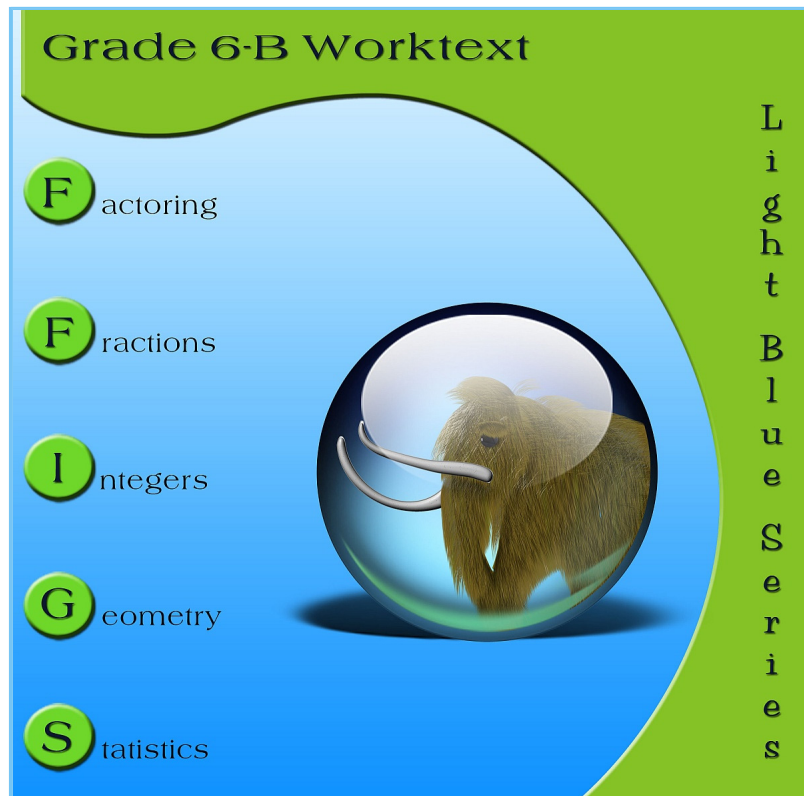


Math Mammoth

Grade 6-B Worktext



By Maria Miller

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The Sieve of Eratosthenes and Prime Factorization

To find all the prime numbers less than 100 we can use the *sieve of Eratosthenes*.

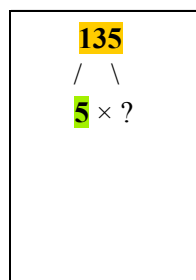
1. Cross out 1, as it is not considered a prime.
2. Cross out all the even numbers except 2.
3. Cross out all the multiples of 3 except 3.
4. You do not have to check multiples of 4. Why?
5. Cross out all the multiples of 5 except 5.
6. You do not have to check multiples of 6. Why?
7. Cross out all the multiples of 7 except 7.
8. You do not have to check multiples of 8 or 9 or 10.
9. The numbers left are **primes**. The numbers you crossed out are **composite numbers**.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

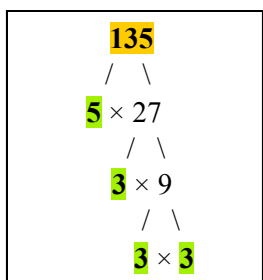
List the primes you found: 2, 3, 5, 7,

Why do you not have to check numbers that are bigger than 10? Let's think about multiples of 11. The following multiples of 11 have already been crossed out: 2×11 , 3×11 , 4×11 , 5×11 , 6×11 , 7×11 , 8×11 , and 9×11 . The multiples of 11 that have not been crossed out are 10×11 and onward... but they are not on our chart! Similarly, the multiples of 13 that are less than 100 are 2×13 , 3×13 , ..., 7×13 , and all of those have already been crossed out when you crossed out multiples of 2, 3, 5, and 7.

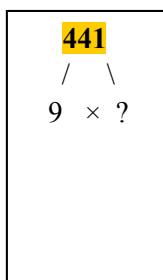
Use the various divisibility tests when building a factor tree for a composite number.



