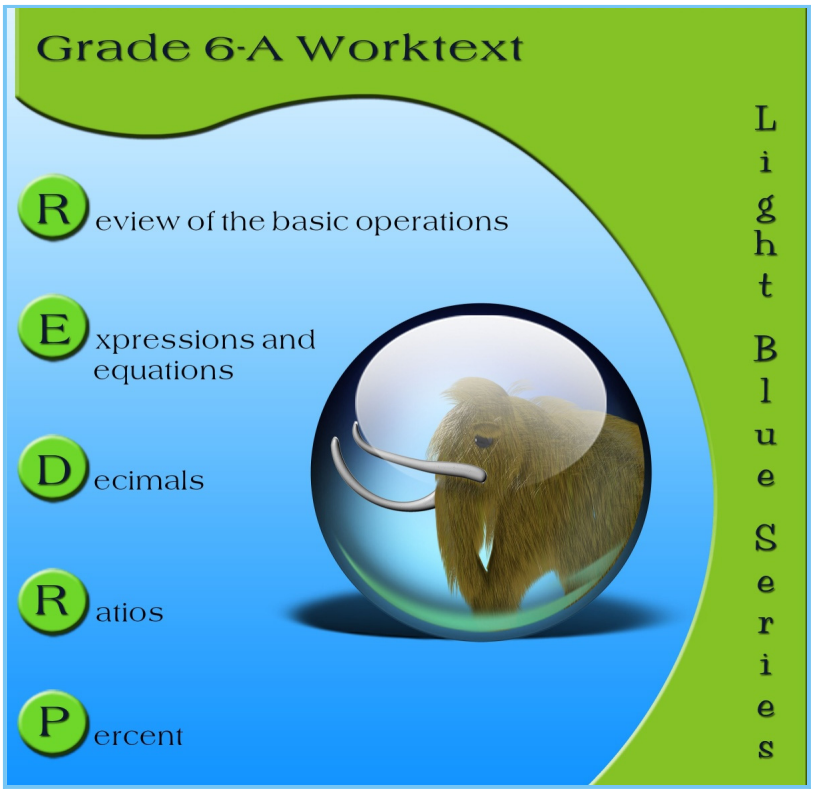
Math Mammoth
Grade 6-A Worktext



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Foreword

Math Mammoth Grade 6 comprises a complete math curriculum for the sixth grade mathematics studies. The curriculum meets and exceeds the Common Core standards.

In sixth grade, we have quite a few topics to study. Students are familiar with some of them, such as fractions and decimals, but many others are introduced for the first time (e.g. exponents, ratios, percent, integers). The main areas of study in Math Mammoth Grade 6 are:

- An introduction to several algebraic concepts, such as exponents, expressions, and equations;
- Rational numbers: fractions, decimals, and percents;
- Ratios, rates, and problem solving using bar models;
- Geometry: area, volume, and surface area;
- Integers and graphing;
- Statistics: students learn to describe distributions using measures of center and variability.

This book, 6-A, covers a review of the four operations, plus exponents (chapter 1), expressions and equations (chapter 2), decimals (chapter 3), ratios (chapter 4), and percent (chapter 5). The rest of the topics are covered in the 6-B worktext.

Some important points to keep in mind when using the curriculum:

- The two books (parts A and B) are like a "framework", but you still have a lot of liberty in planning your child's studies. For the most part, the chapters in the 6th grade curriculum don't have to be studied in the order presented, but you can choose, for example, to study integers before decimals, or statistics right after ratios.

  Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. However, you can still use it in a *spiral* manner, if you prefer. Simply have your child study in 2-3 chapters simultaneously. This type of flexible use of the curriculum enables you to truly individualize the instruction for your child.

- Don't automatically assign all the exercises. Use your judgment, trying to assign just enough for your child's needs. You can use the skipped exercises later for review. For most children, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For review, the curriculum includes a worksheet maker (Internet access required), mixed review lessons, additional cumulative review lessons, and the word problems continually require usage of past concepts. Please see more information about review (and other topics) in the FAQ at **https://www.mathmammoth.com/faq-lightblue.php**

I heartily recommend that you view the full user guide for your grade level, available at **https://www.mathmammoth.com/userguides/**

Lastly, you can find free videos matched to the curriculum at **https://www.mathmammoth.com/videos/**

I wish you success in teaching math!

