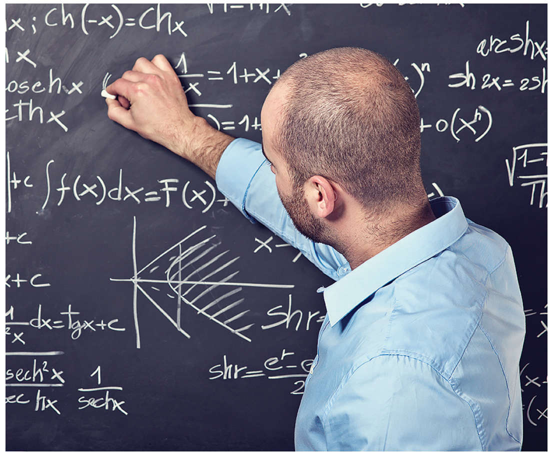

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A mathematician teaching problem solving

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A scientist conducting research

The uses of mathematics are wide-ranging. For example, mathematics may be used to solve scientific problems, design industrial projects, and carry out business transactions.

Mathematics

Mathematics is one of the most useful and fascinating divisions of human knowledge. It includes many topics of study. For this reason, the term *mathematics* is difficult to define. It comes from a Greek word meaning "inclined to learn."

Most of the basic mathematics taught in school involves the study of number, quantity, form, measurement, and relations. *Arithmetic,* for example, concerns problems with numbers. *Algebra* involves solving *equations* (mathematical statements of equality) in which letters represent unknown quantities. *Geometry* concerns the properties and relationships of figures in space.

The most important skills in mathematics are careful analysis, clear reasoning, defining terms precisely, and pattern recognition. Along with rigorous and creative thinking, these skills can help us solve some of the deepest puzzles the world presents. Mathematics is based upon *axioms* (fundamental statements assumed to be true) and logic. Starting from widely accepted statements, mathematicians use logic to draw conclusions and develop mathematical systems.

The importance of mathematics

The work of mathematicians may be divided into *pure mathematics* and *applied mathematics.* Pure mathematics seeks to advance mathematical knowledge for its own sake rather than for any immediate practical use. For example, a mathematician may create a system of geometry for an imaginary world where objects have more dimensions than just length, width, and depth. Applied mathematics seeks to develop mathematical techniques for use in science, *technology,* and other fields. Technology consists of the tools, materials, techniques, and sources of power that make our lives and work easier.

The boundary between pure and applied mathematics is not always clear-cut. Ideas developed in pure mathematics often have far-reaching applications, and work in applied mathematics frequently leads to research in pure mathematics.

Nearly every part of our lives involves mathematics. Most electronic gadgets, such as watches, cell phones, game consoles, personal computers, and ATM's, have mathematical foundations. Mathematics plays an essential role in the development of modern technology.

In everyday life, we use mathematics for such simple tasks as telling time from a clock or counting our change after making a purchase. We also use mathematics for such complex tasks as making up a household budget or figuring our income tax. Cooking, driving, gardening, sewing, and many other common activities involve mathematical calculations. Mathematics is also a key aspect of many games, hobbies, and sports.

In science. Mathematics is an essential part of the scientific world. It helps scientists describe natural phenomena, design experiments, and analyze data. Scientists use mathematical formulas to express their findings precisely and to make predictions based on these findings.

The physical sciences, such as astronomy, chemistry, and physics, rely heavily on mathematics. Biological processes are best studied and understood using mathematical models and analysis. Such social sciences as economics, psychology, and sociology also depend greatly on statistics and other kinds of mathematics. For example, some economists create mathematical models of economic systems. These models are sets of formulas used to predict how a change in one part of the economy might affect other parts.

In industry. Mathematics helps industries design, develop, and test products and manufacturing processes. Mathematics is necessary in designing bridges, buildings, dams, highways, tunnels, and other architectural and engineering projects.

In business. Mathematics is used in transactions that involve buying and selling. Mathematics is used to help increase profits and lower costs. Businesses need mathematics to keep records of such things as inventory and employees' hours and wages. Bankers use mathematics to handle and invest funds. Mathematics helps insurance companies calculate risks and compute the rates charged for insurance coverage.

Branches of mathematics

Mathematics has many branches. They may differ in the types of problems involved and in the practical application of their results. However, mathematicians working in different branches often use many of the same basic concepts and operations. This section discusses several of the main kinds of mathematics.

Arithmetic includes the study of whole numbers, fractions and decimals, and the operations of addition, subtraction, multiplication, and division. It forms the foundation for other kinds of mathematics by providing such basic skills as counting and grouping objects, and measuring and comparing quantities. See **Addition; Arithmetic; Division; Multiplication; Subtraction**.