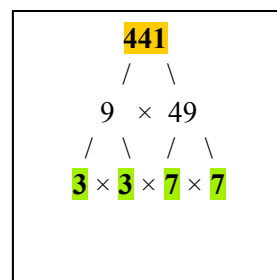
$$\begin{array}{r} 27 \\ 5 \overline{)135} \\ \underline{-10} \\ 35 \\ \underline{-35} \\ 0 \end{array}$$



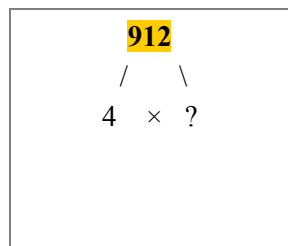
We start out by noticing that 135 is **divisible by 5**. From long division, we know that $135 = 5 \times 27$. The final factorization is $135 = 3 \times 3 \times 3 \times 5$ or $3^3 \times 5$.



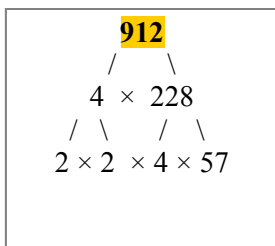
$$\begin{array}{r} 49 \\ 9 \overline{)441} \\ \underline{-36} \\ 81 \\ \underline{-81} \\ 0 \end{array}$$



Adding the digits of 441, we get 9, so it is **divisible by 9**. We divide to get $441 = 9 \times 49$. The end result is $441 = 3 \times 3 \times 7 \times 7$ or $3^2 \times 7^2$.

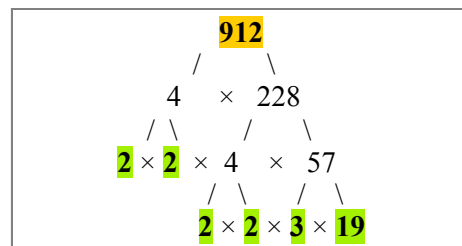


$$\begin{array}{r} 228 \\ 4 \overline{)912} \\ \underline{-8} \\ 11 \\ \underline{-8} \\ 32 \\ \underline{-32} \\ 0 \end{array}$$



The last two digits of 912 are "12" so it is **divisible by 4**.

228, too, is **divisible by 4** (its last digits are "28").



Lastly, 57 is 3×19 . The prime factorization of 912 is $2^4 \times 3 \times 19$.

1. Find the prime factorization of these composite numbers. Use a notebook for long divisions.

<p>a. 124</p> $\begin{array}{r} / \quad \backslash \\ 2 \times \underline{\quad} \\ / \quad \backslash \end{array}$	<p>b. 260</p> $\begin{array}{r} / \quad \backslash \\ 10 \times \underline{\quad} \\ / \quad \backslash \quad / \quad \backslash \end{array}$	<p>c. 96</p> $\begin{array}{r} / \quad \backslash \\ 3 \times \underline{\quad} \\ / \quad \backslash \end{array}$
<p>d. 90</p>	<p>e. 165</p>	<p>f. 95</p>
<p>g. 80</p>	<p>h. 240</p>	<p>i. 272</p>
<p>j. 76</p>	<p>k. 126</p>	<p>l. 104</p>

Chapter 7: Fractions

Introduction

This chapter begins with a review of fraction arithmetic from fifth grade—specifically, addition, subtraction, simplification, and multiplication of fractions. Then it focuses on the new topic: division of fractions.

The introductory lesson on the division of fractions presents the concept of reciprocal numbers and ties the reciprocity relationship to the idea that division is the appropriate operation to solve questions of the form, “How many times does this number fit into that number?” For example, we can write a division from the question, “How many times does $1/3$ fit into 1?” The answer is, obviously, 3 times. So we can write the division $1 \div (1/3) = 3$ and the multiplication $3 \times (1/3) = 1$. These two numbers, $3/1$ and $1/3$, are reciprocal numbers because their product is 1.

