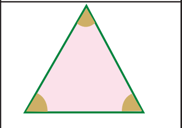
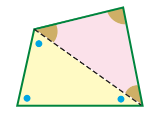
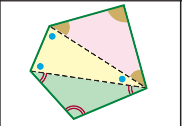
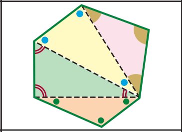
5 ★ 3 polygons | ANGLES

Main Theory & Concept

① Sum of interior angles

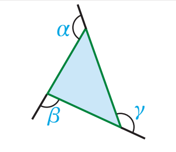
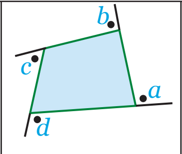
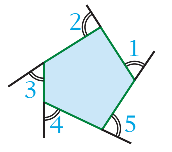
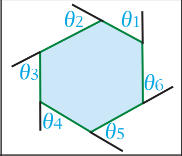
➪ Number of triangles in the polygon × 180°

Triangle	Quadrilateral	Pentagon	Hexagon
			
1 triangle	2 triangles	3 triangles	4 triangles
180°	2 × 180° = 360°	3 × 180° = 540°	4 × 180° = 720°

……

Sum of the interior angles in n-sided polygon	$(n-2) \times 180°$

② Sum of exterior angles

			
$\alpha+\beta+\gamma$ $= 360°$	$a+b+c+d$ $= 360°$	$1+2+3+4$ $+5 = 360°$	$\theta_1+\theta_2+\theta_3+\theta_4$ $+\theta_5+\theta_6 = 360°$

……

Sum of the exterior angles in n-sided polygon	360°

③ Regular n-sided polygon

ⓐ One(each) interior angle ➪ $\theta = [(n-2) \times 180°] \div n$

$$= 180° - \frac{360°}{n}$$

ⓑ One(each) exterior angle ➪ $\alpha = 360° \div n$

Tip from Top

❶ Sum of the interior angles

n sides ⇒ $n+1$ sides

+1 side

Increased by 180°

❷ Sum of exterior angles

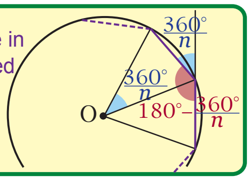
At a angle of a polygon
Interior angle + Exterior angle = 180°

❸ Regular n-sided polygons

≪Each interior/ exterior angle≫

Side	Name	Interior	Exterior
3	*Triangle*	60°	120°
4	*Quadrilateral*	90°	90°
5	*Pentagon*	108°	72°
6	*Hexagon*	120°	60°
7	*Heptagon*	$128\frac{4}{7}°$	$51\frac{3}{7}°$
8	*Octagon*	135°	45°
9	*Nonagon*	140°	40°
10	*Decagon*	144°	36°
11	*Undecagon*	$147\frac{3}{11}°$	$22\frac{8}{11}°$
12	*Dodecagon*	150°	30°

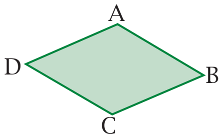
Exterior angle in regular n-sided polygon



Simple Sample

1. If the sign is a regular octagon as shown, then what is each angle of the sign?
(A) 150° (B) 144° (C) 135°
(D) 120° (E) 108°

2. In rhombus ABCD, ∠A = $(10x+6)°$ and ∠B = $(5x-6)°$. What is the measure of ∠C?
(A) 96° (B) 108° (C) 112°
(D) 126° (E) 132°



3. If the measure of each interior angle of a regular polygon is 162°, then how many sides does the polygon have?
(A) Twelve (B) Sixteen (C) Eighteen
(D) Twenty (E) Twenty four

Essence

1. Since the sum of eight interior angles is $(8-2) \times 180° = 1080°$, each angle of the regular octagon is $\frac{1080°}{8} = 135°$

Ans (C)

2. Since $\overline{AD} \parallel \overline{BC}$, $\angle A + \angle B = 180°$
$\therefore (10x+6)° + (5x-6)° = 180°$
$\therefore \cancel{15}x° = \cancel{180°}_{12}$ $\therefore x = 12°$
$\therefore \angle C = \angle A = 10 \times 12° + 6° = 126°$

Ans (D)

3. Since each interior angle of the regular polygon is 162°, each exterior angle is $180° - 162° = 18°$
∴ The number of sides of the polygon is $\frac{360°}{18°} = 20$ **Ans** (D)

pattern guide | Ace of Base

① Sum of interior angles

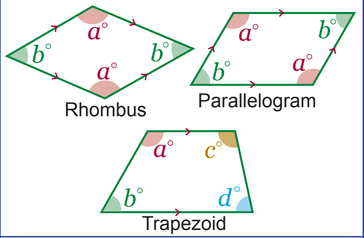
Sum of the interior angles in n-sided polygon

$(n-2)\times 180^\circ$

② Special quadrilateral

Supplementary angle

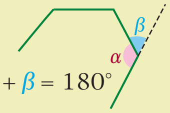
Sum of co-interior angles = 180°



$a^\circ + b^\circ = c^\circ + d^\circ = 180^\circ$

③ Interior/Exterior Angles

Interior angle + Exterior angle at a vertex point = 180°



$\alpha + \beta = 180^\circ$

Sum of the exterior angles of any-sided polygon = 360°

④ Regular polygons

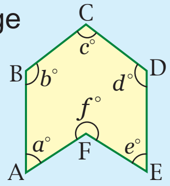
Each interior angle in n - sided regular polygon

$$\theta = \frac{(n-2)\times 180^\circ}{n} = 180^\circ - \frac{360^\circ}{n}$$

In hexagon ABCDEF, what is the average (arithmetic mean) of a, b, c, d, e, and f?

(A) 60 (B) 90

(C) 120 (D) 150



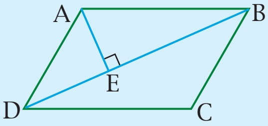
(Sol) Since the polygon ABCDEF is a hexagon, the sum of the six interior angles is $a^\circ + b^\circ + c^\circ + d^\circ + e^\circ + f^\circ$
$= (6-2)\times 180^\circ = 4\times 180^\circ = 720^\circ$
$\therefore$ The average (arithmetic mean) of a, b, c, d, e, and f is $720 \div 6 = 120$

Ans (C)

In parallelogram ABCD, ∠C = 120° and ∠BDC = 24°. What is the measure of ∠DAE?

(A) 24° (B) 36°

(C) 48° (D) 54°



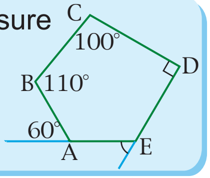
(Sol) In parallelogram ABCD, $\overline{AD} \parallel \overline{BC}$. $\therefore \angle C + \angle D = 180^\circ$
$\therefore \angle D = 180^\circ - \angle C = 180^\circ - 120^\circ = 60^\circ$ (← Co-interior angle)
$\therefore \angle ADE = \angle D - \angle BDC = 60^\circ - 24^\circ = 36^\circ$
In right triangle ADE, $\angle DAE = 90^\circ - 36^\circ = 54^\circ$

Ans (D)

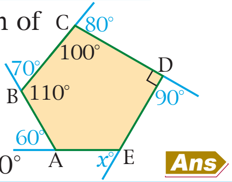
In pentagon ABCDE, what is the measure of the exterior angle at E?

(A) 60° (B) 80°

(C) 120° (D) 240°



(Sol) If the exterior angle at E is x°, the sum of the exterior angles is $60^\circ + (180-110)^\circ$
$+ (180-100)^\circ + (180-90)^\circ + x^\circ$
$= 60^\circ + 70^\circ + 80^\circ + 90^\circ + x^\circ = 360^\circ$
$\therefore 300^\circ + x^\circ = 360^\circ$ $\therefore x^\circ = 360^\circ - 60^\circ = 300^\circ$



Ans (A)

What is the measure each exterior angle of a regular polygon with 15 sides?

(A) 144° (B) 156°

(C) 36° (D) 24°

(Sol) Since the sum of exterior angles of any-sided polygon is 360°, each exterior angle of the polygon is
$360^\circ \div 15 = 24^\circ$

Ans (D)

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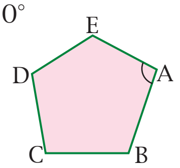
ANSWERS ··· P. 103

1 Sum of interior angles

(1) What is the sum of the interior angles of a trapezoid?

(A) 180° (B) 240° (C) 360° (D) 480°



(2) In the pentagon ABCDE, ∠B = 108°, ∠C = 100°, ∠D = 112° and ∠E = 120°. What is the degree measure of ∠A?

(A) 90 (B) 100 (C) 110 (D) 120

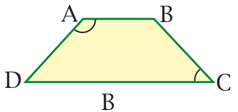
(3) What is the average(arithmetic mean) of the interior angles of a nonagon?

(A) 114° (B) 135° (C) 140° (D) 150°

(4) If the sum of interior angles of a polygon is 2,700°, what is the number of sides of the polygon?

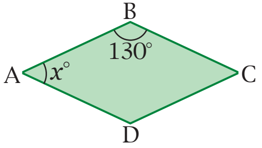
(A) 15 (B) 17 (C) 19 (D) 21

2 Special quadrilateral



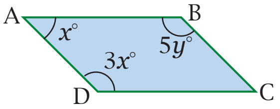
(1) In isosceles trapezoid ABCD, $\overline{AB} \parallel \overline{CD}$ and AD = BC. What is the sum of measures of ∠A and ∠C?

(A) 90° (B) 100° (C) 180° (D) 200°



(2) In rhombus ABCD, ∠B = 130°. What is the value of x?

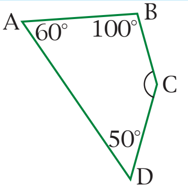
(A) 30 (B) 50 (C) 60 (D) 70



(3) In parallelogram ABCD, ∠A = x°, ∠B = $5y$° and ∠D = $3x$°. What is the value of y?

(A) 27 (B) 25 (C) 24 (D) 15

3 In / Exterior angles

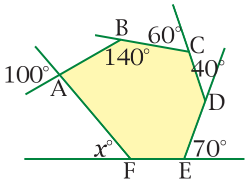


(1) In quadrilateral ABCD, ∠A = 60°, ∠B = 100°, ∠D = 50°. what is the measure of exterior angle of ∠C?