Algebra, unlike arithmetic, is not limited to work with specific numbers. Algebra involves solving problems with equations in which letters, such as x and y, stand for unknown quantities. Algebraic operations also use negative numbers and *imaginary numbers* (the square roots of negative numbers). See **Algebra; Square root** (Square roots of negative numbers).

Geometry is concerned with the properties and relationships of figures in space. *Plane geometry* deals with squares, circles, and other figures that lie on a plane. *Solid geometry* involves such figures as cubes and spheres, which have three dimensions.

About 300 B.C., Euclid, a Greek mathematician, stated the definitions and assumptions of the system of geometry that describes the world as we usually experience it. But later mathematicians developed alternative systems of geometry that rejected Euclid's assumption about the nature of parallel lines. Such *non-Euclidean geometries* have proved useful, for example, in the theory of relativity—one of the outstanding achievements of scientific thought. See **Geometry**.

Analytic geometry and trigonometry. Analytic geometry relates algebra and geometry. It provides a way to represent an algebraic equation as a line or curve on a graph. Analytic geometry also makes it possible to write equations that exactly describe many curves. For example, the equation $x = y^2$ describes a curve called a *parabola.*

Trigonometry is used widely by astronomers, navigators, and surveyors to calculate angles and distances when direct measurement is impossible. It deals with the relations between the sides and angles of triangles, especially *right triangles* (triangles that have a 90° angle). Certain relations between the lengths of two sides of a right triangle are called *trigonometric ratios.* Using trigonometric ratios, a person can calculate the unknown angles and lengths in a triangle from the known angles and lengths. Formulas involving trigonometric ratios describe curves that physicists and engineers use to analyze the behavior of heavenly bodies, heat, light, sound, and other natural phenomena. See **Trigonometry**.

Calculus and analysis have many practical uses in engineering, physics, and other sciences. Calculus provides a way of solving many problems that involve motion or changing quantities. *Differential calculus* seeks to determine the rate at which a varying quantity changes. It is used to calculate the slope of a curve and the changing speed of a rocket or drone. *Integral calculus* tries to find a quantity when the rate at which it is changing is known. It is used to calculate the area of a curved figure or the amount of work done by a varying force. Unlike algebra, calculus includes operations with *infinitesimals* (quantities that are not zero but are smaller than any assignable quantity). See **Calculus**.

Analysis involves various mathematical operations with infinite quantities and infinitesimals. It includes the study of *infinite series,* sequences of numbers or algebraic expressions that go on indefinitely. The concept of infinite series has important applications in such areas as the study of heat and of vibrating strings. See **Series** (Working with infinite series).

Probability and statistics. Probability is the mathematical study of the likelihood of events. It is used to determine the chances that an uncertain event may occur. For example, using probability, a person can calculate the chances that three tossed coins will all turn up heads. See **Probability**.

Statistics is the branch of mathematics concerned with the collection and analysis of large bodies of data to identify trends and overall patterns. Statistics relies heavily on probability, computer-assisted computations, and data analysis. Statistical methods provide information to government, business, and science. For example, physicists use statistics to study the behavior of the many molecules in a sample of gas. Social scientists use statistics to better understand *quantitative* (numerical) relationships between income and education levels, which then helps inform policy decisions. See **Statistics**.

Set theory and logic. Set theory deals with the nature and relations of *sets.* A set is a collection of items, which may be numbers, ideas, or objects. The study of sets is important in investigating most basic mathematical concepts. See **Set theory**.

In the field of logic—the branch of philosophy that deals with the rules of correct reasoning—mathematicians have developed *symbolic logic.* Symbolic logic is a formal system of reasoning that uses mathematical symbols and methods. Mathematicians have devised various systems of symbolic logic that have been important in the development of computers.

History

Early civilization. Prehistoric people probably first counted with their fingers. They also had various methods for recording such quantities as the number of animals in a herd or the days since the full moon. To represent such amounts, they used a corresponding number of pebbles, knots in a cord, or marks on wood, bone, or stone. They also learned to use regular shapes when they molded pottery or carved arrowheads.

By about 3000 B.C., mathematicians of ancient Egypt used a *decimal system* (a system of counting in groups of 10) without place values. The Egyptians pioneered in geometry, developing formulas for finding the area and volume of simple figures. Egyptian mathematics had many practical applications, ranging from surveying fields after the annual floods to making the intricate calculations necessary to build the pyramids.

Around the same time, the people of ancient Mesopotamia, in what is now Iraq, developed a *sexagesimal system*—a system based on groups of 60. Today, we use such a system to measure time in hours, minutes, and seconds. Historians do not know exactly how this system developed. They think it may have arisen from the use of weights and measures based on groups of 60. The system had important uses in astronomy, and also in commerce, because 60 can be divided easily.

