a number stands for. It links the number 4 with the amount four.

Materials - Any small manipulative

Ask the student to count three blocks into your hand. You hide one block and ask, "**How many blocks are hiding?**" The student answers "one." Hide a different amount. Be sure to hide all of the blocks and none of the blocks. The student should be able to answer how many are being hidden. If she says the amount that is in the open hand, say, "**Yes, there are x here. How many are hiding in this hand?**" Point to the hand with the hidden blocks. If the student:

- A) Confidently says the correct number. Increase the amount of blocks until she is saying the wrong amount. *Do not say* yes that is right, or no that is wrong. Do not show the hidden blocks. Nod and act the same with each guess. Circle the number at which she starts having difficulty.
- B) Counts on her fingers. Write down that she can figure out the solution manipulatively only. Record which number she needed to start using manipulatives. See Lesson below.
- C) Misses the amount of blocks that are hidden. Write down the number with which she started having difficulty. If three is the number she is working on, open your hand and have her count the blocks again to make sure she remembers how many blocks she started with. Again hide blocks to see if she now understands the task better. If this doesn't work, drop the blocks and ask her to put two blocks in your hand. Have the student count the blocks in your open palm. Then hide one block and ask, "**How many are hiding in the closed hand?**" If she does not say one, end the test. She has very little understanding of numbers. Work on counting activities. See Lesson below.

Lesson: See *Mathematics Their Way*, Chapter 7

One-to-one Correspondence - Each object is counted one by one. There is a direct link between each object and its numeric name.

Materials - Any counting object such as beans, buttons, blocks, etc.

Put out five blocks. Ask, "**How many blocks are there?**" If the student:

- A) Says the number five immediately: Pick up those blocks and put down three blocks. Ask, "**How many now?**" If the correct answer is given, put out four blocks. Then six, then start adding two or three blocks at a time until there are about fourteen blocks OR the student starts miscounting. If there is any hesitancy in the voice, ask her to count them again. Tell her she can touch the blocks if she wants to. Ask, "**Are you sure?**" When she counts the correct amount, have her recount to prove she knows the answer, even when she is correct. Record the highest number she can count correctly.
- B) Counts the number of blocks mentally or by touching each one: Pick up those blocks and put down three blocks. Ask, "**How many now?**" If the correct answer is given, put out four blocks. Then six, then start adding two or three blocks at a time until there are about fourteen blocks OR the student starts miscounting. If the student can successfully count this many blocks, this test is complete.
- C) Guesses the wrong amount: Have the student count the blocks carefully. Try having the student count objects by touching each object as she counts them. Record that she needs to work on one-to-one correspondence.
- D) Counts the wrong amount: Have the student count the blocks carefully. If she can't count them one by one successfully, she needs work on one-to-one correspondence.

Lesson: *Mathematics Their Way*, Chapter 4; *Patterns in Arithmetic*: General Math - Booklet 1, Circle Game and One-to-one Correspondence on the Internet: Interactive STEM, Mathematics in Early Grades: Counting and Cardinality

Conservation of Number: Conservation - the total remains the same no matter how a number is expanded: $2 + 3 = 5 = 2 + 1 + 2$.

Materials - Any small manipulative

Have the student count out five to seven blocks into your hand, the amount depends upon the sophistication of the student you are working with. Make a judgment based on what the student has done up until now. Close your hands over all the blocks, mix them up. Leave some blocks in one hand and some in the other one. Open your hands palms up and ask, "**How many now?**" Sometimes you have to ask, "**How many are in both hands?**" If the student:

- A) Says the correct amount: Mix the blocks up and put a different combination in each hand but use the same amount of blocks. Again ask how many blocks. If the student says the correct amount, add another block and repeat with a couple of different combinations. If the student says the correct amount, make it a higher number (but raise the amount only by two or three blocks). Mix the blocks up again and ask how many. Have her look and tell you how many blocks there are in total with an increasing amount of blocks. Record the highest number she does correctly or stop at about twelve blocks. The student understands conservation.
- B) Stops and counts each block or guesses the wrong amount: Have her count the blocks one by one. Mix them up and have her count again. Ask, "**How many blocks altogether?**" If she counts by touching each block and does not automatically say the total amount of blocks, it means she does not understand Conservation. Try a lower number of blocks. Record the highest number she can do correctly. She does not understand that it is always the same amount (with five or fewer blocks) no matter how many ways they are mixed up.

Lesson: See *Mathematics Their Way*, Chapter 4: Counting - Invariance or Conservation of Number or *Patterns in Arithmetic: Addition - Booklet 1*, Breaking Up Numbers - Three Circles

Symbolic Concept of Number/Written Number

Have the student put out three blocks and write the numeral that goes with that amount. Repeat with a few random numbers up to about twelve. Record the highest number.

Lesson: See *Mathematics Their Way*, Chapter 9

Instant Recognition/Subitizing

Materials - Any counting material.

During the test for one-to-one correspondence note how many blocks the student can instantly recognize before counting them one by one either by touching each block or nodding as she counts. Circle the number in the Instant Recognition recording space.

