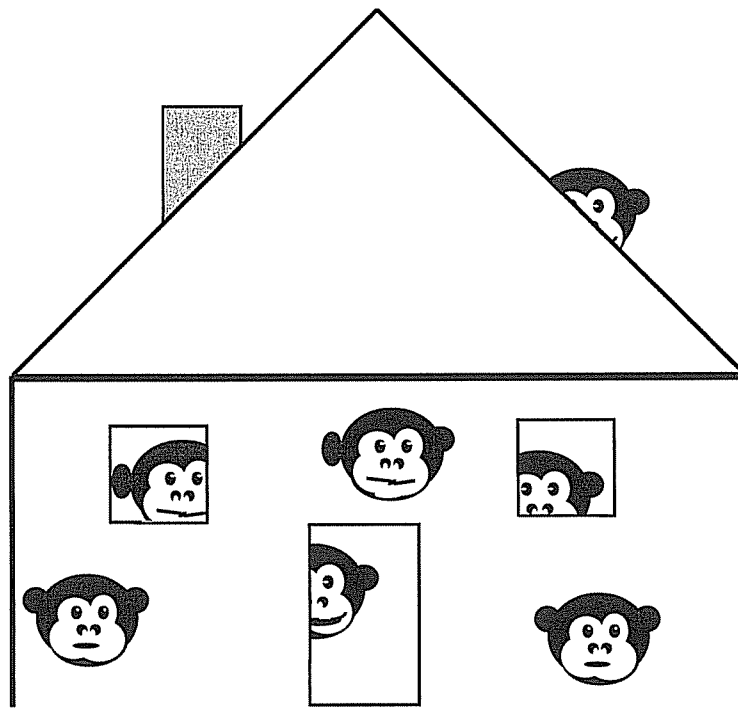


Patterns in Arithmetic
Division - Booklet 1 PDF
Basic Concepts
Parent/Teacher Guide



By Alysia Krafel, Susan Carpenter, and Suki Glenn

Illustrations by Karen Minns and Suki Glenn

Based on methods developed by Prof. Michael Butler at the

UCI Farm Elementary School
University of California, Irvine

and

Chrysalis Charter School

Division: Booklet 1 - PDF - Basic Concepts

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This booklet is dedicated to Paul Krafel. His dedication and leadership at Chrysalis Charter School has kept the principles of the Farm School alive and flourishing.

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For many years Farm School teachers, students, parents, and staff have shared their unfailing delight in learning. Thank you for your support and dedication.

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The cover mandala and many delightful illustrations are by Karen Marie Christa Minns. Other illustrations are by Suki Glenn and ClickArt by T/Maker.

To all of the mathematicians, from antiquity to the present, who discovered the principles of mathematics goes our heartfelt appreciation for your dedication.

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Put a question mark next to any question you don't know how to do.

1. Start with eight blocks.  Draw the blocks.

- a. How many groups of two can you make? _____
- b. How many groups of four can you make? _____
- c. How many groups of one can you make? _____
- d. How many groups of eight can you make? _____

2. Gracie had six puppies and some big boxes. She put three puppies in each box.

- a. How many boxes did she use? _____
- b. Draw a picture.


3. Use Cuisenaire Rods to build if needed.

This is a 12 white rod train. Record leftovers as remainders.

Circle groups of: Example: Number of Groups

2  ÷ 2 = 6

a. 3  ÷ 3 = _____

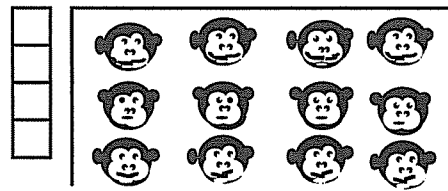
b. 4  ÷ 4 = _____

c. 5  ÷ 5 = _____

d. 7  ÷ 7 = _____

Put a question mark next to any question you don't know how to do.

1. Mr. Zoo had 12 monkeys and some cages.
Each cage holds 4 monkeys.
How many cages does Mr. Zoo need for his monkeys?



2. Write the division number sentence for problem one.

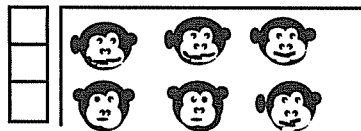
	\div	$=$
--	--------	-----

3. a. What question does this number sentence ask?

24	\div	4	$=$
----	--------	---	-----

- b. What is the answer? _____

4. a. Draw the answer.



	\div	$=$
--	--------	-----

- b. Write the division number sentence.

5. a.

$$2 \overline{) 8}$$

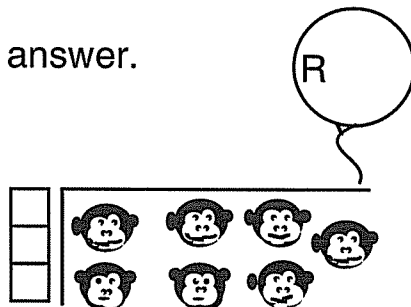
- b.

$$5 \overline{) 15}$$

- c.

$$4 \overline{) 32}$$

6. a. Draw the answer.

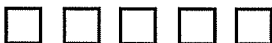



	\div	$=$
--	--------	-----

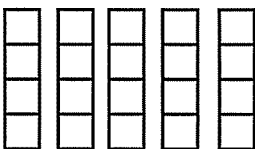
- b. Write the division number sentence.

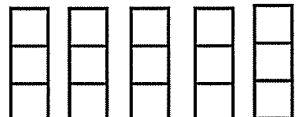
7. a. $4 \overline{)18}$ b. $5 \overline{)32}$ c. $3 \overline{)23}$

8. In the problem $20 \div 4 = 5$, 5 is the 'answer.'
 Draw a box around any drawings below that show what the 5 means.

a. 

b. 

c. 

d. 

9. Make a Family of Facts with these three numbers.

a. $\underline{3} \times \underline{6} = 18$
 $\underline{\quad} \times \underline{\quad} =$
 $\underline{\quad} \div \underline{\quad} =$
 $\underline{\quad} \div \underline{\quad} =$

b. $\begin{array}{r} 3 \\ \times 6 \\ \hline 18 \end{array}$ $\begin{array}{r} \underline{\quad} \\ \times \underline{\quad} \\ \hline \underline{\quad} \end{array}$

3	6	18
---	---	----

$\begin{array}{r} \underline{\quad} \\ \hline \end{array}$ $\begin{array}{r} \underline{\quad} \\ \hline \end{array}$

10. These are hard.
 Fill in the missing numbers.

a. $8 \div 2 = \square$

b. $15 \div \square = 3$

c. $\square \div 4 = 5$

Assessment Guide

Purpose The purpose of this guide is to assess the fundamental knowledge necessary for success in this booklet. Pre-Assessment: Part 1 is a preview of the material presented in *Patterns in Arithmetic: Book 2* and is used to set the baseline for what the student already knows at the beginning of instruction. If your student is new to the *Patterns in Arithmetic* program Pre-Assessment: Part 1 is an appropriate starting place.

Pre-Assessment: Part 2 is a preview of the new material presented in this booklet.

The Post-Assessment is administered to determine if the student learned the material that was presented in this booklet. A comparison of the score on Pre-Assessment: Part 2 to the score on the Post-Assessment will give both you and your student a sense of growth. The Post-Assessment is the same as Pre-Assessment: Part 2.

Prerequisites *Patterns in Arithmetic: Multiplication - Booklet 1*

Materials Division: Booklet 1, Pre-Assessment: Part 1, page 1 and Pre-Assessment: Part 2, pages 53 and 54.
Pre-Assessment: Part 1, Pre-Assessment: Part 2 and Post-Assessment Score Sheets
Counters
Cuisenaire Rods

Instructions Instruct the student to attempt all the problems. If he does not know how to do a problem, he should put a question mark by it. This will let you know he looked at the item and decided he could not do it. Do the assessment in two parts. Give Pre-Assessment: Part 1 and check it for readiness for this booklet. If he is not ready for this booklet, there is no point in giving Pre-Assessment: Part 2. If he passes all the readiness items, then give Pre-Assessment: Part 2.

It is acceptable to read the items to a student. We are assessing math, not reading. Do not explain any items to him. If he does not know what the question means, tell him to put a question mark on that item.

Assessment Guide This Assessment Guide explains what concept each item on the test is assessing. The item numbers match the item numbers on the student test page. The title of the lesson and Booklet number tell you where the concept is taught. In the Assessment Guide, under each lesson title are several assessment criteria. Each criterion is labeled with capital letters 'A,' 'B,' etc. These criteria tell you what to look for in the student work. On the student test, sometimes multiple problems are used to test a concept. These multiple problems are labeled with small letters 'a,' 'b,' etc.

Score sheets that match the Assessment Guide for Pre-Assessment: Part 1 and Pre-Assessment: Part 2, and the Post-Assessment follow.

Assessment Criteria for Pre-Assessment: Part 1

Can the student:

1. How Many Stacks? (*Patterns in Arithmetic: Book 2*)
 - A. divide a set amount of counters into equal groups?
 - B. draw a picture of the solution?
 - C. record the answers?
2. Word Problems (*Patterns in Arithmetic: Book 2*)
 - A. give the correct answer to a basic division word problem?
 - B. draw a picture of the problem?
3. Trains on Tracks (*Patterns in Arithmetic: Book 2*)
 - A. divide twelve by circling equal groups?
 - B. record the number of groups?
 - C. record leftover rods as a remainder?

Assessment Criteria for Pre-Assessment: Part 2

Can the student:

1. Word Problems (Division: Booklet 1)
 - A. give the correct answer to a basic division word problem?
 - B. use a graphic to find the answer?
2. Number Sentence
 - A. write the division number sentence for $12 \div 4 = 3$?
3. Number Sentence
 - A. write 'How many groups of four are in twenty-four?'?
 - B. give the correct answer 6?
4. Monkeys on the Roof (Basic division problem)
 - A. draw the answer?
 - B. write the division number sentence?
5. Recording (Basic division problem)
 - A. give the correct answer on two of three problems?
6. Monkeys on the Roof (Basic division problem with remainders)
 - A. draw the answer?
 - B. write the division number sentence?
7. Remainders (Division problem with remainders)
 - A. give the correct answer on two of three problems?
8. Number Sentence
 - A. circle one correct representation of $20 \div 4 = 5$? The 5 can be either the number of groups as shown in c or the number in each group as shown in b.
 - B. circle another correct representation of $20 \div 4 = 5$?
9. Family of Facts: Patterns
 - A. fill in all the numbers in set a?
 - B. fill in all the numbers in set b?
10. Missing Factors
 - A. supply the missing number in two of three problems?

Can the student:

1. How Many Stacks?

- Yes No A. divide a set amount of counters into equal groups for three of four answers?
- Yes No B. draw a picture of the solution?
- Yes No C. record the answers for three of four answers?

2. Word Problems

- Yes No A. give the correct answer to a basic division word problem?
- Yes No B. draw a picture of the problem?

3. Trains on Tracks

- Yes No A. divide twelve by circling equal groups in three of four problems?
- Yes No B. record the number of groups in three of four problems?
- Yes No C. record leftover rods as a remainder in one of two problems?

Items Correct = ____ = _____% 75% needed to begin Division: Booklet 1

Items Possible = 8

If the student scores 75% or better, this is six or more Yes items, proceed to Pre-assessment - Part 2.

If the student scores less than 75%, this is five or less Yes items, proceed to review Division in *Patterns in Arithmetic: Book 2*.

Can the student:

1. Word Problems (Monkey problem)

- Yes No A. give the correct answer to a basic division word problem?
 Yes No B. use a graphic to find the answer?

2. Number Sentence

- Yes No A. write the division number sentence for $12 \div 4 = 3$?

3. Number Sentence

- Yes No A. write 'How many groups of four are in twenty-four?'
 Yes No B. give the correct answer of 6?

4. Monkeys on the Roof (Basic division problem)

- Yes No A. draw the answer?
 Yes No B. write the division number sentence?

5. Recording (Basic division problem)

- Yes No A. give the correct answer on two of three problems?

6. Monkeys on the Roof (Basic division problem with remainders)

- Yes No A. draw the answer?
 Yes No B. write the division number sentence?

7. Remainders (Division problem with remainders)

- Yes No A. give the correct answer on two of three problems?

8. Number Sentence

- Yes No A. circle one correct representation of $20 \div 4 = 5$?
 Yes No B. circle another correct representation of $20 \div 4 = 5$?

9. Family of Facts: Patterns

- Yes No A. fill in all the numbers in set a?
 Yes No B. fill in all the numbers in set b?

10. Missing Factors

- Yes No A. supply the missing number in two of three problems?

Items Correct = _____ = _____% 80% needed to begin Division: Booklet 2.
 Items Possible = 16 This is 13 or more Yes items.

Can the student:

1. Word Problems (Monkey problem)

- Yes No A. give the correct answer to a basic division word problem?
 Yes No B. use a graphic to find the answer?

2. Number Sentence

- Yes No A. write the division number sentence for $12 \div 4 = 3$?

3. Number Sentence

- Yes No A. write 'How many groups of four are in twenty-four?'?
 Yes No B. give the correct answer of 6?

4. Monkeys on the Roof (Basic division problem)

- Yes No A. draw the answer?
 Yes No B. write the division number sentence?

5. Recording (Basic division problem)

- Yes No A. give the correct answer on two of three problems?

6. Monkeys on the Roof (Basic division problem with remainders)

- Yes No A. draw the answer?
 Yes No B. write the division number sentence?

7. Remainders (Division problem with remainders)

- Yes No A. give the correct answer on two of three problems?

8. Number Sentence

- Yes No A. circle one correct representation of $20 \div 4 = 5$?
 Yes No B. circle another correct representation of $20 \div 4 = 5$?

9. Family of Facts: Patterns

- Yes No A. fill in all the numbers in set a?
 Yes No B. fill in all the numbers in set b?

10. Missing Factors

- Yes No A. supply the missing number in two of three problems?

Items Correct = _____ = _____% 80% needed to begin Division - Booklet 2.
 Items Possible = 16 This is 13 or more Yes items.

Concept


Purpose	The purpose is to review the concept of division physically and to link division to successive subtraction.
Prerequisites	Understanding of multiplication
Materials	All of the students in a classroom
Lesson	<p>"Stand up! Quickly." Have all the students get into groups of two.</p> <p>"How many groups? Are there any left over?"</p> <p>Now have the students quickly get into groups of three.</p> <p>"How many groups? Are there any left over?"</p> <p>"Are there more or less groups than before?"</p> <p>"Predict if there will be fewer or more groups of four. Predict how many groups of four. Quickly get into groups of four."</p> <p>"How were your predictions? Who would like to share their thoughts about the predictions?"</p> <p>"Predict if there will be fewer or more groups of five. Predict how many groups of five. Quickly get into groups of five."</p> <p>"How were your predictions? Who would like to share their thoughts about the predictions?"</p> <p>Have the class skip count by fives to one hundred. Skip counting can be done together as a chant. "Five, ten, fifteen, twenty ... one hundred."</p> <p>Next, skip count backwards from one hundred.</p>
Worksheets	Garden Word Problems, Worksheet 1 and 2, pages 3 and 4

Number Sentence

Purpose The purpose is to develop language associated with division and to develop the division number sentence: $x \div y = z$.