The Greeks. Ancient Greek scholars became the first people to explore pure mathematics, apart from practical problems. They made important advances by introducing the concepts of logical deduction and proof to create a systematic theory of mathematics. According to tradition, one of the first to provide mathematical proofs based on deduction was the philosopher Thales, who worked in geometry about 600 B.C.

The Greek philosopher Pythagoras, who lived about 550 B.C., explored the nature of numbers, believing that everything could be understood in terms of whole numbers or their ratios. However, about 400 B.C., the Greeks discovered *irrational numbers* (numbers that cannot be expressed as a ratio of two whole numbers), and they recognized that Pythagorean ideas were incomplete. About 370 B.C., Eudoxus of Cnidus, a Greek astronomer, formulated a theory of proportions to resolve problems associated with irrational numbers. He also developed the *method of exhaustion,* a way of determining areas of curved figures, which foreshadowed integral calculus.

Euclid, one of the foremost Greek mathematicians, wrote the *Elements* about 300 B.C. In this book, Euclid constructs an entire system of geometry by means of basic axioms, abstract definitions, and logical deductions. During the 200's B.C., the Greek mathematician Archimedes extended the method of exhaustion. Using a 96-sided figure to approximate a circle, he calculated a highly accurate value for *pi* (the ratio of a circle's circumference to its diameter). Ptolemy, an astronomer in Alexandria, Egypt, applied geometry and trigonometry to astronomy about A.D. 150 in a 13-part work on planetary motions. It became known as the *Almagest,* meaning *the greatest.*

Chinese mathematics originally developed to aid record keeping, land surveying, and building. By the 100's B.C., the Chinese had devised a decimal system of numbers that included fractions, zero, and negative numbers. They solved arithmetic problems with the aid of special sticks called *counting rods.* The Chinese also used these devices to solve equations—even groups of simultaneous equations in several unknowns.

Perhaps the best-known early Chinese mathematical work is the *Jiu Zhang Suan Shu (Nine Chapters on the Mathematical Art),* a handbook of practical problems that was compiled in the first two centuries B.C. In 263 A.D., the Chinese mathematician Liu Hui wrote a commentary on the book. Among Liu Hui's greatest achievements was his analysis of a mathematical statement called the Gou-Gu theorem. The theorem, known as the Pythagorean theorem in the West, describes a special relationship that exists between the sides of a right triangle. Liu Hui also calculated the value of pi more accurately than ever before. He did so by using a figure with 3,072 equal sides to approximate a circle.

Solving problems for fun

The following problems all require the use of important mathematical skills, including careful analysis of situations and reasoning to reach solutions. Try to solve the problems and then compare your work with the solutions provided.

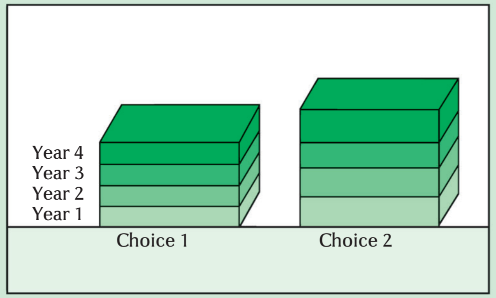
The drawings in this article were prepared for *World Book* by Zorica Dabich and Garri Budynsky.

1. *Which salary would you choose?* Your boss offers you one of two salary arrangements. "Which would you prefer," she asks, "a salary starting at $32,000 a year with a $1,600 increase each year, or one starting at $16,000 for a half year with a $400 increase each half year?" Which choice offers the higher salary?

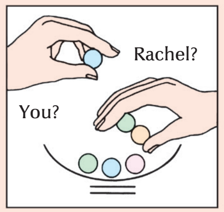
Make a chart to show the two choices of salary over a number of years.

Year	Choice 1	Choice 2
First year	$32,000	$16,000 + $16,400 = $32,400
Second year	$33,600	$16,800 + $17,200 = $34,000
Third year	$35,200	$17,600 + $18,000 = $35,600
Fourth year	$36,800	$18,400 + $18,800 = $37,200

Choice 2 gives you $400 more each year than choice 1 does.



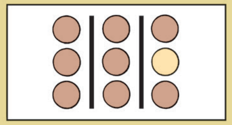
2. *Whose turn to win?* Your friend Rachel challenges you to a game. There are six marbles in a bowl. At a turn, a player may take either one or two marbles. The player who takes the last marble wins the game. Rachel offers to let you take the first turn. If you accept, who wins?

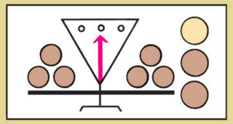


Assume that each player will always make the best move. The winner will take the winning turn when either one or two marbles are left. Therefore, the winner will be the player who leaves the opponent three marbles. If you make the first move and take one marble, Rachel can take two—leaving three for you and winning on her next turn. If, on the first move, you take two marbles, then Rachel can take one—again leaving three for you and winning on her next move. If you accept her kind offer to take the first turn, Rachel will win the game!

