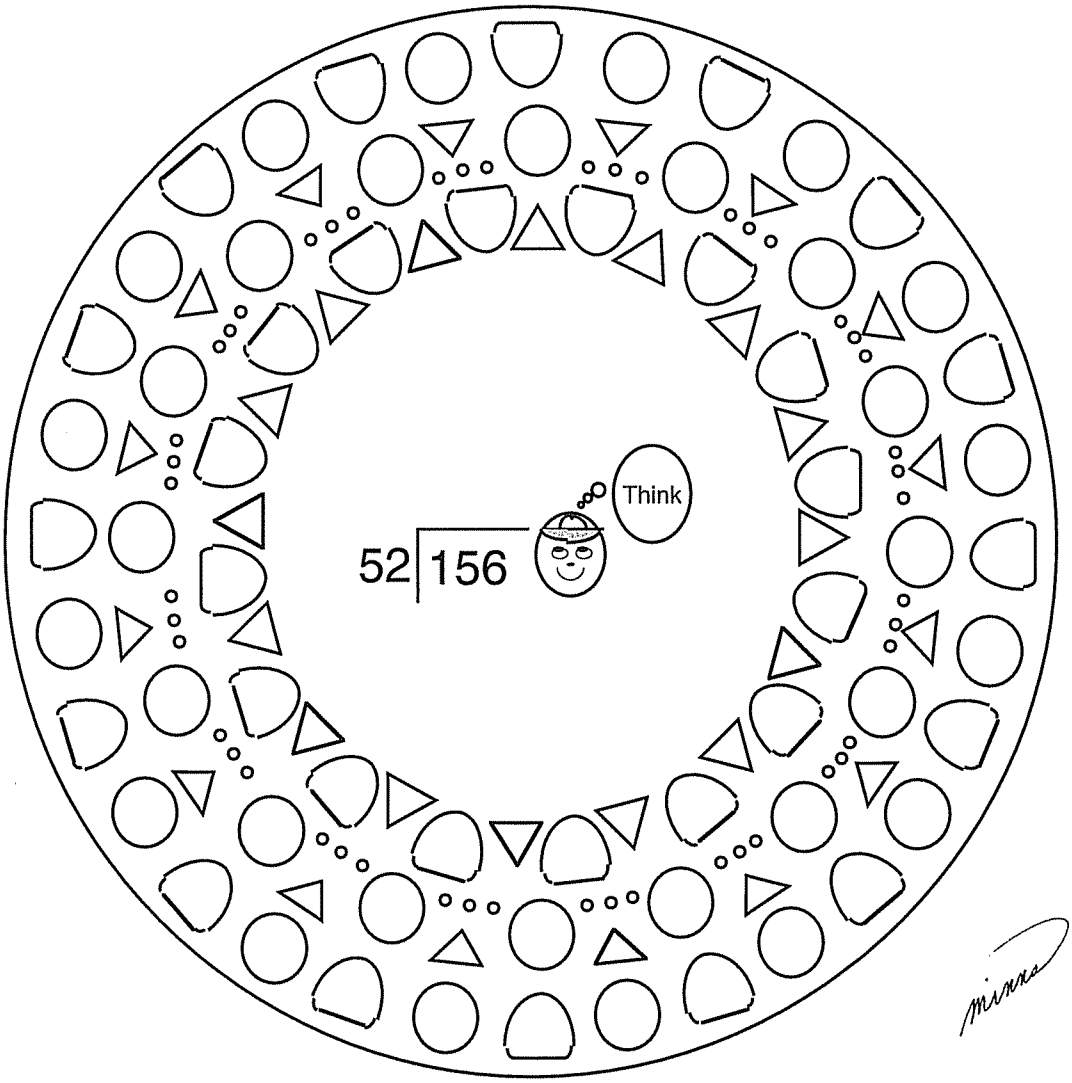
Patterns in Arithmetic

Division - Booklet 3 PDF

Working with Double Digit Division

Parent/Teacher Guide



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Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Division: Booklet 3 - PDF - Working with Double Digit Division

Contents

We dedicate this booklet to Jon Rehmas and Willie Herdon for their gifted teaching in the Big Kids' House at the Farm School Thank you for your help with this math program.

Acknowledgments

The knowledge, patience, and dedication of Professor Michael Butler made the UCI Farm Elementary School and this mathematics program possible. Special thanks go to Alysia Krafel and Susan Carpenter, who helped develop much of the math materials based on the teachings, ideas, and insights of Professor Butler.

For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

The books would never have been completed if the students at Chrysalis Charter School in Redding, California, under the guidance of Alysia and Paul Krafel, hadn't needed them. Thank you for your patience through all of the draft copies.

Susan Carpenter edited, added her wise words, useful suggestions, and helped make the Answer Keys a reality. Karán Founds-Benton contributed her meticulous editing skill and knowledge. Diligent and thorough copy editing was done by Jacqueline Logue.

The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

Patterns in Arithmetic: Division - Booklet 3 PDF
Parent/Teacher Guide

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Pre-Assessment: Part 1 and Part 2 Name _____ Date _____

Put a question mark next to any problem you do not know how to do.

Part 1

1. a. $5\overline{)965}$ b. $8\overline{)4,096}$ c. $9\overline{)1,377}$

d. $4\overline{)384}$ e. $5\overline{)728}$ f. $4\overline{)349}$

2. a. $\begin{array}{r} 34 \\ \times\ 18 \\ \hline \end{array}$ b. $\begin{array}{r} 72 \\ \times\ 84 \\ \hline \end{array}$ c. $\begin{array}{r} 354 \\ \times\ 67 \\ \hline \end{array}$

Part 2

1. Show the remainders as fractions.

a. $4\overline{)15}$ b. $5\overline{)16}$ c. $6\overline{)16}$

2. Show the remainders as decimals. Round off to the nearest hundredth if needed.

a. $6\overline{)213}$ b. $9\overline{)165}$ c. $7\overline{)109}$

Pre-Assessment: Part 2 continued Name _____ Date _____

3. Find and record the missing number.

a. ◯ x 4 = 1,600

b. 80 x ◯ = 5,600

c. Prove your answer is true in problem b by showing a factoring by tens factor tree.

5,600

d. ◯ x 60 = 180,000

4. a. $3\overline{)150}$ b. $30\overline{)150}$ c. 30 x _____ = 150

d. $3\overline{)1{,}500}$ e. $30\overline{)1{,}500}$ f. 30 x _____ = 1,500

5. a. $700 \div 20 =$ _____ b. $1{,}260 \div 30 =$ _____

c. $1{,}380 \div 60 =$ _____ d. $4{,}860 \div 90 =$ _____

6. If the dividend stays the same, $50\overline{)2{,}950}$

and the divisor goes up a little,

I expect the quotient to go _____ a little.

↓ down or ↑ up

7. a. $49\overline{)343}$ b. $47\overline{)282}$ c. $62\overline{)310}$

8. a. $52\overline{)1{,}248}$ b. $54\overline{)972}$ c. $46\overline{)4{,}922}$

Assessment Guide

Purpose

The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Pre-Assessment: Part 1 is review material from the last booklet and is used to determine student readiness for this booklet. Pre-Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

Post-Assessment: Part 2 of the Pre-Assessment is re-administered to determine if the student learned the material that was presented in this booklet. A comparison of the score on Part 2 of the pre-assessment to the score on the post-assessment will give both you and your student a sense of growth.

Prerequisites

Patterns in Arithmetic: Division - Booklet 2 or *Chrysalis Charter School*: Base Ten Division, or other previous successful instruction in the basic division concept with and without remainders, how to calculate the quotient with a single digit divisor and a dividend into the thousands. Example: $3456 \div 6 =$ ____
Patterns in Arithmetic: Multiplication: Booklet 4 or similar instruction on problems such as $23 \times 58 =$ _____
Patterns in Arithmetic: Fractions: Booklet 2 or other basic instruction in fractional units
Keys To Decimals: Book 1 - Decimal Place Value or similar instruction

Materials

Division: Booklet 3 - Pre-Assessment, pages i and ii, Post-Assessment, pages 46 and 47. Copies of both Pre- and Post-Assessments are included in this lesson.

Instructions

Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it.

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

Do the assessment in two parts. Give Pre-Assessment: Part 1 and check it for readiness for this booklet. If he is not ready for this booklet, there is no point in giving Pre-Assessment: Part 2. If he passes all the readiness items, then give Pre-Assessment: Part 2.

After scoring Pre-Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

Assessment Guide

This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion

is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc. Score sheets that match the Assessment Guide for Pre-Assessment: Part 1, Pre-Assessment: Part 2, and the Post-Assessment follow.

Assessment Criteria for Pre-Assessment: Part 1

Can the student:

1. Single Digit Divisor (Division: Booklet 2)
   A. calculate correctly the quotient of five of six single digit division problems? If No, then go to Item B.
   B. calculate correctly the quotient of four of six single digit division problems?

2. Standard Procedure (Multiplication: Booklet 4)
   A. calculate correctly the product of two of three double digit multiplication problems?

Booklet Selection Guide based on results of Pre-Assessment: Part 1

If Yes on Items 1A and 2A, proceed to Pre-Assessment: Part 2.
If Item 1A is a No, but Item 1B is a Yes, proceed to Pre-Assessment: Part 2 of the assessment. Give additional practice. The first ten pages of Division - Booklet 3 use only single digit divisors. This review may be enough to rekindle previous knowledge. If not, reteach before going on.

If Items 1A and 1B are No, the student is not ready for this booklet. Reteach single digit division before going on. If your student is new to *Patterns in Arithmetic* but has had previous instruction in division, consider beginning with *Chrysalis Charter Schoo*l: Base Ten Division to build concrete understanding of this difficult procedure. If your student has never had instruction in division and knows the multiplication table, begin with Division: Booklet 2.
If Item 2 is left blank, go to Multiplication: Booklet 4 before continuing with this booklet.
If Item 2 is a No, but you can see the student knows how to do the procedure, give more practice. Consider testing for understanding using the assessment for Multiplication: Booklet 4. Understanding of double digit multiplication is critical for algebra as well as for doing division.

Assessment Criteria for Pre-Assessment: Part 2

Can the student:

1. Remainders as Fractions (Division: Booklet 3)
   A. correctly show the remainder as a fraction in two of the three problems?

2. Remainders as Decimals (Division: Booklet 3)
   A. insert a decimal point after the whole number and add zeros?
   B. give correct decimal remainder in problem a; decimal comes out even?
   C. give correct decimal remainder in problem b; the decimal repeats/uses a bar?
   D. give correct decimal remainder in problem c; the answer must be taken to the thousandths place and rounded off to the nearest hundredth.

3. Missing Factors Puzzles (Division: Booklet 3)
   A. show the correct missing number in both problems a and b?
   B. show the tens factor tree used to prove the answer to problem b?

C. show the correct missing number in problem d?

If the student gets a Yes on Items A and C and a No on Item B, it indicates possible lack of understanding of how the place value of the missing factor is determined. The student may be counting zeros with no understanding of why that pattern works. Review/teach in Multiplication: Booklet 4.

4. Double Digit: Beginning (Division: Booklet 3)

A. use place value patterns to determine the place value of the quotient in problem b using the correct answer from problem a?

B. prove the answer to problem b with the missing factor in problem c?

C. use place value patterns to determine the place value of the quotient in problem e using the correct answer from problem d?

D. prove the answer to problem e with the missing factor in problem f?

5. Double Digit: Beginning (Division: Booklet 3)

A. give the correct answer in three of the four problems with division with a two digit divisor that is a factor of ten? This requires the student to be able to use the standard division procedure and make place value adjustments to the quotient.

6. Sequences (Division: Booklet 3)

A. answer that if the divisor goes up a little bit, the quotient will go down a little bit? If the student is able to do one or two of the problems correctly on the Pre-Assessment, begin with the Double Digit section of the booklet to achieve mastery.

7. Double Digit Division - Single Digit Quotients (Division: Booklet 3)

A. use the correct procedure to do this kind of problem?

B. find a single digit quotient with a double digit divisor and get the correct answer two out of three times?

8. Practice (Division: Booklet 3)

A. use the correct procedure to do this kind of problem?

B. find a double or triple digit quotient with a double digit divisor and get the correct answer two out of three times?

Booklet Selection Guide based on results of Pre-Assessment: Part 2

Students who score 80% or more do not need this booklet. Remediate any weak areas and move on to your next topic: *Keys to Decimals:* Book 3 - Division with Decimals.

Students who get Yes on Items 1 - 6, 7A, and 8A but No on Items 7B and 8B need only practice to gain accuracy. This booklet is not needed.

Students who score Yes on Items 1, 2, 3A, 3C, 5, 7A, 7B, 8A, and 8B but No on Items 3B, 4, and 6 are showing you they can do the problems but lack understanding of the concepts the procedure is based on. Begin with lessons on Place Value Patterns, Missing Factors Puzzles, and Sequences and retest.