Prerequisites Understanding of multiplication

Materials Number Sentence - Worksheets 1 - 3, pages 5 - 7
Unifix cubes, blocks, or tiles
Cuisenaire Rods are helpful in Session 3.
A large piece of paper or tag board for making a chart, and a pencil

Lesson Session 1 Warm up with a review of number sentences. The teacher builds: 
"What addition problem is this?" "Two plus two plus two."
"Write that as a number sentence." " $2 + 2 + 2 = 6$."
"Say it as a multiplication sentence." "Two times three equals six."
"Write that as a number sentence." " $2 \times 3 = 6$."
"Explain how you get the numbers two, three, and six from the model." "Two are in each stack, there are three stacks, which equals a total of six."

Take out twelve blocks.

"How many even groups of two can you make out of the twelve blocks?" *Give enough time to manipulate the blocks.* "Six."

"How many even groups of three can you make?" "Four."

"How many even groups of four can you make?" "Three."

"How many even groups of five can you make?" "You can't make even groups of five." Or, "Two groups with two left over."

"How many even groups of six can you make?" "Two."

"How many even groups of one can you make?" "Twelve."

"How many even groups of twelve can you make?" "One."

Repeat the same process with nine blocks. Have the student make groups of one, three, and nine.

Take out ten blocks. **"Find all the ways to make equal groups from ten blocks. No leftovers are allowed."** "One group of ten, two groups of five, five groups of two, and ten groups of one."

If she has difficulty finding all the ways, tell her that there are four ways to group ten. Encourage students working in a group to help each other.

Repeat the same questions a few times using different numbers of blocks.

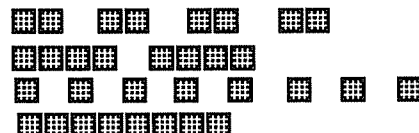
Worksheets

Number Sentence: Worksheets 1 and 2, pages 5 and 6 Solve Set 1 together.

Problem Set 1

Start with eight blocks.

- How many groups of two can you make?
- How many groups of four can you make?
- How many groups of one can you make?
- How many groups of eight can you make?



**Session 2
Materials**

This section is better done with cube blocks, tiles, or beans instead of Cuisenaire Rods, because the student can more easily see what is happening to the individual units.

Have the student bring the completed Number Sentence - Worksheet 1 to the lesson.

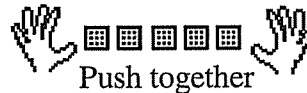
Warm up by building models of different number sentences. Call attention to your hands. Take two blocks, then add three blocks and push all the blocks together.



Start with two



Add three



Push together

"What number sentence goes with what I just did with the blocks?" Repeat the action. $2 + 3 = 5$. Start with two blocks and add three more to make five."

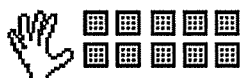
"What number sentence is this?" Put two blocks in a group. Then add three more *groups* of two blocks.

$2 \times 4 = 8$. You start with two and make it four times."



Take a group of two. Make three more groups of two. End with four groups of two.

"What number sentence is this?" Take ten blocks and remove four.



Start with ten.



Take away four.



End up with six.

$10 - 4 = 6$. Start with ten blocks and take away four and have six blocks left."

"Now we'll change each multiplication number sentence that you did on Worksheet 1 into a division number sentence." Begin with problem A in Set 1.

"How many blocks did you start with?" "Eight."

"What size of group did you make?" "Two in each group."

"How many groups of two did you end up with?" "Four groups." Make sure that she says the word groups and not just four. If she says four ask, **"Four what?"**

"These questions identify the three numbers in the division number sentence.

What is the sign for division?" \div Show her this convention if she doesn't know it. Have her draw a few.

To help her grasp the format, make a chart that looks like this:

	\div	$=$
--	--------	-----

"What does the first number of a division number sentence tell you?"

"What you start with." Write that in the first box.

The number that you start with.	\div	$=$
---------------------------------	--------	-----

"What happens in the second box?" "It tells you what to do."

"For a division problem, the numeral after the \div sign tells you, 'The size of the groups you make.'" Write that in the second box.

The number that you start with.	\div The size of the groups you make.	$=$
---------------------------------	---	-----

"What goes in the third box?" "The answer or what you ended up with."

"If I make groups of two out of six blocks, what do I end up with?" "Three groups of two."

"In which box do I write the three?" "In the third box."

In the third box write, "The number of groups you make."

The number that you start with.	\div The size of the groups you make.	$=$ The number of groups you make.
---------------------------------	---	------------------------------------

Have her draw three boxes to the right of the question for Problem Set 1, Problem A.

"In Problem A, how many blocks did you start with?" "Eight."

Have her write 8 in the first box.

"What size of group did you make?" "Two in each group."

Have her write $\div 2$ in the second box.

"How many groups of two did you end up with?" "Four groups."

Have her write $= 4$ in the third box.

The completed problem will look like this:

A. How many groups of two can you make? 4 

8	$\div 2$	$= 4$
---	----------	-------

Repeat the same procedure with the other three problems in Set 1. The number sentences are as follows:

A. $8 \div 2 = 4$ B. $8 \div 4 = 2$ C. $8 \div 1 = 8$ D. $8 \div 8 = 1$

If she has difficulty with C and D, refer to the chart that you made together. Remind her that the first numeral in the sentence is the number that you start with. The numeral right after the \div tells the size of the group she made. The numeral after the $=$ sign is the number of groups.

You might want to redo the problems with the blocks as you place numbers in the

boxes. Do not rely on the pictures to help if a student is having trouble or if the pictures are poorly drawn. For some students (the younger, the truer), a leap to a picture representation in the second session is too fast. For most eight-year-old students, the leap is a stretch but does not usually cause problems. If any confusion arises, use the blocks.

Session 3 Materials





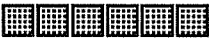



Division Number Sentence - Worksheet 3, page 7
Blocks or Cuisenaire Rods

In this session, students are given a starting number and search for all the even groupings they can find for that number. This time they will draw each grouping and record it as a number sentence. Do this with them until they can proceed in the search and recording alone. Some students enjoy coloring in the block pictures.

This activity is fun to do with Cuisenaire Rods. Have them find all the possible single color 'trains' of a given length, draw and color each train. Drawing on centimeter grid paper makes the recording faster. A template makes the drawing extra fun, and many students love templates.

If you are lucky enough to have both blocks and rods, use them both. The blocks offer a sharp visual image of what is happening to the individual pieces being divided; the rods offer a subtle guidance in developing a method for finding all the ways to group a number. Students using rods will often try the one rod, then the two rod, then the three rod, and so on. Students using blocks will have to work harder to come up with an orderly search method.

A completed problem set looks like this:
Find all the ways of grouping six.

blocks		<table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="width: 30px; height: 30px;">6</td> <td style="width: 30px; height: 30px;">÷ 2</td> <td style="width: 30px; height: 30px;">= 3</td> </tr> </table>	6	÷ 2	= 3	Cuisenaire Rods	 red rods
6	÷ 2	= 3					
		<table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="width: 30px; height: 30px;">6</td> <td style="width: 30px; height: 30px;">÷ 3</td> <td style="width: 30px; height: 30px;">= 2</td> </tr> </table>	6	÷ 3	= 2		 green rods
6	÷ 3	= 2					
		<table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="width: 30px; height: 30px;">6</td> <td style="width: 30px; height: 30px;">÷ 6</td> <td style="width: 30px; height: 30px;">= 1</td> </tr> </table>	6	÷ 6	= 1		 a dark green rod
6	÷ 6	= 1					
		<table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="width: 30px; height: 30px;">6</td> <td style="width: 30px; height: 30px;">÷ 1</td> <td style="width: 30px; height: 30px;">= 6</td> </tr> </table>	6	÷ 1	= 6		 white rods
6	÷ 1	= 6					

Worksheets

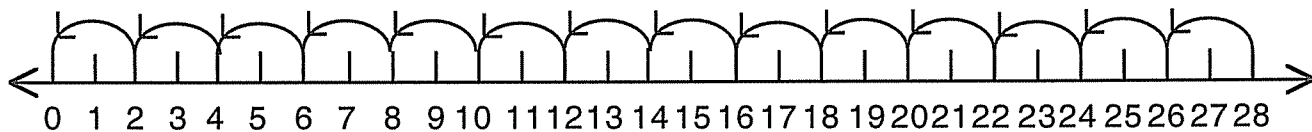
Number Sentence - Worksheet 4, page 8, uses Cuisenaire Rods. Number Sentence - Worksheets 5 - 8, pages 9 -12, uses rods or tiles. The student writes the number sentence and draws a picture of all the different ways of grouping different numbers.

Session 4 Number Lines

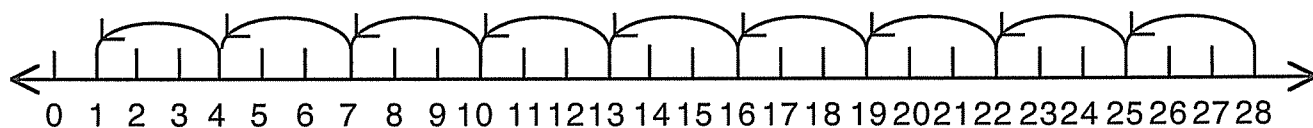
Start with the total number of students in the class or a set of counters and skip count backwards by twos, threes, fours, and fives.

Set up a number line showing how many students are in the class or the set of counters. Record the skip counting on the number line for each problem above. Compare them. **"How many groups? Are there any left over?"**

Skip count backwards by twos from twenty-eight. There are fourteen groups of two.



Skip count backwards by threes from twenty-eight. There are nine groups of three with one left over.



Worksheet

Number Sentence - Worksheet 9, page 13 This page links division to successive subtraction using a number line.

Session 5

In this session, word problems are introduced that correspond to the physical model we have been building .

Start with problem 1 on Number Sentence - Worksheet 10, page 14.

Jill had six kittens and some boxes. She put two kittens into each box. How many boxes did she use?

"Read the problem and look at the picture. Write the number sentence that goes with the problem. Put the numerals into the boxes on the chart. You can look at the chart." Give the student time to think about it.

"What is Jill dividing?" "The kittens."

"What numeral goes into the first box?" "6."

"Why?" "Because that's the number of kittens she started with."

"What numeral goes into the second box?" "2."

"Why?" "Because that's the number of kittens she put into each box (or group)."

"What numeral goes into the third box?" "Three."

"Why?" "Because that's the number of boxes she used (or groups she made)."

Do the second problem the same way. Remind her to draw a picture of the problem before she writes the number sentence. Have her work independently as soon as she understands. A student who wants more help should stay with you.

Worksheets

Number Sentence - Worksheets 11 and 12, pages 15 and 16

In the next session, go over each problem and have the student share the problems that she wrote. If a student writes a problem that uses the other formulation of a word problem (as discussed in the lesson Word Problems: Two Formats), accept it, and move on. If you call attention to it, some students will get anxious that they missed something or haven't understood the lesson. At this point the distinction between

the two kinds of division problems is too difficult for most students to handle. In this lesson, the basic units of a division problem are introduced. The next lesson works even more deeply on the meanings of the positions of the numbers in the problem.

**Test for
Understanding**

1. **"What question does the number sentence $10 \div 5 = \underline{\quad}$ ask?"**
"How many groups of five are there in ten?"
2. **"What numeral tells me what to start with?"** "The first numeral, ten."
3. **"What does the \div five tell me to do?"** "Make groups of five."
4. **"What do you end up with?"** "Two groups of five."
5. **"What problem is this?"** You start with six blocks on the table and arrange them into groups of three. " $6 \div 3 = 2$." You may also ask, **"What multiplication problem is this?"** The completed division problem shows six blocks arranged into two groups with three blocks in each group. That is the physical model for the multiplication problem $3 \times 2 = 6$. She should be able to recognize the number sentence from the physical model of the problem. In this problem, you put six blocks on the table and proceeded to make groups of three. Do several more like this. If she can't do that easily, she is not ready to move to the next division lesson.
6. Have her write a division word problem.

Can the student formulate a division problem that might occur in the world? Don't be concerned if the problem she writes uses the alternate form discussed in the lesson Word Problems: Two Formats.

Monkeys on the Roof

Purpose The purpose of this lesson is to develop a physical model of partitioning a division problem using the division symbol (querel): $\overline{\hspace{1cm}}$
This is the "Guzzinda" method.*

Prerequisites Number Sentence

Materials Monkeys on the Roof - Worksheets 1 - 5, pages 17 - 24
Unifix cubes
Scissors
Paste

Lesson Begin with a few division number sentences such as $12 \div 3 = \underline{\hspace{1cm}}$. Have the student solve the problem with blocks or prove the answers with blocks. Test for understanding of a division sentence.
"What question does $12 \div 3 = \underline{\hspace{1cm}}$ ask?" "How many groups of three are there in twelve."
"Which numeral tells me how many blocks to start with?" "The twelve."
"What does the three tell you?" "The size of the group I am going to make."
"What is the answer?" "Four."
"Four what?" Or, "What does the four stand for?" "Four groups of three (in twelve).

Then take out fifteen blocks and, slowly, make groups of five. While you are doing that, ask, "What problem am I doing?" "Fifteen divided by five." Do several problems like this. Then have each student take the teacher's place and the other students identify the problem they are doing.

The worksheets use a story to demonstrate the use of the division symbol: $\overline{\hspace{1cm}}$

Worksheets Monkeys on the Roof - Worksheets 1 - 5, pages 17 - 24


It is essential that you do not skimp on time in this lesson. This concept seems easy and the lesson proceeds smoothly for most students. But an incomplete understanding of what the numbers in a division problem actually mean (number you start with, number of groups or size of group) leads to major conceptual problems later. The mental picture formed in this lesson will be drawn on again and again when you are working with long or difficult problems. It will be important later, when the students have dropped the physical manipulation of the blocks, to review this lesson now and then. It is very easy for students to write numbers on worksheets and forget what the numbers mean.

*Partitioning (Goes Into or Guzzinda): You have twelve cookies. You want to give each of your friends two cookies. How many friends can you give cookies to? $\underline{\hspace{1cm}}$ The number sentence is $12 \div 2 = 6$. This is also called 'goes zin da.' As in two 'goes zin da' twelve how many times?

Recording

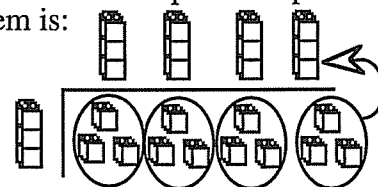
Purpose The purpose is to record the division number sentence linking the blocks (concrete) with drawings (representational) and numbers (abstract).

