Math Mammoth
Grade 7-B Worktext

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Ratios and proportions

Percent

Geometry

Pythagorean theorem

Probability

Statistics

Light Blue Series

By Maria Miller

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Foreword

Math Mammoth Grade 7 comprises a complete math curriculum for the seventh grade mathematics studies. This is a pre-algebra course, so students can continue to an algebra 1 curriculum after studying this.

The curriculum meets and actually exceeds the Common Core Standards (CCS) for grade 7. The two major areas where it exceeds those standards are linear equations (chapter 5) and the Pythagorean Theorem (chapter 9). Linear equations are covered in more depth than the CCS requires, and the Pythagorean Theorem belongs to grade 8 in the CCS. You can access a document detailing the alignment information either on the Math Mammoth website or in the download version of this curriculum.

The main areas of study in Math Mammoth Grade 7 are:

- The basics of algebra (expressions, equations, inequalities, graphing);
- Integers;
- Ratios, proportions, and percent;
- Geometry;
- Probability and statistics.

This book, 7-B, covers ratios and proportions (chapter 6), percent (chapter 7), geometry (chapter 8), the Pythagorean Theorem (chapter 9), probability (chapter 10), and statistics (chapter 11). The rest of the topics are covered in the 7-A worktext.

Some important points to keep in mind when using the curriculum:

- The two books (parts A and B) are like a "framework", but you still have a lot of liberty in planning your student's studies. The five chapters in part 7-A are best studied in the order presented. However, you can study the chapters on geometry, probability, and statistics at most any point during the year. The chapters on ratios & proportions and percent (in part 7-B) are best left until the student has learned to solve one-step equations (in chapter 3).

  Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. However, you can still use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. This type of flexible use of the curriculum enables you to truly individualize the instruction for the student.

- Don't automatically assign all the exercises. Use your judgment, trying to assign just enough for your student's needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For review, the curriculum includes a worksheet maker (Internet access required), mixed review lessons, additional cumulative review lessons, and the word problems continually require usage of past concepts. Please see more information about review (and other topics) in the FAQ at
  https://www.mathmammoth.com/faq-lightblue.php

I heartily recommend that you view the full user guide for your grade level, available at
https://www.mathmammoth.com/userguides/

Lastly, you can find free videos matched to the curriculum at **https://www.mathmammoth.com/videos/**

I wish you success in teaching math!

Maria Miller, the author

Chapter 6: Ratios and Proportions
Introduction

Chapter 6 reviews the concept, which has already been presented in previous grades, of the ratio of two quantities. From this concept, we develop the related concepts of a rate (so much of one thing per so much of another thing) and a proportion (an equation of ratios). We also study how tables of equivalent ratios can help to solve problems with rates, and how cross-multiplying can help to solve problems with proportions.

The lesson *Unit Rates* defines the concept of the unit rate, shows how to calculate one, and gives practice at doing so, including practice with complex fractions. We also consider rates as two quantities that vary, graph the corresponding equation in the coordinate grid, and tie in the concept of unit rate with the concept of slope.

The concept of direct variation is introduced in the lesson *Proportional Relationships*. Writing and graphing equations gives a visual understanding of proportionality. In two following lessons, students also practice solving rate problems in different ways, using the various methods they have learned throughout the chapter.

The lessons *Scaling Figures, Floor Plans*, *and Maps* give useful applications and more practice to master the concepts of proportions.

Before the *Chapter Review* there is also an optional lesson, *Significant Digits*, that deals with the concept of the accuracy of a measurement and how it limits the accuracy of the solution. It is optional because significant digits is not a standard topic for seventh grade, yet the concept in it is quite important, especially in science.

Keep in mind that the specific lessons in the chapter can take several days to finish. They are not "daily lessons." Instead, use the general guideline that seventh graders should finish about 12 pages a week in order to finish the curriculum in about 40 weeks. Also, I recommend not assigning all the exercises by default, but that you use your judgment, and strive to vary the number of assigned exercises according to the student's needs.

Please see the user guide at https://www.mathmammoth.com/userguides/ for more guidance on using and pacing the curriculum.

There are free videos matched to the curriculum at https://www.mathmammoth.com/videos/ (choose 7th grade).

The Lessons in Chapter 6

Ratios and Rates

A **ratio** is a comparison of two numbers, or quantities, using division.

For example, to compare the hearts to the stars in the picture, we say that the ratio of hearts to stars is 5:10 (read "five to ten").