Students who score less than 50% overall should start at the beginning of this booklet.

Pre-Assessment: Part 1 Score Sheet

Name _____ Date _____

Can the student:

Part 1

1. Single Digit Divisors (Division: Booklet 2)

Yes No A. answer correctly five of six problems?

Yes No B. get four of six problems correct?

Circle any items that were missed: a b c d e f

2. Double Digit Multiplication (Multiplication: Booklet 4)

Yes No A. answer correctly two of three problems?

Items Correct = _____ = _____%

Items Possible = 2

Pre-Assessment: Part 2 Score Sheet

Can the student:

1. Remainders as Fractions (Division: Booklet 3)

Yes No A. show the remainder as a fraction correctly in two of the three problems?

2. Remainders as Decimals (Division: Booklet 3)

Yes No A. insert a decimal point after the whole number and add zeros?

Yes No B. give correct decimal remainder in problem a; decimal comes out even?

Yes No C. give correct decimal remainder in problem b; the decimal repeats/uses a bar?

Yes No D. give correct decimal remainder in problem c; the decimal taken to the thousandths place and rounded off to the nearest hundredth?

3. Missing Factors Puzzles (Division: Booklet 3)

Yes No A. show the correct missing number in both problems a and b?

Yes No B. show the tens factor tree used to prove the answer to problem b?

Yes No C. show the correct missing number in problem d?

4. Double Digit: Beginning (Division: Booklet 3)

Yes No A. use place value patterns to determine the place value of the quotient in problem b using the correct answer from problem a?

Yes No B. prove the answer to problem b using problem c?

Yes No C. use place value patterns to determine the place value of the quotient in problem e using the correct answer from problem d?

Yes No D. prove the answer to problem e using problem f?

5. Double Digit: Beginning (Division: Booklet 3)

Yes No A. give the correct answer in three of the four problems?

6. Sequences (Division: Booklet 3)

Yes No A. answer that if the divisor goes up a little bit, the quotient will go down a little bit?

7. Double Digit Division - Single Digit Quotients

Yes No A. use the correct procedure to do this kind of problem?

Yes No B. find a single digit quotient with a double digit divisor and get the correct answer two out of three times?

8. Practice (Division: Booklet 3)

Yes No A. use the correct procedure to do this kind of problem?

Yes No B. find a double or triple digit quotient with a double digit divisor and get the correct answer two out of three times?

Items Correct = _____ = _____%
Items Possible = 18

Post-Assessment Score Sheet Name _____ Date _____

Can the student:

1. Remainders as Fractions (Division: Booklet 3)

   Yes No A. show the remainder as a fraction correctly in two of the three problems?

2. Remainders as Decimals (Division: Booklet 3)

   Yes No A. insert a decimal point after the whole number and add zeros?

   Yes No B. give correct decimal remainder in problem a; decimal comes out even?

   Yes No C. give correct decimal remainder in problem b; the decimal repeats/uses a bar?

   Yes No D. give correct decimal remainder in problem c; the decimal taken to the thousandths place and rounded off to the nearest hundredth?

3. Missing Factors Puzzles (Division: Booklet 3)

   Yes No A. show the correct missing number in both problems a and b?

   Yes No B. show the tens factor tree used to prove the answer to problem b?

   Yes No C. show the correct missing number in problem d?

4. Double Digit: Beginning (Division: Booklet 3)

   Yes No A. use place value patterns to determine the place value of the quotient in problem b using the correct answer from problem a?

   Yes No B. prove the answer to problem b using problem c?

   Yes No C. use place value patterns to determine the place value of the quotient in problem e using the correct answer from problem d?

   Yes No D. prove the answer to problem e using problem f?

5. Double Digit: Beginning (Division: Booklet 3)

   Yes No A. give the correct answer in three of the four problems?

6. Sequences (Division: Booklet 3)

   Yes No A. answer that if the divisor goes up a little bit, the quotient will go down a little bit?

7. Double Digit Division - Single Digit Quotients

   Yes No A. use the correct procedure to do this kind of problem?

   Yes No B. find a single digit quotient with a double digit divisor and get the correct answer two out of three times?

8. Practice (Division: Booklet 3)

   Yes No A. use the correct procedure to do this kind of problem?

   Yes No B. find a double or triple digit quotient with a double digit divisor and get the correct answer two out of three times?

Items Correct = _____ = _____% 16 correct are needed to pass without remediation
Items Possible = 18

Post-Assessment - Worksheet 1 Name _____ Date _____

1. Show the remainders as fractions.

a. $4\overline{)15}$

b. $5\overline{)16}$

c. $6\overline{)16}$

2. Show the remainders as decimals. Round off to the nearest hundredth if needed.

a. $6\overline{)213}$

b. $9\overline{)165}$

c. $7\overline{)109}$

3. Find and record the missing number.

a. $\bigcirc \times 4 = 1{,}600$

b. $80 \times \bigcirc = 5{,}600$

c. Prove your answer is true in problem b by showing a factoring by tens factor tree.

5,600

/ \

d. $\bigcirc \times 60 = 180{,}000$

4. a. $3\overline{)150}$

b. $30\overline{)150}$

c. $30 \times \_\_\_\_ = 150$

d. $3\overline{)1{,}500}$

e. $30\overline{)1{,}500}$

f. $30 \times \_\_\_\_ = 1{,}500$

Post-Assessment - Worksheet 2 Name _____ Date _____

5. a. $700 \div 20 =$ _____ b. $1{,}260 \div 30 =$ _____

c. $1{,}380 \div 60 =$ _____ d. $4{,}860 \div 90 =$ _____

6. If the dividend stays the same,

$50\overline{)2{,}950}$

and the divisor goes up a little,

I expect the quotient to go _____ a little.

↓ or ↑

down up

7. a. $49\overline{)343}$ b. $47\overline{)282}$ c. $62\overline{)310}$

8. a. $52\overline{)1,\ 248}$ b. $54\overline{)972}$ c. $46\overline{)4{,}922}$

Remainders as Fractions

Purpose The purpose of this lesson is to develop the concept that a remainder can be expressed as a fraction of the divisor. The lesson uses cookies as models since students are familiar with sharing fractional parts of that last cookie on the plate. The lesson continues the development of the concept of a remainder as a fraction of the divisor with Cuisenaire Rods. Students can observe that the value of the white rod changes when the divisor changes.

Note The early lessons of this Booklet can be taught concurrently with the practice phase at the end of Division - Booklet 2.

Prerequisites Fluent in basic division and the concept of the Changing Wholes as taught in Fractions: Booklet 2. If the Changing Wholes concept has not been taught, it can be taught with this lesson but less deeply.

Materials
Remainders as Fractions: Worksheets 1 - 7, pages 1 - 7
Large soft cookies (optional but fun); paper ones can be substituted
Knife or scissors
Prism Fractions Circles
Cuisenaire Rods for Part 2
Calculator for Part 3

Part 1
Warm Up

Take out three cookies.
"How can two people share three cookies?" "Each person gets one."
"Are you just going to leave that last remaining cookie sitting on the plate? It's really good!" "No, I would cut the last cookie in half so each person would get 1 ½ cookies."

Put down four cookies. **"What if there are three people and four cookies? Cut up any remaining cookies to make it so each person gets the same amount."** "I will have to cut that remaining cookie into thirds." ○○○☮

$$\begin{array}{r} 1 \\ 3\overline{)\,4} \\ \underline{3} \\ 1 \end{array}$$

"So how much does each person get?" "One and one third cookie."
"Let's show that as a division problem. Do the division. $3\overline{)\,4}$
You are not going to show a remainder of 1 like you have in the past. You are not going to leave that remaining cookie on the plate. So you will write the answer as a mixed number showing how much cookie each person gets." Write this on the paper.

$$\begin{array}{r} 1\,\frac{1}{3} \\ 3\overline{)\,4} \\ \underline{3} \\ 1 \end{array}$$

Take out the Fraction Circles or your paper cookies. Try this challenge problem.
"How much would each person get if there were three people and two cookies to be divided exactly evenly? How could you cut those up?" *Wait.* Let her figure this out on her own. What she has to realize is that each cookie must be

cut up into thirds, with one-third of each cookie given to each person. ☮☮
So each person gets two-thirds of a cookie.

Here is a challenge problem: Seven cookies and four children. The Fraction Circles will really help here. Again the main problem to be solved is that each remaining cookie must be cut into fourths and then distributed.

The best solution I ever saw to this problem was a student who converted the remaining three cookies into an improper fraction of twelve fourths and divided the twelve by four to get three fourths. He did this in his head. A great creative response.

If you get an answer and she did it mentally ask, **"How did you get that answer?"** If you are teaching a class, not every one in the class will be ready to understand this solution, so don't push it.

Lesson

Remainders as Fractions: Worksheets 1 and 2, pages 1 and 2

Work through the two worksheets together. If the warm-up activity was successful, the worksheet will pose no difficulty. Beginning with the problems showing the chocolate chip cookies, all remainders should be shown as fractions to show that the remaining cookie was cut up.

Worksheets

None for Part 1

Give this Gus the Bus Driver problem.

Gus took a special job driving a dog pound truck for the Humane Society. His truck has six cages. He goes to a house to pick up a load of abandoned dogs. There are eight. How many dogs per cage?

Test for Understanding

This test is to see if the student recognizes that fractional remainders are not always appropriate. You can not cut dogs up and put part of a dog in each cage. In this case a remainder is appropriate.

The solution could be to put one dog in each cage, then add another cage, or to make a second trip.

Part 2

Warm Up

Take out the Cuisenaire Rods and let her play with them for ten or fifteen minutes. Give these problems to warm her up to do division problems with the rods.

Use Remainders as Fractions: Worksheet 3, page 3, for this part of the lesson.

Lesson

This lesson reteaches the concept of a remainder as a fraction using a different manipulative. The instructions read, 'Use each color of Cuisenaire Rod to make this a twelve train.' What that means is shown in the graphics in the main body of the worksheet.

On the first problem, she is asked to use an orange ten rod. One orange rod and two white rods fit into the twelve centimeter long box. Thus, the actual problem is $12 \div 10$, but that is not stated to her at this time. Have her color the space orange where the orange rod would fit. Have her note that the end of the orange rod and each white rod was traced. Leave the white spaces white. How she is to record is shown with the numbers and words above the box. In this first problem, the recording is done for her.

Problem 2 asks her to use the blue nine rod. Have her place the blue rod in the twelve centimeter box and trace where the block ends. She then colors in that space blue and records that one blue rod fits in the box. She will then fill in the rest of the space with white rods. She should trace the ends of each box as is shown in the example.

Note

Do not let her use one green rod to fill the remaining space. Doing so creates the wrong visual image.

When she gets to the dark green rod, it will take exactly two dark green rods and no remainders. Have her finish the page on her own. Check the work.

Worksheet

Do Remainders as Fractions: Worksheet 4, page 4, together. Use the Answer Key to be sure you are answering the questions correctly.

The question asked is, "The two white rods are what fraction of the orange rod?" The essential understanding here is that she sees that the value of the remainder is determined by the divisor. This concept harkens back to the idea of the Changing Whole taught in Fractions: Booklet 2. This concept is critical in understanding remainders in general but especially in division of fractions. There is more on this in Fractions: Booklet 8 on the subject of the division of fractions.

She is then asked to answer the statement, "The blue rod goes into the twelve, ____ and ____ time."

"What part of the blue rod is covered by the three remaining white cubes?"
"Three out of nine, or three ninths." She might also say one-third.

"In the first problem the whites were tenths and now you say they are ninths. How can that be? Both times you are using the exact same white rods."

Wait. Give her time to think about this. "Because in the first problem we are comparing the size of the whites to the orange rod; in the second problem we are comparing the whites to the size of the blue rod. So the fraction will change. It takes ten whites to cover an orange, so a white is one tenth, but it takes only nine whites to cover the blue rod, so now each white is a ninth."

Complete Remainders as Fractions: Worksheets 4 and 5, pages 4 and 5, together.

Practice Worksheets

Remainders as Fractions: Worksheets 6 and 7, pages 6 and 7
On Worksheet 6, there is a graphic at the top of the page that will help if help is needed. On the second problem 12 ÷ 5, she would count twelve squares on the graphic and make a light end-mark in pencil. Then she would place two yellow five rods over the squares and see that there are two squares remaining and cover those with two white rods. The two remaining white rods are a remainder of two-fifths. Make sure she knows how to record the answers. Have her do the top two rows and then check the work.

Test for Understanding

Watch to see when she stops using the blocks and uses the pattern of writing the remainder as the numerator of the fraction and the denominator of the remaining fraction from the divisor. If she has not seen this pattern by the end of Worksheet 6, draw her attention to it before she begins Worksheet 7 the next day.

"How do you know what number to write as your remaining fraction?" "The denominator is the size of the group you are using, so you use the divisor as the denominator. The remainder is a fractional part of the divisor, so it is the numerator."