Prerequisites Number Sentence and Monkeys on the Roof

Materials Division: Recording - Worksheet 1, page 25
Blocks
A large drawing of the division symbol (querel) 

Lesson Session 1 In this lesson the student builds the block model, then draws a picture representation to record on the worksheet. The first problem is:

$$12 \div 3 = ?$$

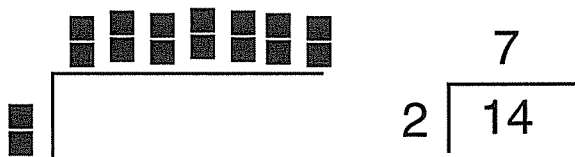


Session 2

Division: Recording, Worksheet 2 - 4, pages 26 - 28

The student solves it with blocks, adds a picture representation, and records what she is doing on the worksheet. She records it in pictures at each step. Finally she records the numbers on the worksheet.

$$14 \div 2 = 7$$



Notes

Using Worksheet 3 format, continue to practice this lesson for several weeks while you work on other things. Give number sentences with problems that come out evenly. Keep the numbers below thirty-six. Counting blocks past that number becomes burdensome and often inaccurate. Twenty problems per week is usually adequate.

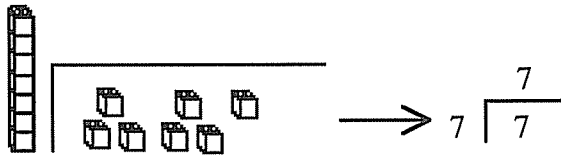
If you are working with a large group, have the students work independently whenever they begin to operate correctly on their own. Students who need help should continue to work with you for as long as they need.

Students will drop the block models whenever they wish to. They usually will ask to do so when they feel ready. After a week or two of practice, if the student has not dropped the blocks independently, give her the challenge of trying to do a division problem without using blocks.


Test for Understanding

divisor $\overline{)$ $\begin{array}{r} \text{quotient} \\ \text{dividend} \end{array}$

The basic understanding being worked on in this lesson is the identification of the meaning of the positions of the numbers in a division problem. The numeral under the line, called the *dividend*, is the number of things you start with. It is a counting of single units. The number to the left of the line, called the *divisor*, is an operator. It gives the instruction to make groups of a certain size out of the number of things you start with. The answer, called the *quotient*, is written above the line and represents the number of groups that were made. (The divisor can also be the number of groups and the quotient the size of the groups, but we will not deal with this switch until later.) It is important that the student understands the difference between the number in the dividend and the number in the quotient. The first is a number of individual objects, the second is a number of groups of objects. It is important to have the student clearly articulate these differences in order to establish her level of understanding.


1. "Is this problem done correctly?" 

"No, the answer should be 1."

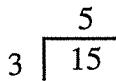
2. "What part of the division problem is this?" 

"The answer or quotient."

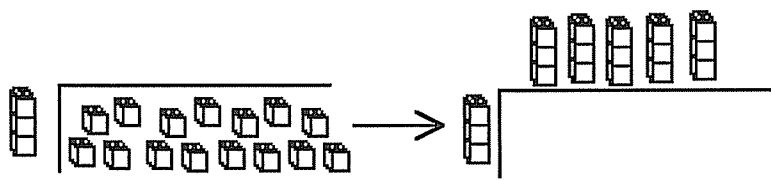
"How can you tell?" "Because the twelve blocks are already arranged in groups of three. If it was the divisor it would be a number of ungrouped blocks."


3. "Is this problem done correctly?" "No." 

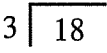
"How do you know? Can you build it?" "The four and the twelve are in the wrong places. The number above the line tells the number of groups of three in the number below the line. There can't possibly be twelve groups of three in four."

4. "Is this problem done correctly?"  "Yes."

"How do you know? Can you build it?" "Because there are three groups of five in fifteen."




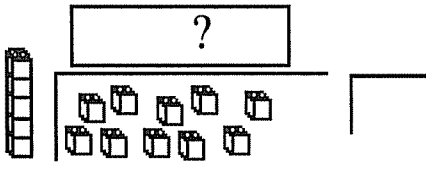
5. "Change this number sentence problem $? \div 3 = 6$ into a division problem in this format: 

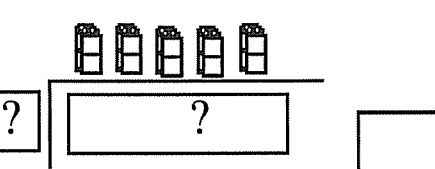
"The sentence is $18 \div 3 = 6$." 

"What numeral would go under the line?" "In the new format, the eighteen goes under the line."

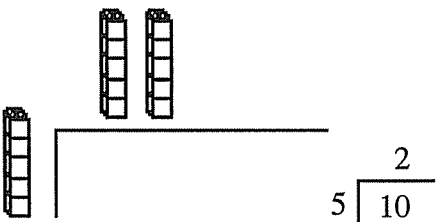
"How do you figure that out?" "If you make groups of three and you end up with six groups, then you have to start with eighteen. The numeral that tells how many you start with is under the line."

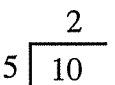
6. Draw the right number of blocks into the square with a question mark in it and record the problem with numerals in the new format: 

A. 

B. 

Answers.

A. 



B. 



Missing Factors

Purpose

The purpose is to lay the foundation for students to relate multiplication and division as inverse operations.

In multiplication, the two numbers that are multiplied together to obtain a product (the answer) are called *factors*. In these problems one of the factors is missing. You are given one factor and the product. From these two numbers, you can figure out what the missing factor is. The question asked by the missing factor problem, " $6 \times \underline{\quad} = 30$, how many sixes make thirty?" is the same question asked by this division problem $30 \div 6 = \underline{\quad}$? In the multiplication problem $6 \times 5 = \underline{\quad}$, you start with the six and add it five times to get thirty as the product of the repeated addition. In the division problem $30 \div 6 = \underline{\quad}$, you start with the thirty and find out how many groups of six are in it, subtract it five times until you get to zero. In the problem $6 \times \underline{\quad} = 30$, you are trying to find out how many groups of six are in thirty, the same thing you do in a division problem. So, the missing factor problem $6 \times \underline{\quad} = 30$ is actually the division problem $30 \div 6 = \underline{\quad}$, written in the familiar form of a multiplication problem.

Using the known to build a bridge to the unknown is a powerful tool for developing understanding. Using familiar multiplication problems to solve unfamiliar division problems introduces this tool. It also helps develop understanding of the connection between multiplication and division. Later, using the known tool will be used to unravel difficult division problems that involve large numbers, fractions or decimals. It is also an essential tool needed to solve algebraic problems.

Prerequisites

Firm conceptual understanding of multiplication and knowing Times Tables

Materials

Cuisenaire Rods and meter stick (or cube blocks or tiles)
Multiplication Facts Chart completed in Multiplication: Booklet 1, page 15, (see the example on the following page)
Missing Factors - Worksheets 1 - 3, pages 29 - 31

Lesson

"Build the problem $4 \times 6 = \underline{\quad}$?" The student uses six purple rods and lines them up along the meter stick to read twenty-four as the answer.

Give this problem: $6 \times \underline{\quad} = 30$. **"What question does this problem ask?"** "Six how many times equals thirty?"

"How can you use the rods to figure out the answer?" One possible way: The student selects the six rod (dark green) and lines them up along the meter stick until they reach the thirty mark. He counts the number of rods used. If you are using tiles, he counts out thirty tiles and arranges them into rows of six and then counts the number of rows.

"Complete all of the problems on Worksheet 1 with the rods or tiles."

If you have both Cuisenaire Rods and tiles you might want the student to use both because the image is slightly different.

Worksheets Check the Missing Factors - Worksheets 1 - 3 to make sure they are done correctly. Some students may find the Multiplication Facts Chart useful for figuring out the missing factor. Have the student solve the last section on Worksheet 4, page 32 by drawing a picture. If the student is unable to do the problems without the manipulatives, this indicates that he has not internalized the image of the problem and needs more time with manipulatives. Continue work on these types of problems until he can do them without a manipulative.

Timespan Give several missing factor problems each week until you reach the lesson on Families of Facts.

Test for Understanding Have him do a missing factor problem without a manipulative or multiplication tables chart. Give an easy problem such as: $5 \times \underline{\quad} = 15$. Can he figure it out? Most students will add fives until they get to fifteen. If he can't figure out the problem, it indicates that he doesn't understand the relationships between the numbers and he needs further practice with these problems. Check his understanding of the Multiplication Concept.

"How did you find the answer to a missing factor problem?" Most students will tell you that they added fives until they got to fifteen. Some will use images of the rods to help them figure out what to do. Other students may use the Multiplication Facts Chart to find the answer.

"How do you know the number you have chosen is the right one?" "The answer is right because three fives make fifteen." In other words $5 \times 3 = 15$.

"How did you check your answers?" Since the missing factor problem $5 \times \underline{\quad} = 15$ is the same as the division problem $15 \div 5 = \underline{\quad}$, the problem used to check it would be $5 \times 3 = 15$. This is an example of using multiplication to check a division problem. Students who have a strong concept of the operations they are working on know intuitively this relationship exists. In later lessons, the use of inverse operations for checking will be developed formally.

Multiplication Facts Chart

X	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

Remainders

Purpose The purpose is to introduce and build a model for the concept of remainders in a division problem.

Prerequisites Concept of Division, Number Sentence, and Missing Factors

Materials Remainders - Worksheet 1, page 33
Cuisenaire Rods
A centimeter ruler

Lesson Session 1 **"Make as many twenty-four centimeter long 'trains' as you can."** Have the student draw pictures of the trains freehand, on grid paper or with a template. Tell her that there are six different trains possible. She will make trains of three brown eight rods, four dark green rods, six purple four rods, eight light green three rods, twelve red two rods, and twenty-four white one rods.

"Why didn't you use any orange ten rods?" "Because the orange rods don't fit into a twenty-four train, (or some variation of this idea)."

"How many orange ten rods would fit into a twenty-four train?" "Two."

"How many white rods would you need to add on to the orange rods to make a twenty-four train?" "Four."

The reason for using only white Cuisenaire Rods to show the remainder of four instead of one purple four rod is that some students will see the single purple rod and write remainder of one. Using the white rods gives the same image for every remainder. They also show the remainders as single units rather than as a group as the use of the colored rods would do.

"How can you make a twenty-four train with the blue nine rods?" "Two blues and six whites."

"How can you make a twenty-four train with the black seven rods?" "Three blacks and three whites."

"How can you make a twenty-four train with the yellow five rods?" "Four yellows and four whites."

Worksheets Do the first problem together on Remainders - Worksheet 1 to be sure she understands how to record the answers. Remainders - Worksheets 2 and 3, pages 34 and 35 can be done independently. Remainders - Worksheet 5, page 37, is a self-correcting practice page.

Session 2 Remainders - Worksheet 4, page 36. Have the student build the problem: $5 \overline{)12}$
"How many groups of five are in twelve?" "Two."
"Record that." $5 \overline{)12} \quad \text{"Are there any remainders?"}$ "Two."
 $5 \overline{)12} \quad \text{"The remainder is recorded with an R 2 next to the first answer."}$
 $5 \overline{)12} \quad \text{"Do another one so I can tell you how to record the remainder the new way."}$

Review Worksheets Remainders - Worksheet 6, page 38, Cuisenaire Rods Review - Division by Four; and Remainders - Worksheet 7, page 39 - Number Lines Review

Families of Facts: Patterns

Purpose	The purpose is to relate multiplication and division as inverse operations.
Prerequisites	Commutative Property of Multiplication, Multiplication and Division Number Sentences, Basic Division, and Times Tables.
Materials	Families of Facts: Patterns - Worksheets 1 - 6, pages 40 - 45 Colored cube blocks or tiles
Lesson Session 1	<p>Remember, in these books the conventional way to write the multiplication number sentence is different from most books. Refer to the box at the top of Worksheet 1. Families of Facts: Patterns - Worksheets 1 and 2, pages 40 and 41, review the multiplication number sentence. Families of Facts: Patterns - Worksheets 3 and 4 review the division number sentence. The pictures are different because they have different numbers of groups and a different number of blocks in each group. The pictures are the same because they have the same numbers in them and they have the same answers. If a student notices this, it is good mathematical thinking.</p> <p>Play 'What Division Problem Is This?'</p> <p>Start with all the blocks in a pile and then separate the blocks into groups. <i>Do not start by arranging the blocks into groups as you draw them from the pile.</i></p> <p>For example, place eight blocks on the table.</p> <p>Then divide the blocks into two groups of four.</p> <p>"What division problem is this?" $8 \div 4 = 2.$</p> <p>Do several of these.</p>
Session 2	<p>Families of Facts: Patterns - Worksheet 1 Have the student solve the first two problems by drawing in the boxes.</p> <p>"Why are the two problems not the same?" "Because they have different start with numbers."</p> <p>"How are these two problems different?" "One makes groups of two, the other makes groups of three."</p> <p>"How are these two problems the same?" "They both use six blocks. Or they both have the same answer."</p> <p>Now have the student write out the answers to this set of questions. Require complete sentences.</p> <p>Families of Facts: Patterns - Worksheet 2</p> <p>Problem 1 "What do you think the picture is showing?" "It shows a group of six apples. Then it goes on to show a total of three identical groups of six apples. The arrow shows that you are to 'Bulldozer' or push together all three groups to get the total." The language may not be this precise. Help the student articulate this if needed. Explain if nothing else works.</p> <p>"This picture can be written with a number sentence. How do you know which number to write on the first line?" "The first number is the start with number. You start with six apples. So you write a 6 on the first line."</p> <p>"How do you know what to write after the x?" "You count the number of times you see the group of six apples. You write a 3 after the x sign."</p> <p>"What goes after the = sign?" "The total number of apples, which is eighteen."</p> <p>"What number sentence fits the second set of pictures?" $4 \times 6 = 24.$</p> <p>Have the student finish the page.</p>

Families of Facts: Patterns - Worksheet 3, page 42

Draw the student's attention to the number sentence in the box. Have her solve the problem with blocks and then draw in the blocks in the box that is given. Be sure she is making groups of two and not two equal groups. Have her solve the second problem which makes groups of three.

See Word Problems: Two Formats on Partitioning and Sharing, on page 20.

If the student remarks that these pictures look just like the pictures she drew on Worksheet 1, she has made an excellent observation of an important mathematical truth. Multiplication and division are deeply related. Ask her to explain why she thinks the pictures are the same. The answer is that she starts with the same number in multiplication is the size of the group and in division becomes the divisor, which is also the size of groups. Nice piece of mathematics if she can explain this. Most students can not at this time.

"Why are the two problems not the same?" "Because they have different grouping numbers."

"How are these two problems different?" "One makes groups of two, the other makes groups of three." (Notice that the answer to this question on Worksheet 3 is the same answer as given on Worksheet 1.)

"How are these two problems the same?" "They both start with six blocks."

Now have the student write out the answers to this set of questions. Require complete sentences.