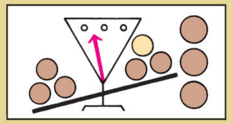
3. *Find the counterfeit.* You have nine rare and valuable coins. Although they appear to be identical, you know that one is counterfeit and weighs less than the others. Using a balance scale only twice, how can you find the fake coin?

First, divide the coins into three groups of three coins.



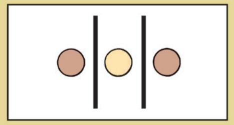


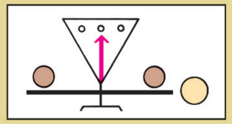
Weigh one group against another. If they balance, then the fake, or light, coin is in the group you have not weighed.



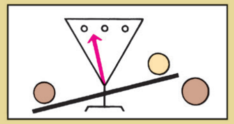
If the two groups of coins do not balance, then the fake coin is in the lighter group on the scale.

Next, take the group of three coins that contains the fake.



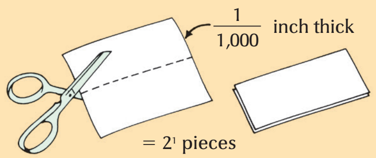


Weigh two coins from this group against each other. If they balance, then the coin you have not weighed is the fake.

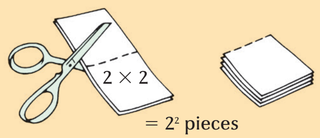


If the two coins do not balance, then the lighter coin on the scale is the counterfeit.

4. *A little pile of paper.* Imagine that you have a huge sheet of paper only $\frac{1}{1,000}$ inch (0.025 millimeter) thick. You cut the sheet in half and put one piece of paper on top of the other.



You cut these two pieces in half and put the resulting four pieces together in a pile. Then you cut the pile of four pieces in half and put the resulting eight pieces in a pile. Suppose that you can continue in this manner until you have cut the pile in half 50 times, each time piling up the resulting pieces. How high do you think the final pile would be?

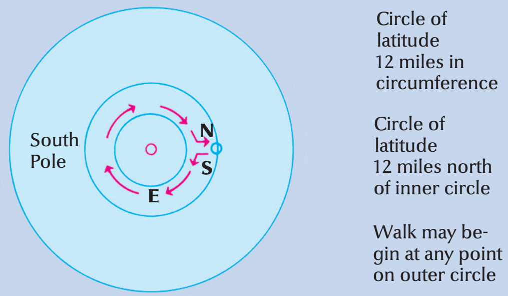


After the first cut, you have 2 pieces. After the second cut, you have 2 × 2, or 2^2 pieces. Following the third cut, you have 2 × 2 × 2 or 2^3 pieces. Therefore, after the 50th cut, you should have 2^{50} pieces. Two multiplied by itself 50 times is about 1,126,000,000,000,000. Because there are 1,000 pieces of paper to the inch, divide by 1,000 to find the number of inches in the pile. Next, divide the number of inches by 12 to find the number of feet in a pile. Then divide the number of feet by 5,280 to find the number of miles. You may be surprised to find that the pile of paper is about 17,770,000 miles (28,600,000 kilometers) high!

5. *Where to start?* From what point or points on Earth's surface could you walk 12 miles (19 kilometers) due south, then walk 12 miles due east, then walk 12 miles due north, and find yourself back at your starting point?

The usual answer to this old riddle is the North Pole. But Earth actually has an infinite number of points from which you could begin such a walk.

In theory, the equator forms a circle of latitude around the middle of Earth. As one goes north or south from the equator, the circumference of the circles of latitude gets progressively smaller until you reach the poles. Near the South Pole, there is a circle of latitude exactly 12 miles in circumference. Twelve miles north of this first circle is a second circle of latitude. You can start your walk at any point on this outer circle. You walk 12 miles south and find yourself on the inner circle. Then you walk 12 miles east around this circle—that is, you walk once around Earth, which is only 12 miles in circumference at this latitude. You then walk 12 miles north and arrive back at your starting point.



An infinite number of other points also can serve as your starting place. There are circles of latitude closer to the South Pole that have a circumference of 6 miles, 3 miles, 2 miles, and so forth. By starting at any point 12 miles north of any of these circles, you could take the walk described. Suppose, for example, you start 12 miles north of the circle of latitude that has a circumference of 1 mile. First you walk 12 miles south. Then you walk 12 miles east—that is, you go around the inner circle 12 times. Then, after walking north for 12 miles, you will arrive at your starting point on the outer circle.

