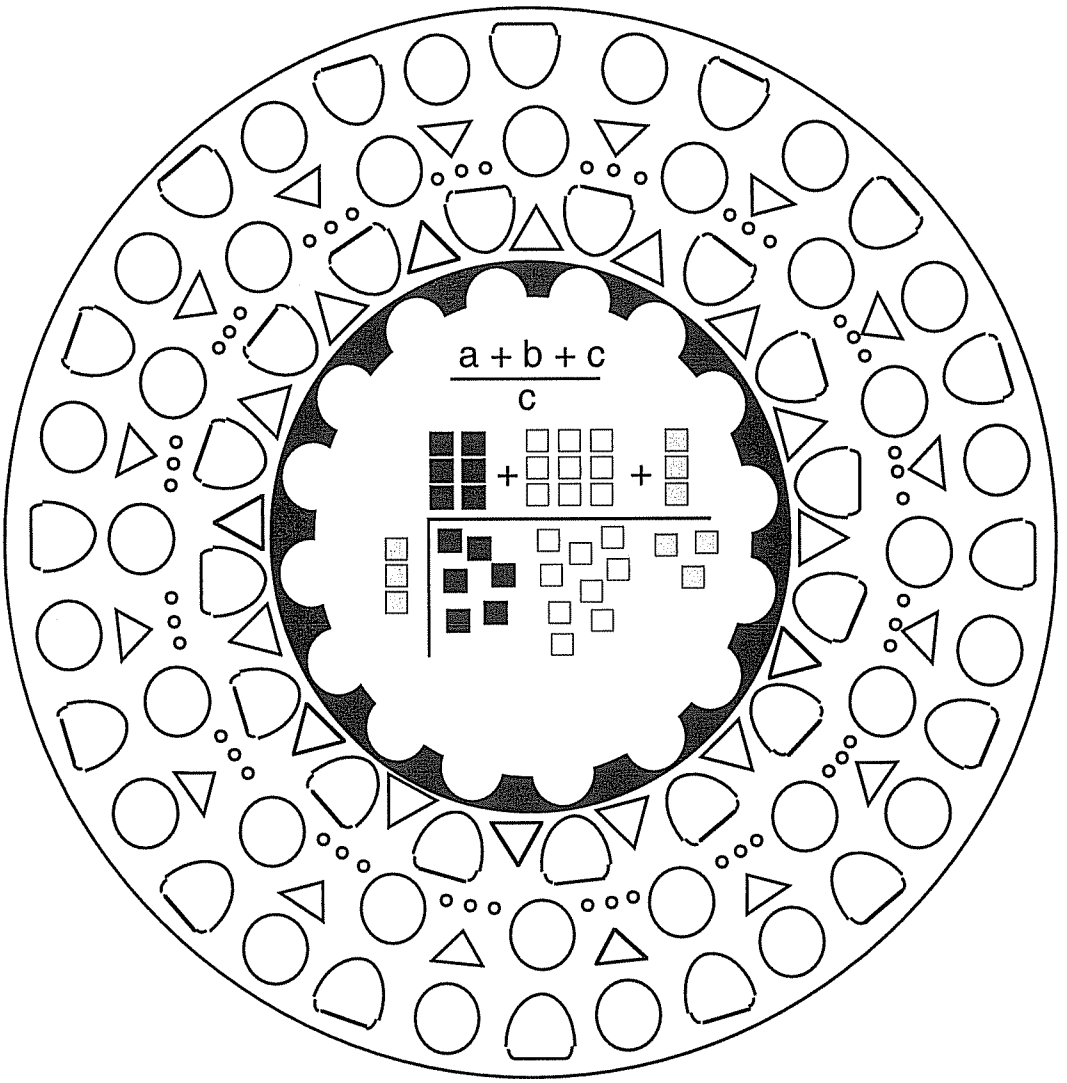
Patterns in Arithmetic

Division - Booklet 2 PDF

Breaking Up Division

Parent/Teacher Guide



By Alysia Krafel, Susan Carpenter, and Suki Glenn

Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the
UCI Farm Elementary School
University of California, Irvine

Division: Booklet 2 - PDF - Breaking Up Division

Contents

The circle division series is dedicated to Albion Lawrence who, as an eight-year-old whiz kid, taught Alysia how to break up numbers for division.

Acknowledgments

The knowledge, patience, and dedication of Professor Michael Butler made the UCI Farm Elementary School and this mathematics program possible. Special thanks go to Alysia Krafel and Susan Carpenter, who helped develop much of the math materials based on the teachings, ideas, and insights of Professor Butler.

For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

The books would never have been completed if the students at Chrysalis Charter School in Redding, California, under the guidance of Alysia and Paul Krafel, hadn't needed them. Thank you for your patience through all of the draft copies.

Susan Carpenter edited, added her wise words, useful suggestions, and helped make the Answer Keys a reality. Kárán Founds-Benton contributed her meticulous editing skill and knowledge. Diligent and thorough copy editing was done by Jacqueline Logue.

The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

Patterns in Arithmetic: Division - Booklet 2 PDF
Parent/Teacher Guide

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Pre-Assessment - Part 1 Name_____ Date _____

Put a question mark next to any problem you do not know how to do.

1. Susan has eight chocolate chip cookies. She put two cookies on each plate. How many plates did she need? _____

a. Draw a picture of the problem.

b. Write the number sentence that goes with the story.

___	÷	___	=	___

c. Record.

___
___ ⟌ ___

2. Use only the three numbers 4, 7, and 28 to make four Families of Facts number sentences.

a. ◯ x ◯ = ◯ ◯ ÷ ◯ = ◯

◯ x ◯ = ◯ ◯ ÷ ◯ = ◯

b. x ____ = ____

x ____ = ____

3. Fill in the missing numbers.

a. ◯ ÷ 4 = 8 b. 15 ÷ ◯ = 3 c. 35 ÷ 5 = ◯

d. ◯ ÷ 10 = 2

e. ☐ ⟌ 48 (6)

f. 8 ⟌ ☐ (4)

g. 2 ⟌ 10 (☐)

4. Solve.

a. 86 − 39 = ____

b. 347 − 156 = ____

c. 435 − 279 = ____

5. Solve.

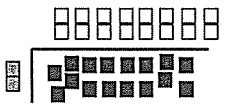
a. 138 x 6 = ____

b. 2,375 x 4 = ____

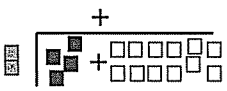
c. 6,598 x 4 = ____

Pre-Assessment: Part 2 - Worksheet 1 Name_____ Date _____

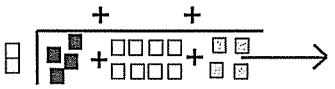
1.

a.  $2\overline{)16}$ Study this picture. What does this 8 mean?_____

b. Draw the blocks. Fill in the blanks.

 $2\overline{)4 + 12}$ ___ + ___ = ___ ← Final answer.

c. Fill in the blanks.

 ___ $\overline{)}$ ___ + ___ + ___ ___ + ___ + ___ = ___

d. What do you notice about the answers in a, b, and c? _____

_____

2.

a. Build 16 this way. Record the number sentence. Arrange the blocks in each set into groups of 2.  $2\overline{)5 + 11}$ ___ + ___ = ___

b. Build 20 this way. Record the number sentence. Arrange the blocks in each set into groups of 4.  ___ $\overline{)}$ ___ + ___ + ___ ___ + ___ + ___ = ___

c. Build 19 this way. Record the number sentence.  $4\overline{)6 + 8 + 5}$ ___ + ___ + ___ = ___

d. Break the 38 into three numbers and divide by 5. $5\overline{)38}$ ___ $\overline{)}$ ___ + ___ + ___ ___ + ___ + ___ = ___

3. Solve each problem two different ways.

a. $4\overline{)46}$ ___ $\overline{)}$ ___ + ___ ___ + ___ = ___ ___ $\overline{)}$ ___ + ___ + ___ ___ + ___ + ___ = ___

b. $4\overline{)38}$ ___ $\overline{)}$ ___ + ___ ___ + ___ = ___ ___ $\overline{)}$ ___ + ___ + ___ ___ + ___ + ___ = ___

c. $4\overline{)56}$ ___ $\overline{)}$ ___ + ___ ___ + ___ = ___ ___ $\overline{)}$ ___ + ___ + ___ ___ + ___ + ___ = ___

Pre-Assessment: Part 2 - Worksheet 2 Name_____ Date _____

4. Tony was shooting baskets every day. Here is a table of the number of baskets he made each day.

Monday	12 baskets made
Tuesday	6 baskets made
Wednesday	9 baskets made
Thursday	14 baskets made
Friday	14 baskets made

a. How many baskets were made in all? _____

b. How many days were there? _____

c. What was the average number of baskets per day? _____

5. Solve. Show your work on how you break up each number.

a. $8\overline{)94}$ ___ ___ + ___ + ___ = ___ / ___ + ___ + ___

b. $9\overline{)117}$ ___ ___ + ___ + ___ = ___ / ___ + ___ + ___

6. Solve these two problems.

a. $3\overline{)30 + 30 + 12}$ ___ + ___ + ___ = ___

b. $3\overline{)18 + 36 + 10 + 8}$ ___ + ___ + ___ + ___ = ___

c. Which problem was easier to solve? ____ d. Explain why. _____

_____

7. Fill in the missing numbers.

a. $3\overline{)51}$ 10 + 7 = ___ / 3 ___ + ___

b. $4\overline{)92}$ 20 + 3 = ___ / 4 ___ + ___

8. Make the answers come out in tens and ones with remainders in the ones place only.

a. $3\overline{)96} = 3\overline{)\ \ }$ 0 + ___ = ___ / ___ + ___

b. $4\overline{)184} = 4\overline{)\ \ }$ 0 + ___ = ___ / ___ + ___

c. $5\overline{)385} = 5\overline{)\ \ }$ 0 + ___ = ___ / ___ + ___

9. Solve each problem the long way, showing how the problem is broken up to get the answer to come out in hundreds, tens, and ones. Then do each problem the short way.

a. $3\overline{)3,609} = 3\overline{)\ \ }$ _____+_____+_____ = _____ / _____+_____+_____

b. $3\overline{)477} = 3\overline{)\ \ }$ _____+_____+_____ = _____ / _____+_____+_____

c. $3\overline{)2,124} = 3\overline{)\ \ }$ _____+_____+_____ = _____ / _____+_____+_____

d. $4\overline{)3,625} = 4\overline{)\ \ }$ _____+_____+_____ = _____ / _____+_____+_____

Assessment Guide

Purpose

The purpose of this guide is to determine if this booklet is the correct starting place. Pre-Assessment: Part 1 assesses the fundamental knowledge necessary for success in this booklet.

Pre-Assessment: Part 1 is review material from the last booklet and is used to determine student readiness for this booklet. Pre-Assessment: Part 2 is a preview of the new material presented in this booklet and is used to set the baseline for what the student already knows at the beginning of instruction.

The Post-Assessment is administered to determine if the student learned the material that was presented in this booklet. A comparison of the score on Pre-Assessment: Part 2 to the score on the Post-Assessment will give both you and your student a sense of growth. The Post-Assessment is the same as Pre-Assessment: Part 2.

Prerequisites

Patterns in Arithmetic: Division - Booklet 1, Multiplication: Booklet 1, Multiplication: Booklet 2, and Subtraction: Booklet 3

Materials

Patterns in Arithmetic: Division - Booklet 2
Pre-Assessment: Part 1 and Pre-Assessment: Part 2
Score sheets
Cuisenaire Rods and Unifix cubes

Instructions

Do the assessment in two parts. Give Pre-Assessment: Part 1 and check it for readiness for this booklet. If the student is not ready for this booklet, there is no point in giving Pre-Assessment: Part 2. If he passes all the readiness items, then give Pre-Assessment: Part 2.

After scoring Pre-Assessment: Part 2, use the Booklet Selection Guide to determine the correct booklet for your student based on the results of the assessment.

Assessment Guide

This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc. Score sheets that match the Assessment Guide for Pre-Assessment: Part 1, Pre-Assessment: Part 2, and the Post-Assessment follow.

Assessment Criteria for Pre-Assessment: Part 1
Can the student:
1. Number Sentence (Division: Booklet 1)
    A. place correctly the numbers from a word problem into a division number sentence?

B. place the divisor, the dividend, and the quotient in the correct locations in the ⌐‾ notation?
C. draw a correct picture of the word problem?

2. Families of Facts (Division: Booklet 1)
A. write four Families of Facts for the three given numbers?
B. place the division number sentences in the ⌐‾ correctly?

3. Family of Facts: Connections (Division: Booklet 1)
Does the student understand the relationship between dividends, divisors, and quotients? It is common for students to have difficulty with this concept on the pre-test. It is introduced in Division: Booklet 1 but generally not mastered. This concept is retaught in Division: Booklet 2.

A. fill in the correct numbers on items c and e?

Missing Quotient - These two items test comprehension of the basic division number sentence with the quotient missing.

B. fill in the correct numbers on Items b and g?

Missing Divisor–These two items tests comprehension of the relationship between the dividend and the quotient. Item g also tests understanding of the notation in division.

C. fill in the correct numbers on items a, d, and f?

Missing Dividend–These items test comprehension of the relationship of the divisor and the quotient. Many students will do the problem shown below incorrectly. If the number 2 is placed in the empty space on item f, but the number sentence in item a, ___ ÷ 4 = 8 is done correctly; the student is confused about the notation. He is dividing the four into the eight instead of multiplying. This confusion is common. $4\overline{)8}$

4. Subtraction with Regrouping (Subtraction: Booklet 3)
A. give correct answers in two of the three subtraction problems?

5. Expanded Multiplication and Short Notation (Multiplication: Booklet 2)
A. give correct answers in two of the three multiplication problems?

Booklet Selection Guide based on results of Pre-Assessment: Part 1

If the student receives a Yes on 1A, B, and C, 2B, 4A, and 5A, give Pre-Assessment: Part 2 and begin Division: Booklet 2. If the student received a No on 3A, B, or C, pay particular attention to this topic when you meet it again in Division: Booklet 2.

If the student receives a No on three or more items in problems 1 and 2, begin with Division: Booklet 1. Do not give Pre-Assessment: Part 2 of this assessment.

If the student receives a No on 4A, begin with Subtraction: Booklet 3 or Base Ten Subtraction. Subtraction is an important part of the division procedure. Do not give Pre-Assessment: Part 2 of this assessment.

If the student receives a No on 5A, begin with Multiplication: Booklet 2. Single digit multiplication is an important part of the division procedure. Do not give Pre-Assessment: Part 2 of this assessment.

Assessment Criteria for Pre-Assessment: Part 2

All items in this section are taught in Division: Booklet 2. Students who have not completed Division: Booklet 2 will be unable to complete most of the sections of the Pre-Assessment.

Can the student:

1. Breaking-Up: Manipulative
   - A. 2 points identify the eight as standing for the number of groups of two in sixteen?
     1 point identify the eight as the answer?
   - B. draw correctly in the missing blocks in item b and record the quotient with numbers?
   - C. fill in the correct missing numbers to match the picture of the blocks in problem c?

These two items test to see if the student can recognize a representation for breaking up the dividend and put numbers to those representations.

D. 2 points "The answers are the same because it is the same problem all three times only broken up differently."

1 point "They all have the same answer."

Can the student explain that the quotients are the same even though dividends were broken up differently?

2. Breaking Up: Remainders
   - A. record correctly the quotients and remainders in problem a?
   - B. fill in correctly the numbers in the divisor and dividend for problem b? This problem has remainders in the break up, but the final quotient does not have a remainder.
   - C. fill in correctly the numbers in the divisor and dividend for problem c? This problem has remainders in the break up and in the final quotient.
   - D. use the break up procedure in problem d with no pictures or blocks used?
   - E. give the correct answer on problem d?

Students who have not completed Division: Booklet 2, may give the correct final quotient with the correct remainder to the simple problem but not break up the dividend or not show the remainders in the parts of the broken up dividend. This will tell you that they can work with remainders, but do not know how to work with them when breaking up a dividend.

3. Breaking Up Recording
   - A. give the correct answer on two of the three problems in this set?
   - B. demonstrate that he can do two of the three problems two different ways? Give a Yes here even if there are arithmetic errors. It is the process you are looking for. Does he know to break up the dividend two different ways and divide each section by the divisor?
   - C. give evidence of strategies to make the work easier by using multiples of the divisor or tens and ones?

If he can give only the correct answer on the short notation problem and is unable to demonstrate two different ways to do the problem, understanding of the use of the Distributive Property applied to division is lacking.

4. Averaging
   - A. calculate a simple average by giving the correct number in item c?

5. Breaking Up
   - A. give the correct answer on one of the two problems using any notation?

B. break up the numbers into three parts and divide each part by the divisor? Give a Yes if the procedure is correct even if there are arithmetic errors.

C. use multiples of the divisor in the break up?

6. Discovering Easier Ways

A. do both problems correctly?

B. 2 points Problem a is easier because it uses multiples of the divisor, or multiples of three to break up the dividend. Problem b uses numbers not in the three times tables and so has remainders.

1 Point Problem a is easier because it uses the three times tables and problem b does not. The second part of this sentence can be missing to get a Yes, but not the first.

7. Discovering Easier Ways

A. fill in the correct missing dividends by using the quotient at the top in both problems?

8. Breaking Up: Tens and Ones

A. break up the dividend to create quotients that come out in tens and ones in two of the three problems?

B. give the correct quotient in two of the three problems?

9. Expanded Division and Short Notation

A. show the correct expansion in three of the four problems?

B. show the correct notation for the short procedure in three of the four problems?

C. give the correct quotient in three of the four problems?

Booklet Selection Guide based on results of Pre-Assessment: Part 2

If a student scores 21 or more points on the Pre-Assessment, this booklet is not needed. Proceed to Multiplication: Booklet 4 and Division: Booklet 3.

If a student scores a Yes on 1A, 2E, 4A, 5A, 8B, 9B, and 9C and No on the other items it indicates that he knows how to divide using the standard short procedure. He may not understand why his procedure works. Most students memorize the procedure with no understanding of what is physically happening to the numbers. He probably does not understand how the Distributive Property applies to division. This is an important concept to understand for success in algebra. You can use Base Ten Division to help him understand why the standard procedure works the way it does. The Distributive Property is used but not explicitly taught. The Base Ten Division Booklet is a good choice for students who struggle with division or for average and above-average students who do not understand why the procedure works the way it does and time is a constraint. It is better to use this booklet, Division: Booklet 2, quickly, doing only the parts on Breaking Up, Discovering Easier Ways and Expanded Division. This is the best choice for average and above-average students if time is not a constraint.

If the student passes Pre-Assessment: Part 1 of the assessment and gets No on most of Pre-Assessment: Part 2, this is the correct booklet for this student.

Pre-Assessment: Part 1 Score Sheet

Name _____ Date _____

Can the student:

1. Number Sentences (Division: Booklet 1)
   - Yes No A. place the correct numbers from a word problem into a division number sentence?
   - Yes No B. place the divisor, the dividend, and the quotient in the correct locations in the ⌐‾ (querel)?
   - Yes No C. draw a correct picture of the word problem?

2. Families of Facts (Division: Booklet 1)
   - Yes No A. write four Families of Facts for the three given numbers?
   - Yes No B. place the division number sentences in the ⌐‾ (querel) correctly?

3. Families of Facts: Connections (Division: Booklet 1)
   - Yes No A. fill in the correct numbers on items c and e? Missing Quotient
   - Yes No B. fill in the correct numbers on items b and g? Missing Divisor
   - Yes No C. fill in the correct numbers on items a, d, and f? Missing Dividend

4. Subtraction with Regrouping (Subtraction: Booklet 3)
   - Yes No A. give correct answers in two of the three subtraction problems?

5. Expanded Multiplication and Short Notation (Multiplication: Booklet 2)
   - Yes No A. give correct answers in two of the three multiplication problems?

Items Correct = _____ = _____%

Items Possible = 10

Pre-Assessment: Part 2 Score Sheet

Name _____ Date _____

Can the student:

1. Breaking Up: Manipulative
   - 2, 1, 0 points A. identify the eight as standing for the number of groups of two in sixteen? 2 points identify the eight as the answer? 1 point
   - Yes No B. draw in the correct missing blocks in item b and record the quotient with numbers?
   - Yes No C. fill in the correct missing numbers to match the picture of the blocks in problem c?
   - 2, 1, 0 points D. clearly explain why the problems have the same answer?

2. Breaking Up: Remainders
   - Yes No A. record the correct quotients and remainders in problem a?
   - Yes No B. fill in the correct numbers for problem b?
   - Yes No C. fill in the correct numbers for problem c?
   - Yes No D. use the break up procedure in problem d with no pictures?
   - Yes No E. give the correct answer on problem d?

3. Breaking Up: Recording
   - Yes No A. give the correct answer on two of the three problems in this set?
   - Yes No B. demonstrate that he can do two of the three problems two different ways? Give a Yes here even if there are arithmetic errors.
   - Yes No C. give evidence of strategies to make the work easier by using multiples of the divisor or tens and ones?

4. Averaging
   - Yes No A. calculate a simple average by giving the correct number in item c?

5. Breaking Up
   - Yes No A. give the correct answer on one of the two problems using any notation?
   - Yes No B. break up the numbers into three parts and divide each part by the divisor? Give a Yes if the procedure is correct even if there are arithmetic errors.
   - Yes No C. use multiples of the divisor in the break up?

6. Discovering Easier Ways
   - Yes No A. do both problems correctly?
   - 2, 1 0 points B. give a clear explanation of why problem a is easier to do?

7. Discovering Easier Ways
   - Yes No A. fill in the correct missing dividends by using the quotient at the top in both problems?

8. Breaking Up: Tens and Ones
   - Yes No A. break up the dividend to create quotients that come out in tens and ones in two of the three problems?
   - Yes No B. give the correct quotient in two of the three problems?

9. Expanded Division and Short Notation
   - Yes No A. show the correct expansion in three of the four problems?
   - Yes No B. show the correct notation for the short procedure in three of the four problems?
   - Yes No C. give the correct quotient in three of the four problems?

Items Correct = _____ = _____%

Items Possible = 27

Post-Assessment Score Sheet

Name _____ Date _____

Can the student:

1. Breaking Up: Manipulative
   - 2, 1, 0 points — A. identify the eight as standing for the number of groups of 2 in 16? 2 points identify the eight as the answer? 1 point
   - Yes No — B. draw in the correct missing blocks in item b and record the quotient with numbers?
   - Yes No — C. fill in the correct missing numbers to match the picture of the blocks in problem c?
   - 2, 1, 0 points — D. clearly explain why the problems have the same answer?

2. Breaking Up: Remainders
   - Yes No — A. record the correct quotients and remainders in problem a?
   - Yes No — B. fill in the correct numbers for problem b?
   - Yes No — C. fill in the correct numbers for problem c?
   - Yes No — D. use the break up procedure in problem d with no pictures?
   - Yes No — E. give the correct answer on problem d?

3. Breaking Up: Recording
   - Yes No — A. give the correct answer on two of the three problems in this set?
   - Yes No — B. demonstrate that he can do two of the three problems two different ways? Give a Yes here even if there are arithmetic errors.
   - Yes No — C. give evidence of strategies to make the work easier by using multiples of the divisor or tens and ones?

4. Averaging
   - Yes No — A. calculate a simple average by giving the correct number in item c?

5. Breaking Up
   - Yes No — A. give the correct answer on one of the two problems using any notation?
   - Yes No — B. break up the numbers into three parts and divide each part by the divisor? Give a Yes if the procedure is correct even if there are arithmetic errors.
   - Yes No — C. use multiples of the divisor in the break up?

6. Discovering Easier Ways
   - Yes No — A. do both problems correctly?
   - 2, 1, 0 points — B. give a clear explanation of why problem a is easier to do?

7. Discovering Easier Ways
   - Yes No — A. fill in the correct missing dividends by using the quotient at the top in both problems?

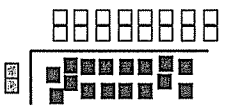
8. Breaking Up: Tens and Ones
   - Yes No — A. break up the dividend to create quotients that come out in tens and ones in two of the three problems?
   - Yes No — B. give the correct quotient in two of the three problems?

9. Expanded Division and Short Notation
   - Yes No — A. show the correct expansion in three of the four problems?
   - Yes No — B. show the correct notation for the short procedure in three of the four problems?
   - Yes No — C. give the correct quotient in three of the four problems?

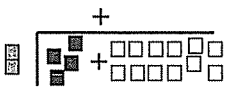
Items Correct = _____ = _____%
Items Possible = 27

Post-Assessment - Worksheet 1 Name_____ Date _____

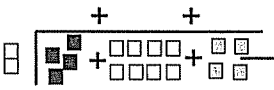
1.

a.  $\longrightarrow$ $2\overline{)16}^{\,8}$ Study this picture. What does this 8 mean?_____

b. Draw the blocks. Fill in the blanks.

 $\longrightarrow$ $2\overline{)4 + 12}$ ___ + ___ = ___ ← Final answer.

c. Fill in the blanks.

 $\longrightarrow$ ___$\overline{)}$ ___ + ___ + ___ ___ + ___ + ___ = ___

d. What do you notice about the answers in a, b, and c? _____

2.

a. Build 16 this way. Record the number sentence. Arrange the blocks in each set into groups of 2.  $2\overline{)5 + 11}$ ___ + ___ = ___

b. Build 20 this way. Record the number sentence. Arrange the blocks in each set into groups of 4.  ___$\overline{)}$ ___ + ___ + ___ ___ + ___ + ___ = ___

c. Build 19 this way. Record the number sentence.  $4\overline{)6 + 8 + 5}$ ___ + ___ + ___ = ___

d. Break the 38 into three numbers and divide by 5. $5\overline{)38}$ ___$\overline{)}$ ___ + ___ + ___ ___ + ___ + ___ = ___

3. Solve each problem two different ways.

a. $4\overline{)46}$ ___$\overline{)}$ ___ + ___ ___ + ___ = ___ ___$\overline{)}$ ___ + ___ + ___ ___ + ___ + ___ = ___

b. $4\overline{)38}$ ___$\overline{)}$ ___ + ___ ___ + ___ = ___ ___$\overline{)}$ ___ + ___ + ___ ___ + ___ + ___ = ___

c. $4\overline{)56}$ ___$\overline{)}$ ___ + ___ ___ + ___ = ___ ___$\overline{)}$ ___ + ___ + ___ ___ + ___ + ___ = ___

Post-Assessment - Worksheet 2 Name_____ Date _____

4. Tony was shooting baskets every day. Here is a table of the number of baskets he made each day.

a. How many baskets were made in all? _____

b. How many days were there? _____

c. What was the average number of baskets per day?_____

Monday	12 baskets made
Tuesday	6 baskets made
Wednesday	9 baskets made
Thursday	14 baskets made
Friday	14 baskets made

5. Solve. Show your work on how you break up each number.

a. $8\overline{)94}$ ___ + ___ + ___ = ___

___ = ___ + ___ + ___

b. $9\overline{)117}$ ___ + ___ + ___ = ___

___ = ___ + ___ + ___

6. Solve these two problems.

a. $3\overline{)30 + 30 + 12}$ ___ + ___ + ___ = ___

b. $3\overline{)18 + 36 + 10 + 8}$ ___ + ___ + ___ + ___ = ___

c. Which problem was easier to solve? ____ d. Explain why. _____

_____

7. Fill in the missing numbers.

a. $3\overline{)51}$ 10 + 7 = ___

3 | ___ + ___

b. $4\overline{)92}$ 20 + 3 = ___

4 | ___ + ___

8. Make the answers come out in tens and ones with remainders in the ones place only.

a. $3\overline{)96} = 3 \times$ ___ + ___ = ___

b. $4\overline{)184} = 4 \times$ ___ + ___ = ___

c. $5\overline{)385} = 5 \times$ ___ + ___ = ___

9. Solve each problem the long way, showing how the problem is broken up to get the answer to come out in hundreds, tens, and ones. Then do each problem the short way.

a. $3\overline{)3,609} = 3 \times$ _____+_____+_____ = _____

b. $3\overline{)477} = 3 \times$ _____+_____+_____ = _____

c. $3\overline{)2,124} = 3 \times$ _____+_____+_____ = _____

d. $4\overline{)3,625} = 4 \times$ _____+_____+_____ = _____

Number Sentence: Review

Purpose

The purpose of this lesson is to develop language associated with division and to develop the division number sentence: $x \div y = z$.

