MATH NOW

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List

Approved

Teacher Resource

MATH NOW Grades 1– 8

Grade 1	Grade 2	Grade 3	Grade 4
Chapter 1 *(Sample included)* What is Quantity?	**Chapter 1** Count Me In	**Chapter 1** Numbers, Numbers Everywhere!	**Chapter 1** Number Sense
Chapter 2 Information Station	**Chapter 2** From Sorts to Graphs	**Chapter 2** Data in My Life	**Chapter 2** Data Literacy
Chapter 3 Patterns All Around	**Chapter 3** Patterns Please	**Chapter 3** A World of Patterns!	**Chapter 3** Patterns and Relationships
Chapter 4 Quantities to 50	**Chapter 4** We Value Numbers	**Chapter 4** By the Hundreds!	**Chapter 4** Addition and Subtraction
Chapter 5 Moving Through Space and Time	**Chapter 5** On the Move	**Chapter 5** On the Move!	**Chapter 5** Location and Movement
Chapter 6 Sharing Quantities Fairly	**Chapter 6** Together or Apart?	**Chapter 6** Fair Play	**Chapter 6** Multiplication and Division
Chapter 7 Balancing Act	**Chapter 7** Balancing Quantities	**Chapter 7** Equivalent Relationships	**Chapter 7** *(Sample included)* Variables, Equations and Inequalities
Chapter 8 All Shapes and Sizes	**Chapter 8** Shapes All Around	**Chapter 8** I Spy Shapes	**Chapter 8** Geometric Reasoning
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Chapter 10 Understanding Sequence in Coding	**Chapter 10** Writing Instructions in Coding	**Chapter 10** Making Loops in Coding	**Chapter 10** Using Variables in Coding

Grade 5	Grade 6	Grade 7	Grade 8
Chapter 1 Number Sense	**Chapter 1** Number Sense	**Chapter 1** *(Sample included)* Number Sense	**Chapter 1** Number Sense
Chapter 2 Data Literacy	**Chapter 2** Data Literacy	**Chapter 2** Data Literacy	**Chapter 2** Data Literacy
Chapter 3 Patterns and Relationships	**Chapter 3** Patterns and Relationships	**Chapter 3** Patterns and Relationships	**Chapter 3** Patterns and Relationships
Chapter 4 Addition and Subtraction	**Chapter 4** Addition and Subtraction	**Chapter 4** Part-whole Relationships	**Chapter 4** Working with Rational Numbers
Chapter 5 Location and Movement	**Chapter 5** Location, Transformations, Measurement	**Chapter 5** Two-Dimensional Shapes and Measurement	**Chapter 5** Geometric Relationships and Movements
Chapter 6 Multiplication and Division	**Chapter 6** Multiplication and Division	**Chapter 6** Working with Rational Numbers	**Chapter 6** Multiplying and Dividing Rational Numbers
Chapter 7 Variables, Equations and Inequalities	**Chapter 7** Variables, Equations and Inequalities	**Chapter 7** Variables, Equations and Inequalities	**Chapter 7** Variables, Equations and Inequalities
Chapter 8 Geometric Reasoning	**Chapter 8** Geometric Reasoning	**Chapter 8** Three-Dimensional Shapes and Measurements	**Chapter 8** Geometric Reasoning and Measurement
Chapter 9 Probability	**Chapter 9** Probability	**Chapter 9** Probability	**Chapter 9** Probability
Chapter 10 Introducing Conditional Statements in Coding	**Chapter 10** Enhancing Your Skills in Coding	**Chapter 10** Writing Subprograms in Coding	**Chapter 10** Making Decisions in Coding

LESSON 3

Describing Five

Curriculum Expectations

B. NUMBER	
Whole Numbers	
B1.1	read and represent whole numbers up to and including 50 and describe various ways they are used in everyday life
B1.2	compose and decompose whole numbers up to and including 50, using a variety of tools and strategies, in various contexts
B1.5	count to 50 by 1s, 2s, 5s and 10s using a variety of tools and strategies
Math Facts	
B2.2	recall and demonstrate addition facts for numbers up to 10 and related subtraction facts
C. ALGEBRA	
Patterns	
C1.4	create and describe patterns to illustrate relationships among whole numbers up to 50

Materials

- math counters
- connecting cubes
- Canadian coin manipulatives (optional)
- cardstock, glue and scissors (optional)

Multimedia

Video: *Five Little Monkeys Jumping on the Bed*, nursery rhyme (also in books), may be used for concepts of 5, counting back from 5 by 1s, addition/subtraction math facts for 5 https://www.youtube.com/watch?v=uTR6GPmT8BE

Blackline Masters

- **1BLM 1.3** Count and Write to 5 *(sample included)*
- **1BLM 1.4** Five-Frames
- **1BLM 1.5** Fun with Five-Frames
- **1BLM 1.6** Instant Numbers to 5! (optional)

Assessment Tools

- **1AS 1.3** Describe 5 Assessment Checklist *(sample included)*
- **ASLS-1** Learning Skills and Work Habits Assessment Checklist - Collaboration

Lesson Introduction

Learning Goal:

We can recognize and count quantities of 5.

Involve students and identify success criteria for this learning goal. Sample guiding questions are provided in the link.

In this lesson, students will identify, read and represent whole numbers to 5 and explore counting patterns by 1s and 5s to 30. They will begin to demonstrate conservation of number as they compose and decompose 5 into different quantity combinations. At this stage, these combinations will be recorded using the words *is* and *and*, not using the symbols = or +, for example, "5 is 3 and 2" and "5 is 4 and 1."

The numbers 5 and 10 are considered benchmark or anchor numbers around which math facts to 10 relate. This concept will be further developed and assessed in Lesson 4.

Begin by asking students if they can think of a time where they use the number 5 in their lives. Listen for all suggestions, for example:

- I am 5 years old. I will be 6 soon.
- There are 5 people in my family.
- We have the number 5 in our address.

Read the Introduction in the Student Resource together to discuss how numbers have quantity and can be represented using symbols. Number words have been included to show students that we can read and say a number symbol and what we say can also be written as a number word.

Allow them to personalize the other numbers from 0 to 4 by having students use each number to tell something about themselves, for example:

- I have zero brothers.
- I have 1 sister.
- There is a 2 in our phone number.
- My brother is 3 years old.
- I have four grandparents.

Have the students practise counting and writing number symbols using **1BLM 1.3: Count and Write to 5**.

Sample Probing Questions

- What are the two ways you see numbers represented?
  [Look for students' understanding that numbers can be represented as words and as symbols and that they mean the same quantity no matter what way it is represented.]
- How do you think number symbols can help you when you are writing numbers?
  [Look for the idea that symbols help us to record things quicker or faster.]
- Does zero describe a quantity? Use it in a sentence.
  [Have students think about other words that we use to describe zero, such as nothing, none, empty.*]*
- Explain how 1/2/3/4/5 is a quantity by using it in a sentence.
  [This questions allows students to reinforce their understanding of numbers as quantities that are related to their own everyday lives.]

Exploring – Fun with 5

Provide students with **1BLM 1.4: Five-Frames** and counters to use throughout this lesson. Allow students time to provide a variety of suggestions to the question in the Student Resource, "How much is 5?" Students may say:

- Five is how many fingers I have on one hand.
- Five is how many people are in my family.
- I have five pencils.

Have students point to the empty 5-frame in the Student Resource and count the squares. Reinforce how the 5-frame has 0 or no quantity of counters on it.

You may wish to have students view the video and learn the song: *Five Little Monkeys Jumping on the Bed*, *https://www.youtube.com/watch?v=uTR6GPmT8BE*.

Students may review counting by 1s forward to 30 and backward by 1s from 30 using the 5-frames. Allow time for students to Think-Pair-Share any counting patterns they see. *[Sample pattern: Each number below a number ends in the same number, for example, 2, 12, 22; 9, 19, 29.]*

In order to help students understand how counting by 5s means saying every fifth number, have the class do this together.

Whisper or mouth four numbers and say the next number (i.e., 5, 10, 15, 20, 25, 30) out loud.

Talk about the patterns they see when counting forward by 5s and backward by 5s.

Have students work in pairs with their eyes closed and practise reciting counting forward and backward by 1s and by 5s. Tell them that they may need to peek for the first few times and encourage them to get to a point where they can do it without peeking.

