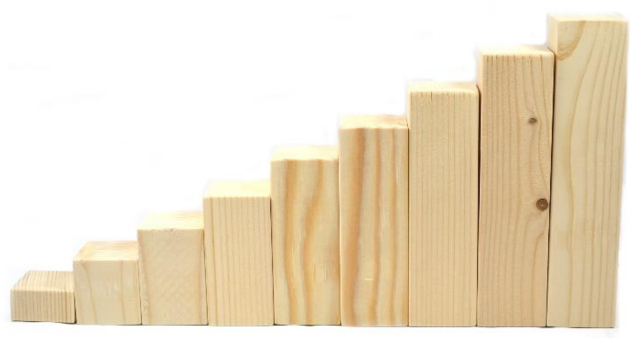
GARAGE PHYSICS

by eisco



# Magic Square Puzzle Kit

## GP00005

# Guide for Educators

We've designed this kit to enable an engaging back and forth for almost every age of child. Young children can use the kit to explore the natural numbers. Elementary school children can use them to investigate addition, subtraction, multiplication, as well as units of measure, and the associative property. They can also try to solve simplified versions of the magic square puzzle by trying to make two stacks, three stacks, or five stacks of the same height. Middle school students can use the puzzle to explore the magic square in depth (in which all rows, columns and the two diagonals should stack to the same height), as well as arithmetic sequences (including the first few elements of the Fibonacci sequence). Beyond working out the complete solution to the magic square, high school students can use the kit to explore counting in binary up to 15 and, in classroom settings, counting in ternary up to 26.

## Contents of Kit

**9 blocks of increasing height - Set of stickers for labeling the blocks**
**Ball-point pen**

# Background

**The magic square has been part of the mathematical history of nearly every major civilization, including China, Persia, Arabia, India, and Europe. The magic square is defined as a square grid of unique integers in which  the rows, columns, and diagonals all add to the same value. This block set allows children and students to explore the 3x3 magic square as well as other principles of mathematics both visually and kinesthetically.**

# Young Children

### Free exploration

Allow the child to simply play with the blocks, exploring different ways to stack them.

### Order the blocks

You can lead the child in this direction by giving the child two blocks and asking, “Which is bigger?” Or you can ask the child to see if they can make some stairs. Though seemingly simple, this is the first step for understanding ordered sequences, a critical operation for counting and addition later on.

**Count the blocks**

Count them from smallest to largest saying the numbers out loud as you point. This helps promote the idea that when you count, the number you speak is associated with a single item. At this level it is unwise to insist the child do it right - rather, praise their efforts and always demonstrate the correct way to do it. They'll catch on when they're ready.

# Elementary School

**Fundamental Magic Square Principles**

In working with our own young children, we found that the full magic square puzzle was difficult to clearly understand. It is better therefore to pose smaller puzzles that lead toward understanding the more complex elements of the magic square. Try not to move faster than the child is able. We find it productive to look at the following questions as prompts for discussion rather than problems to solve.

1) Can you make two stacks of the same height? Three stacks? Four? Five?
2) Can you use up all the blocks to make three stacks of three blocks each which are the same height? How high are the stacks? Each stack should be 15 units high.

4) Can you take 1 block from each stack of 15 to make a new stack of the same height? Can you do this more ways than one? How many different ways can you do this?

5) Arrange the original stacks of 15 on the paper grid as shown below. Explain what a row and column are, and show how you can stack blocks in a row, or stack blocks in a column. In which way (row or column) do the blocks stack to the same height?

6) Without changing the blocks in each row that stack to 15, can you rearrange the blocks in a given row so that the columns stack to different heights? Can you make a stack of 24? Can you make a stack of 6? Can you confirm that the original rows still stack to 15?

7) Can you make the stacks in each row and column stack to 15?

8) Show that the blocks can also be stacked along a diagonal. Is there anyway to change how the diagonals stack without changing how the rows and columns stack? Can you make it so the diagonals also stack to 15?

## Order and label the blocks

The kit includes sheets of sticky labels that you can use to label the blocks. When you've had a chance to play with the blocks for a while you can show the child the stickers and suggest they can use them to label the blocks. Most children - even kindergarten children - will be able to quickly order them and, with a little help for younger children, label them.

## Units of Measure

Can the blocks be used to measure things? Try measuring various objects in the room. What is the tallest thing you can measure? What is the shortest thing you can measure? How is your unit of measure (the 1-block) related to an inch?