Families of Facts: Patterns - Worksheet 4, page 43. Put Worksheet 2 adjacent to Worksheet 4.

"What pattern do you see when you look at both pages?" "There are apples, faces and flags on both pages." A great connection is that the end number on the multiplication page is the start with number on the division page.

Look at problem 1 on Worksheet 4.

"This picture can be written with a number sentence. How do you know which number to write on the first line?" "The first number is the start with number. You start with eighteen apples. So you write an 18 on the first line."

"How do you know what to write after the ÷?" "You count the number in each group. Here we are making groups of six."

"What goes after the = sign?" "The number of groups we can make, which is three. So the number sentence for the apples is $18 \div 6 = 3$."

"What number sentence fits the second set of pictures?" " $24 \div 4 = 6$."

Have the student finish the page.

Session 3

Review the work done in the last session. Tell the student that Families of Facts: Patterns - Worksheets 5 and 6 are puzzles. They have to look at the picture and tell if the problem is a multiplication problem or a division problem.

In Worksheet 5, page 44, the student differentiates between multiplication and division problems.

Look at the picture on problem 5.

"How can you tell if this is a division problem or a multiplication problem?"

"You can tell it is a division problem because you have a large group of fish being separated into smaller groups. The start with number is one big group."

"What division problem is shown in problem 5?" "Start with eighteen fish and divide them into groups of six. There will be three bowls needed." $18 \div 6 = 3$

Look at the picture on problem 6.

"How can you tell if this is a division problem or a multiplication problem?"

"You can tell it is a multiplication problem because you start with several equal small

groups and then you push them all into one large group."

"What multiplication problem is shown in problem 6?" "Start with six fish in a bowl. Then make two more bowls just like it."

"How many fish do you end up with?" " $6 \times 3 = 18$."

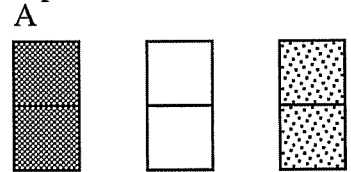
"Do problems 7 and 8 on your own."

Session 4

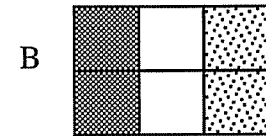
Many students will notice that the division problems on Worksheet 3 are the same as the multiplication problems on Worksheet 1. If they do notice it, ask, "How are the problems different? Why is one a multiplication problem and the other a division problem?" This is a Test for Understanding question.

"A multiplication problem starts with a small group and adds it repeatedly to make a larger total. Division starts with a large group and breaks it down into several smaller, equal groups." *If they do not notice it, do not point it out.* A student who has difficulty with Worksheet 5 may be confused on this point.

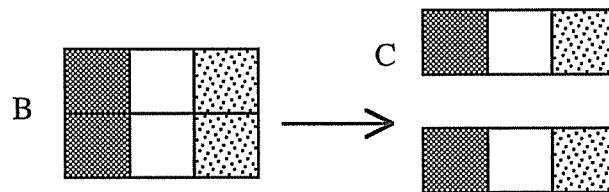
This lovely manipulative pattern demonstrates the Families of Facts. Build two, three times with the blocks. Each group should be shown in a different color. "What multiplication problem is shown by the blocks?" "Two times three equals six or $2 \times 3 = 6$."



Push the blocks together to show the answer of six.



Start with the blocks pushed together and separate them to end looking like Picture C.



"Now what problem am I doing?" As you say that, separate the blocks into two groups of three by sliding down one row of the blocks as a unit. There will now be one block of each color in each group.

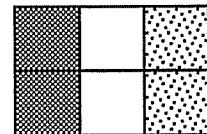
"Six divided by three equals two or $6 \div 3 = 2$."

Then ask them to look at the blocks as they are now (as the two groups of three).

"What multiplication problem is now shown with the blocks (Picture C)?"

"Three times two equals six or $3 \times 2 = 6$."

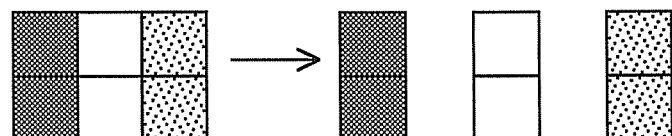
Push the blocks back together to show the answer of six.



Start with the six again and end with Picture A.

"Now what problem am I doing?" As you say that, separate the blocks into three, two times by sliding apart as a unit the blocks of the same color.

"Six divided by two equals three or $6 \div 2 = 3$."

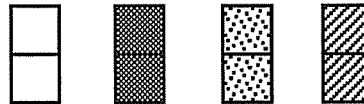


You are now back where you started from. Visually this is a beautiful pattern. It also demonstrates the relationship between multiplication and division facts that use the same three numbers.

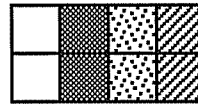
Repeat the same sequence again. This time have the student record the four problems as they are built. They are $2 \times 3 = 6$, $6 \div 3 = 2$, $3 \times 2 = 6$ and $6 \div 2 = 3$. **"What patterns do you see?"** "All of the problems use the same three numbers. There are two multiplication problems in which the two and the three trade places." **"This set of four number sentences is called a 'Family of Facts.'"**

Repeat the process with a different set of numbers, such as four, two, and eight. Have the student build several sets with different numbers. Have the student write the four problems on a piece of paper and look for patterns.

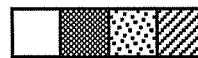
Build two, four times.
 $2 \times 4 = 8$



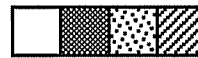
Push the blocks together.



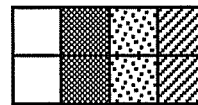
Move the blocks apart to show eight divided by four.
 $8 \div 4 = 2$



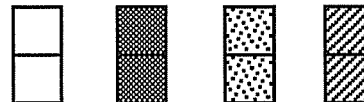
The blocks now show four, two times
 $4 \times 2 = 8$



Push the blocks together.



Move the blocks apart to show eight divided by two.
 $8 \div 2 = 4$



After some experimenting, ask, **"Do you think that same pattern will happen again if you use different numbers?"** Answers and certainty levels will vary.

Families of Facts: Patterns - Worksheet 6, page 45, is helpful to do together while the student builds with blocks.

"Write a multiplication problem and its answer." " $5 \times 3 = 15$." **"Write the other three sentences that would be in that same family."** Or, **"Write three more number sentences that use those same three numbers."** See if she can do it without the blocks. If she can't, have her use the blocks in the same way she did in Session 2.

Families of Facts: Patterns - Worksheet 6, page 45

Problem 9 – Use bits of colored paper if you do not have colored blocks.

This is a neat geometric way of looking at Families of Facts that works well for your right-brained people. Multiplication problem is $2 \times 5 = 10$.

Division problem is $10 \div 5 = 2$.

Be aware of an important issue here. Two Division Formats: Partitioning (Goes Into or Guzzinda) and Divvy-up (Sharing). See Word Problems: Two Formats, page 20.

Problem 10 shows $5 \times 2 = 10$ and $10 \div 2 = 5$.

Problem 11 Shows the Family of Facts: $2 \times 5 = 10$ $5 \times 2 = 10$
 $10 \div 2 = 5$ $10 \div 5 = 2$

Notes

In Session 2 there may be a temptation to say to the student, "Look, see how all the problems go together. See how $2 \times 3 = 6$ is related to $6 \div 2 = 3$? See how you can go forward and backward?" The pattern shows on the worksheet, but it is not explicitly outlined for the student. Try to refrain from saying this. It won't help the student and it steals away any opportunity for the student to recognize it on her own. This concept of the reverse of operations seems simple and straightforward, but it isn't. It takes many experiences with the numbers and the blocks for the student to grasp the idea that you can move back and forth between a multiplication and division problem in the same family, that in fact they are the same problem done in different orders. This is a major mathematical idea. The whole process of solving equations in algebra rests on this concept: The understanding must come from within the student if it is to be useful. Please be patient.

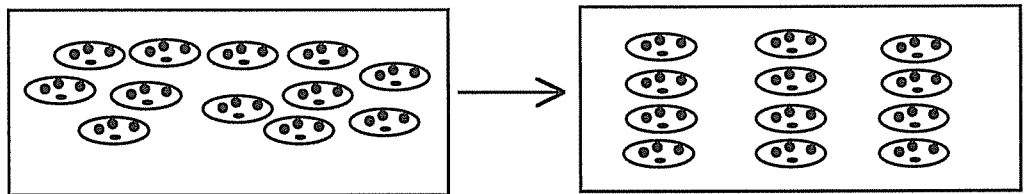
Word Problems: Two Formats

Purpose The purpose is to build division story problems with concrete objects (Partitioning, which is also referred to as the Goes Into or Guzzinda method). quotient

Materials Any counter divisor dividend
 $\text{dividend} \div \text{divisor} = \text{quotient}$ number of groups
 start with \div size of the group = number of groups size of group start with

Lesson Have each student take a handful of counters. Read the following story problems while the students act out each story with the manipulatives. After the first story ask the students to explain their way of showing it.

1. "Maria has twelve cookies to share with her friends. Each friend receives four cookies. How many friends can have four cookies?" Three.



2. "John has nine marbles. He wants to give three marbles to each of his friends. How many friends can he give marbles to?" "Three."
3. "Shirley collected twenty seashells at the seashore. She has ten cats. She wants to give three shells each to her cats. How many cats can she give shells to?" Six. "How many shells are left over?" "Two."
4. "George has a flea circus. He has sixteen fleas and some tiny boxes for the fleas to jump into. He wants two fleas to jump into each tiny box. How many tiny boxes will he need?" "Eight."
5. "Lola loves lime lollipops. She has eight lime lollipops. Her mother says she can eat four lime lollipops a day. How many days will Lola be able to eat lime lollipops?" "Two."
6. "Lola decides to eat only two a day. How many days will the lime lollipops last now?" "Four."

"Gus, the bus driver, has sixteen children to take to the zoo. His small blue bus has nine seats. Two children can sit on each seat. How many seats will he need for the children? "Eight." How many seats will be left over?" "One."

Make up more stories with your students following this format.

Worksheets Word Problems - Worksheets 1 and 2, pages 48 and 49

Word Problems: Two Formats Partitioning (Goes Into or Guzzinda) and Divvy-up (Sharing)

Division word problems come in two varieties. Partitioning (Goes Into or Guzzinda) is introduced first in the Lesson Number Sentence because it helps the connection with multiplication happen faster. The questions, "How many threes are in twelve?" and, "three how many times makes twelve?" are very useful questions for beginning dividers. Divvy-up (Sharing) is investigated in the Base Ten Division Booklet. Introducing both at the same time confuses most students. Divvy-up is the method. 'Sharing' is the language used when giving directions and working with students.

This information is for you to be able to discern the different formats because word problems in the world come both ways. Here are two word problems that show the two ways of thinking about division problems.

Partitioning (Goes Into or Guzzinda): You have 12 cookies. You want to give each of your friends two cookies. How many friends can you give cookies to? ____ The number sentence is $12 \div 2 = 6$. This is also called 'goes zin da.' As in two 'goes zin da' twelve how many times?

Divvy-up (Sharing): You have twelve cookies. You and a friend decide to split the cookies between you. How many cookies do each of you get? __ Six. The number sentence is $12 \div 2 = 6$. (One for you, one for me until all twelve are divided, six to each person.)

Note that in both cases the number sentence is $12 \div 2 = 6$. In the first problem, the two represents the *size* of the group, two cookies, and the six represents the *number* of people eating two cookies, Partitioning (Guzzinda). In the second problem, the two represents the *number of* groups, two people, and the six, the *size* of the groups or the number of cookies each person gets, Divvying-up. The size of the group and the number of groups change places and are not an easy distinction to make.

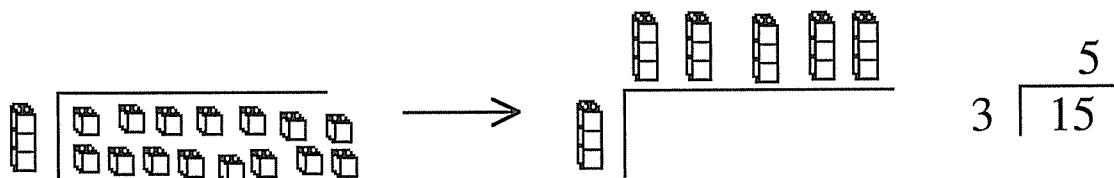
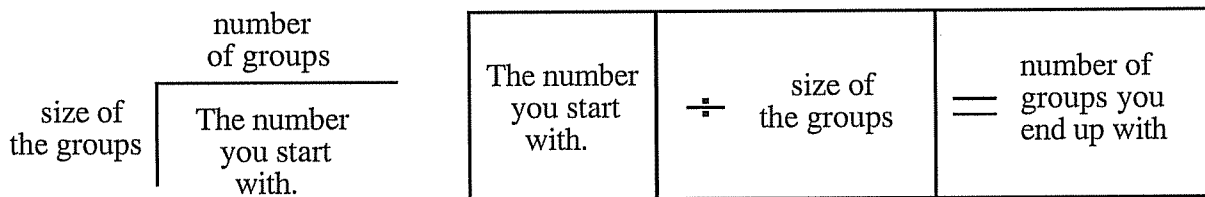
Understanding the difference between these two types of problems is best taught to students nine years or older. Younger students usually have difficulty grasping the conceptual difference and will get frustrated and confused. However, when the younger student writes her own division word problems, it is not uncommon for her to use both formats unconsciously. This is because both types of problems occur in her experience. Do not point out the difference. Simply accept whichever form she uses.

You will see both Partitioning and Divvy-up problems in other mathematics resource books. For instance, *Picturing Multiplication and Division* has the student making three groups from twelve blocks (Divvy-up). The "Line Them Up" lesson in *Mathematics Their Way* is Divvy-up. Cuisenaire books have the student find out how many groups of three are in twelve, which is Partitioning. Other books will mix the two, especially in the word problem section.

Be careful to notice which format is used when you assign practice pages from other sources.