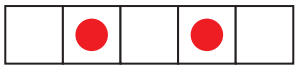
Sample Probing Questions

- What number patterns do you see when you count forward by 1s to 30?
- What number patterns do you see when you count back by 1s from 30?
  [If needed, prod students to notice how each decade uses a 1 to 9 (counting forward) pattern and a 9 to 1 (counting backward) pattern. Look for students to notice that all the teens start with 1, all the twenties start with 2 and thirties start with 3.]
- What number patterns do you see when you count forward by 5s?
  [Look for descriptions of a pattern of counted numbers ending with 5 and 0 alternately; every fifth number ends with a 5, then a 0, a 5, then a 0 and so on.]
- What number patterns do you see when you count back by 5s?
  [Look for descriptions of a pattern of counted numbers ending with 0 and 5 alternately; every fifth number ends with a 0, then a 5, a 0, then a 5 and so on.]
- How can you use these patterns to help you count by 1s and by 5s without looking?
  [Look for ideas that the number patterns can help them to figure out what comes next in the counting.]

Math Clinic – Fun with Five-Frames

Provide students with **1BLM 1.5: Fun with 5-Frames** along with blank 5-frames (**1BLM 1.4: Five-Frames**) and counters so they may also make a hands-on version of the numbers with manipulatives that correspond to the pictures on the worksheet. Students will review/practise how numbers 1 to 5 are represented with counters on a 5-frame (starting from the left and moving to the right). Encourage students to think about how many more counters are needed to make 5 for each representation on the worksheet.

After they complete the worksheet, have students use a blank 5-frame to show a number from 1 to 4 in the usual format (left to right). Then have them move those counters to different squares on the same 5-frame; for example, have them place 2 counters on a 5-frame in the usual format (left to right in the first two squares); then have them move the 2 counters to different squares on the same 5-frame.

Example: 

Ask students, "After you moved these counters to different squares on the 5-frame, how many counters do you have?" and "Does the arrangement of counters on a 5-frame affect the quantity of the counters?"

> *[Look for students who understand that the quantity does not change (students who are conserving number) and students who have to count the counters each time to verify that the quantity has not changed when their arrangement on the 5-frame changed (students who are not yet conserving). This is a developmental concept; you may encourage this logical thinking by providing activities that compose and decompose quantities of items. More opportunity to observe a task of this sort will be found in Reflecting below.]*

Practising Fundamentals – Making 5

Provide students with 5-frames (**1BLM 1.4: Five-Frames**) and counters. The first question is done with dotted lines as an example; have students fill in the counters and write the number to model what they need to do for the rest of the questions.

1. *[4]* 2. *[1]* 3. *[3]* 4. *[2]* 5. *[5]* 6. *[0]*

Problem Solving – The Same or Different Quantities?

Introduce the task by asking students who has played soccer, if they are in a league, how the game is played, and so on. The soccer ball formations in #1 require students to decompose a quantity of 4 into two, three and four parts to encourage students to expand their thinking from just the 2-part decompositions used with the 5-frames in Practising Fundamentals. Students will then transfer this expanded knowledge to decomposing 5 in as many different ways as they can (i.e., 2, 3, 4 and even 5 parts) using connecting cubes.

Facilitator Suggestions

1. Project the soccer ball images in the Student Resource for all to see. Allow some time for students to study the pictures before asking what they see. *[Each picture shows 4 soccer balls.]* You may wish to record the following sentences as students discuss each part:

   a) 4 is 4 and 0.

   b) 4 is 2 and 2.

   c) 4 is 1 and 1 and 2.

   d) 4 is 1 and 1 and 1 and 1.

   Using a Think-Pair-Share strategy, ask students if they can think of any other combinations of numbers that make a quantity of 4. Have students discuss this with a partner (or in small groups) to include and record:
   4 is 3 and 1.

Beside each of the five number statements you recorded, draw soccer balls in those formations to clearly connect each statement to the corresponding picture. Students can use this as a model when they record their findings in #2.

This provides another demonstration of conservation of number. Some students may not yet be conserving number. Guide their thinking in the direction that although the arrangement of the 4 soccer balls looks different, the quantity (4) remains the same. One-to-one counting may be required each time to help consolidate this thinking.

2. Students will work in small groups to transfer their understanding of decomposing numbers, once again 5, but this time decomposing it in many different ways and using a different manipulative, linking cubes. Supply each small group with paper on which to record their combinations using pictures, words and numbers (as modelled in the first task).

   Allow groups ample time to explore different combinations that make 5 with the linking cubes. While students are working, circulate to note the combinations they find and to keep them focussed on the goals of learning to work collaboratively to find as many ways as possible to decompose 5. Expand all students' thinking by having volunteers demonstrate the combinations of 5 they found. If possible, select volunteers such that all the possible combinations will be shared.

3. Provide time for students to practise recalling number facts by describing to a partner how many ways they can make 5.

   [Look for all of the following combinations of 5 that may be discussed with the whole class:

   - *2 parts: 5 is 1 and 4; 5 is 2 and 3*
   - *3 parts: 5 is 1 and 1 and 3; 5 is 1 and 2 and 2*
   - *4 parts: 5 is 1 and 1 and 1 and 2*
   - *5 parts: 5 is 1 and 1 and 1 and 1 and 1*

   Students may use any order of numbers in any of the above number combinations for 4 or 5; for example, instead of 5 is 1 and 1 and 3, they may use 5 is 3 and 1 and 1, and they may see these as two different combinations for 5. Students will be exploring the commutative property of addition in a later Chapter. At this stage, you may accept a different order of the same numbers as two separate or different combinations and list them as such when suggested by the students.]

Sample Probing Questions

- How many ways can you take 5 apart into 2 parts?
- Have you thought about using 3 parts? 4 parts? 5 parts?
- How can you record what you made with the linking cubes using pictures?
- What words and numbers would you write to go with those pictures?
  [Look for understanding of composing and decomposing 5 in a variety of ways, i.e., into two, three, four, and even five parts. Students should be able to use the modelling done in the soccer ball example to record their findings using pictures, words and numbers. The linking cubes may be represented by simple squares or any other pictures that correctly correspond to the number decomposition recorded.
  Example:

Reflecting – What I Know About 5

The first question will demonstrate consolidation of students' skills and knowledge of

- counting by 5s
- cardinality (the final number in a count represents the amount or quantity of items)
- conservation of number.

Conservation refers to a logical thinking ability that allows a person to determine that a certain quantity will remain the same despite adjustment of the container, the shape, or apparent size. This is a developmental concept so, if a student is not yet conserving, gently encourage this logical thinking by continuing to provide activities that compose and decompose quantities of items. You might, for example, move the objects to all different locations on the work surface or even cover all or part of the objects and keep asking, "How many are there now?" Count them with students every time (by 5s to practise) to convince students that the quantity has not changed.

SEL: Stress Management and Coping

This SEL reflecting question focuses on students being able to identify and explain a strategy that helped them count by 5s. Their responses can indicate their awareness that other strategies could also be used and to identify one that they would try.

Differentiated Instruction

Readiness

- Have students make pictures of quantities 1 to 5 and write the numbers on the pictures. Use the pictures in random order each day, having students read the numbers using the picture clues.
- Have students create 6 trains of 5 connecting cubes each. Have students skip count forward by 5s as they connect the cube trains and count back by 5s as they disconnect the trains.
- Make a game of having students use a rhythmic sing-song voice as they count by 5s during everyday events, for example, while lining up in groups of 5 for recess, coming to the carpet and so on.
- Have students regularly count back from any number less than 30 using everyday events; for example, as the first half of the class leave their seats one at a time to line up at the door, the students may count 13, 12, 11, 10, and so on.

Extensions

- Subitizing is a way of instantly counting. This exercise allows students to practise perceptual subitizing, or perceiving a number of objects (up to 5) immediately. Use **1BLM 1.6: Instant Numbers to 5!** to further consolidate different compositions/decompositions of quantities up to 5.
- Use Canadian coin manipulatives to talk about the value of a nickel (5 cents). Then count by 5s to find the value of 2, 3, 4, 5 and 6 nickels.

English Language Learners (ELL)

a) Glue **1BLM 1.6: Instant Numbers to 5!** onto a sheet of cardstock. When dry, cut along the dotted lines to create individual picture cards. Have ELL students write the number symbol on each card. Use these flash cards for practice.

b) After a period of time, cover or cut off the number symbols and have students recognize and read what's left—dot number formations.

Lesson 3 Assessment

Throughout the lesson, observe students to assess how they are:

- using words and visuals to make their thinking visible
- recognizing numbers in everyday contexts
- counting by 1s and 5s to 30
- relating numbers to the benchmark of 5 (composing/decomposing 5)
- using models and manipulatives to represent mathematical ideas

1AS 1.3: Describing 5 Assessment Checklist to assess students' knowledge and understanding of 5 as a quantity that can be counted, represented using words and symbols, and composed and decomposed into math facts.