The two numbers in the ratio are called the **first term** and the **second term** of the ratio. The order in which these terms are mentioned does matter! For example, the ratio of stars to hearts is *not* the same as the ratio of hearts to stars. The former is 10:5 and the latter is 5:10.

We can write this ratio in several different ways:

- The ratio of hearts to stars is 5:10.
- The ratio of hearts to stars is 5 to 10.
- The ratio of hearts to stars is $\frac{5}{10}$.
- For every five hearts, there are ten stars.

Note that we are not comparing two numbers to determine which one is greater (as in 5 < 10). The comparison is relative as in a multiplication problem. For example, the ratio 5:10 can be simplified to 1:2, and it indicates to us that there are twice as many stars as there are hearts.

We **simplify ratios** in exactly the same way we simplify fractions.

Example 1. In the picture at the right, the ratio of hearts to stars is 12:16. We can simplify that ratio to 6:8 and even further to 3:4. These three ratios (12:16, 6:8, and 3:4) are called **equivalent ratios**.

The ratio that is simplified to lowest terms, 3:4, tells us that for every three hearts, there are four stars.

1. Write the ratio and then simplify it to lowest terms.

   The ratio of triangles to diamonds is _____ : _____ = _____ : _____ .

   In this picture, there are _____ triangles to every _____ diamonds.

2. **a.** Draw a picture with pentagons and circles so that the ratio of pentagons to the total of all the shapes is 7:9.

   b. What is the ratio of circles to pentagons?

3. **a.** Draw a picture in which (1) there are three diamonds for every five triangles, and (2) there is a total of 9 diamonds.

   b. Write the ratio of all the diamonds to all the triangles, and simplify this ratio to lowest terms.

4. Write the equivalent ratios.

a. 5 to 45 = 1 to _____	**b.** 3 : _____ = 9 : 60	**c.** 280 : 420 = 2 : _____	**d.** $\frac{5}{13} = \frac{\square}{65}$

We can also form **ratios using quantities that have units**. If the units are the same, they cancel.

Example 2. Simplify the ratio 250 g : 1.5 kg.

First we convert 1 kg to grams and then simplify: $\dfrac{250 \text{ g}}{1.5 \text{ kg}} = \dfrac{250 \text{ g}}{1,500 \text{ g}} = \dfrac{250}{1,500} = \dfrac{1}{6}$.

5. Use a fraction line to write ratios of the given quantities as in the example. Then simplify the ratios.

a. 5 kg and 800 g $\dfrac{5 \text{ kg}}{800 \text{ g}} =$	**b.** 600 cm and 2.4 m
c. 1 gallon and 3 quarts	**d.** 3 ft 4 in and 1 ft 4 in

We can generally **convert ratios with decimals or fractions into ratios of whole numbers**.

Example 3. Because we can multiply both terms of the ratio by 10, $\dfrac{1.5 \text{ km}}{2 \text{ km}} = \dfrac{15 \text{ km}}{20 \text{ km}}$.

Then: $\dfrac{15 \text{ km}}{20 \text{ km}} = \dfrac{15}{20} = \dfrac{3}{4}$. So the ratio 1.5 km : 2 km is equal to 3:4.

You can also see that the ratio is 3:4 by noticing that both 1.5 km and 2 km are evenly divisible by 500 m.

Example 4. Simplify the ratio ¼ mile to 5 miles.

First, ¼ mi : 5 mi = ¼ : 5. Multiplying both terms of the ratio by 4, we get ¼ : 5 = 1:20.

6. Use a fraction line to write ratios of the given quantities. Then simplify the ratios to integers.

a. 5.6 km and 3.2 km	**b.** 0.02 m and 0.5 m
c. 1.25 m and 0.5 m	**d.** 1/2 L and 7 1/2 L
e. 1/2 mi and 3 1/2 mi	**f.** 2/3 km and 1 km

Chapter 7: Percent
Introduction

In this chapter we review the concept of percent as "per hundred" or as hundredth parts and how to convert between fractions, decimals, and percents. The lesson *Solving Basic Percentage Problems* is intended for review of sixth grade topics, focusing on finding a known percentage of a number (such as 21% of 56) or finding a percentage when you know the part and the total.

We take a little different perspective of these concepts in the lesson *Percent Equations*. Students write simple equations for situations where a price increases or decreases (discounts). This lesson also explains what a percent proportion is. Personally, I prefer *not* to use percent proportion but to write the percentage as a decimal and then write an equation. I feel that approach adapts better to solving complex problems than using percent proportion.