Students learn to solve questions like that through using visual models and writing division sentences that match them. The eventual goal is to arrive at the shortcut for fraction division—that each division can be changed into a multiplication by taking the reciprocal of the divisor, which is often called the “invert (flip)-and-multiply” rule.

However, that “rule” is just a shortcut. It is necessary to memorize it, but memorizing a shortcut doesn’t help students make sense conceptually out of the division of fractions—they also need to study the concept of division and use visual models to better understand the process involved.

In two lessons that follow, students apply what they have learned to solve problems involving fractions or fractional parts. A lot of the problems in these lessons are review in the sense that they involve previously learned concepts and are similar to problems students have solved earlier, but many involve the division of fractions, thus incorporating the new concept presented in this chapter.

Consider mixing the lessons from this chapter (or from some other chapter) with the lessons from the geometry chapter (which is a fairly long chapter). For example, the student could study these topics and geometry on alternate days, or study a little from both each day. Such, somewhat spiral, usage of the curriculum can help prevent boredom, and also to help students retain the concepts better.

The Lessons in Chapter 7

	page	span
Review: Add and Subtract Fractions and Mixed Numbers	34	4 pages
Add and Subtract Fractions: More Practice	38	3 pages
Review: Multiplying Fractions 1	41	3 pages
Review: Multiplying Fractions 2	44	3 pages
Dividing Fractions: Reciprocal Numbers	47	5 pages
Divide Fractions	52	4 pages
Problem Solving with Fractions 1	56	3 pages
Problem Solving with Fractions 2	59	3 pages
Mixed Review Chapter 7	62	2 pages
Fractions Review	64	3 pages

Review: Add and Subtract Fractions and Mixed Numbers

Example 1. Add $\frac{5}{6} + 2\frac{5}{8}$.

We need to convert unlike fractions into equivalent fractions that have a common denominator before we can add them. The common denominator must be a **multiple of both 6 and 8** (a *common* multiple).

Naturally, $6 \times 8 = 48$ is one common multiple of 6 and 8. We could use 48. However, it is better to use 24, which is the *least* common multiple (LCM) of 6 and 8, because it leads to easier calculations.

The common denominator is 24:

$$\begin{array}{r} \frac{5}{6} + 2\frac{5}{8} \\ \downarrow \quad \downarrow \\ \frac{20}{24} + 2\frac{15}{24} = 2\frac{35}{24} = 3\frac{11}{24} \end{array}$$

1. Write the missing addition sentences.

<p>a.</p>	<p>b. Common denominator 36</p> $\frac{3}{4} + \frac{1}{9}$	<p>c. Common denominator 20</p> $\frac{7}{10} + \frac{1}{4}$
------------------	--	---

2. Find a common denominator (c.d.) that will work for adding or subtracting these fractions. Remember that the *best* possible choice for the common denominator (but not the only one) is the LCM of the denominators.

Fractions	c.d.
a. $\frac{5}{16}$ and $\frac{1}{6}$	
d. $\frac{1}{12}$ and $\frac{2}{5}$	

Fractions	c.d.
b. $\frac{1}{12}$ and $\frac{4}{9}$	
e. $\frac{11}{15}$ and $\frac{13}{20}$	

Fractions	c.d.
c. $\frac{5}{6}$ and $\frac{3}{8}$	
f. $\frac{45}{100}$ and $\frac{9}{20}$	

3. Add and subtract. Use the common denominator you found in the previous exercise.

a. $\frac{5}{16} + \frac{1}{6}$	b. $3\frac{1}{12} + 1\frac{4}{9}$	c. $\frac{5}{6} - \frac{3}{8}$
d. $2\frac{5}{12} + \frac{4}{5}$	e. $5\frac{11}{15} - 2\frac{3}{20}$	f. $\frac{45}{100} + \frac{9}{20}$

Regroup in subtraction, if necessary.

Example 2. Here we regroup **one** as 13/13. This leaves 9 wholes. There is already 1/13 in the column of the fractional parts, so in total we get 14/13.

$$\begin{array}{r} 9 \frac{14}{13} \\ - 5 \frac{5}{13} \\ \hline 4 \frac{9}{13} \end{array}$$

We can use the same idea (regrouping) when the fractions are written horizontally.