ASLS-1: Learning Skills and Work Habits Assessment Checklist - Collaboration to assess students' skills when working with a partner or in a small group, such as responding positively to others, listening to each other.

SEL: Stress Management and Coping Reflecting Question to assess students' understanding that their resourcefulness in using various strategies to respond to stress is helping them build personal resilience.

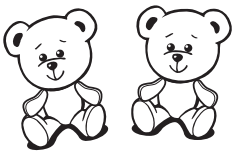
Name: _____

Count and Write to 5

1.

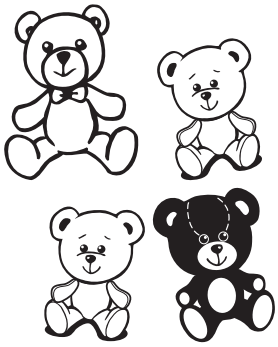
1 one

2.



3.

4.



5.

6. Fill in the blanks.

1	one
	two
3	
4	
	five

1AS 1.3: Describing 5 Assessment Checklist

Name: _____ Date: _____

☐ Reads and represents symbols and words to 5 (five) ☐ Counts forward to 30 by 1s and 5s ☐ Counts backward from 30 by 1s and 5s ☐ Describes counting patterns by 1s and 5s to 30 ☐ Composes and decomposes numbers to 5 ☐ Recalls number facts to 5	

1AS 1.3: Describing 5 Assessment Checklist

Name: _____ Date: _____

☐ Reads and represents symbols and words to 5 (five) ☐ Counts forward to 30 by 1s and 5s ☐ Counts backward from 30 by 1s and 5s ☐ Describes counting patterns by 1s and 5s to 30 ☐ Composes and decomposes numbers to 5 ☐ Recalls number facts to 5	

1AS 1.3: Describing 5 Assessment Checklist

Name: _____ Date: _____

☐ Reads and represents symbols and words to 5 (five) ☐ Counts forward to 30 by 1s and 5s ☐ Counts backward from 30 by 1s and 5s ☐ Describes counting patterns by 1s and 5s to 30 ☐ Composes and decomposes numbers to 5 ☐ Recalls number facts to 5	

LESSON 3

Inequalities

Curriculum Expectations

C. ALGEBRA	
Equalities and Inequalities	
C2.3	solve inequalities that involve addition and subtraction of whole numbers up to 20, and verify and graph the solutions
Mathematical Modelling	
C4	apply the process of mathematical modelling to represent, analyse, make predictions and provide insight into real-life situations
F. FINANCIAL LITERACY	
Financial Management	
F1.4	explain the relationship between spending and saving, and describe how spending and saving behaviours may differ from one person to another

Materials

- chart paper
- manipulatives (interlocking cubes, centimetre cubes, colour tiles, counters, etc.)
- markers, assorted colours
- whiteboard, large for projection
- scale, double pan balance (optional)

Multimedia

- Video: Silly School Songs video, The Greater Than Less Than Song, (2'45"), *https://www.youtube.com/watch?v=ka9zbPcqXBI* (accessed 2023-01-19) (optional)
- Video: Math Antics – Basic Inequalities, (show to 8'37") *https://www.youtube.com/watch?v=mgHO-bsCDrA* (accessed 2023-01-20) (optional)

Blackline Masters

- **BLM-B1** Number Lines (Horizontal)
- **BLM-B17** The Process of Mathematical Modelling
- **BLM-B53** Inequality Symbols
- **BLM-B54** Graphing Inequalities on Number Lines *(sample included)*

Assessment Tools

- **4AS 7.1** Financial Plan Assessment Rubric
- **AS-B7** Mathematical Modelling Assessment Rubric
- **AS-B88** Inequalities Assessment Checklist
- **AS-B89** Inequalities Assessment Rubric
- **ASLS-1** Learning Skills and Work Habits Assessment Checklist - Collaboration *(sample included)*
- **ASLS-3** Learning Skills and Work Habits Assessment Checklist - Initiative
- **ASLS-5** Learning Skills and Work Habits Assessment Checklist - Organization

Lesson Introduction

Learning Goal:

We can solve inequalities using addition and subtraction, and verify and graph the solutions.
We understand the relationship between spending and saving to meet a financial goal.

Involve students and identify success criteria for this learning goal. Sample guiding questions are provided in the link.

This lesson introduces inequalities using vocabulary and symbols for greater than, less than, greater than and equal to and less than and equal to. This is a new concept. In Grade 3 students identified equivalent relationships. Now they solve inequalities using guess and check, balance model strategies and number lines to graph their solutions.

Qualifying Time

The qualifying time scenario is introduced with a description about sprinting. Students may not be familiar with the difference between a sprint race and a regular race. You may need to explain the meaning of "qualifying time," and that qualifying time may be greater than, or less than, the given time, depending on the particular situation, for example,

- to enter a race, the qualifying time must be less than or equal to the given time because the goal is speed.
- to enter certain types of competitions, such as video-game competitions, the qualifying time (played) might need to be greater than a given number so that all of the players have enough experience to make it a fair competition.

Activate students' prior knowledge about quantity values using a strategy like Think-Pair-Share for students to understand the qualifying time scenario and what the values represent.

Project the number line in the Student Resource. Focus on time as a value in whole seconds. Facilitate a class discussion and encourage students to share how they can use a number line to tell if a student qualifies for the track-and-field race.

[Look for: If the student's race time is 16 seconds or to the left of 16 on the number line, the student qualifies for the event. Otherwise, the student does not qualify.]

Use questions to probe students' thinking about comparing time on a number line.

a) Will more students qualify if we move the qualifying time higher, e.g., to 17 seconds? *[yes]*

b) Are more students qualified if we move the qualifying time lower, e.g., to 15 seconds? *[no]*

Number lines will be used throughout the lesson as a graphing tool to investigate the concepts of inequalities.

Sample Probing Questions

- What is the relationship between 100 m and 16 seconds in the qualifying time scenario?
  [Look for an understanding that these two values provide information to solve the problem. 100 m is a constant. 16 seconds is the maximum amount of time that a student has to qualify for the sprint. Make sure that students do not consider 16 seconds as an absolute time but rather that students can qualify if they sprint 16 seconds or less than 16 seconds, and they do not qualify if their sprint takes longer than 16 seconds.]
- Look at these numbers: 3, 9, 5, 2. How many ways can you order these numbers? Explain your process.
  [Look for an understanding that to order numbers, the numbers have to be compared to each other. Students may order numbers in increasing or decreasing order.]
- Using the same set of numbers, where would you place the number 9 in an increasing order? Explain why.
  [Look for understanding that number 9 is greater than any of the other numbers and should be placed at the end of an increasing order.]
- Using the same set of numbers, how many numbers will you place before the number 9 in an increasing order? Explain why.
  [Look for students' understanding that three numbers (2, 3 and 5) are all smaller numbers than the number 9. The order would be 2, 3, 5, 9.]
- What is a number line?
  [Students have had prior experiences with number lines and should understand that they are visual representations of numbers that are placed at equal intervals on a straight line. The line can be vertical or horizontal.]
- Represent the set 2, 3, 5, 9 on a number line.
  [Look for an appropriate scale for their number line (0 to 10) with equal intervals of 1; intervals are labelled; the numbers 2, 3, 5 and 9 are identified.]

Exploring – Understanding Inequalities

This section is scaffolded to help students understand the concept of inequalities. There are two parts. Part 1 introduces students to the inequality signs. Some students may be familiar with these signs and may have used "<" and ">" to compare whole numbers in previous grades. Part 2 shows how we can use number lines to graph inequalities.

Part 1 – Inequality Signs

Explain that, in mathematics, we use symbols and signs in specific ways just as we do with English or any other languages, for example, when we use the "+" sign, we know that we are adding values. Ask students what other math signs they are familiar with. *[Look for the four operators, +, –, ×, ÷ and the = sign.]*

Mention that, in this lesson, they will learn more mathematical signs. Have students work with a partner and provide each pair a copy of **BLM-B53: Inequality Symbols**. Allow time for them to read and discuss with each other what they understand.

As a whole class, write the variable *t* on a whiteboard or chart paper. Mention that *t* can have any value depending on the context. Ask how students would write a math statement if *t* represents the number of days in a week. [$t = 7$]

Explain that not all variables need to represent a specific amount. Sometimes, we look for values that are greater than a given amount or less than a given amount.

