GARAGE PHYSICS

by eISCO

Standing Arch Kit

GP00001

Arches have been used in construction for the past several millennia. Their systematic use began with the ancient Romans who used them to build both bridges and buildings.

With this kit, we will be exploring an arch of a particular shape, the catenary arch. Its shape fits the mathematical equation described by a curve called the catenary.

Next Generation Science Standards (NGSS):

PS2 Motion and Stability: Forces and Interactions

The Catenary:

In physics and geometry, the catenary is the shape described by a hanging chain or string when supported only at its own ends.

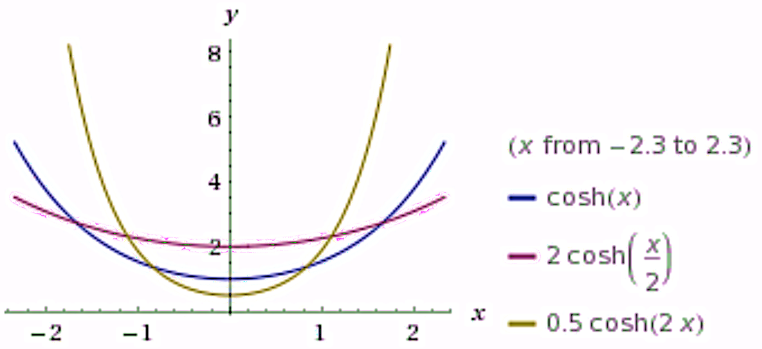
The curves described by catenaries are used in architecture in the design of bridges and arches so that the forces do not result in shearing stresses on the stones.

Sir Robert Hooke described this shape in 1675, "… as hangs a flexible cable so, inverted, stand the touching pieces of an arch."

The equation of a catenary in Cartesian coordinates has the form:

$$y = a\cosh\left(\frac{x}{a}\right) = \frac{1}{2}\left(e^{x/a} + e^{-x/a}\right)$$

In the above equation, cosh is the hyperbolic cosine function. All catenary curves are similar to each other, changing the parameter is equivalent to a uniform scaling of the curve.



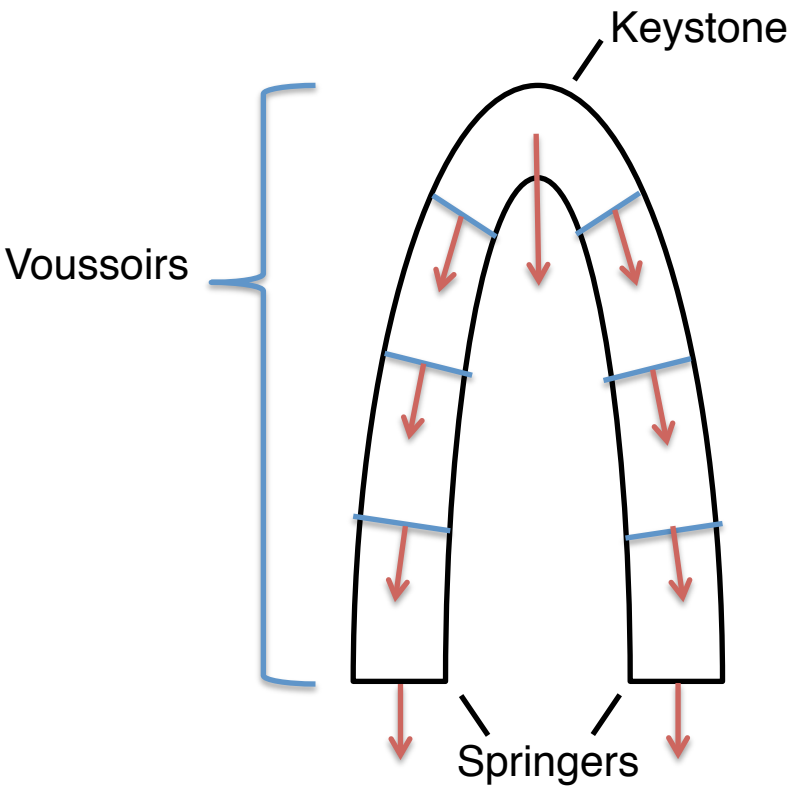
Above are plotted catenary curves for several values of *'a'*. What would you approximate the value of *'a'* for the arch in this kit?

To try: Discover the shape of the catenary for yourself. Hold the two ends of the included string about 11 inches apart. This is the approximate shape of the arch in this kit, although upside down!

Forces at Play:

The goal of the shape of the arch is to redistribute the forces

An arch is made up of a number of voussoirs (which comes from the French verb 'to turn'), which are wedge shaped elements, usually made of stone when it comes to bridge-building. In the figure to the right, observe how the force from the keystone is 'turned', redirecting the downward force along the transverse direction of each voussoir.



Parts of the Arch:

Two types of voussoirs are of more importance, the springers and the keystone.

Keystone:
A keystone is the wedge-shaped stone piece at the apex of a masonry vault or arch, which is the final piece placed during construction and locks all stones into position, allowing the arch to bear weight.

Although a masonry arch or vault cannot be self-supporting until the keystone is placed, the keystone experience the least stress of any of the voussoirs, due to its position at the apex.

Springers: The springers are the lowermost voussoirs, and experience the brunt of the force from the weight above.

Construction:

Try building the arch on your own. Can you complete the arch with just two hands, or do you need someone to help hold the voussoirs in place while you insert the keystone?

By now you will have found that it is a difficult task to build an arch on your own by hand.

The blocks of this arch cannot support itself until the keystone (the center block) is in place.

The blocks in this kit are each labeled with a number of sunken holes. What order did you find worked to build your arch? Can you construct the arch in any order other than 1-4 from bottom to top?

If an arch can not stand freely without the keystone in place, then how does one build an arch?

By using a temporary framework, that is removed after the completion. A frame is constructed in the shape of the arch, usually out of wood, just below where the arch is to be built. After the keystone is inserted, the framework is disassembled and the arch remains in place.

Quantitative Accessories:

With the optional add-on hanging mass kit (**GP00002)**, see how much weight you can hang from the keystone. This simulates a live load; the amount of weight that a structure or bridge must support on top of its own weight (such as cars on top of a bridge).

Do you think there is weight that is too great for the arch to support? Why would the arch fail?

Alternatively, the amount of weight that can be supported can be measured using a **NeuLog Force Sensor** (**NUL211 & USB200**).

Hang the Force Sensor from the hook to the eyelet under the keystone. Pull on the bottom of the force sensor to see the maximum force you can exert on the arch. To convert Newtons to a measure of weight, divide the observed force by 9.81 m/s^2 to find the weight in kilograms.