Word Problems: Two Formats

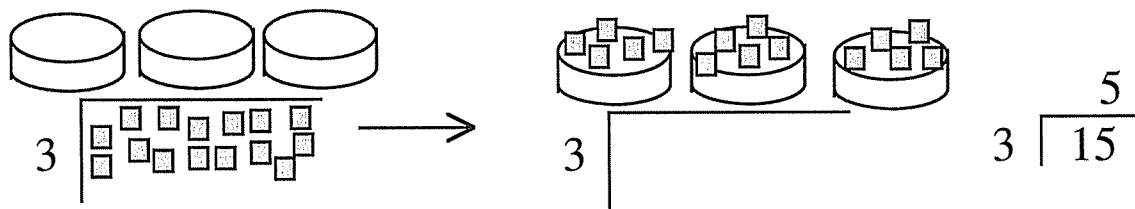
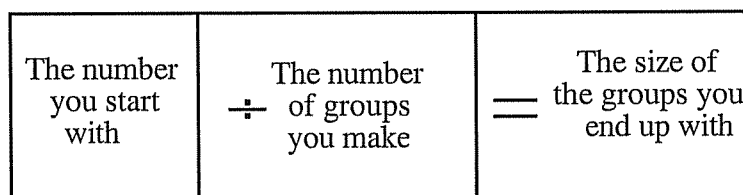
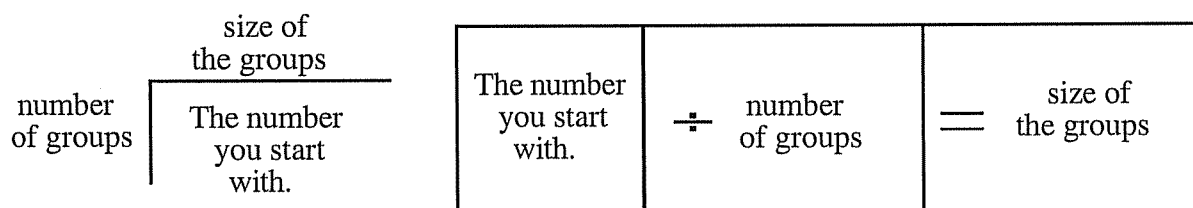
Partitioning: size of group (Goes Into or Guzzinda)



The number you start with \div size of the groups = number of groups $15 \div 3 = 5$

Story Problem: Susan has eight chocolate chip cookies. She will put **two cookies into each bag**. How many bags of two cookies can she make? $8 \div 2 = 4$

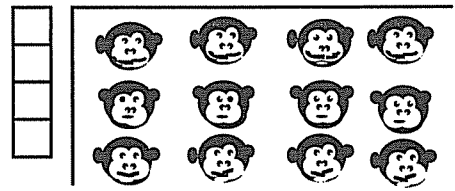
Divvy-up: One for you, one for me. (Sharing)



The number you start with \div number of groups = size of the groups $15 \div 3 = 5$

Story Problem: Susan has eight chocolate chip cookies **to share** fairly with her sister Mary. How many cookies does each girl get? $8 \div 2 = 4$

1. Mr. Zoo had 12 monkeys and some cages. Each cage holds 4 monkeys. How many cages does Mr. Zoo need for his monkeys?
- _____



2. Write the division number sentence for problem one.

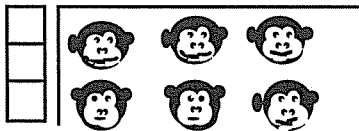
	\div	$=$
--	--------	-----

3. a. What question does this number sentence ask?

24	\div	4	$=$
------	--------	-----	-----

- b. What is the answer? _____

4. a. Draw the answer.



- b. Write the division number sentence.

	\div	$=$
--	--------	-----

5. a.

$$2 \overline{) 8}$$

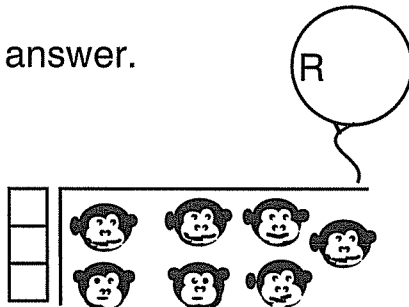
- b.

$$5 \overline{) 15}$$

- c.

$$4 \overline{) 32}$$

6. a. Draw the answer.




- b. Write the division number sentence.


	\div	$=$
--	--------	-----

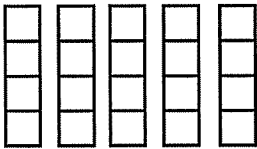
7. a. $4 \overline{)18}$ b. $5 \overline{)32}$ c. $3 \overline{)23}$

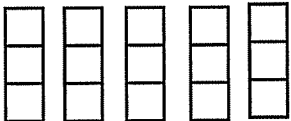
8. In the problem $20 \div 4 = 5$, 5 is the 'answer.'

Draw a box around any drawings below that show what the 5 means.

a. 

b. 

c. 

d. 

9. Make a Family of Facts with these three numbers.

a. $\underline{3} \times \underline{6} = 18$
 $\underline{\quad} \times \underline{\quad} =$
 $\underline{\quad} \div \underline{\quad} =$
 $\underline{\quad} \div \underline{\quad} =$

b. $\begin{array}{r} 3 \\ \times 6 \\ \hline 18 \end{array}$ $\begin{array}{r} \underline{\quad} \\ \times \underline{\quad} \\ \hline \underline{\quad} \end{array}$

3	6	18
---	---	----

$\begin{array}{r} \underline{\quad} \\ \underline{\quad} \end{array}$ $\begin{array}{r} \underline{\quad} \\ \underline{\quad} \end{array}$

10. These are hard.

Fill in the missing numbers.

a. $8 \div 2 = \square$

b. $15 \div \square = 3$

c. $\square \div 4 = 5$

Patterns in Arithmetic

Division: Booklet 1

Basic Concepts

Answer Key

for the

Student Workbook

By Suki Glenn and Susan Carpenter

Answer Key Legend

AWV = answer(s) will vary Cuisenaire Rods
BUWV = break up will vary 1 w = white
OWV = order will vary 2 r = red
3 lg = light green
Pattern Blocks 4 p = purple
r = red trapezoid 5 y = yellow
g = green triangle 6 dg = dark green
y = yellow hexagon 7 bk = black
o = orange square 8 bn = brown
b = blue parallelogram 9 bl = blue
t = tan rhombus 10 o = orange

Note: Some items and pages are left out of the answer key.

- 1) Some pages in which the answers are open-ended or will vary.
- 2) Make your own problems. Since students create their own problems and solutions, these sections give valuable information about the level of confidence and competence. It can be a useful source of curriculum for other students.
- 3) Blank practice pages
- 4) Workboards
- 5) Games
- 6) Self correcting pages
- 7) Instructions only pages

Patterns in Arithmetic: Division - Booklet 1
Answer Key for the Student Workbook
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Fallbrook, CA 92088

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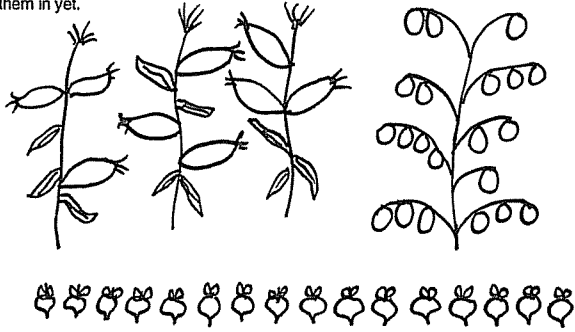


ISBN 978-1-935559-01-6

Division - Book 1

Garden Word Problems - Worksheet 1

Use a pencil to draw a garden scene: 1 tomato plant with 21 tomatoes, 3 corn plants with 3 ears of corn on each plant. Draw 16 small radish plants. Do not color them in yet.



1. Alysia had 21 tomatoes to give away. Color in 21 tomatoes in the picture. Count out 21 counters.

She put three tomatoes in each basket.

Build it. Put your counters into groups of three. How many baskets did she fill? 7 baskets.

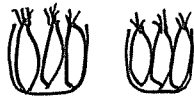


Draw the baskets.
Draw the tomatoes in each basket.

2. John wanted to give 7 ears of corn away. Color in 7 ears of corn in the picture. Build it.

He put the ears of corn into two baskets. But to be fair, each basket had to have the same number of ears in it.

Draw the two baskets in the space to the right.



3

Number Sentences - Worksheet 1

Example:

Start with six blocks.

A. How many groups of three can you make? 2

B. How many groups of one can you make? 6

Problem Set 1

Draw your blocks.

Start with eight blocks.

A. How many groups of two can you make? 4

B. How many groups of four can you make? 2

C. How many groups of one can you make? 8

D. How many groups of eight can you make? 1

Problem Set 2

Start with nine blocks.

A. How many groups of three can you make? 3

B. How many groups of one can you make? 9

C. How many groups of nine can you make? 1

Problem Set 3

Start with fifteen blocks.

A. How many groups of three can you make? 5

B. How many groups of five can you make? 3

C. How many groups of one can you make? 15

D. How many groups of fifteen can you make? 1

5

Garden Word Problems - Worksheet 2

How many ears of corn went into each basket? 3 ears

Are there any ears left over? yes, 1

3 ears per basket with 1 left

3. A rabbit ate the tops off of half of the radish plants.

If a rabbit ate half of the radishes, how many would be left? 8 radishes would be left.

Color the eaten plants brown. Color the ones that are left nice and green. Draw in the rabbit too.

4. Draw in another plant with fruit on it into your garden picture. Write a problem to go along with the picture. Have someone find the answer to your problem.

AWV

Skip count and record to forty-five by threes.

0 3 6 9 12 15 18 21 24 27 30 33 36 39 42

Skip count and record to sixty by fours.

0 4 8 12 16 20 24 28 32 36 40 44 48 52 56

Skip count backwards by twos from thirty and record.

30 28 26 24 22 20 18 16 14 12 10 8 6 4 2 0

Skip count backwards by fives from seventy-five and record.

75 70 65 60 55 50 45 40 35 30 25 20 15 10 5 0

4

Number Sentences - Worksheet 2

Problem Set 4

Draw your blocks.

Start with eighteen blocks.

A. How many groups of one can you make? 18

B. How many groups of two can you make? 9

C. How many groups of three can you make? 6

D. How many groups of six can you make? 3

E. How many groups of nine can you make? 2

F. How many groups of eighteen can you make? 1

Problem Set 5

Start with twenty-four blocks.

A. How many groups of one can you make? 24

B. How many groups of two can you make? 12

C. How many groups of three can you make? 8

D. How many groups of four can you make? 6

E. How many groups of six can you make? 4

F. How many groups of eight can you make? 3

G. How many groups of twelve can you make? 2

H. How many groups of twenty-four can you make? 1

Challenge

With your twenty-four blocks try making other groups. Try groups of five or nine.

What happens? The groups are not equal. There are leftover blocks!

6

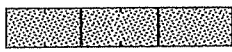
Number Sentences - Worksheet 3 O.W.V

Use Cuisenaire Rods and crayons.

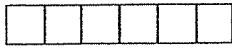
The length of the train.	÷	The length of the car you use	=	The number of cars you end up with.
(start with)		(size of group)		(number of groups)

Problem Set 1

How many six centimeter trains can you make? Each train must use only one color. Draw and color each rod train. Then in the big boxes, write the number sentence that goes with that train. One is done for you.



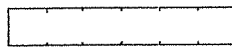
$$6 \div 2 = 3$$



$$6 \div 1 = 6$$



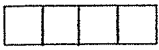
$$6 \div 3 = 2$$



$$6 \div 2 = 3$$

Problem Set 2

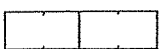
Show all the ways of grouping four blocks.



$$4 \div 1 = 4$$



$$4 \div 2 = 2$$



$$4 \div 4 = 1$$

7

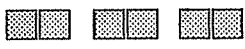
Number Sentences - Worksheet 5 O.W.V

Take out blocks.

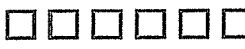
The number you start with.	÷	The size of each group you make.	=	The number of groups you make.
----------------------------	---	----------------------------------	---	--------------------------------

Problem Set 1

Show all the ways of grouping six blocks evenly. Draw a picture of each way below. In the boxes below, write the number sentence that goes with each picture. One way is done for you.



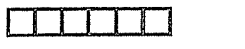
$$6 \div 2 = 3$$



$$6 \div 1 = 6$$



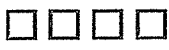
$$6 \div 3 = 2$$



$$6 \div 6 = 1$$

Problem Set 2

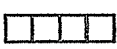
Show all the ways of grouping four blocks evenly.



$$4 \div 1 = 4$$



$$4 \div 2 = 2$$



$$4 \div 4 = 1$$

9

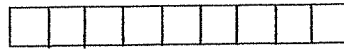
Number Sentences - Worksheet 4 O.W.V

Use Cuisenaire Rods and crayons.

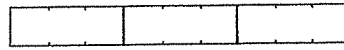
9	÷	9	=	1
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Problem Set 3

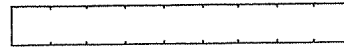
Show all the nine centimeter trains.



$$9 \div 1 = 9$$



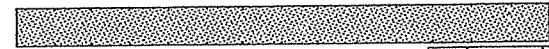
$$9 \div 3 = 3$$



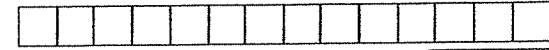
$$9 \div 9 = 1$$

Problem Set 4

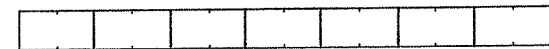
Show all the fourteen centimeter trains. Since you don't have a fourteen rod, choose a color that stands for fourteen and color your one rod train that color. The problem that uses the one fourteen rod is done for you this time. Color it in.



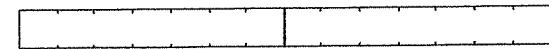
$$14 \div 14 = 1$$



$$14 \div 1 = 14$$



$$14 \div 2 = 7$$



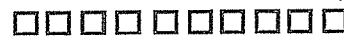
$$14 \div 7 = 2$$

8

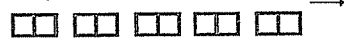
Number Sentences - Worksheet 6 O.W.V

Problem Set 3

Show all the ways to group ten blocks evenly. Draw a picture of each way.



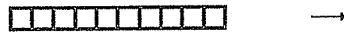
$$10 \div 1 = 10$$



$$10 \div 2 = 5$$



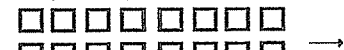
$$10 \div 5 = 2$$



$$10 \div 10 = 1$$

Problem Set 4

Show all the ways to group sixteen blocks evenly. Draw a picture of each way.



$$16 \div 1 = 16$$



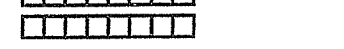
$$16 \div 2 = 8$$



$$16 \div 4 = 4$$



$$16 \div 8 = 2$$



$$16 \div 16 = 1$$



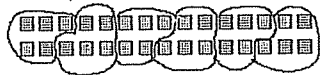






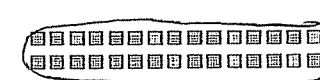
$$16 \div 16 = 1$$

10

Number Sentences - Worksheet 7

Problem Set 5
Show all the ways to group thirty blocks.
Circle the blocks to show your groups.
One is done for you.
Example:

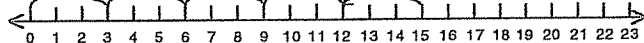
OWV


 $30 \div 5 = 6$

 $30 \div 2 = 15$

 $30 \div 3 = 10$

 $30 \div 6 = 5$

 $30 \div 1 = 30$

 $30 \div 10 = 3$

 $30 \div 15 = 2$

 $30 \div 30 = 1$

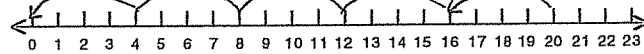
Problem Set 6
On another piece of paper show all the ways of grouping thirty-six.