Part 3 Extension Warm Up

Give her the calculator and show her how to divide with it and how to record the answer 15 ÷ 5 = 3. She will know this is correct. Then do a few more problems.

Remainders as Fractions: Worksheet 8 and 9 is an investigation of the pattern of remainders and the resulting fractions and decimals. Make several copies of these two worksheets to investigate different numbers. Starting with five works very well because the decimal answers will show a pattern and the pattern of the remainders shows up quickly.

If you are making groups of five, the worksheet would be filled out as follows.

# objects	# groups of 5	Leftovers	Fraction	Patterns	Decimal
1	0	1	1/5		0.2
2	0	2	2/5		0.4
3	0	3	3/5		0.6
4	0	4	4/5		0.8
5	1	0	5/5		1.0
6	1	1	1 and 1/5		1.2
7	1	2	1 and 2/5		1.4
8	1	3	1 and 3/5		1.6
9	1	4	1 and 4/5		1.8
10	2	0	2		2.0
11	2	1	2 and 1/5		2.2

The patterns visible are the constant denominator, the connection between the remainder and the numerator, the sequence of the remainders, and the sequence of the decimals.

Remainders as Decimals

Purpose The purpose of this lesson is to teach the student to express remainders as decimals by continuing to divide until the thousandths place. Money is used to help the student understand that the remainders are being regrouped into ever smaller units, thereby increasing their number to a value that is greater than that of the divisor. The student will work with money, then with a money division problem using both recording and the money to keep things straight and then finally with abstract division problems with abstract decimal remainders. Finally, the student will compare the fractional remainder to the decimal remainder and check the match by dividing the denominator into the numerator.

Prerequisites The student must be fluent in basic division and in decimal place value.

Materials Remainders as Decimals - Worksheets 1 and 2, pages 10 and 11, Remainders as Fractions - Worksheets 6 and 7, pages 6 and 7
Dollars, dimes, and pennies (real ones if possible)

Warm Up Divide sums of money between two, three and four people. Have one person count out $5.28 and place it on the table.

Lesson **"How can two people divide up $5.28?"** "I can give each person two dollars. Then I have to trade the last dollar for ten dimes. That will give me twelve dimes. I can give each person six dimes. Then I split up the eight pennies into four each. So each person gets two dollars, six dimes and four pennies, or $2.64."

Note If he does not know to trade the dollar for dimes, start tearing up the dollar bill, or pretend to. He will react strongly. You don't tear up dollar bills; you make change. He may want to make change with quarters. Tell him when working with decimals at this point, he is only allowed to make change with dimes and pennies. This usually gets him started in the right direction.

"What about three people splitting the same amount?" "I can give each person one dollar. Then I have to trade the other two dollars for twenty dimes. That will give me twenty-two dimes. I can give each person seven dimes. There will be one dime left over. I will trade them for ten pennies. That will give me eighteen pennies in all. Then I split up the eighteen pennies into six each. So each person gets one dollar, seven dimes and six pennies, or $1.76."

"After all those trades, do you still have $5.28?" "Yes, but it is now in three stacks of $1.76 each."

"Why did you have to trade the dime in for pennies?" "Because I had only one coin and I needed three to divide equally. So I traded in the dime for more pennies so I could keep dividing." *This is a key concept.*

"How do you know when you have to trade?" "If the number of coins you have is less than the number of people (the divisor) you are sharing with, then you have to trade to get more coins."

"What about four people splitting the same amount?" "I can give each person one dollar. Then I have to trade the other dollar for ten dimes. That will give me twelve dimes. I can give each person three dimes. There will be no dimes left over. I will then give two pennies to each person. So each person gets one dollar, three dimes, and two pennies, or $1.32."

Note

It is important to have him work the problems that appear on Worksheet 1 before he sees the page. He will understand the examples much better if he has already worked the problems.

Lesson Part 1

Now let's write these out as division problems. Have him drop the decimal point as he works down so he can see the place value and understand that he is trading when the number of coins is less than the divisor.

$2.64	$1.76	$1.32
2⟌5.28	3⟌5.28	4⟌5.28
- 4.00	- 3.00	- 3.00
1.28	2.28	1.28
- 1.20	- 2.10	- 1.20
.08	.18	.08

A possible conversation for recording the sharing of $5.28 between three people. (Redo the manipulations as you record.)

"What about three people splitting the same amount?" "I can give each person one dollar."

"Where do you record that one dollar per person?" "Above the five."

"How much money have you given out so far?" "Three dollars."

"Where do you record that?" "Under the 5.28."

"How do you calculate how much money you have left to divide up?" "I subtract the three dollars from the five dollars and twenty-eight cents. That gives me two dollars and twenty-eight cents left."

"Now what do you have to do?" "I have to trade the two dollars for twenty dimes."

"So, how many dimes do you have now?" "Twenty-two."

Note

It is important to have him underline the twenty-two. The number goes over the decimal point, so it must be pointed out to him that even though there is a two in the

one dollar place, he is now thinking of it as twenty-two dimes. This understanding is very important. The decimal point gets in the way of students seeing that as 22 instead of 2.2.

"What now?" "I can give each person seven dimes. There will be one dime left over."

"Where do you record the seven dimes given to each person?" "Above the 2."

"How do you know these are dimes?" "I know because the seven is in the tenths place, and dimes are one tenth of one dollar."

"So how do you calculate how much you have left to divide still?" "I have to figure out how much I used that would be twenty-one dimes and subtract that from two dollars and twenty-eight cents. That would leave me with eighteen cents."

"Now what?" "Then I split up the eighteen pennies into six each. So each person gets one dollar, seven dimes, and six pennies, or $1.76."

Now give $93 ÷ 5 people. This is a problem from Remainders as Decimals - Worksheet 1, page 10.
Then give this problem: $7.00 ÷ 3 = ___

In this problem, he will end up with one cent left over.
"How can you split up one cent between three people?" "You can't."

"Right, but in decimals you can keep going and keep dividing. When you are working with money, this last part does not matter, but if you are working with chemicals or measurements in science, that last part does matter."

Show him how to add another two zeros onto the end of the problem and divide again and again. He will see that the answer becomes a repeating decimal. Show him how we draw a line over the last number in the answer to indicate which number repeats.

Give the problem $106 ÷ 6. This is one from Remainders as Decimals - Worksheet 1, page 10.

Try eleven dollars shared by nine people. He will get a repeating two.
Finally, give one hundred six dollars shared by seven people, $106 ÷ 7. Tell him when working with decimal remainders a standard procedure is to have three numbers after the decimal point. Use zeros to fill in any missing numbers. So, $106 ÷ 7 will be written like this:

$$7\overline{)106.000}$$

Tell him to divide until he gets three digits after the decimal and then round his

answer to the nearest one hundredth.

This kind of problem comes up most often at the gas pump. The amount you buy is shown to the thousandth of a cent. The machine then rounds to the nearest cent. If the machine rounds down, the customer got a tiny bit of free gas. If the machine rounds up, the customer paid for a tiny bit of gas he did not get. It comes out about even over time. But imagine how much money would be lost by gas stations if you took all the gas sold over the whole country in a day and the pump did not measure to the nearest thousandth but only to the nearest cent. The gas stations would sell millions of gallons of gas for which they would not get paid.

Go over Remainders as Decimals - Worksheet 1, page 10, examples. Point out that the examples do not carry the decimal down as he did when he was working the problems with money. This is because it is easier and less confusing to not pay attention to the place value while working the problem.
Do several problems from Remainders as Decimals - Worksheet 2, page 10.

Practice Worksheets

Finish Remainders as Decimals - Worksheet 2. Check answers.

Redo all the problems Remainders as Fractions - Worksheets 6 and 7 and show all the remainders as decimals.

Lesson Part 3

Changing a fraction to a decimal
Go back to Remainders as Fractions - Worksheet 6, page 6. The fractional remainder is 2/5. The decimal remainder is 0.4. Remind him that 2/5 means two divided by five. Have him try $5\overline{)2}$ to remind him to put a decimal point and add three zeros then divide as normal. What pattern does he see? $5\overline{)2.000}$

Test for Understanding

1. You are dividing 27÷ 7 and get three with a remainder of six. List the steps you must now take to show that remainder as a decimal.

2. You have been doing a division problem and the quotient is now 23.45 and there is still a remainder. What should you do now?

3. You are dividing by four and you get a remainder of two. What decimal can you use to show that remainder? _____ How do you know?
Answers:
1. First, put a decimal point at the end of the dividend and add three zeros. Continue to divide until it comes out even, repeats or you divide to the thousandth place and round off to the nearest one hundredth.

2. Bring down another zero and divide one more time. Then round off to the nearest hundredth.

3. ".5" **"How do you know?"** "Because two is half of four, so the remainder can be expressed as $\frac{1}{2}$, which is .5 as a decimal."

Place Value Patterns and Missing Factors

Purpose

The purpose of this lesson is to remind the student of the concept of factoring to find the place value of an answer. In division, the missing number is the quotient. The quotient is the missing number in a multiplication problem 3 x ? = 12. To find this missing number, factor the twelve into three times four. The four is the missing number. In long division the problem can look like this: 300 x ? = 120,000. It is obvious that the quotient contains a four, but in what place value is that four? This lesson asks the student to prove how the solution is found. (Counting zeros is a trick. What is that trick based on?)

Prerequisites

Fluent in basic division, factoring, previous experience with factoring by tens, Multiplication - Booklet 3

Materials

Place Value Patterns, page 12, Place Value Patterns: Missing Numbers, page 13, Missing Factors Puzzles - Worksheets, pages 14 - 17

Warm Up

Review Factoring into Tens as it was taught in Multiplication - Booklet 3. Do some problems the long way in the fourth, the fifth, and the sixth columns. Then allow the student to use the patterns. The pattern is that you multiply the first digit in each number together and add the total number of zeros in the numbers above the line to the answer below the line. Simple. To prove that this pattern works, factoring by tens is required.

Practice Worksheets

The worksheet Factoring into Tens from Multiplication - Booklet 3 and a blank practice page to copy are on pages 21 and 22 in this booklet.

Place Value Patterns: Missing Numbers, page 13

Lesson

Teacher Options:
Option 1 - Use the zero pattern to complete the Missing Numbers pages.

Option 2 - Do the Missing Factors Puzzles to encourage the student to prove the results of the zero pattern and demonstrate where the place value of the answer comes from. The Missing Factors Puzzles are fairly difficult. You may decide the time to do them is not worth the additional understanding the student will get. Talented math students should definitely do these puzzles. What follows are the instructions for teaching the puzzles.

Do Missing Factors Puzzles, page 14, together. This page reviews the concept that the order of combination of addition and multiplication problems can be changed without changing the answer. This is a key concept for using factoring to solve problems.

"Which two numbers were combined to create the 10,000 on the right side of this equation 7 x 10 x 1,000 = ___ x 10,000?" "The 10 and the 1,000."

Check the answers with the Answer Key.

Note

The teacher must study the Missing Factors Puzzles Example and instructions on page 23, in this booklet before doing the lesson with a student. The goal is to use factoring to find the value of M. The symbol # indicates where the first digit of the number is written. PV means place value. The place value of the first digit is shown with a factor of ten. The numbers in the circles tell you in what order the numbers are recorded.

Missing Factors Puzzles, pages 15 - 17

Worksheets

Use the blank page, page 24, in this booklet, and copy the numbers from the example page working through the process. Then try 80 x M = 24,000.

Here is the solution:

A 80 x M = 24,000
B (8 x 10) x M = 24 x 1,000
C = (8 x 3) x (10 x 100)
D (8 x 10) x M = (8 x 10) x (3 x 100)
E 80 x M = 80 x 300
F 80 x 300 = 24,000

Practice this with her until she can do a problem easily on her own. What she sees written out is the thought process behind a missing factor problem and the proof for where the zeros come from.

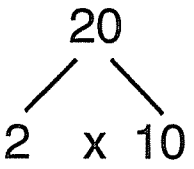
Test for Understanding

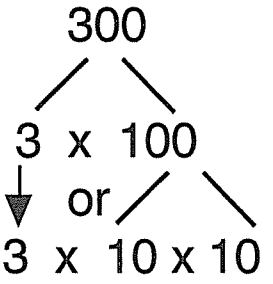
Have her write out the process for solving M x 60 = 180,000.

Factoring into Tens - Worksheet 1 Date_____

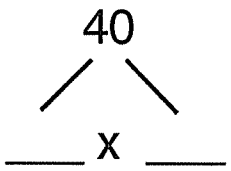
Sometimes factoring a number into multiples of ten is more useful than factoring it with prime numbers.