Prerequisites

Understanding the multiplication and division lessons in Division: Booklet 1, in particular, Monkeys on the Roof (also included in this booklet on page 18 for teacher instructions and in the Addendum, pages 1 - 7.

Materials

Number Sentence: Review, Worksheets 1 and 2, pages 4 and 5
Unifix cubes, blocks, or tiles
Cuisenaire Rods for Division with Cuisenaire Rods, page 6

Lesson Part 1

Review this chart:

The number you start with	÷ The size of the groups	= The number of groups

Give Number Sentence: Review, page 4. The first problem is:
Susan has eight chocolate chip cookies. She put two cookies on each plate. How many plates will she need?

"Read the problem. Build it with the counters and plates. Write the number sentence that goes with the problem. Put the numerals into the boxes on the chart." Give her time to do this.

"What is Susan dividing?" "The cookies."

"What numeral goes into the first box?" "Eight."

"Why?" "Because that's the number of cookies she started with."

"What numeral goes into the second box?" "Two."

"Why?" "Because that's the number of cookies she put on each plate (or group)."

"What numeral goes into the third box?" "Four."

"Why?" "Because that's the number of plates (or groups) she made."

"Remember 'Monkeys on the Roof'? (From Division: Booklet 1) **See the house to the right side of the number sentence box? That is where all the monkeys started out. Can you put the numbers in the correct place?"**

"Where does the dividend, the Start With number, go?" "It goes in the house, under the roof, or under the line." Have her write the eight in the dividend spot.

"Where does the divisor go? This was the number of cages in each unit that

Note

the zookeeper had." "It goes in front." Have her put the two in the divisor location. Draw her attention to the fact that when you compare the numerals in the number sentence to the ones written in the house, the numerals switch places. This is a source of confusion for many students.

"Where does the quotient go? This used to be the number of cages on the roof." "Above the line."
Have her solve the next problem on her own. Give help only if needed.

Practice Worksheets

Number Sentence: Review - Worksheet 2, page 5
Give Division with Cuisenaire Rods, page 6 and a set of Cuisenaire Rods.

Lesson Part 2

"Turn to page 6 now. Take out twenty-four white rods and place them 'in the house.' These are the monkeys in the house." Wait while she does this.

"Now bring the zookeeper's cages; color the divisor light green to show which rod the cage is." Wait while she does this.

"Get a light green rod. Put three monkeys on it. Then slide it up onto the roof." Wait until she completes this.

"How many cages are now on the roof?" "Eight."

"In the problem on the right, where does the eight go?" "Above the line."

"Please write it in now. Do you see how to do the other problems?" She may or may not need the rods to solve these. The groups can be seen in the graphic. Do not insist that she use rods unless she demonstrates some unsureness as to where the numbers go.

Have her complete Division with Cuisenaire Rods, page 6.

Test for Understanding

1. **"What question does the number sentence ten ÷ five = ___ ask?"** "How many groups of fives are there in ten?"

2. **"What numeral tells me what to start with?"** "The first one, ten."

3. **"What does the ÷ five tell me to do?"** "Make groups of five."

4. **"What do you end up with?"** "Two groups of five."

Families of Facts

Purpose

The purpose is to build a physical model of multiplication and division Families of Facts. To demonstrate the importance of a consistent manner in physically representing multiplication and division. Learning Families of Facts facilitates the discovery of using multiplication to check division answers and vice versa. The concept of fact families is important for learning algebra in later studies.

Prerequisites

Monkeys on the Roof from Division: Booklet 1 (also included; see page 18 for instructions.) Student pages are at the end of this booklet. Previous work in addition and subtraction number sentences is helpful.

Materials

Unifix cubes or other stacking manipulative - beans in little bowls work fine.

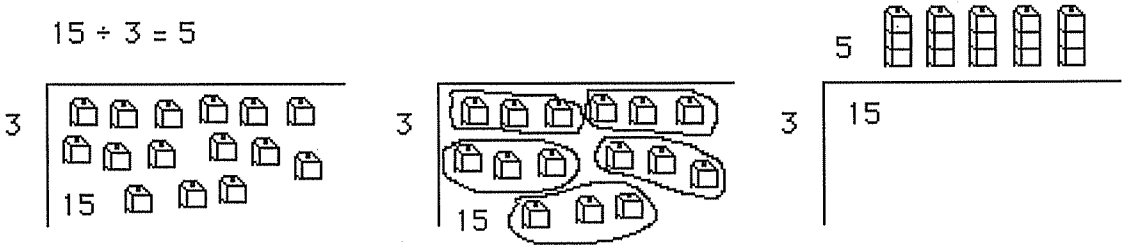
Lesson Part 1 Review

Have the student build the problem 3 x 5. It will look like this: 

"How can you make this into a division problem?" Play around with the materials to discover what the division number sentence is.

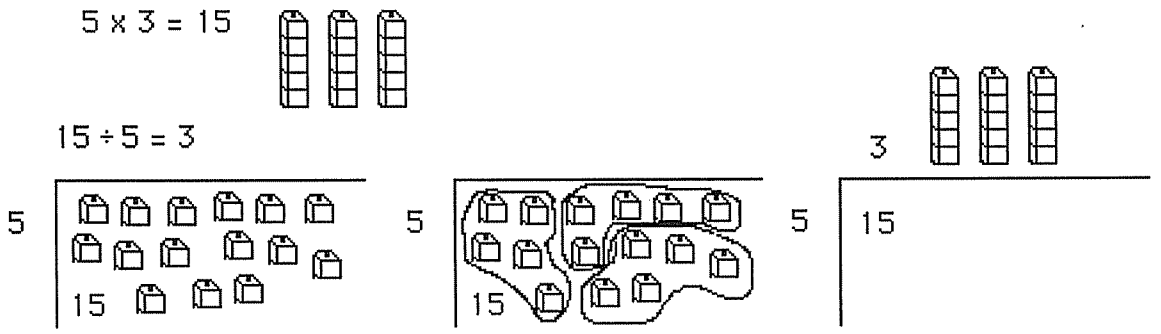
A question to figure out is whether the number sentence is 15 ÷ 3 or 15 ÷ 5. The solution is found through using the manipulatives. The way the model is built all depends upon what the physical model for 3 x 5 looks like. In this math series 3 x 5 is physically represented this way. 

Therefore the division number sentence will look like this: 15 ÷ 3 = 15.



The manipulative sequence is to have the fifteen blocks put into groups of three (which is the divisor). In order for the manipulative sequences throughout the program to be consistent, it is important that the Families of Facts be represented in this manner.

The continuation of the Families of Facts looks like this:



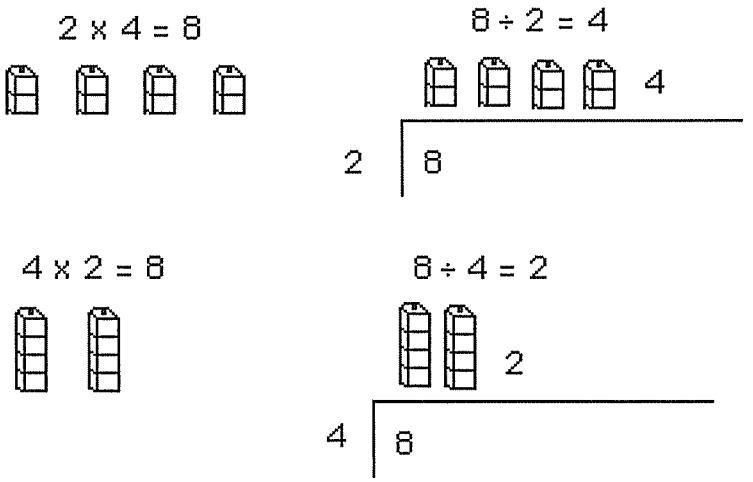
Have the student write out each number sentence and build the model for each. Have her decide which number sentence goes with which physical model and which problems are the converse of each other in the Families of Facts.

$3 \times 5 = 15 \quad 15 \div 3 = 5$
$5 \times 3 = 15 \quad 15 \div 5 = 3$

Test for Understanding

1. Give a problem, such as 2 x 4, and have the student build the models and write the number sentences. Below is a possible problem and its solution.

$2 \times 4 = 8 \quad 8 \div 2 = 4$
$4 \times 2 = 8 \quad 8 \div 4 = 2$



2. Look at the following number sentences. Build a model of each.
$24 \div 4 = 6 \quad 4 \times 6 = 24 \quad 6 \times 4 = 24 \quad 24 \div 6 = 4$
Draw a picture of each model.
Write the number sentence that goes with each model.

3. Use Families of Facts to check your work. Children can discover this important relationship by generalizing from Families of Facts. **"How could you use the Families of Facts to check your answer?"**

4. Give a problem such as 35 - 16 = 14. (This is wrong but don't tell her that.)
"What are the other problems in this fact family?"
Have her check the problem 16 + 14 with counters or any other way she knows.
"What is wrong? Why don't the numbers match?" "Because 16 + 14 is not 35."

Part 2

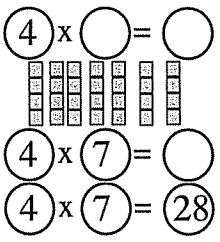
Materials

Families of Facts - Worksheets 1 - 3, pages 7 - 9
Cuisenaire Rods and Unifix cubes

Lesson

Give Families of Facts - Worksheet 1, page 7

Lesson Part 2



"Notice that you are given the numbers 4, 7, and 28. Make a stack of four cubes. This is your Start With number. Write 4 in the first circle of the top multiplication problem. Now build six more of these stacks of four." Give him time to do this.

"Where do you write the seven?" "In the second circle after the times sign."

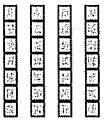
"What do you put in the last circle after the equal sign?" "The twenty-eight."

$$\begin{array}{r} 4 \\ \times 7 \\ \hline \end{array}$$

"The worksheet asks you to write this problem vertically also. How do you do this?" "You write the four on the top and the seven next to the times sign and the answer below the line."

7 x 4 = 28

"What is the other multiplication number sentence in this family? Change your stacks to show the other number sentence." He should build a stack of seven and build four in all.



"Write this number sentence below the first one."
"Break up all your stacks. How many cubes should you have?" "Twenty-eight cubes."

28 ÷ 7 = 4
28 ÷ 4 = 7

"What two division problems can you make in this family with your cubes?" "I can make groups of four and groups of seven."
Have him do that and record the number sentences and write the numbers in the 'house' also.

Note

Many students find it helpful to color code each position in the number sentence and stay with that color scheme. If your student did Families of Facts in *Patterns in Arithmetic*: Book 2, the color pattern used was yellow + blue = green. In that case, translucent colored chips were used to add and subtract the colors. In Division: Booklet 1, a similar color scheme was used. Try purple for the dividend, blue for the divisor and red for the quotient. Have him color in the circles to match the positions of the numbers so he can see the pattern more easily. The colors will show him that the color pattern never changes even when the numbers change.

Practice Worksheets

Families of Facts - Worksheets 2 and 3, pages 8 and 9 Do not assign him to do all three worksheets in one day. Space them out over three days, doing one each day. Be sure he does the 'Make your own' problems.

Test for Understanding

Use the 'Make your own' on Families of Facts - Worksheet 3 as an assessment. The problems below are an assessment of his understanding of the meanings of the positions of the numbers when they are written in the 'house' format.
If he shows uncertainty on the bottom problems, have him build the problems with Unifix cubes or with rods. Help him if needed. This concept will be worked on in Missing Numbers.

Monkeys on the Roof: Review

Purpose

The purpose of this lesson is to develop a physical model of partitioning a division problem using the division symbol (querel): ⌐‾
This is the 'Guzzinda' method.*

Prerequisites

Number Sentence: Review

Materials

Monkeys on the Roof - Worksheets 1 - 5 pages 1 - 5 which are located in the Addendum at the back of this book after page 56.
Unifix cubes
Scissors
Paste

Lesson

Begin with a few division number sentences such as $12 \div 3 = \_\_$. Have the student solve the problem with blocks or prove the answers with blocks. Test for understanding of a division sentence.
"What question does $12 \div 3 = \_\_$ ask?" "How many groups of three are there in twelve."

"Which numeral tells you how many blocks to start with?" "The twelve."

"What does the three tell you?" "The size of the group I am going to make."
"What is the answer?" "Four."

"Four what?" Or, **"What does the four stand for?"** "Four groups of three (in twelve)."

Then take out fifteen blocks and, slowly, make groups of five. While you are doing that, ask, **"What problem am I doing?"** "Fifteen divided by five." Do several problems like this. Then have each student take the teacher's place and the other students identify the problem they are doing.

Worksheets

The worksheets use a story to demonstrate the use of the division symbol: ⌐‾

Monkeys on the Roof - Worksheets 1 - 5

It is essential that you do not skimp on time in this lesson. This concept seems easy and the lesson proceeds smoothly for most students. But an incomplete understanding of what the numbers in a division problem actually mean (number you start with, number of groups, or size of group) leads to major conceptual problems later. The mental picture formed in this lesson will be drawn on again and again when you are working with long or difficult problems. It will be important later, when the students have dropped the physical manipulation of the blocks, to review this lesson now and then. It is very easy for students to write numbers on worksheets and forget what the numbers mean.

*Partitioning (Goes Into or Guzzinda): You have twelve cookies. You want to give each of your friends two cookies. How many friends can you give cookies to? The number sentence is $12 \div 2 = 6$. This is also called 'goes zin da'. As in two 'goes zin da' twelve how many times?

Missing Numbers

Purpose

The purpose of this lesson is to strengthen the concept of the relationships between the dividend, the divisor and the quotient in a division problem. In the problems one of the three components is missing. The problems are given in both number sentence form and the 'house' form. This is a difficult lesson for many students.

Prerequisites

Previous lessons

Materials

Missing Numbers: Worksheets 1 - 4, pages 10 - 13
Unifix cubes
Post-it Notes
Crayons

Warm Up

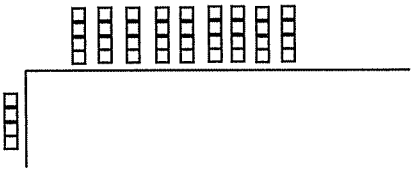
Do a now familiar Monkeys on the Roof problem. **"There are thirty-two baby monkeys in the house (dividend). The cages hold four monkeys (divisor). How many cages are on the roof (quotient)?"** Tell the story and have the student manipulate the cubes. Remind her of the names of the numbers: the dividend, the number of monkeys in the house; the divisor, the zookeeper's cages; and the quotient, the number of cages on the roof.

Now change the story. **"One night a zookeeper was walking home from work. To his astonishment there were cages on the roof of a house! Someone had forgotten to pick up the monkeys! Fortunately there were no poor monkeys hanging onto balloons in the dark sky. He quickly counted the cages. He saw there were nine cages on the roof. Each cage held four monkeys. How many monkeys had been in the house that day?"**

roof
$\square \div 4 = 9$

Write this on a piece of paper or a board. Draw a house around the missing number.
_____ ÷ 4 monkeys per cage = 9 cages.

"If there are nine cages with four monkeys in each cage, how many monkeys were in the house to begin with?" Have her build the nine cages with four cubes in each cage.
If she says, "I don't have to build it. I already know there were thirty-six monkeys in the house."
Ask her, **"How do you know this?"** "Because nine times four is thirty-six." If she can do this, have her fill in the missing number on the paper, write the four number sentences that belong to this group of three numbers, and let her finish the worksheet on her own.
If not, have her build.



Have her fill in the missing number (36 ÷ 4 = 9) and write the Family of Facts that fit with these three numbers. Give another problem of the same type and repeat the

same process.
"What number sentence goes with this problem?" Two possible answers: "4 x 9 = 36" or "36 ÷ 4 = 9."

Lesson Part 1

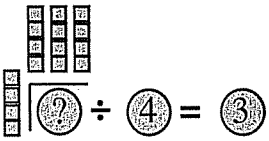
"What are the number sentences that go with this family of facts?"
"4 x 9 = 36, 9 x 4 = 36, 36 ÷ 4 = 9 and 36 ÷ 9 = 4."
"Which number sentence gives you the answer thirty-six?"
"4 x 9 or 9 x 4."
The number sentence that matches this story is ____ ÷ 4 = 9.

Give Missing Numbers - Worksheet 1, page 9. Here's the story: **One night a zookeeper was walking home from work. To his astonishment there were cages on the roof of a house! Someone had forgotten to pick up the monkeys! He saw there were three cages on the roof. Each cage held four monkeys. How many monkeys had been in the house that day?"**

Have her draw a little house around the first circle and color in the circle with the color you used for the dividend. How many monkeys were in the house is the question.

Note

Don't tell her to just multiply the three times the four. It is important that she constructs this idea herself. This concept is a foundation for solving equations in algebra. It is not an easy concept for most students. Continue with this conversation.



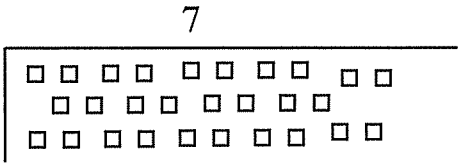
"What size cages are they in?" "Cages with four monkeys in each cage." Make a cage of four monkeys with the Unifix Cubes.
"This is the divisor. Color it in with the divisor color."
"How many cages were there on the roof?" "Three." Build three cages. This is the quotient.
"How many monkeys must have been in the house then?" "Twelve."
"Write the number 12 in the first circle."
"List all the number sentences for this family." "3 x 4 = 12, 4 x 3 = 12, 12 ÷ 3 = 4 and 12 ÷ 4 = 3."
"Which one of these number sentences could you use to get twelve for an answer to solve the problem you had?" "4 x 3 = 12 or 3 x 4 = 12."

Repeat this process for one or two more of these problems and then let her finish the page on her own. Allow her to build with the cubes to solve the problems. Notice if she is using the multiplication number sentence to find the answer. This indicates understanding if she constructed it herself.

Lesson Part 2

Next story:
"The next day the head zookeeper got a call from a zookeeper at a house. This is what she heard. 'I have twenty-eight monkeys in this house. Please send seven cages right away!' Then the caller's cell phone died. What size cages does should the head zookeeper send to the house?"

Draw a house. The caller said there were twenty-eight monkeys in the house. Put the twenty-eight inside the house. The caller said he needed seven cages, so he will have seven cages on the roof. Put a 7 on the roof.



"Wait! He didn't say what size cage. How do we figure this out?"

Wait to see if she can figure this one out. Most students will need to build this. To do this, she needs to realize that she needs to make seven groups. Many students will make groups of seven. That is not what the zookeeper asked for. He asked for seven cages. She will then most likely rearrange the groups into groups of four. This problem is a head twister. Let her struggle with it a bit.

"Which number is missing?" "The size of the cage, the divisor."

"What do you know?" "That there are twenty-eight monkeys and I need seven cages."

Hint

Offer a Post-it Note to act as a place holder for a cage. Let her distribute the cubes onto the Post-its. She will see there will be four on each note. So the cage must be a four monkey cage.

The number sentence that matches this story is $28 \div \_\_\_\_ = 7$.

Missing Numbers: Worksheets 2 - 4, pages 11 - 13. Worksheet 2 mixes up all three types of problems in the same format: $\_\_\_ \div 5 = 3$, $12 \div 4 = \_\_\_$, and $18 \div \_\_\_ = 6$. Worksheets 3 and 4 have the standard division format with a querel. ⌐‾

Test for Understanding

Give these five problems orally or in writing. See if the student can calculate the answers. The problems are easy enough to see the student do them in her head, or at least use the division workboard independently.

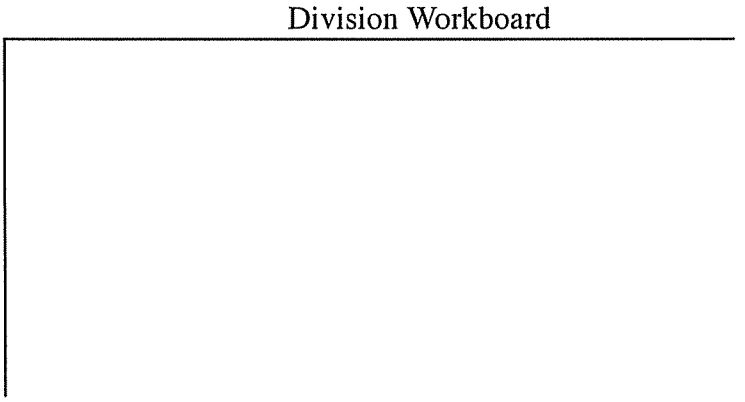
$16 \div \_\_\_\_ = 8$

$\_\_\_\_ \div 5 = 3$

$20 \div \_\_\_\_ = 5$

$9 \div 3 = \_\_\_\_$

$\_\_\_\_ \div 2 = 10$



Breaking Up: Manipulative

Purpose

The purpose of this lesson is to develop a physical model of the Distributive Property as it applies to division.

Prerequisites

Understanding of multiplication and the Distributive Property in multiplication (*Patterns in Arithmetic:* Multiplication - Booklet 2), basic division and division with remainders (*Patterns in Arithmetic:* Division - Booklet 1).

Materials

Breaking Up: Manipulative - Worksheets 1 - 4, pages 14 - 17
Tiles (preferred) or cube blocks
Colored pencils or fine tip markers
Paper

Lesson Session 1

Have the student take out twelve blocks. **"How many groups of three are in twelve?"**
He will make four groups of three.
Then have him take twelve blocks and arrange them into a group of nine and a group of three. It is helpful if the nine blocks are all one color and the three blocks are another color.

■ ■ ■ ■ ■ ■ ■ ■ ■ □ □ □
9 3

"Do you still have twelve blocks?" "Yes."

"How many groups of three were there in twelve?" "Four."

"How have we broken up the twelve?" "Into a nine and a three."

"How many groups of three are there in the nine?" "Three."
"Show it with the blocks."

■ ■ ■ ■ ■ ■ ■ ■ ■ three groups of three

"How many groups of three are there in the three?" "One."

"Show it with the blocks."

□ □ □ one group of three

"Do you still have four groups of three?" "Yes."

■ ■ ■ ■ ■ ■ ■ ■ ■ □ □ □ four groups of three

Now have them arrange the twelve blocks into two groups of six. Use a different color for each group.

■ ■ ■ ■ ■ ■ □ □ □ □ □ □
6 6

"Do you still have twelve blocks?" "Yes."

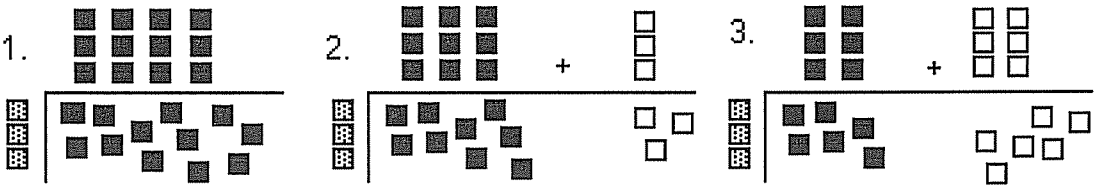
"How many groups of three are in the first group of six blocks?" "Two."

"How many groups of three are in the second group of six blocks?" " Two."

Worksheet

Breaking Up: Manipulatives - Worksheet 1, page 14 This worksheet shows pictures of the problems that were just done. When the student looks at the pictures, repeat the questions you used with the blocks. Have him draw in the answers, the groups of three, as shown below. Have the student use colored pencils or markers

to draw the answers. Try to have the drawings done in the same color as the blocks he used.



"What pattern do you notice?" or **"What pattern do you see in the answers?"**
"That the answers are always the same."

Say in mock protest, **"But how can that be? The twelve was broken up differently each time."** He will probably say something such as, "Because you always had twelve blocks and you always made groups of three."

Now have him show how many groups of four there are in twelve.
He will show three groups of four.
Have him arrange twelve blocks into a group of eight and a group of four. Use different colors for each group.

"Do you still have twelve blocks? "Yes."

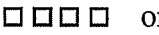
"How many groups of four were there in twelve?" "Three."

"How have we broken up the twelve?" "Into an eight and a four."

"How many groups of four are there in the eight?" " Two." Have him show it with the blocks.

 two groups of four

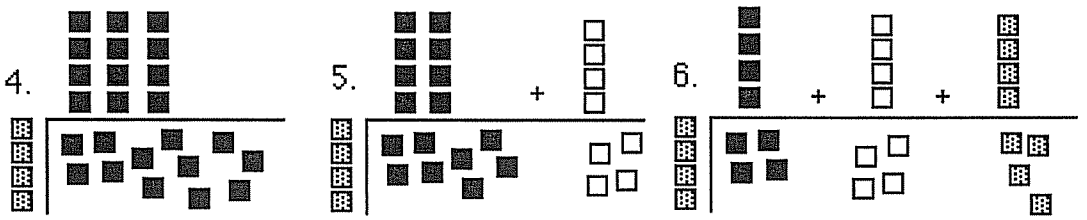
"How many groups of four are there in the four?" "One." Have him show it with the blocks.

 one group of four

"Do you still have three groups of four?" "Yes."

 three groups of four

Have him do problems 4 and 5 on Student Worksheet 1, page 14. Then have him try arranging the blocks like the picture in problem 6. The picture shows the twelve broken up into three groups with four blocks in each group. Then have him do the problem. From each group of blocks, the student will make one group of four. The completed problems are shown below.



"What patterns do you see in the answers?" "The answers are all the same. There are always three groups of four."

Now have him take out eighteen blocks. Have him show how many groups of three there are in eighteen. They will make six groups of three.
Have him arrange eighteen blocks into a group of six, a group of nine, and a group of three. Use different colors for each group.

"Do you still have eighteen blocks?" "Yes."

"How is the eighteen broken up?" "Into six, nine, and three."

"How many groups of three are in the six?" "Two." Have him show it with the blocks.

"How many groups of three are in the nine?" "Three." Have him show it with the blocks.

"How many groups of three are in the three?" "One." Have him show it with the blocks.

"How many groups of three are there altogether?" "Six."

"Does that agree with what you got when you made groups of three with the eighteen blocks the first time?" "Yes."

This time break the eighteen up into a group of twelve and a group of six. Repeat the same questions as above. Then have him do problems 7, 8, 9, and 10 on the worksheet.