(A) 30° (B) 80° (C) 260° (D) 330°

(2) What is the average(arithmetic mean) of the exterior angles of an octagon?

(A) 45° (B) 40° (C) 22.5° (D) 16°



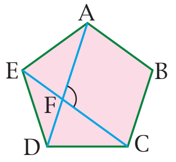
(3) In hexagon ABCDEF, what is the value of x?

(A) 50 (B) 60 (C) 70 (D) 80

4 Regular polygon

(1) Which of the following is the measure of an interior angle in a regular decagon (10-sided polygon)?

(A) 36° (B) 48° (C) 135° (D) 144°



(2) In regular pentagon ABCDE, what is the measure of ∠AFC?

(A) 108° (B) 114° (C) 120° (D) 136°

(3) If each exterior angle of a regular polygon is 20°, what is the number of sides of the polygon?

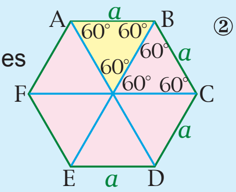
❶ (1) C (2) B (3) C (4) B ❷ (1) C (2) B (3) A ❸ (1) A (2) A (3) C ❹ (1) D (2) A (3) 18

Polygons

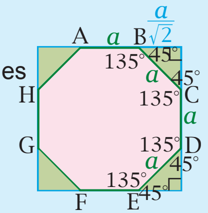
n - sided polygon ⇨ ⓐ Sum of the interior angles = $(n-2) \times 180^\circ$
ⓑ Sum of the exterior angles = 360°

spark! Special regular polygons

① Regular hexagon
⇨ 6 equilateral triangles

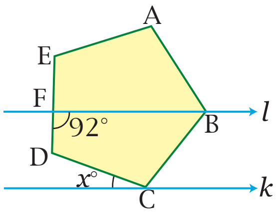


② Regular octagon
⇨ Square – 4 isosceles right triangles



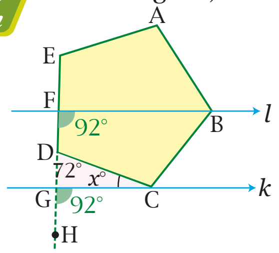
CAP In the figure, if two parallel lines l and k pass two vertices of a regular pentagon ABCDE, then what is the value of x?

(A) 12 (B) 18 (C) 20
(D) 22 (E) 28



Accent One exterior angle of regular n-sided polygon $= \dfrac{360^\circ}{n}$

Simple Solution In the figure, since an exterior angle of a regular pentagon is $\dfrac{360^\circ}{5} = 72^\circ$,



$\angle CDG = 72^\circ$

Since two lines l and k are parallel,

$\angle CGH = \angle BFD = 92^\circ$ (← Corresponding angles)

In triangle CDG, $\angle DCG + \angle CDG = \angle CGH$

$\therefore x^\circ + 72^\circ = 92^\circ \quad \therefore x^\circ = 92^\circ - 72^\circ = 20^\circ$

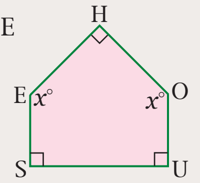
Ans (C)

training

ANSWERS ··· P. 104

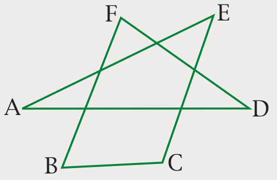
(1) In pentagon HOUSE, $\angle H$, $\angle U$ and $\angle S$ are right angles. If $\angle O$ and $\angle E$ are x° respectively, then what is the value of x?

(A) 95 (B) 115 (C) 125
(D) 135 (E) 145



(2) In the figure, what is the sum of the measures, in degrees, of the six angles A, B, C, D, E and F?

(A) 270 (B) 360 (C) 450
(D) 540 (E) 630



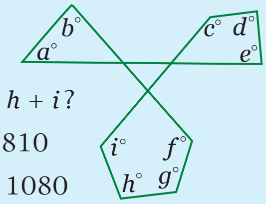
Super Model

1

Sum of interior angles

➪ $(n-2)\times180^\circ$

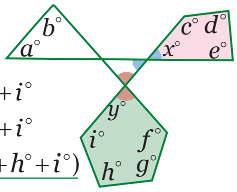
In the figure, what is the value of $a+b+c+d+e+f+g+h+i$?



(A) 720 (B) 810

(C) 900 (D) 1080

In the figure,



$a^\circ+b^\circ=x^\circ+y^\circ$

$\therefore a^\circ+b^\circ+c^\circ+d^\circ+e^\circ+f^\circ+g^\circ+h^\circ+i^\circ$

$=x^\circ+y^\circ+c^\circ+d^\circ+e^\circ+f^\circ+g^\circ+h^\circ+i^\circ$

$=\underbrace{(x^\circ+c^\circ+d^\circ+e^\circ)}_{\text{quadrilateral}}+\underbrace{(y^\circ+f^\circ+g^\circ+h^\circ+i^\circ)}_{\text{pentagon}}$

$=(4-2)\times180^\circ+(5-2)\times180^\circ=2\cdot180^\circ+3\cdot180^\circ$

$=360^\circ+540^\circ=900^\circ$

Ans (C)

2

Sum of interior angles

➪ $(n-2)\times180^\circ$

The interior angles of a polygon, measured in degrees, form an arithmetic progression. If the smallest angle is 126° and the greatest angle is 174°, then what is the number of sides of the polygon?

(A) 10 (B) 12

(C) 14 (D) 16

If the polygon has n sides, the sum of interior angles of the polygon is

$\frac{n}{2}(126^\circ+174^\circ)=(n-2)\times180^\circ$

$\therefore \frac{n}{2}\times\overset{5}{\cancel{300}}{}^\circ=(n-2)\times\overset{3}{\cancel{180}}{}^\circ$

$\therefore \frac{n}{2}\times5=(n-2)\times3 \quad \therefore 5n=6(n-2)$

$\therefore 5n=6n-12 \quad \therefore n=12$

Ans (B)

3

Sum of an interior angle and its exterior angle

➪ Interior angle + Exterior angle = 180°

If the measures of the angles of a pentagon are in the ratio of 2 : 3 : 4 : 5 : 6, then what is the smallest exterior angle of the pentagon?

(A) 18° (B) 20°

(C) 24° (D) 27°

If the five interior angles are $2k^\circ, 3k^\circ, 4k^\circ, 5k^\circ$ and $6k^\circ$,

$2k^\circ+3k^\circ+4k^\circ+5k^\circ+6k^\circ=(5-2)\times180^\circ$

$\therefore \cancel{20}k^\circ=3\times180^\circ=\overset{27}{\cancel{540}}{}^\circ \quad \therefore k^\circ=27^\circ$

$\therefore$ Since the largest interior angle is $6k^\circ$ $=6\times27^\circ=162^\circ$, the smallest exterior angle is $180^\circ-162^\circ=18^\circ$

Ans (A)

4

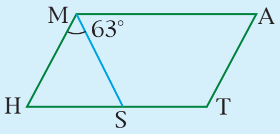
Special quadrilateral

➪ Rhombus and Parallelogram ⇒ Sum of two consecutive angles = 180°

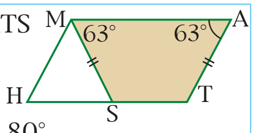
In parallelogram MATH, MS = AT and ∠SMA = 63°, what is the measure of ∠HMS?

(A) 42° (B) 54°

(C) 63° (D) 76°



Since the quadrilateral MATS is an isosceles trapezoid,



$\therefore \angle A=\angle AMS=63^\circ$

Since $\overline{MA}\parallel\overline{ST}$, $\angle A+\angle T=180^\circ$

$\therefore \angle T=180^\circ-63^\circ=117^\circ \quad \therefore \angle M=\angle T=117^\circ$

$\therefore \angle HMS=117^\circ-63^\circ=54^\circ$

Ans (B)

the melting zone

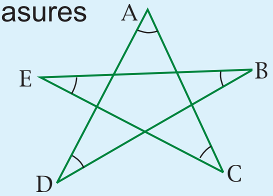
ANSWERS ··· P. 105

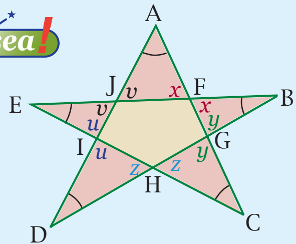
SHOW CASE

1 In the figure, what is the average (arithmetic mean) of the measures of acute angles of ∠A, ∠B, ∠C, ∠D and ∠E?

(A) 24° (B) 30° (C) 36°

(D) 40° (E) 54°





From the five exterior triangles of pentagon FGHIJ,

$\angle A + \angle B + \angle C + \angle D + \angle E + 2(x + y + z + u + v) = 5 \times 180^\circ$

Since the sum of the exterior angles of a pentagon is 360°,

$x + y + z + u + v = 360^\circ$

$\therefore\ \angle A + \angle B + \angle C + \angle D + \angle E + 2 \times 360^\circ = 5 \times 180^\circ$

$\therefore\ \angle A + \angle B + \angle C + \angle D + \angle E = 900^\circ - 720^\circ = 180^\circ$

∴ The average of five angles ∠A, ∠B, ∠C, ∠D and ∠E is 180° ÷ 5 = 36°

Ans (C)

2 What is the number of sides of a polygon for which the sum of the measures of its interior angles is 2160°?

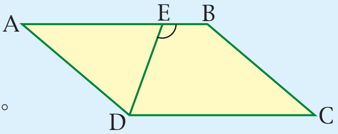
(A) 12 (B) 14 (C) 16

(D) 18 (E) 20

3 In parallelogram ABCD, $\angle A = 3x^\circ - 20^\circ$, $\angle C = 7x^\circ - 100^\circ$ and $\overline{DE}$ bisects ∠ADC. What is the measure of ∠DEB?

(A) 110° (B) 120° (C) 130°

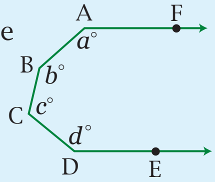
(D) 140° (E) 150°



4 In the figure, if two rays $\overrightarrow{AF}$ and $\overrightarrow{DE}$ are parallel, then what is the average (arithmetic mean) of a, b, c and d?