In the Middle East. Starting in 750, an Islamic empire called the Abbāsid caliphate ruled much of the Middle East. Scholars in the caliphate translated and preserved the works of ancient Greek mathematicians and made their own original contributions as well. A book written about 825 by the Persian mathematician al-Khwārizmī described a numeration system developed in India. This decimal system, which used place values and zero, became known as the Arabic (or Hindu-Arabic) numeral system. Al-Khwārizmī also wrote an influential book about algebra. The word *algebra* comes from the Arabic title of this book. In addition, the word *algorithm* is derived from his name.

In the mid-1100's, a Latin translation of al-Khwārizmī's book on arithmetic introduced the Arabic numeral system to Europe. In 1202, Leonardo Fibonacci, an Italian mathematician, published a book on algebra that helped promote this system. Arabic numerals gradually replaced Roman numerals in Europe.

Middle Eastern astronomers of the 900's made major contributions to trigonometry. In the 1000's, an Arab physicist known as Alhazen applied geometry to optics. The Persian poet and astronomer Omar Khayyam wrote an important book on algebra about 1100. In the 1200's, Nasir al-Din al-Tusi, a Persian mathematician, created ingenious mathematical models for use in astronomy.

The Renaissance. During the 1400's and 1500's, European explorers sought new overseas trade routes, stimulating the application of mathematics to navigation and commerce. Artists created a system of mathematical perspective that gave their paintings an illusion of depth and distance. The invention of printing with movable type in the mid-1400's resulted in speedy and widespread communication of mathematical knowledge.

The Renaissance also brought major advances in pure mathematics. In a book published in 1533, a German mathematician known as Regiomontanus established trigonometry as a field separate from astronomy. French mathematician François Viète made advances in algebra in a book published in 1591.

Mathematics and the scientific revolution. By 1600, the increased use of mathematics and the growth of the experimental method were contributing to revolutionary advances in knowledge. In 1543, Nicolaus Copernicus, a Polish astronomer, published an influential book in which he argued that the sun, not Earth, is the center of the universe. In 1614, John Napier, a Scottish mathematician, published his discovery of *logarithms,* numbers that can be used to simplify such complicated calculations as those used in astronomy. Galileo, an Italian astronomer of the late 1500's and early 1600's, found that many types of motion can be analyzed mathematically.

In 1637, French philosopher René Descartes proposed mathematics as the perfect model for reasoning. He invented analytic geometry. Another French mathematician of the 1600's, Pierre de Fermat, founded modern number theory. He and French philosopher Blaise Pascal explored probability theory. Fermat's work with infinitesimals helped lay a foundation for calculus.

The English scientist Sir Isaac Newton invented calculus in the mid-1660's. His discovery was published in 1687. Working independently, the German philosopher and mathematician Gottfried Wilhelm Leibniz also invented calculus in the mid-1670's. He published his findings in 1684 and 1686.

Important dates in mathematics

c. 3000 B.C. The Egyptians used a system based on groups of 10 and developed basic geometry and surveying techniques.
c. 370 B.C. Eudoxus of Cnidus developed the method of exhaustion, foreshadowing integral calculus.
c. 300 B.C. Euclid constructed a system of geometry by means of logical deduction from basic statements called *axioms* and *postulates.*
Mid-1100's A translation of al-Khwārizmī's book on arithmetic introduced the Arabic numeral system to Europe.
1614 John Napier published his discovery of logarithms, an aid in simplifying calculations.
1637 René Descartes published his discovery of analytic geometry, proposing mathematics as the perfect model for reasoning.
Mid-1680's Sir Isaac Newton and Gottfried Wilhelm Leibniz published their independent discoveries of calculus.
Early 1800's Carl F. Gauss, Janos Bolyai, and Nikolai Lobachevsky separately developed non-Euclidean geometries.
Early 1820's Charles Babbage developed mechanical computing machines.
1854 George Boole published his system of symbolic logic.
Late 1800's Georg Cantor developed set theory and a mathematical theory of the infinite.
1910-1913 Alfred North Whitehead and Bertrand Russell published *Principia Mathematica,* which argues that all mathematical propositions can be derived from a few axioms.
Early 1930's Kurt Gödel showed that in any system of axioms, there are statements that cannot be proved.
Late 1950's and 1960's A new system of teaching called the *new mathematics* was introduced in the United States.
1970's and 1980's Computer-based mathematical models came into wide use in studies in business, industry, and science.
1995 Andrew Wiles published a proof of Fermat's Last Theorem.
2016 Maryna Viazovska proved the most efficient way to pack spheres in a space containing either 8 or 24 dimensions.

Developments in the 1700's. A remarkable family of Swiss mathematicians, the Bernoullis, made many contributions to mathematics during the late 1600's and the 1700's. Jakob Bernoulli did pioneering work in analytic geometry and wrote about probability theory. Jakob's brother Johann also worked in analytic geometry and in mathematical astronomy and physics. Johann's son Nicolaus helped advance probability theory. Johann's son Daniel used mathematics to study the motion of fluids and the properties of vibrating strings.