You may have students Brainstorm situations in real life where they need to identify the following:

- values that are greater than a given amount (e.g., minimum height to go on a ride at an amusement park)
- values that are less than a given amount (e.g., paying for an item with the cash you have)
- values that are between two amounts (e.g., using 4 to 6 bananas to make a cake)

Refer students to BLM-B53 and review the information and symbols for less than, less than or equal to, greater than and greater than or equal to. Use the images of the balance model to reinforce these values.

Introduce the term inequalities. Write "inequality and inequalities" on the whiteboard or chart paper along with this definition:

Inequalities are math statements used to show the relationship between two values that are not equal.

Write the terms "equation," "equal" and "inequality" beside each other. Use a strategy like Think-Pair-Share for students to compare how an inequality is different from the other two terms.

> *[Look for students' understanding that equations always have an equal sign, and both sides of the equation must be equivalent. An inequality may or may not have two equal expressions. One expression can be less than, greater than, less than or equal to or greater than or equal to the other expression.]*

Use the Qualifying Time scenario in the Introduction and describe how the four inequality symbols apply or do not apply. Students may work on this with a partner or in small groups before you debrief as a whole class.

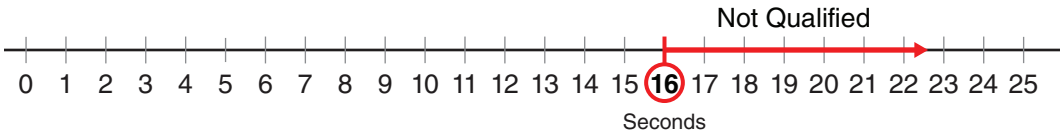
Symbol	***Meaning***	***In the Qualifying Time scenario***
$>$	*greater than*	$t > 16$, *not qualified*
$\geq$	*greater than or equal to*	$t \geq 16$, *not applicable* *A student who ran 16 seconds is qualified, but anyone who ran more than 16 seconds is not. The greater-than part is "not qualified," and the equal-to part is "qualified."*
$<$	*less than*	$t < 16$, *qualified*
$\leq$	*less than or equal to*	$t \leq 16$, *qualified*

Part 2 – Inequalities and Number Lines

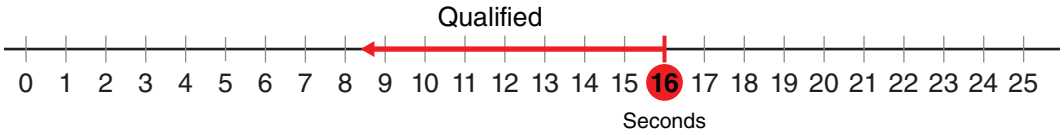
Project the number line from the Introduction on the whiteboard. Describe how a number line can visually represent the set of values in an inequality. In the qualifying time scenario, students must sprint 16 seconds or less to qualify.

Students will understand the arrows on the number line. Introduce them to two additional symbols used on number lines when graphing inequalities.

- An open dot (or circle) on the number line represents "<" and ">" relationships; the number you compare on the number line is not included in the set of values.



- A closed dot represents "≤" and "≥" relationships; the number you compare on the number line is included in the set of values.



This convention is reinforced in the Math Clinic.

Sample Probing Questions

- How are algebraic equations and inequalities similar and different?
  [Look for understanding that they both indicate the relationship between two expressions. The two expressions are connected by a 'comparing symbol or sign' (e.g., =, >, <, ≥, ≤) indicating the different relationships.]
- How is the value of the variable or unknown different when solving for it in an algebraic equation and an inequality?
  [Look for understanding that specific values can be determined in solving algebraic equations. In inequalities, the solution is represented by a set of values, e.g., $a > 6$, in which the solution is true with any values 7, 8, 9 and so on.]
- How do we know if the solution includes an inequality? Give an example to explain.
  [Look for an understanding of word prompts such as maximum, minimum, at most, at least and understand the limit of an event, e.g., the maximum highway speed is 100 km/hour. When comparing whether a car is travelling at the speed limit, the legal speed will be speed ≤ 100 km/h, and the illegal speed will be speed > 100 km/h. Driving at 100 km/h is legal.]
- Do you think we can have inequalities with algebraic expressions involving addition or subtraction? Use an example to justify your answer.
  [Yes, for example, $h + 2 \leq 5 + 2$, where h could represent the number of days students get homework in one week.]
- Do you think we can we simplify the expressions in an inequality before solving the problem?
  [Yes, we can simplify the expressions on both sides before determining the solution, for example, the previous inequality, $h + 2 \leq 5 + 2$, can be simplified to $h + 2 \leq 7$.]

Math Clinic – Solving Inequalities

Summarize what students have learned about solving inequalities and how they are similar to solving equations. When solving an inequality, we can use guess and check and balance model strategies. Provide students with manipulatives to use with a balance-model strategy when solving inequalities.

When solving a word problem, key words such as at least, greater than and equal to are some of the clues that indicate you are working with an inequality. They help you identify the symbol to use.

Emphasize the meaning of the four inequality symbols and how they are represented on a number line. Understanding the role of the closed versus open dot when graphing inequalities on a number line is extremely important. Making the connection between using/seeing an open dot and a < or > symbol and understanding that the starting value is not included in the possible results cannot be overstated.

Use **BLM-B54: Graphing Inequalities on Number Lines** as a reference.

Do a Think Aloud with students using the example of the fundraising event in the Student Resource.

If students need more practice have them solve the inequality, $y + 9 > 16$ showing their strategy and the solution on a number line. (A sample solution is provided below.) Provide students with **BLM-B1: Number Lines (Horizontal)**.

[Solving $y + 9 > 16$

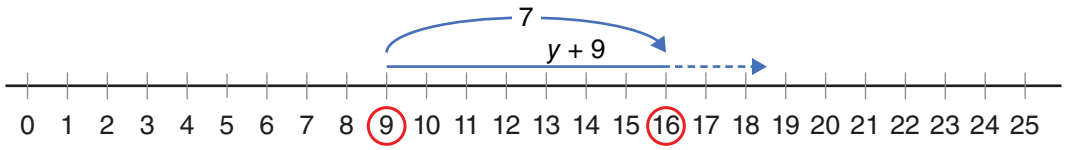
Using the balance model, isolate y by taking away 9 from the expression on the left side; this means we also need to subtract 9 from the expression on the right side.

- *$y + 9 > 16$*
- *$y + 9 - 9 > 16 - 9$*
- *$y > 7$*

Verifying $y > 7$ or $y = 8$, $y = 9$, $y = 10$ and so on

- *Substituting 8 for y, $8 + 9 > 16$. This statement is true.*
- *Substituting 9 for y, $9 + 9 > 16$. This statement is true.*
- *Substituting 10 for y, $10 + 9 > 16$. This statement is true.*
- *Substituting 7 for y, $7 + 9 > 16$. This statement is false.*
- *Therefore, the solution $y > 7$ is correct.*

Graphing $y + 9 > 16$



Practising Fundamentals – Working with Inequalities

1. *[D: 8]*

2. a) *[$p > 4$]* b) *[$z \geq 10$]* c) *[$11 < b$]* d) *[$15 \leq x$]*

3. a) *[$(10) - 3 > 6$ is True.]* b) *[$13 > (18) - 5$ is False.]* c) *[$16 < (5) + 9$ is False.]*

4. a) *[$n \geq 10$]* b) *[$a > n$]* c) *[$f \leq 15$]*

5. *$c > 4$*

6. *$7 \leq t$*

Problem Solving – Creating a Financial Plan

In this Problem Solving task, students apply their understanding of inequalities within the context of creating a financial plan. This task also addresses expectation F1.4. As students work on the task, they apply the process of mathematical modelling, which they have used in the previous lesson. Display **BLM-B17: The Process of Mathematical Modelling** or provide students with copies of the blackline master to help them work through the task. They create their own situation that requires a financial plan to meet a goal. In working on this task, students will demonstrate their understanding of spending and saving relationships and mathematical ideas of inequalities. They will write algebraic expressions and compare the expressions using inequality signs. They will graph the set of values to meet their financial goal on a number line. Students use numbers ≤ 20.

Facilitator Suggestions

Review, with the whole class, terminology and relationships between earnings, savings, expenses and spending.

Ask students to think about taking a trip. It can be a small trip in their neighbourhood or a larger trip. Be prepared for a wide-range of responses, as students come from diverse backgrounds. Ensure students are respectful of all responses.

Introduce the concept of planning a trip with the end goal of being at the final location.

You could use Hamas' scenario in the Lesson 2 Problem Solving task as an example.

Brainstorm planning considerations (e.g., why take the trip, how long is the trip, how much money is needed to pay before the trip, what money will be spent during the trip and so on).