(A) 120 (B) 125 (C) 135

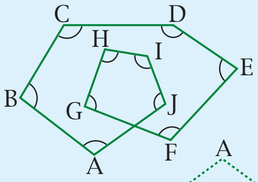
(D) 150 (E) 165



5 In the figure, what is the sum of the measure of ten angles of $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G + \angle H + \angle I + \angle J$?

(A) 720° (B) 900° (C) 1,080°

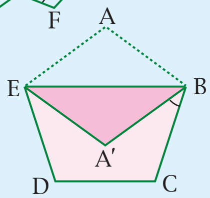
(D) 1,260° (E) 1,440°



6 If the regular pentagon ABCDE is folded as shown, then what is the measure of ∠A′BC?

(A) 24° (B) 28° (C) 32°

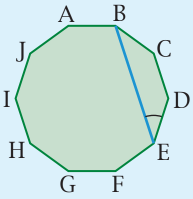
(D) 36° (E) 40°



7 In the regular decagon ABCDEFGHIJ (10-sided polygon), what is the measure of ∠BED?

(A) 20° (B) 24° (C) 30°

(D) 36° (E) 48°

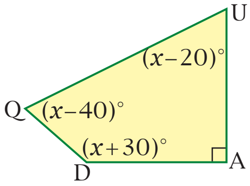


JUMP

ANSWERS ··· P. 105

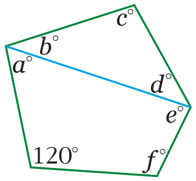
1 In the figure, what is the measure of the greatest angle in quadrilateral QUAD?

(A) 100°
(B) 110°
(C) 120°
(D) 130°
(E) 140°



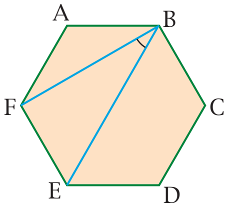
2 In the figure, what is the average(arithmetic mean) of a, b, c, d, e and f?

(A) 105
(B) 84
(C) 75
(D) 70
(E) 55



3 In regular hexagon ABCDEF as shown, what is the degree measure of ∠FBE?

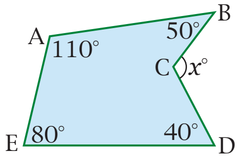
(A) 15
(B) 17.5
(C) 20
(D) 22.5
(E) 30



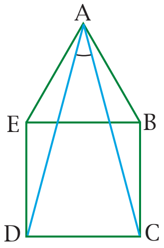
4 Which type of polygon has an interior angle that is always four times the measure of its exterior angle?

(A) A regular pentagon
(B) A regular hexagon
(C) A regular octagon
(D) A regular decagon
(E) A regular dodecagon

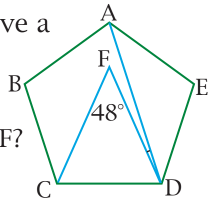
5 In the figure, ∠A = 110°, ∠B = 50°, ∠D = 40° and ∠E = 80°. What is the value of x?



6 An equilateral triangle and a square have a common side as shown. What is the measure of ∠DAC?

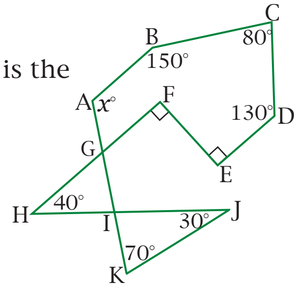


7 In the figure, a regular pentagon ABCDE and an isosceles triangle CFD have a common side $\overline{CD}$ as shown. If FC = FD and ∠CFD = 48°, what is the measure of ∠ADF?

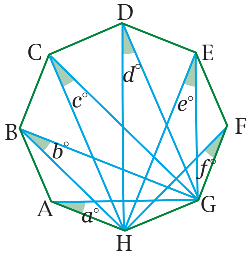


Math Mania!

8 In the figure, what is the value of x?



9 In regular octagon ABCDEFGH, what is the sum of $a + b + c + d + e + f$?



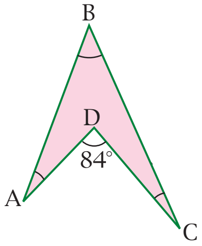
All-Round Checks

EXAMINATION

ANSWERS ··· P. 106

1 In the polygon ABCD as shown, what is the average(arithmetic mean) of a, b and c?

(A) 6
(B) 14
(C) 21
(D) 28
(E) 42

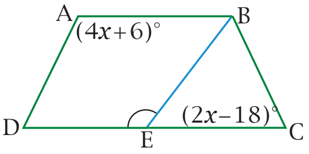


2 In a quadrilateral ABCD, $\angle A = 3x^\circ + 10^\circ$, $\angle B = x^\circ - 15^\circ$, $\angle C = 4x^\circ - 45^\circ$ and $\angle D = 2x^\circ + 50^\circ$. What is the measure of the greatest angle in quadrilateral ABCD?

(A) 118°
(B) 122°
(C) 128°
(D) 135°
(E) 140°

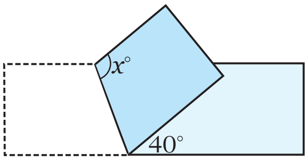
3 In an isosceles trapezoid ABCD with AD = BC, $\angle A = (4x + 6)^\circ$, $\angle C = (2x - 18)^\circ$ and $\overline{BE}$ bisects ∠B. What is the measure of ∠BED?

(A) 113°
(B) 112°
(C) 111°
(D) 110°
(E) 109°



4 If the rectangular sheet is folded as shown, then what is the value of x?

(A) 140
(B) 130
(C) 120
(D) 110
(E) 100

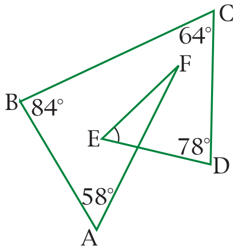


5 If the sum of all but one of the interior angles of a convex polygon is 2450°, what is the value of the remaining angle in the polygon?

(A) 50°
(B) 70°
(C) 110°
(D) 130°
(E) 150°

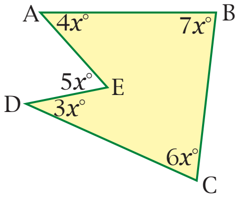
6 In the figure, ∠A = 58°, ∠B = 84°, ∠C = 64° and ∠D = 78° and ∠E : ∠F = 3 : 1. What is the measure of ∠E?

(A) 48°
(B) 51°
(C) 54°
(D) 57°
(E) 60°



7 In five sided-polygon ABCDE, $\angle A = 4x^\circ$, $\angle B = 7x^\circ$, $\angle C = 6x^\circ$, $\angle D = 3x^\circ$ and $\angle DEA = 5x^\circ$. What is the value of x?

(A) 12
(B) 12.5
(C) 15
(D) 16
(E) 18

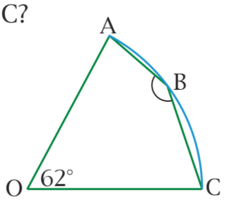


8 Which of the following cannot be the measure of an exterior angle of a regular polygon?

(A) 18°
(B) 24°
(C) 32°
(D) 45°
(E) 72°

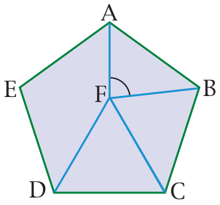
9 In the figure, O is the center of a circle and a quadrilateral OABC is inscribed in the sector. If $\angle AOC = 62^\circ$, then what is the measure of $\angle ABC$?

(A) 128°
(B) 132°
(C) 148°
(D) 149°
(E) 162°



10 In regular pentagon ABCDE, triangle FCD is an equilateral triangle. What is the measure of $\angle AFB$?

(A) 60°
(B) 74°
(C) 78°
(D) 84°
(E) 86°

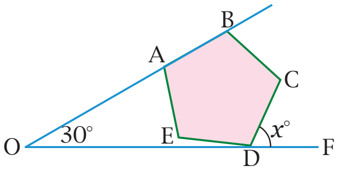


11 If the average(arithmetic mean) of the measures of the interior angles of a n-sided polygon is x°, what is the sum of the interior angles of a $n+3$ sided polygon?

(A) $3x^\circ + 270^\circ$
(B) $3x^\circ + 360^\circ$
(C) $nx^\circ + 270^\circ$
(D) $nx^\circ + 540^\circ$
(E) $nx^\circ + 1{,}080^\circ$

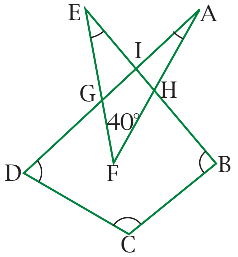
12 In regular pentagon ABCDE, $\angle AOD = 30^\circ$. What is the value of x?

(A) 84
(B) 72
(C) 68
(D) 66
(E) 64



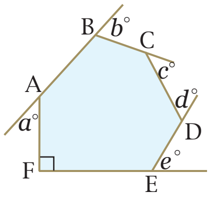
13 In the figure, if $\angle F = 40^\circ$, what is the average (arithmetic mean) of degree measures of $\angle A$, $\angle B$, $\angle C$, $\angle D$ and $\angle E$?

(A) 60
(B) 64
(C) 72
(D) 75
(E) 80



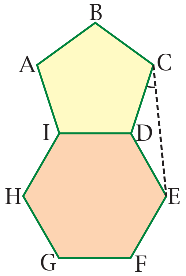
14 In the figure, what is the average (arithmetic mean) of a, b, c, d and e?

(A) 27
(B) 36
(C) 42
(D) 54
(E) 60



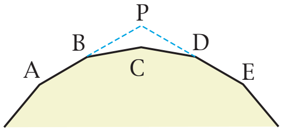
15 A regular pentagon ABCDI and a regular hexagon IDEFGH have a common side $\overline{ID}$ as shown. What is the measure of $\angle DCE$?

(A) 18°
(B) 20°
(C) 24°
(D) 32°
(E) 36°

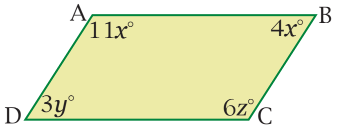


16 In the figure, A, B, C, D and E are vertices of a regular polygon. Two sides $\overline{AB}$ and $\overline{DE}$ are extended to meet at P as shown. If $\angle BPC = 140^\circ$, how many sides does the regular polygon have?