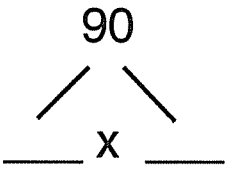
Two examples:

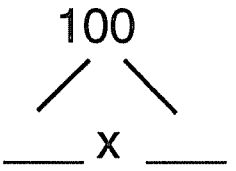


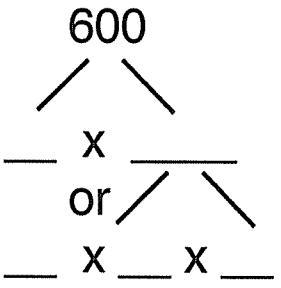


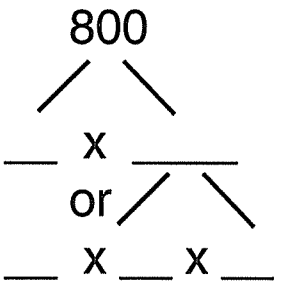
Factor these numbers by tens.

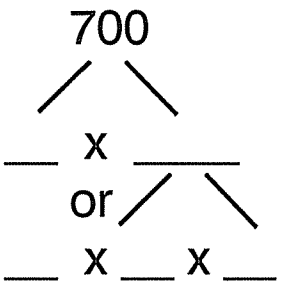




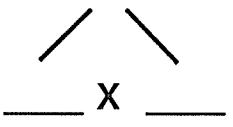


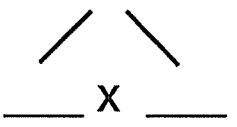






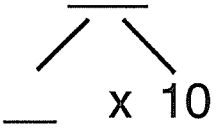
Make your own.

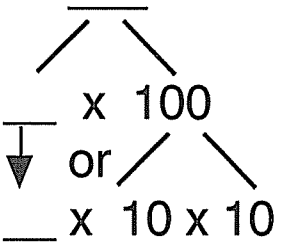




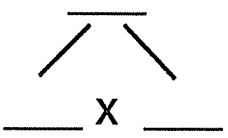
Factoring into Tens - Worksheet 2 Date_____

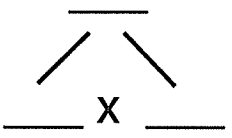
Write in your own numbers.

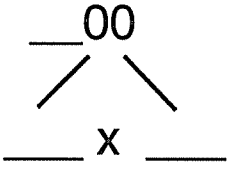


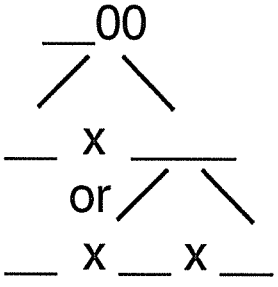


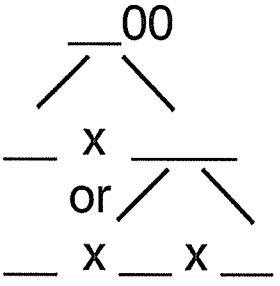
Factor these numbers by tens.

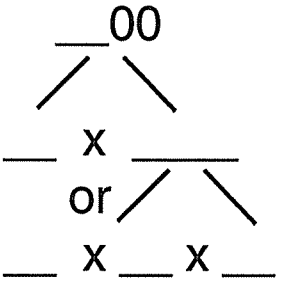


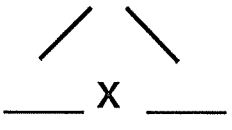


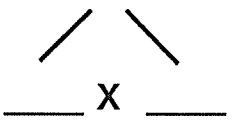












Missing Factors Puzzles Example

A 40 x M = 1,200 .

B ①(4 x 10) x M = (12 x 100 ②)
(4: #, 10: PV; 12: ③ / #, 100: PV)

C = (4 x 3)(10 ④ x 10 ⑥)
(4: ⑦ #, 3: #, 10: PV, 10: PV)

D (4 x 10) x M = (4 x 10)(3 x 10)
(4: #, 10: PV, 3: #, 10: PV)

E 40 X ⑧ M = 40 X 30

F 40 X 30 = 1,200

Here is how you do this kind of problem.

On **Line A** you are given the problem 40 x M = 1,200

On **Line B** find the ① next to the first set of ( ), write the factors of the number above (in this case the 40) on the lines, 4 x 10 in that order.

Also on Line B, you will find the ②. Factor the 1,200 into 12 x 100 and write 12 x 100 on the lines in the ( ).

Now find **Line C**. On this line you will factor the 12 and the 100.

There are two pairs of factors for 12. 2 and 6, 3 and 4. You will choose the pair that has a number that matches the first number factor at ①. In this case, you have 4 x 10 at ①. This means you should factor the 12 into 4 x ③.

Write the 4 at ③ and the 3 at ⑤. Now factor the 100. This one is easy because 100 factors into 10 x 10. Write those numbers at 4 and ⑥.

On **Line D**, in the first ( ), on the left, simply drop the numbers from Line B, following the arrows, into the left side of Line D. Now find the ⑦. We need to construct the 40 that appears on the left side of the equation. So take the four from the ③ and the ten from the 4 and write it at ⑦. Take the three from the ⑤ and the ten from the ⑥ and place them in the second set of ( ). Use the blank page, page 17, in the student workbook and copy the numbers from the example page, working through the process. Then try 880 x M = 24,000.

On **Line E**, multiply all the pairs in the ( ). ⑧ This leaves you with 40 x (missing number) = 40 x 30. Obviously the missing number is the 30.

Line F gets you back where you started from with your missing factor identified.

Missing Factors Puzzles

Practice - make multiple copies.

#1

A. _____ x M = _____

B. (___ x ___) x M = (_____ x _____)
   # PV # PV

C. = (_____ x ___)(_____ x ___)
   # # PV PV

D. (___ x ___) x M = (_____ x ___)(_____ x ___)
   # PV # PV

E. _____ X M = _____ X _____

F. _____ X _____ = _____

#2

A. _____ x M = _____

B. (___ x ___) x M = (_____ x _____)
   # PV # PV

C. = (_____ x ___)(_____ x ___)
   # # PV PV

D. (___ x ___) x M = (_____ x ___)(_____ x ___)
   # PV # PV

E. _____ X M = _____ X _____

F. _____ X _____ = _____

Double Digit: Beginning

Purpose

The purpose of this lesson is to make the student aware of the place value patterns that affect the quotients when switching from a single digit divisor to a double digit divisor. The pattern language you want to develop is 'increasing the divisor by a factor of ten decreases the quotient by a factor of ten.' We are working directly with the concept of inverse relationships so critical to understanding division and, hence, fractions. If the divisor goes up, the quotient goes down by the same multiple. Students will see this as 'the number stays the same, but the number of zeros changes.'

Prerequisites

Completion of Division: Booklet 2, Algebraic Division, or Base Ten Division

Materials

Double Digit: Beginning, Worksheets 1 - 7, pages 18 - 24

Warm Up

Review Missing Factors by giving the problems that show up on Worksheet 1 as missing factors problems. Example: 30 x ___ = 180. Plug in the factoring format used in the last lesson. Example: 30 x M = 180

$$3 \times 10 \times M = 18 \times 10$$
$$(3 \times 10) \times M = (3 \times 6) \times 10$$
$$= (3 \times 10) \times 6$$

Therefore M = 6

This thought process will be rapid at this point for most students. Most will be able to do this mentally. But be sure the student understands.

Lesson Part 1

Double Digit: Beginning - Worksheet 1, page 18. Have the student work the problems moving left to right. You will see that the problem on the left hand column matches the problem on the right except for a place value change.

The students does the problem 180 ÷ 3. **"The answer is sixty. When you divide by thirty instead of three, how will the quotient change?"** Many students will say they are not sure. Make it more concrete for them.

"If you have 180 kids and you ask them to get in groups of three, how many groups will there be?" "Sixty."

"Well, if you then tell them to get into groups of thirty, will the number of groups go up or down?" "Down."

"By how much? Try it and see." "180 ÷ 3 = 60 and 180 ÷ 30 = 6. The answer went down."

"By what factor? How many times less is the second answer than the first?" "Six is ten times less than sixty."

Have him write out the missing factor strings for both problems

$3 \times 60 = 180$ $30 \times 6 = 180$

$3 \times 6 \times 10$ is the same as $3 \times 10 \times 6$

"What property of multiplication are you using here?" "The associative because all I am doing is changing the order I put the numbers in."
Repeat this process as you move together down the page. Be sure to have him write in his own words at the bottom of the page what patterns he sees.
End the session and continue the next day.

Worksheets

Give Double Digit: Beginning, Worksheet 2, page 19.
Begin with the problem $1{,}500 \div 5$ and $1{,}500 \div 50$. Have him do the problems using the missing factor format. He can see that the tens just reorganize.

$5 \times M = 1{,}500$
$5 \times M = 15 \times 100$
$= 5 \times 3 \times 10 \times 10$
$= 5 \times (3 \times 10 \times 10)$
$M = 300$

$50 \times M = 1{,}500$
$5 \times 10 \times M = 15 \times 100$
$= 5 \times 3 \times 10 \times 10$
$= (5 \times 10) \times (3 \times 10)$
$M = 30$

Note

This may seem like the hard way to do all this. I agree, it is easier to just count the zeros, but you are building mathematical fluency along with the ability to understand how dividing with a double digit divisor works.

Have him finish the page alone. Check the work as soon as he completes it. Be sure he does the missing factors problems in the far right hand column.

Lesson Part 2

Double Digit: Beginning - Worksheet 3, page 20
Do the left hand column of the worksheet together. The first two problems are a repeat of what has been done before. Add in this time the standard division notation on the second problem.

$$\begin{array}{r} 50 \\ 30\overline{)1{,}500} \\ \underline{-1{,}500} \\ 0 \end{array} \qquad \begin{array}{r} 51 \\ 30\overline{)1{,}530} \\ \underline{-1{,}500} \\ 30 \end{array}$$

When he gets to $1{,}620 \div 30$, he will need to begin using the old way of thinking about division when he was doing single digit division and combine that with missing factor thinking to deal with the place value changes. If he gets stuck, have him do $1{,}620 \div 3$ first and then do $1{,}620 \div 30$. Most students do not have difficulty with this transition. Observe closely and help when needed. Have him do the second column for independent practice.

Worksheets

Check the work as soon as he finishes.

Test for Understanding

Double Digit: Beginning, Worksheets 4 - 7, pages 21 - 24.

"If the answer to $1{,}530 \div 5$ is 306, what do you think the answer to $1{,}530 \div 50$ will be?" This question tests his understanding of the place value issue. This problem is made difficult by the fact that the answer to $1{,}530 \div 5$ has a zero in the quotient. The quotient is 306. If he writes that the answer will be 30.6, he has nailed it. He may write 30 with a remainder of 30. This is a less strong answer. Ask him to put in a decimal point and show the remainder as a decimal. If he writes 36, it shows that his grasp of the place value changes are not solid yet. Give more problems of this type, having him show the remainders as decimals.

Sequences

Purpose The purpose of this lesson is to connect the place value patterns developed in the previous lesson with the procedure already learned with single digit divisors to that of double digit divisors. This lesson uses a sequence in which the answer is predictable. The dividend is held steady and the divisor is increased by one in each problem. The student will be reminded that when the divisor goes up a little bit, the quotient will go down a little bit. We are working directly with the concept of inverse relationships so critical to understanding division and, therefore, fractions.

Prerequisites Previous lessons in this booklet

Materials Sequences - Worksheets 1 - 3, pages 25 - 27

Warm Up Give the problems: $1{,}530 \div 5$ $1{,}530 \div 50$ showing remainders as decimals.
$1{,}827 \div 3$ $1{,}827 \div 30$
For a challenge, try $1{,}800 \div 300$ and then $1{,}827 \div 300$
Investigate this sequence before beginning the worksheet.

	$1{,}536 \div 2$	$1{,}536 \div 3$	$1{,}536 \div 4$	$1{,}536 \div 5$
Answers	768	512	384	307.2

"What happens to the quotient as you inch up the size of the divisor?" "The quotient goes down."

Expansion For an advanced student: Continue the sequence up to $1{,}536 \div 9$. Then take the difference between each quotient. $768 - 512 = 256$, $512 - 384 = 128$ $384 - 307.2 = 76.8$

Test for Understanding **"Why are the differences between the quotients going down as the divisor goes up?"** This is a very difficult concept for a student at this age. The differences are going down because when you divide by two, you are cutting something in half, by three in thirds, by four in fourths, as you cut the dividend into smaller and smaller pieces, the differences between the sizes of those pieces goes down.

Lesson Sequences - Worksheet 1, page 25. In the margin of the page,have the student solve the top problem: $2{,}950 \div 50$.

"Based on what you saw in the sequence you did in the Warm Up, what do you expect to happen to the quotient to this problem if we raise the divisor from fifty to fifty-one?" "I would expect the quotient to go down a little."

"What do you estimate the quotient might be?" Answers will vary.