During the mid-1700's, Swiss mathematician Leonhard Euler advanced calculus by showing that the operations of differentiation and integration were opposites. Beginning in the late 1700's, French mathematician Joseph-Louis Lagrange worked to develop a firmer foundation for calculus. He was suspicious of relying on assumptions from geometry and, instead, developed calculus entirely in terms of algebra. There were also great advances in the study of differential equations, in which the solution involves a *function* (relation between two variables) or set of functions rather than a number or set of numbers. *Calculus of variations,* used for finding a function or curve that optimizes a particular situation, was also developed. For example, the *tautochrone problem* finds the curve for which two balls, when placed on the curve, roll to the bottom of it in the same amount of time no matter how high up either ball is originally placed.

In the 1800's, public education expanded rapidly, and mathematics became a standard part of university education. Many of the great works in mathematics of the 1800's were written as textbooks. In the 1790's and early 1800's, French mathematician Adrien Marie Legendre wrote particularly influential textbooks and did work in calculus, geometry, and number theory. Important calculus textbooks by French mathematician Augustin Louis Cauchy were published in the 1820's. Cauchy and Jean Baptiste Fourier, another French mathematician, made significant advances in mathematical physics.

Carl Friedrich Gauss, a German mathematician, proved the fundamental theorem of algebra, which states that every equation has at least one root. His work with imaginary numbers led to their increased acceptance. In the 1810's, Gauss developed a non-Euclidean geometry but did not publish his discovery. Working separately, Janos Bolyai of Hungary and Nikolai Lobachevsky of Russia also developed non-Euclidean geometries. They published their discoveries about 1830. In the mid-1800's, Georg Friedrich Bernhard Riemann of Germany developed another non-Euclidean geometry.

During the early 1800's, the works of German mathematician August Ferdinand Möbius helped develop a study in geometry that became known as *topology.* Topology explores the properties of a geometrical figure that do not change when the figure is bent or stretched.

In the late 1800's, German mathematician Karl Theodor Wilhelm Weierstrass worked to establish a more solid theoretical foundation for calculus. In the 1870's and 1880's, his student Georg Cantor developed set theory and a mathematical theory of the infinite.

Much exciting work in applied mathematics was done in the 1800's. In the United Kingdom, Charles Babbage developed early mechanical computing machines, and George Boole created a system of symbolic logic. During the late 1800's, French mathematician Henri Poincaré contributed to probability theory, celestial mechanics, and the study of electromagnetic radiation.

Philosophies of mathematics. Many mathematicians have shown concern for the philosophical foundations of mathematics. To eliminate contradictions, some mathematicians have used logic to develop mathematics from a set of axioms. Two British philosophers and mathematicians, Alfred North Whitehead and Bertrand Russell, promoted a philosophy of mathematics called *logicism.* In their three-volume work, *Principia Mathematica* (1910-1913), they argued that all *propositions* (statements) in mathematics can be derived logically from just a few axioms.

David Hilbert, a German mathematician of the early 1900's, was a *formalist.* Formalists consider mathematics to be a purely formal system of rules. Hilbert's work led to the study of imaginary spaces with an infinite number of dimensions.

Beginning in the early 1900's, Dutch mathematician Luitzen Brouwer championed *intuitionism.* He believed people understand the laws of mathematics by *intuition* (knowledge not gained by reasoning or experience).

In the early 1930's, Austrian mathematician Kurt Gödel demonstrated that for any logical system, there are always theorems that cannot be proved either true or false by the axioms within that system. He found this to be true even of basic arithmetic.

Mathematicians made major advances in the study of abstract mathematical structures during the 1900's. One such structure is the *group.* A group is a collection of items, which may be numbers, and rules for some operation with these items, such as addition or multiplication. Group theory is useful in many areas of mathematics and such fields as subatomic physics.

Since 1939, a group of mathematicians, most of whom are French, have published an influential series of books under the pen name Nicolas Bourbaki. This series takes an abstract approach to mathematics, using axiom systems and set theory.

New areas of mathematical specialization arose during the 1900's, including systems analysis and computer science. Advances in mathematical logic have been essential to the development of electronic computers. Computers, in turn, enable mathematicians to complete long and complicated calculations quickly. Since the 1970's, computer-based mathematical models have become widely used to study weather patterns, economic relationships, and many other systems.

Other new areas of mathematical specialization in the late 1900's included *fractal geometry* and *chaos theory.* Fractal geometry deals with complex shapes called *fractals.* These shapes consist of small-scale and large-scale structures that resemble one another. Certain fractals are also similar to natural objects, such as coastlines or branching trees. Although fractals seem irregular, they have a simple organizing principle. Chaos theory tries to find underlying patterns in variations that seem random, such as changes in the weather or the stock market.