Go through ideas that are part of making a plan (purpose, goal, time, costs, assumptions, etc.)

Introduce the Problem Solving task in the Student Resource. Read with the class the first part of the task including the first six points and the sentence that follows them. Ensure that students are aware they need to create a financial plan for a specific financial goal that they want to reach. Make sure students understand that they are applying their understanding of inequalities in creating their plan. If students are unsure of how inequalities relate to the financial plan, ask them to suggest an aspect involved in a financial plan (e.g., the money saved to pay for a goal, which costs $18) and a variable to represent the money saved (e.g., s).

Then ask, "What amount of money could s represent? *[Look for: s = $0, $1, $2, … $17]*

How would you describe the money saved before you reach your financial goal?"
[Look for: using the inequality, s < $18 or s ≤ $17] Encourage students to consider whether other situations in their financial plan could involve inequalities rather than just equations.

Organize students into small groups who then meet and read the rest of the task, discussing the five remaining criteria among themselves.

Provide each group with chart paper and markers to record their financial plan, their work demonstrating how inequalities relate to the plan, their solutions and number lines. Provide students with **BLM-B1: Number Lines (Horizontal)** as needed.

As you circulate among the groups, have groups share their financial goal, sources of earnings and spending that may be needed, etc. Review their plans for reasonableness and make note of groups that used different strategies and approaches, to share with the class.

Example:

- purpose of the plan *[to save money to make a donation]*
- goal *[variable: t]*
- duration to meet the goal *[n weeks]*
- sources of income *[doing errands, babysitting, etc., at $3 per week]*
- expenses that may be required *[buying a meal at $1]*

Possible expressions related to the example:

» *Income: 3n*

» *Expenses: $1 to buy lunch*

» *Savings: 3n – 1*

» *Inequality: 3n – 1 ≥ t*

Look for understanding that students will have to set the target to solve the inequality.

In this example, the goal could be to donate at least $8:

$3n - 1 \geq 8$

Isolate 3n:

$3n - 1 + \mathbf{1} \geq 8 + \mathbf{1}$ *(Add 1 to both sides – balance model.)*

$3n \geq 9$ *(Use math fact: $3 \times 3 = 9$.)*

$n \geq 3$

*A student has to work **3 or more weeks** to donate at least $8.*

Verify the solution:

$3 \times 3 - 1 \geq 8$ *The statement is true.*

Graph the solution on a number line:

Upon completion, have groups present their work with the rest of the class using a strategy like a Gallery Walk, Math Forum, Show and Tell or Math Congress. Call upon any groups that you noted in approaching the task to share their ideas with the class.

Students actively listen and reflect on how other groups approached the task. Encourage them to share their comments about each others' work. Ensure that their comments offer constructive criticism and suggestions.

Sample Probing Questions

- What is a financial plan?
  [Look for an understanding that a financial plan is developed to meet a specific goal and specific actions to achieve the goal. This includes ideas for savings and spending, and outlines a schedule of when the goal will be met.]
- What inequalities are relevant to your plan?
  [Look for an understanding that the financial plan has a goal that needs to be met or exceeded. The plan needs to take into account inequalities that provide solutions to meet or succeed the goal. Inequalities may involve money, time and so on.]

- What scenario and goal did you decide for your financial plan? Explain how your goal can be an inequality.
  [Look for a plausible goal given the scenario. Students should justify their decision. Look for an understanding that the goal can be met or exceeded (greater than or equal to) or have a maximum requirement (less than).]
- What assumptions did you make in developing your financial plan and meeting the goal?
  [Look for plausible assumptions related to earning, savings, etc.]
- How are spending and saving money related? What can affect people's decisions about their spending or their saving?
  [Look for students' understanding that the more one spends the less money they have for savings. Students should be able to come up with some suggestions that influence spending and saving such as income, expenses, needs, health, culture, etc.
  Record their ideas and ask the class how each example affects spending and/or saving. Sample responses include:
  - » *"Earnings affect spending, because if you earn a little money, you don't have a lot of money to spend on expenses, unless you already saved up money and want to spend that money."*
  - » *"Health affects spending because if someone needs something to make their health better, they would try to buy it because it's important for them."]*
- What would you do differently if you were to do this task again?
  [Look for plausible suggestions on any recommended actions.]

Reflecting - What I Know About Inequalities

[Look for students' understanding that the seesaw may be used as an analogy to model inequalities as compared to the balance model, which is used for equations. We use these symbols, $>$, $\geq$, $<$, $\leq$ to show the relationship of two expressions with unequal values as in a tilted seesaw. Students may say that the lower side of a seesaw has a greater mass just like one side of an inequality may have a greater value (and vice versa); the inequalities $\geq$ and $\leq$ include a situation of equivalence just like a seesaw may be balanced or one side may be higher/lower.]

SEL: Stress Management and Coping

This SEL reflecting question requires students to apply their understanding of inequalities by making connections between the mathematics that they are learning and how it applies to financial literacy.

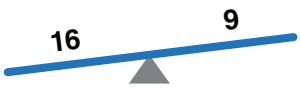
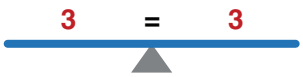
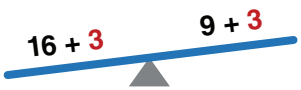
Differentiated Instruction

Readiness

- Students can listen to the Silly School Songs video, The Greater Than Less Than Song, https://www.youtube.com/watch?v=ka9zbPcqXBI, to review the symbols for greater than, less than and equal.
- Use a Think Aloud to review **BLM-B54: Graphing Inequalities on Number Lines**.
  Make sure students can differentiate the inclusion and exclusion of the values that they are comparing when graphing inequalities on a number line:
  - » Visually, inclusion of a value is represented by a closed dot, and exclusion is represented by an open dot on the number line.

- » We use an arrow line to represent the set of values that is true for the inequality. The arrow line represents all the values on the number line in the direction of the arrow.
- » Look for words such as most, maximum, least, minimum, etc., to determine whether the "or equal to" value applies and the closed dot is used.

- Use the Math Antics video, Basic Inequalities, https://www.youtube.com/watch?v=mgHO-bsCDrA, as a tutorial to review key concepts being taught in this lesson. Show to 8'37". Up to this point, the video explains the inequality symbols and how number lines can be used to graph inequalities, including when to use a closed or open dot. The rest of the video discusses double or compound inequalities. Although these are clearly explained, they are not part of the Grade 4 expectation.
- Use visuals to explain why we can add or subtract to remove numbers in inequality expressions.

	Inequality 16 > 9
	If we add/subtract a quantity to/from the left side of a balance scale and add/subtract the same quantity to/from the right side, the scale will remain balanced. Ask students what they expect to see if we change both 3s to 1s. *[The scale will remain balanced.]* Reiterate, "When you perform an operation on the left side of an equation and perform the same operation on the right side, the equation will continue to hold (to be true)."
	In 16 > 9, if you add 3 to both side of the > sign, the inequality will still be true, 16 + 3 > 9 + 3. *[Students can evaluate the expressions for further confirmation: 19 > 12.]*

Extensions

- Students create a chart of antonyms or opposites showing how the prefix 'in' meaning 'not' is used in everyday language and, just like in math, the words equal and inequal have opposite meanings. *[Some sample pairs could be: ability/inability, accessible/ inaccessible, accurate/inaccurate, adequate/inadequate, capable/incapable, sensitive/insensitive.]*
- Challenge students to use variables for all of the conditions in the financial model Problem Solving task and use real numbers to test the model.

  [Example:
  Goal: g
  Income: i per week
  Duration: w weeks
  Expenses: e
  Math statement: $wi - e \geq g$
  Students substitute numbers for any 3 of the 4 variables to determine a solution.]

English Language Learners (ELL)

- Assess students' understanding of terms and symbols related to inequalities (e.g., greater than, less than, equal to, greater than or equal to, less than or equal to). Use drawings or illustrations as visual prompts to have students remember the symbols < and > (e.g., BIG > small). It is important that students use

the words to describe the inequality and not only use the symbols (e.g., *h* is greater than 5). Add the words to the word wall. If necessary, pre-teach vocabulary prior to the lesson.

- Use ELL suggested strategies from Lessons 1 and 2, to support students' understanding of word problems.
- The Silly School Songs video, The Greater Than Less Than Song, *https://www.youtube.com/watch?v=ka9zbPcqXBI* has visuals to represent values for the inequality symbols < and >, and the equal symbol, =.
- Use manipulatives to show visual representations of the four inequality symbols. Model language to describe comparisons of values.