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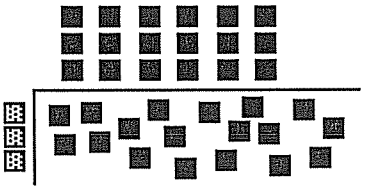

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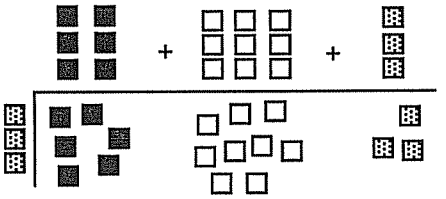

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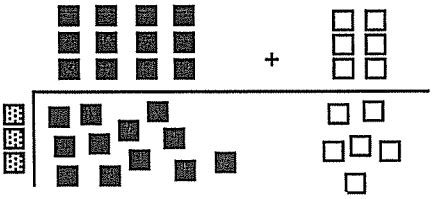

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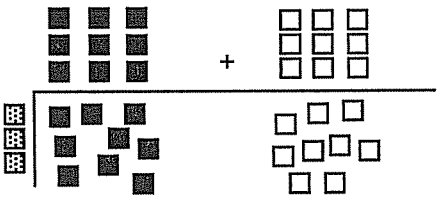


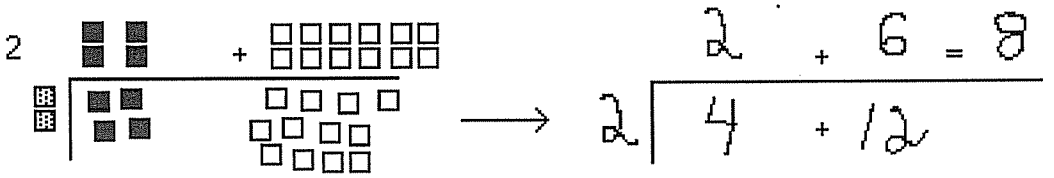
The completed problems will look like this. Call their attention to the answers. Ask him what pattern does he see. The answers are all the same even though the eighteen was broken up differently each time.

Session 2

Give Breaking Up: Manipulatives - Worksheet 2, page 15, and the tiles or blocks. Have him look at the picture in problem 1. Ask him to build that problem with the blocks. He will take out sixteen blocks and make groups of two. He will get eight groups of two. The entire problem is already drawn on the worksheet. Next to the block picture is an arrow pointing to the number representation of the same problem. Have him answer the question, **"What does the eight stand for?"** "Eight *groups* of two."

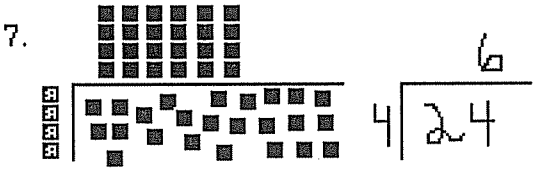
Then have him build and solve problem 2. The problem is $16 \div 2$ arranged in a new way. (*Don't tell him what to do.* See if he can do it alone before you give any assistance.)

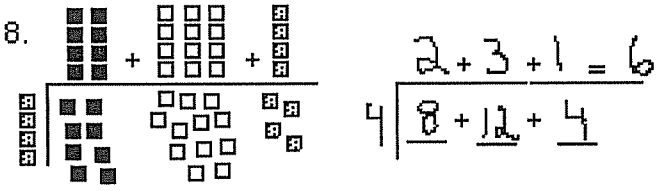
The picture shows the sixteen blocks arranged into a group of four and a group of twelve. He should solve the problem and record the answer in the same way he did in Session 1. Next to the picture is the number representation of the problem. Some of the numbers are put in for him. He should enter the numbers that express the answer. The completed problem will look like this:

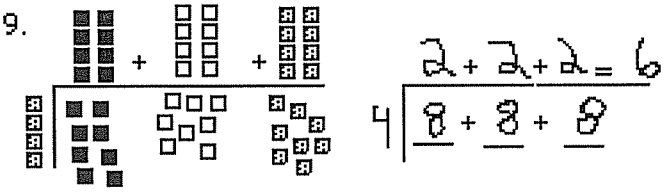


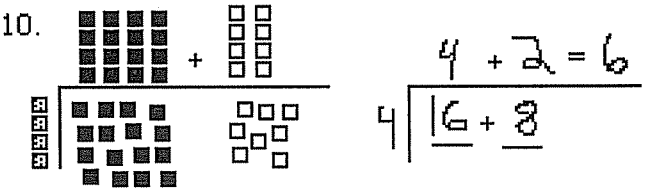
Have him complete the rest of this worksheet and Breaking Up: Manipulative - Worksheet 3, page 16. Have him build each problem, make the groups indicated by the picture of the divisor, and draw the picture of the answer. Then he should show the problem with the numbers. He simply counts the blocks in the pictures and writes the number in the same position as the block pictures appear.

The completed problems will look like this:









Worksheets Give Breaking Up: Manipulative - Worksheet 4, page 17.

Breaking Up: Remainders

Purpose The purpose of this lesson is to investigate what happens if non-multiples are used to break up the dividend.

Prerequisites Breaking Up: Manipulative

Materials
Breaking Up: Manipulative - Worksheet 2, page 15 (for review)
Breaking Up: Remainders - Worksheets 1 - 5, pages 18 - 22
Tiles (preferred) or cube blocks
Colored pencils or fine tip markers

Warm Up Have the student take out sixteen blocks and Breaking Up: Manipulative - Worksheet 2, page 15.

"What happens when you break up the number that you are dividing? Is that an 'OK' thing to do?" "Yes. The way you break up a problem does not change the answer."

Look at problems 2 and 3 on Breaking Up: Manipulative - Worksheet 2.
"Are there any numbers used in the breaking up that two can not go into evenly?"
" No, two goes evenly into four, eight and twelve, the numbers used to break up the problems."

"What would happen if two wouldn't go evenly into a number?" Encourage discussion.

Lesson Session 1 Give Breaking Up: Remainders - Worksheet 1, page 18.

"Look at problem 1 on page 18. Does two go evenly into nine or seven?" "No."

"Let's try using a nine and a seven to break up the sixteen." The answers will vary; most students will say they are unsure what will happen whether or not the answer will still come out to be eight.

Have a student take out nine blocks of one color and seven of another color. **"Make groups of two from the nine blocks."** He will get four groups of two with one block left over.
Then have him make groups of two from the seven blocks. He will get three groups of two with one block left over. The blocks will look like the second picture in the illustration on the next page.

"How many groups of two do you have?" " Seven." (Some will anticipate the next question and say eight.)

"But I thought there were eight groups of two in sixteen. Where is the other group of two?" "In the remainders or blocks left over. The two remaining blocks are put together to make the eighth group of two." Neat, isn't it? See the third picture in the illustration on the next page.

If the student noticed on his own that there were still eight groups of two, say, **"Show me the last group of two. Where does it come from? How do you think you could record this kind of problem?"**

Let him think of ways. He may come up with a perfectly satisfactory way. The way we found worked the best was as follows. On the next page are the pictures and the number recordings of the problem above.

Student does this with the blocks. Then writes this on the paper:

Makes 16 with a group of 9 and a group of 7 blocks.

$2\overline{)16}$

then

$$2\overline{)9 + 7} \qquad + \quad =$$

Makes groups of 2.

then

$$2\overline{)9 + 7} \qquad 4^{R1} + 3^{R1} = 7^{R2}$$

Makes a new group of 2 from the keftover blocks.

then

$$2\overline{)9 + 7} \qquad 4^{R1} + 3^{R1} = 7^{R2} = 8$$

It is important to have the student record the total remainder (7 **r 2**) before making the additional group of two from it. This prevents the confusion of the number of groups made and the remainder. If a student writes 9 as the answer, there is confusion between the quotient and the remainder.

Finish the session with Breaking Up: Manipulative - Worksheet 1. Problem 1 on the worksheet is the same problem done in the lesson. You may want to do several problems with him until you are satisfied he can proceed alone. There are a few problems in which the student breaks up the dividend his own way. If you notice a student taking pains to find numbers to use in the dividend that will not have any remainders, ask him why he is doing that. The student who replies, "Because it's easier that way," has made a major discovery. It is true that you can break up the dividend in any way you choose, but some ways are easier to work with than others.

Give Breaking Up: Remainders - Worksheet 2, page 19 to complete on his own.

The completed problems will look like this:

2. $2\overline{)5 + 11}$ $\quad 2 + 5^{r1} = 7^{r1} = 8^{r2}$

3. $3\overline{)8 + 10}$ $\quad 2^{r2} + 3^{r1} = 5^{r3} = 6$

4. $3\overline{)5 + 13}$ $\quad 1^{r2} + 4^{r1} = 5^{r3} = 6$

6. $4\overline{)9 + 11}$ $\quad 2^{r1} + 2^{r3} = 4^{r4} = 5$

7. $4\overline{)6 + 10 + 4}$ $\quad 1^{r2} + 2^{r2} + 1 = 4^{r4} = 5$

Problems 5, 8, and 9 are created by the student. Expansions will vary.

Some students will need a week or so of practicing this material before going on to Session 2.

Give problems that come out evenly with no remainders in the final quotient. You can have the student give himself the problems if he uses a multiplication facts chart and chooses his dividends from the numbers in the same row as the divisor he plans to use. It doesn't matter if he gets the final answer from the chart, he still has to break up the numbers and solve the problem in several different ways. Knowing what the final answer should be will not hinder the practice at all. Have him use only divisors under six and dividends under fifty. Use the Practice Sheet.

Session 2

This session introduces problems with remainders in the final quotient.
Go over Breaking Up: Remainders - Worksheet 2, page 19, to make sure it is done correctly. Have the student compare his solutions to problem 8. The dividends will not be broken up the same way, but the answers should all match.

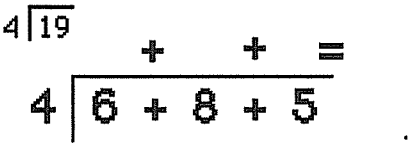
"Why do the answers come out the same even though you broke them up differently?" "Because if you do not change the divisor or the dividend, the answer has to be the same every time. Twenty divided by four is always five no matter how you do the problem."

"What if you do the same problem two ways and the answers don't match, then what?" "You need to do the problems again because you made a mistake somewhere."

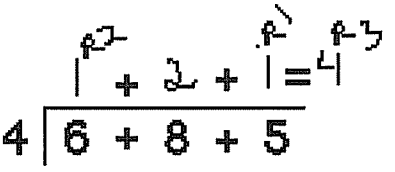
Now give Breaking Up: Remainders - Worksheet 3, page 20. Have him try problem 1, 19 ÷ 4 = ___. Have him make groups of four from the nineteen blocks so he knows what the answer is supposed to be. Then build the problem as it is shown in the picture. Then have him write in the quotients in the *Record* section. In this problem the total remainder will be three, not enough to make another group of four.
See the illustration that follows:

Make 19 this way.

Record



Now arrange the blocks in each set into groups of 4.
The students will do this with the blocks.



Watch quietly to see what he does. When he is finished or if he is having trouble ask, **"What happens if the total remainder is too small to make another group of four?"** "You leave it as a remainder."

"What is the total remainder here?" "Three."

Now try problem 2 on Worksheet 3, page 20. Have him build it as it is shown on the worksheet. Let him play with it. Watch and see what he does. There are two new complications to face in this problem. First is the division of four into three. He may or may not have encountered this in the lesson on remainders. The

solution to this difficulty is to record the answer as 0 groups of four with a remainder of three or 0 r 3.

If he has trouble, ask, **"How many groups of four can you make with three blocks?"** He will almost certainly say, "None."

"How is none written in math language?" "0."

"What then do you do with the three blocks left over?" "Record as a remainder of three."

The second new situation is that the total *remainder* will be seven. This is enough to make one more group of four but not enough for a second group (of four). The solution here is to make one more group of four and leave a remainder of three. This changes the answer from three groups of four with a remainder of seven (3 r 7) to four groups of four with a remainder of three (4 r 3).

It sounds more complicated than it is. A student will see right away what to do with the remainder of seven if he is using blocks. If he doesn't see the solution after several minutes of pondering it or if he leaves the answer as 3 r 7, ask, **"Can you make another group of four from the remainder?"**

See the illustration that follows. Have him record it like this:

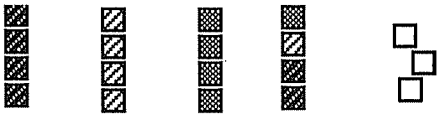
Problem 2
Now break up the 19 this way.

$4\overline{)6+5+5+3}$

Make groups of 4 again.

$4\overline{)6+5+5+3}$ with $1+1+1+0=3$ above the dividend

Putting the remainders together we get...



$4\overline{)6+5+5+3}$ with $1+1+1+0=3=4$ above the dividend

Have him complete Breaking Up: Remainders - Worksheets 4 and 5. Worksheet 5 drops the use of blocks and pictures. If difficulty arises, have the student use the blocks. He will drop the use of the blocks when he is ready. The completed problems on Worksheet 4 will look like this. Problems 1 and 2 are shown above.

4. $3+2+1=6=7$ (r2 r2 r4 r1) $3\overline{)9+8+5}$

6. $1+2+1=4=5$ (r3 r1 r4 r8 r3) $5\overline{)8+11+9}$

7. $2+2+1+0=\ \ =5$ (r3 r3) $5\overline{)10+10+5+3}$

Problems 3, 5, 8, and 9 are created by the student. Expansions will vary.

The student should practice doing these kinds of divisions regularly until he reaches the lesson on Breaking Up Division: Recording. Allow him to use blocks if he needs them. Four problems done three ways each is enough for one week's practice. Use the Breaking Up: Remainders - Practice Sheet on the following page.

Keep the dividends in the practice problems below fifty and the divisors below six. Have the student make up four problems to do. This is important because it teaches the student that there is nothing magic about making up problems. It gives him power over his work. You will need a practice sheet for each two problems. The student should write the problem at the top of the page where it says, "My problem for today is... ."
Have him write a word problem that goes with the problem of the day.

Students enjoy using a calculator to check their own work.

Notes

If you have a large group and time for checking is a problem, have four students make up one problem each. The students in the group can take turns giving the problems of the week. Then everyone is doing the same problems.

Another way to deal with the checking problem in a very large group is to have four students write one problem each Since each problem is to be done three ways, twelve additional students can then break up the dividend for one problem each. These could then be written on a practice sheet and copied for the whole class. This way all the answers will be identical and easily checked. *This method is not the best way* but is better than not doing the practice at all for lack of teacher checking time.

If you use cooperative learning groups, you can have each group make up their four problems, solve them independently, and have the work checked by the group members.

Students are capable of checking each other's work if you teach them how.

Teach them to check by working backwards. *First the checker looks to see if the answer is correct.* If it is, he circles it. (Even if the answer given is correct, he should go through all the checking steps. Often the answer will be correct, but some of the work in the problem may not be. This is especially true for the second and third solving of a problem. Since he knows that the answer is the same as the first problem, there is a tendency to get sloppy on the later problems and just write in the correct answer.) If the answer is incorrect, then he goes through the steps *with the person who made the error* to find where the error occurred.

After checking the final answer, he *checks the addition of the numbers in the quotient, including the remainders,* followed by *checking the addition in the dividend.* Finally, he *checks the divisions.* He lightly circles all the correct work as he checks. He then signs the paper at the bottom. An error on any step should be corrected.

In one class, the students did two problems per week and then checked another person's problems as part of their weekly work. *Sometimes*, they were allowed to use a calculator to do the checking. Accurate checking was rewarded with ample praise.

Breaking Up: Remainders - Practice Sheet

Date _____

Solve each problem three ways. If you get stuck, take out blocks.

Problem 1 My problem today is ___ ⟌ ___

___ + ___ + ___ =

___ + ___ + ___

___ + ___ + ___ =

___ + ___ + ___

Break it up into four numbers this time.

___ + ___ + ___ + ___ =

___ + ___ + ___ + ___

Do ALL the answers match? _____

Problem 2 My problem today is ___ ⟌ ___

___ + ___ + ___ =

___ + ___ + ___

___ + ___ + ___ =

___ + ___ + ___

Break it up into four numbers this time.

___ + ___ + ___ + ___ =

___ + ___ + ___ + ___

Do ALL the answers to problem 2 match? _____

Breaking Up: Recording

Purpose

The purpose of this lesson is to investigate the results of breaking up the dividend into multiples of the divisor.

Prerequisites

Breaking Up: Manipulative and Breaking Up: Remainders, recent practice on the types of problems done in both lessons

Materials

Breaking Up: Recording - Workboard, pages 23 - 26 or a large piece of paper with three or four adjacent circles drawn on it (one for each student)
Unifix cubes (preferred), or any type of counter

Warm Up

"Solve 38 ÷ 4 two different ways." Have counters available. Look for uses of tens and ones or use of multiples of the divisor in the expansions. These would indicate that the student is finding systematic, easier ways to do the problems. If any difficulties are noted, remediate them before proceeding. Review lessons in Breaking Up: Manipulative and Breaking Up: Remainders.

Lesson Session 1

Have the student bring a box with fifty-six counters in it to the lesson.
Write the problem 56 ÷ 4 on a piece of paper or the board. **"What question is this problem asking?"** "How many groups of four are in fifty-six."

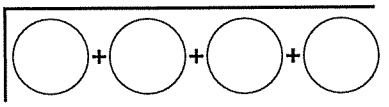
"Guess and record how many groups of four are in fifty-six."

"This is a larger number than you have divided before. How could you find the answer to this problem?" Accept any method she suggests.

She may or may not want to use the counters. Have the papers with the circles on them on the table.
If in a group, have all the students work on the problem individually. Encourage them to share ideas or get help from each other.

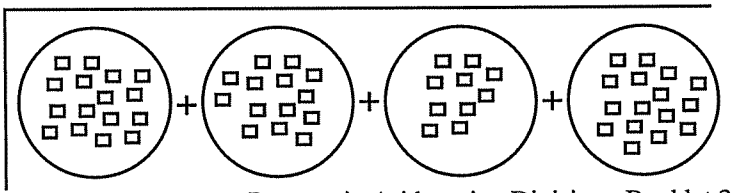
Many students will use the counters by arranging them into groups of four and counting the groups. Some students will grab handfuls of counters and place them on the circle papers like they did in Breaking Up: Manipulative. Other students will break up the fifty-six and do the problem on paper. If they do it on paper, have them build it with the counters to check their work. When they have finished discuss their solutions.

Ask her to do the problem again this way:



Have the student place the fifty-six counters on the table. Give her a piece of paper with three or four large circles on it. Have her place the counters into the circles any way she chooses.

Example:



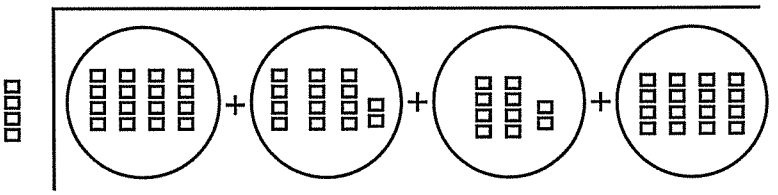
"Count how many counters are in each circle." The teacher records those numbers on a piece of paper.

Example:

$$4 \overline{)\ 16 + 14 + 10 + 16\ } \quad \underline{\quad + \quad + \quad + \quad} = \underline{\quad}^{R} = \underline{\quad}$$

"Arrange the counters in each circle into groups of four." The student counts up the number of groups of four and the number of leftover counters in each circle. The teacher records what the student reports.

Example:



The student counts up the total number of groups of four in all the circles and the total number of remaindered counters. The teacher records:

$$4 \overline{)\ 16 + 14 + 10 + 16\ } \quad \underline{4} + \underline{3}^{R2} + \underline{2}^{R2} + \underline{4} = \underline{\quad}^{R} = \underline{\quad}$$

$$4 \overline{)\ 16 + 14 + 10 + 16\ } \quad \underline{4} + \underline{3}^{R2} + \underline{2}^{R2} + \underline{4} = \underline{13}^{R4} = \underline{\quad}$$

Finally, any remaindered counters will be grouped into sets of four. The new total number of fours are recorded by the teacher.

$$4 \overline{)\ 16 + 14 + 10 + 16\ } \quad \underline{4} + \underline{3}^{R2} + \underline{2}^{R2} + \underline{4} = \underline{13}^{R4} = \underline{14}$$

Repeat the same problem three or four times using the same procedure but with different arrangements of the counters in the circles.

The third time around ask the student if she thinks she could record the problem. If in a group, have one student manipulate and another record. A few students will not be ready to record yet. That is fine. Continue to record for them until they feel comfortable doing it themselves.

"Explain why the answers to the different problems are the same." "Because you always had fifty-six blocks and they were grouped into fours every time. How you break them up does not change the answer."

"Solve 68 ÷ 4 three different ways. Record how you solved each way." You may want to have her do this as independent work or continue to work with you at the table. What choice you make here will depend on the level of understanding. Save this

paper to bring to Session 2.

The student can drop the use of the counters if she wishes to. Have her use the counters to check one of the problems. If a student is having difficulty, work with her until she feels comfortable working alone. If the difficulty is major, change to smaller dividends.

Repeat the session using these problems: $65 \div 5$ and $80 \div 5$, each done three ways. Repeat a third time using $78 \div 6$ and $84 \div 6$.

Session 2

Materials

Breaking Up: Recording Workboard
The paper she used to record the solutions to $68 \div 4$.
Sixty-eight counters for the lesson
Scratch paper

Check over the division problems done independently. If in a group, have the students share the different ways they found the answers.

Using the counters, ask the student to build a number that can be divided into groups of four with no remainders (left overs). If she builds a number greater than four, have her demonstrate that the number can be divided into groups of four.

"On the same paper you used to solve $68 \div 4$ (in the previous session), list the ten numbers between one and forty that can be arranged into groups of four with no remainders." Some will think of using the four times table, some will not. Allow her time to find the numbers with the counters. Have her watch for patterns that will allow her to predict the numbers.
Answer. 4, 8, 12, 16, 20, 24, 28, 32, 36 and 40.

"Do you recognize this group of numbers?" "It's the four times table."

If more than thirtybminutes are taken with this list making, stop here and continue the next day.

"Solve $68 \div 4$ one more time but this time with the constraint that when breaking up the sixty-eight there are *no* remainders anywhere in the problem." In order to satisfy this constraint, the student must use multiples of four to break up the sixty-eight.

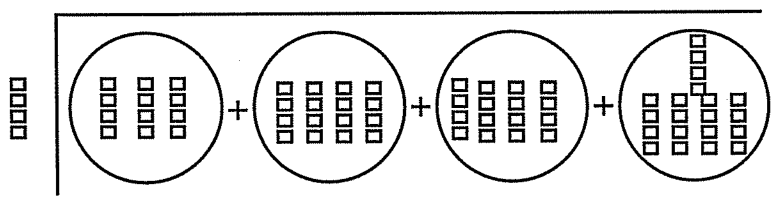
Do not tell her to use the list of numbers on the paper. See if she figures that out alone. Give her time to fiddle around and try a few numbers to see what happens. Do not intervene unless the student asks for help or is becoming very frustrated.

If she has trouble, tell her to use the counters and the circle paper. Have her place the sixty-eight counters in the circles so that the counters in each circle can be grouped into even sets of four with no remainders. Have her use the list. When this

is done, she can arrange the counters in each circle into groups of four and count up the total. Then have her try to record the work if she feels she can. If she does not feel she can, you record what she manipulates as was done in the first session of this lesson.
Some students will be able to solve the problems without any counters at all. If you are working with a group, pair up students who are having difficulty with ones who can work symbolically. Have this second student record what the first student does.

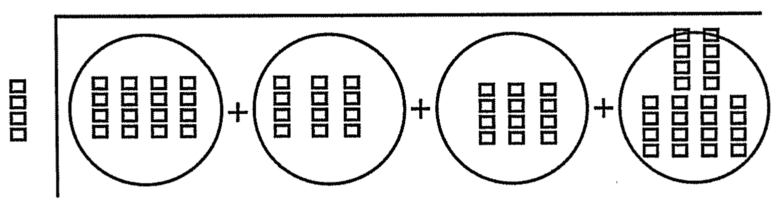
Give a new problem. Have the student solve 64 ÷ 4 two different ways using the new rule. Have her compare her answers to the answers she got for the same problem in the first session.

Below are three examples of how the problems might be done.



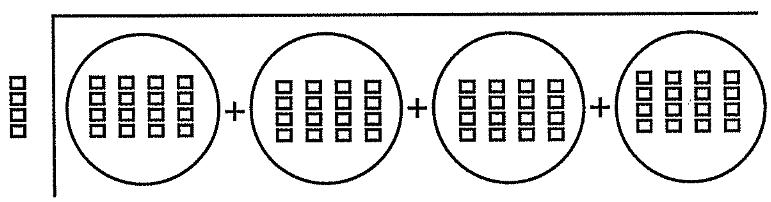
3 + 4 + 4 + 5 = **16**

4 | 12 + 16 + 16 + 20



4 + 3 + 3 + 6 = **16**

4 | 16 + 12 + 12 + 24



4 + 4 + 4 + 4 = **16**

4 | 16 + 16 + 16 + 16

Repeat this session a second time using groupings of five. Have her list the ten numbers between one and fifty that can be evenly divided by five. Give these problems: 65 ÷ 5 and 80 ÷ 5.

Repeat a third time using groupings of six. List the numbers from one to sixty that are evenly divisible by six. Give 78 ÷ 6 and 84 ÷ 6.

Session 3

Worksheet

Give the student Breaking Up: Recording - Worksheet 1, page 27.
Do Worksheet 1 together. If she has difficulty, continue working together. If no problems arise, she can complete Breaking Up: Recording - Worksheet 2 - 3, pages 28 and 29, independently.

Test for Understanding

Breaking Up: Test for Understanding - Worksheets 1 and 2, pages 31 and 32

Breaking Up: Test for Understanding - Worksheet 1. You can do the test questions on the first page without the worksheet if you wish. This works well in a tutorial setting. If you are working with a large group, use the worksheets. Students may need clarification of the instructions.

In problem 1, the student is asked to identify how the twelve is broken up. In problem a, it is broken up into 6 + 6, in problem b, 9 + 3 and in problem c, 3 + 6 + 3.

In problem 2, the student is asked to identify the number of groups of three there are in each of the sections the twelve was broken up into. In problem d, both sections have two groups of three. In problem e, the first section has three groups of three and the second section has one group of three. In problem f, the first section has one group of three, the second two groups of three, and the third section one group of three.

In problem 3, the student is asked to give the total number of groups of three there were for each of the three problems in part 2. All three problems have four groups of three. The student must notice this and explain why it makes sense that all three should have the same answer.

The representation used here, one of the twelve blocks in a rectangular array that is simply separated in different ways, makes it easier for the student to see why the answer is always the same. There are always four groups of three because there are always twelve blocks. The fact that you change the arrangement of the blocks does not change the number of groups of three it has in it. It is important that the student clearly understands this. On this understanding will be based his freedom to tinker with the arrangements of the dividends. Later this tinkering will help him find ways to make the division of large numbers easier.

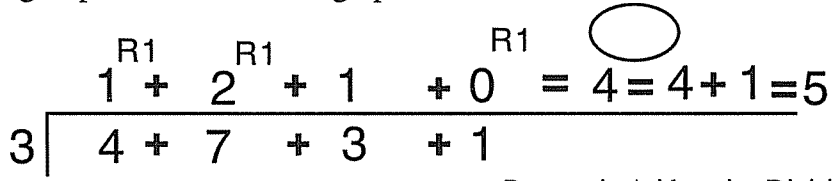
Breaking Up: Test for Understanding - Worksheet 2 tests the student's understanding of the procedure of division by breaking up. It asks him to spot errors in problems and explain what is wrong with the problem.

In problem 4, the expanded dividend (the broken up dividend) should have totaled to twenty-four, not twenty-seven. The error is in the original expansion of the twenty-four.

In problem 5, the error is in the division of the four into the twelve. The small quotient should be three, not four. The final quotient should be five, not six.

In problem 6, there is an error in the calculation of the remainder in the second number. Eight divided by three is 2 r 2, not 2 r 1. The correct answer is 6, not 5 r 2.