## Mathematical Properties

Associative property:

1) Does the height of a stack of blocks depend on the order of the blocks?
2) How many different ways can you stack two blocks?
3) How many different ways can you stack three blocks?

# Middle School

Middle school students can solve the standard magic square puzzle with minimal coaching once the problem has been clearly explained. In addition, they can explore sequences and sums of sequences described below.

**Sequences**

The blocks can be used to make the concept of sequences physical. For example, the integer sequence {1, 2, 3, 4,…} has a nice staircase shape when implemented with the blocks. Another sequence to try with the blocks is double stairs, {1, 3, 5,…}, or triple stairs {1, 4, 7,…}. Explain that for these sequences you can use the visible difference between neighboring blocks to determine the next block.

Some sequences depend on the previous two blocks, such as the Fibonacci sequence. This sequence defines each step as the sum of the previous two blocks. In classroom settings the students can pair up and start with two 1-blocks (even though the Fibonacci sequence really starts with 0 and 1). Otherwise they can start with the 1-block and the 2-block. How far in the Fibonacci sequence can you go with 1 set? 2 sets? 3 sets? 4 sets?

Addition is not the only way to build sequences and they don't have to start with the 1-block. You can start with any block and use any mathematical operation. Some sequences to try (some of these require pairs of students):

1) Two up, one down {1, 3, 2, 4, 3, 5, 4, 6, 5,…}.
2) Three up, one down {1, 4, 3, 7, 6, 9, 8,…}.

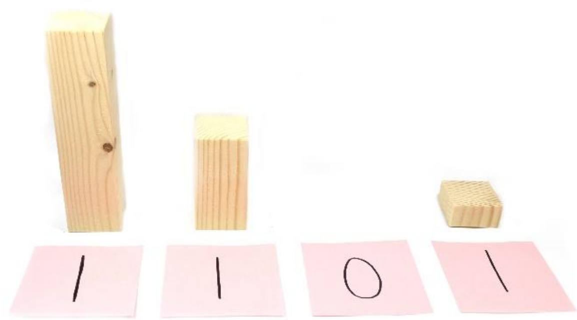
3) Minus one {9, 8, 7, 6,…}
4) A+1, B+1 {5, 3, 6, 4, 7, 5,…}
5) A+1, B-1 {1, 9, 2, 8,…}

Have the students play 'Sequence Challenge' by coming up with sequences for the educator and/or other students to guess. You can transition to paper to allow for sequences requiring numbers higher than 9.

# High School

**Alternate Counting Systems**

Show that with just three blocks you can make stacks to every number from 1 to 6. Can you make stacks of every number from 1 to 7 with just three blocks? How high can you count with only 4 blocks? This is called a binary counting system because each block in the counting set is twice the previous block.



If you have multiple sets in the classroom, you can group students in pairs and ask how high they can count with 6 blocks (at least up to 26, with two 1's, two 3's and two 9's).
This is a ternary counting system because each unique block in the counting set is three times the previous block.
We find it helpful for this exercise to have the students write out the numbers they are trying make and check them off as they go.

The blocks make it easy to explore binary and ternary representation. For example, in binary, 0001 represents block 1, 0010 represents block 2, 0100 represent block 4, and 1000 represents block 8. A number like 1101 means they should stack blocks 1, 4, and 8 to make 13. In ternary, 001 represents one 1-block, 002 represents two 1-blocks, 010 represents one 3-block, 020 represents two 3-blocks, and so on. A number like 012 represents a stack of one 3-block and two 1-blocks to make 5. Use the blocks to write out the numbers from 1 to 15 in binary representation. Write out the numbers from 1 to 26 in ternary representation.

**Sums of sequences**

*Sum of integers:* $1+2+...+N = \frac{N(N+1)}{2}$

Ask the student to pick a number between 2 and 9. Then ask them to stack the blocks from 1 to their number. Tell them to figure out how tall the stack is, and that they are free to rearrange the blocks any way they want. Ask them to pick a different number and do it again. They should very quickly settle on the following procedure: if N is even they put the 1-block on the largest block, the 2-block on the second largest block, and so on, making N/2 stacks that are each (N+1) units tall (see the left picture below for N = 6). This gives the formula

N(N+1)/2. If N is odd they put the 1-block on the second largest block, the 2-block on the third largest block, and so on, making (N+1)/2 stacks which are each N units tall (see the right picture above for N = 7). This also gives the same formula.

This strategy can be adapted to try to figure out formulas for other sums of sequences (see the middle school section).