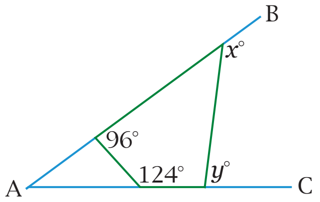
(A) 12
(B) 16
(C) 20
(D) 24
(E) 27



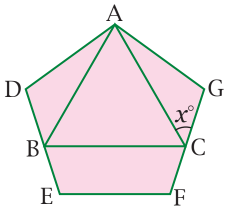
17 In parallelogram ABCD, $\angle A = 11x^\circ$, $\angle B = 4x^\circ$, $\angle C = 6z^\circ$ and $\angle D = 3y^\circ$. What is the value of $y + z$?



18 In the figure, what is the value of $x + y$?

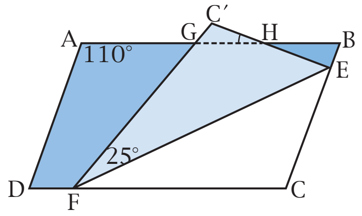


19 (1) An equilateral triangle ABC is inscribed in regular pentagon ADEFG. If $\overline{BC} \parallel \overline{EF}$, then what is the value of x?



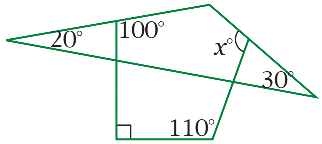
(2) Point P is inside regular octagon ABCDEFGH so that triangle ABP is equilateral. How many degrees are in angle BCP?

20 A parallelogram sheet ABCD is folded as shown. If $\angle A = 110^\circ$ and $\angle GFE = 25^\circ$, what is the measure of $\angle C'HG$?

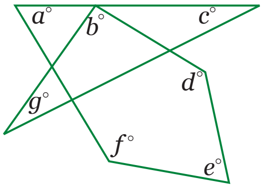


21 How many regular polygons are there that have integer degrees as their interior angles?

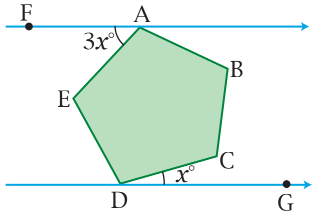
22 (1) In the figure, what is the value of x?



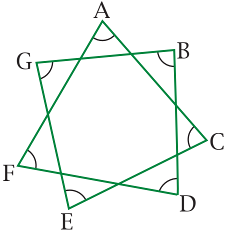
(2) In the figure, what is the value of $a + b + c + d + e + f$?



23 In regular pentagon ABCDE, two lines $\overleftrightarrow{FA}$ and $\overleftrightarrow{DG}$ are parallel and $\angle FAE = 3x^\circ$ and $\angle CDG = x^\circ$. What is the value of x?



24 In the figure, what is the sum of the measures of seven angles $\angle A$, $\angle B$, $\angle C$, $\angle D$, $\angle E$, $\angle F$, and $\angle G$?



18. In triangle EBH, $\angle BHC = 26^\circ + 36^\circ = 62^\circ$

In triangle HGC,

$\angle HGF = 34^\circ + 62^\circ = 96^\circ$

In triangle ADG,

$x^\circ + y^\circ + \angle AGD = 180^\circ$

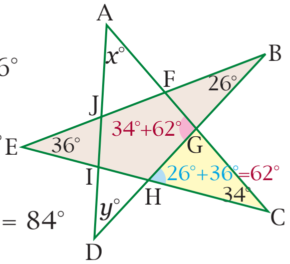
$\therefore x^\circ + y^\circ + 96^\circ = 180^\circ$

$\therefore x^\circ + y^\circ = 180^\circ - 96^\circ = 84^\circ$

Since $x : y = 4 : 3$,

if $x = 4k^\circ$ and $y = 3k^\circ$, $x^\circ + y^\circ = \cancel{7}k^\circ = \overset{12}{\cancel{84}}^\circ$

$\therefore k^\circ = 12^\circ \quad \therefore x^\circ = 4k^\circ = 4 \times 12^\circ = 48^\circ$



Ans 48

19. In rectangle ABCD, $\angle D = 90^\circ$

In right triangle AED, $2x^\circ + 3x^\circ = 90^\circ$

$\therefore \cancel{5}x^\circ = \overset{18}{\cancel{90}}^\circ \quad \therefore x^\circ = 18^\circ$

And $\angle BEC = \angle ABE = 4x^\circ$

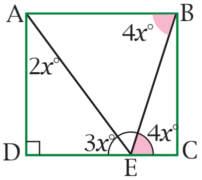
(← Alternate angles)

$\therefore 3x^\circ + \angle AEB + 4x^\circ = 180^\circ$

$\therefore \angle AEB + 7x^\circ = 180^\circ$

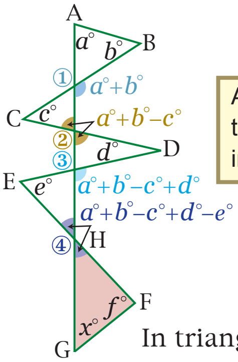
$\therefore \angle AEB = 180^\circ - 7x^\circ = 180^\circ - 7 \times 18^\circ$

$= 180^\circ - 126^\circ = 54^\circ$



Ans 54°

20.



An exterior angle is equal to the sum of the other two interior angles

In triangle FGH,

$(a^\circ + b^\circ - c^\circ + d^\circ - e^\circ) + f^\circ + x^\circ = 180^\circ$

$\therefore \angle G = x^\circ = 180^\circ - (a^\circ + b^\circ - c^\circ + d^\circ - e^\circ) - f^\circ$

$= 180^\circ - (a^\circ + b^\circ + d^\circ + f^\circ) + (c^\circ + e^\circ)$

$= 180^\circ - 250^\circ + 100^\circ = 30^\circ$

Ans 30°

21. In the figure, $\angle DAB = \angle ABC$

(← Alternate angles)

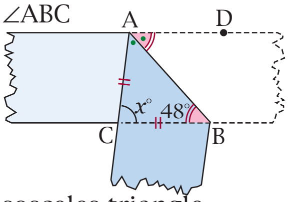
and $\angle DAB = \angle CAB$

(← Symmetric angles)

$\therefore \angle CAB = \angle CBA$

∴ Triangle CAB is an isosceles triangle.

$\therefore x^\circ = 180^\circ - 2 \times 48^\circ = 180^\circ - 96^\circ = 84^\circ$



Ans 84°

22. In the figure, if $\angle ABC = x^\circ$ and $\angle ACB = y^\circ$,

in triangle ABC,

$x^\circ + y^\circ = 180^\circ - 54^\circ = 126^\circ$

If $\angle CBD = a^\circ$ and $\angle BCD = b^\circ$,

$(2a^\circ + x^\circ) + (2b^\circ + y^\circ) = 360^\circ$

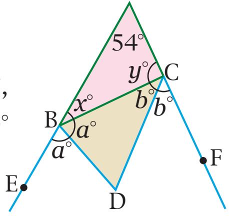
(← Two straight angles)

$2a^\circ + 2b^\circ = 360^\circ - (x^\circ + y^\circ)$

$= 360^\circ - 126^\circ = 234^\circ \quad \therefore a^\circ + b^\circ = \frac{234^\circ}{2} = 117^\circ$

In triangle BCD, $\angle BDC = 180^\circ - (a^\circ + b^\circ)$

$= 180^\circ - 117^\circ = 63^\circ$



Ans 63°

23. Since triangle ABD is isosceles,

$\angle ADB = \angle A = 62^\circ$.

$\therefore \angle ABD = 180^\circ - 2 \times 62^\circ$

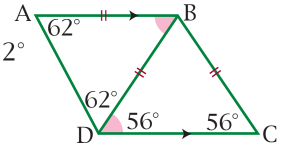
$= 180^\circ - 124^\circ = 56^\circ$.

Since $\overline{AB} \parallel \overline{DC}$,

$\angle BDC = \angle ABD = 56^\circ$ (← Alternate angles)

Since triangle BCD is isosceles,

$\angle DBC = 180^\circ - 2 \times 56^\circ = 180^\circ - 112^\circ = 68^\circ$



Ans 68°

24. In the figure, if $\angle BAD = x^\circ$ and $\angle ABD = y^\circ$,

in triangle ADB, $x^\circ + y^\circ + 134 = 180^\circ$

$\therefore x^\circ + y^\circ = 180^\circ - 134^\circ = 46^\circ$

In triangle ABC,

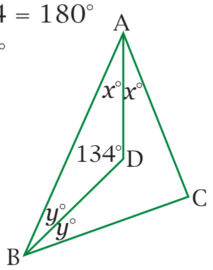
$2x^\circ + 2y^\circ + \angle C = 180^\circ$

$\therefore 2(x^\circ + y^\circ) + \angle C = 180^\circ$

$\therefore \angle C = 180^\circ - 2(x^\circ + y^\circ)$

$= 180^\circ - 2 \times 46^\circ$

$= 180^\circ - 92^\circ = 88^\circ$



Ans 88

5★3 polygons

P.211 pattern drill

❶ (1) Since a trapezoid is a quadrilateral, the sum of the interior angles is $(4 - 2) \times 180^\circ$

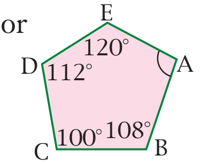
$= 2 \times 180^\circ = 360^\circ$

Ans (C)

(2) Since the sum of five interior angles of the pentagon is

$\angle A + \angle B + \angle C + \angle D + \angle E$

$= (5 - 2) \times 180^\circ$



$\therefore \angle A + 108^\circ + 100^\circ + 112^\circ + 120^\circ = 3 \times 180^\circ$

$\therefore \angle A + 440 = 540 \quad \therefore \angle A = 540^\circ - 440^\circ = 100^\circ$

Ans (B)

(3) Since a nonagon is a 9-sided polygon, the average (arithmetic mean) of the interior angles is [(9 – 2) × 180°] ÷ 9

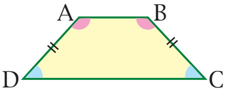
$= \dfrac{7 \times \cancel{180^\circ}^{20}}{\cancel{9}} = 7 \times 20^\circ = 140^\circ$ **Ans** (C)

(4) If the polygon has n sides, the sum of interior angles is $(n - 2) \times 180^\circ = 2{,}700^\circ$,

$n - 2 = \dfrac{\cancel{2{,}700^\circ}^{30}}{\cancel{180^\circ}_{2}} = \dfrac{30}{2} = 15$

$\therefore n = 15 + 2 = 17$ **Ans** (B)

❷ (1) Since quadrilateral ABCD is isosceles trapezoid,



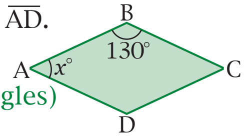
$\angle A = \angle B$ and $\angle C = \angle D$

Since $\overline{AB} \parallel \overline{DC}$, $\angle A + \angle D = 180^\circ$

(← Sum of co-interior angles)

$\therefore \angle A + \angle C = 180^\circ$ **Ans** (C)

(2) In rhombus ABCD, $\overline{BC} \parallel \overline{AD}$.



