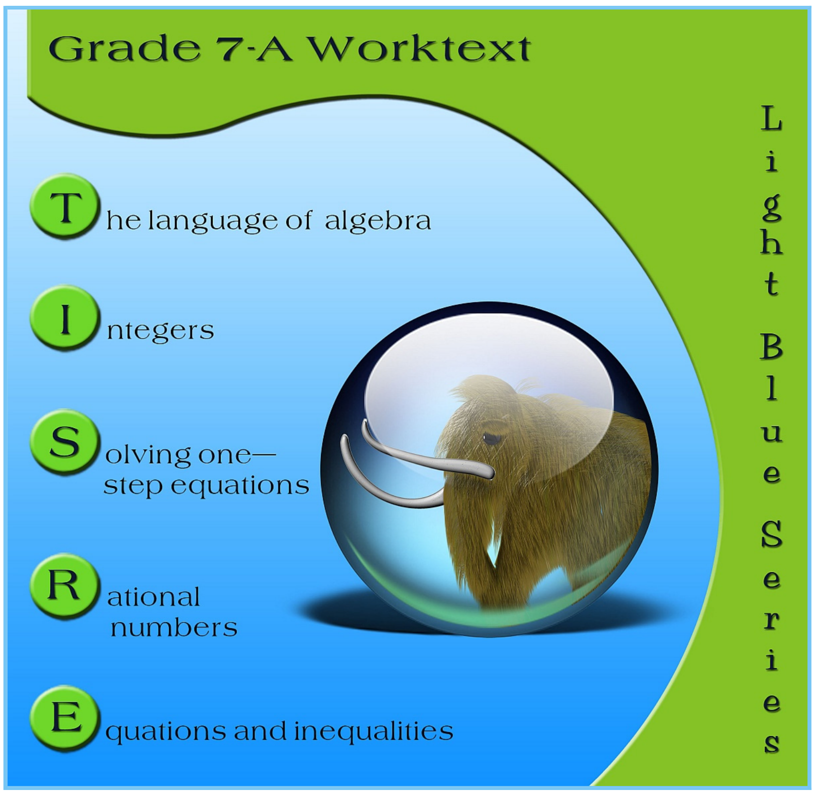
Math Mammoth
Grade 7-A Worktext



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Foreword

Math Mammoth Grade 7 comprises a complete math curriculum for the seventh grade mathematics studies. This is a pre-algebra course, so students can continue to an algebra 1 curriculum after studying this.

The curriculum meets and actually exceeds the Common Core Standards (CCS) for grade 7. The two major areas where it exceeds those standards are linear equations (chapter 5) and the Pythagorean Theorem (chapter 9). Linear equations are covered in more depth than the CCS requires, and the Pythagorean Theorem belongs to grade 8 in the CCS. You can access a document detailing the alignment information either on the Math Mammoth website or in the download version of this curriculum.

The main areas of study in Math Mammoth Grade 7 are:

- The basics of algebra (expressions, equations, inequalities, graphing);
- Integers;
- Ratios, proportions, and percent;
- Geometry;
- Probability and statistics.

This book, 7-A, covers the language of algebra (chapter 1), integers (chapter 2), one-step equations (chapter 3), rational numbers (chapter 4), and equations and inequalities (chapter 5). The rest of the topics are covered in the 7-B worktext.

Some important points to keep in mind when using the curriculum:

- The two books (parts A and B) are like a "framework", but you still have a lot of liberty in planning your student's studies. The five chapters in part 7-A are best studied in the order presented. However, you can study the chapters on geometry, probability, and statistics at most any point during the year. The chapters on ratios & proportions and percent (in part 7-B) are best left until the student has learned to solve one-step equations (in chapter 3).

  Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. However, you can still use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. This type of flexible use of the curriculum enables you to truly individualize the instruction for the student.