Maria Miller, the author

Warm-up: Mental Math

To **multiply** 2,000 × 120, simply multiply 2 × 12, and "tag" four zeros on the answer: 2000 × 120 = 240,000	Solve **division** by thinking of multiplication "backwards": 5,600 ÷ 70 = ? Think what number times 70 will give you 5,600. Since 70 × 80 = 5,600, then 5,600 ÷ 70 = 80.	You can **add in parts**. 76 + 120 + 65 = ? First add 70 + 120 + 60 = 250. Then, 6 + 5 = 11. Lastly, 250 + 11 = 261.
The **order of operations** is: 1. Parenthesis; 2. Exponents; 3. Multiplication and division; 4. Addition and subtraction.		
To calculate 9 × 80 − 10 × 70, first solve 9 × 80 and 10 × 70. Subtract only after that. 9 × 80 − 10 × 70 = 720 − 700 = 20	In the expression 4,500 ÷ (5 + 45) × 80, solve 5 + 45 first. Then, divide. 4,500 ÷ (5 + 45) × 80 = 4,500 ÷ 50 × 80 = 90 × 80 = 7,200	

1. Solve using mental math.

a. 410 + 2 × 19 =	**b.** 3 × 50 + 4 × 150 =	**c.** 70 × 80 − 40 × 50 =
d. 14 + (530 − 440) =	**e.** 45 + 56 + 35 =	**f.** 300 ÷ 5 − 400 ÷ 10 =

2. Solve using mental math.

a. 17 + _____ = 110	**b.** 345 + _____ = 1,000	**c.** 3 × 40 + _____ = 500

3. Divide. Remember that division can be written using a fraction line as well.

a. $\frac{240}{4}$ = **c.** $\frac{72}{9}$ = **e.** $\frac{5{,}600}{10}$ = **g.** $\frac{420}{20}$ = **i.** $\frac{420}{70}$ =

b. $\frac{7{,}200}{100}$ = **d.** $\frac{450}{9}$ = **f.** $\frac{8{,}000}{200}$ = **h.** $\frac{10{,}000}{50}$ = **j.** $\frac{7{,}200}{800}$ =

4. Solve. Notice carefully which operation(s) are done first.

a. $500 - 40 - 3 \times 50 =$ _____	**b.** $1{,}020 - (40 - 10) \times 20 =$ _____
c. $42{,}000 - 12{,}000 + 3 \times 5{,}000 =$ _____	**d.** $(70 - 20) \times 70 =$ _____
e. $\frac{210}{2} + 3 \times 15 =$ _____	**f.** $250 \times 4 + \frac{6{,}300}{70} =$ _____

5. Find a number that fits in place of the unknown.

a. $x \div 70 = 40$	**b.** $20 \times M = 1200$	**c.** $500 - y = 320$

6. Find the rule that is used in the table and fill in the missing numbers.

n	130	250	360	410	775	820	1,000
$n -$ ____		215		375			

7. Find the rule that is used in the table and fill in the missing numbers.

n	3	5	12	15	25	35	60
		200			1,000		

8. Rick cut off a 50-cm piece from his 6-m board, and then he divided the rest of the board into five equal pieces. How long was each piece?

9. **a.** Eve works 8 hours a day and earns $104 daily. What is her hourly wage?

b. How much does Eve earn in a five-day work week?

How much does she earn in 3 months (which is 13 weeks)?

10. Alexis and Mia baked cookies for a bake sale. They used this recipe, but they needed to triple it:

2 1/4 cups of flour
1/3 cup of honey
1/2 cup of butter
3/4 teaspoon of nutmeg
1 1/2 teaspoons of cinnamon
1/2 teaspoon of ground cloves
3/4 cup of walnuts
Makes 2 1/2 dozen cookies.

a. Triple the recipe for them.

b. How many cookies did they bake?

Chapter 2: Expressions and Equations

Introduction

In this chapter we concentrate on two important concepts: expressions and equations. We also touch on inequalities and graphing on a very introductory level. In order to make the learning of these concepts easier, the expressions and equations in this chapter do not involve negative numbers (as they typically do when studied in pre-algebra and algebra). The study of negative numbers is in part 6-B.

