INTERMEDIATE LOGIC

Mastering Propositional Arguments

TEACHER: THIRD EDITION

Canon Logic Series

canonpress
Moscow, Idaho

PUBLISHER'S NOTE from CANON PRESS

SCHEDULES

It's up to you to choose the pace for working through *Intermediate Logic*. If you're comfortable with moving at a quick pace, and can schedule three to five classes per week, you can work through the course in one semester. Those who prefer a more leisurely pace can plan to complete the course in a year with one to three class meetings per week. On the following pages, we have provided four sample schedules—one semester or a full year, including or not including the optional Unit 5. Use these as a guideline, and adapt as needed to meet the needs of your class or homeschool. Just cover the material listed for each week in as many days as you have, and you'll finish on time. Or tighten or expand either schedule to suit your students' pace and the time you have allotted for the course.

PAGE NUMBERS

This Teacher text contains the entire Student version as well—with the same page numbers as the students you'll be teaching. The Arabic numerals (on single-columned pages) are the same in both texts. Your teacher notes (double-columned pages) are numbered with Roman numerals.

DAILY LESSON PLANS

Each student lesson in the Teacher edition is accompanied by double-columned teaching notes: objectives, step-by-step teaching instructions, assignments, and more. You can decide whether you want to read through the lesson with the students out loud, have the students read through it alone and then teach through it, teach through it without reading it…whatever suits your personal teaching style best.

GRADING

This Teacher Edition contains all the answers you need for all exercises, quizzes, and tests. For many lessons, answers may vary depending on the imagination and creativity of your students. Expect this; you'll still be able to grade the differing answers fairly if you, as teacher, thoroughly understand the principles involved. We've included point values for each quiz or exercise question to help with this. Consider giving partial credit for incorrect answers that have a piece of the final answer right. If you mark an answer wrong, but a student thinks it is *not* wrong, consider allowing them to try to argue the point back, in writing. This gives them practice arguing, and they just might be right.

DVD COURSE

If you can take advantage of the fantastic DVD course companion, we'd suggest that you watch the day's lesson first (let our teacher's years of experience do the hard work), and then you can answer any questions as your students work on the exercises. The DVD works through every "Form B" Test, so the DVD can be especially helpful for practice tests.

As always, if you've got questions, ideas, or just want to get in touch, call 208-892-8074 or find us online at www.canonpress.com. We'd love to help you as you teach the mastery of propositional arguments.

SCHEDULE OPTION 1A: ONE SEMESTER (NOT INCLUDING UNIT 5)

This schedule will allow you to cover the contents of *Intermediate Logic* meeting daily over the course a sixteen-week semester. Adapt as needed if you meet fewer days per week.

Week	Day	Text	Assignment
1	Mon	Lesson 1	Exercise 1
	Tues	Lesson 2	Exercise 2
	Wed	Quiz Day	*Quiz One*
	Thur	Lesson 3	Exercise 3, 1–3
	Fri		Exercise 3, 4–15
2	Mon	Lesson 4	Exercise 4, 1–2
	Tues		Exercise 4, 3–15
	Wed	Quiz Day	*Quiz Two*
	Thur	Review for Test	Review Questions
	Fri	Test Day	*Test One*
3	Mon	Lesson 5	Exercise 5
	Tues	Lesson 6	Exercise 6
	Wed	Quiz Day	*Quiz Three*
	Thur	Lesson 7	Exercise 7a, 1–10
	Fri		Exercise 7a, 11–14
4	Mon		Exercise 7b
	Tues	Quiz Day	*Quiz Four*
	Wed	Review for Test	Review Questions
	Thur	Test Day	*Test Two*
	Fri	Lesson 8	Exercise 8, 1–6
5	Mon		Exercise 8, 7–12
	Tues	Lessons 9	Exercise 9
	Wed	Quiz Day	*Quiz Five*
	Thur	Lesson 10	Exercise 10
	Fri	Lesson 11	Exercise 11
6	Mon	Lesson 12	Exercise 12
	Tues	Quiz Day	*Quiz Six*
	Wed	Review for Test	Review Questions
	Thur	Test Day	*Test Three*
	Fri	Lesson 13	Exercise 13
7	Mon	Lesson 14	Exercise 14a, 1–9
	Tues		Exercise 14a, 10–16
	Wed	Quiz Day	*Quiz Seven*
	Thur		Exercise 14b
	Fri	Lesson 15	Exercise 15a, 1–6
8	Mon		Exercise 15a, 7–16
	Tues	Quiz Day	*Quiz Eight*
	Wed		Exercise 15b
	Thur	Review for Test	Review Questions
	Fri	Test Day	*Test Four*

Week	Day	Text	Assignment
9	Mon	Lesson 16	Exercise 16, 1–10
	Tues		Exercise 16, 11–18
	Wed	Quiz Day	*Quiz Nine*
	Thur	Lesson 17	Exercise 17a, 1–6
	Fri		Exercise 17a, 7–9
10	Mon		Lesson 17a, 10–14
	Tues		Exercise 17b
	Wed	Quiz Day	*Quiz Ten*
	Thur	Review for Test	Review Questions
	Fri	Test Day	*Test Five*
11	Mon	Lesson 18	Exercise 18, 1–4
	Tues		Exercise 18, 5–9
	Wed	Lesson 19	Exercise 29
	Thur	Quiz Day	*Quiz Eleven*
	Fri	Lesson 20	Exercise 20, 1–2
12	Mon		Exercise 20, 3–7
	Tues	Lesson 29	Exercise 21
	Wed	Quiz Day	*Quiz Twelve*
	Thur	Review for Test	Review Questions
	Fri	Test Day	*Test Six*
13	Mon	Lesson 22	Exercise 22
	Tues	Lesson 23	Exercise 23
	Wed	Lesson 24	Exercise 24
	Thur	Quiz Day	*Quiz Thirteen*
	Fri	Lesson 25	Exercise 25
14	Mon	Lesson 26	Exercise 26
	Tues	Lesson 27	Exercise 27
	Wed	Quiz Day	*Quiz Fourteen*
	Thur	Review for Test	Review Questions
	Fri	Test Day	*Test Seven*
15	Mon	Lesson 28	Exercise 28a
	Tues		Exercise 28b
	Wed		Exercise 28c, 1–4
	Thur		Exercise 28c, 5–9
	Fri	Quiz Day	*Quiz Fifteen*
16	Mon	Review for Comprehensive Exam	
	Tues	Review for Comprehensive Exam	
	Wed	Exam	*Comprehensive Exam*

SCHEDULE OPTION 1B: ONE SEMESTER (INCLUDING UNIT 5)

This schedule will allow you to cover the contents of *Intermediate Logic* meeting daily over the course a twenty-week semester. Adapt as needed if you meet fewer days per week.