Lesson 3 Assessment

Throughout this lesson, observe students to assess how they are:

- identifying inequality symbols and using words to describe their meanings
- solving inequalities involving addition and subtraction of whole numbers up to 20
- verifying solutions to inequalities involving addition and subtraction of whole numbers up to 20
- graphing inequality solutions on a number line
- explaining the relationships between spending and saving when developing a financial plan
- using strategies, manipulatives and drawings to solve problems involving inequalities

4AS 7.1: Financial Plan Assessment Rubric to assess students' understanding of the relationship between spending and saving when creating a financial plan.

AS-B7: Mathematical Modelling Assessment Rubric to assess students' application of the processes of mathematical modelling to create a financial plan to meet a goal.

AS-B88: Inequalities Assessment Checklist to track students' understanding of representing, solving and verifying inequalities, and graphing the solutions on a number line.

AS-B89: Inequalities Assessment Rubric to assess students' knowledge, understanding, skills and applications of solving inequalities involving whole numbers up to 20.

ASLS-1: Learning Skills and Work Habits Assessment Checklist - Collaboration to assess students' skills when working with a partner or in small groups such as contributing an equitable share of work, responding positively to others, etc.

ASLS-3: Learning Skills and Work Habits Assessment Checklist - Initiative to assess students' approaches to new ideas and opportunities with a positive attitude and demonstrating an interest in learning.

ASLS-5: Learning Skills and Work Habits Assessment Checklist - Organization can be used to assess individual students' skills in following a plan and managing time for completing the tasks.

SEL: Critical and Creative Thinking Reflecting Question to assess students' awareness to make connections between the math they are learning about inequalities and how it can apply to real-world examples within the context of financial literacy.

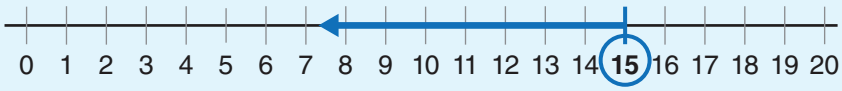
Name: _____

Graphing Inequalities on Number Lines

Open Dot (Start value not included)

$n > 10$

n is greater than 10. The arrow covers all numbers to the right of 10 on the number line.



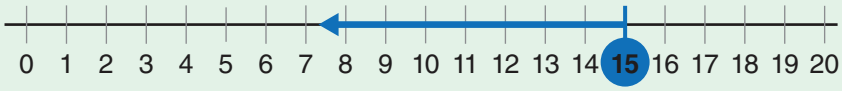
$n < 15$

n is less than 15. The arrow covers all numbers to the left of 15 on the number line.

Closed Dot (Start value included)

$n \geq 10$

n is greater than or equal to 10. The arrow covers all numbers to the right of, and including, 10 on the number line.



$n \leq 15$

n is less than or equal to 15. The arrow covers all numbers to the left of, and including, 15 on the number line.

ASLS-1: Learning Skills and Work Habits Assessment Checklist - Collaboration

1: rarely observed **2**: sometimes observed **3**: usually observed **4**: consistently observed

Name of Students	Accepts various roles & equitable share of work				Responds positively to ideas of others				Builds healthy peer relationships				Works to resolve conflicts and builds consensus				Shares information, resources and expertise			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Remarks																				
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LESSON 3

Factors

Curriculum Expectations

B. NUMBER	
Properties and Relationships	
B2.1	use the properties and order of operations, and the relationships between operations, to solve problems involving whole numbers, decimal numbers, fractions, ratios, rates and percents, including those requiring multiple steps or multiple operations
Multiplication and Division	
B2.6	determine the greatest common factor for a variety of whole numbers up to 144 and the lowest common multiple for two and three whole numbers

Materials

- Chart paper
- Whiteboards

Multimedia

- Video: Why do Cicadas Only Show Up at Prime intervals of 13 and 17 Years?, (6 min 32 sec), *https://www.sciencealert.com/watch-why-do-cicadas-only-show-up-at-prime-intervals-of-13-and-17-years* (accessed Feb. 2, 2020)

Blackline Masters

- **BLM-B9** 12 × 12 Multiplication Math Facts Table (optional)
- **BLM-B22** Divisibility Rules (optional) *(sample included)*

Assessment Tools

- **AS-B10** Problem Solving Processes Assessment Rubric
- **AS-B20** Factor Assessment Rubric *(sample included)*
- **7AS 1.3** GCF and LCM Assessment Rubric
- **ASLS-1** Learning Skills and Work Habits Assessment Checklist - Collaboration
- **ASLS-5** Learning Skills and Work Habits Assessment Checklist - Organization

Lesson Introduction

Learning Goal:

We can determine the greatest common factor for a variety of whole numbers up to 144 and the lowest common multiple for sets of two or three whole numbers.

Involve students and identify success criteria for this learning goal. Sample guiding questions are provided in the link.

In this lesson, students build upon their knowledge of factoring and are introduced to the terms greatest common factor (GCF) and lowest common multiple (LCM).

Review students' prior knowledge about prime numbers, composite numbers and factoring. Have students answer the question in the Student Resource.

[Prime factors would be the rooms because they cannot be composed of any smaller space other than the room itself.]

Discuss what makes a number prime and what makes it composite.

Use this discussion to assess students' understanding of prime and composite numbers and note students who require additional instruction or support about these concepts.

Project Ella and Dean's factor trees on the whiteboard or have students examine them from the Student Resource. Working with a partner, decide which factor tree is correct or if they both are. Have students explain their reasoning to the class.

Write the number sentence $2 \times 2 \times 11 = 44$ on a whiteboard or chart paper. Ask students what they notice about the numbers being multiplied. Students should recognize that these are all prime numbers.

*[Both factor trees are correct. The order in multiplication does not matter (associative and commutative properties of multiplication). Both factor trees result in the same factors for the number $44 = 2 \times 2 \times 11$. The prime factors of 44 are 2 and 11. The process is called **prime factorization**. The factors of 44 are 1, 2, 4, 11, 22 and 44.]*

Ask students to define the term prime factorization mentioned in the Student Resource.

[Prime factorization is a process to identify the prime numbers which, when multiplied together, make the original number. Some numbers may appear more than once.]

If students are unclear, provide examples as below:

Is the sentence a prime factorization of the number?

$7 \times 11 \times 11 = 847$	*[Yes. 7 and 11 are prime factors of 847.]*
$2 \times 10 = 20$	*[No. 10 is not a prime number. Prime factorization of 20 is $2 \times 2 \times 5$.]*
$2 \times 5 \times 7 = 70$	*[Yes. 2, 5 and 7 are all prime factors of 70.]*
$3^2 \times 8 = 72$	*[No. 8 is not a prime number. Prime factorization of 72 is $3^2 \times 2 \times 2 \times 2$.]*
$2^2 \times 5^2 = 100$	*[Yes. 2 and 5 are prime factors of 100.]*

Sample Probing Questions

- What is a prime number? Explain, using examples.
- What is a composite number? Explain, using examples.
- What is the difference between factors of prime numbers and of composite numbers? Explain, using examples.
- What are factors and how can we determine them? Explain your answer.
- What are factor trees? Explain, using an example other than Ella and Dean's factor trees. *[Look for an understanding that prime numbers have exactly two factors, 1 and the number itself, and composite numbers have more than two factors. Factor tree is a tool for prime factorization. The last line of the tree identifies all the prime factors of the original number.]*

Exploring - Greatest Common Factor and Lowest Common Multiple

Project the image of the cicada insect on a whiteboard. Students will know it is a cicada from the Student Resource. Ask them if they have ever seen this insect and how we know it is an insect. Students should have prior knowledge about the characteristics of insects (e.g., six legs, 3 body parts).

Watch the video *Why Do Cicadas Only Show Up at Prime Intervals of 13 and 17 Years?* *https://www.sciencealert.com/watch-why-do-cicadas-only-show-up-at-prime-intervals-of-13-and-17-years*, (6 min 32 sec).

Discuss the content of the video and ask why they think it was chosen.

Review factoring.

- Have students work in pairs and identify two different numbers from 100 to 144 to factor. One number must end with the digit 2, 4, 6 or 8 ensuring it is an even composite. The other number must end with a 5 or a 0.
- Each student independently chooses a strategy to factor their numbers (e.g., factor tree, factor rainbow, divisibility rules).
- Student partners compare their factors for each number and prime factorization multiplication equations. Have them discuss if their results were identical or different.

Have students continue to work as partners with the factors for their numbers.