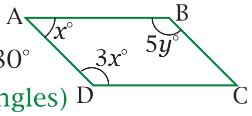
$\therefore \angle A + \angle B = 180^\circ$

(← Sum of co-interior angles)

$\therefore x^\circ + 130^\circ = 180^\circ$

$\therefore x^\circ = 180^\circ - 130^\circ = 50^\circ$ **Ans** (B)

(3) In parallelogram ABCD,



$\overline{AB} \parallel \overline{DC}$ $\therefore \angle A + \angle D = 180^\circ$

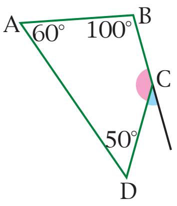
(← Sum of co-interior angles)

$\therefore x^\circ + 3x^\circ = 180^\circ$ $\quad \therefore \cancel{4}x^\circ = \cancel{180^\circ}^{45}$ $\quad \therefore x^\circ = 45^\circ$

Since $\angle D = \angle B$, $3x^\circ = 3 \times 45^\circ = 135^\circ = 5y^\circ$

$\therefore y^\circ = \dfrac{\cancel{135^\circ}^{27}}{\cancel{5}} = 27^\circ$ **Ans** (A)

❸ (1) Since the sum of interior angles of quadrilateral ABCD is 360°,



$60^\circ + 100^\circ + \angle C + 50^\circ = 360^\circ$

$210^\circ + \angle C = 360^\circ$

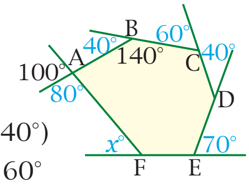
$\therefore \angle C = 360^\circ - 210^\circ = 150^\circ$

∴ The exterior angle of ∠C is

$180^\circ - 150^\circ = 30^\circ$ **Ans** (A)

(2) Since the sum of the exterior angles of any sided polygon is 360°, the average (arithmetic mean) of exterior angles of an octagon is 360° ÷ 8 = 45° **Ans** (A)

(3) Since the sum of the all exterior angles of the hexagon is 360°,



$(180^\circ - 100^\circ) + (180^\circ - 140^\circ)$

$+ 60^\circ + 40^\circ + 70^\circ + x^\circ = 360^\circ$

$\therefore 80^\circ + 40^\circ + 60^\circ + 40^\circ + 70^\circ + x^\circ = 360^\circ$

$\therefore 290^\circ + x^\circ = 360^\circ$ $\quad \therefore x^\circ = 360^\circ - 290^\circ = 70^\circ$

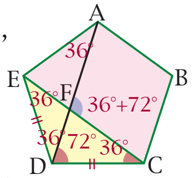
Ans (C)

❹ (1) An interior angle in a regular decagon is

$180^\circ - (360^\circ \div 10) = 180^\circ - 36^\circ = 144^\circ$

Ans (D)

(2) In regular pentagon ABCDE,



$\angle D = 180^\circ - (360^\circ \div 5)$

$= 180^\circ - 72^\circ = 108^\circ$

In isosceles triangle CDE,

$\angle DCE = \dfrac{1}{2}(180^\circ - 108^\circ) = \dfrac{1}{2} \times 72^\circ = 36^\circ$

$\angle FDC = 108^\circ - 36^\circ = 72^\circ$

In triangle CDF, $\angle AFC = 36^\circ + 72^\circ = 108^\circ$

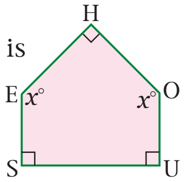
Ans (A)

(3) Since the sum of all exterior angles of n-sided polygon is 360°, each exterior angle of the polygon is $360^\circ \div n = 20^\circ$

$\therefore n = \dfrac{\cancel{360^\circ}^{18}}{\cancel{20^\circ}} = 18$ **Ans** 18

P.212 training

1. Since the sum of the interior angles in a pentagon HOUSE is



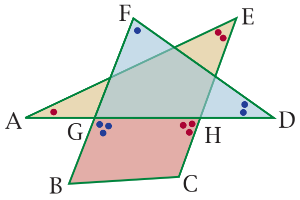
$(5 - 2) \times 180^\circ = 540^\circ$,

$2 \times x^\circ + 3 \times 90^\circ = 540^\circ$

$\therefore 2x^\circ = 540^\circ - 270^\circ = 270^\circ$

$\therefore x = \dfrac{\cancel{270}^{135}}{\cancel{2}} = 135$ **Ans** (D)

2. In triangle AEH,



$\angle A + \angle E = \angle AHC$.

In triangle FDG,

$\angle F + \angle D = \angle DGB$

$\therefore \angle A + \angle B + \angle C + \angle D + \angle E + \angle F$

$= (\angle A + \angle E) + \angle B + \angle C + (\angle D + \angle F)$

$= \angle AHC + \angle B + \angle C + \angle DGB$ (← Sum of interior angles of quadrilateral GHCB)

$= (4 - 2) \times 180^\circ = 360^\circ$ **Ans** (B)

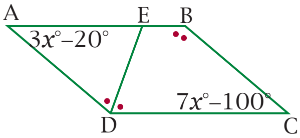
P.214 the melting zone

2. Since the sum of the interior angles in n-sided polygon is $(n-2)\times 180°$,

$(n-2)\times 18\cancel{0}° = 216\cancel{0}°$ $\therefore n-2 = \dfrac{\cancel{216}^{12}}{\cancel{18}}$

$\therefore n-2 = 12$ $\therefore n = 12+2 = 14$ **Ans** (B)

3.



In parallelogram ABCD, $\angle A = \angle C$

$\therefore 3x° - 20° = 7x° - 100°$ $\therefore \cancel{4}x° = \cancel{80}^{20}°$ $\therefore x° = 20°$

$\therefore \angle A = 3x° - 20° = 3\times 20° - 20° = 40°$

Since $\overline{AB} \parallel \overline{DC}$, $\angle A + \angle D = 180°$

(← Sum of co-interior angles)

$\therefore \angle D = 180° - 40° = 140°$

Since DE bisects $\angle ADC$, $\angle EDC = \dfrac{1}{2}\times 140° = 70°$

Since $\angle EDC + \angle DEB = 180°$

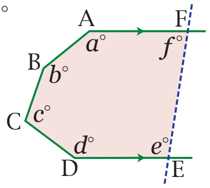
(← Sum of co-interior angles),

$\angle DEB = 180° - \angle EDC = 180° - 70° = 110°$

Ans (A)

4. Since $\overline{AF} \parallel \overline{DE}$, $\angle F + \angle E = 180°$

(← Sum of co-interior angles)



In hexagon ABCDEF,

$a° + b° + c° + d° + (e° + f°)$

$= a° + b° + c° + d° + 180°$

$= (6-2)\times 180°$

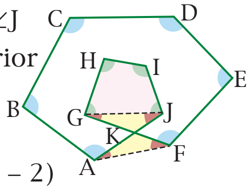
$\therefore a + b + c + d = 4\times 180° - 180°$

$= 720° - 180° = 540°$

$\therefore$ The average(arithmetic mean) of a, b, c and d is $540 \div 4 = 135$ **Ans** (C)

5. In the figure, $\angle JGK + \angle GJK = \angle KAF + \angle KFA$.

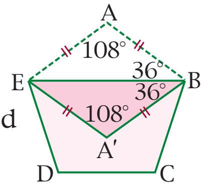
The sum of 10 angles of $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G + \angle H + \angle I + \angle J$ is equal to the sum of interior angles of hexagon ABCDEF and quadrilateral GHIJ.



$\therefore 180°\times(6-2) + 180°\times(4-2)$

$= 180°\times 4 + 180°\times 2 = 720° + 360° = 1080°$ **Ans** (C)

6. In regular pentagon ABCDE,



$\angle A' = \angle A = [(5-2)\times 180°] \div 5$

$= \dfrac{3\times \cancel{180}°^{36}}{\cancel{5}} = 3\times 36° = 108°$ and

$\triangle A'BE$ is an isosceles triangle,

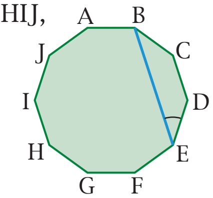
$\angle A'BE = \angle ABE = \dfrac{1}{2}\times(180° - 108°)$

$= \dfrac{1}{2}\times 72° = 36°$

$\therefore \angle A'BG = \angle ABC - \angle ABA' = 108° - 2\times 36°$

$= 108° - 72° = 36°$ **Ans** (D)

7. In regular decagon ABCDEFGHIJ, each interior angle is



$[(10-2)\times 180°] \div 10$

$= \dfrac{8\times 180°}{10} = 8\times 18° = 144°$

If $\angle BED = x°$,

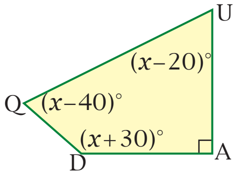
since quadrilateral BCDE is an isosceles trapezoid, $2x° + 2\times 144 = 360°$

$\therefore 2x° = 360° - 288° = 72°$

$\therefore \angle BED = x° = \dfrac{72°}{2} = 36°$ **Ans** (D)