In problem 7, the remainder of three was added to the four (four groups of three) to give an answer of seven groups of three. The answer should be five groups of three. The remainder of three should be made into one additional group of three. That one group of three is added to the four groups of three already recorded. The correct answer is to add four and the one to get the correct answer of five. The five refers to five groups of three. See the graphic below.



Breaking Up with Cuisenaire Rods

Purpose The purpose of this lesson is to strengthen the understanding of the relationship between multiplication and division. The lesson uses the same format as the lessons in Multiplication: Booklet 2 substituting the break up of the number to be multiplied, shown on the top row of the array, as the quotient. It also substitutes the products of the little multiplication problems with the dividend. If the student did not do Multiplication: Booklet 2, you may want to skip this lesson.

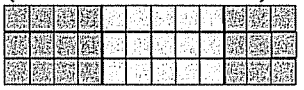
Prerequisites Previous lessons and Multiplication: Booklet 2

Materials Breaking Up with Cuisenaire Rods, Worksheets 1 - 4, pages 37 - 40, and Show You Know, page 41
Cuisenaire Rods
Crayons and one index card

Warm Up If you still have Multiplication: Booklet 2, take it out and review the early lessons on breaking up using Arrays. The problems will look like the one at the top of Breaking Up with Cuisenaire Rods - Worksheet 1. Also, allow her to freely explore with the rods before beginning the lesson. Review the value of each color by making a stair of increasing lengths from one centimeter to ten centimeters.

Lesson Begin with Breaking Up with Cuisenaire Rods - Worksheet 1. Review the Distributive Property of multiplication as shown in the top graphic.
Cover the entire graphic with the index card leaving only the top row of numbers showing. Review the meaning of the top line. It is an equality. The left side of the equation says 12 x 3 =. The right side of the equation says = (4 + 5 + 3) x 3.
"Is 12 x 3 equal to (4 + 5 + 3) x 3? How can you tell without calculating the answers to both sides?" "Yes, they are equal. You can tell because (4 + 5 + 3) is the same as twelve. It is just broken up, so both sides say 12 x 3."
"What is the same on both sides of the equal sign?" "The x 3."
Place an index card over the lower two rows of the array, the large grid. Place the rods that match each section onto the paper. The first one is purple, the second yellow, and the third light green. Have her move the rods upwards and color each section to match the color of the rods.
"Is the rod train of purple, yellow, and light green equal to twelve centimeters?" "Yes."
"So we have the twelve broken up into 4 + 5 + 3. How many times are we adding twelve? Or, how many rows of twelve do you see?" "Three."
Color each section to match the color of the rod at the top.

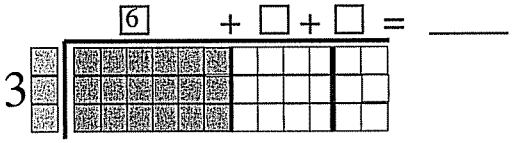
12 x 3 = (4 + 5 + 3) x 3



12 x 3 = (4 x 3) + (5 x 3) + (3 x 3)
12 x 3 = 12 + 15 + 9
12 x 3 = 36

"In the second row of numbers under the picture, it says: 12 x 3 = (4 x 3) + (5 x 3) + (3 x 3). Where is the 4 x 3 in the picture?" "The three purple rods is 4 x 3." Repeat for each color.
"How do you get the answer?" "You add all the answers of the little multiplication problems. You add 12 + 15 + 9 to get 36."
Read the paragraph in the middle of the page.
"When we do division, we start with the answers to the little multiplication problems. Color in the section with the twelve purple. Color in the section with the fifteen yellow, and the one with the nine light green. Notice that it looks just like the multiplication problem above."

"What we are looking for in a division problem is what were the numbers in the very top line. We are starting with the thirty-six squares in columns of three. What we are looking for is the twelve. We want to find out what was in that top line." Point to the numbers in the boxes, the 4, the 5, and the 3. **"These numbers are what we are looking for. We are just working backwards from the problem we did at the top."**
In the problem at the bottom, have her count over six squares and draw a line all the way down.
"In big numbers, write the number of how many squares are in this little array?" "Eighteen." Have her write 18.
"What color rod fits on this part of the array?" "Dark green."
"Color this first array dark green." Wait for her to do this. **"What number goes in the box above the line?** "Six."



"Count over four squares now and draw another line. What color rod fits here?" "The purple." Wait for her to color it in.
"What number goes in the top box?" "Four."
"What color rod fits in the last spot?" "The red." Wait for her to color it in.
"What number goes in the top box?" "Two."
"If you add all the numbers in the top boxes, what do you get?" "Twelve."
"This twelve is what we are looking for when we divide. It is the answer. Where is this same twelve in the multiplication problem at the top of the page?" "It is the top line. It is the way we broke up the twelve."
Go to Breaking Up with Cuisenaire Rods - Worksheet 2, page 38, and repeat this entire process again with a different problem. Use the same questions as were used above.

Practice Worksheets

Stop at this point and continue Breaking Up with Cuisenaire Rods - Worksheets 3 and 4, pages 39 and 40, the following day. On Breaking Up with Cuisenaire Rods - Worksheet 4, the student must break up all the problems alone. Require her to color the problems in. The first division problem in the middle of the page must match the multiplication problem at the top.

Test for Understanding

Use Show You Know on page 41. She should be able to do these problems independently.

Averages and Breaking Up: Practice

Purpose

The purpose of this lesson is to use the division procedure students have just learned to compute averages. Averaging is usually taught as a procedure that tells the student to add all the numbers in the set and divide by the number of sets. This allows for the correct answers but not for understanding exactly what an average is. An average is making all numbers in a set equal, making high places low and low places high. By using cubes arranged in stacks, the student can grasp the idea of making all the stacks equal by moving cubes from the high stacks and adding them to the low stacks to achieve equal sized stacks. When this idea is set, then a shortcut can be developed that takes the total number of cubes in the problem and divides them equally between the stacks. This is what an average is.

This lesson also contains in the practice section notes on what to look for as a student practices breaking up dividends to do division problems. What strategies are being used? Have any shortcuts appeared?

Prerequisites

Previous lessons

Materials

Breaking Up: Practice, Worksheets 1 - 4, pages 46 - 49
Averages: Worksheets, 1 - 4, pages 42 - 45
Show You Know, page 50
Multi step Word Problems, page 51
Unifix cubes work best, but any type of cube or tile will work.

Warm Up

Allow free exploration with the Unifix cubes before doing the lesson. If you are working with a group of students, make up a problem like this. Use a number that will come out evenly. If you have fifteen students, for example, use a multiple of fifteen for the total number of cubes in the problem, for example, forty-five.

"There once was a group of children at a birthday party. There was a bowl of candy. Some children reached in a grabbed whole handfuls before other children even knew there was candy. Some children had lots of candy and some had no candy." Pass out the cubes in such a way as to be an obviously unfair distribution. **"How can you make it fair for everyone?"** The students figure out to move the cubes from one person to the next until everyone has the same amount.

Repeat the problem with a different number of cubes. Again have students move the cubes from one person to another until everyone has an equal number. The children who had lots will say this is mean. Yes, this is called a mean average, because you are taking some from the big stacks and sharing them equally with all the children.
"Can anyone think of a faster way of giving everyone an equal share?"
Often someone in the group will hit on the idea of putting all the cubes together in a pile and dividing them up equally. Thus, the procedure of averaging can be arrived at naturally.

Lesson Part 1

Follow the lesson as written on Averages - Worksheets 1 and 2, pages 42 and 43. On Averages - Worksheet 2, the student is asked to identify how many baskets were made in all, and how many days were used to throw these baskets. The pattern of finding an average is visible in both problems presented on this page.

Note

Do not tell her what the pattern is. Wait until Averages - Worksheet 3 is completed before nudging her to the procedure. This is a good stopping place.

Practice Worksheets

Give Breaking Up: Practice - Worksheets 1 and 2, pages 46 and 47, for independent work.

Pay attention to the strategies she is using to do these problems.
Is she breaking the dividend up into tens and ones and struggling to find the quotients with the remainders? If so, use a constraint that requires her to have no remainders in any of the quotients except the last one. If she has difficulty, suggest looking for useful numbers to use in the Flip Book.
Is she using multiples of the divisor to break up the dividend? This is a good strategy.
Is she selecting quotients first, like ten, and then multiplying by the divisor to find out what that part of the dividend should be? This is a good shortcut.

Lesson Part 2

Make a chart of the data from Averages - Worksheet 2, page 43. Ask if the student sees any patterns in the numbers that would allow her to calculate the average. If not, continue to Averages - Worksheet 3, page 44, adding in the new data to the chart.

	Number of Baskets in All	Number of Days	Average
Problem 1	40	5	8
Problem 2	50	5	10
Problem 3			
Problem 4			

After Averages - Worksheet 3 is completed, have her write out the procedure for taking an average. If she can't at this point, make up more problems and add the data to the chart.
Try problems such as: On one plate there were ten cookies; on the second plate there was only one. On the third plate there were four cookies. This looks stupid. All the plates should have the same number of cookies. How can you figure out how many cookies should have been on each plate? Add the data to the chart. What is the average number of cookies on each plate?
Keep doing problems like this until she realizes that the easiest way to do the problem is to find the total and divide it up equally between all sets.
Averages - Worksheet 4 is a practical use of averaging.

Practice Worksheets

Breaking Up: Practice - Worksheets 3 and 4, pages 48 and 49
If you have not done so already, put the constraint on her that when doing these practice sheets, there can be no remainders in the quotient until the very last step. This will require her to choose multiples of the divisor. Allow her to use her Ex-

panded Tables Flip Book.

Note

At this point, resist telling her to choose the largest possible multiple of the divisor. Allow her time to discover this shortcut on her own. In the next lesson, a constraint will be put on her solutions that require this choice. But not yet.

Test for Understanding

Give the Show You Know assessment on page 50. See the Answer Key.

Give this problem orally. See if she can calculate the average.
Sandy took a series of tests on her multiplication facts. She had thirty seconds to complete each test. Each test had ten problems on it. Here are the numbers of problems she did correctly on each test. What is her average number correct on these tests?

Test Number	1	2	3	4	5	6	7	8	9
Correct Answers	8	7	10	6	7	9	9	10	6

The total is seventy-two correct over nine tests for an average of eight problems answered correctly.

Extension

Multistep Word Problems, page 51. Many students need assistance with this page

The grocery store is a great place to practice averaging. Allow a student to use a calculator because of decimals. What is the average number of apples in a pound of apples? Try different varieties of apples.
What is the average number of bananas in a bunch of bananas?
What is the average cost of one pound of margarine? The possibilities are endless.

Discovering Easier Ways

Purpose and Background Information

The purpose is to have students identify strategies that make breaking up divisors and finding quotients easier. One prime strategy to make division easier is to use multiples of the divisor in the breaking up of the dividend. This eliminates remainders, makes it easier to use multiplication facts, and eventually allows answers to come out in tens and ones for easier addition at the end. The short division procedure we all memorized involves choosing the largest possible multiple of the divisor at each step and recording the quotient in standard place value notation of hundreds, tens. and ones. Example: Consider the problem $147 \div 3$.

$$\begin{array}{r} 49 \\ 3\overline{)147} \\ \underline{12} \\ 27 \end{array} \qquad \begin{array}{r} 40 + 9 \\ 3\overline{)120 + 27} \end{array}$$

When you think, "How many times does three go into fourteen?" you settle on four because the closest multiple of three to fourteen is twelve. The 'twelve' you place under the 'fourteen' in this problem is actually one hundred twenty. You chose the twelve because it is the largest number in the three times table (multiple of the divisor) you could choose. When using the short procedure, the largest multiple of the divisor must be chosen to avoid having to put another ten in the answer.

Example: If nine was chosen instead of twelve, there will be fifty-seven left. There is no way to divide that without messing up the writing of the answer, or by erasing the three and adding the one ten from the nineteen to get that four in the tens place of the answer. In other words, it is a mess, called a mistake. There is no reason you can not do this in the more general long way as shown below, but it will not work when working the short notation. You can get the correct answer, but it will not come out in tens, and ones, which is needed for the short procedure to work.

$$\begin{array}{r} 3\ 19 \\ 3\overline{)147} \\ \underline{9} \\ 57 \end{array} \qquad \begin{array}{r} 30 + 19 = 49 \\ 3\overline{)90 + 57} \end{array}$$

Looking back up at the top example, twelve is chosen because it allows the placement of the four into the tens place. Then subtract to get the two (which is really a twenty) and bring down the seven to make the twenty-seven. Then divide the twenty-seven by three to get nine. The nine is placed in the ones place. The answer reads forty-seven. What actually was done is you chose to break up the one hundred forty-nine into 120 + 27, you just did not know that is what you were doing. Most of us thought that was the only way to do it. As you know now, this is not the only way to do this problem. It is also not the way that is obvious to most children.

In this lesson, Discovering Easier Ways, the constraints are put on how the student can break up the dividend and how the quotient can come out. This will move the

student towards being able to do division the short way and to understand what she is doing. She will understand, though, that this special way is ***a*** way, not ***the*** way to divide. It is a shortcut based on the Distributive Property. It is a lovely shortcut, though. So lovely in fact that just about everyone in the world uses it! That is why it is called the Standard Procedure.

Prerequisites Previous lessons

Materials Discovering Easier Ways: Worksheets 1 - 11, pages 52 - 62
Expanded Tables Flip Book

Warm Up Families of Facts as taught on page 7 of the Student Workbook. Present several problems of that type. Give the problem 51 ÷ 3. Allow her to solve the problem any way she wishes.

Lesson Part 1 Discovering Easier Ways - Worksheet 1, page 52
In this part of the lesson, we want her to get used to the idea that we can choose the break ups of the divisors based on how we want the quotients to come out. This is a new idea for most students. Up until now, most students will have been choosing break ups based on a focus on the dividend, not the quotient. In problem three, there are lots of tens in the answer because adding tens is easy. So how would you break up the divisor if you wanted tens in the quotient?

The first problem given is _____ ÷ 3 = 17
"How do you know what the Start With number, or the dividend, is for this problem?" "It is fifty-one because I know that 3 x 17 is 51. Have her write the 51 in the house.
"Can you figure out by looking at the quotients in the problem on the right how the fifty-one was broken up?"

Wait. See if she can do this on her own.

If not, ask, **"The first quotient is five. What number divided by three will give five as an answer?"** "Fifteen." Have her write the fifteen under the five. Proceed this way over the rest of the problem. She should write the numbers 15 + 12 + 24 in the house. Study the answer key to see how the completed problem should look.
"Does 15 + 12 + 24 add up to fifty-one?" "Yes."
Finish the rest of the problems on Discovering Easier Ways - Worksheet 1 in this same fashion.

Worksheet Turn to Discovering Easier Ways - Worksheet 2, page 53. Point out that there are five different solutions to the problem 64 ÷ 4 being presented. Have her work all the problems on her own.
Have a discussion about which problems were easier to do and why. Common discussion points from students follow.

Most students will say that Amber's way was the easiest because four into twenty

is very easy and the repetition of the twenties makes it even easier. It is also easy to add the fives in the quotient.

Danielle's way is generally rated as pretty easy also because forty and twenty-four are easy multiples to work with and the answer comes out 10 + 6, which is easy to add.

Robert's way is a bit harder unless you happen to already know how many fours are in sixty.

John's way looks easy at first because of all those tens, but dividing tens by four creates a bunch of remainders that are a real hassle to deal with.

The same can be said of **Sue's way**. Sue probably wanted to use 60 + 4 in her break up but did not know how many fours were in sixty, so she broke the sixty into two thirties. But this also creates many remainders.

"How can you avoid remainders if you are dividing by four?" "Choose numbers in the four times table."
This is what we mean when we say it is easier to divide if you choose multiples of the divisor you are using.
Be sure she writes clear explanations of what makes a problem easier or harder to do. Be sure she understands what it means to 'choose a multiple of the divisor.'

Practice Worksheets

For Part 1: Discovering Easier Ways - Worksheets 3 and 4, pages 54 and 55

Test for Understanding

For Part 1: Examine the work she does on Discovering Easier Ways - Worksheet 4. Did she use multiples of the divisor in her break ups? For example, three, six, twelve, thirty, sixty.
Did she use any break ups that would give the answer in tens and ones? 60 + 12
Can she explain why it is generally not a good strategy to break the dividend into tens and ones as was done in Roy's way?

Note

Some students will persist in breaking up the divisor into tens and ones. Tell her that this works really well for addition, subtraction, and multiplication, but not for division because of those nasty remainders.

Warm Up

Review the work done on Discovering Easier Ways - Worksheets 2 and 3. Discuss what makes some ways of doing the problems easier than others.

Lesson Part 2

Discovering Easier Ways - Worksheet 5, page 56

Read the top of the worksheet together and discuss it. Neither example uses a tens and ones expansion in the divisor. This hardly ever works because often the numbers in the tens and ones expansion are not multiples of the divisor and produce many messy remainders when divided.

Have her look at her work on the previous page and see if any of her problems had quotients that came out in tens and ones before she did the final addition.
In the problems on this worksheet, there is the figure __0 + ___ in the space where the quotient is written. We are putting a constraint on how she can do the problems to require that the quotient is expressed in tens and ones only.

Under this constraint, the first number in the quotient needs to be a ten, twenty, or thirty, etc. The first quotient in the first three problems on this worksheet are all ten. If the forty-five is broken up into 30 + 15, the answer will come out 10 + 3. This is the goal.

Note

Recall this constraint focuses on ***a*** way to do the division problem, not ***the*** way.
The dialogue may sound something like this for 52 ÷ 4.
"What numbers could you choose for the first quotient?" "Ten, twenty, or thirty. It has to have a zero at the end."

"Choose one and write it on the __0 spot." "Twenty."

"So what number divided by four would give you twenty?" "Eighty."

"Do you have eighty in your dividend?" "No, I only have fifty-two. So eighty is too large. I will try ten this time."

"So what number divided by four would give you ten?" "Forty. So the first number in the house is forty."

"What is the second number then?" "Fifty-two take away forty is twelve, so the second number must be twelve."

"So what is the answer?" "Thirteen, ten plus three."

Finish the worksheet and go on to Discovering Easier Ways - Worksheet 6, page 57.
You should do the problem given at the top of the page using your standard short procedure. ***Do not explain it to her at this time.***

At the end of this worksheet, tell her that in order to do division the fast way, the answer has to come out in a specific way. The dividend must be broken up using multiples of the divisor, and the quotient must come out in tens and ones.

Practice Worksheets

Discovering Easier Ways, Worksheets 7 - 11, pages 58 - 62

Do the first problem together on Discovering Easier Ways - Worksheet 7. The problem is done two ways. The first problem is the old way where the dividend is broken up using multiples of the divisor only. Notice there are no remainders. Most students will find this way easier than trying to get the answer to come out in tens and ones. So we work backwards from the desired form of the answer to

the way the dividend had to be broken up to get it. The insight she needs is that to get the answer to come out in tens and ones, she must choose the largest round multiple of the divisor. The round multiples are listed in the Expanded Tables Flip Book. A round multiple is 3 x 4 = 12, 30 x 4 = 120, 300 x 4 = 1,200. These are called Expanded Tables.

Note

Do not tell her to use the largest round multiple on Discovering Easier Ways - Worksheet 7, page 58. As usual, give her the opportunity to figure this out on her own. On Discovering Easier Ways - Worksheet 8, suggest using the Expanded Tables Flip Book to help her.

Discovering Easier Ways - Worksheet 8, page 59
There is space to do the problems the old way. Once she knows what the answer is, break the answer into tens and ones, write it in on the problem to the right and work backwards to see how the dividend should have been broken up to obtain the answer in tens and ones. This is a difficult step for most students. At this point, most don't see the point. The Expanded Tables Flip Book will help. Choose when to help her by telling her to find the largest number in her expanded table list that is less than the dividend. Many students need this hint. A few students will figure it out on their own.

Discovering Easier Ways - Worksheet 9, page 60
Give explicit guidance on the use of the Expanded Tables. Allow her to use her Expanded Tables Flip Book.

Discovering Easier Ways - Worksheets 10 and 11, pages 61 and 62. are self explanatory.

Test for Understanding

Use the second column on Discovering Easier Ways - Worksheet 11, page 62, as an assessment. Many students will still need some assistance at this point. The next section works for mastery of this concept of choosing multiples that will give the answer in tens and ones.

Breaking Up: Tens and Ones - Fours, Fives, Sixes, and Sevens

Purpose

The purpose is to develop the connection between the expanded tables and the strategy of having the quotient come out in tens and ones. This strategy allows the use of the shortcut commonly used as the standard procedure generally used for division. The student needs to realize, first that it is necessary to use only multiples of the divisor in breaking up the dividend, and, second, that those multiples must be the largest possible multiples in order to have the quotient come out in tens and ones.

Prerequisites

Previous lessons.

Materials

Breaking Up: Tens and Ones - Fours, Worksheets 1 - 3, pages 63 - 65
Breaking Up: Tens and Ones - Fives, Worksheets 1 - 3, pages 66 - 69
Breaking Up: Tens and Ones - Sixes, Worksheets 1 - 5, pages 70 - 73
Breaking Up: Tens and Ones - Sevens, Worksheets 1 - 4, pages 74 - 77
Expanded Tables Flip Book – filled in for both multiplication and division
Base Ten Blocks available if the student is not fluent with expanded tables.

Warm Up

If the Expanded Tables Flip Book has not been filled out for the four times table, do this now. Otherwise, have him complete the top of Breaking Up: Tens and Ones - Fours, Worksheet 1 and the top section of Breaking Up: Tens and Ones - Worksheet 2.

If he has never worked with expanded tables before, have him build the table with the Base Ten Blocks. This is what the expanded table looks like:

1 x 4 = 4	10 x 4 = 40	100 x 4 = 400
2 x 4 = 8	20 x 4 = 80	200 x 4 = 800
3 x 4 = 12	30 x 4 = 120	300 x 4 = 1,200
4 x 4 = 16	40 x 4 = 160	400 x 4 = 1,600 to 9 x 4, etc.

Use concept of Families of Facts to help the student connect the expanded multiplication table 3 x 4 = 12, 30 x 4 = 120 to the division facts of 12 ÷ 4 = 3 and 120 ÷ 4 = 30. Have him fill out both the multiplication and the division sections of his Flip Book for fours before you begin the division part of the lesson.

Lesson

Review what he learned in Discovering Easier Ways, where he found out that breaking up the dividend into multiples of the divisor makes the division easier. Review what he learned about making the quotient come out in tens and ones.

Note

Do not tell him at this point the key is to find the multiple that is closest to the number in the dividend. As usual, allow him to discover this. There is help later if he does not figure it out on his own.

Have him complete the bottom section of Breaking Up: Tens and Ones - Fours, Worksheet 1. Allow him to use the Expanded Tables Flip Book or the chart at the top of Breaking Up: Tens and Ones - Worksheet 2. On Worksheet 2, the first problem is 44 ÷ 4. He already knows the answer is eleven. On the right hand side of the

page is the tens and ones division template. Have him put 10 + 1 on the quotient line.
"What number has to be in the dividend below the ten?" "Forty."

"How do you know this?" "Because four times ten is forty."

"What number has to be in the dividend below the one?" "Four."

The second problem is 92 ÷ 4. Have him estimate how many fours might be in ninety-two. How many are in one hundred? Have him write that number on the estimate line on the left hand column on the worksheet.

Note

Do not tell him to look for the largest number in the four times table list above. Let him figure that out. Let him work with the problem. He may even do it a different way and work backwards from the known answer. But the one on the page must be set up so that the answer comes out in tens and ones. Let him experiment.
Here is a sample conversation:
"How many fours do you think might be in ninety-two?" "Well, I know that there are twenty-five in one hundred, so it is less than that."

"Do you think the first number of the quotient might still be in the twenties or do you think it will be less?" "I think it will be twenty because ninety-two is very close to one hundred."

"So if the first number in the quotient is twenty, then what number will be below it in the dividend?" "Eighty, because four times twenty is eighty."

"How can you find out what the other number is?" "The two numbers have to add up to ninety-two. So if the first number is eighty, then the second one must be twelve."

"Where do you write the twelve?" "After the 80 +."

"What do you write in the quotient then?" "Three because three times four is twelve. So the answer is twenty-three."

Note

The third problem uses a one hundred, which most students know the answer to when dividing by four. Many students will write one hundred on the first line and try to put twenty-five in the quotient line above. Remind them that in these problems, they are not allowed to put ones, in this case the five, in the first quotient. They can only put numbers such as twenty and thirty.

If he insists on breaking the number into 100 + 16, have him do it off to the side in the right hand margin. He will get 25 + 4 = 29. He can then work backwards to get the answer of 20 + 9, which is what we are after.

"Could the first number in the quotient be thirty?" "No, because four times

thirty is one hundred twenty. So the number must be twenty again."

"So what number goes under the twenty?" "Eighty."

"So how do you figure out what the second number in the break up (dividend) is?" "I have to do one hundred sixteen take away eighty, which is thirty-six."
"So what is the answer?" " Twenty-nine."

Move through the rest of the worksheet with similar conversations. Allow him to do the division problem his old way if needed. Encourage him to use the list up above to help him find useful numbers.

At the end of the page, stop and continue Breaking Up: Tens and Ones - Fours Worksheet 3 the next day.

"How can you use the expanded table to help you find numbers to use?" Answers will vary. You are listening for, "It helps me find numbers that will not give me any remainders" or "I can use the Flip Book to find the biggest number to put into the tens place."

Note

If you reach the sixes and he has not figured out that he can use his Expanded Tables Flip Book to look up the division facts to find the largest possible dividend that is less than the dividend in the problem, help him to see this pattern. It would now be appropriate to suggest that he look for the multiple of the divisor that is closest to the dividend given in the problem.

Practice Worksheets

Breaking Up: Tens and Ones - Fives, pages 67 - 68, and Breaking Up: Tens and Ones - Sixes, pages 69 - 73

Note

Before going on to sevens, do the assessment in Test for Understanding. Determine where the student is. If he is at the Getting It stage without having to look at his Flip Book or at the Extending It stage even with the Flip Book, move to the next lesson and use the Breaking Up: Tens and Ones - Sevens for more practice. See the following page for definitions of these terms.

If he is still in the Developing stage or needs lots of support to get to the tens and ones answers, continue practicing using the Breaking Up: Tens and Ones - Sevens for more practice.

Work on sevens for more practice.
If he has not yet realized that to get the answer to come out in tens and ones, he must choose the largest possible multiple that does not exceed the dividend in the problem, now is the time to show him this.

Example:

1 x 7 = 7	10 x 7 = 70	100 x 7 = 700
2 x 7 = 14	20 x 7 = 140	200 x 7 = 1,400
3 x 7 = 21	30 x 7 = 210	300 x 7 = 2,100

If the problem is $200 \div 7$, look in the expanded table to find the number closest to two hundred without going over. This number is one hundred forty. Two hundred ten is closer, but in this problem two hundred ten is greater than the number being divided, which is two hundred. So one hundred forty must be selected.

Big picture: These division problems can be done many different ways. But to be able to use the short procedure, you must have the quotient come out in tens and ones.

$7\overline{)200}$ $\qquad$ $7\overline{)140 + 60}$ $\quad 20 + 8 \text{ r } 4 = 28 \text{ r } 4$

Test for Understanding

Give the problem $196 \div 4$. Observe the strategies the student uses to solve it.

Here is a rubric of attributes for various stages of understanding.