Here is a quick example to show the difference between the two methods. Let's say an item is discounted by 22% and it now costs $28. Then, the new price is 78% of the original. If we let p be the price of the item before the discount, we can write the percent proportion $28/p = 78/100$ and solve for p. If, we write the percentage 78% as the decimal 0.78, we get the equation $0.78p = \$28$. Personally, I consider percent proportion to be an optional topic, and the reason I have included it here is to make this curriculum fully meet the Common Core Standards for seventh grade.

The lesson *Circle Graphs* provides students a break from new concepts and allows them to apply the concept of percent in a somewhat familiar context. Next, we delve into the percentage of change. Students sometimes view the percentage of change as a totally different concept as compared to other percentage topics, but it is not that at all. To calculate the percentage of change, we still use the fundamental idea of *percentage = part/total*, only this time, the "part" is how much the quantity in question changes (the difference) and the "total" is the original quantity.

Tying in with percentage of change, students also learn to compare values using percentages, such as how many percent more or less one thing is than another. Once again, this is not really a new concept but is based on the familiar formula *percentage = part/total*. The percentage difference (or relative difference) is the fraction (*actual difference*)/(*reference value*).

Simple Interest is a lesson on the important topic of interest, using as a context both loans and savings accounts. Students learn to use the formula $I = prt$ in a great variety of problems and situations.

The text concludes with a review lesson of all of the concepts taught in the chapter.

You can find matching videos for topics in this chapter at http://www.mathmammoth.com/videos/ (grade 7).
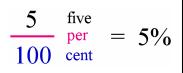
The Lessons in Chapter 7

Review: Percent

Percent (or **per cent**) means *per hundred* or "divided by a hundred."
(The word "cent" means one hundred.) So, simply put, percent means
a hundredth part.

To convert percentages into fractions, simply read the "per cent" as "per
100." Thinking of hundredths, you can also easily write them as decimals.

Therefore, 8% = 8 per cent = 8 per 100 = 8/100 = 0.08.

Similarly, 167% = 167 per 100 = 167/100 = 1.67.

$$\frac{5}{100} \quad \begin{array}{l} \text{five} \\ \text{per} \\ \text{cent} \end{array} = 5\%$$

1. Write as percentages, fractions, and decimals.

a. 52% = ▭ = _____	**b.** _____% = ▭ = 0.07	**c.** _____% = $\frac{59}{100}$ = _____
d. 109% = ▭ = _____	**e.** _____% = $\frac{382}{100}$ = _____	**f.** 200% = ▭ = _____

A decimal number with two decimal digits is in hundredths, so it can easily
be written as a percentage. For example, 0.56 = 56%. But even if we have 3 or
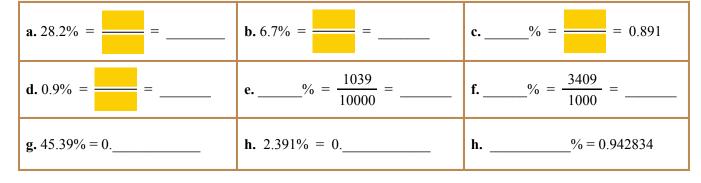more decimals, we can still convert into percent.

Example 1. The number 0.564 is 564 thousandths. As a percentage,
0.564 = 56.4%. Compare this to 0.56 = 56%. The decimal digit "4" that
follows the digits "56" is in the thousandths place, so it becomes 4 tenths
of a percent (56.**4**%).

$$0.091 = 9.1\% = \frac{91}{1000}$$

$$0.387 = 38.7\% = \frac{387}{1000}$$

This is how to convert percentages with even more decimal digits:

decimal	percentage		decimal	percentage
0.38429 =	38.429%		3.0281930 =	302.81930%

Think of it this way. Since 0.38 = 38%, any decimal digits that we have beyond 0.38 (the digits 429) simply
become decimal digits for the percentage. In effect, we move the decimal point two places to the right.

2. Write as percentages, fractions, and decimals.

a. 28.2% = ▭ = _____	**b.** 6.7% = ▭ = _____	**c.** _____% = ▭ = 0.891
d. 0.9% = ▭ = _____	**e.** _____% = $\frac{1039}{10000}$ = _____	**f.** _____% = $\frac{3409}{1000}$ = _____
g. 45.39% = 0._____	**h.** 2.391% = 0._____	**h.** _____% = 0.942834