Example 3. Take one of the 7 wholes, think of it as 9/9, and regroup that with the fractional parts (with 2/9). Instead of 7 wholes, we are left with 6, and instead of 2/9, we get 11/9.

$$\begin{array}{r} 7 \frac{2}{9} - 3 \frac{8}{9} \\ \downarrow \quad \downarrow \\ 6 \frac{11}{9} - 3 \frac{8}{9} = 3 \frac{3}{9} \end{array}$$

4. Subtract.

<p>a.</p> $\begin{array}{r} 7 \frac{3}{9} \\ - 2 \frac{7}{9} \\ \hline \end{array}$	<p>b.</p> $\begin{array}{r} 18 \frac{1}{10} \\ - 5 \frac{9}{10} \\ \hline \end{array}$	<p>c.</p> $\begin{array}{r} 10 \frac{1}{15} \\ - 3 \frac{8}{15} \\ \hline \end{array}$	<p>d.</p> $16 \frac{3}{9} - 9 \frac{8}{9}$
			<p>e.</p> $7 \frac{3}{14} - 2 \frac{10}{14}$

5. Subtract. First write equivalent fractions with the same denominator.

<p>a.</p> $\begin{array}{r} 3 \frac{3}{4} \\ - 1 \frac{1}{6} \\ \hline \end{array} \rightarrow \begin{array}{r} 3 \frac{\text{yellow}}{\text{yellow}} \\ - 1 \frac{\text{yellow}}{\text{yellow}} \\ \hline \end{array}$	<p>b.</p> $\begin{array}{r} 3 \frac{3}{8} \\ - 1 \frac{5}{12} \\ \hline \end{array} \rightarrow \begin{array}{r} \\ - \\ \hline \end{array}$	<p>c.</p> $\begin{array}{r} 8 \frac{9}{11} \\ - 5 \frac{1}{2} \\ \hline \end{array} \rightarrow \begin{array}{r} \\ - \\ \hline \end{array}$
--	---	---

6. Figure out and explain how these subtractions were done!

Emma's way:

$$9 \frac{2}{17} - 3 \frac{8}{17}$$

$$= (9 - 3) + \left(\frac{2}{17} - \frac{8}{17} \right) = 6 - \frac{6}{17} = 5 \frac{11}{17}$$

Joe's method:

$$5 \frac{3}{14} - 2 \frac{9}{14}$$

$$\downarrow$$

$$5 \frac{3}{14} - 2 \frac{3}{14} - \frac{6}{14}$$

$$= 3 - \frac{6}{14} = 2 \frac{8}{14}$$

Chapter 8: Integers

Introduction

In chapter 8, students are introduced to integers, the coordinate plane in all four quadrants, and integer addition and subtraction. The multiplication and division of integers will be studied in seventh grade.

Integers are introduced using the number line to relate them to the concepts of temperature, elevation, and money. We also study briefly the ideas of absolute value (an integer's distance from zero) and the opposite of a number.

Next, students learn to locate points in all four quadrants and how the coordinates of a figure change when it is reflected across the x or y -axis. Students also move points according to given instructions and find distances between points with the same first coordinate or the same second coordinate.

Adding and subtracting integers is presented through two main models: (1) movements along the number line and (2) positive and negative counters. With the help of these models, students should not only learn the shortcuts, or "rules," for adding and subtracting integers, but also understand *why* these shortcuts work.

A lesson about subtracting integers explains the shortcut for subtracting a negative integer from three different viewpoints (as a manipulation of counters, as movements on a number line, and as a distance or difference). There is also a roundup lesson for addition and subtraction of integers.

Note: Addition and subtraction of integers are not included in the Common Core standards for sixth grade. I have included them because I feel students are ready to study them, at least to some extent, at the same time as they study the concepts of integers, absolute value, and ordering integers. In seventh grade, we will study all operations with integers.