- Don't automatically assign all the exercises. Use your judgment, trying to assign just enough for your student's needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For review, the curriculum includes a worksheet maker (Internet access required), mixed review lessons, additional cumulative review lessons, and the word problems continually require usage of past concepts. Please see more information about review (and other topics) in the FAQ at **https://www.mathmammoth.com/faq-lightblue.php**

I heartily recommend that you view the full user guide for your grade level, available at **https://www.mathmammoth.com/userguides/**

Lastly, you can find free videos matched to the curriculum at **https://www.mathmammoth.com/videos/**

I wish you success in teaching math!

Maria Miller, the author

The Order of Operations

Let's review! Exponents are a shorthand for writing repeated multiplications by the same number.

For example, $0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9$ is written 0.9^5.

The tiny raised number is called the **exponent**.
It tells us how many times the **base** number is multiplied by itself.

exponent

$$12^4 = 12 \times 12 \times 12 \times 12$$
$$= 20{,}736$$

base

The expression 2^5 is read as "two to the fifth power," "two to the fifth," or "two raised to the fifth power."

Similarly, 0.7^8 is read as "seven tenths to the eighth power" or "zero point seven to the eighth."

The "powers of 6" are simply expressions where 6 is raised to some power: for example, 6^3, 6^4, 6^{45}, and 6^{99} are powers of 6.

Expressions with the exponent 2 are usually read as something "**squared.**" For example, 11^2 is read as "eleven squared." That is because it gives us the area of a square with sides 11 units long.

Similarly, if the exponent is 3, the expression is usually read using the word "**cubed.**" For example, 1.5^3 is read as "one point five cubed" because it is the volume of a cube with an edge 1.5 units long.

1. Evaluate.

a. 4^3 **b.** 10^5 **c.** 0.1^2

d. 0.2^3 **e.** 1^{100} **f.** 100 cubed

2. Write these expressions using exponents. Find their values.

a. $0 \cdot 0 \cdot 0 \cdot 0 \cdot 0$ **b.** $0.9 \cdot 0.9$

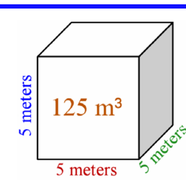
c. $5 \cdot 5 \cdot 5 \;+\; 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

d. $6 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \;-\; 9 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

The expression $(5\text{ m})^3$ means that we multiply 5 meters by itself three times:

$5\text{ m} \cdot 5\text{ m} \cdot 5\text{ m} = 125\text{ m}^3$

Notice that $(5\text{ m})^3$ is different from 5 m^3. The latter has no parentheses, so the exponent (the little 3) applies only to the unit "m" and not to the whole quantity 5 m.



3. Find the value of the expressions.

a. $(2\text{ cm})^3$ **b.** $(11\text{ ft})^2$ **c.** $(1.2\text{ km})^2$

4. Which expression from the right matches with (a) and (b) below?

a. The volume of a cube with sides 2 cm long.

b. The volume of a cube with sides 8 cm long.

(i) 8 cm^3 **(ii)** $(8\text{ cm})^3$ **(iii)** 2 cm^3

The Order of Operations (PEMDAS)	
1) Solve what is within parentheses **(P)**. 2) Solve exponents **(E)**. 3) Solve multiplication **(M)** and division **(D)** from left to right. 4) Solve addition **(A)** and subtraction **(S)** from left to right.	You can remember PEMDAS with the silly mnemonic *Please Excuse My Dear Aunt Sally*. Or make up your own!

5. Find the value of these expressions.

a. $120 - (9 - 4)^2$ **b.** $120 - 9 - 4^2$	**c.** $4 \cdot 5^2$ **d.** $(4 \cdot 5)^2$	**e.** $10 \cdot 2^3 \cdot 5^2$ **f.** $10 + 2^3 \cdot 5^2$
g. $(0.2 + 0.3)^2 \cdot (5 - 5)^4$	**h.** $0.7 \cdot (1 - 0.3)^2$	**i.** $20 + (2 \cdot 6 + 3)^2$

Example 1. Solve $(10 - (5 - 2))^2$.