- Students should write all the factors for each number from the smallest to the largest value in order to make comparisons. Look for their understanding of prime factorization and the factors of a number.
- Have them identify factors that are common to both numbers and the greatest common factor.
- Write the term greatest common factor or GCF on a whiteboard.
- Have students summarize their findings and define GCF.

Have students work with their partner and write the multiples of 2 *[2, 4, 6, 8, …]* and of 3 *[3, 6, 9, 12, …]*. They can use the number lines in the Student Resource.

- Identify the multiples that are common to both numbers. *[6, 12, 18, …]*
- Write the term "lowest common multiple or LCM" on a whiteboard.

- Have students identify the LCM of 2 and 3. *[6]*
- Have students summarize their findings and define LCM.

As a whole class discuss how scientists determined periodical cicadas with 13-year and 17-year life cycles would emerge together every 221 years.

[Look for an understanding that 221 is the LCM of the two life spans.]

STEM Connection: This topic relates to the grade 7 science & technology strand, Life Systems, when students are learning about populations of living things and about biotic and abiotic factors.

Sample Probing Questions

- What strategy did you use to determine the factors of your numbers? Did you consider another strategy? Why or why not?
- What do these numbers have in common: 17, 23, 31, 47, 59, 73, 97? How do you know?
  [They are prime factors and not divisible by any factors other than 1 and the number.]
- What is the smallest number that has 3 and 4 as factors? What do you notice about this number?
  [The LCM of 3 and 4 is their product ($3 \times 4 = 12$).]
- Is 24 the LCM of 4 and 6? Explain.
  [No. Twelve is the lowest common multiple of 4 and 6 because 12 is the smallest number that divides evenly by 4 and by 6.]
- Does the order of prime numbers in a multiplication sentence matter? [No]
- How do you determine which numbers are in both lists? Describe your strategy.
- How do you determine the multiples of a number?

Math Clinic - Reviewing Factors and Multiples

This Math Clinic reviews what students have been learning about factoring and how to determine GCFs and LCMs. It can be used as a reference for students to check their understanding of prime factorization, greatest common factors and lowest common multiples.

Factor Tree and Factor Rainbow are reviewed to strengthen students' skills in factorization.

Have students recognize how knowing multiplication facts can help to determine factors. Project **BLM-B9: 12 × 12** Multiplication Math Facts Table on a whiteboard or provide students with online or print copies.

Ask how knowing multiplication facts helps to determine factors.

Students should be able to demonstrate their skills in reading from the times table to determine the prime factors of a number. Look for steps in their findings:

[For example, pick the number 56 and read from the times table:

- *$56 = 7 \times 8$ (7 is a prime number.)*
- *$8 = 2 \times 4$ (Have students read from the times table to reduce the number a step at a time.)*
- *$4 = 2 \times 2$*

Therefore, $56 = 2 \times 2 \times 2 \times 7$]

Practising Fundamentals - Determining GCFs and LCMs

1. *[Multiples of 13: 13, 26, 39, 52, 65, 78, 91, 104, 117, 130, 143, 156, 169, 182, etc.*
   Multiples of 7: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105, 112, 119, 126, 133, 140, etc.]

   a) *[The first common multiple is 91, the next common multiple will be $2 \times 91 = 182$.]*

   b) *[LCM is 91. Representations of the multiples of 7 and 13 to determine the LCM will vary.]*

2. *[Strategies will vary. Look for accuracy.*
   Factors of 54: 1, 2, 3, 6, 9, 18, 27, 54; Factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72]

   a) *[GCF of 54 and 72 is 18.]*

   b) *[LCM of 54 and 72 is 216.]*

3. *[The number is 110. Known factors: 1, 22, 55.*

   $22 = 2 \times 11$ therefore 2 and 11 are factors.

   $55 = 5 \times 11$ therefore 5 is a factor. We already know 11 is a factor.

   If 2 and 5 are factors, $2 \times 5 = 10$ is also a factor.

   Therefore, the factors are 1, 2, 5, 10, 11, 22, 55, n.

   The possible prime factors of n are: 2, 5, 11. Therefore, n = 110]

4. a) [The LCM of 4 and 6 is 12, therefore, they will clap at the same time on the 12th beat.]

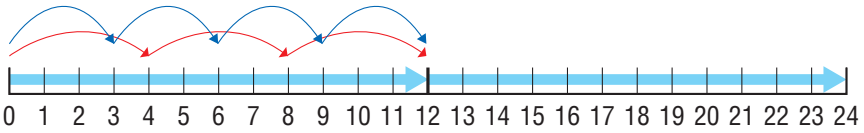
Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Jona				x				x				x				x				x				x
Roma						x						x						x						x

   b) *[The LCM of 4, 6, and 8 is 24, therefore, they will clap at the same time on the 24th beat.]*

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Jona				x				x				x				x				x				x
Roma						x						x						x						x
Toma								x								x								x

   c) *[GCF is 2.]*

5. *[They will see each other in 12 days. LCM of 4 and 3 is 12.*
James goes to the gym again in 4, 8, 12, 16, 20, 24 days and so on. These are multiples of 4. Simon goes to the gym again in 3, 6, 9, 12, 15, 18, 21, 24 days and so on. These are multiples of 3.]



6. *[No. 10 is not a prime number. Prime factorization of 140 = 2 × 2 × 5 × 7]*

Problem Solving - How Many in the Bucket?

This problem requires students to use their knowledge of factors and multiples to determine how many wings are in the Biggest Bucket O'Wings.

Facilitator Suggestions

Have students work in small groups of 4. Provide each group with chart paper or whiteboards to show their work and solve the problem.

Review the problem as a whole class. Have students restate the problem for clarity.

Provide time to solve the problem in class. Circulate to each group and ask questions to probe their ideas about how they are solving the problem. If students are struggling to begin, have them read each hint in the Student Resource and explain the information they get from that hint.

As you circulate to each group, observe strategies they are using to solve the problem. Identify 3 or 4 groups with similar or different approaches and call on them to present their work and solutions.

Provide time for the class to comment on the presentations using Two Stars and a Wish. After the presentations and discussions, have each group meet to discuss how the work that was presented is similar and/or different from their own. Each group can summarize what they thought went well when solving their problem and what they could improve next time.

Possible solution: *[Students can analyze all the hints as follows:*

i) *"If they are taken out seven at a time, no wings will be left." Therefore, the number of wings must be a multiple of 7.*
List multiples of 7 up to 140.

7	14	21	28	35
42	49	56	63	70
77	84	91	98	105
112	119	126	133	140

ii) *"If the wings are removed two at a time, one wing will be left." Therefore, the number must be an odd number. This will eliminate all even numbers. Numbers remaining are: 7, 21, 35, 49, 63, 77, 91, 105, 119, and 133.*

iii) *"If they are removed three at a time, two wings will remain." Therefore, the number must be one less than a multiple of three. This eliminates the numbers 7, 21, 49, 63, 91, 105, 133. Numbers remaining are: 35, 77, and 119.*

iv) *"If the wings are removed four, five or six at a time, then three, four and five wings, respectively, will remain". Therefore, the number must be one less than a multiple of four, five, and six. Only 119 remains.*

~~7~~	~~14~~	~~21~~	~~28~~	(~~35~~)
~~42~~	~~49~~	~~56~~	~~63~~	~~70~~
(~~77~~)	~~84~~	~~91~~	~~98~~	~~105~~
~~112~~	(119)	~~126~~	~~133~~	~~140~~

The smallest number of wings that could be in the bucket is 119. We can check by walking through the instructions in the problems and making sure we get the correct number of wings remaining.

Alternatively, the students can deduce from all the steps that the number is a multiple of seven as well as a number that is one less than a multiple of 2, 3, 4, 5, and 6.
The smallest multiple of these 5 numbers is 2 x 2 x 3 x 5 = 60.

119 is divisible by 7, so the number 119 is the smallest number of wings that could be in the bucket.]

Multiple of 60	Multiple of 60 – 1
60	59
120	119
180	179
240	239

Sample Probing Questions

- What is the ultimate question to solve in this problem? State it in your own words.
- What math concepts and skills are involved in this problem?
- What information is provided in the situation? How can you use this information to solve the problem?
- What strategies are you using to solve this problem? Why did you choose these?
- How are you going to show your work so that others can understand your ideas?
- What major challenges did you have in solving the problem and working as a group? Explain how you overcame these.
- Convince us that your solution is correct.
  [Responses will vary. Look for responses that ensure students are on task, are applying mathematical concepts accurately, demonstrate problem solving skills, can communicate their mathematical ideas, and are successful in completing the task.]

Reflecting - What I Know about Greatest Common Factor and Lowest Common Multiple

Students can choose their strategies and record the steps. Share the following approach with students.