We start out by learning some basic vocabulary used to describe mathematical expressions verbally—terms such as the sum, the difference, the product, the quotient, and the quantity. Next, we study the order of operations once again. A lot of this lesson is review. The lesson *Multiplying and Dividing in Parts* is also partially review and leads up to the lesson on distributive property that follows later.

Then, we get into studying expressions in definite terms: students encounter the exact definition of an expression, a variable, and a formula, and practice writing expressions in many different ways.

The concepts of equivalent expressions and simplifying expressions are important. If you can simplify an expression in some way, the new expression you get is equivalent to the first. We study these ideas first using lengths— it is a concrete example, and hopefully easy to grasp.

In the lesson *More On Writing and Simplifying Expressions* students encounter more terminology: term, coefficient, and constant. In exercise #3, they write an expression for the perimeter of some shapes in two ways. This exercise is once again preparing them to understand the distributive property.

Next, students write and simplify expressions for the area of rectangles and rectangular shapes. Then we study the distributive property, concentrating on the symbolic aspect and tying it in with area models.

The next topic is equations. Students learn some basics, such as, the solutions of an equation are the values of the variables that make the equation true. They use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. I have also included a few two-step equations as an optional topic.

Lastly, students get to solve and graph simple inequalities, and study the usage of two variables and graphing.

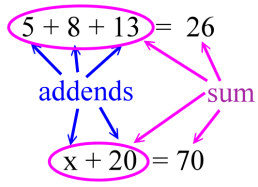
You will find free videos covering many topics of this chapter of the curriculum at https://www.mathmammoth.com/videos/ (choose 6th grade).

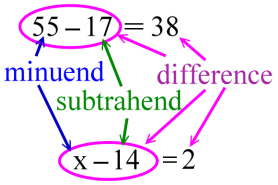
The Lessons in Chapter 2

	page	span
Terminology for the Four Operations	40	*4 pages*
Order of Operations	44	*3 pages*
Multiplying and Dividing in Parts	47	*4 pages*
Expressions	51	*3 pages*
Writing and Simplifying Expressions 1: Length and Perimeter	54	*3 pages*
More on Writing and Simplifying Expressions	57	*3 pages*
Writing and Simplifying Expressions 2: Area	60	*5 pages*
The Distributive Property	65	*4 pages*
Equations	69	*4 pages*
More Equations	73	*4 pages*

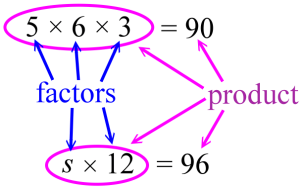
Terminology for the Four Operations

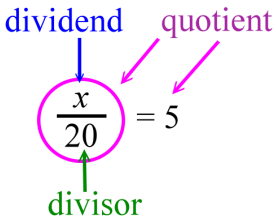
Study carefully the illustrations below to learn the terminology used with the four operations.





The **m**inuend comes before the **s**ubtrahend, just like "M" comes before "S" in the alphabet.





Example 1. The sum of 5, 6, and z is written as $5 + 6 + z$.

Example 2. The expression 1 + 100 + 13 is a sum, even though it has not been calculated yet. You can call it the *sum written*, and the answer 114 you can call the sum that has been *solved* or *calculated*.

Example 3. Similarly, the expression 55 − 40 is a *difference*. Specifically, it is the difference of 55 and 40. The answer, 15, is also called the difference. You can differentiate between the two by saying that 55 − 40 is the *difference written*, and the answer 15 is the difference *solved*.

Example 4. The difference of x and 15 is written as $x - 15$, *not* the other way around.

Example 5. The product of 5, x, and y is written as $5 \cdot x \cdot y$ or as $5xy$. We can use a raised dot · for the multiplication symbol, because × can be confused with the letter x (such as $x \times y$). Normally, however, the multiplication symbol is simply *omitted* between letters (ax) or between a number and a letter ($5x$).

Example 6. The quotient of x and 4 is written as $\frac{x}{4}$ or as $x \div 4$. It is far more common (and recommendable) to use the fraction line and not the symbol ÷ when we involve letters (variables).

Example 7. The quotient of 6 and y is written as $\frac{6}{y}$, not $\frac{y}{6}$.