Week	Day	Text	Assignment
1	Mon	Lesson 1	Exercise 1
	Tues	Lesson 2	Exercise 2
	Wed	Quiz Day	*Quiz One*
	Thur	Lesson 3	Exercise 3, 1–3
	Fri		Exercise 3, 4–15
2	Mon	Lesson 4	Exercise 4, 1–2
	Tues		Exercise 4, 3–15
	Wed	Quiz Day	*Quiz Two*
	Thur	Review for Test	Review Questions
	Fri	Test Day	*Test One*
3	Mon	Lesson 5	Exercise 5
	Tues	Lesson 6	Exercise 6
	Wed	Quiz Day	*Quiz Three*
	Thur	Lesson 7	Exercise 7a, 1–10
	Fri		Exercise 7a, 11–14
4	Mon		Exercise 7b
	Tues	Quiz Day	*Quiz Four*
	Wed	Review for Test	Review Questions
	Thur	Test Day	*Test Two*
	Fri	Lesson 8	Exercise 8, 1–6
5	Mon		Exercise 8, 7–12
	Tues	Lessons 9	Exercise 9
	Wed	Quiz Day	*Quiz Five*
	Thur	Lesson 10	Exercise 10
	Fri	Lesson 11	Exercise 11
6	Mon	Lesson 12	Exercise 12
	Tues	Quiz Day	*Quiz Six*
	Wed	Review for Test	Review Questions
	Thur	Test Day	*Test Three*
	Fri	Lesson 13	Exercise 13
7	Mon	Lesson 14	Exercise 14a, 1–9
	Tues		Exercise 14a, 10–16
	Wed	Quiz Day	*Quiz Seven*
	Thur		Exercise 14b
	Fri	Lesson 15	Exercise 15a, 1–6
8	Mon		Exercise 15a, 7–16
	Tues	Quiz Day	*Quiz Eight*
	Wed		Exercise 15b
	Thur	Review for Test	Review Questions
	Fri	Test Day	*Test Four*
9	Mon	Lesson 16	Exercise 16, 1–10
	Tues		Exercise 16, 11–18
	Wed	Quiz Day	*Quiz Nine*
	Thur	Lesson 17	Exercise 17a, 1–6
	Fri		Exercise 17a, 7–9
10	Mon		Lesson 17a, 10–14
	Tues		Exercise 17b
	Wed	Quiz Day	*Quiz Ten*
	Thur	Review for Test	Review Questions
	Fri	Test Day	*Test Five*
11	Mon	Lesson 18	Exercise 18, 1–4
	Tues		Exercise 18, 5–9
	Wed	Lesson 19	Exercise 29
	Thur	Quiz Day	*Quiz Eleven*
	Fri	Lesson 20	Exercise 20, 1–2
12	Mon		Exercise 20, 3–7
	Tues	Lesson 29	Exercise 21
	Wed	Quiz Day	*Quiz Twelve*
	Thur	Review for Test	Review Questions
	Fri	Test Day	*Test Six*
13	Mon	Lesson 22	Exercise 22
	Tues	Lesson 23	Exercise 23
	Wed	Lesson 24	Exercise 24
	Thur	Quiz Day	*Quiz Thirteen*
	Fri	Lesson 25	Exercise 25
14	Mon	Lesson 26	Exercise 26
	Tues	Lesson 27	Exercise 27
	Wed	Quiz Day	*Quiz Fourteen*
	Thur	Review for Test	Review Questions
	Fri	Test Day	*Test Seven*
15	Mon	Lesson 28	Exercise 28a
	Tues		Exercise 28b
	Wed		Exercise 28c, 1–4
	Thur		Exercise 28c, 5–9
	Fri	Quiz Day	*Quiz Fifteen*
16	Mon	Lesson 29	Exercise 29
	Tues	Lesson 30	Exercise 30
	Wed	Lesson 31	Exercise 31
	Thur	Quiz Day	*Quiz Sixteen*
	Fri	Lesson 32	Exercise 32
17	Mon	Lesson 33	Exercise 33
	Tues	Lesson 34	Exercise 34
	Wed	Lesson 35	Exercise 35
	Thur	Quiz Day	*Quiz Seventeen*
	Fri	Lesson 36	Exercise 36
18	Mon	Lesson 37	Exercise 37, 1–3
	Tues		Exercise 37, 4–6
	Wed	Lesson 38	Exercise 38
	Thur	Quiz Day	*Quiz Eighteen*
	Fri	Lesson 39	Exercise 39
19	Mon	Lesson 40	Exercise 40a
	Tues	Quiz Day	*Quiz Nineteen*
	Wed		Exercise 40b
	Thur		Exercise 40b continued
	Fri		Exercise 40b continued
20	Mon	Review for Test	Review Questions
	Tues	Test Day	*Test Eight*
	Wed	Review for Comprehensive Exam	
	Thur	Review for Comprehensive Exam	
	Fri	Exam	*Comprehensive Exam*

SCHEDULE OPTION 2A: FULL YEAR (NOT INCLUDING UNIT 5)

This schedule will allow you to cover the contents of *Intermediate Logic* meeting three days per week over the course a thirty-week school year. Adapt as needed if you meet fewer days per week.

Week	Day	Text	Assignment
1	1	Lesson 1	Exercise 1
	2	Lesson 2	Exercise 2
	3	Quiz Day	*Quiz One*
2	4	Lesson 3	Exercise 3, 1–3
	5		Exercise 3, 4–15
	6	Lesson 4	Exercise 4, 1–2
3	7		Exercise 4, 3–15
	8	Finish Exercises and Review	
	9	Quiz Day	*Quiz Two*
4	10	Review for Test	Review Questions
	11	Practice Test	*Test 1a*
	12	Test	*Test 1b*
5	13	Lesson 5	Exercise 5
	14	Lesson 6	Exercise 6
	15	Quiz Day	*Quiz Three*
6	16	Lesson 7	Exercise 7a, 1–10
	17		Exercise 7a, 11–14
	18		Exercise 7b
7	19	Quiz/Test Review	*Quiz Four*
	20	Practice Test	*Test 2a*
	21	Test Day	*Test 2b*
8	22	Lesson 8	Exercise 8, 1–6
	23		Exercise 8, 7–12
	24	Lesson 9	Exercise 9
9	25	Quiz Day	*Quiz Five*
	26	Lesson 10	Exercise 10
	27	Lesson 11	Exercise 11
10	28	Lesson 12	Exercise 12
	29	Finish Exercises and Review	
	30	Quiz Day	*Quiz Six*
11	31	Review for Test	Review Questions
	32	Practice Test	*Test 3a*
	33	Test Day	*Test 3b*
12	34	Lesson 13	Exercise 13
	35	Lesson 14	Exercise 14a, 1–9
	36		Exercise 14a, 10–16
13	37	Quiz Day	*Quiz Seven*
	38		Exercise 14b
	39	Lesson 15	Exercise 15a, 1–6
14	40		Exercise 15a, 7–16
	41	Quiz Day	*Quiz Eight*
	42		Exercise 15b
15	43	Review for Test	Review Questions
	44	Practice Test	*Test 4a*
	45	Test Day	*Test 4b*
16	46	Lesson 16	Exercise 16, 1–10
	47		Exercise 16, 11–18
	48	Quiz Day	*Quiz Nine*
17	49	Lesson 17	Exercise 17a, 1–6
	50		Exercise 17a, 7–9
	51		Exercise 17a, 10–14
18	52		Exercise 17b
	53	Finish Exercises and Review	
	54	Quiz Day	*Quiz Ten*
19	55	Review for Test	Review Questions
	56	Practice Test	*Test 5a*
	57	Test Day	*Test 5b*
20	58	Lesson 18	Exercise 18, 1–4
	59		Exercise 18, 5–9
	60	Lesson 19	Exercise 19
21	61	Quiz Day	*Quiz Eleven*
	62	Lesson 20	Exercise 20, 1–2
	63		Exercise 20, 3–7
22	64	Lesson 21	Exercise 21
	65	Finish Exercises and Review	
	66	Quiz Day	*Quiz Twelve*
23	67	Review for Test	Review Questions
	68	Practice Test	*Test 6a*
	69	Test Day	*Test 6b*
24	70	Lesson 22	Exercise 22
	71	Lesson 23	Exercise 23
	72	Finish exercises	
25	73	Lesson 24	Exercise 24
	74	Quiz Day	*Quiz Thirteen*
	75	Lesson 25	Exercise 25
26	76	Lesson 26	Exercise 26
	77	Lesson 27	Exercise 27
	78	Quiz Day	*Quiz Fourteen*
27	79	Review for Test	Review Questions
	80	Practice Test	*Test 7a*
	81	Test Day	*Test 7b*
28	82	Lesson 28	Exercise 28a
	83		Exercise 28b
	84	Finish exercises	
29	85		Exercise 28c, 1–4
	86		Exercise 28c, 5–9
	87	Quiz Day	*Quiz Fifteen*
30	88	Review for Comprehensive Exam	
	89	Review for Comprehensive Exam	
	90	Exam	*Comprehensive Exam*

SCHEDULE OPTION 2B: FULL YEAR (INCLUDING UNIT 5)

This schedule will allow you to cover the contents of *Intermediate Logic* meeting three days per week over the course a thirty-seven-week school year. Adapt as needed if you meet fewer days per week.