Lesson: See *Mathematics Their Way*, Chapter 7

Numeral Form/Written Number

Materials - Paper and pencil. Use the back of the evaluation sheet or a scratch piece of paper.

Say, "**Write the numerals (or numbers) zero to nine.**" If the student:

- A) Can not write some or any of the numerals: Tell her it is OK if she does not know how to write it. How to write numerals is something she will be learning.
- B) Successfully writes the numerals from zero to nine: Look at the form of the numerals, how the pencil is held and how each numeral is made. Record which numerals need just to be practiced more for better penmanship or those that need to be taught how to be formed correctly. For example, are eights made with two circles versus a flowing motion from top to bottom? For fluency and speed, eights are better learned in one fluid motion. Are any numerals drawn backwards or upside down? Are the same numerals always formed in this manner, or is this a random occurrence? Check this at other times to see which is the case. Attach this sheet to the evaluation pack.
- C) Reverses numbers.: Sometimes a student has simply not mastered the correct direction and more practice is needed. In older students (eight and up), forming the letters in reverse can be a sign that there may be learning difficulties, especially if reading is very difficult and a word like "saw" is written or read as "was." A more in-depth reading evaluation would need to be done in this case. Attach this sheet to the evaluation pack.

Lesson: See *Mathematics Their Way*, Chapter 2

Counting Backwards

Ask, "**From what number would you like to start counting backwards?**" You may want to ask if the student has ever counted backwards. Encourage her to try any number she chooses. If the student successfully counts from a number you consider below her ability, ask her to start at a higher number, but not too much higher. If the student easily counts backwards from five, ask her to count from ten. Depending upon her attempt or non-attempt, you can get a glimpse of how much she likes to challenge herself and what kind of a learner she is, e.g., timid, assertive, independent, self-correcting, confident,

dependent, autodidactic, or wants help with each situation.

Lesson: See *Mathematics Their Way*, Chapter 4

Numeral Recognition

Materials - Paper and pencil

After the student has written her numerals, tell her it is your turn to write some numerals. Start with any number under ten and randomly write each number. Say, "**Read the numeral or number.**" Do not write the numerals in order, as she will just recite the names without having to recognize them. If she has no difficulty with 0 - 9, write a few numbers in the teens. After 20 write one number from each set of ten, e.g., 44, 76, 37, 58, 83, 68. If she still can say each one accurately, go on to 100. Then go on to 106, 110, 132. Stop at this point so neither one of you is bored. Stop earlier if she is having difficulty. Record any difficulties and the highest number.

Lesson: See Place Value: Base Ten in *Patterns in Arithmetic: Place Value - Booklet 1*

Pattern

Materials - Any material with several different colors or shapes. Unifix cubes work well.

Make a pattern, such as red, blue, red, blue, red, blue, red. Ask, "**What is the next color in my pattern?**" Answer, blue. Have her place the block. Ask, "**What would be next in the pattern?**" Answer, red. Again have her place the block. Next say, "**Make your own pattern.**" Record what she does. Does she make another one like yours or does she make up a different pattern? An easy way to record is to write down the initial letter of the color she used, e.g., r, b, r, b. Does she make a more complicated pattern, such as yellow, yellow, green, yellow, yellow, green? Or does she make a random row of blocks? If you are uncertain what she is doing, ask her to explain the pattern to you. Do not interfere or try to have her make any particular pattern. Let her make whatever she wants and record the results.

Lessons: See Pattern in *Patterns in Arithmetic: General Math - Booklet 1*, and/or *Mathematics Their Way*, Chapter 2

Order

Materials - Five circles of different diameters.

Say, "**Put the circles in order by size.**" Write down what she does. Very few students have difficulty with this task. Some get confused if different thicknesses of circles are used, but that is a good test of her degree of observation and how she thinks about the world.

In my experience, all children have been successful. Thus, this exercise provides the student confidence early in the session.

Shapes

If during the tests a student does not know the shape of any object, write down the shape that needs to be studied in the future.

No lessons in shape are in this book, as these lessons focus on the school-age student.

Which Is More?

Materials - About twenty beans or small blocks

While she is doing the "ordering the circles" test, choose a small pile of beans or blocks (about five or six) and a large pile of beans (around eleven or twelve). Ask, "**Which is more?**" Record the results. Most students can tell which is more or less at a glance. If she has to stop and count, record this.

Lesson: Comparing Lesson in *Patterns in Arithmetic: General Math - Booklet 1*. Also see *Mathematics Their Way*, Chapter 5: Comparing

Estimating Large Numbers

Materials

Jar A - 35 small blocks (1 cm cube size)

Jar B - 100 lima beans

Jar C - 250 pinto beans

Have three small jars the same size (baby food jars work well) with different amounts of different objects in them. Be sure to seal the jars tightly. Say, "**Guess how many are in Jar A.**" Record the results by circling the number she said or write the number if it isn't on the sheet. Repeat with Jar B and Jar C. Ask, "**Which jar has the most things in it?**" Record which one. Sometimes the student will state the reason. It is interesting to record this too. It will give you insight into how the student thinks.

