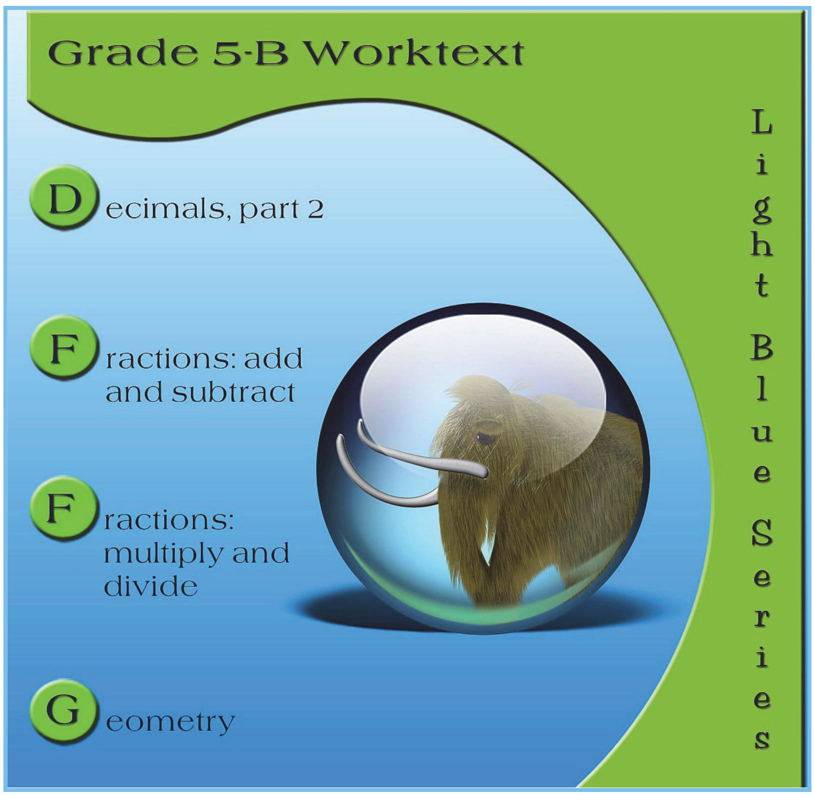
Math Mammoth
Grade 5-B Worktext



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Foreword

Math Mammoth Grade 5-B Worktext covers the second half of fifth grade mathematics studies. In part 5-A, students have studied the four operations with whole numbers, large numbers, problem solving, decimal arithmetic, and statistical graphs. In this part, 5-B, we study more about decimals, a lot about fractions and fraction arithmetic, and geometry.

Chapter 6 continues our study of decimals. The focus is on multiplying decimals by decimals, dividing decimals by decimals, and conversions between measuring units.

Chapter 7 covers the addition and subtraction of fractions—another topic of focus for 5th grade, besides decimals. The most difficult topic of this chapter is adding and subtracting unlike fractions, which is done by first converting them to equivalent fractions with a common denominator.

In chapter 8, we study the multiplication and division of fractions from various angles.

Chapter 9 takes us to geometry, starting with a review of angles and polygons. From there, students will learn to draw circles, to classify triangles and quadrilaterals, and the concept of volume in the context of right rectangular prisms (boxes).

I wish you success with teaching math!

Maria Miller, the author

Chapter 6: Decimals, Part 2
Introduction

In this chapter, we focus on decimal multiplication and division, and conversions between measurement units.

We start out with the topic of multiplying decimals by decimals. This is typically a fairly easy topic, as long as students remember the rule concerning the decimal digits in the answer. This rule could be confused with the other rules of decimal arithmetic that we also study in this chapter. In 6th grade, I provide a proof for the rule using fraction multiplication. I didn't include it here, because the chapter already contains so many new topics for students that including the justification for the rule may just cause an overload, plus, students haven't studied fraction multiplication yet.

Then we learn about multiplication as *scaling*. We cannot view decimal multiplications, such as 0.4×1.2, as repeated addition. Instead, they are viewed as scaling—shrinking or enlarging—the number or quantity by a scaling factor. So, 0.4×1.2 is thought of as scaling 1.2 by 0.4, or as four-tenths *of* 1.2. You may recognize this as the same as 40% of 1.2.