Week	Day	Text	Assignment
1	1	Lesson 1	Exercise 1
	2	Lesson 2	Exercise 2
	3	Quiz Day	*Quiz One*
2	4	Lesson 3	Exercise 3, 1–3
	5		Exercise 3, 4–15
	6	Lesson 4	Exercise 4, 1–2
3	7		Exercise 4, 3–15
	8	Finish Exercises and Review	
	9	Quiz Day	*Quiz Two*
4	10	Review for Test	Review Questions
	11	Practice Test	*Test 1a*
	12	Test	*Test 1b*
5	13	Lesson 5	Exercise 5
	14	Lesson 6	Exercise 6
	15	Quiz Day	*Quiz Three*
6	16	Lesson 7	Exercise 7a, 1–10
	17		Exercise 7a, 11–14
	18		Exercise 7b
7	19	Quiz/Test Review	*Quiz Four*
	20	Practice Test	*Test 2a*
	21	Test Day	*Test 2b*
8	22	Lesson 8	Exercise 8, 1–6
	23		Exercise 8, 7–12
	24	Lesson 9	Exercise 9
9	25	Quiz Day	*Quiz Five*
	26	Lesson 10	Exercise 10
	27	Lesson 11	Exercise 11
10	28	Lesson 12	Exercise 12
	29	Finish Exercises and Review	
	30	Quiz Day	*Quiz Six*
11	31	Review for Test	Review Questions
	32	Practice Test	*Test 3a*
	33	Test Day	*Test 3b*
12	34	Lesson 13	Exercise 13
	35	Lesson 14	Exercise 14a, 1–9
	36		Exercise 14a, 10–16
13	37	Quiz Day	*Quiz Seven*
	38		Exercise 14b
	39	Lesson 15	Exercise 15a, 1–6
14	40		Exercise 15a, 7–16
	41	Quiz Day	*Quiz Eight*
	42		Exercise 15b
15	43	Review for Test	Review Questions
	44	Practice Test	*Test 4a*
	45	Test Day	*Test 4b*
16	46	Lesson 16	Exercise 16, 1–10
	47		Exercise 16, 11–18
	48	Quiz Day	*Quiz Nine*
17	49	Lesson 17	Exercise 17a, 1–6
	50		Exercise 17a, 7–9
	51		Exercise 17a, 10–14
18	52		Exercise 17b
	53	Finish Exercises and Review	
	54	Quiz Day	*Quiz Ten*
19	55	Review for Test	Review Questions
	56	Practice Test	*Test 5a*
	57	Test Day	*Test 5b*

Week	Day	Text	Assignment
20	58	Lesson 18	Exercise 18, 1–4
	59		Exercise 18, 5–9
	60	Lesson 19	Exercise 19
21	61	Quiz Day	*Quiz Eleven*
	62	Lesson 20	Exercise 20, 1–2
	63		Exercise 20, 3–7
22	64	Lesson 21	Exercise 21
	65	Finish Exercises and Review	
	66	Quiz Day	*Quiz Twelve*
23	67	Review for Test	Review Questions
	68	Practice Test	*Test 6a*
	69	Test Day	*Test 6b*
24	70	Lesson 22	Exercise 22
	71	Lesson 23	Exercise 23
	72	Finish exercises	
25	73	Lesson 24	Exercise 24
	74	Quiz Day	*Quiz Thirteen*
	75	Lesson 25	Exercise 25
26	76	Lesson 26	Exercise 26
	77	Lesson 27	Exercise 27
	78	Quiz Day	*Quiz Fourteen*
27	79	Review for Test	Review Questions
	80	Practice Test	*Test 7a*
	81	Test Day	*Test 7b*
28	82	Lesson 28	Exercise 28a
	83		Exercise 28b
	84	Finish exercises	
29	85		Exercise 28c, 1–4
	86		Exercise 28c, 5–9
	87	Quiz Day	*Quiz Fifteen*
30	88	Lesson 29	Exercise 29
	89	Lesson 30	Exercise 30
	90	Lesson 31	Exercise 31
31	91	Quiz Day	*Quiz Sixteen*
	92	Lesson 32	Exercise 32
	93	Lesson 33	Exercise 33
32	94	Lesson 34	Exercise 34
	95	Lesson 35	Exercise 35
	96	Quiz Day	*Quiz Seventeen*
33	97	Lesson 36	Exercise 36
	98	Lesson 37	Exercise 37
	99	Lesson 38	Exercise 38
34	100	Quiz Day	*Quiz Eighteen*
	101	Lesson 39	Exercise 39
	102	Lesson 40	Exercise 40a
35	103	Quiz Day	*Quiz Nineteen*
	104	Project	Exercise 40b
	105		Exercise 40b continued
36	106	Review for Test	Review Questions
	107	Practice Test	*Test 8a*
	108	Test Day	*Test 8b*
37	109	Review for Comprehensive Exam	
	110	Review for Comprehensive Exam	
	111	Exam	*Comprehensive Exam*

PREFACE to the FIRST EDITION

This text, *Intermediate Logic: Mastering Propositional Arguments,* is designed as a continuation to *Introductory Logic: The Fundamentals of Thinking Well*. Together, these two textbooks should provide sufficient material for a complete course in basic logic.

We have attempted to make this a useable workbook for the logic student. To that end we have included exercises for every lesson, each of which has been developed and used over many years of logic classes. The goal is to keep the text clear and complete, such that an adult could teach himself the fundamentals of logic.

A number of other logic texts were consulted throughout the writing of *Intermediate Logic.* Most helpful was Irving Copi's invaluable *Introduction To Logic* (Macmillan Publishing Co., 1978). While we did not lift material directly from it, of course, that book has so shaped our own understanding of this subject that *Intermediate Logic* undoubtedly echoes much of its format and contents. *Intermediate Logic* has also benefitted from *The Art of Reasoning* by David Kelley (W. W. Norton & Company, Inc., 1990) and *The Logic Book* by Bergmann, Moor, and Nelson (McGraw-Hill, Inc., 1990).

Although we cannot list them here, we are indebted to many people for the completion of this project. We give special credit to the students throughout the years who have been introduced to the beauty and practicality of the world of logic found in the pages of these textbooks. Good students always force teachers to re-evaluate their own understanding of a subject, and such students have contributed more to this book than we or they realize.

PREFACE to the SECOND EDITION

The subject of logic may be divided into two main branches: formal and informal. The definition of logic as "the science and the art of correct reasoning" allows us to distinguish these two branches. Formal logic deals *directly* with reasoning. Reasoning means "drawing conclusions from other information." Whenever we consider how to analyze and write logical arguments—in which conclusions are drawn from premises—we are working in the realm of formal logic. Informal logic, on the other hand, deals more *indirectly* with reasoning. When we argue, we often find ourselves defining terms, determining the truth values of statements, and detecting spurious informal fallacies. While in none of these activities do we concentrate on reasoning in a formal way, we do recognize that such activities are indirectly related to and support the process of reasoning, and are thus best included under informal logic.