P.215 JUMP

1. In quadrilateral QUAD,



$\angle Q + \angle U + \angle A + \angle D = 360°$

$\therefore (x-40)° + (x-20)°$

$+ 90° + (x+30)° = 360°$

$\therefore 3x° + 60° = 360°$

$\therefore 3x° = 360° - 30° = 300°$ $\therefore x° = 100°$

Since $(x-40)° < (x-20)° < (x+30)°$,

the greatest angle in quadrilateral QUAD is

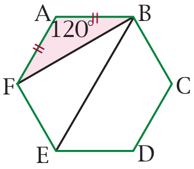
$\angle D = x° + 30° = 100° + 30° = 130°$ **Ans** (D)

2. Since the sum of interior angles of a pentagon is $180°\times(5-2) = 180°\times 3 = 540°$,

$a° + b° + c° + d° + e° + f° = 540° - 120° = 420°$

$\therefore$ The average (arithmetic mean) of a, b, c, d, e and f is $420 \div 6 = 70$. **Ans** (D)

3. Since an interior angle of the regular hexagon is



$[(6-2)\times 180°] \div 6 = \dfrac{4\times \cancel{180}°^{30°}}{\cancel{6}}$

$= 4\times 30° = 120°$, $\angle A = 120°$

Since triangle ABF is isosceles,

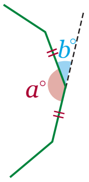
$\angle ABF = \frac{1}{2} \times (180^\circ - 120^\circ)$

$= \frac{1}{2} \times 60^\circ = 30^\circ$ and $\angle ABE = \frac{1}{2} \times 120^\circ = 60^\circ$

$\therefore \angle FBE = \angle ABE - \angle ABF = 60^\circ - 30^\circ = 30^\circ$

Ans (E)

4. In the figure, $a : b = 4 : 1$



If $a = 4k^\circ$, $b = k^\circ$ $\quad \therefore 4k^\circ + k^\circ = 180^\circ$

$\therefore \cancel{5}k^\circ = \cancel{180^\circ}^{36^\circ}$ $\therefore k^\circ = 36^\circ$ $\quad \therefore b^\circ = 36^\circ$

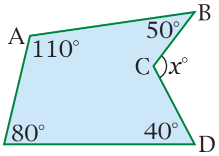
Since the sum of exterior angles of any sided polygon is 360°,

$360^\circ \div n = 36^\circ \quad \therefore n = 10$

∴ The polygon is a regular decagon.

(←10-sided polygon) **Ans** (D)

5. Since the figure ABCDE is a pentagon, the sum of the interior angles is



$110^\circ + 50^\circ + (360^\circ - x^\circ) + 40^\circ + 80^\circ$

$= 280^\circ + (360^\circ - x^\circ) = 640^\circ - x^\circ$

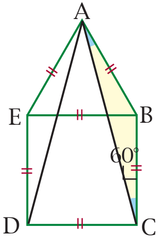
$\therefore 640^\circ - x^\circ = 180^\circ \times (5 - 2) = 3 \times 180^\circ = 540^\circ$

$\therefore x^\circ = 640^\circ - 540^\circ = 100^\circ$ **Ans** 100

6. In the figure, since $\triangle ABE$ is an equilateral triangle and quadrilateral BCDE is a square,

$\angle ABE = 60^\circ$ and $\angle EBC = 90^\circ$.

$\therefore \angle ABC = 60^\circ + 90^\circ = 150^\circ$



Since AB = BC and AE = ED,

$\triangle ABC$ and $\triangle AED$ are isosceles triangles.

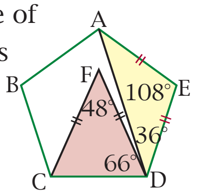
$\therefore \angle BAC = \angle BCA = \angle EAD = \angle EDA$

$= \frac{1}{2}(180^\circ - 150^\circ) = \frac{1}{2} \times 30^\circ = 15^\circ$

$\therefore \angle DAC = \angle A - 2\angle BAC = 60^\circ - 2 \times 15^\circ$

$= 60^\circ - 30^\circ = 30^\circ$ **Ans** 30°

7. In the figure, an interior angle of the regular pentagon ABCDE is



$[(5 - 2) \times 180^\circ] \div 5$

$= \frac{3 \times \cancel{180^\circ}^{36^\circ}}{\cancel{5}} = 3 \times 36^\circ = 108^\circ$

$\triangle AED$ is an isosceles triangle.

$\therefore \angle EDA = \frac{1}{2}(180^\circ - 108^\circ) = \frac{1}{2} \times 72^\circ = 36^\circ$

Since $\triangle CFD$ is an isosceles triangle,

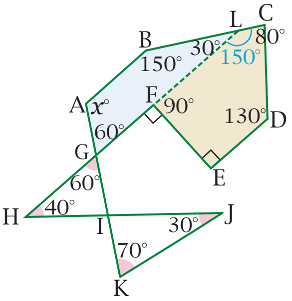
$\angle CDF = \frac{1}{2}(180^\circ - 48^\circ) = \frac{1}{2} \times 132^\circ = 66^\circ$

Since $\angle D = \angle CDF + \angle ADF + \angle EDA = 108^\circ$,

$66^\circ + \angle ADF + 36^\circ = 108^\circ \ \therefore \angle ADF + 102^\circ = 108^\circ$

$\therefore \angle ADF = 108^\circ - 102^\circ = 6^\circ$ **Ans** 6°

8.



In the figure, since $\angle J + \angle K = \angle HGI + \angle H$,

$30^\circ + 70^\circ = \angle HGI + 40^\circ$. $\quad \therefore 100^\circ = \angle HGI + 40^\circ$

$\therefore \angle HGI = 100^\circ - 40^\circ = 60^\circ$

$\therefore \angle AGF = \angle HGI = 60^\circ$ (← Vertical angles)

In pentagon $\angle CDEFL$,

$\angle CLF + 80^\circ + 130^\circ + 90^\circ + 90^\circ = \angle CLF + 390^\circ$

$= 540^\circ$ (← Sum of interior angles in a pentagon)

$\therefore \angle CLF = 540^\circ - 390^\circ = 150^\circ$ (← Straight angle)

$\therefore \angle BLF = 180^\circ - 150^\circ = 30^\circ$ **Ans** 120

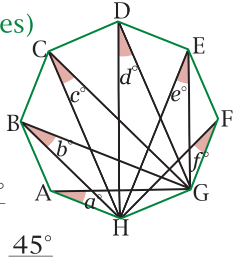
In quadrilateral ABLG,

$x^\circ + 150^\circ + 30^\circ + 60^\circ = 360^\circ$

(← Sum of interior angles in a quadrilateral)

$\therefore x^\circ + 240^\circ = 360^\circ \quad \therefore x^\circ = 360^\circ - 240^\circ = 120^\circ$

9. Since the regular octagon ABCDEFGH is inscribed in a circle, a°, b°, c°, d°, e°, and f° are the same. (← Inscribed angles)



$\angle G = [(8 - 2) \times 180^\circ] \div 8$

$= \frac{\cancel{6}^{3} \times \cancel{180^\circ}^{45^\circ}}{\cancel{8}\cancel{4}} = 3 \times 45^\circ = 135^\circ$

In isosceles triangle HFG,

$f^\circ = \frac{1}{2} \times (180^\circ - 135^\circ) = \frac{45^\circ}{2}$

$\therefore a^\circ = b^\circ = c^\circ = d^\circ = e^\circ = f^\circ = \frac{45^\circ}{2}$

$\therefore a^\circ + b^\circ + c^\circ + d^\circ + e^\circ + f^\circ = \cancel{6}^{3} \times \frac{45^\circ}{\cancel{2}}$

$= 3 \times 45^\circ = 135^\circ$ **Ans** 135

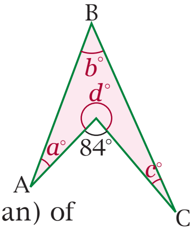
P.216 E·X·A·M·I·N·A·T·I·O·N

1. In quadrilateral ABCD,

$$\begin{cases} a^\circ + b^\circ + c^\circ + d^\circ = 360^\circ \cdots\cdots① \\ 84^\circ + d^\circ = 360^\circ \cdots\cdots② \end{cases}$$

① – ②; $a^\circ + b^\circ + c^\circ - 84^\circ = 0$

$\therefore a^\circ + b^\circ + c^\circ = 84^\circ$



$\therefore$ The average (arithmetic mean) of a, b and c is $(a + b + c) \div 3 = 84 \div 3 = 28$

Ans (D)

2. Since the sum of interior angles in a quadrilateral ABCD is 360°,

$(3x^\circ + 10^\circ) + (x^\circ - 15^\circ) + (4x^\circ - 45^\circ) + (2x^\circ + 50^\circ)$

$= 360^\circ \quad \therefore \cancel{10}x^\circ = \overset{36}{\cancel{360}}{}^\circ \quad \therefore x^\circ = 36^\circ$

$\therefore \angle A = 3 \times 36^\circ + 10^\circ = 72^\circ + 10^\circ = 82^\circ$,

$\angle B = 36^\circ - 15^\circ = 21^\circ$,

$\angle C = 4 \times 36^\circ - 45^\circ = 144^\circ - 45^\circ = 99^\circ$,

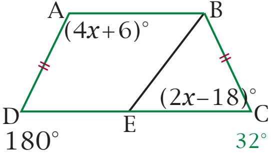
and $\angle D = 2 \times 36^\circ + 50^\circ = 72^\circ + 50^\circ = 122^\circ$

$\therefore 122^\circ$ (= ∠D) is the greatest angle in the quadrilateral ABCD.

Ans (B)

3. In isosceles trapezoid ABCD, $\angle C = \angle D$ and $\angle A + \angle D = 180^\circ$

(← Co-interior angles)



$\therefore \angle A + \angle C = 180^\circ$

$\therefore (4x + 6)^\circ + (2x - 18)^\circ = 180^\circ$

$\therefore 6x^\circ - 12^\circ = 180^\circ \quad \therefore \cancel{6}x^\circ = 180^\circ + 12^\circ = \overset{32^\circ}{\cancel{192}}{}^\circ$

$\therefore x^\circ = 32^\circ$

Since $\overline{BE}$ bisects ∠B, $\angle ABE = \angle EBC = \frac{1}{2}\angle B$

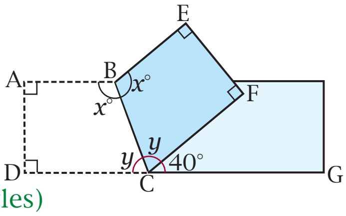
$= \frac{1}{2}\angle A = \frac{1}{2}(4 \times 32 + 6)^\circ = \frac{1}{2} \times 134^\circ = 67^\circ$

Since $\angle ABE + \angle BED = 180^\circ$(←Co-interior angles),

$67^\circ + \angle BED = 180^\circ \quad \therefore \angle BED = 180^\circ - 67^\circ = 113^\circ$

Ans (A)

4.