"Do you think the quotient will still be in the fifties?" Most will be a bit unsure. Walk her through the use of the divisor, which now has a digit in the ones place. Have her assume the first number in the quotient is five in the tens place. But when

she multiplies to find out how much she used, she must multiply by fifty-one not fifty. Have her do this multiplication in the margin.

```
      58 r 42
   ____
51 | 2950
   -2550
    450
    -408
      42
```

51 x 50 = 2550

Now she will have to divide fifty-one into four hundred fifty. She already knows that fifty times nine is four fifty.
"What would fifty-one times eight be? Try it." "It is four hundred eight."

"Does that work?" "Yes."

"So fifty-one goes into four hundred fifty, eight times but not nine times." "Right."

"What is the answer?" "Fifty-eight with a remainder of forty-two."

Note

I usually do not ask students at this point to show the remainders as decimals. It tends to bog down the process of getting used to that second digit. It is an option, though. You can also use a calculator to put that decimal in. Have her round off to the nearest tenth so she can see how fast the numbers are dropping.

"So the quotient to the first problem was fifty-nine and the quotient to the second problem was fifty-eight with a big remainder of forty-two, almost another group. What do you think the quotient of the next problem will be?" " Fifty-seven or fifty-six."

Continue on down the page in this same manner. Use the answer key to check your work. Keep reminding her to multiply by the actual divisor, not fifty. This will be a common mistake at this point.

Optional

You can continue the work on another piece of paper going all the way to sixty as a divisor.

Practice Worksheets

Sequences - Worksheet 2, page 26

Test for Understanding

Sequences - Worksheet 3, page 27 Watch to see how she answers the questions at the top of the page. Does she realize that if we make the divisors go down, the quotients will go up?

Double Digit: Thinking Roundly and Transitions

Purpose The purpose of this lesson is to develop the estimate and test strategy to find the quotient in a double digit division problem. The missing factor strategy will again be employed to get close to a likely quotient.

Prerequisites Previous lessons in this booklet.

Materials Double Digit: Thinking Roundly - Worksheets 1 - 5, pages 30 - 34
Double Digit: Transitions - Worksheets 1 - 3, pages 35 - 37
Double Digit: Practice - Worksheets 1 - 4, pages 38 - 41

Warm Up Using the 32 Times Tables chart on page 29, do two problems for a little challenge: 352 ÷ 32 (answer is 11) and 2,720 ÷ 32 (answer is 85).

Lesson Part 1 Clearly, we do not want to have to write out a complete times table for every double digit number we may want to use as a divisor. So a simpler process needs to be used. The technique used by most people is to use missing factor thinking and the knowledge of the multiplication table to estimate the likely quotient and then test it out with multiplication. It involves trials and errors to get the correct answer.

Please note that the dialogues in most lessons are idealized with a student giving all the correct answers. The dialogues you have with your student will be unique. What's most important is to listen to the student and figure out the model of the world she is presenting. From your understanding of what she says, continue to ask probing questions or statements, such as, **"How did you get that?" "Show me what you mean." "Build a model of that." "Tell me more so I can understand what you are saying."**

Double Digit: Thinking Roundly - Worksheets 1 and 2 teach this strategy. In these first pages, all the quotients are single digit numbers with no remainders. They are designed for student success.

Depending upon the student you may want to step back and see how she solves the problem without imposing this strategy on her. See if her way works for many problems. Her way of solving double digit problems may prove effective. Affirm that. Now the constraint will be on speed. The estimate and test strategy will allow her to become quicker at solving problems. With a different student you may decide to move to using a calculator.

Do this page together. The steps are 1. Round the divisor to the nearest ten. 2. Use your missing factor idea to find a product of the rounded divisor that is as close as possible to the dividend (without going over). 3. Multiply the estimated missing factor by the actual divisor, not the rounded one, to see if it produces a number close to, but not larger, than the dividend.

The example at the top of the worksheet shows this thought process.
'Thinking roundly,' means to round the fifty-two to the nearest ten and use that number in the missing factor number sentence.

"What is fifty-two rounded to the nearest ten?" "Fifty."

"So fifty times what number is going to get us close to or about one hundred fifty-six?" "Three."

"How do you know?" "Because fifty times three is one hundred fifty, and fifty times four would be two hundred, which is too large."

"Now try out three as your quotient by multiplying the real divisor, fifty-two, by the three. What do you get?" "One hundred fifty-six."

The second problem leads the student though the thought process again. The third problem lets her figure out the likely missing number. The correct number is six, because fifty times six is three hundred.

The fourth problem lets her do the rounding and locate the likely missing factor. The rounded divisor is again fifty and the likely missing factor is nine because fifty times nine is four hundred fifty. When she multiplies fifty-two times nine she will get four hundred sixty-eight.

Note

As the actual divisor gets further from the rounded divisor, the less likely she is to get the correct missing factor on the first try. She may have to adjust up or down. On the second problem on Double Digit: Thinking Roundly - Worksheet 2, she could use fifty times five and then multiply fifty-seven times five to get the correct answer. She may correctly round the fifty-seven to sixty. The conversation will go something like this:
"Sixty times what number will get you close to two hundred eighty-five?"
"Four, because sixty times forty is two hundred forty, but sixty times five is three hundred which is too high."

"Try both the four and the five and see which one works."

"Why did the five work when your estimate said that five would be too high?"
"Because I was using sixty, and the real number is only fifty-seven, so the estimate was a bit low."

The third problem on Double Digit: Thinking Roundly - Worksheet 2, $464 \div 58$, deals with this problem of closing in on the best estimate. Let her know that in double digit division, you may have to make several trials to get the correct quotient.

The tricky part of double digit division shows up here. In the third problem, fifty-eight actually rounds off to sixty. If the six times table is used, the estimates are going to be a bit low. If the five times table is used, estimates will tend to be high.

She needs to learn to make trials to find the correct number.

At the bottom of Double Digit: Thinking Roundly - Worksheet 2, the conversation may sound like this:
"Sixty times what is about four hundred sixty-four?" "Sixty times seven is four hundred twenty."

"Write that down in the space and then try seven times fifty-eight. What happens?" "I get four hundred six."

"What would the remainder be?" "Fifty-eight."

"What does that tell you if the remainder is equal to or greater than the divisor?" "That my quotient is too small."

"What number would be good to try next?" "Eight, and that works, so the answer is eight."

Practice Worksheets

Have her complete Double Digit: Thinking Roundly - Worksheets 3 and 4 as independently as she is able. She may need help, especially if arithmetic errors derail her estimates. Encourage her to use the answers to previous problems to solve new ones. Consider allowing her to use a calculator to check her arithmetic. We are working on the process right now, not the arithmetic. One tiny error will cause the final answer to be incorrect. This is frustrating for students.

Require a well-written answer to the question on Double Digit: Thinking Roundly - Worksheet 4, page 33. Have her clearly explain the process.

Lesson Part 2 Warm Up

Have the student read her answer to the essay question on Thinking Roundly - Worksheet 4. Have her follow her written process to do the following problems. 306 ÷ 51 (easy), 228 ÷ 38 (medium), 532 ÷ 76 (challenging). She may need to improve her essay to state each step clearly.

Lesson

All the quotients on Double Digit: Thinking Roundly - Worksheet 5 are two digit quotients. The new part to be added is the estimation of the quotient.

Note

Double Digit: Thinking Roundly - Worksheet 5 is intended to be done by the student and the teacher working together.

"What is the first step?" "Round off the twenty-seven to thirty."

"So thirty times what will get you close to 1,701?" "Thirty times fifty, or maybe sixty."

"What do you write in the estimation circles and boxes?" "30 x 50 = 1,500 and 30 x 60 = 1,800."

"Now what?" "I test 27 x 50 and 27 x 60." "27 x 50 = 1,350 and 27 x 60 is 1,620."

"And now?" "I record a six in the quotient in the tens place and subtract 1,701 - 1,620 and get 81."

"What now?" "I estimate again. 30 x 2 is sixty and 30 x 3 is 90. Then I test those. It is three because 27 x 3 is 81. So the answer is 63."
Repeat with the last three problems.

Lesson Part 3

Double Digit: Transitions - Worksheets 1 - 3, pages 35 - 37 These worksheets use the same divisor on several problems, allowing her to reuse multiplications she has already done.

Note

Do not have her turn the page back and use the times table chart. Tell her it takes too long to write one of those out every time you need to divide a number.

"What round number is nearest to this divisor of thirty-two?" "Thirty."

"Remember when you are using times tables for missing factors, do not forget to use expanded tables. You can use numbers like twenty, thirty, or forty. So thirty times what is going to get you close to four hundred sixteen?" "Thirty times ten will get me to three hundred, thirty times twenty to six hundred, which is too much, so thirty times ten is the best choice."

"Write 30 x 10 = 300 in the second row of your thinking roundly boxes. So now you know that the first number in the quotient is a ten. Over in the check area, multiply 32 x 10. What do you get?" "Three hundred twenty."

"Where do you record that you made ten groups of thirty-two?" "In the quotient space above the line. I write a 1 in the tens place just above the four hundred sixteen."

"What do you do now?" "Subtract the three hundred twenty from the four hundred sixteen to get the remainder of ninety-six and record that under the four hundred sixteen."

```
   1
32|416
 - 320
   96
```

"Now estimate again. Thirty times what is going to get you close to ninety-six?" "Three."

"What do you write in the thinking roundly bubbles." "30 x 3 = 90."

"What do you do now?" "Multiply three times thirty-two over in the check box. This gives me ninety-six, so I know the three works."

"So what is the answer to 416 ÷ 32?" "Thirteen."

Move further down the page and do 672 ÷ 32 together. Then have her complete the pages as independently as she can. Check the work immediately to make sure she is practicing correctly.

It is not uncommon for a student to make many arithmetic errors in this long tedious process. Make sure she is multiplying by the actual divisor and not the estimating divisor.

Note

Some students will estimate the answers and then instead of doing the subtraction and re–division, they will make multiple guesses for the quotient doing sequences that look like this: For this problem, 1,701 ÷ 27, you will see a series of two digit multiplication problems such as 27 x 51, 27 x 53, 27 x 58 and so on–a twenty-seven times table essentially. This is an inefficient strategy. Do not allow her to do this. Students not increase in speed if they use this strategy.

Reteach

Require the student to estimate only the number in the tens place of the quotient; then multiply it by the actual divisor to check that it is not too large. She needs to find the largest possible multiple that is not greater than the dividend. This does take some trial and error. When she finds that number, she should subtract it from the dividend, find the remainder, and repeat the process to find the number in the ones place.

Practice Worksheets

Double Digit: Practice - Worksheets 1 - 4, pages 38 - 41

Test for Understanding

Use Double Digit: Practice - Worksheet 4, page 41 as a Test for Understanding. the divisors are mixed. If she can get 50% correct on her own with no help from you, she understands the process. She will need to continue to practice one or two problems per day to master it.

Go over the work carefully and locate the places where she made errors and have her redo the problems.

There are several sites on the Internet that allow interactive practice where she gets correction immediately.

Gus the Bus Driver: Word Problems

Purpose

The purpose of this lesson is to press the student to use arithmetic to solve a complex set of word problems and to use the answers to those arithmetic problems to determine what combination of buses would be the least expensive for transporting a given number of students. One set of problems also creates the situation in which a remainder can not be used, thus requiring the answer to be rounded up to the next whole number. These problems will let you know if the student understands a real world application of a remainder.

Prerequisites

Previous lessons

Materials

Gus the Bus Driver: Worksheets 1 and 2, pages 42 and 43
A red and a blue crayon or colored pencil
Calculator - for problem 3
Lined notebook paper

Warm Up

This is the Jumbo bus part of the problem presented alone to help the student understand the problem without all the distractions of the different vehicles.

Write this problem on a piece of paper or on the board.
Gus has a Jumbo bus that can carry 120 students. How many Jumbo buses will he need to carry 240 students? 360 students? 480 students? And 600 students?

Lesson

Have him read the problem on Gus the Bus Driver: Worksheet, page 42, at least twice. As he reads the problem a second time, have him use the red crayon to underline the number of students each vehicle carries. Then have him write the number of students each vehicle will carry above the picture of that vehicle.

Have him use the blue crayon to circle each number of student groups shown down the left hand side of the page under Problem 1. There are five student groups, a group of 120 students, 240, 360, 480, and 600 respectively. Color coding will help him understand the parts of the problem.

He has already done the work for the Jumbo bus. Have him enter the number of Jumbo buses it would take to move each student group. Check the answer key to see the numbers in the column under the Jumbo bus.

"If it takes one Jumbo bus to carry one hundred twenty students, how many Large buses will it take to carry that same number of students?" "Two, because sixty is half of one hundred twenty." Or "Two because sixty plus sixty is one hundred twenty." Or "Two because one hundred twenty divided by sixty is two."

"What about two hundred forty students?" "Four."

"What pattern in your numbers can help you finish this column?" "The num-

ber of Large buses is two times the number of Jumbo buses because the large bus is only half as large, so it takes more of them."