With this in mind, several changes were made in 2006 to this second edition of *Intermediate Logic*. If it's of interest, those changes are listed below.

First, in order to present to the student a more logical progression of topics, the section on defining terms from the first edition has been entirely removed from this text and placed at the beginning of *Introductory Logic*, where it is taught along with other branches of informal logic and categorical logic. Consequently, this text now focuses solely on the branch of formal logic called propositional logic, of which formal proofs of validity and truth trees are subsets.

Second, review questions and review exercises have been added to each unit for every lesson in the text, effectively doubling the number of exercises for students to verify their knowledge and develop their understanding of the material. Additionally, some especially challenging problems which relate to the material have been included in the review exercises. Students who can correctly answer all of the review questions demonstrate a sufficient knowledge of the important concepts. Students who can correctly solve the review exercises demonstrate a sufficient understanding of how to apply those concepts.

Third, the definitions of important terms, key points made, and caution signs regarding common errors are now set apart in the margins of the text. This should help students to distinguish the most important topics, as well as aid in their review of the material.

Fourth, every lesson has been reviewed in detail with the goal of improving the clarity of the explanations and correcting several minor errors that were found in the first edition. To all logic students and teachers goes the credit for any improvements that have been made in this second edition; for those remaining errors and defects we take full responsibility.

PREFACE to the THIRD EDITION

Formal logic is a fascinating subject. Students are often intrigued by the concepts and methods of reasoning revealed in it. Truth tables, formal proofs, and other operations of propositional logic challenge their ability to think abstractly, and provide opportunities to practice and develop their puzzle solving skills. Pure logic is fun. But for students to learn how to reason properly, and through the process of reasoning to recognize and discover truth, they must learn how to apply these methods of formal logic to the world around them.

Many teachers and parents want logic to be practical. They want their logic students to be able to employ symbolic logic as a tool of thinking, a tool both powerful and flexible enough to use in many different ways and on many different media.

With these things in mind, in 2014 we added two important sections on the practical applications of propositional logic to the third edition.

First, a new lesson teaches students how to apply the tools that they have learned to actual arguments. Lesson 28 teaches students how to analyze chains of reasoning found in writings such as philosophy and theology. Though some new concepts are introduced, most of this lesson is aimed at teaching students to employ what they have learned in the previous 27 lessons. This one lesson therefore has three corresponding exercises, giving students the opportunity to work through small portions of three ancient texts: Boethius's *The Consolation of Philosophy*, the Apostle Paul's argument on the resurrection from 1 Corinthians 15, and a section on angelic will from Augustine's *City of God*. The additional exercises for this lesson also consider Deuteronomy 22, a portion from Martin Luther's sermon on John chapter 1, and a witty interchange that you may have seen in the 2008 action comedy *Get Smart*.

Second, a longer optional unit has been added on the useful and stimulating topic of digital logic. Unit 5 includes twelve new lessons that unlock the logic of electronic devices. These lessons work through the concepts of digital displays, binary numbers, and the design and simplification of digital logic circuits. Many of the lessons learned earlier in the text are given new and intriguing applications, as students learn how to employ propositional logic to understand the electronic gadgets that they see and use every day. The final exercise gives students the opportunity to design a complex circuit that can convert a binary input to a decimal display output. This unit has become a favorite of many logic students over the past decades.

It is our hope that these additions give students a vision of the power of propositional logic and fulfill teachers' and parents' desires to make propositional logic more practical.

INTRODUCTION

Logic has been defined both as the *science* and the *art* of correct reasoning. People who study different sciences observe a variety of things: biologists observe living organisms, astronomers observe the heavens, and so on. From their observations they seek to discover natural laws by which God governs His creation. The person who studies logic as a science observes the mind as it reasons—as it draws conclusions from premises—and from those observations discovers laws of reasoning which God has placed in the minds of people. Specifically, he seeks to discover the principles or laws which may be used to distinguish good reasoning from poor reasoning. In deductive logic, good reasoning is *valid* reasoning—in which the conclusions follow necessarily from the premises. Logic as a science discovers the principles of valid and invalid reasoning.

Logic as an *art* provides the student of this art with practical skills to construct arguments correctly as he writes, discusses, debates, and communicates. As an art logic also provides him with rules to judge what is spoken or written, in order to determine the validity of what he hears and reads. Logic as a science discovers rules. Logic as an art teaches us to apply those rules.

Logic may also be considered as a symbolic language which represents the reasoning inherent in other languages. It does so by breaking the language of arguments down into symbolic form, simplifying them such that the arrangement of the language, and thus the reasoning within it, becomes apparent. The outside, extraneous parts of arguments are removed like a biology student in the dissection lab removes the skin, muscles and organs of a frog, revealing the skeleton of bare reasoning inside. Thus revealed, the logical structure of an argument can be examined, judged and, if need be, corrected, using the rules of logic.

So logic is a symbolic language into which arguments in other languages may be translated. Now arguments are made up of propositions, which in turn are made up of terms. In categorical logic, symbols (usually capital letters) are used to represent terms. Thus "All men are sinners" is translated "All M are S." In propositional logic, the branch of logic with which this book primarily deals, letters are used to represent entire propositions. Other symbols are used to represent the logical operators which modify or relate those propositions. So the argument, "If I don't eat, then I will be hungry; I am not hungry, so I must have eaten" may appear as $\sim E \supset H,\ \sim H,\ \therefore E$.

Unit 1 of this book covers the translation and analysis of such propositional arguments, with the primary concern of determining the validity of those arguments. Unit 2 introduces a new kind of logical exercise: the writing of formal proofs of validity and related topics. Unit 3 completes propositional logic with a new technique for analyzing arguments: truth trees. Unit 4 considers how to apply these tools and techniques to arguments contained in real-life writings: philosophy, theology, and the Bible itself. Unit 5 introduces digital logic and helps students to unlock the logic of electronic devices.

UNIT 1

TRUTH TABLES

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INTRODUCTION TO PROPOSITIONAL LOGIC

Intermediate Logic, pp. 9–11

STUDENT OBJECTIVES

1. Define "proposition," "logical operator," and "truth functional."
2. Distinguish between simple and compound propositions, and between propositional constants and variables.
3. Complete Exercise 1.

SPECIAL NOTES

1. It is up to you whether you read the chapter aloud with students prior to teaching the material, read through it in chunks and teach as you go, or teach straight through the lesson and have students read the chapter on their own later. The lesson plans are designed to stand alone; they cover all the material in the chapter, so you may find it redundant to read the chapter aloud as well. At some point, however, the students should read through the chapter on their own.
2. Before beginning the lesson, it may be good to refresh some memories. Ask the students a few questions: What are we studying when we study logic? How is logic a science? How is logic an art? Have them refer to the Introduction on p. 5 if they're clueless.

TEACHING INSTRUCTIONS

1. Write "languages" on the board. Have students shout out the names of as many different languages as they can think of (French, Latin, German, Russian, Swahili) and list them on the board. Tell them that there is one language that you don't expect them to think of. Some smarty-pants will probably eventually suggest "logic." Explain that logic is indeed a language: a symbolic language that arguments in every other language can be translated into.
2. Remind students that earlier they studied categorical logic, and in categorical logic they used symbols to represent terms (All M are S), because terms ("bird," "bat," "bandits") were the basic unit of thought. Explain that this semester they will be studying **propositional logic, a branch of logic in which the basic unit of thought is the proposition**. Have somebody take a stab at defining "proposition." Explain that **a proposition is simply a statement**. Write on the board the three sentences from Intermediate Logic p. 9: "God loves the world," "The world is loved by God," and "Deus mundum amat." Explain that even though these are three different sentences, they all have the same meaning, and they therefore represent the same proposition.
3. Explain that "propositional logic" is often called "symbolic logic," because in propositional logic we replace almost all the words with symbols. Letters are used to represent not terms but whole propositions. For example, the proposi-

tion "Women are humans" would be represented not by "All W are H" but just by "W."