Next, we learn about decimal divisions that can be done with mental math. Students divide decimals by whole numbers (such as $0.8 \div 4$ or $0.45 \div 4$) by relating them to equal sharing. They divide decimals by decimals in situations where the divisor goes evenly into the dividend, thus yielding a whole-number quotient (e.g. $0.9 \div 0.3$ or $0.072 \div 0.008$).

In the next lesson, *More Division with Decimals*, we simply review long division with decimals, where the divisor is a whole number.

The following topic is multiplying and dividing decimals by powers of ten. This is presented with the help of place value charts. The actual concept is that the number being multiplied or divided *moves* in the place value chart, as many places as there are zeros in the power of ten. As a shortcut, we can move the decimal point. However, the movement of the decimal point is an “illusion”—that is what seems to happen—but in reality, the number itself got bigger or smaller; thus, its digits actually changed positions in the place value system.

Next, we study the metric system and how to convert various metric units (within the metric system), such as converting kilograms to grams, or dekaliters to hectoliters. The first of the two lessons mainly deals with very commonly used metric units, and we use the meaning of the prefix to do the conversion. For example, centimeter is a hundredth part of a meter, since the prefix “centi” means 1/100. Knowing that, gives us a means of converting between centimeters and meters.

The second lesson deals with more metric units, even those not commonly used, such as dekaliters and hectograms, and teaches a method for conversions using a chart. These two methods for converting measuring units within the metric system are sensible and intuitive, and help students not to rely on mechanical formulas.

Next, we turn our attention to dividing decimals by decimals, which then completes our study of all decimal arithmetic. The principle here is fairly simple, but it is easy to forget (multiply both the dividend and the divisor by a power of ten, until you have a whole-number divisor).

After learning that, students practice measurement conversions within the customary system, rounding measurements, and some generic problem solving with decimals. Recall that not all students need all the exercises; use your judgment.

Multiply Decimals by Decimals

Multiplying decimals is easy! You simply multiply as if there were no decimal points. Then place the decimal point in the answer following this rule:

The answer will have **as many decimal places/digits** as there are, IN TOTAL, in all of the factors.

Example 1. 0.05×0.7

Multiply in your head: $5 \times 7 = 35$. The factor 0.05 has **two** and 0.7 has **one** decimal digit. The answer has to have **three**, so the answer is 0.035.

Example 2. $12 \times 2 \times 0.3 \times 0.2$

Multiply mentally: $12 \times 2 \times 3 \times 2 = 144$. The factors have 0, 0, 1, and 1 decimal digits—a total of 2. The answer has to have 2 decimal digits/places, so the answer is 1.44.

1. Multiply first as if there were no decimal points. Then add the decimal point to the answer.

a. $0.5 \times 0.3 =$ ________	**c.** $0.4 \times 0.08 =$ ________	**e.** $8 \times 0.3 =$ ________
b. $0.9 \times 0.6 =$ ________	**d.** $0.7 \times 0.02 =$ ________	**f.** $0.1 \times 2.7 =$ ________
g. $0.2 \times 0.1 =$ ________	**i.** $0.9 \times 0.01 =$ ________	**k.** $0.7 \times 0.3 =$ ________
h. $0.8 \times 0.1 =$ ________	**j.** $9 \times 0.06 =$ ________	**l.** $7 \times 0.03 =$ ________

The answer to a decimal multiplication may end in one or more *decimal* zeros. That is no problem. You may **simplify the final answer** by dropping the ending decimal zeros.

Example 3. To solve 50×0.006, first multiply in your head $50 \times 6 = 300$.

The factors (50 and 0.006) have 0 and 3 decimal places, so the answer will have **3**. Therefore, the answer is 0.300, but it *simplifies* to 0.3.

Example 4. To solve 400×0.05, we first multiply $400 \times 5 = 2000$. The factors have 0 and 2 decimal digits, so the answer will have **2**.

The answer is 20.00. You can simplify that to 20.

2. Multiply. Simplify your final answer.

a. $0.4 \times 0.5 =$ *0.20* = *0.2*	**e.** $40 \times 0.05 =$ ________ = ________
b. $20 \times 0.06 =$ ________ = ________	**f.** $0.6 \times 0.2 \times 0.5 =$ ________ = ________
c. $3 \times 0.2 \times 0.5 =$ ________ = ________	**g.** $600 \times 0.004 =$ ________ = ________
d. $300 \times 0.009 =$ ________ = ________	**h.** $0.4 \times 0.5 \times 60 =$ ________ = ________

The shortcut to decimal multiplication

1) Multiply as if there were no decimal points.