"The Jumbo bus and the Large bus were easy and you could use addition to solve them. To solve the problem of the Van, which holds twenty-four students, what will you need to do?" "Divide each grouping number by twenty-four."

Note

Have him complete the column independently later. Now move to the mini-van problem. The problem of the Mini-van is an embedded assessment on understanding remainders.

Test for Understanding Part 1

"How many Mini-vans do you need to move one hundred twenty students?" *Wait.*
If he says thirteen with a remainder of three, you know he is not thinking about what the answer means.

"What is the remainder of three? Is it three Mini-vans or three students?" *Wait.*
The answer 13r3 means that it will take thirteen Mini-vans and three students will be left over.

"Does that mean those students do not get to go on the field trip?" "No, you need to bring another Mini-van. So Gus will need fourteen Mini-vans to move one hundred twenty students."

If he says, "Fourteen Mini-van will be needed," he is demonstrating that he is paying attention to what the question and the answer mean and that he understands that a remainder in this situation is inappropriate. You can't have a fraction of a mini-van, nor can you leave the three students behind.

Part 2

Finish Problem 1 and stop. Continue the next day.

Problem 2
Do the cost of Mini-vans first.
"How much does a Mini-van cost?" "$75."

"What did you forget?" "Hmmm, oh, the driver. So I have to add on a hundred dollars for her, so it would be $175 per Mini-van."

"So, how much will it cost to move one hundred twenty students with Mini-vans?" "$175 x 14 = $2,450."
Have him finish the chart. Stop for the day.

Part 3

Problem 3
Sit with him while he works this problem. Help him organize the work using the lined paper. Label each calculation so he can use it to solve the problem. Give him a calculator to do this problem. Observe how he thinks about the problem. Watch

to see if he realizes that he should work with the largest possible vehicle and work backwards. It is the thinking needed here, not the arithmetic. Have him record on another piece of lined paper to help him keep his work organized.

Test for Understanding Part 4

Problem 4 *No* calculator for this one. Do have him record on lined paper but do not help him with this problem. This is an assessment for you to see how well he can handle multiple calculations in a complex problem. Tell him to do his best.

The Make Your Own word problem can also be a Test for Understanding. Exchange these problems and have other students solve them to see how well the problem is understood and solved by someone else.

Patterns in Arithmetic

Division: Booklet 3

Working with Double Digit Division

Answer Key for the Student Workbook

By Suki Glenn and Susan Carpenter

Answer Key Legend

AWV = answer(s) will vary
BUWV = break up will vary
OWV = order will vary

Pattern Blocks
r = red trapezoid
g = green triangle
y = yellow hexagon
o = orange square
b = blue parallelogram
t = tan rhombus

Cuisenaire Rods
1 w = white
2 r = red
3 lg = light green
4 p = purple
5 y = yellow
6 dg = dark green
7 bk = black
8 bn = brown
9 bl = blue
10 o = orange

Note: Some items and pages are left out of the answer key.
1) Some pages in which the answers are open-ended or will vary.
2) Make your own problems. Since students create their own problems and solutions, these sections give valuable information about the level of confidence and competence. It can be a useful source of curriculum for other students.
3) Practice pages.
4) Workboards.
5) The answers are in the Parent/Teacher Guide.

Patterns in Arithmetic: Division - Booklet 3
Parent/Teacher Guide
Answer Key for the Student Workbook

Published by Pattern Press
P.O. Box 2737
Fallbrook, CA 92088

Printed on recycled paper.

ISBN 978-1-935559-23-8

Division - Booklet 3

Remainders as Fractions - Worksheet 1
Share Those Cookies

Let's say there are 5 cookies on one plate.

You and a friend are sharing them.

The division number sentence for this is:

5 cookies ÷ 2 friends = 2 cookies each with a remainder of 1

$$2\overline{)5} \quad 2 \text{ r } 1$$

How many of you would leave that remaining cookie on the plate? Really! What would you do? Cut it in half.

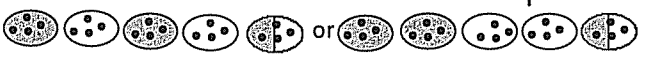
I would split the last cookie and each person would have $2\frac{1}{2}$ cookies.

Remainders are often divided up between the folks sharing it.

Here is a set of cookie problems.

Color the cookies to show how they are shared, then write the number sentence.

Example: 5 cookies and 2 people $5 \div 2 = 2\frac{1}{2}$ cookies each $2\overline{)5}$ $2\frac{1}{2}$



Now, you try one:

7 cookies and 2 people $7 \div 2 = 3\frac{1}{2}$ $2\overline{)7}$ $3\frac{1}{2}$

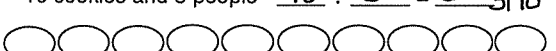
Remainders as Fractions - Worksheet 2
Share Those Cookies

7 cookies and 3 people $7 \div 3 = 2\frac{1}{3}$ $3\overline{)7}$ $2\frac{1}{3}$

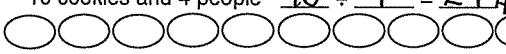
9 cookies and 4 people $9 \div 4 = 2\frac{1}{4}$ $4\overline{)9}$ $2\frac{1}{4}$

11 cookies and 4 people $11 \div 4 = 2\frac{3}{4}$ $4\overline{)11}$ $2\frac{3}{4}$

10 cookies and 3 people $10 \div 3 = 3\frac{1}{3}$ $3\overline{)10}$ $3\frac{1}{3}$

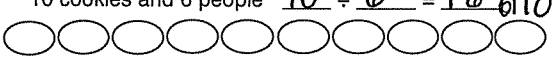


10 cookies and 4 people $10 \div 4 = 2\frac{2}{4}$ $4\overline{)10}$ $2\frac{2}{4}$



Challenge!

10 cookies and 6 people $10 \div 6 = 1\frac{4}{6}$ $6\overline{)10}$ $1\frac{4}{6}$

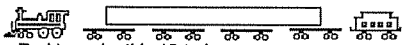


12 cookies and 5 people $12 \div 5 = 2\frac{2}{5}$ $5\overline{)12}$ $2\frac{2}{5}$

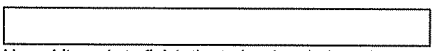
Your turn: AWV

___ cookies and ___ people ___ ÷ ___ = ___

Remainders as Fractions - Worksheet 3
Cuisenaire Rods

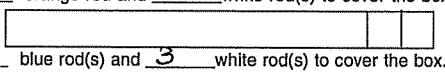


Use each color of Cuisenaire Rod to make this 12 train.

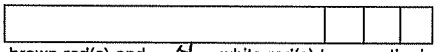


Use white rods to finish the train when it doesn't come out even.
Record your results in the sentence above the picture of the Cuisenaire Rod.
My results:

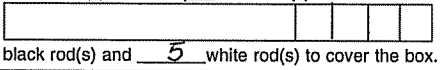
It takes 1 orange rod and 2 white rod(s) to cover the box.



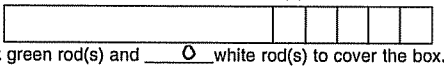
It takes 1 blue rod(s) and 3 white rod(s) to cover the box.



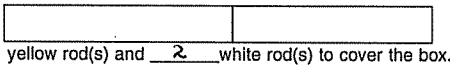
It takes 1 brown rod(s) and 4 white rod(s) to cover the box.



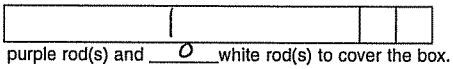
It takes 1 black rod(s) and 5 white rod(s) to cover the box.



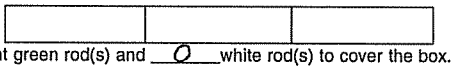
It takes 2 dark green rod(s) and 0 white rod(s) to cover the box..



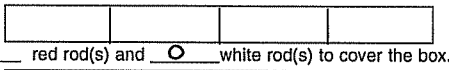
It takes 2 yellow rod(s) and 2 white rod(s) to cover the box.



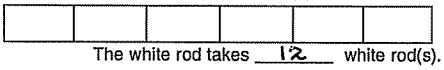
It takes 3 purple rod(s) and 0 white rod(s) to cover the box.



It takes 4 light green rod(s) and 0 white rod(s) to cover the box.

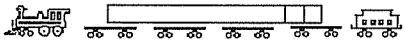


It takes 6 red rod(s) and 0 white rod(s) to cover the box.

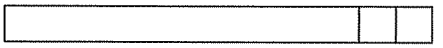


The white rod takes 12 white rod(s).

Remainders as Fractions - Worksheet 4
Cuisenaire Rods

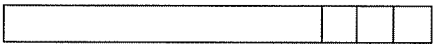


One orange rod two white rods.



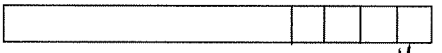
The 2 white rods are what fraction of the orange rod? 2 out of 10
Think before you look.

So, the orange rod (10) goes into 12, 1 and $\frac{2}{10}$ time. $12 \div 10 = 1\frac{2}{10}$



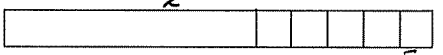
The white rods are what fraction of the blue rod?

The blue rod goes into 12, 1 and $\frac{3}{9}$ or $\frac{1}{3}$ time. $12 \div 9 = 1\frac{1}{3}$



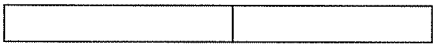
The 4 white rods are what fraction of the brown rod? $\frac{4}{8}$ or $\frac{1}{2}$

The brown rod goes into 12, 1 and $\frac{4}{8}$ or $\frac{1}{2}$ time. $12 \div 8 = 1\frac{1}{2}$



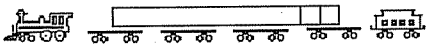
The 5 white rods are what fraction of the black rod? $\frac{5}{7}$

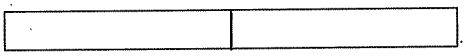
The black rod goes into 12, 1 and $\frac{5}{7}$ time. $12 \div 7 = 1\frac{5}{7}$



The dark green rod goes into 12, 2 times. $12 \div 6 = 2$

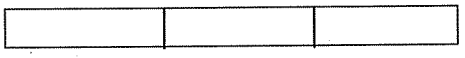
Remainders as Fractions - Worksheet 5
Cuisenaire Rods



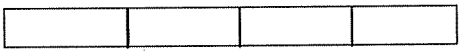


The 2 white rods are what fraction of the yellow rod? 2/5

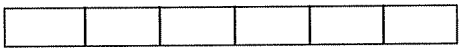
The yellow rod goes into 12, 2 and 2/5 time. 12 ÷ 5 = 2 2/5



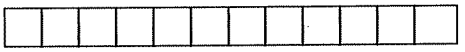
The purple rod goes into 12, 3 time(s). 12 ÷ 4 = 3



The light green rod goes into 12, 4 time(s). 12 ÷ 3 = 4



The red rod goes into 12, 6 time(s). 12 ÷ 2 = 6

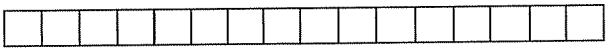


The white rod goes into 12, 12 time(s). 12 ÷ 1 = 12

5

Remainders as Fractions - Worksheet 6
Cuisenaire Rods

Show each remainder as a fraction. Hint: To answer these problems ask yourself, "The remainder is what fraction of the groups I am making?" Use Cuisenaire Rods if you want to make a model.



4) 12 → 3	5) 12 → 2 2/5	6) 12 → 2
4) 13 → 3 1/4	5) 13 → 2 3/5	6) 13 → 2 1/6
4) 14 → 3 2/4 = 1/2	5) 14 → 2 4/5	6) 14 → 2 2/6 = 1/3
4) 15 → 3 3/4	5) 15 → 3	6) 15 → 2 3/6 = 1/2
4) 16 → 4	5) 16 → 3 1/5	6) 16 → 2 4/6 = 2/3

6

Remainders as Fractions - Worksheet 7
Cuisenaire Rods

Show the remainders as fractions.

5) 90 → 18	6) 170 → 28 2/6 = 1/3	9) 137 → 15 2/5
4) 68 → 17	5) 123 → 24 3/5	4) 76 → 19
6) 213 → 35 3/6 = 1/2	3) 136 → 45 1/3	6) 226 → 37 4/6 = 2/3
7) 109 → 15 4/7	9) 165 → 18 3/9 = 1/3	8) 100 → 12 4/8 = 1/2
4) 147 → 36 3/4	7) 112 → 16	6) 166 → 27 4/6 = 2/3

7

Remainders as Decimals - Worksheet 2

Show the remainders as decimals.