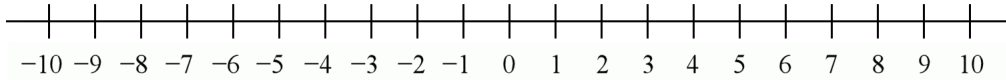
The last topic in this chapter is graphing. Students will plot points on the coordinate grid according to a given equation in two variables (such as $y = x + 2$), this time using also negative numbers. They will notice the patterns in the coordinates of the points and the pattern in the points drawn in the grid and also work through some real-life problems.

The Lessons in Chapter 8

	page	span
Integers	71	3 pages
Coordinate Grid	74	4 pages
Coordinate Grid Practice	78	3 pages
Addition and Subtraction as Movements	81	3 pages
Adding Integers: Counters	84	3 pages
Subtracting a Negative Integer	87	2 pages
Add and Subtract Roundup	89	4 pages
Graphing	91	4 pages
Mixed Review Chapter 8	95	2 pages
Integers Review	97	3 pages

Integers

When we continue the number line towards the left from zero, we come to the **negative numbers**.



The **negative whole numbers** are $-1, -2, -3, -4$, and so on.

The **positive whole numbers** are $1, 2, 3, 4$, and so on. You can also write them as $+1, +2, +3$, etc.

Zero is neither positive nor negative.

All of the negative and positive whole numbers and zero are called **integers**.

Read -1 as “negative one” and -5 as “negative five.” Some people read -5 as “minus five.” That is very common, and it is not wrong, but be sure that you do not confuse it with subtraction.

Put a “ $-$ ” sign in front of negative numbers. This sign can also be elevated: $\bar{5}$ is the same as -5 .

Often, we need to put parentheses around negative numbers in order to avoid confusion with other symbols. Therefore, $\bar{5}$, -5 , and (-5) all mean “negative five.”

Negative numbers are commonly used with temperature. They are also used to express debt. If you owe \$5, you write that as $-\$5$. Another use is with elevation below sea level. For example, just as 200 m can mean an elevation of 200 meters above sea level, -100 m would mean 100 meters *below* sea level.

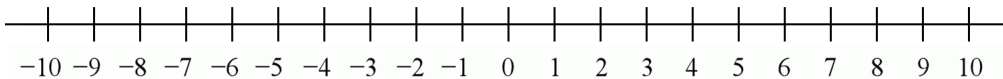
1. Plot the integers on the number line.

a. -7

b. $+6$

c. -4

d. -2



2. Write an integer appropriate to each situation.

a. Daniel owes \$23.

b. Mary earned \$250.

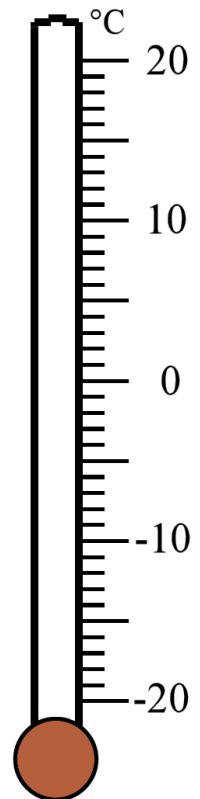
c. The airplane flew at the altitude of 8,800 meters.

d. The temperature in the freezer is 18 degrees Celsius below zero.

e. A dolphin dove 30 ft below sea level.

3. The temperature changed from what it was before. Find the new temperature. You can draw the mercury on the thermometer to help you.

before	1°C	2°C	-2°C	-4°C	-12°C	-8°C
change	drops 3°C	drops 7°C	drops 1°C	rises 5°C	rises 4°C	rises 3°C
now						



Chapter 9: Geometry

Introduction

The focus topics of this chapter are:

- the area of triangles
- the area of polygons
- nets and the surface area of prisms and pyramids
- the volume of prisms with sides of fractional length

However, the chapter starts out with some review topics from earlier grades. We review the different types of quadrilaterals and then students do some basic drawing exercises. In these drawing problems, students will need a ruler to measure lengths and a protractor to measure angles.