Mathematics flourishes by solving problems and creating new ones. Some of the most difficult and longstanding mathematical problems have been solved since the mid-1900's. For example, the four color theorem, which states that only four colors are needed to distinguish separate areas on any two-dimensional map, was proved with the aid of computers by American mathematician Kenneth Appel and German American mathematician Wolfgang Haken in 1974. The proof of Fermat's Last Theorem, which states that, if n is greater than 2, there are no whole-number solutions to the equation $x^n + y^n = z^n$, was published by British mathematician Andrew Wiles in 1995. In 2016, Ukrainian mathematician Maryna Viazovska proved a longstanding *conjecture* (unproven statement) about the most efficient way to pack spheres in a space containing either 8 or 24 dimensions.

Many interesting conjectures are still awaiting proof or disproof. For example, *Goldbach's conjecture* states that every even integer greater than 2 is the sum of two prime numbers (see **Prime number**). Another example is the *twin prime conjecture*, which states that there are infinitely many prime numbers that differ by just two. Also, no one has been able to prove that there are infinitely many *Mersenne primes,* which are prime numbers of the form *2p – 1* where *p* is a prime number.

Trends in teaching mathematics. Before the 1950's, most math courses in elementary, junior high, and high schools in the United States stressed the development of basic computational skills. During the late 1950's and the 1960's, *new mathematics* was introduced. New mathematics is a way of teaching that stresses understanding concepts rather than memorizing rules and performing repetitious drills. In the 1970's and 1980's, educators continued to use new mathematics, but they gave added emphasis to problem solving and computational skills. More recently, educators have become aware of different learning styles and of the importance of taking a student's life and mathematics background into account when applying different teaching techniques. Today, educators make much greater use of technology and computer programs to assist in learning. Group activities in the classroom have largely replaced the roll of lectures and rote learning techniques.

At the college level, educators have moved away from teaching mathematics in the same way to all students. Instead, colleges and universities offer more courses in specialized applications of mathematics in such fields as economics, engineering, and physics. Long-term projects have also been added to further student understanding as well as to incorporate research and communication skills.

Careers

A strong background in mathematics is excellent preparation for a wide variety of careers. Students who wish to study mathematics in college should take high school courses in algebra, geometry, trigonometry, statistics, and calculus, if available. These courses also are useful for study in architecture, engineering, *linguistics* (the study of language), and all the natural and social sciences.

In college, the basic courses for a major in mathematics include advanced calculus, graph theory, numerical analysis, probability and statistics, theories of real and complex variables, topology, and differential equations. Courses in logic and computer programming also are useful in preparing for many careers.

Mathematicians teach at all levels. High school mathematics teachers must have at least a bachelor's degree in mathematics. Many mathematicians with a doctor's degree teach at colleges and universities.

Large numbers of mathematicians work in business, government, or industry. People with a mathematics degree may find work as accountants and auditors, actuaries, data and computer scientists, engineers, financial and stock analysts, medical scientists, software developers, and statisticians. Those with advanced degrees conduct research for the communications, energy, manufacturing, or transportation industries. Mathematicians also work in the computer industry as programmers or as systems analysts who determine the most efficient use of a computer in any given situation. Insurance companies employ mathematicians as actuaries to calculate risks and help design policies. Mathematicians also work as software developers, as data analysts, and in the field of computer science. Some mathematicians work as *cryptanalysts* (people who work with codes and ciphers) to keep the flow of information secure (see **Codes and ciphers**). Peter Schumer

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- G. Set theory and logic

III. History

IV. Careers

Mather, Cotton (1663-1728), was a leading Puritan minister and theologian in colonial New England. His grandfather Richard Mather and his father, Increase Mather, were also famous colonial religious leaders.

Mather involved himself in all the affairs of New England. He wrote about 450 published works, most of them sermons. Many scholars consider him one of the first distinctly American thinkers. Mather spent most of his energy preaching on how to know and serve God. He taught that individuals must have an inward, personal experience with God to be saved and that they must lead lives devoted to acts of goodness.

Among Mather's best-known works is the *Magnalia Christi Americana* (1702), a history of Christianity in New England in the 1600's. *Bonifacius* (1710) is a book of essays on how to be good. Mather gained fame and criticism for his support of smallpox inoculation during an epidemic in Boston in 1721. He blended his interest in science with a fascination with the supernatural, especially witchcraft and Bible prophecies about the end of the world. Some people accused him of prompting a hysteria over witches that resulted in the conviction and execution of 19 people as witches at Salem, Massachusetts, in 1692. Today most historians dispute the charge. See **Salem witchcraft trials; Witchcraft** (History).