5) 90 → 18	6) 170 → 28.3	9) 137 → 15.2
4) 68 → 17	5) 123 → 24.6	4) 76 → 19
6) 213 → 35.5	3) 136 → 45.3	6) 226 → 37.6
7) 109 → 15.57	9) 165 → 18.3	8) 100 → 12.5
4) 147 → 36.75	7) 112 → 16	6) 166 → 27.6

11

Place Value Patterns

5	50	500	50	500	5,000
x 3	x 3	x 3	x 30	x 30	x 30
15	150	1,500	1,500	15,000	150,000
3	30	300	30	300	3,000
x 5	x 5	x 5	x 50	x 50	x 50
15	150	1,500	1,500	15,000	150,000
4	40	400	40	400	4,000
x 3	x 3	x 3	x 30	x 30	x 30
12	120	1,200	1,200	12,000	120,000
3	30	300	30	300	3,000
x 4	x 4	x 4	x 40	x 40	x 40
12	120	1,200	1,200	12,000	120,000
9	90	900	90	900	9,000
x 7	x 7	x 7	x 70	x 70	x 70
63	630	6,300	6,300	63,000	630,000

What patterns did you _____

Find an easy way to do these problems.

Place Value Patterns: Missing Numbers

Multiply. Use the pattern you figured out on the previous page.

(4)	(40)	(400)	(40)	(400)	(4,000)
x 4	x 4	x 4	x 40	x 40	x 40
16	160	1,600	1,600	16,000	160,000
4	40	400	40	400	4,000
x 5	x 5	x 5	x 50	x 50	x 50
20	200	2,000	2,000	20,000	20,000
5	50	(500)	50	(500)	5,000
x 3	x (3)	x 3	x (30)	x 30	x 30
15	150	1,500	1,500	15,000	150,000
3	(30)	300	30	300	(3,000)
x (6)	x 6	x (6)	x 60	x (60)	x 60
18	180	1,800	1,800	18,000	180,000
8	80	(800)	80	(800)	8,000
x 7	x (7)	x 7	x (70)	x 70	x 70
56	5,600	5,600	560	56,000	560,000

Missing Factors Puzzles

These puzzles are a tool you can use to do large division problems.
Learning to solve them can be difficult, but you can learn it.
You must solve them by using factoring, logic, and working backwards.

Solve these easy ones. Fill in the blanks. Try not to calculate. Use your logic.
The two sides of any equation must be equal. What number will make it so?

4 x 3 x 6 = 6 x 4 x 3
How do you know the missing number is 6? Associative Property

Sometimes in these problems just the order has been changed around.
Fill in the blanks.

(2 + 4) x 5 = 5 x (4 + 2) **6 + 8 = 8 + 6**

5 x 7 x 10 = 7 x 10 x 5

Sometimes in these problems the form has been changed. Two numbers that stood alone on one side of the equation are multiplied on the other. Figure out which number goes in the blanks.

7 x 10 x 1,000 = 7 x 10, 000

4 x 9 x 2 = 36 x 2

Sometimes both the order and the form change. Fill in the blanks.

2 + 3 + 5 = 8 + 2 **10 + 5 - 4 = 6 + 5**

320 = 4 x 10 x 8 **5 x 10 x 3 = 15 x 10**

6 x 10 x 9 x 10 = 63 x 10 x 10

400 x 3 = 100 x 3 x 4

Double Digit: Beginning Worksheet 1

60	6
3 | 180	30 | 180
600	60
3 | 1,800	30 | 1,800
40	4
4 | 160	40 | 160
400	40
4 | 1,600	40 | 1,600
50	5
5 | 250	50 | 250
500	50
5 | 2,500	50 | 2,500

What patterns did you notice? two digits | one digit | three digits
one digit | two digits | three digits
two digits | four digits

Double Digit: Beginning - Worksheet 2

50 / 3 | 150	5 / 30 | 150	30 x 5 = 150
500 / 3 | 1,500	50 / 30 | 1,500	30 x 50 = 1,500
30 / 4 | 120	3 / 40 | 120	40 x 3 = 120
300 / 4 | 1,200	30 / 40 | 1,200	40 x 30 = 1,200
30 / 5 | 150	3 / 50 | 150	50 x 3 = 150
300 / 5 | 1,500	30 / 50 | 1,500	50 x 30 = 1,500
70 / 6 | 420	7 / 60 | 420	60 x 7 = 420
700 / 6 | 4,200	70 / 60 | 4,200	60 x 70 = 4,200

Double Digit: Beginning - Worksheet 3

500 / 3 | 1,500	900 / 2 | 1,800
50 / 30 | 1,500	90 / 20 | 1,800
51 / 30 | 1,530	91 / 20 | 1,820
52 / 30 | 1,560	92 / 20 | 1,840
53 / 30 | 1,590	93 / 20 | 1860
54 / 30 | 1,620	94 / 20 | 1880
55 / 30 | 1,650	95 / 20 | 1,900
56 / 30 | 1,680	96 / 20 | 1920

What patterns did you notice? The ones digit increases by one in the quotient of both sets of problems.

Double Digit: Beginning - Worksheet 4

5,000 / 3 | 15,000	3,000 / 2 | 18,000
500 / 30 | 15,000	300 / 20 | 18,000
501 / 30 | 15,030	301 / 20 | 18,060
510 / 30 | 15,300	310 / 20 | 18,600
511 / 30 | 15,330	311 / 20 | 18,660
512 / 30 | 15,360	312 / 20 | 18,720
520 / 30 | 15,600	313 / 20 | 18,780
521 / 30 | 15,630	314 / 20 | 18,840

What patterns did you notice? 2 digit divisors into 5 digit dividend equal 3 digit quotient

Double Digit: Beginning -Worksheet 5

50 / 5 \| 250 → 5 / 50 \| 250

710 / 5 \| 3,550 → 71 / 50 \| 3,550

Notice any patterns in the answers? yes

What did you see? Decrease by 10 times

440 / 4 \| 1,760 → 44 / 40 \| 1,760

6,970 / 4 \| 27,880 → 697 / 40 \| 27,880

3,010 / 7 \| 21,070 → 301 / 70 \| 21,070

Double Digit: Beginning - Worksheet 6

Code

700 ÷ 20 35 → A

1,260 ÷ 30 42 → R

1,120 ÷ 20 56 → E

2,480 ÷ 40 62 → A

3,700 ÷ 50 74 → L

1,560 ÷ 20 78 → G

2,340 ÷ 60 39 → O

2,910 ÷ 30 97 → O

1,380 ÷ 60 23 → D

2,380 ÷ 70 34 → J

2,000 ÷ 80 25 → O

4,860 ÷ 90 54 → B

Do the work **NEATY** on another piece of paper.

Match the answer to the letter. Write the letter on the line.

You will get a message at the end. It will answer this question:

What have you done?

_____

Answer Code

A	J	G	E	O	L	R	O	A	D	B	O
35	34	78	56	39	74	42	97	62	23	54	25

Double Digit: Beginning - Worksheet 7

Watch your Place Value. Code

23,750 ÷ 10 = 2,375 → R

90,320 ÷ 20 = 4,516 → U

151,410 ÷ 30 = 5,047 → D

103,480 ÷ 40 = 2,587 → O

327,450 ÷ 50 = 6,549 → N

59,220 ÷ 60 = 987 → E

3,111,080 ÷ 70 = 44,444 → Y

9,876,480 ÷ 80 = 123,456 → E

Challenge!

10,100,970 ÷ 90 = 112,233 → T

Solve this one in your head.

48,500 ÷ 100 = 485 → ?

Answer Code

?	N	E	T	Y
485	6,549	123,456	112,233	44,444
U	O	R	D	E
4,516	2,587	2,375	5,047	987

Sequences Worksheet 1

quotient

divisor | dividend

If the dividend, stays the same, 50 | 2,950 → 59

and the divisor goes up a little, 51 | 2,950 → 57 R43

Use the **5 X** tables to estimate.

I expect the quotient to go down a little.

down ↓ or ↑ up

52 | 2,950 → 56 R38

53 | 2,950 → 55 R35

54 | 2,950 → 54 R34

55 | 2,950 → 53 R35

Begin using the **6 X** tables to estimate answers.

Sequences Worksheet 2

If the dividend, stays the same, 30 | 1,620

and the divisor goes up a little, 31 | 1,620

I expect the quotient to go down a little.

32 | 1,620 → 50 R20

33 | 1,620 → 49 R3

34 | 1,620 → 47 R12

35 | 1,620 → 46 R10

Sequences
Worksheet 3

If the dividend, stays the same, $40\overline{)2{,}640}$ = 66

and the divisor goes down a little, $39\overline{)2{,}640}$ = 67 R27

I expect the quotient to go up a little.

$38\overline{)2{,}640}$ = 69 R18

$37\overline{)2{,}640}$ = 71 R13

$36\overline{)2{,}640}$ = 73 R12

$35\overline{)2{,}640}$ = 75 R15

Double Digit: Times Tables - Worksheet 1

Solve these first.

25 x 1 = 25	25 x 2 = 50	25 x 3 = 75	25 x 4 = 100	25 x 5 = 125
25 x 6 = 150	25 x 7 = 175	25 x 8 = 200	25 x 9 = 225	25 x 10 = 250

Check the answers to the multiplication problems with a calculator before going on.

Estimate	Break up
$25\overline{)550}$ = 22	= $25\overline{)500 + 50}$ = 20 + 2 = 22
$25\overline{)825}$ = 33	= $25\overline{)750 + 75}$ = 30 + 3 = 33
$25\overline{)1{,}100}$ = 44	= $25\overline{)1{,}000 + 100}$ = 40 + 4 = 44
Make your own. AWV	
$25\overline{\quad}$	= $\overline{\quad + \quad}$ = $\overline{\quad + \quad}$

Double Digit: Times Tables - Worksheet 2

Solve these first.

32 x 1 = 32	32 x 2 = 64	32 x 3 = 96	32 x 4 = 128	32 x 5 = 160
32 x 6 = 192	32 x 7 = 224	32 x 8 = 256	32 x 9 = 288	32 x 10 = 320

Check the answers to the multiplication problems with a calculator before going on.

Estimate	Break up
$32\overline{)384}$ = 12	= $32\overline{)320 + 64}$ = 10 + 2 = 12
$32\overline{)544}$ = 17	= $32\overline{)320 + 224}$ = 10 + 7 = 17
$32\overline{)768}$ = 24	= $32\overline{)640 + 128}$ = 20 + 4 = 24
Make your own. AWV	
$32\overline{\quad}$	$32\overline{\quad}$

Double Digit: Thinking Roundly - Worksheet 1

Every time you divide you don't want to have to write out a times table. One way to find a quotient is to round off the divisor, estimate, and then check by multiplication. Here is an example: ≈ means is about)

$52\overline{)156}$ Think Roundly: 50 x ? ≈ 156; 50 x 3 = 150

Check: 52 x 3 = 156

Yes! The answer is 3. $52\overline{)156}$ = 3

Try it again with a different problem.

$52\overline{)364}$ Think Roundly: 50 x ? ≈ 364; 50 x 7 = 350

Check: 52 x 7 = 364

Now you try it.

$52\overline{)312}$ = 6 Think Roundly: 50 x ? ≈ 312; 50 x 6 = 300

Check: 52 x 6 = 312

$52\overline{)468}$ = 9 Think Roundly: 50 x ? ≈ 468; 50 x 9 = 450

Check: 52 x 9 = 468

Now with different divisors.

$54\overline{)324}$ = 6 Think Roundly: 50 x ? ≈ 324; 50 x 6 = 300

Check: 54 x 6 = 324

Double Digit: Thinking Roundly - Worksheet 2

$31\overline{)248}$ Think Roundly: $30 \times ? \approx 248$; $30 \times 8 = 240$. Check: $31 \times 8 = 248$

$57\overline{)285}$ Think Roundly: $50 \times ? \approx 285$; $50 \times 5 = 250$. Check: $57 \times 5 = 285$

Sometimes you have to experiment to find the best round number.

$58\overline{)464}$ Think Roundly: $50 \times ? \approx 464$; $50 \times 9 = 450$. Check: $58 \times 9 = 522$

Did 58 x 9 work? no
Why not? too big

Try again. $60 \times ? \approx 464$; $60 \times 8 = 480$. Check: $58 \times 8 = 464$

Try again with a different round number. You might need to try several different combinations.

$58\overline{)464}$ $60 \times ? \approx 464$; $60 \times 7 = 420$. Check: $58 \times 7 = 406$

$60 \times ? \approx 464$; $60 \times 9 = 540$. Check: $58 \times 9 = 540$

$60 \times ? \approx 464$; $60 \times 8 = 480$. Check: $58 \times 8 = 464$

Which estimate was the fastest or easist? AWV

Double Digit: Thinking Roundly - Worksheet 3

$26\overline{)208}$ Think Roundly: $20 \times ? \approx 208$; $20 \times 10 = 200$; $30 \times 6 \approx 300$; $20 \times 8 = 160$. Check: $26 \times 10 = 260$; $26 \times 6 = 156$; $26 \times 8 = 208$

$26\overline{)182}$ Think Roundly: $20 \times ? \approx 182$; $20 \times 9 = 180$; $30 \times 6 \approx 180$; $30 \times 7 = 210$. Check: $26 \times 9 = 234$; $26 \times 7 = 182$

$26\overline{)234}$ Think Roundly: $20 \times ? \approx 234$; $20 \times 11 = 220$; $20 \times 9 \approx 180$; $30 \times 7 = 210$. Check: $26 \times 7 = 182$; $26 \times 8 = 208$; $26 \times 9 = 234$

$26\overline{)286}$ Think Roundly: $20 \times ? \approx 286$; $20 \times 14 = 280$; $30 \times 9 \approx 270$; $30 \times 11 = 330$. Check: $26 \times 9 = 234$; $26 \times 11 = 286$

Double Digit: Thinking Roundly - Worksheet 4

Try two round numbers again.