4. Explain that propositional or symbolic logic deals with **truth-functional propositions, propositions in which the truth value of the proposition depends on the truth value of the component parts.** Reassure students that if that makes no sense to them yet, that's okay, it will soon. Write on the board "There are stars in the sky." Ask how many component parts (statements) this proposition has (one). Explain that **if a proposition has only one component part it is a simple proposition. If it has more than one component part, or is somehow modified, it is a compound proposition.** Ask students how they might turn the simple proposition on the board into a compound proposition (something like: "There are stars in the sky but there is no moon.") Write this compound proposition on the board. Circle the word "but." Tell students that **words like "and," "but," and "or," which combine or modify simple propositions to make compound propositions, are called logical operators.**

5. Explain that the new sentence on the board is a truth-functional compound proposition. It is truth-functional because whether or not it is true depends on whether or not the two statements inside it are true. If it is actually the case *both* that there are stars in the sky and that there is no moon, then the whole proposition is true as well. But if either of those statements are false—there aren't any stars out or there is in fact a moon—the whole proposition is false.

6. Go back to the simple proposition "There are stars in the sky." Explain that there is another logical operator we can add to this simple proposition to make it a compound proposition: "It is false that." Write "It is false that there are stars in the sky." Make sure students can see that this is in fact compound: it contains a statement that could stand alone ("There are stars in the sky"). Explain that this new proposition is also truth-functional; the truth value of "It is false that there are stars in the sky," depends on the truth value of "There are stars in the sky." Ask students what the truth value of the whole proposition is if "There are stars in the sky" is false. If it's true?

7. Write on the board "I believe that there are stars in the sky." Ask students if this statement is truth-functional. Does the truth or falsity of the component part ("There are stars in the sky") effect the truth or falsity of the whole? Make sure students can see that because "I believe that there are stars in the sky" is a self-report, it can be considered true whether or not the component part is true; it could be true that you believe there are stars even if its false that there are any. Thus, self-reports are not truth-functional.

8. Explain that just as in syllogisms we abbreviated terms to make them easier to work with, in compound propositions we abbreviate the component parts—the simple propositions—so that we can handle them more easily. Explain that **a propositional constant is an uppercase letter that represents a single, given proposition**. Usually we choose propositional constants that have some connection with the propositions they symbolize: i.e., "The walrus ate me" would probably be abbreviated "W," and "Stephanie ate a cookie" would be abbreviated "S." But if you had the propositions "Stephanie ate a cookie" along with "Stephanie drank milk," you would probably abbreviate them as "C" and "M," respectively. Emphasize that within one argument or compound proposition it is important to use the same propositional constant to represent the same proposition every time it appears. Make sure students understand that a simple proposition cannot be represented by more than one constant: you are abbreviating not the terms but the whole statement.

9. Explain to students that we only use uppercase letters when we are dealing with particular, known propositions. If we want to emphasize the form of an argument and not the content of it, we use **propositional variables: lowercase letters that represent any proposition**. Explain that using propositional variables in logic is a lot like using "x" and "y" in algebra: just like x and y represent an unlimited number of possible numbers, propositional variables represent any unlimited number of possible propositions. Explain that for some reason when using propositional variables, we usually begin with the letter "p" and continue from there (q, r, s, . . .). Who knows what those original logicians had against the first half of the alphabet.

10. Tell students to take a deep breath: you are about to make this all rather more complicated. While a simple proposition cannot be represented by more than one constant or variable, a compound proposition doesn't have to be represented by more than one. A compound proposition can be represented by one constant or variable. Write on the board "It is false that if the Walrus eats Stephanie, then, if Stephanie does not eat a cookie, then the Walrus will not get indigestion." Explain that you could abbreviate this very compound proposition as *~[S ⊃ (~C ⊃ ~I)]*. Or you could abbreviate it simply as "F." Explain that how you decide to abbreviate a compound proposition depends on the context, and that students will learn how to do this in the next few lessons.

ASSIGNMENT

Have students complete Exercise 1, and go over it with them.

LESSON 1

INTRODUCTION TO PROPOSITIONAL LOGIC

Propositional logic is a branch of formal, deductive logic in which the basic unit of thought is the propositon. A **proposition** is a statement, a sentence which has a truth value. A single proposition can be expressed by many different sentences. The following sentences all represent the same proposition:

God loves the world.
The world is loved by God.
Deus mundum amat.

These sentences represent the same proposition because they all have the same meaning.

In propositional logic, letters are used as symbols to represent propositions. Other symbols are used to represent words which modify or combine propositions. Because so many symbols are used, propositional logic has also been called "symbolic logic." Symbolic logic deals with **truth-functional propositions**. A proposition is truth-functional when the truth value of the proposition depends upon the truth value of its component parts. If it has only one component part, it is a **simple proposition**. A categorical statement is a simple proposition. The proposition *God loves the world* is simple. If a proposition has more than one component part (or is modified in some other way), it is a **compound proposition**. Words which combine or modify simple propositions in order to form compound propositions (words such as *and* and *or*) are called **logical operators**.

For example, the proposition *God loves the world and God sent His Son* is a truth-functional, compound proposition. The word *and* is the logical operator. It is truth functional because its truth value depends upon the truth value of the two simple propositions which make it up. It is in fact a true proposition, since it is true that God

DEFINITIONS

Propositional logic is a branch of formal, deductive logic in which the basic unit of thought is the proposition. A ***proposition*** is a statement.

KEY POINT

One proposition may be expressed by many different sentences.

DEFINITIONS

A proposition is ***truth-functional*** when its truth value depends upon the truth values of its component parts.

If a proposition has only one component part, it is a ***simple proposition***. Otherwise, it is ***compound***.

DEFINITIONS

Logical operators are words that combine or modify simple propositions to make compound propositions.

A ***propositional constant*** is an uppercase letter that represents a single, given proposition.

A ***propositional variable*** is a lowercase letter that represents any proposition.

loves the world, and it is true that God sent His Son. Similarly, the proposition *It is false that God loves the world* is compound, the phrase *it is false that* being the logical operator. This proposition is also truth-functional, depending upon the truth value of the component *God loves the world* for its total truth value. If *God loves the world* is false, then the proposition *It is false that God loves the world* is true, and vice versa.

However, the proposition *Joe believes that God loves the world,* though compound (being modified by the phrase *Joe believes that),* is *not* truth-functional, because its truth value does not depend upon the truth value of the component part *God loves the world.* The proposition *Joe believes that God loves the world* is a self-report and can thus be considered true, regardless of whether or not *God loves the world* is true.

When a given proposition is analyzed as part of a compound proposition or argument, it is usually abbreviated by a capital letter, called a **propositional constant**. Propositional constants commonly have some connection with the propositions they symbolize, such as being the first letter of the first word, or some other distinctive word within the proposition. For example, the proposition *The mouse ran up the clock* could be abbreviated by the propositional constant M. On the other hand, *The mouse did not run up the clock* may be abbreviated *~M* (read as *not M*). Within one compound proposition or argument, the same propositional constant should be used to represent a given proposition. Note that a simple proposition cannot be represented by more than one constant.