In the figure,

$\angle ABC = \angle EBC = x^\circ$

(← Symmetric angles)

If $\angle FCB = \angle DCB = y^\circ$(← Symmetric angles),

$y^\circ + y^\circ + 40^\circ = 180^\circ$(← Straight angle)

$\therefore \cancel{2}y^\circ = 180^\circ - 40^\circ = \overset{70^\circ}{\cancel{140}}{}^\circ \quad \therefore y^\circ = 70^\circ$

In trapezoid ABCD,

$x^\circ + y^\circ = 180^\circ$ (← Co-interior angles)

$\therefore x^\circ + 70^\circ = 180^\circ \quad \therefore x = 180^\circ - 70^\circ = 110^\circ$

Ans (D)

5. If the convex polygon has n sides, the sum of all interior angles is $(n - 2) \times 180^\circ$.

$\therefore (n - 2) \times 180 > 2450$

$\therefore n - 2 > \frac{245\cancel{0}}{18\cancel{0}} = 13.6\cdots\cdots$

$\therefore n > 2 + 13.6\cdots\cdots \quad \therefore n > 15.6\cdots\cdots \quad \therefore n = 16$

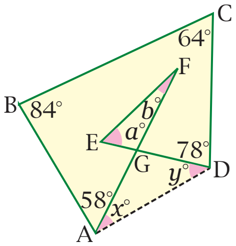
$\therefore$ The remaining angle of the polygon is

$(16 - 2) \times 180^\circ - 2450^\circ = 14 \times 180^\circ - 2450^\circ$

$= 2520^\circ - 2450^\circ = 70^\circ$

Ans (B)

6.



In the figure, if $\angle E = a^\circ$, $\angle F = b^\circ$ $\angle GAD = x^\circ$ and $\angle GDA = y^\circ$, $a^\circ + b^\circ = x^\circ + y^\circ$.

$\therefore$ In quadrilateral ABCD,

$(x^\circ + 58^\circ) + 84^\circ + 64^\circ + (78^\circ + y^\circ) = 360^\circ$

(← Sum of interior angles in quadrilateral ABCD)

$\therefore x^\circ + y^\circ + 284^\circ = 360^\circ$

$\therefore x^\circ + y^\circ = 360^\circ - 284^\circ = 76^\circ \quad \therefore a^\circ + b^\circ = 76^\circ$

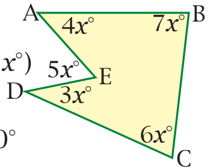
If $a^\circ = 3k^\circ$ and $b^\circ = k^\circ$,

$a^\circ + b^\circ = 3k^\circ + k^\circ = 4k^\circ = 76^\circ \quad \therefore k^\circ = 19^\circ$

$\therefore a^\circ = 3k^\circ = 3 \times 19^\circ = 57^\circ \quad \therefore \angle E = 57^\circ$

Ans (D)

7. In pentagon ABCDE, the sum of the interior angles is



$4x^\circ + 7x^\circ + 6x^\circ + 3x^\circ + (360^\circ - 5x^\circ)$

$= (5 - 2) \times 180^\circ$

$\therefore 15x^\circ + 360^\circ = 3 \times 180^\circ = 540^\circ$

$\therefore \cancel{15}x^\circ = 540^\circ - 360^\circ = \overset{12}{\cancel{180}}{}^\circ \quad \therefore x = 12^\circ$

Ans (A)

8. Since the sum of the exterior angles of any sided regular polygon is 360°, an exterior angle of a regular polygon is a factor of 360°

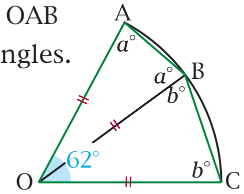
(A) $18^\circ = 360^\circ \div 20$ (B) $24^\circ = 360^\circ \div 15$

(C) $32^\circ \neq 360^\circ \div n$ (D) $45^\circ = 360^\circ \div 8$

(E) $72^\circ = 360^\circ \div 5$

Ans (C)

9. In the figure, two triangles OAB and OBC are isoscelces triangles.



In quadrilateral OABC,

if $\angle OAB = \angle OBA = a^\circ$

and $\angle OBC = \angle OCB = b^\circ$,

$2(a^\circ + b^\circ) + 62^\circ = 360^\circ$

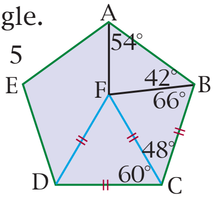
(← Sum of interior angles in a quadrilateral)

$\therefore\ 2(a^\circ + b^\circ) = 360^\circ - 62^\circ = 298^\circ$

$\therefore\ a^\circ + b^\circ = 298^\circ \div 2 = 149^\circ$

$\therefore\ \angle ABC = a^\circ + b^\circ = 149^\circ$ **Ans** (D)

10. In the figure, △FCD is an equilateral triangle.



$\therefore\ \angle FCD = 60^\circ$ and DC = FC = BC.

$\therefore$ △BCF is an isosceles triangle.

Since $\angle C = [(5-2) \times 180^\circ] \div 5$

$= \dfrac{3 \times \cancel{180^\circ}^{36}}{\cancel{5}} = 3 \times 36^\circ = 108^\circ$,

$\angle FCB = 108^\circ - 60^\circ = 48^\circ$

In isosceles triangle CBF,

$\angle CBF = \dfrac{1}{2}(180^\circ - 48^\circ) = \dfrac{1}{2} \times 132^\circ = 66^\circ$

$\therefore\ \angle ABF = 108^\circ - 66^\circ = 42^\circ$

In triangle ABF,

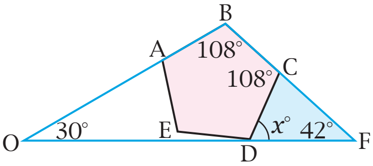
$\angle AEB = 180^\circ - (\dfrac{1}{2} \times 108^\circ + 42^\circ)$

$= 180^\circ - (54^\circ + 42^\circ) = 180^\circ - 96^\circ = 84^\circ$

Ans (D)

11. Since the sum of the interior angles of a n-sided polygon is $n \times x^\circ$, the sum of the interior angles of a $(n + 3)$-sided polygon is $n \times x^\circ + 3 \times 180^\circ = nx^\circ + 540^\circ$ **Ans** (D)

12.



Since $\angle B$ is an interior angle of a regular pentagon ABCDE is $[(5-2) \times 180^\circ] \div 5$

$= \dfrac{3 \times \cancel{180^\circ}^{36^\circ}}{\cancel{5}} = 3 \times 36^\circ = 108^\circ$

In triangle OBF, $\angle BFO = 180^\circ - (30^\circ + 108^\circ)$

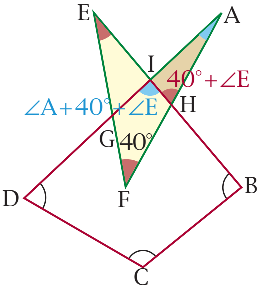
$= 180^\circ - 138^\circ = 42^\circ$

In triangle CDF, $x^\circ + 42^\circ = 108^\circ$

$\therefore\ x^\circ = 108^\circ - 42^\circ = 66^\circ$ **Ans** (D)

13. In triangle EFH, $\angle IHA = 40^\circ + E^\circ$

In triangle AHI, $\angle HIG = \angle A + (40^\circ + \angle E)$



In quadrilateral BCDI,

$\angle B + \angle C + \angle D + (\angle A + 40^\circ + \angle E) = 360^\circ$

$\therefore\ \angle B + \angle C + \angle D + \angle A + \angle E = 360^\circ - 40^\circ$

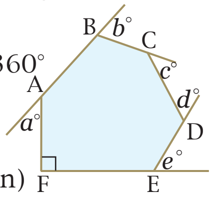
$= 320^\circ$

$\therefore$ The average (arithmetic mean) of $\angle A$, $\angle B$, $\angle C$, $\angle D$ and $\angle E$ is

$(\angle A + \angle B + \angle C + \angle D + \angle E) \div 5 = 320^\circ \div 5$

$= 64^\circ$ **Ans** (B)

14. Since the sum of all exterior angles of the polygon is 360°,



$a^\circ + b^\circ + c^\circ + d^\circ + e^\circ + 90^\circ = 360^\circ$

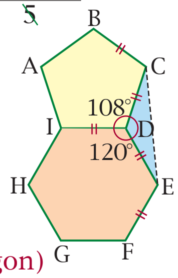
$\therefore\ a^\circ + b^\circ + c^\circ + d^\circ + e^\circ$

$= 360^\circ - 90^\circ = 270^\circ$

The average(arithmetic mean) of a, b, c, d and e is $270 \div 5 = 54$ **Ans** (D)

15. In regular pentagon ABCDI,

$\angle IDC = [(5-2) \times 180^\circ] \div 5 = \dfrac{3 \times \cancel{180^\circ}^{36^\circ}}{\cancel{5}}$



$= 3 \times 36^\circ = 108^\circ$

In regular hexagon IDEFGH,

$\therefore\ \angle IDE = [(6-2) \times 180^\circ] \div 6$

$= \dfrac{4 \times \cancel{180^\circ}^{30^\circ}}{\cancel{6}} = 4 \times 30^\circ = 120^\circ$

$\therefore\ \angle CDE = 360^\circ - (108^\circ + 120^\circ)$

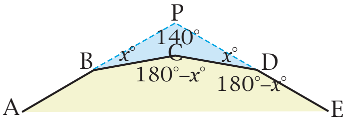
$= 360^\circ - 228^\circ = 132^\circ$ (← Perigon)

Since CD = DE, △DEC is an isosceles triangle.