$49\overline{)343}$ Think Roundly: $40 \times ? \approx 343$; $40 \times 8 = 320$; $50 \times 6 \approx 300$; $50 \times 7 = 350$. Check: $49 \times 8 = 392$; $49 \times 7 = 343$

$47\overline{)282}$ Think Roundly: $40 \times ? \approx 282$; $40 \times 6 = 240$; $50 \times 5 \approx 250$; 50×6 300. Check: $47 \times 5 = 235$; $47 \times 6 = 282$

$62\overline{)310}$ Think Roundly: $60 \times ? \approx 310$; $60 \times 5 = 300$. Check: $62 \times 5 = 310$

What is the best way you found to solve these problems?

Multiply the round number to get close. Then change the second number to get exact. Or if the divisor in the ones digit is less than 5, round down to the closest multiple of 10. If the divisor in the ones place is 5 or greater than 5, round up to the next higher number.

Double Digit: Thinking Roundly - Worksheet 5

$27\overline{)1,701}$ Think Roundly: $30 \times ? \approx 1,701$; $30 \times 50 = 1,500$; $30 \times 60 \approx 1,800$; ◯ x ◯ = . Check: $27 \times 50 = 1,350$; $27 \times 63 = 1,701$

$27\overline{)2,025}$ Think Roundly: $20 \times ? \approx 2,025$; $20 \times 60 = 1,200$; $30 \times 70 \approx 2,100$; ◯ x ◯ = . Check: $27 \times 71 = 1,917$; $27 \times 75 = 2,025$

$27\overline{)1,134}$ Think Roundly: $20 \times ? \approx 1,134$; $30 \times 40 = 1,200$; $30 \times 42 \approx 1,260$; ◯ x ◯ = . Check: $27 \times 40 = 1,080$; $27 \times 42 = 1,134$

$27\overline{)1,431}$ Think Roundly: $30 \times ? \approx 1,431$; $20 \times 50 = 1,000$; $30 \times 50 \approx 1,500$; $27 \times 53 = 1,431$. Check: $27 \times 50 = 1,500$; $27 \times 53 = 1,431$

Double Digit: Transitions - Worksheet 1

$32\overline{)416}$ = 13

Think Roundly

30 x ? ≈ 416

30 x 10 = 300

40 x 10 ≈ 400

30 x 12 = 360

30 x 13 = 390

Check

32 x 12 = 384

32 x 13 = 416

$32\overline{)352}$ = 11

$32\overline{)512}$ = 16

$32\overline{)672}$ = 21

$33\overline{)800}$ = 25

Make your own. AWV

$32\overline{)736}$ = 23

$32\overline{)\ \ }$

Double Digit: Transitions - Worksheet 2

$31\overline{)1,891}$ = 61

Think Roundly

30 x ? ≈ 1,891

30 x 60 = 1,800

30 x 61 ≈ 1,830

31 x 61 = 1,891

Check

31 x 60 = 1800

31 x 61

Check by multiplication.

$31\overline{)2,170}$ = 70

$31\overline{)1,984}$ = 64

$31\overline{)2,201}$ = 71

$31\overline{)2,232}$ = 72

Double Digit: Transitions - Worksheet 3

$52\overline{)156}$ = 3

Check: 52 x 3 = 156

$52\overline{)3,692}$ = 71

Check: 71 x 52 = 142, 355, 3,692

$52\overline{)1,248}$ = 24

Check: 52 x 24 = 1,248

$52\overline{)26,468}$ = 509

Check: 509 x 52 = 26,468

$52\overline{)676}$ = 13

Check: 52 x 13 = 676

$52\overline{)53,040}$ = 1,020

Check: 1,020 x 52 = 53,040

Make your own.

$52\overline{)3,536}$ = 68

Check: 68 x 52 = 3,536

$52\overline{)\ \ }$ x ____

Double Digit: Practice - Worksheet 1

$32\overline{)928}$ = 29

-640

288

-288

0

$32\overline{)352}$ = 11

$32\overline{)1,728}$ = 54

$32\overline{)2,784}$ = 87

$32\overline{)1,216}$ = 38

$32\overline{)12,160}$ = 380

Double Digit: Practice - Worksheet 2

$35\overline{)420}$ = 12, 420 − 350 = 70, 70 − 70 = 0

$54\overline{)972}$ = 18

$35\overline{)595}$ = 17

$54\overline{)1,242}$ = 23

$35\overline{)840}$ = 24

$35\overline{)630}$ = 1

$54\overline{)1,944}$ = 36

$35\overline{)805}$ = 23

Make your own AWV

$54\overline{)\quad}$

$35\overline{)980}$ = 28

Double Digit: Practice - Worksheet 3

$46\overline{)828}$ = 18

$46\overline{)966}$ = 21

$46\overline{)1,702}$ = 37

$46\overline{)2,438}$

$46\overline{)3,864}$ = 84

$46\overline{)4,922}$ = 107

$46\overline{)1,610}$ = 35

Make your own AWV

$46\overline{)\quad}$

Double Digit: Practice - Worksheet 4

$42\overline{)714}$ = 17

$48\overline{)768}$ = 16

$43\overline{)817}$ = 19

$43\overline{)946}$ = 22

$42\overline{)1,722}$ = 41

$45\overline{)2,385}$ = 53

$43\overline{)3,827}$ = 89

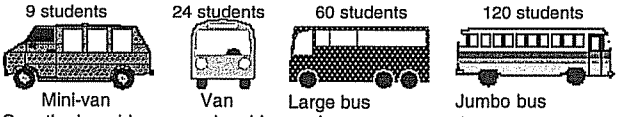
$43\overline{)4,859}$ = 113

$44\overline{)1,540}$ = 35

Make your own AWV

$46\overline{)\quad}$

Gus the Bus Driver - Worksheet 1



Gus, the bus driver, now has his own bus company. His Mini-van carries 9 students, the Van carries 24 students, the Large bus carries 60 students, and the Jumbo bus carries 120 students.

1. How many buses are needed for each grouping of students?

	Mini-van	Van	Large bus	Jumbo bus
120	13 R3	5	2	1
240	26 R6	10	4	2
360	40	15	6	3
480	53 R3	20	8	4
600	66 R6	25	10	5

2. Each vehicle rents for $75, $150, $200, or $300. Each vehicle needs a driver for $100.
How much would it cost to transport each grouping of students?

Cost of bus / Students	$75 Mini-van	$150 Van	$200 Large bus	$300 Jumbo bus
120	$2,450	$1,250	$600	$400
240	4,725	2,500	1,200	800
360	7,000	3,750	1,800	1,200
480	9,450	5,000	2,400	1,600
600	11,725	6,250	3,000	2,000

Gus the Bus Driver - Worksheet 2

Gus has one Jumbo bus that carries 120 students, four Large buses that carry 60 students, one Van carries 24 students, and one Mini-van that carries 9 students. Each vehicle needs a driver for $100. Gus wants to use the fewest drivers possible for each field trip.

3. Figure out the least expensive way to transport 391 students.
Cost of the field trip and vehicles used: $2025 1 jumbo bus, 4 large buses, 1 mini-van, 1 van
Record your work on lined paper.

$400
1200
250
175
2025

391 − 120 = 271
271 − 240 = 31

31 − 24 = 7

4. Figure out the least expensive way to transport 250 students.
Cost of the field trip and vehicles used: $1,250 1 jumbo bus, 2 buses, 1 van
Record your work on lined paper.

5. Challenge. If 250 students share equally in the cost of the busses, how much does each ticket cost? $5.00

Answer to 4.

250 − 120 = 130 jumbo $400
130 − 120 = 10 2 buses 600
10 1 van 250
$1,250

Practice - Worksheet 1

$5\overline{)900}$ = 180

$8\overline{)3,824}$ = 478

$9\overline{)1,308}$ = 145 R3

$4\overline{)272}$ = 68

$5\overline{)623}$ = 124 R3

$4\overline{)306}$ = 76 R2

$12\overline{)672}$ = 56

$13\overline{)754}$ = 58

$15\overline{)180}$ = 12

$21\overline{)273}$ = 13

$14\overline{)336}$ = 24

$18\overline{)414}$ = 23

$24\overline{)3,696}$ = 154

$17\overline{)1,955}$ = 115

$15\overline{)1,665}$ = 111

Practice - Worksheet 2

$5\overline{)965}$ = 193

$8\overline{)4,096}$ = 512

$9\overline{)1,377}$ = 153

$4\overline{)384}$ = 96

$5\overline{)728}$ = 145 R3

$4\overline{)349}$ = 87 R1

$12\overline{)792}$ = 66

$13\overline{)936}$ = 72

$15\overline{)360}$ = 24

$21\overline{)1,134}$ = 54

$14\overline{)758}$ = 54 R2

$18\overline{)522}$ = 29

$24\overline{)3,816}$ = 159

$17\overline{)2,397}$ = 141

$15\overline{)1,890}$ = 126

Pre-Assessment - Part 1

Part 1

1. a. $5\overline{)965}$ = 193 b. $8\overline{)4,096}$ = 512 c. $9\overline{)1,377}$ = 153

d. $4\overline{)384}$ = 96 e. $5\overline{)728}$ = 145 R3 f. $4\overline{)349}$ = 87 R1

2. a. 34 × 18 = 612 b. 72 × 84 = 6,048 c. 354 × 67 = 23,713

Part 2

1. Show the remainders as fractions.
   a. $4\overline{)15}$ = $3\frac{3}{4}$ b. $5\overline{)16}$ = $3\frac{2}{10}$ or $3\frac{1}{5}$ c. $6\overline{)16}$ = $2\frac{4}{6}$ or $2\frac{2}{3}$

2. Show the remainders as decimals. Round to the nearest hundredth if needed.
   a. $6\overline{)213}$ = 35.5 b. $9\overline{)165}$ = 18.33 c. $7\overline{)109}$ = 15.57

Pre-Assessment - Part 2 continued

3. Find and record the missing number.
   a. 400 × 4 = 1,600 b. 80 × 70 = 5,600
   c. Prove your answer is true in problem b by showing a factoring by tens factor tree.
   5,600 → 560 × 10 → 56 × 10
   d. 3,000 × 60 = 180,000

4. a. $3\overline{)150}$ = 50 b. $30\overline{)150}$ = 5 c. 30 x 5 = 150

   d. $3\overline{)1,500}$ = 500 e. $30\overline{)1,500}$ = 50 f. 30 x 50 = 1,500

5. a. 700 ÷ 20 = 35 b. 1,260 ÷ 30 = 42

   c. 1,380 ÷ 60 = 23 d. 4,860 ÷ 90 = 54

6. If the dividend stays the same, $50\overline{)2,950}$
   and the divisor goes up a little, $51\overline{)2,950}$
   I expect the quotient to go down (↓ down or ↑ up) a little.

7. a. $49\overline{)343}$ = 7 b. $47\overline{)282}$ = 6 c. $62\overline{)310}$ = 5

8. a. $52\overline{)1,248}$ = 24 b. $54\overline{)972}$ = 18 c. $46\overline{)4,922}$ = 107

Post-Assessment - Worksheet 1

1. Show the remainders as fractions.
   a. $4\overline{)15}$ = $3\frac{3}{4}$ b. $5\overline{)16}$ = $3\frac{3}{10}$ or $3\frac{3}{5}$ c. $6\overline{)16}$ = $2\frac{4}{6}$ or $2\frac{2}{3}$

2. Show the remainders as decimals. Round to the nearest hundredth if needed.
   a. $6\overline{)213}$ = 35.5 b. $9\overline{)165}$ = 18.33 c. $7\overline{)109}$ = 15.57

3. Find and record the missing number.
   a. 400 × 4 = 1,600 b. 80 × 70 = 5,600
   c. Prove your answer is true in problem b by showing a factoring by tens factor tree.
   5,600 → 560 × 10 → 56 × 10
   d. 3000 × 60 = 180,000

4. a. $3\overline{)150}$ = 50 b. $30\overline{)150}$ = 5 c. 30 x 5 = 150

   d. $3\overline{)1,500}$ = 500 e. $30\overline{)1,500}$ = 50 f. 30 x 50 = 1,500

Post-Assessment - Worksheet 2

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