Developing: The student works the problem the old way, using multiples of the divisor from his charts. Then he transfers the answer to the tens and ones answer format and fills in the dividend needed. He has not realized that to get the tens and ones in the quotient directly he must look for the largest possible multiple of the divisor in the extended table list that is as close to the dividend as possible without going to a number greater than the dividend.

$4\overline{)196}$ $\qquad$ $4\overline{)100 + 80 + 16}$ $\quad 25 + 20 + 4 = 49$ $\qquad$ $4\overline{)160 + 36}$ $\quad 40 + 9 = 49$

Getting It: The student has realized he can use the list to find the largest possible multiple of the divisor that gets you as close as possible to the dividend without going over. He must be doing this on his own with no coaching or direction from you.

$4\overline{)160 + 36}$ $\quad 40 + 9 = 49$

Extending: The student begins to spot the multiple needed and writes it under the dividend instead of in the tens and ones format given on the right and subtracts under the original problem. When a student begins to do this, move him to the next lesson.

Example:

$$\begin{array}{r} 49 \\ 4\overline{)196} \\ -160 \\ \hline 36 \end{array} \qquad 4\overline{)160 + 36} \quad 40 + 9 = 49$$

Breaking Up: Tens and Ones - Eights and Nines

Purpose

The purpose of this lesson is to move the student to the standard short notation commonly used in division. Once the student can break the dividend into a form that will result in the quotient coming out in tens and ones, she is ready to drop the expanded notation and use the standard short notation.

Prerequisites

Getting It or Extending in the Test for Understanding in the last lesson.

Materials

Breaking Up: Tens and Ones - Eights, Worksheets 1 - 5, pages 78 - 82
Breaking Up: Tens and Ones - Nines, Worksheets 1 - 3, pages 83 - 85
Breaking Up: Tens and Ones - Sevens, Worksheets 1 - 3 if they were not completed in the last lesson.
Expanded Tables Flip Books - only if needed. Try to switch to a standard multiplication chart if she does not know her tables by heart. She needs to begin to internalize the expanded table and to be able to do it in her head or from just a fact chart that gives the basic fact 6 x 7 = 42, for example, and to be able to know she also can use 420, 4,200, and so on.

Warm Up

Have her fill out the top section of Breaking Up: Tens and Ones - Eights, Worksheet 1, page 78. Allow her to use her Flip Book to do the division if she needs it.

Lesson Part 1

Breaking Up: Tens and Ones - Eights, Worksheet 2, page 79 Have her fill out the top on her own. We want her to do these easily. Try to avoid using the Expanded Tables Flip Book if possible. If she does not know her eights, let her use the top of Worksheet 1 to help. She should know by now that to do the problems so that the answer comes out in tens and ones she must choose the multiple of eight that is closest to the dividend. The problems on this page are still pretty easy and repetitive. The first four problems are all going to use ten in the first space of the quotient.

Add this new piece if she is not already doing this: Have her write the number to be subtracted, the eighty, under the one hundred four.

$$8\overline{)104} \qquad \begin{array}{r} 10+3 \\ 4\overline{)80+24} \end{array}$$

New Step
This time, have her record eighty under the one hundred four and subtract.

$$\begin{array}{r} 8\overline{)104} \\ \underline{-80} \\ 24 \end{array}$$

Now have her place the twenty-four in the customary second space of the dividend and finish the problem.

$$\begin{array}{r} 10+3=13 \\ 4\overline{)80+24} \end{array}$$

Doing the subtraction with the notation above instead of off to the side, or in the head, will set her up for understanding the standard procedure and notation for division. We are beginning the process of bridging from the general process of division that she already knows to ***the way*** we all memorized in school. Only, she will

understand it as many of us did not.

Worksheets

Complete Breaking Up: Tens and Ones - Eights, Worksheets 2 and 3, pages 79 and 80. End Part 1 of the lesson here.

Lesson Part 2

Breaking Up: Tens and Ones - Eights Worksheets 4 and 5, pages 81 and 82

We are now going to complete the process of transferring what she already knows to the standard short procedure notation.

You are going to do what we call a one to one transfer. You will do a step using the familiar long way of recording on the right hand side of the worksheet, stop, and then repeat the same step using the short way notation. Steps 1, 3, and 5 are shown at the top of Breaking Up: Tens and Ones - Worksheet 4 as arrows. Below is a more detailed explanation of each of the steps.

	Short way	Tens and Ones
Step 1	$8\overline{)312}$	$4\overline{)\underline{240} + \_\_\_}$

Begin as you have before. Find the largest multiple of the divisor and write it in the tens and ones format on the right.

Step 2	$\begin{array}{r} 3 \\ 8\overline{)312} \end{array}$	$\begin{array}{r} 30 + \\ 4\overline{)\underline{240} + \_\_\_} \end{array}$

After you have recorded the 30 on the tens and ones, record a 3 in the tens place. In the short notation, the 30 is shown with a 3 in the tens place and no 0.

Step 3	$\begin{array}{r} 3 \\ 8\overline{)312} \\ \underline{-240} \\ 72 \end{array}$	$\begin{array}{r} 30 + \\ 4\overline{)\underline{240} + \underline{72}} \end{array}$

Record the subtraction of the 240 on the short notation just like you did in Part 1 of this lesson. Do not write 24; write 240. This keeps her in touch with the place value. Record the 72 on the tens and ones expanded problem on the right.

Step 4	$\begin{array}{r} 39 \\ 8\overline{)312} \\ \underline{-240} \\ 72 \end{array}$	$\begin{array}{r} 30 + \ 9 \\ 4\overline{)\underline{240} + \underline{72}} \end{array}$

Divide 72 by 8 and record the 9 on both problems.

$$\begin{array}{r} 39 \\ 8\overline{)312} \\ \underline{-240} \\ 72 \\ \underline{-72} \\ 0 \end{array} \qquad \begin{array}{r} 30 + \ 9 = 39 \\ 4\overline{)\underline{240} + \underline{72}} \end{array}$$

Step 5

In the short way only, multiply 8 x 9 to get seventy-two and write it under the first seventy-two and subtract to show that you have no remainder. On the expanded tens and ones problem, show the total as thirty-nine.

Big Picture Reminder: These division problems can be done many different ways but to be able to use the short procedure, you must have the quotient come out in tens and ones. Occasionally, ask her to solve the problem the old way, not using tens and ones or the short notation.

Worksheets

Breaking Up: Tens and Ones - Nines, Worksheets 1 - 3, pages 83 - 85
Breaking Up: Tens and Ones - Sevens, Worksheets 1 - 3, pages 74 - 76, if they were not completed in the previous lesson.

Test for Understanding

Show You Know on page 86
The answers are in the Answer Key.

Families of Facts

Purpose The purpose of this lesson is to review the Families of Facts and insert some practice in factoring and the use of the expanded tables.

Prerequisites Previous lessons with Families of Facts. If this is unfamiliar to the student see Division - Booklet 1.

Materials Families of Facts - page 87
Multiplication fact chart, if needed

Warm Up Have the student write the Family of Facts for 3 x 5 = 15. It is 3 x 5 = 15, 5 x 3 = 15, 15 ÷ 3 = 5, 15 ÷ 5 = 3.

Lesson Do the first problem on the Families of Facts worksheet, which is 3, 150, and 50. **"Which two numbers are being multiplied? How can you tell?"** "Three times fifty are being multiplied to make one hundred fifty. I can tell because one hundred fifty is the largest number."

"What about the next one? One of the numbers is missing. How will you know which number to put there?" There are two possible answers. Allow either one. "The other one is 3,240 because that is what five hundred forty times six is." Or "The other number is ninety. I know because six times nine is fifty-four. So six times ninety is five hundred forty."

On the third one, he must multiply the eighty and the six because eighty is not a factor of six. On the last two, remind him about factoring to help him solve the problem.

Worksheets There are no practice pages.

Test for Understanding Watch him do the 'For you' problem. He should be able to compose a Family of Facts fairly quickly on his own. For a bit of extra challenge, require both of the numbers in the multiplication problem to have zeros at the end, no fair using just ten.

Example: 20 x 700 = 14,000
700 x 20 = 14,000
14,000 ÷ 20 = 700
14,000 ÷ 700 = 20

Expanded Division: Practice

Purpose The purpose is to extend what the student already knows about tens and ones division into longer problems with hundreds, tens, and ones in the quotient. At the same time, the student is practicing the standard short notation for division.

Prerequisites Previous lessons

Materials Expanded Division: Practice - Worksheets 1 - 7. pages 88 - 94
Expanded Tables Flip Book

Warm Up Using Breaking Up: Tens and Ones - Nines, Worksheet 1, page 83, as a guide, give this problem - 2,799 ÷ 9. Have her look at the page and find the problem 2700 ÷ 9 = 300. She already knows the answer to this problem. Have her break up the problem like this: $9\overline{)2700+90+9}$

This time the answer will come out in hundreds, tens, and ones. The answer is three hundred eleven.

Lesson **"The first problem at the top of page 88 is $3\overline{)2,349}$. Please use the blank $3\overline{)\_\_+\_\_+\_\_}$ to break up the first problem and make the quotient come out in hundreds, tens, and ones."** The answer is eight hundred thirteen.

"How did you break it up and why did you break it up this way?" The answer will be unique.

Note If she needs help she can use the Expanded Tables Flip Book on the first two or three pages of this lesson.

"What is the closest multiple of three to 2,439?" "2,400."
"Write that on the first line of the dividend." $3\overline{)2400+\_\_0+\_\_}$

As she works through the problem, have her use the one-to-one transfer process used in the last lesson to do the problem the short way. She should have little difficulty at this point. It will look like this when it is finished:

$$\begin{array}{r} 813 \\ 3\overline{)2,439} \\ -2,400 \\ \hline 39 \\ -30 \\ \hline 9 \\ -9 \\ \hline 0 \end{array} \qquad\qquad \begin{array}{r} 800+10+3 \\ 3\overline{)2400+30+9} \end{array}$$

Have her complete Expanded Division: Practice - Worksheet 1 with your supervision.

Practice Worksheets

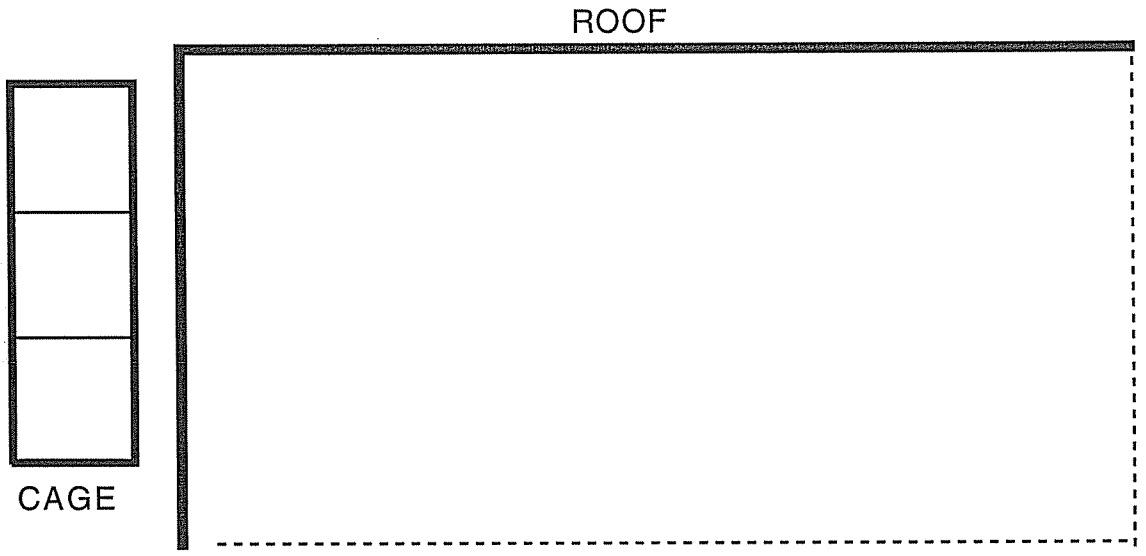
Expanded Division: Practice - Worksheets 2 - 7, pages 89 - 94
Have her do one page each day until the booklet is completed. Remove the Expanded Table Flip Book after Expanded Division: Practice - Worksheet 3. She can use a regular multiplication chart if she has not mastered the times tables. If she has not mastered the times tables, please work on it.

In your lessons, try extending the problems to thousands, hundreds, tens, and ones. Here are a few problems that will do that: $6{,}369 \div 3$, $3{,}024 \div 3$, $3{,}388 \div 4$

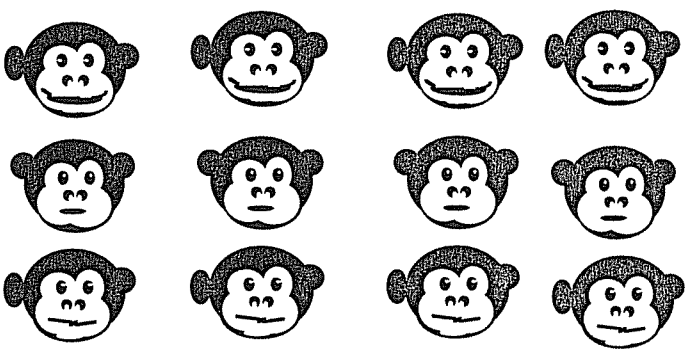
Addendum

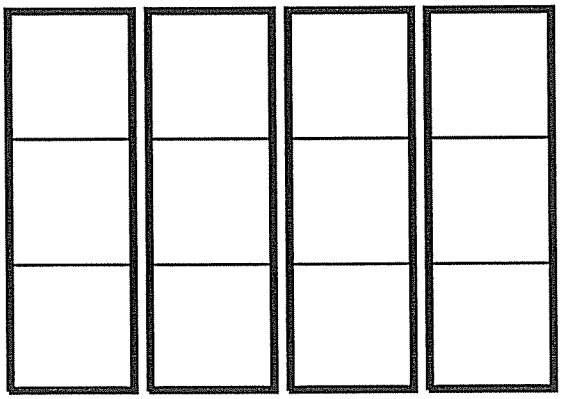
Monkeys on the Roof - Worksheet 1 Date_____

Once upon a time there was a whole batch of baby monkeys who got loose from a zoo and found their way into Mrs. Querel's house. She called the zookeeper, who came over with some cages. The cages looked like little towers with three little doors. The zookeeper went into the house and put three baby monkeys into each cage. Then he called a helicopter to come and pick up the cages. The helicopter pilot wanted the cages on the roof of the house so he could get them easily with his hook. So the zookeeper tossed the cages up on the roof. The pilot grabbed them with a hook and took them all back to the zoo. Mrs. Querel's house was all messed up but empty of baby monkeys. The zookeeper stayed to help clean up the mess.



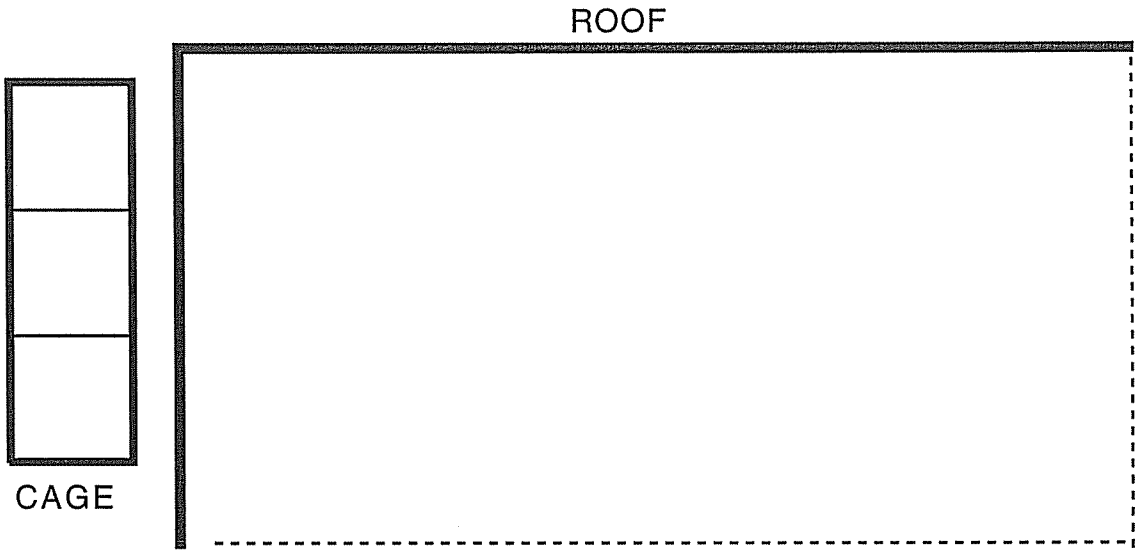
Cut out the12 baby monkeys. Put all the baby monkeys into the house. Now cut out a cage. Paste three baby monkeys into each cage and put the cage on the roof. Keep doing that until all those frisky monkeys are in cages on the roof.





Monkeys on the Roof - Worksheet 2 Date_____

Oh no! The baby monkeys came back! This time they brought their friends too. Now there are 15 monkeys. How many cages will the zookeeper need this time?



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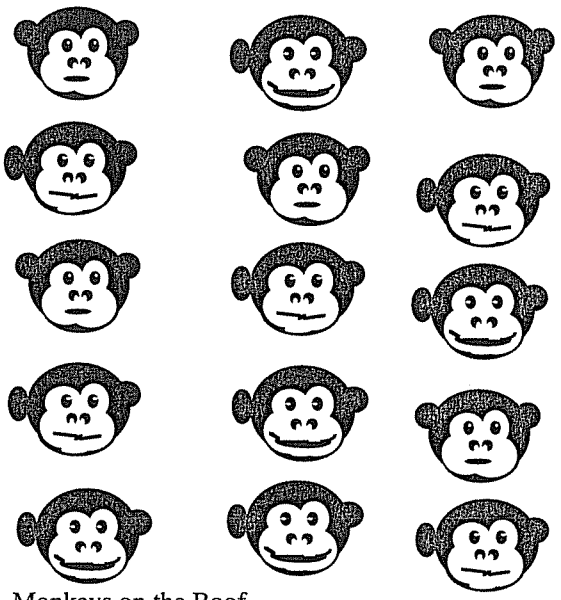
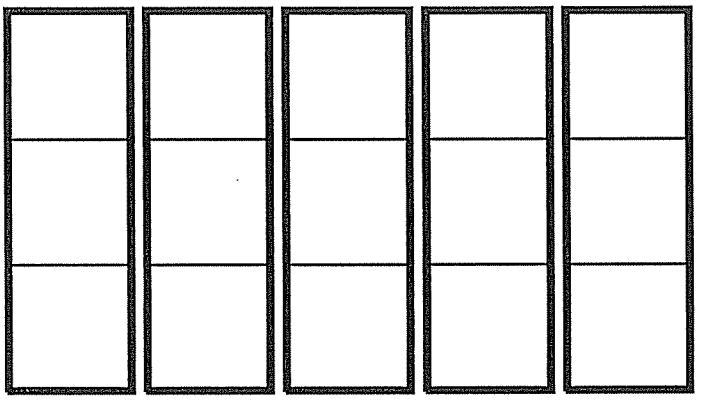




Monkeys on the Roof - Worksheet 3 Date_____

Use Unifix cubes. These will stand for the monkeys. To put them in cages, snap them together in groups of three and put them on the roof. Record how many cages are on the roof.

Number of Cages on the Roof

Cage Size | Number of Monkeys in the house

6 Cages
3 | 18
Monkeys

Cages
3 | 24
Monkeys

Cages
3 | 21
Monkeys

Cages
3 | 30
Monkeys

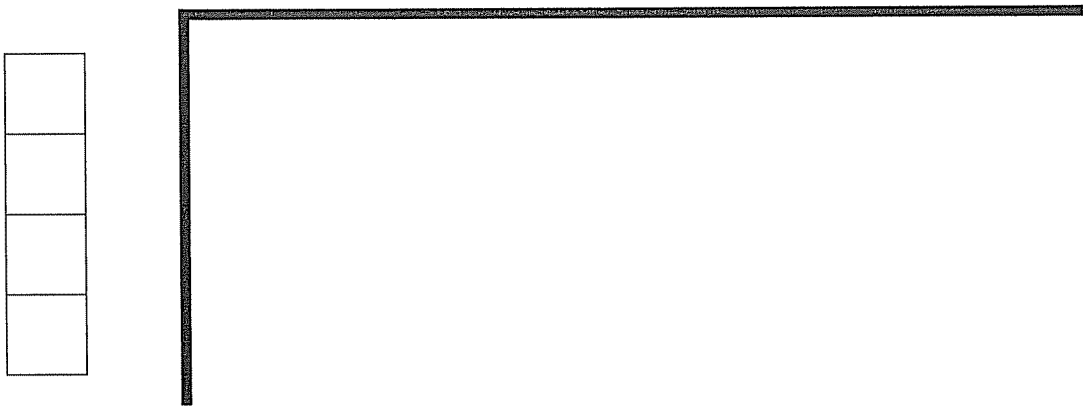
Cages
3 | 9
Monkeys

Cages
3 | 27
Monkeys

Cage
3 | 33
Monkeys

Monkeys on the Roof - Worksheet 4 Date_____

The zookeeper got in some new cages. These cages hold four monkeys. Again, use your Unifix cubes to stand for monkeys. In the house, the monkeys are not snapped together.



Number of Cages on the Roof

Cage Size | Number of Monkeys in the house

4 | 12

4 | 16

4 | 24

4 | 32

4 | 8

4 | 20

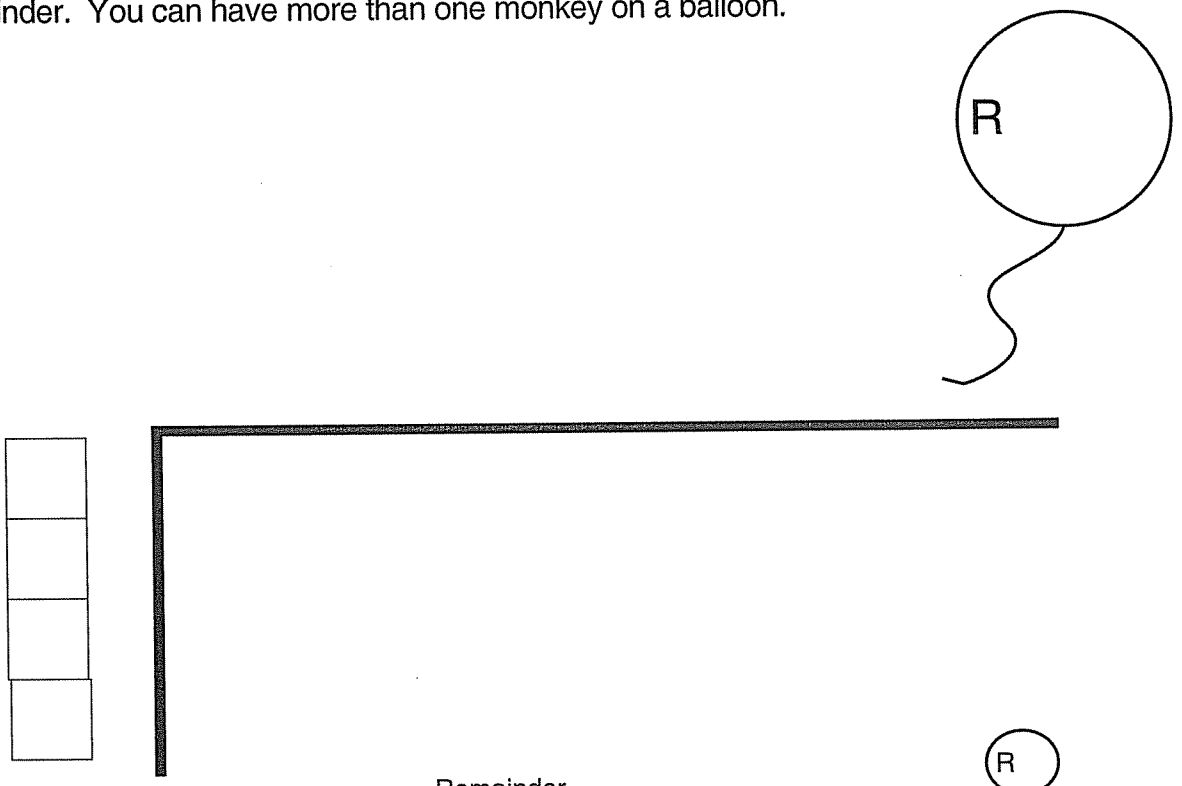
4 | 4

Monkeys on the Roof - Worksheet 5

Date_____

Oops! The monkeys on the roof don't all fit in the cages.

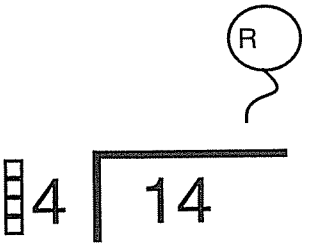
What if there are 13 monkeys? You can get 12 of the monkeys into the cages, but there is one left over. The zookeeper did not want to use another cage. So he tied the baby monkey to a balloon and floated it up to the helicopter. The monkey on the balloon is called a remainder. You can have more than one monkey on a balloon.

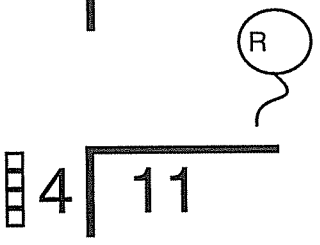


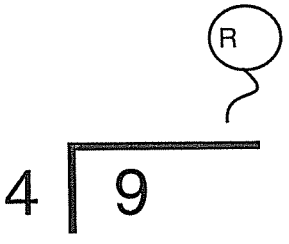
Remainder

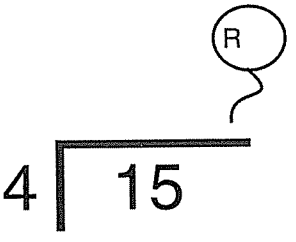
Number of Full Cages on the Roof

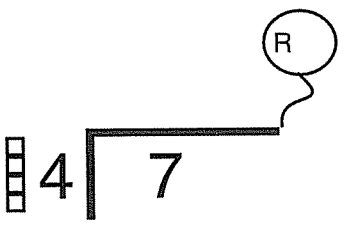
Cage Size | Number of Monkeys in the house

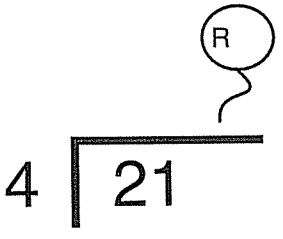


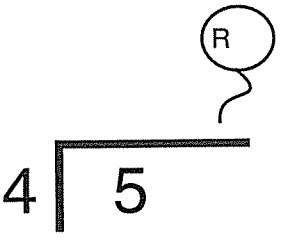












Patterns in Arithmetic

Division: Booklet 2

Answer Key for the Student Workbook

By Suki Glenn and Susan Carpenter

Answer Key Legend

AWV = answer(s) will vary
BUWV = break up will vary
OWV = order will vary

Pattern Blocks
r = red trapezoid
g = green triangle
y = yellow hexagon
o = orange square
b = blue parallelogram
t = tan rhombus

Cuisenaire Rods
1 w = white
2 r = red
3 lg = light green
4 p = purple
5 y = yellow
6 dg = dark green
7 bk = black
8 bn = brown
9 bl = blue
10 o = orange

Note: Some items and pages are left out of the answer key.
1) Some pages in which the answers are open-ended or will vary.
2) Make your own problems. Since students create their own problems and solutions, these sections give valuable information about the level of confidence and competence. It can be a useful source of curriculum for other students.
3) Blank practice pages
4) Workboards
5) Games
6) Self correcting pages
7) Instructions only pages

Patterns in Arithmetic: Division - Booklet 2
Parent/Teacher Guide
Answer Key for the Student Workbook

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Division - Booklet 2

Pre-Assessment - Part 1

1. Susan has eight chocolate chip cookies. She put two cookies on each plate. How many plates did she need? 4

   b. Write the number sentence that goes with the story.