One focus of the chapter is the area of polygons. To reach this goal, we follow a step-by-step development. First, we study how to find the area of a right triangle, which is very easy, as a right triangle is always half of a rectangle. Next, we build on the idea that the area of a parallelogram is the same as the area of the related rectangle, and from that we develop the usual formula for the area of a parallelogram as the product of its base times its height. This formula then gives us a way to generalize finding the area of any triangle as *half* of the area of the corresponding parallelogram.

Finally, the area of a polygon can be determined by dividing it into triangles and rectangles, finding the areas of those, and summing them up. Students also practice their new skills in the context of a coordinate grid. They draw polygons in the coordinate plane and find the lengths of their sides, perimeters, and areas.

Nets and surface area is another major topic. Students draw nets and determine the surface area of prisms and pyramids using nets. They also learn how to convert between different area units, not using conversion factors or formulas, but using logical reasoning where they learn to determine those conversion factors themselves.

Lastly, we study the volume of rectangular prisms, this time with edges of fractional length. (Students have already studied this topic in fifth grade for prisms with edges that are a whole number long.) The basic idea is to prove that the volume of a rectangular prism *can* be calculated by multiplying its edge lengths even when the edges have fractional lengths. To that end, students need to think how many little cubes with edges $\frac{1}{2}$ or $\frac{1}{3}$ unit go into a larger prism. Once we have established the formula for volume, students solve some problems concerning the volume of rectangular prisms.

There are quite a few videos available to match the lessons in this chapter at <https://www.mathmammoth.com/videos/> (choose 6th grade).

The Lessons in Chapter 9

	page	span
Quadrilaterals Review	104	2 pages
Drawing Problems	106	2 pages
Area of Right Triangles	108	2 pages
Area of Parallelograms	110	3 pages
Area of Triangles	113	3 pages
Area of Polygons	116	3 pages
Polygons in the Coordinate Grid	119	3 pages
Area and Perimeter Problems	122	2 pages

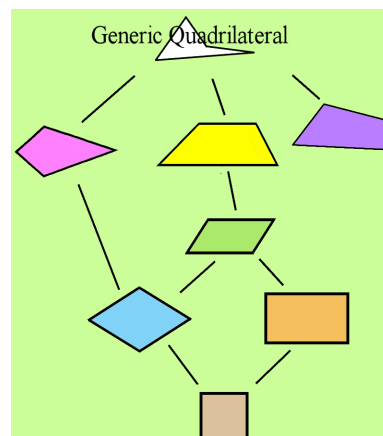
Quadrilaterals Review

The chart shows the seven different types of quadrilaterals as a “family,” descending from the generic quadrilateral at the top.

If a quadrilateral is listed under another, it means the two have like a “parent-child” relationship: the quadrilateral listed lower (the child) has its parent’s characteristics.

Number in the chart the following types of quadrilaterals:

- | | |
|--------------|--------------------------|
| 1. rhombus | 5. scalene quadrilateral |
| 2. kite | 6. trapezoid |
| 3. rectangle | 7. parallelogram |
| 4. square | |



1. Find the correct type of quadrilateral for each definition.

- A quadrilateral with four congruent sides.
- A quadrilateral where the opposite sides are parallel.
- A quadrilateral with one pair of parallel sides.
- A quadrilateral with two pairs of congruent sides, where each pair of congruent sides are touching.
- A quadrilateral with four congruent sides and four right angles.

2. Think of the “parent-child” relationships as shown in the chart, and answer the questions:

- Is a rhombus also a kite?
- Is a rectangle also a parallelogram?
- Is a trapezoid also a rectangle?

3. Reflect the parallelograms in the coordinate grid.

<p>a.</p> <p style="text-align: center;">Reflect the parallelogram in the x-axis.</p>	<p>b.</p> <p style="text-align: center;">Reflect the parallelogram in the y-axis.</p>	<p>c.</p> <p style="text-align: center;">Draw your own parallelogram. Then reflect it both in the x-axis and in the y-axis.</p>
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