1. Fill in the table, calculating the sum, difference, product, and quotient of the numbers.

numbers	sum	difference	product	quotient
a. 50 and 2				
b. 5 and 3				

2. *Write* the sum, difference, product, and quotient of the numbers and letters (you *cannot* calculate anything).

numbers/letters	sum	difference	product	quotient
a. x and 6				
b. z and w				

3. Match the expressions.

a. the product of 5 and T	$5 - T$	29	**e.** the difference of T and 5
b. the quotient of 5 and T	$T - 5$	$\frac{T}{5}$	**f.** the sum of T and 5
c. the product of 6 and 5	30	$5T$	**g.** the sum of 5, 15, and 9
d. the quotient of T and 5	$5 \div T$	$T + 5$	**h.** the difference of 5 and T

4. Write an equation, and find the part that is not given.

Statement	Equation
a. The quotient is 5, the divisor is 8, the dividend is _____.	_____ ÷ _____ = _____
b. The subtrahend is _____, the difference is 15, and the minuend is 45.	
c. The factors are 5, 6, and 8, and the product is _____.	
d. The addends are 7, 8, and _____, and the sum is 100.	

5. **a.** Write a multiplication problem, using three factors, where the product is 0.

b. Write a division problem where the dividend is 120 and the quotient is less than 15.

c. Using four addends, write a sum that is less than 9.

In these two problems, the *minuend* is unknown:	_____ − 56 = 67	$x - 400 = 1{,}200$
To find it, use "addition backwards." This means you *add* the difference and the subtrahend.	_____ − 56 = 67 **Add:** 67 + 56 = 123. Solution: **123** − 56 = 67.	$x - 400 = 1{,}200$ **Add:** 1,200 + 400 = 1,600, Solution: **1600** − 400 = 1,200

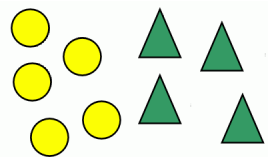
6. Find a strategy that always works for finding an unknown *subtrahend*. Use it to solve these two problems.

a. 56 − _____ = 19

b. 4,203 − x = 553

Ratios and Rates

A **ratio** is simply a *comparison* of two numbers or other quantities.



To compare the circles to the triangles in the picture, we say that the *ratio of circles to triangles* is 5:4 (read "five to four").

We can write this ratio (in text) in many different ways:

- The ratio of circles to triangles is 5:4 (read "5 to 4").
- The ratio of circles to triangles is 5 to 4.
- The ratio of circles to triangles is $\frac{5}{4}$.
- For each five circles, there are four triangles.

The two numbers in the ratio are called the **first term** and the **second term** of the ratio.

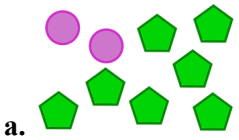
In this picture, the ratio of males to females is 4:3. However, the ratio of *females to males* is **3:4**. The order in which the terms are mentioned does matter!

We can also compare a part to the whole. The ratio of males to everyone is 4:7.

Moreover, you can use fractions to describe the same image: $\frac{4}{7}$ of the people are males, and $\frac{3}{7}$ are females.

1. Describe the images using ratios and fractions.

a.



The ratio of circles to pentagons is ____ : ____

The ratio of pentagons to all shapes is ____ : ____

$\frac{\quad}{\quad}$ of the shapes are pentagons.

b.

The ratio of hearts to stars is ____ : ____

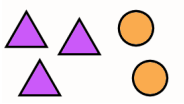
The ratio of stars to all shapes is ____ : ____

$\frac{\quad}{\quad}$ of the shapes are stars.

2. **a.** Draw a picture: There are hearts and circles, and the ratio of hearts to all the shapes is 1:3.

b. What is the ratio of hearts to circles?

3. Look at the picture of the triangles and circles. If we drew more triangles and circles in the same ratio, how many circles would there be …



a. … for 9 triangles?

b. … for 15 triangles?

c. … for 300 triangles?

Chapter 5: Percent
Introduction

The concept of percent builds on the student's understanding of fractions and decimals. Specifically, students should be very familiar with the idea of finding a fractional part of a whole (such as finding 3/4 of $240). Students who have used Math Mammoth have been practicing that concept since 4th grade. One reason why I have emphasized finding a fractional part of a whole so much in the earlier grades is specifically to lay a groundwork for the concept of percent. Assuming the student has mastered how to find a fractional part a whole, and can easily convert fractions to decimals, then studying the concept of percent should not be difficult.