8	÷ 2	= 4

   c. Record. $2\overline{)8}$ 4

   a. Draw a picture of the problem.

2. Use only the three numbers 4, 7, and 28 to make four Families of Facts number sentences.

   a. 4 × 7 = 28 ; 28 ÷ 4 = 7 ; 7 × 4 = 28 ; 28 ÷ 7 = 4

   b. $4\overline{)28}$ 7 ; $7\overline{)28}$ 4 ; 4 × 7 = 28 ; 7 × 4 = 28

3. Fill in the missing numbers.

   a. 32 ÷ 4 = 8 b. 15 ÷ 5 = 3 c. 35 ÷ 5 = 7

   d. 20 ÷ 10 = 2 e. $6\overline{)48}$ 8 f. $4\overline{)32}$ 8 g. $5\overline{)10}$ 2

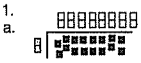
4. Solve.

   a. 86 − 39 = 47 b. 347 − 156 = 191 c. 435 − 279 = 156

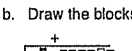
5. Solve.

   a. 138 × 6 = 828 b. 2,375 × 4 = 9,500 c. 6,598 × 4 = 26,392

Pre-Assessment - Part 2 - Worksheet 1

1. a.  → $2\overline{)16}$ 8 Study this picture. What does this 8 mean? There are 8 groups of 2.

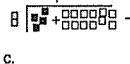
   b. Draw the blocks. Fill in the blanks.



   2 + 6 = 8 ← Final answer.

   $2\overline{)4 + 12}$

   c. Fill in the blanks.



   2 + 4 + 2 = 8

   $2\overline{)4 + 8 + 4}$

   d. What do you notice about the answers in a, b, and c? They are all the same.

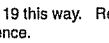
2. a. Build 16 this way. Record the number sentence. Arrange the blocks in each set into groups of 2.

   $2\overline{)5 + 11}$ $2^{R1} + 5^{R1} = 7^{R2} = 8$

   b. Build 20 this way. Record the number sentence. Arrange the blocks in each set into groups of 4.

   $4\overline{)\_ + \_ + \_}$ $1^{R2} + 2^{R2} + 1 = 4^{R4} = 5$

   c. Build 19 this way. Record the number sentence.



   $4\overline{)6 + 8 + 5}$ $1^{R2} + 2 + 1^{R1} = 4^{R3}$

   d. Break the 38 into three numbers and divide by 5. $5\overline{)38}$

   $5\overline{)\_ + \_ + \_}$ BUWV $= 7^{R3}$

3. Solve each problem two different ways.

   a. $4\overline{)46}$ 11^{R2} $4\overline{)\_ + \_}$ $= 11^{R2}$ $4\overline{)\_ + \_ + \_}$ $= 11^{R2}$

   b. $4\overline{)38}$ 9^{R2} $4\overline{)\_ + \_}$ $= 9^{R2}$ $4\overline{)\_ + \_ + \_}$ $= 9^{R2}$

   c. $4\overline{)56}$ 14 $4\overline{)\_ + \_}$ $= 14$ $4\overline{)\_ + \_ + \_}$ $= 14$

Pre-Assessment - Part 2 - Worksheet 2

4. Tony was shooting baskets every day. Here is a table of the number of baskets he made each day.

Monday	12 baskets made
Tuesday	6 baskets made
Wednesday	9 baskets made
Thursday	14 baskets made
Friday	14 baskets made

   a. How many baskets were made in all? 55

   b. How many days were there? 5

   c. What was the average number of baskets per day? 11

5. Solve. Show your work on how you break up each number. BUWV

   a. $8\overline{)94}$ 11^{R6} $8\overline{)\_ + \_ + \_}$ $= 11^{R6}$

   b. $9\overline{)117}$ 13 $9\overline{)\_ + \_ + \_}$ $= 13$

6. Solve these two problems.

   a. $3\overline{)30 + 30 + 12}$ $10 + 10 + 4 = 24$

   b. $3\overline{)18 + 36 + 10 + 8}$ $6 + 12 + 3^{R1} + 2^{R2} = 23^{R3} = 24$

   c. Which problem was easier to solve? a. d. Explain why. The addition was easier in the quotient, no remainders.

7. Fill in the missing numbers.

   a. $3\overline{)51}$ 17 $3\overline{)30 + 21}$ $10 + 7 = 17$

   b. $4\overline{)92}$ 23 $4\overline{)80 + 12}$ $20 + 3 =$ ___

8. Make the answers come out in tens and ones with remainders in the ones place only.

   a. $3\overline{)96} = 3\overline{)90 + 6}$ 32 $30 + 2 = 32$

   b. $4\overline{)184} = 4\overline{)160 + 24}$ 44 $40 + 6 = 46$

   c. $5\overline{)385} = 5\overline{)350 + 35}$ 77 $70 + 7 = 77$

9. Solve each problem the long way, showing how the problem is broken up to get the answer to come out in hundreds, tens, and ones. Then do each problem the short way.

   a. $3\overline{)609} = 3\overline{)3{,}000 + 600 + 9}$ 1,203 $1{,}000 + 200 + 3 = 1{,}203$

   -3, -6, -6, 9

   b. $3\overline{)477} = 3\overline{)300 + 150 + 27}$ 159 $100 + 50 + 9 = 159$

   3, 17, -15, 27

   c. $3\overline{)2{,}124} = 3\overline{)2{,}100 + 0 + 24}$ 708 $700 + 0 + 8 = 708$

   -21, 2, -0, 24

   d. $4\overline{)3{,}625} = 4\overline{)3200 + 400 + 25}$ 906^{R1} $800 + 100 + 6^{R1} = 906^{R1}$

   -36, 2, -0, 25, -24, 1

Number Sentence: Review - Worksheet 1

dividend	divisor	quotient
The number you start with	÷ The size of the group	= The number of groups

size of the group $\overline{\text{number you start with}}$ number of groups

divisor $\overline{\text{dividend}}$ quotient

In the boxes below, write the number sentence that goes with the story.

Susan has eight chocolate chip cookies. She puts two cookies on each plate. How many plates will she need?

8	÷2	=4

$2\overline{)8}$ 4

Suki has one hundred bees. Each cage needs five bees. How many cages can be filled?

100	÷5	=20

$5\overline{)100}$ 20

One hundred fifty queen honeybees were ordered. Each shipping box holds fifty queens. How many shipping boxes will be needed?

150	÷50	= 3

$50\overline{)150}$ 3

Number Sentence: Review - Worksheet 2

quotient / divisor | dividend

Dawn needs to swim one thousand yards for her swim test. Each lap is twenty-five yards. How many laps will she need to swim? Show your work. Explain how you solved this problem.

1,000 ÷ 25 = 40

$25\overline{)1{,}000}$ 40

AWV

Many students know there are four twenty-fives in one hundred. So there would be forty twenty-fives in one thousand.

Write two problems like these on your own. Remember the divisor should be the size of groups you are making.

Answer will be unique.

[ ] ÷ [ ] = [ ]

[ ] ÷ [ ] = [ ]

Division with Cuisenaire Rods - Groups of Three

Use Cuisenaire Rods to build these problems.

8 length — 3 groups — $3\overline{)24}$ 8

6 length — 3 groups — $3\overline{)18}$ 6

7 length — 3 groups — $3\overline{)21}$ 7

11 length — 3 groups — $3\overline{)33}$ 11

12 length — 3 groups — $3\overline{)36}$ 12

Families of Facts - Worksheet 1

Use only the three numbers in the box to make four number sentences.

4 7 28

4 × 7 = 28 — 7 × 4 = 28 — 28 ÷ 4 = 7 — 28 ÷ 7 = 4

$4\overline{)28}$ 7 — $7\overline{)28}$ 4

2 4 8

2 × 4 = 8 — 4 × 2 = 8 — 8 ÷ 2 = 4 — 8 ÷ 4 = 2

2 16 8

2 × 8 = 16 — 8 × 2 = 16 — 16 ÷ 2 = 8 — 16 ÷ 8 = 2

6 30 5

6 × 5 = 30 — 5 × 6 = 30 — 30 ÷ 6 = 5 — 30 ÷ 5 = 6

3 6 18

3 × 6 = 18 — 6 × 3 = 18 — 18 ÷ 3 = 6 — 18 ÷ 6 = 3

14 2 7

2 × 7 = 14 — 7 × 2 = 14 — 14 ÷ 2 = 7 — 14 ÷ 7 = 2

Make your own.

[ ] × [ ] = [ ]

[ ] × [ ] = [ ]

[ ] ÷ [ ] = [ ]

[ ] ÷ [ ] = [ ]

Families of Facts - Worksheet 2

Fill in the blank space with a number that completes a Family of Facts.
Use only the three numbers in the box to make four number sentences.

3 15 5 (or 45)

3 × 5 = 15 — 5 × 3 = 15 — 15 ÷ 3 = 5 — 15 ÷ 5 = 3

9 54 6

9 × 6 = 54 — 6 × 9 = 54 — 54 ÷ 9 = 6 — 54 ÷ 6 = 9

8 48 6

8 × 6 = 48 — 6 × 8 = 48 — 48 ÷ 8 = 6 — 48 ÷ 6 = 8

42 6 7

6 × 7 = 42 — 7 × 6 = 42 — 42 ÷ 6 = 7 — 42 ÷ 7 = 6

72 24 3

24 × 3 = 72 — 3 × 24 = 72 — 72 ÷ 3 = 24 — 72 ÷ 24 = 3

For you:

[ ] × [ ] = [ ]

[ ] × [ ] = [ ]

[ ] ÷ [ ] = [ ]

[ ] ÷ [ ] = [ ]

Families of Facts - Worksheet 3

3 8 24

3 × 8 = 24 — 3 × 8 = 24

8 × 3 = 24 — 8 × 3 = 24

24 ÷ 3 = 8 — 3 | 24 → 8

24 ÷ 8 = 3 — 8 | 24 → 3

7 35 5

5 × 7 = 35 — 5 × 7 = 35

7 × 5 = 35 — 7 × 5 = 35

35 ÷ 5 = 7 — 5 | 35 → 7

35 ÷ 7 = 5 — 7 | 35 → 5

8 7 56

8 × 7 = 56 — 8 × 7 = 56

7 × 8 = 56 — 7 × 8 = 56

56 ÷ 8 = 7 — 8 | 56 → 7

56 ÷ 7 = 8 — 7 | 56 → 8

Make your own. AWV

◯ × ◯ = ◯ — ___ × ___

◯ × ◯ = ◯ — ___ × ___

◯ ÷ ◯ = ◯ — ⌐

◯ ÷ ◯ = ◯ — ⌐

3 | 27 → 9

4 | 32 → 8

7 | 63 → 9

Missing Numbers - Worksheet 1

Fill in the missing numbers (dividends).
This asks you: If you make four groups or stacks with three in each stack, how many blocks did you start with? Use Unifix cubes to help you.

12 ÷ 4 = 3 — ▯▯▯▯

16 ÷ 8 = 2

10 ÷ 5 = 2 — groups / in each group.

9 ÷ 3 = 3

24 ÷ 6 = 4

18 ÷ 6 = 3

21 ÷ 3 = 7

27 ÷ 9 = 3

35 ÷ 7 = 5

40 ÷ 8 = 5

30 ÷ 5 = 6

36 ÷ 9 = 4

Make your own problems and solve.

◯ ÷ ◯ = ◯ AWV

◯ ÷ ◯ = ◯

◯ ÷ ◯ = ◯

◯ ÷ ◯ = ◯

Missing Numbers - Worksheet 2
Worksheet 2

Fill in the missing numbers.

12 ÷ 4 = 3 — 4 | 12 → 3

14 ÷ 7 = 2 — 7 | 14 → 2

20 ÷ 10 = 2 — 10 | 20 → 2

18 ÷ 9 = 2 — 9 | 18 → 2

12 ÷ 4 = 3 — 4 | 12 → 3

15 ÷ 3 = 5 — 3 | 15 → 5

12 ÷ 2 = 6 — 2 | 12 → 6

6 ÷ 2 = 3 — 2 | 6 → 3

35 ÷ 7 = 5 — 7 | 35 → 5

49 ÷ 7 = 7 — 7 | 49 → 7

42 ÷ 7 = 6 — 7 | 42 → 6

54 ÷ 9 = 6 — 9 | 54 → 6

Missing Numbers - Worksheet 3

Example:

3 Groups

6 | Start With 18

In Each Group

or

Example:
Use an array model to solve these problems.

3 length

6 width

4 | 36 → 9

4 | 32 → 8

9 | 36 → 4

7 | 49 → 7

3 | 27 → 9

15 | 45 → 3

5 | 55 → 11

4 | 48 → 12

12 | 36 → 3

8 | 40 → 5

8 | 48 → 6

8 | 64 → 8

7 | 21 → 3

1 | 8 → 8

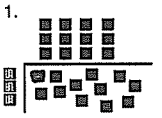
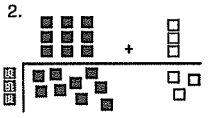
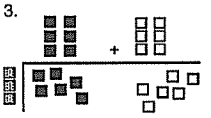
7 | 63 → 9

Missing Numbers - Worksheet 4

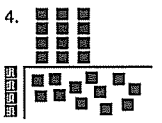
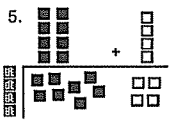
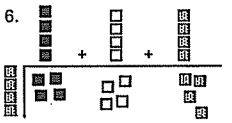
$4\overline{)20}$ → 5	$6\overline{)24}$ → 4	$5\overline{)40}$ → 8
$2\overline{)16}$ → 8	$5\overline{)15}$ → 3	$6\overline{)42}$ → 7
$3\overline{)21}$ → 7	$8\overline{)56}$ → 7	$9\overline{)27}$ → 3
$5\overline{)35}$ → 7	$4\overline{)32}$ → 8	$9\overline{)36}$ → 4
$4\overline{)32}$ → 8	$6\overline{)48}$ → 8	$8\overline{)72}$ → 9
$7\overline{)49}$ → 7	$2\overline{)16}$ → 8	$6\overline{)54}$ → 9

Breaking Up: Manipulative - Worksheet 1

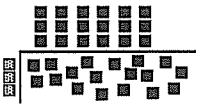
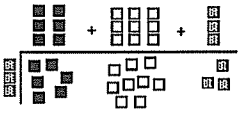
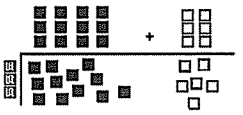
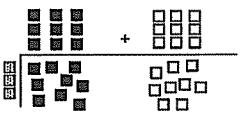
Solve these problem with your teacher.

1. 
2. 
3. 

What patterns do you see? Divisor is always 3. Quotients are stacks of 3. Dividend is broken up into groups divisible by 3 (the divisor). Quotients are the same in all problems.

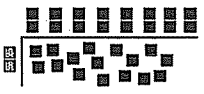
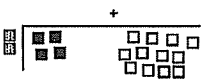
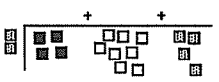
4. 
5. 
6. 

What patterns do you see? Divisors are always 4. Quotients are always the same.

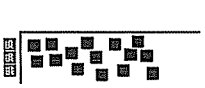
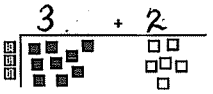
7. 
8. 
9. 
10. 

Breaking Up: Manipulative - Worksheet 2

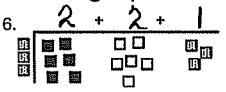
Draw the answers and then record the problem with numbers.

1.  → $2\overline{)16}$ → 8 What does this 8 stand for?
2.  → $2\overline{)4 + 12}$ → $2 + 6 = 8$
3.  → $4 + 8 + 4$ → $2 + 4 + 2 = 8$

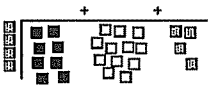
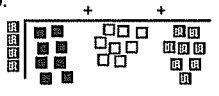
What do you notice about the answers to problems 1, 2, and 3?
they are all the same, eight.

4.  → $3\overline{)6+3+6}$ → $2+1+2=5$
5.  → $8\overline{)9 + 6}$ → $3 + 2 = 5$

Breaking Up: Manipulative - Worksheet 3

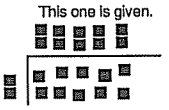
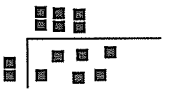
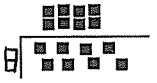
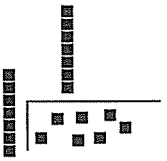
6.  → $3\overline{)6 + 6 + 3}$ → $2 + 2 + 1 = 5$

Do the answers to problems 4, 5, and 6 agree with each other? yes
Explain. Dividend always 15, just broken up different ways. Divisor always 3.

7.  → $4\overline{)24}$ → 6 What does this 6 stand for? 6 groups of 4
8.  → $4\overline{)8 + 12 + 4}$ → $2 + 3 + 1 = 6$
9.  → $4\overline{)8 + 8 + 8}$ → $2 + 2 + 2 = 6$
10.  → $4\overline{)16 + 8}$ → $4 + 2 = 6$

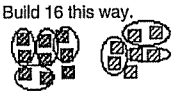
Breaking Up: Manipulative - Worksheet 4

This time you will be given either the drawing of the blocks or the number recording. You put in the missing one and write the number sentence.

Number sentence	Drawing of your blocks	Number recording
Example:	This one is given.	You write this too.
$10 \div 2 = 5$ You write this.		$2\overline{)10}$ quotient 5
11. $6 \div 2 = 3$		$2\overline{)6}$ quotient 3
12. $22 \div 2 = 11$		$2\overline{)22}$ quotient 11
13. $8 \div 2 = 4$		$2\overline{)8}$ quotient 4
14. $7 \div 7 = 1$		$7\overline{)7}$ quotient 1

Breaking Up: Remainders - Worksheet 1

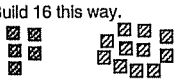
1. Build 16 this way.



$2\overline{)9 + 7}$ $4^{R1} + 3^{R1} = 7^{R2} = 8$

Arrange the blocks in each set into groups of 2.

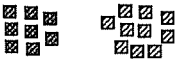
2. Build 16 this way.



$2\overline{)5 + 11}$ $2^{R1} + 5^{R1} = 7^{R2} = 8$

Arrange the blocks in each set into groups of 2.

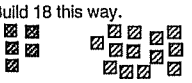
3. Build 18 this way.



$3\overline{)8 + 10}$ $2^{R2} + 3^{R1} = 5^{R3} = 6$

Arrange the blocks in each set into groups of 3.

4. Build 18 this way.



$3\overline{)5 + 13}$ $1^{R2} + 4^{R1} = 5^{R3} = 6$

Arrange the blocks in each set into groups of 3.

5. Build 18 your own way. Draw a picture of your blocks. AWV

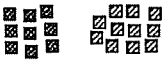
$3\overline{)\_ + \_}$ $+ \quad =$

 Arrange the blocks in each set into groups of 3.

Breaking Up: Remainders - Worksheet 2

Record

6 Build 20 this way.



$4\overline{)9 + 11}$ $2^{R1} + 2^{R3} = 4^{R4} = 5$

Arrange the blocks in each set into groups of 4.

7. Build 20 this way.

$4\overline{)6 + 10 + 4}$ $1^{R2} + 2^{R2} + 1 = 4^{R4} = 5$

Arrange the blocks in each set into groups of 2.

8. Build 20 your own way. Draw a picture of your blocks here. BUWV

$4\overline{)\_ + \_}$ $+$

Arrange the blocks in each set into groups of 4.

9. Build 24 two different ways. The first time, build it with two numbers. The second time, build it with three numbers. Divide the sets into groups of 6. Use blocks but you don't need to draw pictures.

$6\overline{)\_ + \_}$ $+ \quad = 4$

$6\overline{)\_ + \_ + \_}$ $+ \quad + \quad = 4$

Did your answers come out the same? _____

Should they? yes

Breaking Up: Remainders - Worksheet 3

Record

1. Build 19 this way.

$4\overline{)6 + 8 + 5}$ $1^{R2} + 2 + 1^{R1} = 4^{R3}$ (quotient 4, remainder 3)

Now arrange the blocks in each set into groups of 4.

2. Build 19 this way.

$4\overline{)6 + 5 + 5 + 3}$ $1^{R2} + 1^{R1} + 1^{R1} + 0^{R3} = 3^{R7} = 4^{R3}$

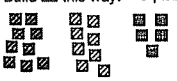
Now arrange the blocks in each set into groups of 4.

3. Build 19 with any two numbers you choose. Draw your blocks here. BUWV

$4\overline{)\_ + \_}$ $+ \quad =$

Arrange the blocks in each set into groups of 4.

4. Build 22 this way. $3\overline{)22}$



$3\overline{)9 + 8 + 5}$ $3 + 2^{R2} + 1^{R2} = 6^{R4} = 7^{R1}$

Arrange the blocks in each set into groups of 3.

5. Build 22 with any two numbers you choose. Arrange the blocks in each set into groups of 3. BUWV

$3\overline{)\_ + \_ + \_}$ $\_ + \_ + \_ \quad =$

Breaking Up: Remainders - Worksheet 4

6. Build 28 this way.

Record

$5\overline{)28}$ $1^{R3} + 2^{R1} + 1^{R4} = 4^{R8} = 5^{R3}$

$5\overline{)8 + 11 + 9}$

Now arrange the blocks in each set into groups of 5.

7. Build 28 this way.

$2 + 2 + 1 + 0^{R3} = 5^{R3}$

$5\overline{)10 + 10 + 5 + 3}$

Now arrange the blocks in each set into groups of 5.

8. Build 28 with any two numbers you choose. Draw your blocks here.

BUWV

__ + __ = ____

$5\overline{)\_\_ + \_\_}$

Arrange the blocks in each set into groups of 5.

9. Take out 35 blocks. $4\overline{)35}$ 8^{R3}

Break the 35 blocks into three sets.
How did you break up the 35?

BUWV

__ + __ + __ =

Arrange the blocks in each set in to groups of 4.
Record your work below.

__ + __ + __ = 8^{R3}

$4\overline{)\_\_ + \_\_ + \_\_}$

BUWV

Do the same problem again.
Break up the 35 a different way.

__ + __ + __ = 8^{R3}

$4\overline{)\_\_ + \_\_ + \_\_}$

Breaking Up: Remainders - Worksheet 5

Break up each problem two different ways. Make a model with blocks or drawings.

1. Break up the 26 into three numbers. Divide by 3.

$3\overline{)26}$ 8^{R2}

BUWV

__ + __ + __ = 8^{R2}

$3\overline{)\_\_ + \_\_ + \_\_}$

Do your answers match? Circle the easier one.

Break up it a different way.

__ + __ + __ = 8^{R2}

$3\overline{)\_\_ + \_\_ + \_\_}$

2. Break up the 38. Divide by 5.

$5\overline{)38}$ 7^{R3}

__ + __ + __ = 7^{R3}

$\_\_ + \_\_ + \_\_$

Do your answers match? Circle the easier one.

__ + __ + __ = 7^{R3}

$\_\_ + \_\_ + \_\_$

3. Break up the 36. Divide by 4.

$4\overline{)36}$ 9

__ + __ + __ = 9

$\_\_ + \_\_ + \_\_$

__ + __ + __ = 9

$\_\_ + \_\_ + \_\_$

Do your answers match? Circle the easier one.

Breaking Up: Recording - Worksheet 1

Take out 60 counters and the circle division workboards on the following pages.
Use them to solve these problems. For the three circle problems cover up one circle.

1. $6\overline{)60}$ Use the four circle workboard. O+O+O+O

Example:

$5 + 4 + 1 + 0 = 10^{R0} = 10$

$6\overline{)30 + 24 + 6 + 0}$

2. $6\overline{)60}$ Use the three circle workboard. O+O+O

$3 + 3^{R2} + 3^{R4} = 9^{R6} = 10$

$6\overline{)18 + 20 + 22}$

Example

This number means that I have 10 groups of 6

Do the answers to problems 1 and 2 match? _____

3. $6\overline{)60}$ Challenge! **No remainders.** Use three or four circles.

$3 + 3 + 4 = 10$

$6\overline{)18 + 18 + 24}$

Example

Breaking Up: Recording - Worksheet 2

Take out 60 counters and the three and four circle division workboards.
Use them to solve these problems.

1. Use the four circle workboard. O+O+O+O

$5\overline{)60}$ Means that I start with 60 objects and make groups of 5.

__ + __ + __ = $\_\_^{R} = 12$

$5\overline{)\_\_ + \_\_ + \_\_ + \_\_}$

2. $5\overline{)60}$ Use the three circle workboard. O+O+O

__ + __ = $\_\_^{R} = 12$

$5\overline{)\_\_ + \_\_ + \_\_}$

This number means that I have 12 groups of 5

Do the answers to problems 1 and 2 match? _____

3. $5\overline{)60}$ Challenge! **No remainders.** Use three or four circles.

$4 + 4 + 4 = 12$

$5\overline{)20 + 20 + 20}$

Example

Breaking Up: Recording - Worksheet 3

Take out 60 counters and the three and four circle division workboards.
Use them to solve these problems. BUWV

1. 4⟌60 Use the four circle workboard. O+O+O+O

___ + ___ + ___ + ___ = ___ R ___ = 15

4⟌ ___ + ___ + ___ + ___

2. 4⟌60 Use the three circle workboard. O+O+O

___ + ___ + ___ = ___ R ___ = 15

4⟌ ___ + ___ + ___

↑ This number means that I have 15 groups of 4.

Do the answers to problems 1 and 2 match? ______

3. 4⟌60 Challenge! **No remainders.** Use three or four circles.

___ + ___ + ___ = 15

4⟌ ___ + ___ + ___

Breaking Up: Recording - Worksheet 4

Take out 60 counters and the three and four circle division workboards.
Use them to solve these problems. BUWV

1. 3⟌60 Use the four circle workboard. O+O+O+O

___ + ___ + ___ + ___ = ___ R ___ = 20

3⟌ ___ + ___ + ___ + ___

2. 3⟌60 Use the three circle workboard. O+O+O

___ + ___ + ___ = ___ R ___ = 20

3⟌ ___ + ___ + ___

↑ This number means that I have 20 groups of 3.

Do the answers to problems 1 and 2 match? ______

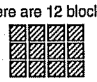
3. 3⟌60 Challenge! **No remainders.** Use three or four circles.

___ + ___ + ___ = 20

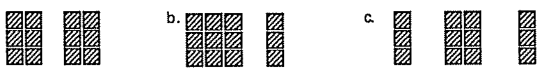
3⟌ ___ + ___ + ___

Breaking Up: Test for Understanding - Worksheet 1

1. Here are 12 blocks.



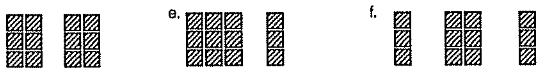
Here are the 12 blocks broken up in different ways.
Write the number sentence for each problem.



a. 6 + 6 = 12 b. 9 + 3 = 12 c. 3 + 6 + 3 = 12

2. On the lines above each little section of blocks, write the number of **Groups of 3** there are in that part. The first one is partly done for you.

2 + 2 = 4 3 + 1 = 4 1 + 2 + 1 = 4



d. e. f.