2) Place the decimal point in the answer. The **number of decimal digits** in the answer is the **SUM** of the number of decimal digits in the factors.

Example 5. To solve 0.81 × 2.5, multiply **as if it was 81 × 25**. In other words, ignore the decimal points. (Also, “081” is the same as 81 so we can ignore the zero, too.)

Since 0.81 has *two* decimal digits, and 2.5 has *one*, the answer will have *three*. The final answer is therefore 2.025.

$$\begin{array}{r} 8\ 1 \\ \times\ \ 2\ 5 \\ \hline 4\ 0\ 5 \\ +\ 1\ 6\ 2\ 0 \\ \hline 2\ 0\ 2\ 5 \end{array}$$

Does that make sense? We can estimate:

0.81 × 2.5 ≈ 0.8 × 3 = 2.4
or 0.81 × 2.5 ≈ 0.8 × 2.5 = 2.

Yes, a final answer of 2.025 makes sense, since it is close to our estimates.

Example 6. This time, we include the decimal points when writing calculating 1.49 × 0.7, but even so, we multiply **as if it was 149 × 7**. Imagine the decimal points are not there! And there is NO need to align the decimal points like in addition and subtraction.

The final answer has *three* decimal places, since the factors have two and one, respectively.

$$\begin{array}{rl} 3\ 6\ \ & \\ 1.4\ 9 & \leftarrow \textbf{two} \text{ decimal places} \\ \times\ \ 0.7 & \leftarrow \textbf{one} \text{ decimal place} \\ \hline 1.0\ 4\ 3 & \leftarrow \textbf{three} \text{ decimal places} \end{array}$$

Estimate: 1.49 × 0.7 ≈ 1.5 × 0.7 = 1.05.

If the estimate was *not* close to our final answer, there would probably be an error somewhere.

3. Solve with long multiplication. Also, estimate. Write the longer number on top.

a. 0.3 × 1.19

Estimate: ___________

b. 0.9 × 51.7

Estimate: ___________

c. 204.5 × 0.4

Estimate: ___________

d. 2.2 × 0.72

Estimate: ___________

e. 5.6 × 2.8

Estimate: ___________

f. 3.34 × 4.2

Estimate: ___________

Example 7. Strawberries cost $5.15/kg, and you buy 0.7 kg. What is the total cost?

If you were to buy 3 kilograms of strawberries, you would multiply 3 × $5.15 to find the total price. When you buy 0.7 kg, you do the SAME: **multiply the price by 0.7.** The total cost is **0.7 × $5.15 = 3.605** (see the multiplication on the right).

$$\begin{array}{r} {}^{1\ 3} \\ 5\ 1\ 5 \\ \times \quad 7 \\ \hline 3\ 6\ 0\ 5 \end{array}$$

Since this is a money amount, it needs rounded to two decimals: $3.61.

Note that the answer also makes sense: 0.7 kg of strawberries should cost less than 1 kg of strawberries, which was $5.15.

4. Find the total cost. Write a multiplication.

a. Ribbon costs $1.10 per meter, and you buy 0.4 meters.

Cost: ________ × ________ = _____________

b. Nuts cost $8 per pound. You buy 0.3 pounds.

Cost: ________ × ________ = _____________

c. A phone call costs $7 per hour. You talk for 1.2 hours.

Cost: ________ × ________ = _____________

d. Lace costs $2.20 per meter, and you buy 1.5 meters.

Cost: ________ × ________ = _____________

5. Lauren went to a farmer's market, and bought 0.4 kg of blueberries at $9.20/kg, 3.5 kg of potatoes at $1.20/kg, and 1.2 kg of carrots at 1.85/kg. Find the total cost.

6. You're trying to find out how much it costs to feed your pet parrot for a year.

You have figured that your parrot eats about 90 grams, which is 0.09 kg, of parrot feed in a day. Also, parrot feed costs $7.08/kg. So, what is the cost of feeding your parrot for a year (not just a day)? Round your answer to the nearest dollar.

Fraction Terminology

As we study fraction operations, it is important that you understand the terms, or words, that we use. This page is for reference. You can post it on your wall or even make your own fraction poster based on it. Some of the terms below you already know; some we will study in this chapter.