The first lesson, *Percent*, practices the concept of percent as a hundredth part and how to write fractions and decimals as percentages. Next, we study how to find a percentage when the part and the whole are given (for example, if 15 out of 25 club members are girls, what percentage of them are girls?).

The following two lessons have to do with finding a certain percentage of a given number or quantity. First, we study how to do that using mental math techniques. For example, students find 10% of $400 by dividing $400 by 10. Next, students find a percentage of a quantity using decimal multiplication, both manually and with a calculator. For example, students find 17% of 45 km by multiplying 0.17×45 km.

I prefer teaching students to calculate percentages of quantities using decimals, instead of using percent proportion or some other method (such as changing 17% into the fraction 17/100 for calculations). That is because using decimals is simpler: we simply change the percentage into a decimal and multiply, instead of having to build a proportion or use fractions. Also, decimals will be so much easier to use later on when solving word problems that require the usage of equations.

Next is a lesson about discounts, which is an important application from everyday life. Then we go on to the lesson *Practice with Percent*, which contrasts the two types of problems students have already studied: questions that ask to calculate a given percentage of a number and questions that ask to find the percentage. For example, the first type of question could be "*What is 70% of $380?*" and the second type could be "*What percentage is $70 of $380?*"

The last lesson lets students find the total when the percentage and the partial amount are known. For example: "Three-hundred twenty students, which is 40% of all students, take PE. How many students are there in total?" We solve these with the help of bar models.

As a reminder, it is not recommended that you assign all the exercises by default. Use your judgment, and strive to vary the number of assigned exercises according to the student's needs. Some students might only need half of even less of the available exercises, in order to understand the concepts.

I have made several videos to match these lessons: **http://www.mathmammoth.com/videos/percent.php**

The Lessons in Chapter 5

	page	span
Percent	170	*4 pages*
What Percentage...?	174	*2 pages*
Percentage of a Number (Mental Math)	176	*3 pages*
Percentage of a Number: Using Decimals	179	*3 pages*
Discounts	182	*2 pages*
Practice with Percent	184	*3 pages*
Finding the Total When the Percent Is Known	187	*2 pages*
Mixed Review Chapter 5	189	*2 pages*
Review: Percent	191	*2 pages*

Percent

Percent (or **per cent**) means *per hundred* or "divided by a hundred."
That is because the word "cent" means one hundred.

The symbol for percent is **%** .

When you divide by 100, you get one hundredth (1/100).
Therefore, 8% means 8 per 100, which is 8/100.
Similarly, 67% means 67 divided by 100, or 67/100.

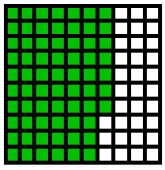
$$\frac{5}{100} \begin{matrix}\text{five}\\ \text{per}\\ \text{cent}\end{matrix} = 5\%$$

Since percentages are just hundredths, we can very easily write them as fractions and as decimals.

$$63\% = \frac{63}{100} = 0.63 \qquad 9\% = \frac{9}{100} = 0.09$$

1. Write the shaded part and the unshaded part as fractions, as decimals, and as percentages.

a.



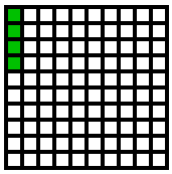
shaded

$\square = \_\_\_\_\_ = \_\_\_\_\_ \%$

unshaded

$\square = \_\_\_\_\_ = \_\_\_\_\_ \%$

b.



shaded

$\square = \_\_\_\_\_ = \_\_\_\_\_ \%$

unshaded

$\square = \_\_\_\_\_ = \_\_\_\_\_ \%$

2. Write as percentages, fractions, and decimals.

a. $28\% = \frac{28}{100} = 0.28$	**b.** $17\% = \square = \_\_\_\_\_$	**c.** $\_\_\_\_\_\% = \square = 0.89$
d. $60\% = \square = \_\_\_\_\_$	**e.** $\_\_\_\_\_\% = \frac{5}{100} = \_\_\_\_\_$	**f.** $\_\_\_\_\_\% = \square = 0.08$

3. Typically, seven out of every 100 babies born in the River Creek Hospital have a birth defect, most of them minor defects.

a. What typical percentage of the babies have birth defects?

b. What typical percentage of the babies do *not* have birth defects?

c. About how many babies with birth defects would you expect to find in a group of 500 babies?

d. About how many babies with birth defects would you expect to find in a group of 1,300 babies?