3. How many groups of 3 are there all together in picture d. 4
How many groups of 3 are there all together in picture e. 4
How many groups of 3 are there all together in picture f. 4
What pattern do you see in these answers? ↑ The answers are all the same.
Why does it make sense that it would be that way? Even though twelve is broken up differently, twelve is still the total amount being divided.

Breaking Up: Test for Understanding - Worksheet 2

Circle the mistake in each of these problems.
Explain the mistake the person made.

4. The problem is 24 ÷ 3 = 8.

3 + 3 + 3 = (9)
3⟌ 9 + 9 + 9

Should have broken up the dividend differently.
For example:
3 + 3 + 2 = 8
3⟌ 9 + 9 + 6

5. The problem is 20 ÷ 4 = 5.

(4) + 2 = (6) — corrected: 3 + 2 = 5
4⟌ 12 + 8

12 ÷ 4 = 3 not 4
Wrong answer because of division error.

6. The problem is 18 ÷ 3 = 6.

3 R1 + (2 R1) = (5 R2) — marked R3
3⟌ 10 + 8

8 ÷ 3 = 2 R2
Should have been
3 R1 + 2 R2 = 5 R3 = 6

7. The problem is 15 ÷ 3 = 5.

1 R1 + 2 R1 + 1 + 0 R1 = 4 (R3) = 4 + 1 = 5
3⟌ 4 + 7 + 3 + 1

Correct answer.
The remainder of 3 was added to the 4 groups of 3.

Multistep Word Problems

1. Mimi got $25 for her birthday. She spent $13.15 and paid with a $20 bill. She had left the $5 bill at home on her desk. When she got home she counted her money and realized she had only $6.15 in her wallet. How much money did the cashier short Mimi when she made the change? short $.70

$20.00 − 13.15 = $6.85

$6.85 − 6.15 = $.70

2. Shania went shopping with Marissa. They bought $34.99 worth of food to at school. They then bought $14.79 worth of pencils, pens, and erasers to sell too. After they sold everything, they had a total of $88.92. If they split the profit evenly, how much money will each girl receive? $19.57

Spent $49.78

88.92 − 49.78 = $39.14

39.14 ÷ 2 = 19.57

3. Dorothy, Jesse's mom, took Jesse, Samantha, and Emily shopping for a new basketball. Each child put in $5. The ball cost $18.21. They borrowed money from Dorothy. How much money does each child owe Dorothy? $1.07

3 x 5 = 15

$18.21 − 15.00 = $3.21

3.21 ÷ 3 = 1.07

Breaking Up: Practice - Worksheet 1
Dividing by Three

$34 \div 3 = 11\ R1$ — $3\overline{)30 + 4}$: $10 + 1^{R1} = 11^{R1}$ — ___ + ___ + ___ =

Example:

$38 \div 3 = 12\ R2$ — $3\overline{)33 + 5}$: $11 + 1^{R2} = 12^{R2}$ — ___ + ___ + ___ =

$45 \div 3 = 15$ — ___ + ___ = — ___ + ___ + ___ =

$54 \div 3 = 18$ — ___ + ___ = — ___ + ___ + ___ =

$60 \div 3 = 20$ — ___ + ___ = — ___ + ___ + ___ =

Breaking Up: Practice - Worksheet 2
Dividing by Four BUWV

$46 \div 4 = 11\ R2$ — ___ + ___ = — ___ + ___ + ___ =

$38 \div 4 = 9\ R2$ — ___ + ___ = — ___ + ___ + ___ =

$56 \div 4 = 14$ — ___ + ___ = — ___ + ___ + ___ =

$67 \div 4 = 16\ R3$ — ___ + ___ = — ___ + ___ + ___ =

$80 \div 4 = 20$ — ___ + ___ = — ___ + ___ + ___ =

Breaking Up: Practice - Worksheet 3
Dividing by Five BUWV

$37 \div 5 = 7\ R2$ — ___ + ___ = — ___ + ___ + ___ =

$56 \div 5 = 11\ R1$ — ___ + ___ = — ___ + ___ + ___ =

$45 \div 5 = 9$ — ___ + ___ = — ___ + ___ + ___ =

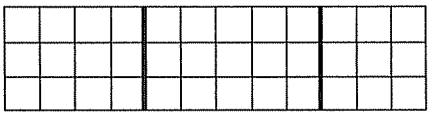
$62 \div 5 = 12\ R2$ — ___ + ___ = — ___ + ___ + ___ =

$60 \div 5 = 12$ — ___ + ___ = — ___ + ___ + ___ =

Breaking Up Division with Cuisenaire Rods
Worksheet 1

Study this problem.

12 x 3 = (4 + 5 + 3) x 3

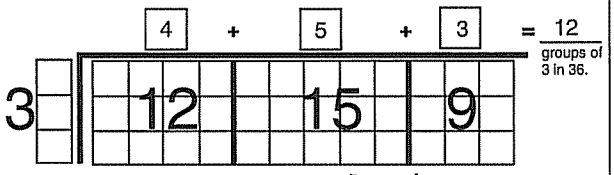


12 x 3 = (4 x 3) + (5 x 3) + (3 x 3)

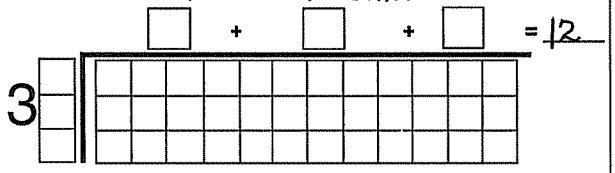
12 x 3 = (12 + 15 + 9)

12 x 3 = 36

In this multiplication problem we want to know what 12, three times is. The answer is 36. When the 36 is divided and we want to know how many threes can fit into it. This can be done by breaking the 36 into sections just like in multiplication.
The number sentence is now 36 ÷ 3 = 12. $3\overline{)36}^{12}$
The number in the box tells how many 3s are in each small section below.

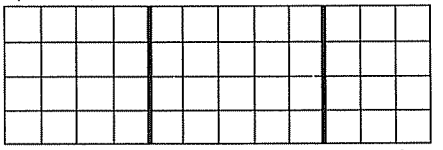


Break it up a different way. BUWV



Breaking Up Division with Cuisenaire Rods
Worksheet 2

12 x 4 = (4 + 5 + 3) x 4

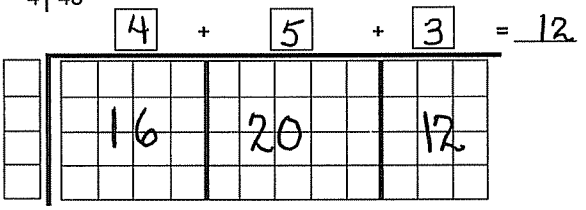


12 x 4 = (4 x 4) + (5 x 4) + (3 x 4)

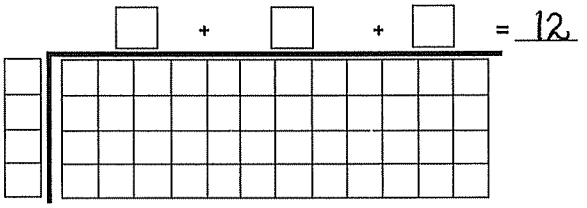
12 x 4 = (16 + 20 + 12

12 x 4 = 48

$4\overline{)48}^{12}$

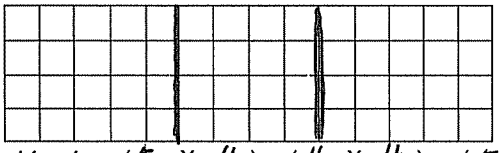


Do the division problem again a different way. BUWV



Breaking Up Division with Cuisenaire Rods
Worksheet 3 Examples:

14 x 4 = (5 + 4 + 5) x 4



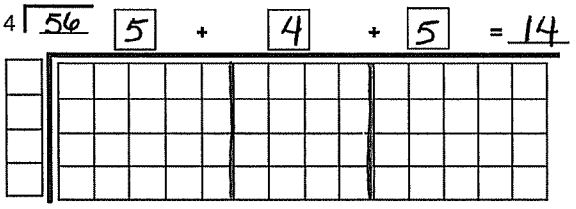
14 x 4 = (5 x 4) + (4 x 4) + (5 x 4)

14 x 4 = (20 + 16 + 20

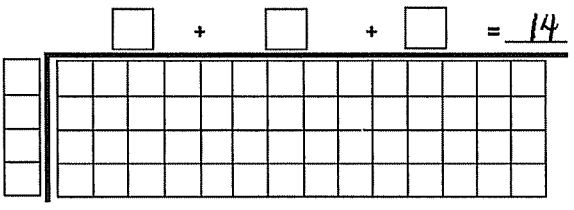
14 x 4 = 56

Break up this division problem the same way you did in the problem above.

$4\overline{)56}$

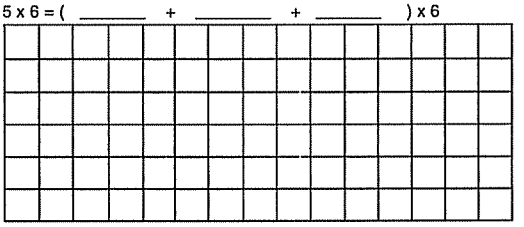


Do the division problem again a different way. BUWV



Breaking Up Division with Cuisenaire Rods
Worksheet 4 BUWV

15 x 6 = (____ + ____ + ____) x 6

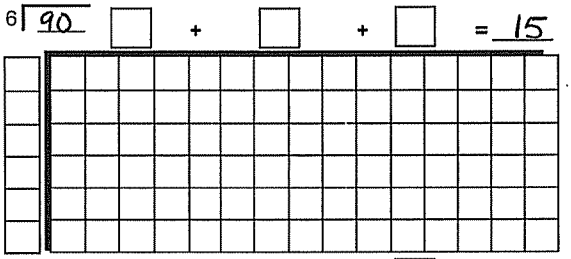


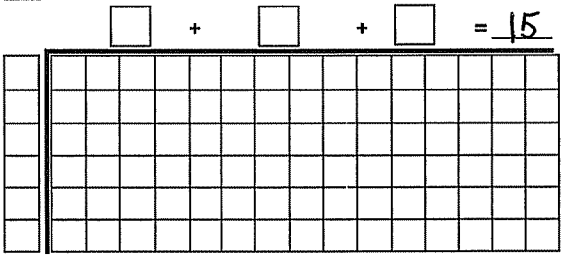
15 x 6 = (___ x ___) + (___ x ___) + (___ x ___)

15 x 6 = (____ + ____ + ____)

15 x 6 = 90

$6\overline{)90}$

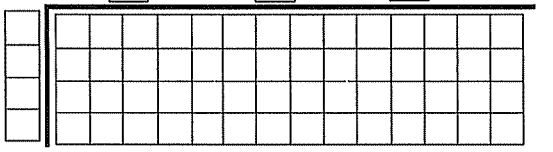




Breaking Up Division with Cuisenaire Rods Assessment - Show You Know

1. Break up this division problem.

$4\overline{)56}$ = ☐ + ☐ + ☐ = 14



2. Solve this problem two ways. $4\overline{)84}$ = 21

3. Are the answers to problem 2 the same? yes Explain why this is so.

Because the problem is 84÷4 both times, so the answer must be the same. It doesn't matter how the 84 is broken up, as long as the total equals 84.

Averages - Worksheet 1

Here is a common problem in the world. Let's say you are learning to shoot baskets. Each day you shoot 15 baskets. You keep track of how many baskets you make. Then someone asks you how you are doing. Do you tell them what you got today? Do you tell them what you got yesterday or two days ago? Most people would ask, "What is your average?"

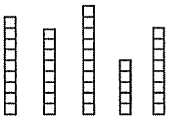
What they mean when they ask that is: If each day you made the same number of baskets, what would that number be? It means you took all your scores and evened them out so that they were all the same; that number is what an average is.

Let's do one.
You will need some blocks. Unifix cubes are really good for this.

Let's say you kept track of how many baskets you made each day for a week. Your record looks like this:

Monday	9 baskets made	Make a stack of 9 blocks.
Tuesday	8 baskets made	Make a stack of 8 blocks.
Wednesday	10 baskets made	Make a stack of 10 blocks.
Thursday	5 baskets made	Make a stack of 5 blocks.
Friday	8 baskets made	Make a stack of 8 blocks.

Your stacks should look like this:



Now keep the number of stacks the same. Five stacks for five days.
But move the blocks around until all the stacks are the same size.

How many are in each stack now? 8

There should be eight in each stack.
So the average number of baskets you made this week is eight.
The average is the number in a stack after the stacks are all the same size. Do more problems on the next page.

Averages - Worksheet 2

Let's look at this data again in numbers.

Monday	9 baskets made	Make a stack of 9 blocks.
Tuesday	8 baskets made	Make a stack of 8 blocks.
Wednesday	10 baskets made	Make a stack of 10 blocks.
Thursday	5 baskets made	Make a stack of 5 blocks.
Friday	8 baskets made	Make a stack of 8 blocks.

After the block stacks were made equal, there were eight in each stack.
So the average number of baskets you made this week is eight.

How many baskets were made in all? 40

How many days (stacks) were there? 5

What was the average number of baskets made per day? 8

Do the same thing with this data.

Monday	5 arrows on target	Make a stack of 5 blocks.
Tuesday	8 arrows on target	Make a stack of 8 blocks.
Wednesday	18 arrows on target	Make a stack of 18 blocks.
Thursday	9 arrows on target	Make a stack of 9 blocks.
Friday	10 arrows on target	Make a stack of 10 blocks.

Answer these questions.
What is the total number of arrows that hit the target? 50

How many days (stacks) did you shoot? 5

What was the average arrows on target a day? 10

Search for a pattern in the numbers. What do you see? 5×8=40 and 40÷5=8
5×10=50 and 50÷5=10

Averages - Worksheet 3

Let's do an average again with different numbers.
Try to use the pattern you noticed to predict what the average will be before you do it with the blocks.

Monday	12 baskets made
Tuesday	6 baskets made
Wednesday	9 baskets made
Thursday	14 baskets made
Friday	14 baskets made

How many baskets were made in all? 55

How many days (stacks) were there? 5

What was the average per day? 11

Did the pattern show up again? yes

Write the pattern as a number sentence. 55÷5=11

Use your number sentence to predict what the average is for this data.
Note that this time the number of days is 4 instead of 5. You will have only 4 stacks this time.

Monday	15 arrows on target
Tuesday	12 arrows on target
Wednesday	18 arrows on target
Thursday	15 arrows on target

What is the total number of arrows that hit the target? 60

How many days (stacks) did you shoot? 4

What is the average? 15

Explain how to calculate an average. If you can not calculate yet, make up more problems like these and use the blocks or a drawing to help you find the pattern.

Add up all the numbers (baskets made) to be averaged. Divide by the amount of numbers (days).

Breaking Up: Practice - Worksheet 1

Use the problem on the top of each column to help you do the problems. Example:

Problem	Answer	Break up
4)48	12	4)20 + 20 + 8 → 5 + 5 + 2 = 12
6)60	10	6)___ + ___ + ___ → + + =
4)55	13 R3	4)___ + ___ + ___ → + + =
6)67	11 R1	6)___ + ___ + ___ → + + =
4)76	19	4)___ + ___ + ___ → + + =
6)69	11 R3	6)___ + ___ + ___ → + + =
4)91	22 R3	4)___ + ___ + ___ → + + =
6)96	16	6)___ + ___ + ___ → + + =
4)100	25	4)___ + ___ + ___ → + + =
6)100	16 R4	6)___ + ___ + ___ → + + =

Breaking Up: Practice - Worksheet 2

Problem	Answer	Break up
6)97	16 R1	6)___ + ___ + ___ → + + =
3)68	22 R2	3)___ + ___ + ___ → + + =
5)48	9 R3	5)___ + ___ + ___ → + + =
4)82	20 R2	4)___ + ___ + ___ → + + =
8)76	9 R4	8)___ + ___ + ___ → + + =
5)67	13 R2	5)___ + ___ + ___ → + + =
7)91	13	7)___ + ___ + ___ → + + =
8)46	5 R6	8)___ + ___ + ___ → + + =
5)89	17 R4	5)___ + ___ + ___ → + + =
5)62	12 R2	5)___ + ___ + ___ → + + =

Breaking Up: Practice - Worksheet 3

Problem	Answer	Break up
3)95	31 R2	3)___ + ___ + ___ → + + =
4)104	26	4)___ + ___ + ___ → + + =
7)62	8 R6	7)___ + ___ + ___ → + + =
6)108	18	6)___ + ___ + ___ → + + =
4)116	29	4)___ + ___ + ___ → + + =
5)71	14 R1	5)___ + ___ + ___ → + + =
6)94	15 R4	6)___ + ___ + ___ → + + =
8)136	17	8)___ + ___ + ___ → + + =
4)92	23	4)___ + ___ + ___ → + + =
5)95	19	5)___ + ___ + ___ → + + =

Breaking Up: Practice - Worksheet 4

Problem	Answer	Break up
4)129	32 R1	4)___ + ___ + ___ → + + =
5)106	21 R1	5)___ + ___ + ___ → + + =
6)86	14 R2	6)___ + ___ + ___ → + + =
7)89	12 R5	7)___ + ___ + ___ → + + =
8)156	19 R4	8)___ + ___ + ___ → + + =
3)78	26	3)___ + ___ + ___ → + + =
7)133	19	7)___ + ___ + ___ → + + =
8)94	11 R6	8)___ + ___ + ___ → + + =
9)117	13	9)___ + ___ + ___ → + + =
6)112	18 R4	6)___ + ___ + ___ → + + =

Show You Know

Solve these two problems.

$$3 \overline{)30+30+12} \quad 10+10+4=24$$

$$3 \overline{)18+36+10+8} \quad 6+12+3^{R1}+2^{R2}=23^{R3}=24$$

Explain why the answers to both of these problems are the same even though the dividend is broken up diferently.

Because the total amount being broken up is the same in both problems.

Explain what an average is.

The middle amount. Averages are found by adding a list of numbers and dividing by the size of the list.

Multistep Word Problems


1. Shyla got $4.25 from her mom, $10. 37 from her grandfather and $3.50 from her piggy bank. She bought a birthday present for Michela for $13. 95. How money does she have left? $4.17


2. Elijah went shopping with Chaska . Chaska had $15.05 and Elijah had $17.16. They bought supplies for Micky's birthday party. They bought a cake for $ 16. 35, a package of balloons for $2.50, and a gallon of ice cream for $4.49 .They want to buy a party set of games and toys for $10. Do they have enough money to buy it? no

If they do have enough, how much money will they have left after they buy the toys? _____

If they do not have enough, how much more money do they need to buy the toys? $1.28

3. Kris, Keala and Donovan put their money together to buy a movie for Micky. They each had a $10 bill. The movie cost $21.90. How much money did each of them have left over? $2.70

Discovering Easier Ways - Worksheet 1

Fill in the missing numbers.

$$3 \overline{)51} = 17 \qquad 3 \overline{)\_+\_+\_} = 5+4+8=17$$

$$4 \overline{)72} = 18 \qquad 4 \overline{)\_+\_+\_} = 7+7+4=18$$

$$5 \overline{)175} = 35 \qquad 5 \overline{)50+50+50+25} = 10+10+10+5=35$$

$$6 \overline{)90} = 15 \qquad 6 \overline{)30+30+30} = 5+5+5=15$$

$$7 \overline{)91} = 13 \qquad 7 \overline{)70+21} = 10+3=13$$

Discovering Easier Ways - Worksheet 2

Here are five ways students solved 64 ÷ 4. Solve each one. Fill in the blanks. $4 \overline{)64}$

1. Amber's way: $4 \overline{)20+20+20+4}$ $5+5+5+1=16$

2. John's way: $4 \overline{)10+10+10+10+10+10+4}$ $2^{R2}+2^{R2}+2^{R2}+2^{R2}+2^{R2}+2^{R2}+1=13^{R12}=16$

3. Danielle's way: $4 \overline{)40+24}$ $10+6=16$

4. Sue's way: $4 \overline{)30+30+4}$ $7^{R2}+7^{R2}+1=15^{R4}=16$

5. Robert's way: $4 \overline{)60+4}$ $15+1=16$

Whose way was easiest for you? AWV

Put these problems in order of the easiest way to the hardest way.

easiest ____ ____ ____ ____ ____ hardest AWV

What made the problem easier? Less arithmetic and no remainders.

What made the problem harder? _____

Discovering Easier Ways - Worksheet 3

Here is another problem done three ways.
Do each one. Fill in the blanks.

$3\overline{)72}$ 24

1. Amber's way

$3\overline{)30+30+12}$ $10+10+4=24$

Amber used 3 x 10 = 30 for her expansion.

2. Roy's way

$3\overline{)70+2}$ $23+0=23^{R3}=24$ ($R1$, $R2$)

Roy used a standard tens and ones expansion. This is an easy way to expand the dividend, but what problem does this cause when you divide? It causes remainders, three doesn't go into every number evenly.

3. James's way

$3\overline{)60+12}$ $20+4=24$

James used an expansion that would give the answer or quotient in tens and ones.

Do all three problems.

Which way do you like the best? AWV but most like James's way.

Why? _____

_____

Will all three ways give the correct answer? yes

Explain why. The total amount of 72 is the same amount being broken up in all three problems.

Discovering Easier Ways - Worksheet 4

Break up each problem two different ways. BUWV

$5\overline{)55}$ 11

$3\overline{)69}$ 23

$3\overline{)93}$ 31

Challenge!

$4\overline{)128}$ 32

$3\overline{)153}$ 51

Discovering Easier Ways - Worksheet 5

James and Danielle both chose numbers for their expansions (in the dividend) to make the answer (quotient) come out in tens and ones.

Did they use a tens and ones expansion in the dividend? yes

quotient / divisor | dividend

$4\overline{)40+24}$ $10+6=16$ or $3\overline{)60+12}$ $20+4=24$

Look at how you did the problems on the previous page.
Circle the ones whose answers came out in tens and ones?

Try to get these problems to have the quotients or answers come out in tens and ones. You may have to do it several times to find the right expansion.

$3\overline{)45}$ 15 $=3\overline{)30+15}$ $10+5=15$

$4\overline{)52}$ 13 $=4\overline{)40+12}$ $10+3=13$

$5\overline{)65}$ 13 $=5\overline{)50+15}$ $10+3=13$

Challenge!

$3\overline{)84}$ 28 $=3\overline{)60+24}$ $20+8=28$

Discovering Easier Ways - Worksheet 6

Ask one of your parents to solve this problem the way they normally do it. (Parents, don't talk to students while you do it. Just solve it, without explaining.)

$3\overline{)255}$

Don't have them explain how they did it. But notice that they didn't break the numbers up like this:

$3\overline{)255}$ $3\overline{)90+90+90+60+15}$

They didn't expand the dividend at all. This is called short division. It's fast because you don't have to expand the dividend number. To learn to do this, you must learn to do your division problems the way James (Worksheet 3) and Danielle (Worksheet 2) did it.

How are James's and Danielle's ways the same? They both broke up the dividend, divided, and added the numbers to get the quotient.

How did their answers come out? In tens and ones.

_____

_____

Discovering Easier Ways - Worksheet 7

Fill in the missing numbers.

$3\overline{)15+12+24}$ $\quad 5+4+8=17$

$3\overline{)30+21}$ $\quad 10+7=17$

$4\overline{)40+40+12}$ $\quad 10+10+3=23$

$4\overline{)80+12}$ $\quad 20+3=23$

$4\overline{)40+40+20}$ $\quad 10+10+5=25$

$4\overline{)80+20}$ $\quad 20+5=25$

$5\overline{)25+25+25+20}$ $\quad 5+5+5+4=19$

$5\overline{)50+45}$ $\quad 10+9=19$

$5\overline{)100+100+25}$ $\quad 20+20+5=45$

$5\overline{)200+25}$ $\quad 40+5=45$

Discovering Easier Ways - Worksheet 8

Solve each problem two ways.
One way must have the answer come out in tens and ones.

$5\overline{)75}$ $\quad 15$

$5\overline{)\quad}$ Buwv

$5\overline{)50+25}$ $\quad 10+5$

$5\overline{)95}$ $\quad 19$

$5\overline{)\quad}$

$5\overline{)50+45}$ $\quad 10+9$

$5\overline{)125}$ $\quad 25$

$5\overline{)\quad}$

$5\overline{)100+25}$ $\quad 20+5$

$5\overline{)155}$ $\quad 31$

$5\overline{)\quad}$

$5\overline{)150+5}$ $\quad 30+1$

$5\overline{)186}$ $\quad 37^{R1}$

$5\overline{)\quad}$

$5\overline{)150+36}$ $\quad 30+7^{R1}$

Discovering Easier Ways - Worksheet 9

3 x 1 = 3	3 x 5 = 15	3 x 10 = 30	3 x 6 = 18	3 x 8 = 24
3 x 4 = 12	3 x 2 = 6	3 x 3 = 9	3 x 7 = 21	3 x 9 = 27

$3\overline{)27}$ 9	$3\overline{)240}$ 80	$3\overline{)2,700}$ 900
$3\overline{)21}$ 7	$3\overline{)270}$ 90	$3\overline{)300}$ 100
$3\overline{)18}$ 6	$3\overline{)150}$ 50	$3\overline{)1,200}$ 400
$3\overline{)12}$ 4	$3\overline{)90}$ 30	$3\overline{)600}$ 200
$3\overline{)24}$ 8	$3\overline{)180}$ 60	$3\overline{)1,500}$ 500

Discovering Easier Ways - Worksheet 10

Solve each problem two ways.
One way must have the answer come out in tens and ones.

$3\overline{)57}$ $\quad 19$

$3\overline{)36+21}$ $\quad 12+7$

$3\overline{)30+27}$ $\quad 10+9$

$3\overline{)69}$ $\quad 23$

$3\overline{)\quad+\quad}$ Buwv

$3\overline{)60+9}$ $\quad 20+3$

$3\overline{)39}$ $\quad 13$

$3\overline{)\quad+\quad}$

$3\overline{)30+9}$ $\quad 10+3$

$3\overline{)159}$ $\quad 53$

$3\overline{)\quad+\quad}$

$3\overline{)150+9}$ $\quad 50+3$

$3\overline{)216}$ $\quad 72$

$3\overline{)\quad+\quad}$

$3\overline{)210+6}$ $\quad 70+2$

$3\overline{)288}$ $\quad 96$

$3\overline{)\quad+\quad}$

$3\overline{)270+18}$ $\quad 90+6$

Discovering Easier Ways - Worksheet 11

Solve these problems. Use your expanded tables to help you.