GCF of 30, 36 and 60:	
1. List the factors of each number. You can use a factor rainbow to help.	**Step 1** 30: 1, 2, 3, 5, 6, 10, 15, 30 36: 1, 2, 3, 4, 6, 9, 12, 18, 36 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
2. Compare the numbers between the lists and identify factors that are in every list.	**Step 2** 30: 1, 2, 3, ~~5~~, 6, ~~10~~, ~~15~~, ~~30~~ 36: 1, 2, 3, ~~4~~, 6, ~~9~~, ~~12~~, ~~18~~, ~~36~~ 60: 1, 2, 3, ~~4~~, ~~5~~, ~~6~~, ~~10~~, ~~12~~, ~~15~~, ~~20~~, ~~30~~, ~~60~~
3. Identify the greatest factor that is common to all the lists.	**Step 3** The greatest common factor is 6.

LCM of 30, 36 and 60:	
1. List the multiples of each number. Use repeated addition or multiplication, starting with the number. Stop listing multiples when you find one that is in each list.	**Step 1** 30: 30, 60, 90, 120, 150, 180, 210 36: 36, 72, 108, 144, 180 60: 60, 120, 180
2. Identify the lowest multiple that is common to all the lists.	**Step 2** The lowest common multiple is 180.

SEL: Critical and Creative Thinking

This SEL question focuses on students being able to make connections between the math that they learned in this lesson and everyday life contexts (examples can be found in many word problems) and applications, such as the periodical cicada life cycles discussed in this lesson.

Differentiated Instruction

Readiness

- The Problem Solving task is a very engaging activity. Use counters or other manipulatives to represent the chicken wings and have students start using trial and error before they work through the hints. Encourage participation. Circle around and join each group in their discussions. Allow time for each students to do his/her trial count.
- Review divisibility rules or use BLM-B22: Divisibility Rules to help students identify factors of numbers.

Extensions

- Have students decode this Anonymous ancient poem to describe how to determine prime numbers in their own words. Students may be familiar with the Sieve of Eratosthenes from earlier grades.

  Sift the Two's and Sift the Three's,
  The Sieve of Eratosthenes.
  When the multiples sublime,
  The numbers that remain are Prime.

English Language Learners (ELL)

- This lesson has many mathematical terms. Go over the terms and their meanings. Have students identify ones they are familiar with and add any new terms to their personal bilingual dictionaries. Mathematical terms used in this lesson: factors, prime numbers, composite numbers, factor tree, factor rainbow, prime factorization, multiples, greatest common factor, GCF, lowest common multiple, LCM.
- Determine students' prior knowledge about factoring. Have them show you what they know, using a familiar strategy. Be open to the strategy as students may have learned about a different way of factoring in their previous school.
- Encourage students to use number cards to help communicate their ideas in presenting their factor tree or factor rainbow strategies.

Lesson 3 Assessment

Throughout this lesson, observe students to assess how they are:

- representing composite numbers as a product of their prime factors
- using strategies such as factor trees to identify factors of composite numbers
- determining the greatest common factor for a variety of whole numbers
- determining the lowest common multiple for two and three whole numbers
- solving problems to connect their mathematical ideas to a conceptual understanding of factoring, greatest common factors and lowest common multiples
- communicating their mathematical ideas effectively

AS-B10: Problem Solving Processes Assessment Rubric to assess students' problem-solving strategies and skills.

AS-B20: Factor Assessment Rubric to assess students' understanding about factoring.

7AS 1.3: GCF and LCM Assessment Rubric to assess students' understanding about greatest common factors and lowest common multiples.

ASLS-1: Learning Skills and Work Habits Assessment Checklist - Collaboration to assess students' skills when working in small groups during the Problem Solving task.

ASLS-5 Learning Skills and Work Habits Assessment Checklist - Organization to assess individual students' contributions to solving problems either individually or in small groups.

SEL: Critical and Creative Thinking Reflecting Question to assess students making connections between the math that they learned in this lesson and everyday life contexts and applications.

Name: _____

Divisibility Rules

Divisor	Rule
2	All even numbers are divisible by 2. Even numbers end in the digits 0, 2, 4, 6 or 8.
3	A number is divisible by 3, if the sum of the digits is divisible by 3.
4	A number is divisible by 4, if the last two digits are divisible by 4.
5	A number is divisible by 5, if the last digit is either 0 or 5.
6	A number is divisible by 6, if it is divisible by 2 AND it is divisible by 3.
8	A number is divisible by 8, if the last three digits are divisible by 8.
9	A number is divisible by 9, if the sum of the digits is divisible by 9.
10	A number is divisible by 10, if the last digit is 0.

AS-B20: Factor Assessment Rubric

Name: _____ Date: _____

Categories	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding	Demonstrates limited understanding of the relationship between prime numbers and composite numbers	Demonstrates some understanding of the relationship between prime numbers and composite numbers	Demonstrates considerable understanding of the relationship between prime numbers and composite numbers	Demonstrates thorough understanding of the relationship between prime numbers and composite numbers
Thinking	Uses a variety of tools and strategies to decompose numbers to all of their factors with limited effectiveness	Uses a variety of tools and strategies to decompose numbers to all of their factors with some effectiveness	Uses a variety of tools and strategies to decompose numbers to all of their factors with considerable effectiveness	Uses a variety of tools and strategies to decompose numbers to all of their factors with a high degree of effectiveness
	Uses a factor tree to decompose a number to all of its prime number factors with limited effectiveness	Uses a factor tree to decompose a number to all of its prime number factors with some effectiveness	Uses a factor tree to decompose a number to all of its prime number factors with considerable effectiveness	Uses a factor tree to decompose a number to all of its prime number factors with a high degree of effectiveness
Communication	Uses conventions, vocabulary, and terminology related to factors with limited effectiveness	Uses conventions, vocabulary, and terminology related to factors with some effectiveness	Uses conventions, vocabulary, and terminology related to factors with considerable effectiveness	Uses conventions, vocabulary, and terminology related to factors with a high degree of effectiveness
Application	Identifies examples of where knowledge of factors can be applied to other contexts with limited effectiveness	Identifies examples of where knowledge of factors can be applied to other contexts with some effectiveness	Identifies examples of where knowledge of factors can be applied to other contexts with considerable effectiveness	Identifies examples of where knowledge of factors can be applied to other contexts with a high degree of effectiveness

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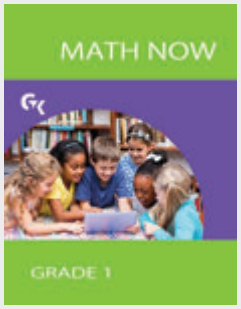

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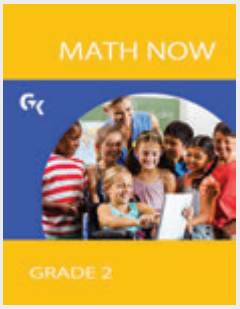

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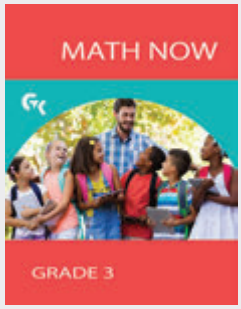

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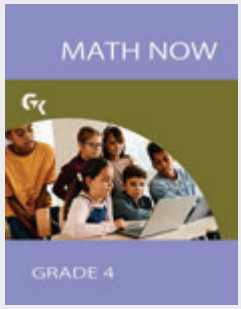

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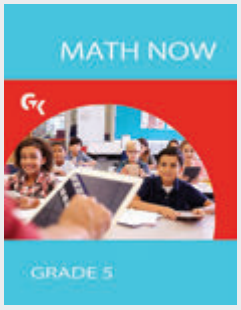

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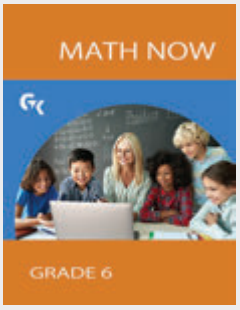

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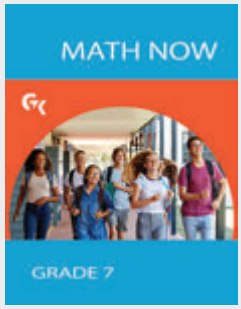

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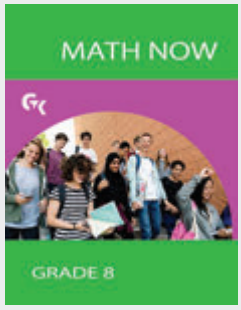

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