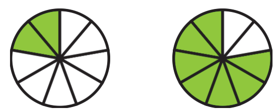
$\frac{3}{11}$ The top number is the **numerator**. It *enumerates,* or numbers (counts), *how many* pieces there are. The bottom number is the **denominator**. It *denominates,* or names, *what kind* of parts they are.

A mixed number has two parts: a whole-number part and a fractional part.

For example, $2\frac{3}{7}$ is a mixed number. Its whole-number part is 2, and its fractional part is $\frac{3}{7}$.

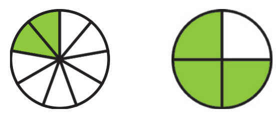
The mixed number $2\frac{3}{7}$ actually means $2 + \frac{3}{7}$.

Like fractions have the same denominator. They have the same kind of parts. It is easy to add and subtract like fractions, because all you have to do is look at *how many* of that kind of part there are.



$\frac{2}{9}$ and $\frac{7}{9}$ are like fractions.

Unlike fractions have a different denominator. They have different kinds of parts. It is a little more complicated to add and subtract unlike fractions. You need to first change them into like fractions. Then you can add or subtract them.

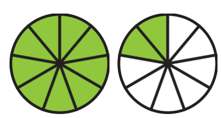


$\frac{2}{9}$ and $\frac{3}{4}$ are unlike fractions.

A proper fraction is a fraction that is less than 1 (less than a whole pie). 2/9 is a proper fraction.

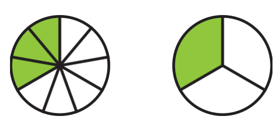
$\frac{2}{9}$ is a proper fraction.

An improper fraction is more than 1 (more than a whole pie). Being a *fraction*, it is written as a fraction and *not* as a mixed number.



$\frac{11}{9}$ is an improper fraction.

Equivalent fractions are equal in value. If you think in terms of pies, they have the same amount of "pie to eat," but they are written using different denominators, or are "cut into different kinds of slices."



$\frac{3}{9}$ and $\frac{1}{3}$ are equivalent fractions.

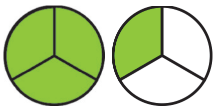
Simplifying or reducing a fraction means that, for a given fraction, you find an equivalent fraction that has a "simpler," or smaller, numerator and denominator. (It has fewer but bigger slices.)

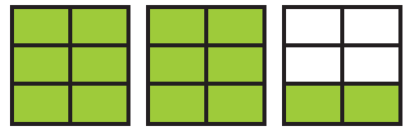
$\frac{9}{12}$ simplifies to $\frac{3}{4}$.

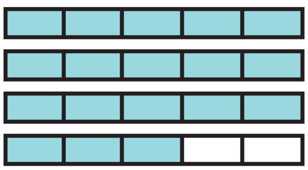
Review: Mixed Numbers

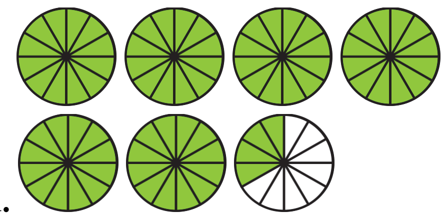
This lesson should be mostly review. However, please don’t go on to the lessons about adding and subtracting mixed numbers until you *thoroughly* understand the concepts in this lesson.

1. Write the mixed numbers that these pictures illustrate.

a. 

b. 

c. 

d. 

2. Draw pictures of “pies” that illustrate these mixed numbers.

a. $4\frac{2}{3}$

b. $2\frac{3}{5}$

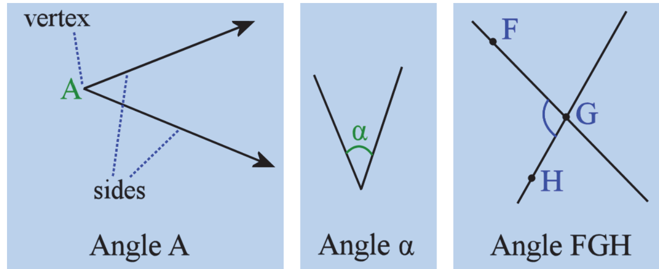
c. $3\frac{2}{6}$

d. $4\frac{7}{8}$

e. $6\frac{8}{10}$

Review: Angles

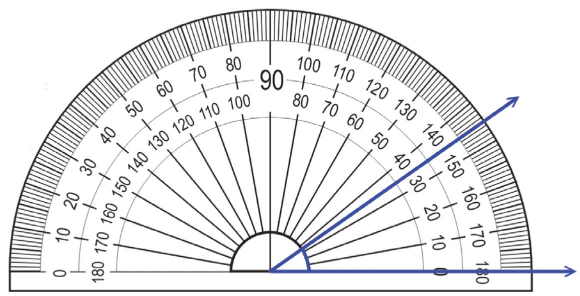
An angle is a figure formed by two **rays** that have the same beginning point. That point is called the **vertex**. The two rays are called the sides of the angle.