Lesson: See *Mathematics Their Way*, Chapter 11, Estimating and Checking, page 308.

Sorting

Materials - A bag of attribute blocks - about twenty blocks of different shapes, sizes, and colors

This is often the last test for young students ages five or six. It will push the student to the limit, which is interesting to observe and allows you to record a short note about her stamina. Pour the attribute blocks out and say, "**Put the blocks into groups that are somehow the same.**" Use your time observing how she sorts. After she has sorted the blocks, point to each group and ask, "**How are these all the same?**" Record how the blocks were sorted. If she sorts by two attributes, such as blue and thick, write down all the criteria she used. Sometimes students will just put the blocks in designs and name the designs. Make a note of this. Push all the blocks back into one pile and say, "**Sort the blocks a different way.**" Be sure to be encouraging, even if she didn't sort the blocks the first time. Often students have never come across this task before and are not really sure what is meant by sorting. After each time the blocks have been sorted, point to each of the groups and ask, "**How are the pieces the same? How are the groups different?**" Repeat until the student can no longer think of any more ways. Usually five different ways is the most any student can think of. Notice where the student's limits are, but do it in a light way. Give encouragement even if the sorts aren't that different. The student has done many tests by now and may be reaching the limit of concentration, patience, ability to focus, and stamina. Note how she is reacting and how she is able to continue with the task at hand.

Lesson: See Sorting and What's the Rule in *Patterns in Arithmetic: General Math - Booklet 1* and *Mathematics Their Way*, Chapter 3

Addition

Materials - Any manipulative, paper, and pencil

Write the equation $2 + 3 = \underline{\quad}$ or written vertically: $\begin{array}{r} 2 \\ + 3 \\ \hline \end{array}$

Ask, "**Have you done a problem like this before? How do you solve it?**" If she is able to record five as the sum, have her build or prove the answer with blocks. Say, "**Show me what this means.**" If the student:

- A) Puts out two blocks for the two and three blocks for the three, pushes all the blocks together and says the answer is five: Give her a more difficult problem. For example, $7 + 5$. If she can successfully do this problem, ask her to write three problems: a very easy one, a very hard problem, and a problem that is not too hard but also not too easy. If the hard problem is too hard, watch and see how she deals with it. Tell her she doesn't have to answer it right now. You are getting a range of her ability. It will tell you where to start working and what size problem she is currently working on. Staple this paper to her evaluation sheet.
- B) Tells you two plus three equals five or is blank, have her do the problem in her head, with blocks, or on her fingers: If she is not sure what to do and can't read the equation, have her build it. Say, "**Put out this many blocks.**" Point to the two. Have her do this. Say, "**Put out this many blocks.**" Point to the three. Have her do this. Ask, "**How many blocks in all?**" She should be able to count them one by one or say five immediately. This may make her remember having done this before but without the symbols.
- C) Is uncertain about what she is doing: End the evaluation.

Lessons: *Patterns in Arithmetic: Addition - Booklet 1*, *Patterns in Arithmetic: Place Value - Booklet 1*. Review *Mathematics Their Way*, Chapters 7, 8, and 9.

Basic Mathematical Knowledge

Patterns in Arithmetic: Addition - Booklet 1 assumes that the student has learned what is typically taught to a five-year-old or in kindergarten. A student should have the following basic mathematical knowledge before beginning this program:

Counting - count out loud to about 20.

Concept of Number/Cardinality - to about 6.

One-to-one Correspondence - to 8.

Conservation - up to 5.

Symbolic Concept of Number - to 10.

Instant Recognition - to 4.

Count Backwards - from 10; but it is not a prerequisite and can be learned during the year.

Numeral Form - write 0 - 9; reversals will need to be practiced.

Numeral Recognition - up to about 25.

Pattern - if a student doesn't know, it will be learned.

Order - can order a set of circles by size.

Which Is More? - can tell which pile of blocks is more.

Estimating Large Numbers - if a student doesn't know, it will be learned.

Sorting - can sort attribute blocks into different set. If a student doesn't know, it will be learned.

Shapes - can recognize shapes: circle, triangle, square, or rectangle.

Addition - none, it will be learned.

Evaluation

Name _____ Date of Birth _____ Date _____

Counting

Concept of Number/Cardinality 4 5 6 7 8 9

1:1 Correspondence _____ Highest number successfully counted

Conservation _____ 1-5 _____ 6-10 _____ 11-18

Symbolic Concept of Number/Written Number

Instant Recognition/Subitizing 2 3 4 5	Numeral Form
Count Backwards	Numeral Recognition

Pattern ABABAB Student

Order	large to small	small to large	Shapes
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Which is more: _____ or _____?

Estimating large numbers Which has the most? _____

- A 5 10 15 20 25 30 35 40 45 50 55 60 65 70
 B 0 20 40 60 80 100 120 140 160 180 200
 C 0 50 100 150 200 250 300 350 400 450 500

Sorting				
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Addition

Comments