Mather was born on Feb. 12, 1663, in Boston. He graduated from Harvard College at 15. In the early 1680's, he joined his father on the staff of the Second Church in Boston, where he remained until his death on Feb. 13, 1728. Richard W. Pointer

Additional resources

Middlekauff, Robert. *The Mathers: Three Generations of Puritan Intellectuals, 1596-1728.* 1971. Reprint. Univ. of Calif. Pr., 1999.
Silverman, Kenneth. *The Life and Times of Cotton Mather.* 1984. Reprint. Welcome Rain, 2002.

Mather, Increase (1639-1723), was the most influential among the second generation of Puritan ministers in colonial New England. The Puritans were Protestant reformers who advocated simpler forms of worship and stricter morals. Richard Mather, his father, and his son Cotton Mather were also Puritan leaders.

Mather wrote many books and pamphlets on a wide variety of topics. He believed there was a general spiritual and moral decline in New England. He repeatedly denounced this trend in sermons. He interpreted a witchcraft scare in Salem in 1692 as God's judgment on a disobedient New England. Among Mather's most celebrated causes was opposing the more lenient standards of Congregational minister Solomon Stoddard on church membership and Holy Communion.

Mather was born on June 21, 1639, in Dorchester, Massachusetts. In 1664, he accepted the position of teacher at the Second Church, or Old North Church, in Boston. He remained there until his death on Aug. 23, 1723, even while serving as president of Harvard College from 1685 to 1701. His successful diplomatic efforts in England from 1688 to 1692 secured a new charter for Massachusetts. Mather's religious conservatism led to his resignation from Harvard and gained him a reputation as a staunch defender of traditional Puritan ways. Richard W. Pointer

Mather, Richard (1596-1669), was an important Puritan minister among the first generation of English settlers in the Massachusetts Bay Colony. He was the father of Increase Mather and the grandfather of Cotton Mather, two leading colonial religious leaders.

Mather was born in Lowtown, England, near Liverpool. He was an ordained minister in the Church of England from 1619 until he was suspended in 1633 because of his Puritan views (see **Puritans**). Mather chose to move with his family to Massachusetts in 1635, and a year later he became pastor of a newly founded church in Dorchester. He held that position until his death on April 22, 1669.

Mather was active in the Massachusetts colony's religious life and urged that each congregation should control its own affairs. He wrote a series of essays on the subject and was a major figure in writing the *Cambridge Platform,* the Congregational Church's official rules on discipline and government, adopted in 1648. Earlier, he had aided Puritan worship by helping to compile the hymnal called the *Bay Psalm Book* (1640), the first book printed in colonial America. Richard W. Pointer

Mathewson, Christy (1880-1925), was one of baseball's greatest right-handed pitchers. He won 373 games, 372 of them with the New York Giants from 1900 to 1916. Mathewson became the first pitcher in the 1900's to win 30 games a season three years in a row. He also pitched 20 or more victories for 12 consecutive seasons.

Christopher Mathewson was born on Aug. 12, 1880, in Factoryville, Pennsylvania. He became famous for developing a reverse curve pitch that was called a *fadeaway.* Today the pitch is known as a *screwball.* Mathewson pitched three shutouts in the 1905 World Series. In 1908, he won 37 games. Mathewson led the league in strikeouts for five seasons—1903, 1904, 1905, 1907, and 1908. He died on Oct. 7, 1925. Mathewson became one of the first five players elected to the National Baseball Hall of Fame in 1936. Jack Lang

See also **Baseball** (picture).

Matisse, *mah TEES,* **Henri,** *ahn REE* (1869-1954), a French painter, was one of the most influential artists of the 1900's. He is best known as the father of the modern art movement called Fauvism. The Fauves used large flat areas of bright color and strong lines in expressive and decorative ways while remaining faithful to traditional subjects, such as landscapes, still lifes, interiors, and portraits. Matisse executed his experiments with form and color in several mediums, including painting, sculpture, drawing, prints, and paper cutouts.

Henri Émile Benoît Matisse was born on Dec. 31, 1869, in Le Cateau, near Cambrai. In 1891, he moved to Paris, where he studied painting and drawing. In 1897, he began to experiment with modern approaches under the influence of the Impressionist painters, who tried to capture fleeting impressions and the effects of light. Matisse gradually learned to paint with lighter colors and bolder brushwork while working outdoors.

Matisse spent the summer months of 1904 and 1905 on the Mediterranean coast. The effects of Mediterranean light made an immediate impact on him. He began to paint in a more abstract style, with color as the dominant feature in his paintings.

Returning to Paris in 1905, he exhibited his new pictures at the annual Autumn Salon. His paintings, along with the works of such fellow French artists as André Derain and Maurice Vlaminck, established the Fauve movement. Matisse's new paintings used brash colors and heavy outlining of form that seemed crude and unruly to the public and critics. One critic called the artists