Number Sentences - Worksheet 9

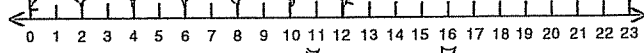
Start on 15. Take away 3s until you reach 0. How many jumps? 5



Start on 20. Jump back by 4s until you reach 0. How many jumps? 5



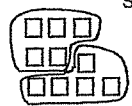
Start on 14. Take away 2s until you reach 0. How many jumps? 7



How many groups of 2 are in 14? 7 $14 \div 2 = 7$

	$14 - 2 = 12$
	$12 - 2 = 10$
	$10 - 2 = 8$
	$8 - 2 = 6$
	$6 - 2 = 4$
	$4 - 2 = 2$
	$2 - 2 = 0$

Subtraction Problem


 $10 - 5 = 5$
 $5 - 5 = 0$

Division Problem

 $10 \div 5 = 2$

How many groups of 5 are in 10? 2

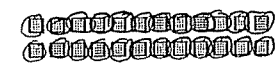






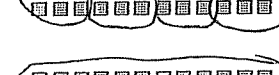
How many groups of 3 are in 12? 4 $12 \div 3 = 4$

	$12 - 3 = 9$
	$9 - 3 = 6$
	$6 - 3 = 3$
	$3 - 3 = 0$

Number Sentences - Worksheet 8

Problem Set 6
Show all the ways to group twenty-four blocks.
Circle the blocks to show your groups.

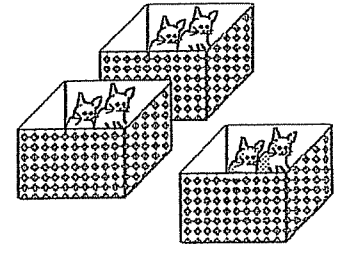
OWV


 $24 \div 1 = 24$

 $24 \div 2 = 12$

 $24 \div 3 = 8$

 $24 \div 4 = 6$

 $24 \div 8 = 3$

 $24 \div 12 = 2$

 $24 \div 6 = 4$

 $24 \div 24 = 1$

Number Sentences - Worksheet 10

Read the problem. Draw a picture to go with the problem.
Then write the number sentence that goes with the problem and
find the answer.

1. Jill had six kittens and some boxes.
She put two kittens in each box.
How many boxes did she use?

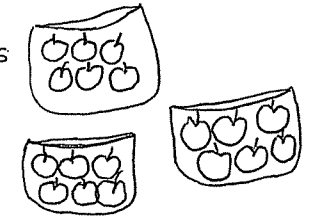
 $6 \div 2 = 3$
 Number sentence


Draw your pictures here.

2. Jack has twelve marbles. He put
three marbles into each bag. How many
bags did he use? 4 bags

 $12 \div 3 = 4$
 Number sentence


3. Jan had eighteen apples.
She put six apples into each bowl.
How many bowls did she have? 3 bowls

 $18 \div 6 = 3$
 Number sentence


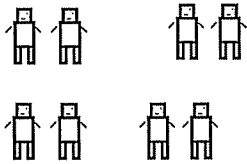
Number Sentences - Worksheet 11

4. Eddie had eight robots. He gave two robots to each person at his party. How many people were at Eddie's party?

Draw your pictures here.

$$8 \div 2 = 4$$

Number sentence



5. You have ten fingers. You have five fingers on each hand. How many hands do you have?

$$10 \div 5 = 2$$

Number sentence



6. If you had twenty fingers with five fingers on each hand, then how many hands would you have?

$$20 \div 5 = 4$$

Number sentence



7. Make your own. **AWV**

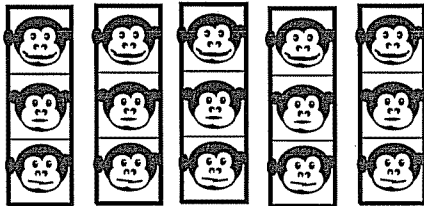
$$\square \div \square = \square$$

Number sentence

15

Monkeys on the Roof - Worksheet 2

Oh no! The baby monkeys came back! This time they brought their friends too. Now there are 15 monkeys. How many cages will the zookeeper need this time?



CAGE

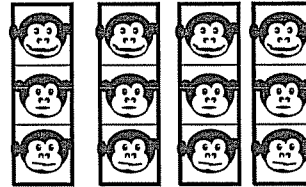
Cut out the 15 baby monkeys. Put all the baby monkeys into the house. Now cut out a cage. Paste three baby monkeys into each cage and put the cage on the roof. Keep doing that until all those frisky monkeys are in cages on the roof.

19

Answer Key: Division - Book 1

Monkeys on the Roof - Worksheet 1

Once upon a time there was a whole batch of baby monkeys who got loose from a zoo and found their way into Mrs. Querel's house. She called the zookeeper, who came over with some cages. The cages looked like little towers with three little doors. The zookeeper went into the house and put three baby monkeys into each cage. Then he called a helicopter to come and pick up the cages. The helicopter pilot wanted the cages on the roof of the house so he could get them easily with his hook. So the zookeeper tossed the cages up on the roof. The pilot grabbed them with a hook and took them all back to the zoo. Mrs. Querel's house was all messed up but empty of baby monkeys. The zookeeper stayed to help clean up the mess.



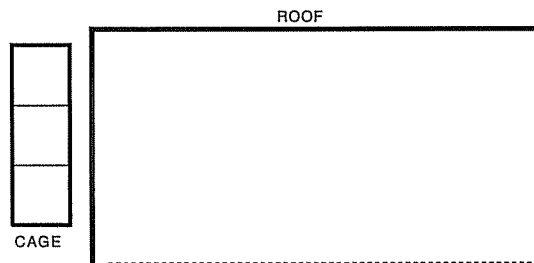
CAGE

Cut out the 12 baby monkeys. Put all the baby monkeys into the house. Now cut out a cage. Paste three baby monkeys into each cage and put the cage on the roof. Keep doing that until all those frisky monkeys are in cages on the roof.

17

Monkeys on the Roof - Worksheet 3

Use Unifix cubes. These will stand for the monkeys. To put them in cages, snap them together in groups of three and put them on the roof. Record how many cages are on the roof.

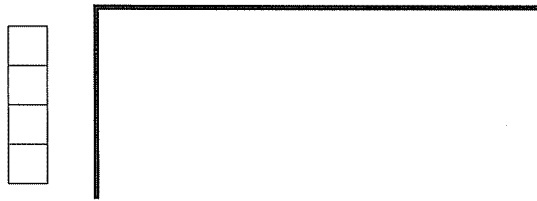


Cage Size	Number of Monkeys in the house	Number of Cages on the Roof
3	18	6 Cages
3	24	8 Cages
3	21	7 Cages
3	30	10 Cages
3	9	3 Cages
3	27	9 Cages
3	33	11 Cages

21

Monkeys on the Roof - Worksheet 4

The zookeeper got in some new cages. These cages hold four monkeys. Again, use your Unifix cubes to stand for monkeys. In the house, the monkeys are not snapped together.



Cage Size	Number of Cages on the Roof	Number of Monkeys in the house
4	3	12
4	4	16
4	6	24
4	8	32
4	2	8
4	5	20
4	1	4

Word Problems: Two Steps

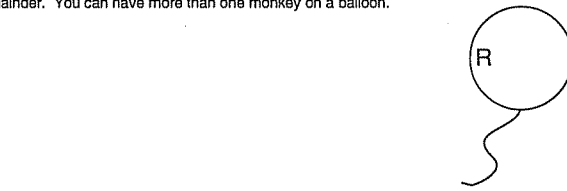
- Cosette baked 3 dozen cookies. Sam baked 2 dozen more. How many cookies are in 1 dozen? 12 cookies
How many cookies did the two bake altogether? 60 cookies
 $3 \times 12 = 36$
 $2 \times 12 = 24$
60
- Kaela went on vacation for 2 weeks. Andrew went on vacation for 4 weeks. How many days in a week? 7 days
How many days were Kaela and Andrew on vacation in all? 42 days
 $2 \times 7 = 14$
 $4 \times 7 = 28$
42
- Sam has 15 marbles. Michael has 12. Trudy has 19 marbles. Karyl has 9 and Cosette has 24. How many marbles in all? 79
 15
 19
 9
 $+24$
79
- How many more marbles do the girls have than the boys? 25 marbles

Sam 15	Trudy 19	52
Michael 12	Cosette 24	-27
<u>27</u>	Karyl 9	<u>25</u>
- Karyl, Kaela, and Ridge each have 1 dozen cookies. They sell 28 cookies at a bake sale. How many cookies do they have left? 8 cookies
 $3 \times 12 = 36$
 -28
8
- The sale price of each of the cookies was 4 cents each. How much money did Karyl, Kaela, and Ridge make? \$1.12
 28
 $\times 4$
\$1.12
- Andrew, Michael, and Cosette bought 5 cookies each. They shared their cookies equally between themselves, Trudy, and Sam. How many cookies did each child get? 3 cookies
 $15 \text{ cookies} \div 5 \text{ children} = 3 \text{ cookies each}$
- Trudy and Sam were so happy that their friends shared with them that they went baked a half dozen cookies for each child in the class, including themselves. There are 8 children in the class. How many cookies did they bake? 48 cookies
 $12 \div 2 = 6$ 6 cookies $\times 8$ children = 48 cookies

Monkeys on the Roof - Worksheet 5

Oops! The monkeys on the roof don't all fit in the cages.

What if there are 13 monkeys? You can get 12 of the monkeys into the cages, but there is one left over. The zookeeper did not want to use another cage. So he tied the baby monkey to a balloon and floated it up to the helicopter. The monkey on the balloon is called a remainder. You can have more than one monkey on a balloon.



Cage Size	Number of Full Cages on the Roof	Number of Monkeys in the house	Remainder
4	3	14	R2
4	2	11	R3
4	2	9	R1
4	3	15	R3
4	1	7	R3
4	5	21	R1
4	1	5	R1

Division: Recording - Worksheet 1

- $12 \div 3 = 4$ Drawing of your blocks: 12 blocks in a row, grouped into 4 groups of 3.
- $12 \div 4 = 3$ Drawing of your blocks: 12 blocks in a row, grouped into 3 groups of 4.
- $12 \div 2 = 6$ Drawing of your blocks: 12 blocks in a row, grouped into 6 groups of 2.
- $12 \div 6 = 2$ Drawing of your blocks: 12 blocks in a row, grouped into 2 groups of 6.
- $15 \div 3 = 5$ Drawing of your blocks: 15 blocks in a row, grouped into 5 groups of 3.
- $15 \div 5 = 3$ Drawing of your blocks: 15 blocks in a row, grouped into 3 groups of 5.
- $18 \div 2 = 9$ Drawing of your blocks: 18 blocks in a row, grouped into 9 groups of 2.
- $18 \div 3 = 6$ Drawing of your blocks: 18 blocks in a row, grouped into 6 groups of 3.
- $18 \div 6 = 3$ Drawing of your blocks: 18 blocks in a row, grouped into 3 groups of 6.
- $18 \div 9 = 2$ Drawing of your blocks: 18 blocks in a row, grouped into 2 groups of 9.
- $8 \div 2 = 4$ Drawing of your blocks: 8 blocks in a row, grouped into 4 groups of 2.
- $20 \div 4 = 5$ Drawing of your blocks: 20 blocks in a row, grouped into 5 groups of 4.

Division: Recording - Worksheet 2

Number sentence	Drawing of your blocks	Record the numbers
1. $14 \div 2 = 7$		$2 \overline{)14} \begin{matrix} 7 \end{matrix}$
2. $20 \div 5 = 4$		$5 \overline{)20} \begin{matrix} 4 \end{matrix}$
3. $21 \div 3 = 7$		$3 \overline{)21} \begin{matrix} 7 \end{matrix}$
4. $16 \div 2 = 8$		$2 \overline{)16} \begin{matrix} 8 \end{matrix}$
5. $16 \div 4 = 4$		$4 \overline{)16} \begin{matrix} 4 \end{matrix}$

Division: Recording - Worksheet 3

Number sentence	Drawing of your blocks	Record the numbers
6. $9 \div 9 = 1$		$9 \overline{)9} \begin{matrix} 1 \end{matrix}$
7. $21 \div 7 = 3$		$7 \overline{)21} \begin{matrix} 3 \end{matrix}$
8. $25 \div 5 = 5$		$5 \overline{)25} \begin{matrix} 5 \end{matrix}$
9. $27 \div 3 = 9$		$3 \overline{)27} \begin{matrix} 9 \end{matrix}$
10. $24 \div 1 = 24$		$1 \overline{)24} \begin{matrix} 24 \end{matrix}$

Division: Recording - Worksheet 4

Number sentence	Drawing of your blocks	Record the numbers
11. $24 \div 4 = 6$		$4 \overline{)24} \begin{matrix} 6 \end{matrix}$
12. $24 \div 6 = 4$		$6 \overline{)24} \begin{matrix} 4 \end{matrix}$
13. $24 \div 8 = 3$		$8 \overline{)24} \begin{matrix} 3 \end{matrix}$
14. $24 \div 12 = 2$		$12 \overline{)24} \begin{matrix} 2 \end{matrix}$
15. $24 \div 2 = 12$		$2 \overline{)24} \begin{matrix} 12 \end{matrix}$

What patterns do you see? All dividends are 24. As the divisor goes up, the quotient goes down.

Missing Factors - Worksheet 1

Use your Cuisenaire Rods and a meter stick to solve these problems. You can also use tiles.

Question	Number Sentence
1. How many 5s make 15? <u>3</u>	$5 \times \underline{3} = 15$ Five and <u>three</u> are factors of 15.
How many 4s make 20? <u>5</u>	$4 \times \underline{5} = 20$ Four and <u>5</u> are factors of 20.
How many 3s make 21? <u>7</u>	$3 \times \underline{7} = 21$ Three and <u>7</u> are factors of 21.
How many 6s make 24? <u>4</u>	$6 \times \underline{4} = 24$ Six and <u>4</u> are factors of 24.
How many 8s make 24? <u>3</u>	$8 \times \underline{3} = 24$ Eight and <u>3</u> are factors of 24.
2. This time you write in the number sentence that goes with the question.	
How many 2s make 16? <u>8</u>	$2 \times \underline{8} = 16$ Two and <u>8</u> are factors of 16.
How many 7s make 28? <u>4</u>	$7 \times \underline{4} = 28$ Seven and <u>4</u> are factors of 28.
How many 4s make 32? <u>8</u>	$4 \times \underline{8} = 32$ Four and <u>8</u> are factors of 32.
How many 9s make 27? <u>3</u>	$9 \times \underline{3} = 27$ Nine and <u>3</u> are factors of 27.
3. This time you write in the question that goes with the number sentence.	
How many 5s make <u>30</u> ? <u>6</u>	$5 \times \underline{6} = 30$
How many <u>9</u> s make <u>36</u> ? <u>4</u>	$9 \times \underline{4} = 36$
How many <u>8</u> s make <u>40</u> ? <u>5</u>	$8 \times \underline{5} = 40$
How many <u>2</u> s make <u>12</u> ? <u>6</u>	$2 \times \underline{6} = 12$
How many <u>3</u> s make <u>15</u> ? <u>5</u>	$3 \times \underline{5} = 15$

Missing Factors - Worksheet 2

This time you write the factors that go with the question. Use blocks or rods to help you find the answers.