First solve what is within the *inner* parentheses: $5 - 2 = 3$. We get $(10 - 3)^2$.

The rest is easy: $(10 - 3)^2 = 7^2 = 49$.

Example 2. Simplify $2 + \frac{1+5}{6^2}$.

Remember, the fraction line works like parentheses as a grouping symbol, grouping both what is above the line and also what is below it. First solve 1 + 5, then the exponent.

$$2 + \frac{1+5}{6^2}$$

$$= 2 + \frac{6}{6^2} = 2 + \frac{1}{6} = 2\frac{1}{6}$$

6. Find the value of these expressions.

a. $(12 - (9 - 4)) \cdot 5$	**c.** $(10 - (8 - 5))^2$
b. $12 - (9 - (4 + 2))$	**d.** $3 \cdot (2 - (1 - 0.4))$

7. Find the value of these expressions.

a. $\frac{4 \cdot 5}{2} \cdot \frac{9}{3}$	**b.** $\frac{4 \cdot 5}{2} + \frac{9}{3}$	**c.** $\frac{4 + 5}{2} + \frac{9}{3 - 1}$

Chapter 2: Integers
Introduction

This chapter deals with integers, which are signed (positive or negative) whole numbers. We begin with a review of addition and subtraction of integers from 6th grade. Then we study in detail multiplication and division of integers and conclude with negative fractions and the order of operations.

The first lesson reviews the concepts of integers, absolute value, the opposite of an integer, and simple inequalities on a number line. The next lessons present the addition and subtraction of integers through two visual models: first as movements on a number line, and then using positive and negative counters. These lessons also endeavor to connect the addition and subtraction of integers with various situations from real life.

The lesson *Subtraction of Integers* includes this important principle: Any subtraction can be converted into an addition (of the number with the opposite sign) and vice versa. This principle allows us to calculate not only subtractions such as $5 - (-7)$ but also problems that contain both addition and subtraction. These mixed problems become simple sums after the subtractions have been converted into additions. Converting subtractions into additions or vice versa is also important when simplifying expressions. For example, $5 + (-x)$ can be simplified to $5 - x$.

Next, we study the distance between two integers. This can be found by taking the absolute value of their difference: the distance between x and y is $|x - y|$. Students learn to use this formula to find distances between integers, and they also compare the result the formula gives to the answer they get by logical thinking.

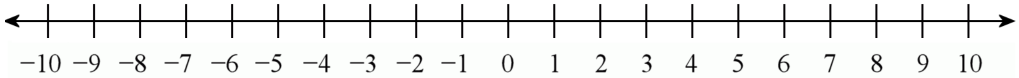
The lesson *Multiplying Integers* not only teaches the mechanics of how to multiply integers but also gives both intuitive understanding and formal justification for the shortcut, "a negative times a negative makes a positive." This formal justification using the distributive property introduces and illustrates the type of careful and precise reasoning that mathematicians use in proofs.

The next lesson, on the division of integers, leads naturally into the topic of negative fractions in the following lesson. The final topic is a review of the order of operations: we perform several operations at a time with integers.

You can find matching videos for the topics in this chapter at
http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#integers

The Lessons in Chapter 2

Integers



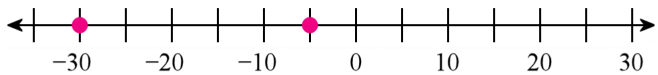
The **counting numbers** are 1, 2, 3, 4, 5, and so on. They work for addition. But counting numbers do not allow us to perform all possible subtractions; for example, the answer to the problem 2 − 7 is not any of them. That is when we come to the *negatives* of the counting numbers: −1, −2, −3, −4, −5, and so on.

Together with zero, all these form the set of **integers**: {..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ...}.
Note: Zero is neither positive nor negative.