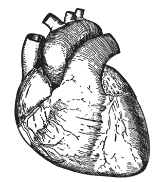
When the *form* of a compound proposition or argument is being emphasized, we use **propositional variables**. It is customary to use lowercase letters as propositional variables, starting with the letter *p* and continuing through the alphabet (*q*, *r*, *s*, . . .). Whereas a propositional constant represents a single, given proposition, a propositional variable represents an unlimited number of propositions.

KEY POINT

A propositional constant or variable can represent a simple proposition or a compound proposition.

It is important to realize that a single constant or variable can represent not only a simple proposition but also a compound proposition. The variable *p* could represent *God loves the world* or it could represent *God loves the world but He hates sin.* The entire compound proposition *It is false that if the mouse ran up the clock, then, if the clock did not strike one, then the mouse would not run down* could be abbreviated by a single constant F, or it could be represented by symbolizing each part, such as ~(M ⊃ (~S ⊃ ~D)). The decision concerning how to abbreviate a compound proposition depends on the purpose for abbreviating it. We will learn how to abbreviate compound propositions in the next few lessons.

SUMMARY



A proposition is a statement. Propositions are truth-functional when the truth value of the proposition depends upon the truth value of its component parts. Propositions are either simple or compound. They are compound if they are modified or combined with other propositions by means of logical operators. Propositional constants are capital letters which represent a single given proposition. Propositional variables are lower case letters which represent an unlimited number of propositions.

EXERCISE 1 (25 points)

What are two main differences between propositional constants and propositional variables? (2 each)

1. Constants represent one given proposition; variables represent any proposition
2. Constants are abbreviated by uppercase letters, variables by lowercase

Modify or add to the simple proposition *We have seen God* to create the following (2 each):

3. A truth-functional compound proposition:
   We have seen God but we will not die.
4. A proposition which is *not* truth-functional:
   I think we have seen God.

Circle S if the given proposition is simple. Circle C if it is compound. (1 each)

5. The Lord will cause your enemies to be defeated before your eyes. **(S)** C
6. There is a way that seems right to a man but in the end it leads to death. S **(C)**
7. The fear of the Lord is the beginning of wisdom. **(S)** C
8. If we confess our sins then He is faithful to forgive us our sins. S **(C)**
9. It is false that a good tree bears bad fruit and that a bad tree bears good fruit. S **(C)**
10. The Kingdom of God is not a matter of talk but of power. S **(C)**

Given that B means *The boys are bad*, M means *The man is mad*, G means *The girls are glad*, S means *The students are sad*

Translate the following compound propositions (2):

11. It is false that B. The boys are not bad. (2)
12. B or G. The boys are bad or the girls are glad. (2)
13. B and M. The boys are bad and the man is mad. (2)
14. If M then S. If the man is mad, then the students are sad. (2)
15. If not M and not S then G. If the man is not mad and the students are not sad, then the girls are glad. (3)

NEGATION, CONJUNCTION, AND DISJUNCTION

Intermediate Logic, pp. 15–18

STUDENT OBJECTIVES

1. Provide the symbols, truth-tables, and English equivalents for the three basic logical operators.
2. Complete Exercise 2.

SPECIAL NOTE

Inform students that there will be a quiz next class over Lessons 1 and 2 (depending on which schedule you're following). Encourage them to study particularly the definitions and key points in the margins of their *Intermediate Logic* textbooks. Tell students about the new quiz/test schedule: every week (after this week) they will have a quiz on the lessons covered since the last quiz. Every other week or so, there will be a test on the material from the last two quizzes.

TEACHING INSTRUCTIONS

1. Remind students that they learned last class about logical operators; see if anyone can remember what they are. If they are all at a loss, explain that **logical operators are the words which combine or modify simple propositions to make compound propositions**. Have students give you some examples of these words ("and," "but," "or," "it is false that"). Explain that in this lesson they will be learning about three fundamental logical operators: negation, conjunction, and disjunction. In this lesson you will be asking and answering three questions for each logical operator (write them at the top of the board): What words in English are abbreviated by it? What is its symbol? How is the truth value of the compounds proposition affected by the truth values of the component parts?
2. Write **negation** on the board, and have students take a stab at guessing what words this logical operator represents ("not," "it is false that," or any other phrase that denies the proposition). See if anyone remembers what symbol represents negation and write it on the board (~). Explain that **negation is the logical operator that denies or contradicts a proposition**. Write a proposition on the board (i.e., "Everyone can read"). Have students abbreviate it (i.e., "E"). Write the negation (~E) and then have students translate it ("Not everyone can read" or "Some people cannot read").
3. Have students remember way back to *Introductory Logic* when they learned about contradictions. The contradiction of an A statement ("All S is P") is "Some S is not P" and vice versa. The contradiction of an E statement ("No S is P") is "Some S is P" and vice versa. Since the logical operator negation "contradicts" a proposition, what was true of contradictions is true of a proposition and its negation: if a proposition is true, its negation is false. If a proposition is false, its negation is true. Make sure that students can see this: if it's true that everyone can read, it must be false that some people can't. Explain

that we can express this relationship between the truth values of a proposition and its negation by making a **truth table**. Draw on the board the negation chart:

p	~p
T	F
F	T

Make sure students follow it: if "*p*" is true (T), "*~p*" (be sure to read it as "not p") must be false (F); if *p* is F, *~p* must be T. Explain that a **truth table** like the one that you have just drawn **is a listing of the possible truth values for a set of one or more propositions**. Explain that this negation truth table is called **a defining truth table,** because **it displays the truth values produced by a logical operator modifying a minimum number of variables**. When students look at you like you just spoke Greek, explain that, in other words, this truth table is the simplest truth table possible for negation.

4. Have students guess what words the logical operator **conjunction** represents ("and," "but," "still," or other similar words), and help them see that this makes sense because "conjunction" is when two things come together. Draw the symbol for conjunction, • (called, shockingly, a dot) on the board. Have students tell you how to abbreviate the compound proposition "It is snowing and I am cold" *(S • C)*. Explain that **conjunction is a logical operator that joins two propositions and is true if and only if both the propositions (conjuncts) are true**. If either conjunct is false, the whole thing is false.

5. Draw this (incomplete) truth table for conjunction on the board, leaving the third column (under *p* • *q*) empty for now:

p	q	p • q
T	T	
T	F	
F	T	
F	F	

Explain that the four rows represent all the possible combinations of truth and falsity for the two propositions: that both are true (i.e., it is true both that it is snowing and that I am cold because I am walking around outside), that the first is true and the second false (i.e., it is true that it is snowing, but it is false that I am cold, because I am inside wrapped in a fuzzy blanket), that the first is false and the second true (i.e., it is false that it is snowing, but it is true that I am cold because I am hiding in the fridge), and that both are false (i.e., it is false that it is snowing and it is false that I am cold, because it is summer and I am playing softball). Now walk students through where the T's and F's should go in the final column of the chart. If *p* is true and *q* is true, then of course the proposition *p* • *q* will be true as well. But if *p* is true and *q* is false, the compound proposition is tainted with falsity and the whole thing is false. So with the third row, and, of course, the fourth. The completed table should look like this:

p	q	p • q
T	T	T
T	F	F
F	T	F
F	F	F

6. Point out that in ordinary English the conjunction does not always appear between two distinct sentences; sometimes it's a little less obvious. "I am cold and wet" is still two separate propositions connected by a conjunction: "I am cold and I am wet." Have students symbolize this conjunction *(C • W)*. Then have them symbolize "You and I are both lost" *(Y•I)*.