Study the example.

Example

How many 5s make 15? First write the problem. $5 \times \square = 15$

Then write the missing factor in the box. $5 \times \boxed{3} = 15$

Question

Number

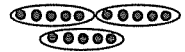
- How many 5s make 25? $5 \times \boxed{5} = 25$
- How many 6s make 18? $6 \times \boxed{3} = 18$
- How many 7s make 28? $7 \times \boxed{4} = 28$
- How many 3s make 30? $3 \times \boxed{10} = 30$
- How many 4s make 16? $4 \times \boxed{4} = 16$
- How many 9s make 36? $9 \times \boxed{4} = 36$
- How many 2s make 14? $2 \times \boxed{7} = 14$
- How many 3s make 12? $3 \times \boxed{4} = 12$
- How many 5s make 20? $5 \times \boxed{4} = 20$
- How many 6s make 42? $6 \times \boxed{7} = 42$

Missing Factors - Worksheet 3

This time you write the question that goes with the problem. Draw pictures to help you find the answers.

Make a drawing

$5 \times \boxed{3} = 15$ How many 5s make 15?



Problem

Question

- $3 \times \boxed{3} = 9$ How many 3s make 9?
- $4 \times \boxed{5} = 20$ How many 4s make 20?
- $6 \times \boxed{5} = 30$ How many 6s make 30?
- $5 \times \boxed{2} = 10$ How many 5s make 10?
- $8 \times \boxed{4} = 32$ How many 8s make 32?
- $7 \times \boxed{2} = 14$ How many 7s make 14?
- $4 \times \boxed{9} = 36$ How many 4s make 36?
- $1 \times \boxed{10} = 10$ How many 1s make 10?
- $2 \times \boxed{11} = 22$ How many 2s make 22?

Missing Factors - Worksheet 4

Use your Cuisenaire Rods or tiles to find the missing factor in each problem

- $3 \times \underline{6} = 18$ $4 \times \underline{2} = 8$ $7 \times \underline{2} = 14$
- $5 \times \underline{5} = 25$ $8 \times \underline{4} = 32$ $9 \times \underline{5} = 45$
- $2 \times \underline{3} = 6$ $3 \times \underline{4} = 12$ $4 \times \underline{3} = 12$

Use your multiplication facts chart to get the answers to these.

- $9 \times \underline{6} = 54$ $5 \times \underline{9} = 45$ $7 \times \underline{7} = 49$
- $3 \times \underline{9} = 27$ $4 \times \underline{7} = 28$ $8 \times \underline{6} = 48$
- $2 \times \underline{8} = 16$ $7 \times \underline{6} = 42$ $9 \times \underline{8} = 72$
- $8 \times \underline{8} = 64$ $6 \times \underline{8} = 48$ $7 \times \underline{8} = 56$
- $6 \times \underline{7} = 42$ $5 \times \underline{8} = 40$ $4 \times \underline{6} = 24$

Do these by drawing a picture. Do your drawing on this paper

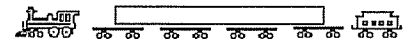
- $2 \times \underline{7} = 14$ $5 \times \underline{6} = 30$
- $6 \times \underline{3} = 18$ $6 \times \underline{6} = 36$
- $4 \times \underline{5} = 20$ $4 \times \underline{9} = 36$

Challenge! Try these. Use your logic.

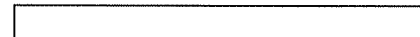
- $\frac{1}{2} \times \underline{2} = 1$ $\frac{1}{2} \times \underline{4} = 2$ $\frac{1}{4} \times \underline{4} = 1$ $\frac{1}{4} \times \underline{8} = 2$

Remainders - Worksheet 1

Cuisenaire Rods



Use each color of Cuisenaire Rod to make this 12 train.



Use white rods to finish the train when it doesn't come out even.

Record your results in the sentence above the picture of the Cuisenaire Rod.

Draw a circle around the remainders in each train.

My results:

The orange rod takes 1 orange(s) and 2 white(s).

Examples:

The blue rod takes 1 blue(s) and 3 white(s).

The brown rod takes 1 brown(s) and 4 white(s).

The black rod takes 1 black(s) and 5 white(s).

The dark green rod takes 2 dark green(s) and 0 white(s).

The yellow rod takes 2 yellow(s) and 2 white(s).

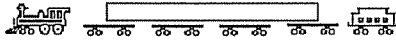
The purple rod takes 3 purple(s) and 0 white(s).

The light green rod takes 4 light green(s) and 0 white(s).

The red rod takes 6 red(s) and 0 white(s).

The white rod takes 12

Remainders - Worksheet 2



Use each color of Cuisenaire Rod to make this 16



Use white rods to finish the train when it doesn't come out even.
Record your results in the sentence above the picture of the Cuisenaire Rod.
Draw a circle around the remainders in each train.

My results:

The orange rod takes 1 orange(s) and 6 white(s).

 The blue rod takes 1 blue(s) and 1 white(s).

 The brown rod takes 2 brown(s) and 0 white(s).

 The black rod takes 2 black(s) and 2 white(s).

 The dark green rod takes 2 dark green(s) and 4 white(s).

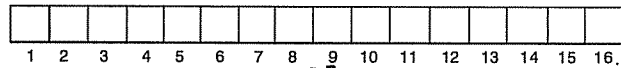
 The yellow rod takes 3 yellow(s) and 1 white(s).

 The purple rod takes 4 purple(s) and 0 white(s).

 The light green rod takes 5 light green(s) and 1 white(s).

 The red rod takes 8 red(s) and 0 white(s).

Remainders - Worksheet 4 Cuisenaire Rods



$$\begin{array}{r} 3 \\ 4 \overline{)12} \end{array} \quad \begin{array}{r} 2 \text{ R } 2 \\ 5 \overline{)12} \end{array} \quad \begin{array}{r} 2 \\ 6 \overline{)12} \end{array}$$

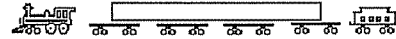
$$\begin{array}{r} 3 \text{ R } 1 \\ 4 \overline{)13} \end{array} \quad \begin{array}{r} 2 \text{ R } 3 \\ 5 \overline{)13} \end{array} \quad \begin{array}{r} 2 \text{ R } 1 \\ 6 \overline{)13} \end{array}$$

$$\begin{array}{r} 3 \text{ R } 3 \\ 4 \overline{)14} \end{array} \quad \begin{array}{r} 2 \text{ R } 4 \\ 5 \overline{)14} \end{array} \quad \begin{array}{r} 2 \text{ R } 2 \\ 6 \overline{)14} \end{array}$$

$$\begin{array}{r} 3 \text{ R } 3 \\ 4 \overline{)15} \end{array} \quad \begin{array}{r} 3 \\ 5 \overline{)15} \end{array} \quad \begin{array}{r} 2 \text{ R } 3 \\ 6 \overline{)15} \end{array}$$

$$\begin{array}{r} 4 \\ 4 \overline{)16} \end{array} \quad \begin{array}{r} 3 \text{ R } 1 \\ 5 \overline{)16} \end{array} \quad \begin{array}{r} 2 \text{ R } 4 \\ 6 \overline{)16} \end{array}$$

Remainders - Worksheet 3



Use each color of Cuisenaire Rod to make a 14



Use white rods to finish the train when it doesn't come out even.
Record your results in the sentence above the picture of the Cuisenaire Rods.
Draw a circle around the remainders in each train.

Number sentence: The orange rod takes 1 orange(s) and 4 white(s).
 $14 \div 10 = \text{R}4$

 The blue rod takes 1 blue(s) and 5 white(s).
 $14 \div 9$

 The brown rod takes 1 brown(s) and 6 white(s).
 $14 \div 8$

 The black rod takes 2 black(s) and 0 white(s).
 14

 The dark green rod takes 2 dark green(s) and 2 white(s).
 14

 The yellow rod takes 2 yellow(s) and 4 white(s).
 14

 The purple rod takes 3 purple(s) and 2 white(s).
 14

 The light green rod takes 4 light green(s) and 2 white(s).
 14

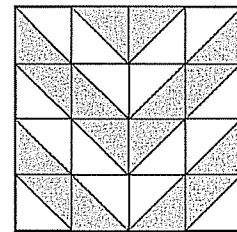
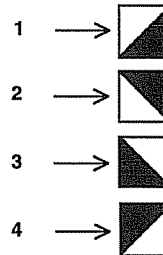
 The red rod takes 7 red(s) and 0 white(s).
 14

Remainders - Worksheet 5

Solve these problems.
If the remainder is:

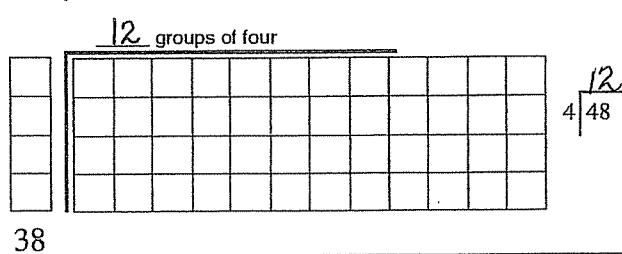
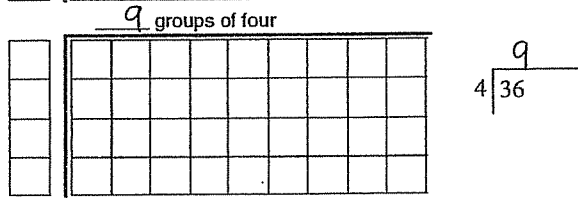
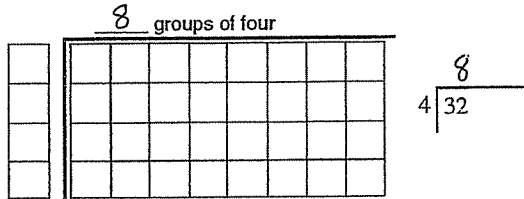
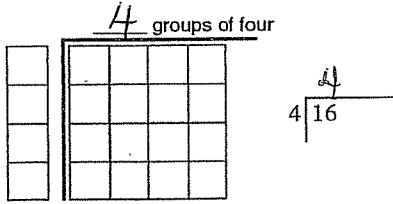
Color the space:

Correct answer makes this design.



$\begin{array}{r} 2 \text{ R } 3 \\ 6 \overline{)15} \end{array}$	$\begin{array}{r} 2 \text{ R } 2 \\ 8 \overline{)18} \end{array}$	$\begin{array}{r} 3 \text{ R } 4 \\ 7 \overline{)25} \end{array}$	$\begin{array}{r} 3 \text{ R } 1 \\ 5 \overline{)16} \end{array}$
$\begin{array}{r} 3 \text{ R } 2 \\ 7 \overline{)23} \end{array}$	$\begin{array}{r} 5 \text{ R } 3 \\ 5 \overline{)28} \end{array}$	$\begin{array}{r} 4 \text{ R } 1 \\ 6 \overline{)25} \end{array}$	$\begin{array}{r} 3 \text{ R } 4 \\ 6 \overline{)22} \end{array}$
$\begin{array}{r} 4 \text{ R } 3 \\ 4 \overline{)19} \end{array}$	$\begin{array}{r} 4 \text{ R } 2 \\ 5 \overline{)22} \end{array}$	$\begin{array}{r} 7 \text{ R } 4 \\ 5 \overline{)39} \end{array}$	$\begin{array}{r} 6 \text{ R } 1 \\ 3 \overline{)19} \end{array}$
$\begin{array}{r} 6 \text{ R } 2 \\ 4 \overline{)26} \end{array}$	$\begin{array}{r} 4 \text{ R } 3 \\ 7 \overline{)31} \end{array}$	$\begin{array}{r} 5 \text{ R } 1 \\ 2 \overline{)11} \end{array}$	$\begin{array}{r} 5 \text{ R } 4 \\ 5 \overline{)29} \end{array}$

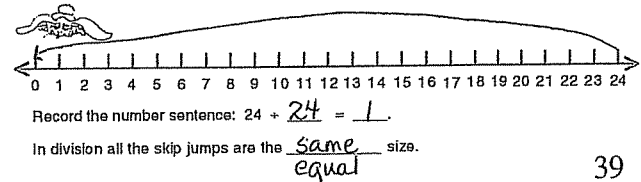
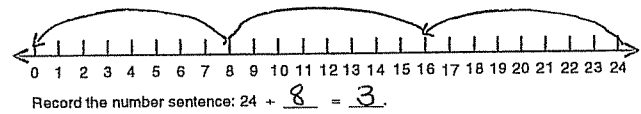
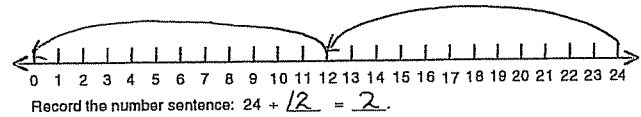
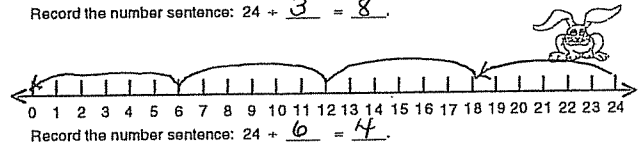
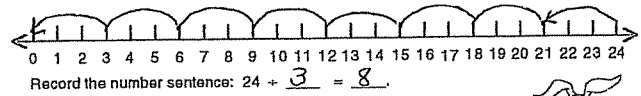
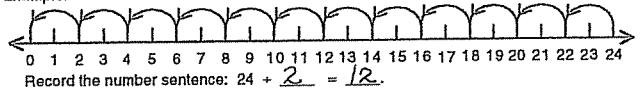
Remainders - Worksheet 6
Cuisenaire Rods Review - Division by Four



Remainders - Worksheet 7 OWV
Number Lines Review



Skip jump to divide twenty-four six different ways on the number line. Start on 24. Take away 2s until you reach 0. How many jumps? 12
How many groups of 2 are in 24? 12
Example:



39

Families of Facts: Patterns - Worksheet 1

Remember this chart?

The number you start with.	The number of times each group is made.	The number of groups you made.
2	$\times 3$	$= 6$

Build $2 \times 3 = 6$ with the blocks. Draw a picture of the blocks in this box.

3	$\times 2$	$= 6$
---	------------	-------

Build $3 \times 2 = 6$ with the blocks. Draw a picture of the blocks in this box. \rightarrow

Are your two drawings the same? no

How are they different?

They have different sizes of groups and numbers of groups.

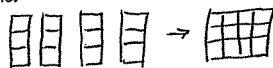
How are they same?