17	10+ 7	15	10+ 5
3 | 57	3 | 30+ 21	3 | 45	3 | 30 + 15
27	20+ 7	25	20+ 5
3 | 81	3 | 60+ 21	3 | 75	3 | 60 + 15
42	40+ 2	32	30+ 2
3 | 126	3 | 120+ 6	3 | 96	3 | 90 + 6
54	50+ 4	43	40+ 3
3 | 162	3 | 150+ 12	3 | 129	3 | 120+ 9
63	60+ 3	54 R2	50+ 4 R2
3 | 189	3 | 180+ 9	3 | 164	3 | 150+ 14

Breaking Up Tens and Ones - Fours - Worksheet 1

4 x 3 = 12	4 x 5 = 20	4 x 2 = 8	4 x 6 = 24	4 x 10 = 40
4 x 4 = 16	4 x 1 = 4	4 x 8 = 32	4 x 7 = 28	4 x 9 = 36

7	50	600
4 | 28	4 | 200	4 | 4,800
5	70	100
4 | 20	4 | 280	4 | 400
4	30	400
4 | 16	4 | 120	4 | 1,600
3	20	200
4 | 12	4 | 80	4 | 800
6	40	300
4 | 24	4 | 160	4 | 1,200

Breaking Up Tens and Ones - Fours - Worksheet 2

1 x 4 = 4	10 x 4 = 40	100 x 4 = 400
2 x 4 = 8	20 x 4 = 80	200 x 4 = 800
3 x 4 = 12	30 x 4 = 120	300 x 4 = 1,200
4 x 4 = 16	40 x 4 = 160	400 x 4 = 1,600
5 x 4 = 20	50 x 4 = 200	500 x 4 = 2,000
6 x 4 = 24	60 x 4 = 240	600 x 4 = 2,400
7 x 4 = 28	70 x 4 = 280	700 x 4 = 2,800
8 x 4 = 32	80 x 4 = 320	800 x 4 = 3,200
9 x 4 = 36	90 x 4 = 360	900 x 4 = 3,600

Estimate			
_____	11	=	10+ 1 = 11
	4 | 44		4 | 40 + 4
_____	23	=	20+ 3 = 23
	4 | 92		4 | 80 + 12
_____	29	=	20+ 9 = 29
	4 | 116		4 | 80 + 36
_____	52	=	50+ 2 = 52
	4 | 208		4 | 200 + 8
_____	54	=	50+ 4 = 54
	4 | 216		4 | 200 + 16
_____	82	=	80+ 2 = 82
	4 | 328		4 | 320 + 8

Breaking Up Tens and Ones - Fours - Worksheet 3

Estimate			
_____	22	=	20+ 2 = 22
	4 | 88		4 | 80 + 8
_____	95	=	90+ 5 = 95
	4 | 380		4 | 360 + 20
_____	53	=	50+ 3 = 53
	4 | 212		4 | 200 + 12
_____	62	=	60+ 2 = 62
	4 | 248		4 | 240 + 8
_____	62 R2	=	60+ 2 R2 = 62 R2
	4 | 250		4 | 240 + 10
_____	75	=	70+ 5 = 75
	4 | 300		4 | 280 + 20
_____	99	=	90+ 9 = 99
	4 | 396		4 | 360 + 36

Breaking Up Tens and Ones - Fives - Worksheet 1

5 x 7 = 35	5 x 6 = 30	5 x 9 = 45	5 x 10 = 50	5 x 5 = 25
5 x 4 = 20	5 x 2 = 10	5 x 3 = 15	5 x 8 = 40	5 x 1 = 5

6	40	900
5 | 30	5 | 200	5 | 4,500
5	70	100
5 | 25	5 | 350	5 | 500
9	30	200
5 | 45	5 | 150	5 | 1,000
3	20	500
5 | 15	5 | 100	5 | 2,500
7	90	300
5 | 35	5 | 450	5 | 1,500

Breaking Up Tens and Ones - Fives - Worksheet 2

10 x 5 = 50	40 x 5 = 200	70 x 5 = 350
20 x 5 = 100	50 x 5 = 250	80 x 5 = 400
30 x 5 = 150	60 x 5 = 300	90 x 5 = 450

Estimate		
____	13 / 5 | 65	= 5 | 10 + 3 = 13 / 50 + 15
____	17 / 5 | 85	= 5 | 10 + 7 = 17 / 50 + 35
____	47^{R1} / 5 | 236	= 5 | $40 + 7^{R1} = 47^{R1}$ / 200 + 36
____	96 / 5 | 480	= 5 | 90 + 6 = 96 / 450 + 30
____	66 / 5 | 330	= 5 | 60 + 6 = 66 / 300 + 30
____	15^{R2} / 5 | 77	= 5 | $10 + 5^{R2} = 15^{R2}$ / 50 + 27

Breaking Up Tens and Ones - Fives - Worksheet 3

50 x 5 = 250	10 x 5 = 50	70 x 5 = 350
80 x 5 = 400	30 x 5 = 150	40 x 5 = 200
60 x 5 = 300	90 x 5 = 450	20 x 5 = 100

Estimate		
____	54 / 5 | 270	= 5 | 50 + 4 = 54 / 250 + 20
____	70 / 5 | 350	= 5 | 70 + 0 = 70 / 350 + 0
____	71 / 5 | 355	= 5 | 70 + 1 = 71 / 350 + 5
____	72 / 5 | 360	= 5 | 70 + 2 = 72 / 350 + 10
____	91 / 5 | 455	= 5 | 90 + 1 = 91 / 450 + 5
____	85 / 5 | 425	= 5 | 80 + 5 = 85 / 400 + 25

Breaking Up Tens and Ones - Sixes- Worksheet 1

6 x 1 = 6	6 x 2 = 12	6 x 3 = 18	6 x 4 = 24	6 x 5 = 30
6 x 6 = 36	6 x 7 = 42	6 x 8 = 48	6 x 9 = 54	6 x 10 = 60

9	50	700
6 | 54	6 | 300	6 | 4,200
6	40	100
6 | 36	6 | 240	6 | 600
8	20	300
6 | 48	6 | 120	6 | 1,800
2	30	400
6 | 12	6 | 180	6 | 2,400
3	80	600
6 | 18	6 | 480	6 | 3,600

Breaking Up Tens and Ones - Sixes- Worksheet 2

10 x 6 = 60	40 x 6 = 240	70 x 6 = 420
20 x 6 = 120	50 x 6 = 300	80 x 6 = 480
30 x 6 = 180	60 x 6 = 360	90 x 6 = 540

Estimate

_____ $6\overline{)65} = 6\overline{)60+5}$ $10^{R5} = 10^{R5}$

_____ $6\overline{)72} = 6\overline{)60+12}$ $12 = 12$

_____ $6\overline{)75} = 6\overline{)60+15}$ $12^{R3} = 12^{R3}$

_____ $6\overline{)132} = 6\overline{)120+12}$ $22 = 22$

_____ $6\overline{)148} = 6\overline{)120+28}$ $24^{R4} = 24^{R4}$

_____ $6\overline{)246} = 6\overline{)240+6}$ $41 = 41$

Breaking Up Tens and Ones - Sixes- Worksheet 3

40 x 6 = 240	10 x 6 = 60	20 x 6 = 120
90 x 6 = 540	30 x 6 = 180	80 x 6 = 480
70 x 6 = 420	60 x 6 = 360	50 x 6 = 300

Estimate

_____ $6\overline{)84} = 6\overline{)60+24}$ $14 = 14$

_____ $6\overline{)126} = 6\overline{)120+6}$ $21 = 21$

_____ $6\overline{)252} = 6\overline{)240+12}$ $42 = 42$

_____ $6\overline{)260} = 6\overline{)240+20}$ $43^{R2} = 43^{R2}$

_____ $6\overline{)500} = 6\overline{)480+20}$ $83^{R2} = 83^{R2}$

_____ $6\overline{)546} = 6\overline{)540+6}$ $91 = 91$

Breaking Up Tens and Ones - Sixes- Worksheet 4

60 x 6 = 360	20 x 6 = 120	90 x 6 = 540
20 x 6 = 60	30 x 6 = 180	80 x 6 = 480
70 x 6 = 420	40 x 6 = 240	50 x 6 = 300
600 x 6 = 3,600	100 x 6 = 600	500 x 6 = 3,000
400 x 6 = 2,400	000 x 6 = 5,400	800 x 6 = 4,800
300 x 6 = 1,800	200 x 6 = 1,200	700 x 6 = 4,200

Estimate

_____ $6\overline{)252} = 6\overline{)240+12}$ $42 = 42$

_____ $6\overline{)276} = 6\overline{)240+36}$ $46 = 46$

_____ $6\overline{)432} = 6\overline{)420+12}$ $72 = 72$

_____ $6\overline{)572} = 6\overline{)540+32}$ $95^{R2} = 95^{R2}$

_____ $6\overline{)426} = 6\overline{)420+6}$ $71 = 71$

_____ $6\overline{)354} = 6\overline{)300+54}$ $59 =$

Breaking Up Tens and Ones - Sixes- Worksheet 5

Estimate

_____ $6\overline{)235} = 6\overline{)180+55}$ $39^{R1} = 30+9^{R1}$

_____ $6\overline{)243} = 6\overline{)240+3}$ $40^{R3} = 40+0^{R3}$

_____ $6\overline{)432} = 6\overline{)420+12}$ $72 = 70+2$

_____ $6\overline{)360} = 6\overline{)360}$ $60 = 60$

Breaking Up Tens and Ones - Sevens - Worksheet 1

7 x 3 = 21	7 x 2 = 14	7 x 7 = 49	7 x 6 = 42	7 x 4 = 28
7 x 5 = 35	7 x 9 = 63	7 x 1 = 7	7 x 8 = 56	7 x 10 = 70

9	50	900
7 | 63	7 | 350	7 | 6,300
6	80	100
7 | 42	7 | 560	7 | 700
8	40	200
7 | 56	7 | 280	7 | 1,400
12	20	700
7 | 84	7 | 140	7 | 4,900
7	90	300
7 | 49	7 | 630	7 | 2,100

Breaking Up Tens and Ones - Sevens - Worksheet 2

Estimate

_____ $7\overline{)49} = 7\overline{)0+7} = 7$; $7\overline{)0+49}$

_____ $7\overline{)77} = 7\overline{)10+1} = 11$; $7\overline{)70+7}$

_____ $7\overline{)84} = 7\overline{)10+2} = 12$; $7\overline{)70+14}$

_____ $7\overline{)91} = 7\overline{)10+3} = 13$; $7\overline{)70+21}$

Breaking Up Tens and Ones - Sevens - Worksheet 3

Estimate

_____ $7\overline{)200} = 28\ R4 = 7\overline{)140+60}$

_____ $7\overline{)210} = 30 = 7\overline{)210+0}$

_____ $7\overline{)420} = 60 = 7\overline{)420+0}$

_____ $7\overline{)560} = 80 = 7\overline{)560+0}$

Breaking Up Tens and Ones - Sevens - Worksheet 4

10 x 7 = 70	40 x 7 = 280	70 x 7 = 490
20 x 7 = 140	50 x 7 = 350	80 x 7 = 560
30 x 7 = 210	60 x 7 = 420	90 x 7 = 630

Estimate

_____ $7\overline{)147} = 21 = 7\overline{)140+7}$

_____ $7\overline{)637} = 91 = 7\overline{)630+7}$

_____ $7\overline{)490} = 70 = 7\overline{)490+0}$

_____ $7\overline{)500} = 71\ R3 = 7\overline{)490+10}$

_____ $7\overline{)245} = 35 = 7\overline{)210+35}$

_____ $7\overline{)574} = 82 = 7\overline{)560+14}$

77

Breaking Up: Tens and Ones - Eights - Worksheet 1

8 x 2 = 16	8 x 10 = 80	8 x 3 = 24	8 x 1 = 8	8 x 7 = 56
8 x 4 = 32	8 x 6 = 48	8 x 8 = 64	8 x 9 = 72	8 x 5 = 40

8 ⟌ 64	300 ⟌ 2,400	200 ⟌ 1,600
6 ⟌ 48	80 ⟌ 640	100 ⟌ 800
9 ⟌ 72	20 ⟌ 160	900 ⟌ 7,200
11 ⟌ 88	40 ⟌ 320	610 ⟌ 4,880
3 ⟌ 24	60 ⟌ 480	70 ⟌ 560

Breaking Up: Tens and Ones - Eights - Worksheet 3

Estimate

_____	8 ⟌ 64 = 8 ⟌ 0 + 64
_____	13 R4 ⟌ 108 = 8 ⟌ 80 + 28 = 13 R4
_____	26 ⟌ 208 = 8 ⟌ 160 + 48 = 20 + 6 = 26
_____	38 R4 ⟌ 308 = 8 ⟌ 240 + 68 = 30 + 8 R4 = 38 R4

Breaking Up: Tens and Ones - Eights - Worksheet 4

Instructiions only

Breaking Up: Tens and Ones - Eights - Worksheet 2

80 x 8 = 640	90 x 8 = 720	30 x 8 = 240
50 x 8 = 400	70 x 8 = 560	60 x 8 = 480
40 x 8 = 320	10 x 8 = 80	20 x 8 = 160

Estimate

_____	11 ⟌ 88 = 8 ⟌ 80 + 8 = 10 + 1 = 11
_____	12 ⟌ 96 = 8 ⟌ 80 + 16 = 10 + 2 = 12
_____	13 ⟌ 104 = 8 ⟌ 80 + 24 = 10 + 3 = 13
_____	18 R6 ⟌ 150 = 8 ⟌ 80 + 70 = 10 + 8 R6 = 18 R6
_____	21 R1 ⟌ 169 = 8 ⟌ 160 + 9 = 20 + 1 R1 = 21 R1
_____	31 ⟌ 248 = 8 ⟌ 240 + 8 = 20 + 1 = 21

Breaking Up: Tens and Ones - Eights - Worksheet 5

Estimate	Short way	Tens and Ones
_____	42 ⟌ 336 − 320 = 16 − 16 = 0	= 8 ⟌ 320 + 16 = 40 + 2 = 42
_____	53 R1 ⟌ 425 − 400 = 25 − 24 = 1	= 8 ⟌ 400 + 25 = 50 + 3 R1 = 53 R1
_____	26 ⟌ 208 − 160 = 48 − 48 = 0	= 8 ⟌ 160 + 48 = 20 + 6 = 26

Breaking Up Tens and Ones - Nines- Worksheet 1

9 x 2 = 18	9 x 4 = 36	9 x 8 = 72	9 x 5 = 45	9 x 7 = 63
9 x 1 = 9	9 x 9 = 81	9 x 6 = 54	9 x 3 = 27	9 x 10 = 90

9 | 63 = 7	9 | 2,700 = 300	9 | 1,800 = 200
9 | 45 = 5	9 | 630 = 70	9 | 900 = 100
9 | 72 = 8	9 | 720 = 80	9 | 8,100 = 900
9 | 99 = 11	9 | 360 = 40	9 | 4,500 = 500
9 | 27 = 3	9 | 810 = 90	9 | 540 = 60

Breaking Up Tens and Ones - Nines- Worksheet 2

10 x 9 = 90	20 x 9 = 180	70 x 9 = 630
30 x 9 = 270	50 x 9 = 450	80 x 9 = 720
40 x 9 = 360	60 x 9 = 540	90 x 9 = 810

Estimate

_____	9 | 99 = 11	9 | 90 + 9 = 10 + 1 = 11
_____	9 | 168 = 18 R6	9 | 90 + 78 = 10 + 8 R6 = 18 R6
_____	9 | 100 = 11 R1	9 | 90 + 10 = 10 + R1 = 11 R1
_____	9 | 200 = 22 R2	9 | 180 + 20 = 20 + 2 R2 = 22 R2
_____	9 | 279 = 31	9 | 270 + 9 = 30 + 1 = 31

Breaking Up Tens and Ones - Nines- Worksheet 3

9 | 468 = 52 = 9 | 450 + 18 = 50 + 2 = 52

9 | 477 = 53 = 9 | 450 + 27 = 50 + 3 = 53

9 | 616 = 68 R4 = 9 | 540 + 76 = 60 + 8 R4 = 68 R4

9 | 909 = 101 = 9 | 900 + 9 = 100 + 1 = 101

9 | 558 = 62 = 9 | 540 + 18 = 60 + 2 = 62

Show You Know

1. Solve this problem.

4 | 40 + 24 + 12 = 76 = 10 6 3 = 19

Break it up a different way. Is the answer the same? yes

BuWV

4 | __ + __ + __ + __ = Why? The dividend is equal to 76 in each problem. It can be broken up any way and there will still be 19 groups of 4 in the whole.

How can you break up the problem so the answer is in tens and ones.

4 | 40 + 36 = 10 + 9 = 19

2. Make your own family of facts. Example:

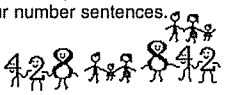
8 x 4 = 32
4 x 8 = 32
32 ÷ 8 = 4
32 ÷ 4 = 8

3. Break up this problem three ways. 8 | 520

Three numbers	Short way	Tens and Ones
BuWV	65	600 + 5 = 65
8 | ___ + ___ +	8 | 520, -480, 40, -40	8 | 480 + 40

Families of Facts - Worksheet 1

Fill in the blank space with a number that completes a family of facts.
Use only the three numbers in the box to make four number sentences.



3 150 50

3 × 50 = 150
50 × 3 = 150
150 ÷ 3 = 50
150 ÷ 50 = 50

90 540 6

90 × 6 = 540
6 × 90 = 540
540 ÷ 90 = 6
540 ÷ 6 = 90

80 480 6

80 × 6 = 480
6 × 80 = 480
480 ÷ 80 = 6
480 ÷ 6 = 80

AWV

420 ___ ___

___ × ___ = ___
___ × ___ = ___
___ ÷ ___ = ___
___ ÷ ___ = ___

720 ___ ___

___ × ___ = ___
___ × ___ = ___
___ ÷ ___ = ___
___ ÷ ___ = ___

For you:

___ × ___ = ___
___ × ___ = ___
___ ÷ ___ = ___
___ ÷ ___ = ___

Expanded Division: Practice - Worksheet 1

BUWV

$3\overline{)2{,}439}$ = 3 [___ + ___ + ___] = ___ + ___ + ___

$3\overline{)477}$ = 159 = 3 [___ + ___ + ___] = ___ + ___ + ___

$3\overline{)2{,}124}$ = 708 = 3 [___ + ___ + ___] = ___ + ___ + ___

$3\overline{)3{,}063}$ = 1,021 = 3 [___ + ___ + ___] = ___ + ___ + ___

$3\overline{)1{,}246}$ = 415 R1 = 3 [___ + ___ + ___] = ___ + ___ + ___

$3\overline{)756}$ = 252 = 7 [___ + ___ + ___] = ___ + ___ + ___

Expanded Division: Practice - Worksheet 2

BUWV

$4\overline{)3{,}625}$ = 906 R1 = 4 [___ + ___ + ___] = ___ + ___ + ___

$4\overline{)756}$ = 189 = 4 [___ + ___ + ___] = ___ + ___ + ___

$4\overline{)1{,}020}$ = 255 = 4 [___ + ___ + ___] = ___ + ___ + ___

$4\overline{)2{,}884}$ = 721 = 4 [___ + ___ + ___] = ___ + ___ + ___

$4\overline{)2{,}509}$ = 627 R1 = 4 [___ + ___ + ___] = ___ + ___ + ___

$4\overline{)2{,}240}$ = 560 = 4 [___ + ___ + ___] = ___ + ___ + ___

Expanded Division: Practice - Worksheet 3

BUWV

$5\overline{)3{,}590}$ = 718 = 5 [___ + ___ + ___] = ___ + ___ + ___

$5\overline{)750}$ = 150 = 5 [___ + ___ + ___] = ___ + ___ + ___

$5\overline{)2{,}535}$ = 507 = 5 [___ + ___ + ___] = ___ + ___ + ___

$5\overline{)2{,}850}$ = 570 = 5 [___ + ___ + ___] = ___ + ___ + ___

$5\overline{)4{,}510}$ = 902 = 5 [___ + ___ + ___] = ___ + ___ + ___

$5\overline{)1{,}955}$ = 391 = 5 [___ + ___ + ___] = ___ + ___ + ___

Expanded Division: Practice - Worksheet 4

$6\overline{)1{,}296}$ = 216 BUWV = $6\overline{)\quad+\quad+\quad}$ = ___+___+___

$6\overline{)5{,}304}$ = 884 = $6\overline{)\quad+\quad+\quad}$ = ___+___+___

$6\overline{)1{,}482}$ = 247 = $6\overline{)\quad+\quad+\quad}$ = ___+___+___

$6\overline{)432}$ = 72 = $6\overline{)\quad+\quad+\quad}$ = ___+___+___

$6\overline{)7{,}380}$ = 1,230 = $6\overline{)\quad+\quad+\quad}$ = ___+___+___

$6\overline{)2{,}814}$ = 469 = $6\overline{)\quad+\quad+\quad}$ = ___+___+___

Expanded Division: Practice - Worksheet 5

$7\overline{)1{,}456}$ = 235 R3 BUWV = $7\overline{)\quad+\quad+\quad}$ = ___+___+___

$7\overline{)1{,}260}$ = 180 = $7\overline{)\quad+\quad+\quad}$ = ___+___+___

$7\overline{)917}$ = 131 = $7\overline{)\quad+\quad+\quad}$ = ___+___+___

$7\overline{)1{,}106}$ = 158 = $7\overline{)\quad+\quad+\quad}$ = ___+___+___

$7\overline{)2{,}149}$ = 178 R2 = $7\overline{)\quad+\quad+\quad}$ = ___+___+___

$7\overline{)1{,}043}$ = 149 = $7\overline{)\quad+\quad+\quad}$ = ___+___+___

Expanded Division: Practice - Worksheet 6

$8\overline{)1{,}160}$ = 145 BUWV = $8\overline{)\quad+\quad+\quad}$ = ___+___+___

$8\overline{)832}$ = 104 = $8\overline{)\quad+\quad+\quad}$ = ___+___+___

$8\overline{)1{,}392}$ = 174 = $8\overline{)\quad+\quad+\quad}$ = ___+___+___

$8\overline{)1{,}480}$ = 185 = $8\overline{)\quad+\quad+\quad}$ = ___+___+___

$8\overline{)1{,}088}$ = 136 = $8\overline{)\quad+\quad+\quad}$ = ___+___+___

$8\overline{)986}$ = 123 R2 = $8\overline{)\quad+\quad+\quad}$ = ___+___+___

Expanded Division: Practice - Worksheet 7

$9\overline{)1{,}134}$ = 126 BUWV = $9\overline{)\quad+\quad+\quad}$ = ___+___+___

$9\overline{)855}$ = 95 = $9\overline{)\quad+\quad+\quad}$ = ___+___+___

$9\overline{)2{,}916}$ = 324 = $9\overline{)\quad+\quad+\quad}$ = ___+___+___

$9\overline{)918}$ = 102 = $9\overline{)\quad+\quad+\quad}$ = ___+___+___

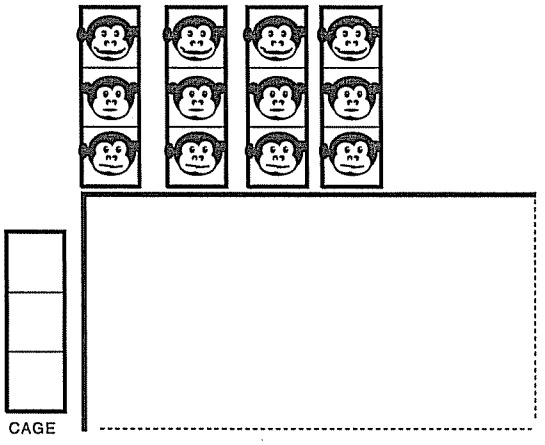
$9\overline{)1{,}097}$ = 121 R8 = $9\overline{)\quad+\quad+\quad}$ = ___+___+___

$9\overline{)585}$ = 65 = $9\overline{)\quad+\quad+\quad}$ = ___+___+___

Pre-Assessment - Part 2 is the same as Post-Assessment

Monkeys on the Roof - Worksheet 1

Once upon a time there was a whole batch of baby monkeys who got loose from a zoo and found their way into Mrs. Querel's house. She called the zookeeper, who came over with some cages. The cages looked like little towers with three little doors. The zookeeper went into the house and put three baby monkeys into each cage. Then he called a helicopter to come and pick up the cages. The helicopter pilot wanted the cages on the roof of the house so he could get them easily with his hook. So the zookeeper tossed the cages up on the roof. The pilot grabbed them with a hook and took them all back to the zoo. Mrs. Querel's house was all messed up but empty of baby monkeys. The zookeeper stayed to help clean up the mess.

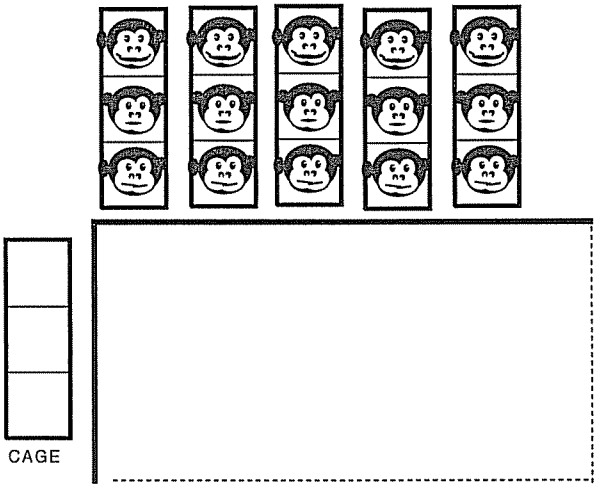


Cut out the12 baby monkeys. Put all the baby monkeys into the house. Now cut out a cage. Paste three baby monkeys into each cage and put the cage on the roof. Keep doing that until all those frisky monkeys are in cages on the roof.

1

Monkeys on the Roof - Worksheet 2

Oh no! The baby monkeys came back! This time they brought their friends too. Now there are 15 monkeys. How many cages will the zookeeper need this time?



Cut out the15 baby monkeys. Put all the baby monkeys into the house. Now cut out a cage. Paste three baby monkeys into each cage and put the cage on the roof. Keep doing that until all those frisky monkeys are in cages on the roof.

2

Monkeys on the Roof - Worksheet 3

Number of Cages on the Roof

Cage Size | Number of Monkeys in the house

$3\overline{)18}$ → 6 Cages, Monkeys

$3\overline{)24}$ → 8 Cages, Monkeys

$3\overline{)21}$ → 7 Cages, Monkeys

$3\overline{)30}$ → 10 Cages, Monkeys

$3\overline{)9}$ → 3 Cages, Monkeys

$3\overline{)27}$ → 9 Cages, Monkeys

$3\overline{)33}$ → 11 Cage, Monkeys

3

Monkeys on the Roof - Worksheet 4

Number of Cages on the Roof

Cage Size | Number of Monkeys in the house

$4\overline{)12}$ → 3

$4\overline{)16}$ → 4

$4\overline{)24}$ → 6

$4\overline{)32}$ → 8

$4\overline{)8}$ → 2

$4\overline{)20}$ → 5

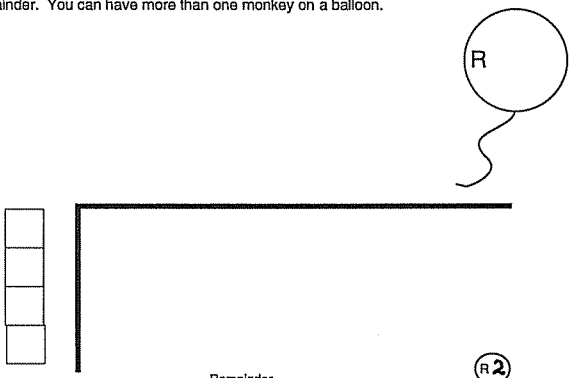
$4\overline{)4}$ → 1

4

Monkeys on the Roof - Worksheet 5

Oops! The monkeys on the roof don't all fit in the cages.

What if there are 13 monkeys? You can get 12 of the monkeys into the cages, but there is one left over. The zookeeper did not want to use another cage. So he tied the baby monkey to a balloon and floated it up to the helicopter. The monkey on the balloon is called a remainder. You can have more than one monkey on a balloon.



Remainder

Number of Full Cages on the Roof

Cage Size | Number of Monkeys in the house

$4\overline{)14}$ → 3 R2

$4\overline{)11}$ → 2 R3

$4\overline{)9}$ → 2 R1

$4\overline{)15}$ → 3 R3

$4\overline{)7}$ → 1 R3

$4\overline{)21}$ → 5 R1

$4\overline{)5}$ → 1 R1

5

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