Read −1 as "negative one" and −5 as "negative five." Some people read −5 as "minus five." That is very common, and it is not wrong, but be sure that you do not confuse it with subtraction.

Often, we need to put parentheses around negative numbers in order to avoid confusion with other symbols. Therefore, $^{-}5$, −5, and (−5) all mean "negative five."

Which is more, −30 or −5?



Which is *warmer*, −30°C or −5°C? Clearly −5°C is. Temperatures get colder and colder the more they move towards the negative numbers. We can write a comparison: −30°C < −5°C.

Similarly, we can write −\$500 < −\$200 to signify that to owe \$500 is a worse situation than to owe \$200.

Or, in elevation, −15 m > − 50 m means that 15 m below sea level is higher than 50 m below sea level.

1. Write comparisons using > , < , and integers. Don't forget to include the units!

a. The temperature at 5 AM was 12°C below zero.
Now, at 9 AM, it is 8°C below zero.

b. I owe my mom \$12, and my sister owes her \$25.

c. The bottom of the Challenger Deep trench is 11,033 m below sea level.
Mt. Everest reaches to a height of 8,848 m.

d. The total electric charge of five electrons is −5*e*.
The total electric charge of 5 protons is +5*e*.
(The symbol *e* means elementary charge, or a charge of a single proton.)

e. Dean has \$55, whereas Jack owes \$15.

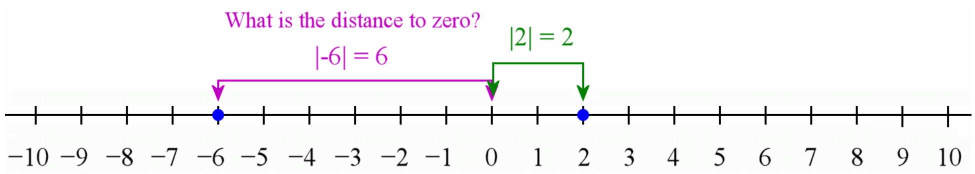
2. Which integer is ...

a. 3 more than −7 **b.** 8 more than −3 **c.** 7 less than 2 **d.** 5 less than −11

3. Write the numbers in order from the least to the greatest.

a. −5 −56 51 −15	**b.** 3 0 −31 −13

The **absolute value** of a number is its distance to zero.



We denote the absolute value of a number by putting vertical bars on either side of it.
So $|-4|$ means “the absolute value of 4,” which is 4. Similarly, $|87| = 87$. In an equation we treat the absolute value bars like parentheses and solve them first.

Example 1. Simplify $|-4| - |1|$. First simplify the absolute values. We get $4 - 1 = 3$.

Let’s say someone’s account balance is −$1,000. That person is in debt. The absolute value of the debt is written as |−$1000| and means that the *size* of the debt is $1,000.

If a diver is at a depth of −22 m, the absolute value |−22 m| tells us how far he is from the surface (22 m).

4. Simplify.

a. $|-11|$	**b.** $|+7|$	**c.** $|0|$	**d.** $|-46|$
e. $|-5| + |-2|$		**f.** $|-5| - |2|$	
g. $|-5| + |-2| + |8|$		**h.** $|5| + |-2| - |-1|$	

5. Answer. Use absolute values to calculate your answers.

a. What is the distance between −153 and zero on a number line?

b. What is the distance between *x* and zero on a number line?

c. What is the distance between −11 and 21 on a number line?

6. Interpret the absolute value in each situation.

a. A fishing net is at the depth of 15 feet. |−15 ft| = _______ft

Here, the absolute value shows ____

b. The temperature is −5°C. |−5°C | = _______°C

Here, the absolute value shows ____

c. Eric’s balance is −$7. |−$7| = $_______

Here, the absolute value shows ____

d. A point is drawn in the coordinate grid at (0, −11) |−11| = _______

Here, the absolute value shows ____