The answer is the same, six blocks.

On these next problems, look at the picture of the blocks.

Write the number sentence for each picture.

Example:



$3 \times 4 = 12$



$4 \times 3 = 12$

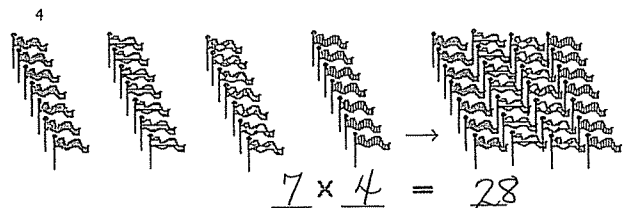
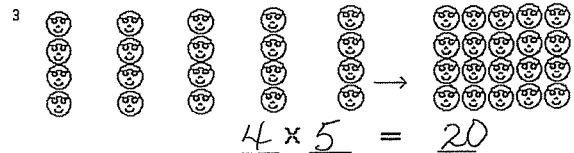
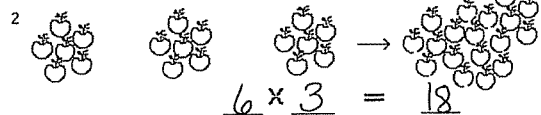
factor factor product

Now you do one.

40

Families of Facts: Patterns - Worksheet 2

Tell what multiplication problem each picture shows.



5. This time the problem is written. You draw the picture to go with it.



$4 \times 4 = 16$

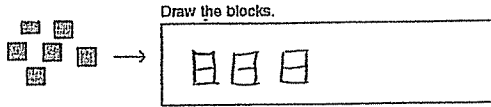
41

Families of Facts: Patterns - Worksheet 3

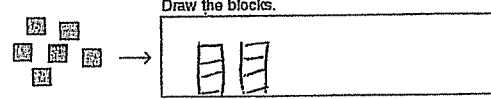
Remember this chart?

The number you start with.	\div	The size of the group you make.	=	The number of groups you made.
6	\div	2	=	3

Take out six blocks. Divide them into groups of two.



Start with six blocks again. This time divide them into groups of three.



6	\div	3	=	2
---	--------	---	---	---

Are your two drawings the same? no How are they different?

There are different sizes of groups and different numbers of groups.

How are they same?

Each problem starts with six blocks.

For each picture write the number sentence.

Example: $8 \div 2 = 4$
dividend divisor quotient

What problem is this? $8 \div 4 = 2$

42

Families of Facts: Patterns - Worksheet 5

Write the number sentence for each picture.

You will have to put in the X or the + signs.

5. $18 \div 6 = 3$

No fishing PLEASE

6. There's something fishy about this! $6 \times 3 = 18$

fish sale 4 sale 5 & each

7. $3 \times 6 = 18$

8. $18 \div 3 = 6$

44

Families of Facts: Patterns - Worksheet 4

Write the number sentence for each picture.

1. $18 \div 6 = 3$

2. $20 \div 4 = 5$

3. $28 \div 7 = 4$

4. Draw the division problem which goes with multiplication problem 4 on Families of Facts: Patterns - Worksheet 2. Write the number sentence.

$16 \div 4 = 4$

43

Families of Facts: Patterns - Worksheet 6

9. With the blocks, build two times five. Each group must be a different color.

What problem is this? $2 \times 5 = 10$

Push the blocks together.

Now divide the blocks so that each group has one block of each color. $10 \div 5 = 2$

What problem does that show?

10. What problem is this?

$5 \times 2 = 10$

Now separate the blocks again so that all the blocks of the same color are together. What problem is this?

$10 \div 2 = 5$

11. Write all four number sentences from this page.

$2 \times 5 = 10$ $10 \div 2 = 5$
 $5 \times 2 = 10$ $10 \div 5 = 2$

45

Families of Facts: Practice - Worksheet 1

Use only the three numbers in the box to make four number sentences.



3 15 5	8 24 3	7 28 4
$3 \times 5 = 15$	$8 \times 3 = 24$	$4 \times 7 = 28$
$5 \times 3 = 15$	$3 \times 8 = 24$	$7 \times 4 = 28$
$15 \div 3 = 5$	$24 \div 8 = 3$	$28 \div 4 = 7$
$15 \div 5 = 3$	$24 \div 3 = 8$	$28 \div 7 = 4$

For you:

30 6 5	9 54 6	AWV
$6 \times 5 = 30$	$9 \times 6 = 54$	$\bigcirc \times \bigcirc = \bigcirc$
$5 \times 6 = 30$	$6 \times 9 = 54$	$\bigcirc \times \bigcirc = \bigcirc$
$30 \div 6 = 5$	$54 \div 9 = 6$	$\bigcirc \div \bigcirc = \bigcirc$
$30 \div 5 = 6$	$54 \div 6 = 9$	$\bigcirc \div \bigcirc = \bigcirc$

21 7 3		
$7 \times 3 = 21$	$\begin{array}{r} 7 \\ \times 3 \\ \hline 21 \end{array}$	$21 \div 7 = 3$
$3 \times 7 = 21$	$\begin{array}{r} 3 \\ \times 7 \\ \hline 21 \end{array}$	$21 \div 3 = \bigcirc$
		$7 \overline{) 21}$
		$3 \overline{) 21}$

Word Problems - Worksheet 1



Use counters to prove your answer. Show how you did the problem. Draw a picture and write a number sentence.

1. You have 15 bubble gum balls in a bag. You want to give 3 gum balls to each friend. How many friends can you give gum to? 5 friends

$15 \div 3 = 5$

2. Of the 15 gum balls, 4 are white and 7 are green gum balls. The rest are yellow. How many yellow gum balls? 4 yellow gum balls

$4 + 7 = 11$ $15 - 11 = 4$

3. You have 20 rats. You are going to give 4 rats per classroom to different classrooms. How many rooms will get rats? 5 rooms

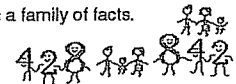
$20 \div 4 = 5$

4. If there were 28 students all together and you wanted to divide the students into four groups, how many students would be in each group? 7 students

$28 \div 4 = 7$

Families of Facts: Practice - Worksheet 2

Fill in the blank space with a number that creates a family of facts. Use only the three numbers in the box to make four number sentences.



4 36 9	4 32 8	7 42 6
$4 \times 9 = 36$	$8 \times 4 = 32$	$6 \times 7 = 42$
$9 \times 4 = 36$	$4 \times 8 = 32$	$7 \times 6 = 42$
$36 \div 4 = 9$	$32 \div 8 = 4$	$42 \div 6 = 7$
$36 \div 9 = 4$	$32 \div 4 = 8$	$42 \div 7 = 6$

For you:

35 5 7	24 12 2	AWV
$5 \times 7 = 35$	$12 \times 2 = 24$	$\bigcirc \times \bigcirc = \bigcirc$
$7 \times 5 = 35$	$2 \times 12 = 24$	$\bigcirc \times \bigcirc = \bigcirc$
$35 \div 5 = 7$	$24 \div 12 = 2$	$\bigcirc \div \bigcirc = \bigcirc$
$35 \div 7 = 5$	$24 \div 2 = 12$	$\bigcirc \div \bigcirc = \bigcirc$

Example: $\begin{array}{r} 24 \ 48 \ 2 \\ 24 \times 2 = 48 \\ 2 \times 24 = 48 \\ 48 \div 24 = 2 \\ 48 \div 2 = 24 \end{array}$

Word Problems - Worksheet 2



5. If each of the 28 students get 3 cookies each, how many cookies will you need for each group? Remember, there are four groups. In problem 4, you figured out how many students in each group. Each group will need 21 cookies. All together, you will need a total of how many cookies? 147 cookies

$21 + 21 + 21 + 21 + 21 + 21 + 21 = 147$

6. You have 6 pet rabbits. Each rabbit had 8 babies. How many baby rabbits in all? 48 baby rabbits

$6 \times 8 = 48$

Challenge.

7. One half of the baby rabbits were white and brown. One fourth were brown one fourth were white. How many were white and brown? 24 rabbits. How many were brown? 12. How many were white? 12

$48 \div 2 = 24$

$48 \div 4 = 12$

$48 \div 4 = 12$

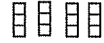
Families of Facts: Connections
Worksheet 1

Use only the three numbers in the box to make four number sentences.

3 4 12

$3 \times 4 = 12$ $12 \div 3 = 4$
 $4 \times 3 = 12$ $12 \div 4 = 3$

If this is 3 x 4,



Then 4 x 3



Build your own model and draw a picture.

$3 \times 4 = 12$ $\begin{array}{r} 3 \\ \times 4 \\ \hline 12 \end{array}$

$4 \times 3 = 12$ $\begin{array}{r} 4 \\ \times 3 \\ \hline 12 \end{array}$

Build your own model and draw a picture

$12 \div 3 = 4$ $3 \overline{)12}$

$12 \div 4 = 3$ $4 \overline{)12}$

Families of Facts: Connections
Worksheet 3

3 8 24

$3 \times 8 = 24$ $\begin{array}{r} 3 \\ \times 8 \\ \hline 24 \end{array}$
 $8 \times 3 = 24$ $\begin{array}{r} 8 \\ \times 3 \\ \hline 24 \end{array}$
 $24 \div 3 = 8$ $3 \overline{)24}$
 $24 \div 8 = 3$ $8 \overline{)24}$

7 35 5

$7 \times 5 = 35$ $\begin{array}{r} 7 \\ \times 5 \\ \hline 35 \end{array}$
 $5 \times 7 = 35$ $\begin{array}{r} 5 \\ \times 7 \\ \hline 35 \end{array}$
 $35 \div 7 = 5$ $7 \overline{)35}$
 $35 \div 5 = 7$ $5 \overline{)35}$

Make your own.

AWV

8 7 56

$8 \times 7 = 56$ $\begin{array}{r} 8 \\ \times 7 \\ \hline 56 \end{array}$
 $7 \times 8 = 56$ $\begin{array}{r} 7 \\ \times 8 \\ \hline 56 \end{array}$
 $56 \div 8 = 7$ $8 \overline{)56}$
 $56 \div 7 = 8$ $7 \overline{)56}$

$\square \times \square = \square$ \times
 $\square \times \square = \square$ \times
 $\square \div \square = \square$ \square
 $\square \div \square = \square$ \square

$3 \overline{)27}$

$4 \overline{)32}$

$7 \overline{)63}$

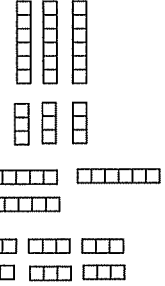
Families of Facts: Connections
Worksheet 2



Draw a picture of each problem.

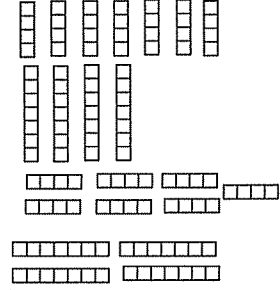
6 3 18

$6 \times 3 = 18$ $\begin{array}{r} 6 \\ \times 3 \\ \hline 18 \end{array}$
 $3 \times 6 = 18$ $\begin{array}{r} 3 \\ \times 6 \\ \hline 18 \end{array}$
 $18 \div 6 = 3$ $6 \overline{)18}$
 $18 \div 3 = 6$ $3 \overline{)18}$



28 4 7

$4 \times 7 = 28$ $\begin{array}{r} 4 \\ \times 7 \\ \hline 28 \end{array}$
 $7 \times 4 = 28$ $\begin{array}{r} 7 \\ \times 4 \\ \hline 28 \end{array}$
 $28 \div 4 = 7$ $4 \overline{)28}$
 $28 \div 7 = 4$ $7 \overline{)28}$



$6 \overline{)18}$ $7 \overline{)28}$ $3 \overline{)18}$
 $5 \overline{)35}$

Make your own. AWV

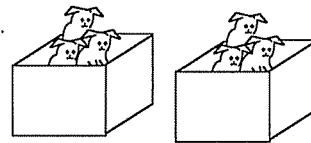
Pre - Assessment - Part 1

1. Start with eight blocks. Draw the blocks.
- How many groups of two can you make? 4
 - How many groups of four can you make? 2
 - How many groups of one can you make? 8
 - How many groups of eight can you make? 1

2. Gracie had six puppies and some big boxes. She put three puppies in each box.

a. How many boxes did she use? 2

b. Draw a picture.



3. Use Cuisenaire Rods to build if needed. This is a 12 white rod train. Record leftovers as remainders.

Circle groups of: Example: Number of Groups

2 $\div 2 = 6$

a. 3 $\div 3 = 4$

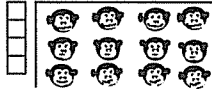
b. 4 $\div 4 = 3$

c. 5 $\div 5 = 2R2$

d. 7 $\div 7 = 1R5$

Pre - Assessment - Part 2 - Worksheet 1

1. Mr. Zoo had 12 monkeys and some cages. Each cage holds 4 monkeys. How many cages does Mr. Zoo need for his monkeys?
3



2. Write the division number sentence for problem one. $12 \div 4 = 3$

3. a. What question does this number sentence ask? $24 \div 4 = 6$
How many groups of 4 are in 24?

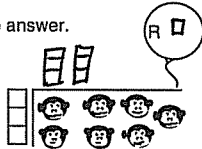
- b. What is the answer? 6

4. a. Draw the answer.

- b. Write the division number sentence. $3 \overline{) 6}$

5. a. $2 \overline{) 8}$ b. $5 \overline{) 15}$ c. $4 \overline{) 32}$

6. a. Draw the answer.



- b. Write the division number sentence. $3 \overline{) 7} R^1$

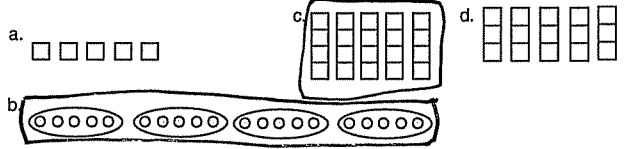
The Post Assessment is the same as the Pre-Assessment-Part 2.

Pre - Assessment - Part 2 - Worksheet 2

7. a. $4 \overline{) 18} R^2$ b. $6 \overline{) 32} R^2$ c. $7 \overline{) 23} R^2$

8. In the problem $20 \div 4 = 5$, 5 is the 'answer.'

Draw a box around any drawings below that show what the 5 means.



9. Make a Family of Facts with these three numbers. $3 \quad 6 \quad 18$

a. $3 \times 6 = 18$ b. $3 \overline{) 18}$ c. $6 \overline{) 18}$
 $6 \times 3 = 18$ $18 \div 3 = 6$ $18 \div 6 = 3$

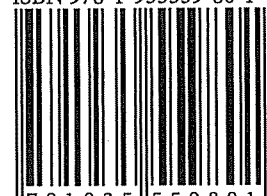
10. These are hard.

Fill in the missing numbers.

- a. $8 \div 2 = 4$ b. $15 \div 5 = 3$ c. $20 \div 4 = 5$

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