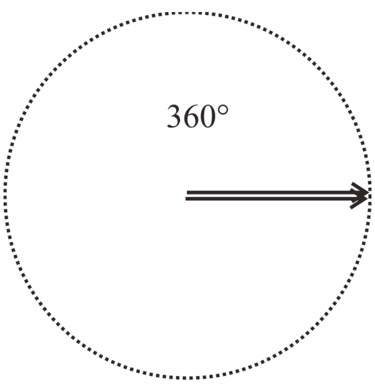
Imagine the two sides as being like two sticks that open up a certain amount. The more they open, the bigger the angle.

An angle can be named (1) after the vertex point, (2) with a letter inside the angle, or (3) using one point on the ray, the vertex point, and one point on the other ray.

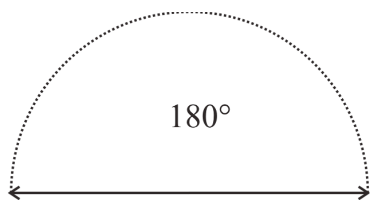
We measure angles in degrees. You can use a protractor like the one at the right to measure angles. The angle in blue measures 35 degrees.



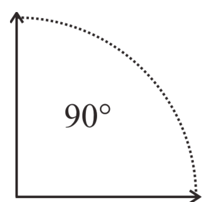
A full angle = 360°



A straight angle = 180°



A right angle = 90°

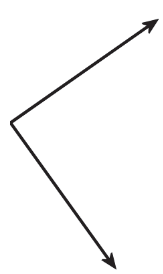


A zero angle = 0°

Angles that are more than 0° but less than 90° are called **acute** (“sharp”) angles.
Angles that are more than 90° but less than 180° are called **obtuse** (“dull”) angles.
(Angles that are more than 180° but less than 360° are called *reflex* angles.)

1. Continue the sides of these angles with a ruler.

   Then, measure them with a protractor.



a. ___________°

b. ___________°

2. In your notebook, draw:

   a. Any acute angle. Measure it. Label the angle as "An acute angle, xx°."

   b. Any obtuse angle. Measure it. Label the angle as "An obtuse angle, xx°."

3. Draw three dots on a blank paper and join them to form a triangle.
   Draw the dots far enough apart so that the triangle nearly fills the page.
   Then, measure the angles of your triangle.

   The angles of my triangle are: ________°, ________°, and ________°.

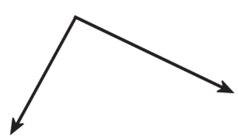
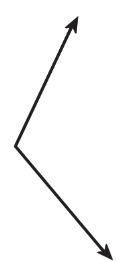
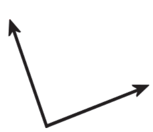
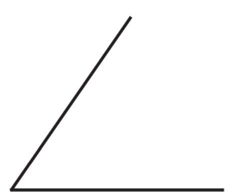
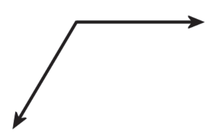
   What is the *sum* of these angle measures? __________°

4. Draw a horizontal line and mark a point on it. This point will be the vertex of an angle.
   Draw the other side of the angle from the vertex so that the angle measures 76°.

5. Follow the procedure above to draw acute angles with the following measures:
   a. 30° **b.** 60° **c.** 45°

6. Draw obtuse angles with the following measures:
   a. 135° **b.** 100° **c.** 150°

7. Now that you have drawn several angles, *estimate* the angle measure of these angles. Write down the estimates on the top lines. Then measure the angles, and write down the measures on the bottom lines. To measure the angles, you will need to continue their sides.

 **a.** ________° ________°	 **b.** ________° ________°	 **c.** ________° ________°
 **d.** ________° ________°	 **e.** ________° ________°	 **f.** ________° ________°

Important Terms	
• *an angle*	• *an acute angle*
• *a zero angle*	• *a right angle*
• *a straight angle*	• *an obtuse angle*