G4G15 Hand-On Workshop
Pentadecagonal Night Light

Answers to Questions

Q: How many distinct {N/k} stars are there for a given N?

A: ⌊N/2⌋ ("floor" of N/2, or the greatest integer ⩽ N/2).

Discounting k=1 (which yields an N-gon, not a star), each k yields a different star until k becomes > N/2. From then on, each star is just a vertical reflection of one of the previous values of k, drawn in the reverse order. When N is even and N/k is an integer, there is a star which can be considered a "degenerate" case, where there are k independent "loops" with 2 points, which appear as diameters of the surrounding circle, like the spokes of a bicycle wheel.

Q: What point angles are possible for an {N/k} star?

A: iπ/N for 1 ⩽ i ⩽ N-4

The N points divide the circle into arcs of 2π/N radians each. Each star point has a vertex on the circle and two chords that span an arc on the opposite side of the circle of some multiple i of these arcs, which span a total arc of angle i*2π/N. The star point itself will measure ½ this angle or iπ/N. The smallest possible angle (when k is largest) has i = 1, and the largest one (when k=2) has i = N-4, since 2 points on each side are skipped.

Q: How many values of s are there for an {N/k} star (that yield visually different stars)?

A: 1 ⩽ s ⩽ ⌈(2k-1)/2⌉

Since each side of each star point) skips k-1 points, there are k-1 points whose edges each cross that side 2 times, for a total of 2(k-1) crossings, dividing it into 2(k-1) + 1 = 2k - 1 segments. Any value of s ⩾ (2k-1)/2 will include all segments and look the same, so only values of s between 1 and (2k-1)/2 will look different. Since s must be an integer, we take the ceiling to include the central segment.

Q: Knowing just N and k (without making the drawing), can you figure out how many loops there will be, and how many points will be contained in each loop?

A: GCD(N,k)=g loops with N/g points each

The number of loops will be the greatest common divisor or g=GCD(N,k). Note that when N and k are relatively prime then g=1 so there will be a single loop, otherwise there will be multiple loops. The number of points in each loop will obviously be N/g. If k has factors of its own, then each loop will be its own {N'/k'} star with N' = g and k' = the other factor of k(!)