7. Have students guess what words the logical operator **disjunction** represents. It's not as obvious: "or," as in "I got us lost or you got us lost." Draw the symbol for disjunction, ∨ (called, believe it or not, a "vee"), on the board. Have students symbolize "I got us lost or you got us lost" *(I ∨ Y)*.

8. Explain that in English, the word "or" is ambiguous: it has two different senses. It can mean "this or that, but not both," as in "We are either lost or not lost"; this is called the "exclusive or," because only one of these options can be true. On the other hand, "or" can also mean "this or that, or both," as in "Forest rangers look for the lost or injured." The rangers will look for you if you are lost, or if you are injured, or if you happen to be both lost and injured, they will look for you too. This is called the "inclusive or." Explain that Latin, clever language that it is, has two separate words for "or": "aut" is exclusive and "vel" is inclusive. Emphasize that **in Logic we always use the disjunction in the *inclusive* sense (this or that or both)**, and that is why we represent it as a "∨"—v for "vel." Explain that **disjunction**, then, **is the logical operator that joins two propositions and is true if and only if one or both of the propositions (disjuncts) is true**.

9. This time, have students tell you how to set up the truth table for disjunction, leaving out the third column for the moment:

p	q	p ∨ q
T	T	
T	F	
F	T	
F	F	

   Explain that once again that first two columns represent all possible combinations of truth and falsity between the two propositions: True and true, true and false, false and true, and false and false. Walk students through determining the final column and fill it in as they figure it out: if *p* is true and *q* is true, *p* ∨ *q* must also be true, because "or" is inclusive. If *p* is true and *q* is false, *p* ∨ *q* is still true because one of the propositions at least is true; it is the same if p is false and q is true. But if both p and q are false, the whole compound proposition is, of course, false. The completed table should look like this:

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

10. Explain that students may find themselves in a situation where they need to use an exclusive "or" instead of an inclusive "or," i.e., "Either the ranger is on his way or he has forgotten us." In that case they will have to symbolize the proposition in a much more complicated way: *(O ∨ F) • ~(O • F)*—that is, "Either the ranger is on his way or he is still at the fort, but he is not both on his way and at the fort." Tell students that they should always assume "or" to be *inclusive* unless otherwise instructed. Keep in mind that sometimes people use the word either to help indicate the exclusive sense of the disjunction, but other times "either *p* or *q*" is just a way of saying "*p* or *q*" with more words.

11. Write on the board *A* ∨ *B* • *C*. Ask students what the problem is with symbolizing the compound proposition this way. Hopefully someone will notice that it is ambiguous: it could mean "*A* or *B*, and *C*" or "*A*, or *B* and *C*." Explain that the way to remove this ambiguity in logic, as in math, is to use parentheses to distinguish which operation should be performed first. Tell students that **in a series of three or more connected propositions parentheses should be used**. Point out that the word "both" is often a good indicator of where to place the parentheses when dealing with conjunctions; in the compound proposition "*O* or *F*, but not both *O* and *F*," the word "both" tells you to place the parentheses around *O* • *F*.

12. Explain that parentheses also help us make the small but **very important distinction between "not both" propositions and "both not" propositions**. Write on the board "Forest rangers and grizzly bears are not both friendly creatures." Explain that this is true and that

you would symbolize it as *~(F • G)*, because the "not" (~) comes before the "both" (•). Now write on the board "Both forest rangers and grizzly bears are not friendly creatures." Explain that this does not mean the same thing at all; it is (for the most part) false, and you would symbolize it as *(~F • ~G)*, because the "both" comes before the "not."

13. Explain that when symbolizing compound propositions that use negation, we usually assume that whatever proposition the negation immediately precedes is the proposition it is negating. Write on the board $\sim p \vee q$ and explain that we just assume this to mean $(\sim p) \vee q$ because the tilde immediately precedes the p. Emphasize that $\sim p \vee q$ means something very different from $\sim(p \vee q)$, just as in math $-5 + 6 = 1$ and $-(5 + 6) = -11$. Make sure students understand that when they want to negate a single variable or constant they don't need parentheses, but if they are negating an entire compound proposition they do need them.

ASSIGNMENTS

1. Have students complete Exercise 2, and go over it with them.
2. Remind students to study for next class's quiz over Lessons 1 and 2.

LESSON 2

NEGATION, CONJUNCTION, AND DISJUNCTION

We will begin our study of abbreviating and analyzing compound propositions by learning about three fundamental logical operators: *negation, conjunction,* and *disjunction.* As we do, we will be answering three questions for each logical operator: What words in English are abbreviated by it? What is its symbol? How is the truth value of the compound proposition affected by the truth values of the component parts?

KEY POINT

Three fundamental logical operators are **negation**, **conjunction**, and **disjunction**.

Negation

Negation is the logical operator representing the words *not, it is false that,* or any other phrase which denies or contradicts the proposition. As we have already seen, the symbol ~ (called a *tilde*) represents negation. If the proposition *All roads lead to Rome* is represented by the propositional constant *R*, then ~*R* means *Not all roads lead to Rome* or *It is false that all roads lead to Rome.* Note that the negation of a proposition is the contradiction of that proposition. Thus ~*R* could also be translated *Some roads do not lead to Rome*. If a proposition is true, its negation is false. If a proposition is false, its negation is true. This can be expressed by the following **truth table**, where T means *true* and F means *false*:

p	~p
T	F
F	T

Truth tables show how the truth value of a compound proposition is affected by the truth value of its component parts. The table above is called the **defining truth table** for negation because it completely defines its operations on a minimum number of variables (in this case, one). The defining truth table for an operator that joins two propositions would require two variables.

DEFINITIONS

Negation (~, *not*) is the logical operator that denies or contradicts a proposition.

A ***truth table*** is a listing of the possible truth values for a set of one or more propositions. A ***defining truth table*** displays the truth values produced by a logical operator modifying a minimum number of variables.

DEFINITIONS

Conjunction (•, *and*) is a logical operator that joins two propositions and is true if and only if both the propositions (*conjuncts*) are true.

Conjunction

When two propositions are joined by *and, but, still,* or other similar words, a **conjunction** is formed. The conjunction logical operator is symbolized by • (called, of course, a *dot*). If *Main Street leads to home* is represented by the constant *H*, then *All roads lead to Rome, but Main Street leads to home* could be represented by *R* • *H* (read as *R dot H,* or *R and H*).

The conjunction is true if and only if its components (called **conjuncts**) are both true. If either conjunct is false, the conjunction as a whole is false. The defining truth table for conjunction is therefore:

p	q	p • q
T	T	T
T	F	F
F	T	F
F	F	F

Thus if *All roads lead to Rome* is false and *Main Street leads to home* is true, then the entire conjunction *All roads lead to Rome but Main Street leads to home* is false, as seen on the third row down.

In ordinary English, the conjunction is not always placed between two distinct sentences. For example, *Paul and Apollos were apostles* could be symbolized *P* • *A*, where *P* means *Paul was an apostle* and A means *Apollos was an apostle.* Similarly, the proposition *Jesus is both God and man* could be represented by *G* • *M*.

DEFINITIONS

Disjunction (∨, *or*) is a logical operator that joins two propositions and is true if and only if one or both of the propositions (*disjuncts*) is true.

Disjunction

A **disjunction** is formed when two propositions are joined by the logical operator *or*, as in *Paul was an apostle or Apollos was an apostle.* The symbol for disjunction is ∨ (called a *vee*). The foregoing disjunction would thus be symbolized *P* ∨ *A* (read simply *P or A*).