$\angle DCE = \dfrac{1}{2} \times (180^\circ - 132^\circ) = \dfrac{1}{2} \times 48^\circ = 24^\circ$

Ans (C)

16.



In the figure, if $\angle PDC = \angle PBC = x^\circ$,

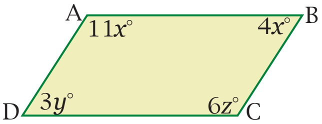
$\angle CDE = 180^\circ - x^\circ$ (← exterior angle)

In quadrilateral BPDC, $x^\circ + 140^\circ + x^\circ$ $= 180^\circ - x^\circ$ $\therefore 3x^\circ = 40^\circ$ $\therefore x^\circ = \frac{40^\circ}{3}$

Since an exterior angle of the regular polygon is $\frac{40^\circ}{3}$, the number of sides of the polygon is $360^\circ \div \frac{40^\circ}{3} = 360^\circ \times \frac{3}{40^\circ} = 9 \times 3$ $= 27$

Ans (E)

17.



Since $\overline{AD} \parallel \overline{BC}$, $\angle A + \angle B = 180^\circ$

(← Sum of co-interior angles)

$\therefore 11x^\circ + 4x^\circ = 180^\circ$ $\therefore \cancel{15}x^\circ = \overset{12}{\cancel{180}}^\circ$ $\therefore x^\circ = 12^\circ$

Since $\angle B = \angle D$, $3y^\circ = 4x^\circ = 4 \times 12^\circ = 48^\circ$

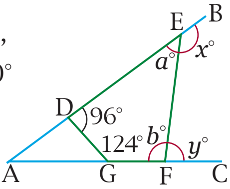
$\therefore y^\circ = \frac{48^\circ}{3} = 16^\circ$

Since $\angle A = \angle C$, $6z^\circ = 11x^\circ = 11 \times 12^\circ = 132^\circ$

$\therefore z^\circ = \frac{132^\circ}{6} = 22^\circ$ $\therefore y^\circ + z^\circ = 16^\circ + 22^\circ = 38^\circ$

Ans 38

18. In quadrilateral DEFG,



if $\angle DEF = a^\circ$ and $EFG = b^\circ$,

$a^\circ + b^\circ + 124^\circ + 96^\circ = 360^\circ$

(← Sum of the interior angles in quadrilateral)

$\therefore a^\circ + b^\circ + 220^\circ = 360^\circ$

$\therefore a^\circ + b^\circ = 360^\circ - 220^\circ = 140^\circ$ ······ ①

Since $a^\circ + x^\circ = 180$ and $b^\circ + y^\circ = 180^\circ$

(← Straight angles).

$\therefore a^\circ + b^\circ + x^\circ + y^\circ = 360^\circ$ ······ ②

① → ②; $140^\circ + x^\circ + y^\circ = 360^\circ$

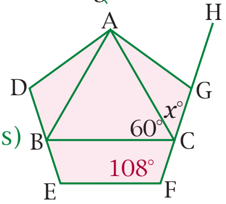
$\therefore x^\circ + y^\circ = 360^\circ - 140^\circ = 220^\circ$

Ans 220

19. (1) In regular pentagon AGFED,

$\angle F = [(5 - 2) \times 180^\circ] \div 5 = \frac{3 \times \cancel{180^\circ}^{36}}{\cancel{5}}$

$= 3 \times 36 = 108^\circ$



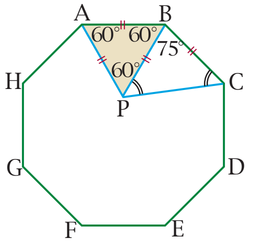
Since $\overline{BC} \parallel \overline{EF}$,

$\angle GCB = \angle F = 108^\circ$

(← Corresponding angles)

$x^\circ = 108^\circ - 60^\circ = 48^\circ$

(2)



In the figure, since triangle ABP is equilateral, $\angle ABP = 60^\circ$

In regular octagon ABCDEFGH,

$\angle B = [(8 - 2) \times 180^\circ] \div 8 = \frac{\cancel{6}^{3} \times \cancel{180^\circ}^{45}}{\cancel{8}\,4}$

$= 3 \times 45^\circ = 135^\circ$

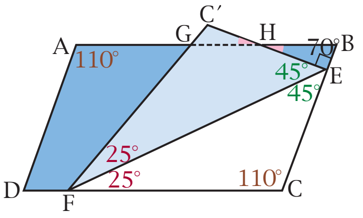
$\therefore \angle PBC = \angle B - \angle ABP = 135^\circ - 60^\circ = 75^\circ$

Since BP = BC, $\triangle BCP$ is an isosceles triangle, $\angle BCP = \frac{1}{2}(180^\circ - 75^\circ)$

$= \frac{1}{2} \times 105^\circ = 52.5^\circ$

Ans (1) 48 (2) 52.5°

20.



In the figure,

$\angle EFC = \angle GFE = 25^\circ$ (← Symmetric angles)

Since $\angle C = \angle A = 110^\circ$, in triangle FEC,

$\angle FEC = 180^\circ - (25^\circ + 110^\circ) = 45^\circ$

Since $\angle C'EF = \angle CEF = 45^\circ$(← Symmetric angles),

$\angle HEB = 180^\circ - 2 \times 45^\circ = 180^\circ - 90^\circ = 90^\circ$

(← Straight angle)

Since $\overline{AD} \parallel \overline{BC}$, $\angle B = 180^\circ - 110^\circ = 70^\circ$

(← Co-interior angles)

In triangle HBE, $\angle BHE = 180^\circ - (90^\circ + 70^\circ)$

$= 180^\circ - 160^\circ = 20^\circ$

$\therefore \angle C'HG = \angle BHE = 20^\circ$ (← Vertical angles)

Ans 20°

21. If an interior angle in a polygon has an integer degree, its exterior angle has an integer degree too. Since the sum of the exterior angles in any sided polygon is 360° and the positive factors of 360 are 1, 2, 3, 4, ······, 180 and 360, each exterior angle of a

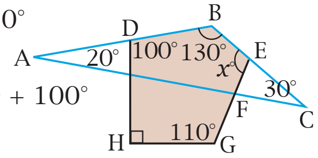
regular polygon could be 1°, 2°, 3°, 4°, ……, 120° (← Except 180° and 360°).
Since the number of positive factors of 360 is (3 + 1)(2 + 1)(1 + 1) = 4 × 3 × 2 = 24 [← $360 = 2^3 \cdot 3^2 \cdot 5$], the number of possible exterior angles of regular polygons is 24 – 2 = 22.

∴ 22 regular polygons have integer degrees as their interior angles.

Ans 22

22. (1) In triangle ABC, ∠B = 180° – (20° + 30°)

= 180° – 50° = 130°



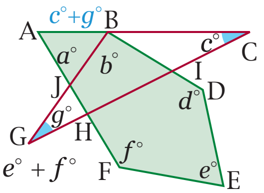
In pentagon BEGHD,

130° + x° + 110° + 90 + 100°

= (5 – 2) × 180°

∴ 430° + x° = 3 × 180° = 540

∴ x° = 540° – 430° = 110°



(2) In triangle CGB,

∠GBA = c° + g°

In pentagon ABDEF,

a° + [(c° + g°) + b°] + d° + e° + f°

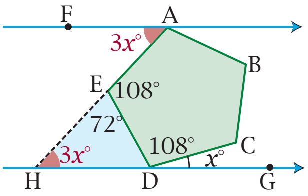
is the sum of all interior angles.

∴ a° + b° + c° + d° + e° + f° + g° = (5 – 2) × 180°

= 3 × 180° = 540°

Ans (1) 110 (2) 540

23.



Since $\overleftrightarrow{FA} \parallel \overleftrightarrow{DG}$, ∠AHD = ∠FAH = 3$x$°

(← Alternate angles)

In regular pentagon ABCDE, ∠AED = ∠EDC

$= [(5 - 2) \times 180°] \div 5 = \frac{3 \times \cancel{180°}^{36°}}{\cancel{5}} = 3 \times 36°$

= 108°

∴ ∠HED = 180° – 108° = 72°

(← Straight angles)

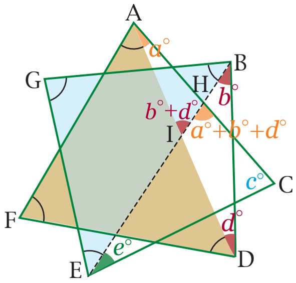
In triangle EDH, ∠EDG = 3x° + 72°

∴ 108° + x° = 3x° + 72°

∴ $\cancel{2}x° = 108° - 72° = \cancel{36°}$ ∴ x° = 18°

Ans 18°

24.



In the figure, the sum of interior angles of two triangles ADF and BEG is 2 × 180° = 360°

In triangle BDI, ∠BIA = b° + d°

In triangle IAH, ∠IHC = a° + (b° + d°)

In triangle HCE, (a° + b° + d°) + e° + c° = 180°

∴ ∠A + ∠B + ∠C + ∠D + ∠E + ∠F + ∠G is the sum of interior angles of the three triangles ADF, BEG and HCE = 3 × 180° = 540°

Ans 540°

6★1 perimeter of polygons

P.223 pattern drill

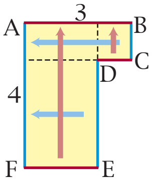
❶ (1) The perimeter of pentagon ABCDE is

AB + BC + CD + DE + EA

= 3 + 4 + 5 + 2 + 4 = 18

Ans (C)

(2) In the figure, CD + EF = AB and BC + DE = AF



∴ The perimeter of the polygon is

AB + BC + CD + DE + EF + AF

= AB + (BC + DE) + (CD + EF) + AF

= AB + AF + AB + AF = 2(AB + AF)

= 2(3 + 4) = 2 × 7 = 14

Ans (B)

(3) Since the sum of lengths of 9 sides is 54, the average(arithmetic mean) of the lengths of 9 sides is 54 ÷ 9 = 6

Ans (B)

❷ (1) Since the 3 sides of an equivalent triangle are the same, each side is

$2\frac{1}{2}$ feet ÷ 3 = $\frac{5}{2}$ feet ÷ 3 = $\frac{5}{6}$ feet

$= \frac{5}{\cancel{6}} \times \cancel{12}^{2}$ inches = 10 inches

Ans (A)