In English, the word *or* is ambiguous. In one sense it can mean "this or that, but not both" (called the *exclusive or*). For example, in the sentence *The senator is either a believer or an unbeliever,* the word *or* must be taken in the exclusive sense; nobody could be both a believer and an unbeliever at the same time in the same way. However, the word *or* can also mean "this or that, or both" (called the *inclusive or*). This is how it should be taken in the sentence *Discounts are given to senior* *citizens or war veterans.* If you were a senior citizen or a war veteran *or both,* you would be allowed a discount.

In Latin, the ambiguity is taken care of by two separate words: *aut,* meaning the "exclusive *or*," and *vel,* meaning the "inclusive *or*." Although it may seem like the exclusive sense of the word *or* is the more natural sense, in logic the disjunction is always taken in the inclusive sense. This is seen in the fact that the symbol ∨ is derived from the Latin *vel.*

The defining truth table for disjunction is therefore:

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

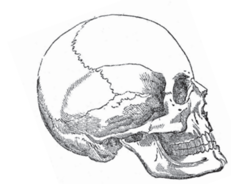
A disjunction is thus considered to be false if and only if both components (called **disjuncts**) are false. If either disjunct is true, the disjunction as a whole is true.

If the context of an argument requires that the word *or* be represented in the exclusive sense, as in *The senator is either a Republican or a Democrat,* it may be translated with the more complicated *(R ∨ D) • ~(R • D)*—that is, "The senator is either a Republican or a Democrat, but not both a Republican and a Democrat." However, you should assume that *or* is meant in the more simple inclusive sense unless instructed otherwise.

As you can see, logic may use parentheses in symbolizing complicated compound propositions. This is done to avoid ambiguity. The compound proposition *A ∨ B • C* could mean *A or B, and C* or it could mean *A, or B and C.* Parentheses remove the ambiguity, as in *(A ∨ B) • C*, which represents *A or B, and C.* This is similar to how parentheses are used in mathematics. Assuming there are no rules about which operation should be performed first, the mathematical expression 5 + 6 × 4 could equal either 44 or 29, depending on whether one adds first or multiplies first. But parentheses would make it clear, as in (5 + 6) × 4. Logic uses parentheses in the same way. Generally, in a series of three or more connected propositions, parentheses should be used.

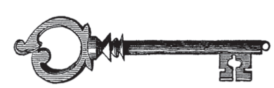
KEY POINT

The logical operator for disjunction is always understood in the inclusive sense: "this or that, or both." If you intend the exclusive *or,* you must specify it explicitly.



CAUTION

Though in English grammar the word *or* is called a conjunction, in logic only *and* (and equivalent words) is a conjunction. *Or* is always called a disjunction.



KEY POINT

Generally, in a series of three or more connected propositions, parentheses should be used to avoid ambiguity.

CAUTION

Do not confuse the propositional meaning of the phrases *not both* and *both not.* Use parentheses to distinguish between them.

The word *both* is often an indicator of how parentheses are to be placed when using conjunctions. The symbolized *exclusive or* in the paragraph above could be read *R or D, but not* ***both*** *R and D,* the word *both* telling us to place parentheses around $R \bullet D$.

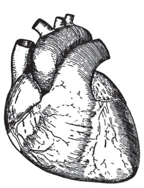
A proper use of parentheses can also help us to distinguish between *not both* and *both not* propositions. For example, the proposition *Cats and snakes are not both mammals* (which is true) would be symbolized as $\sim(C \bullet S)$. The *not* comes before the *both,* so the tilde is placed before the parenthesis. However, *Both cats and snakes are not mammals* (which is false) would be symbolized as $(\sim C \bullet \sim S)$. Note that this second proposition could also be translated *Neither cats nor snakes are mammals.*

KEY POINT

In the absence of parentheses, assume that negation attaches only to the proposition it immediately precedes.

When symbolizing compound propositions which use negation, it is standard practice to assume that whatever variable, constant, or proposition in parentheses the tilde immediately precedes is the one negated. For example, the compound proposition $\sim p \vee q$ is understood to mean $(\sim p) \vee q$, because the tilde immediately precedes the variable p. This is different from $\sim(p \vee q)$. Negation is used in the same way that the negative sign is used in mathematics. The mathematical expression $-5+6$ means $(-5)+6$, which equals 1. This is different from $-(5 + 6)$, which equals -11. So when negating a single variable or constant, you need not use parentheses. But when negating an entire compound proposition, place the tilde in front of the parentheses around the proposition.

SUMMARY



Three common logical operators are negation (*not*, symbolized $\sim$), conjunction (*and*, symbolized $\bullet$), and disjunction (*or*, symbolized $\vee$). These logical operators can be defined by means of truth tables. Negation reverses the truth value of a proposition, conjunction is true if and only if both conjuncts are true, and disjunction is false if and only if both disjuncts are false.

EXERCISE 2 (26 points)

Given: J means *Joseph went to Egypt* F means *There was a famine*
I means *Israel went to Egypt* S means *The sons of Israel became slaves*

Translate the symbolic propositions. (2 each)

1. F • I There was a famine and Israel went to Egypt.
2. ~J ∨ S Joseph did not go to Egypt or the sons of Israel became slaves.
3. ~(J ∨ I) It is false that Joseph or Israel went to Egypt.
4. J • ~S Joseph went to Egypt and the sons of Israel did not become slaves.

Symbolize the compound propositions .

5. Joseph and Israel went to Egypt. J•I (2)
6. Israel did not go to Egypt. ~I (2)
7. Israel went to Egypt, but his sons became slaves. I•S (2)
8. Either Joseph went to Egypt, or there was a famine. J∨F (2)
9. Joseph and Israel did not both go to Egypt. ~(J•I) (2)
10. Neither Joseph nor Israel went to Egypt. ~J•~I (2)
11. Joseph and Israel went to Egypt; however, there was a famine, and the sons of Israel became slaves. (J•I)•(F•S) (3)
12. Israel went to Egypt; but either Joseph did not go to Egypt, or there was a famine. I•(~J∨F) (3)

QUIZ 1 (LESSONS 1–2)

STUDENT OBJECTIVE

Complete Quiz 1.

TEACHING INSTRUCTIONS

1. Give the quiz. Allow 20 minutes for it. Grade quiz with students and spend time reviewing (maybe re-explaining) any problems they struggled with.
2. If you have the time (and the desire) to do so, introduce Lesson 3.

INTERMEDIATE LOGIC | QUIZ 1

Lessons 1–2 (25 points)

Name ______________________________

1. How does *propositional logic* differ from *categorical logic*? (3) In categorical logic, the basic unit of thought is the term. In propositional logic, the basic unit of thought is the proposition.

2. What is a *proposition*? (1) A proposition is a statement.

3. What does it mean that a proposition is *truth functional*? (2) The truth value of the proposition depends upon the truth value of its component parts.

4. Give an example of a proposition that is *not* truth functional. (2)

   I think this is a hard question. (Any self-report, tautology, or self-contradiction).

5. Give an example of a *simple* proposition. (2)

   Jesus is Lord.

Problems 6-12: Given: **V** means *You eat your veggies.*
M means *You eat your meat.*
D means *You get dessert.*

Translate the following symbolic propositions into words.

6. ~ V (1) You do not eat your veggies.
7. M ∨ ~ D (2) You eat your meat or you do not get dessert.
8. ~ (V • M) (2) You do not eat both your veggies and your meat.

Translate the following propositions into symbols.

9. You eat neither your veggies nor your meat. (2) ~ (V ∨ M)
10. You eat your veggies but you do not get dessert. (2) V • ~D

11. You eat your meat or your veggies, but you don't eat both. (3) $(M \vee V) \bullet \sim(M \bullet V)$

12. Are the letters **V**, **M**, and **D** used above *constants* or *variables?* Explain how you know. (3)

They are constants, because they are capital letters that represent given propositions.