Lesson 29

PASSPORT TO ADVANCED MATH

Zeros & Factors of Polynomials

- Frequency (%) -

0 20 40 60 80 100

Most Valuable Points

MVP ❶ Zeros and factors

For polynomial $p(x)$ if a is a real number and $p(a)=0 \Rightarrow$

(1) a is a **zero** of the polynomial $p(x)$

(2) $(x-a)$ is a **factor** of the polynomial $p(x)$

(3) a is a **x-intercept** of the function $y=p(x)$

(4) $x=a$ is a **solution(root)** of the equation $p(x)=0$

MVP ❷ Function interpretations

(1) The point (a, b) lies on the graph of $y=f(x) \Rightarrow b=f(a)$

(2) The intersection point (a, b) of two graphs of $y=f(x)$ and $y=g(x)$

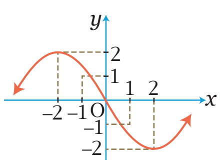
$\Rightarrow$ $x=a$ is the solution of $f(x)=g(x) \Rightarrow$ $f(a)=g(a)=b$

Most Valuable Problems

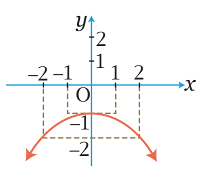
MVP 555 Calculator | ★★☆

For which of the following graphs of h is $h(x)=-|h(x)|$ for all values of x shown?

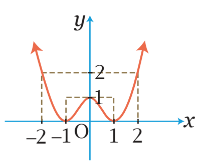
A)



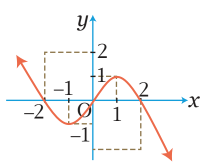
B)



C)



D)



Idea! From $h(x)=-|h(x)|$,

$h(x)+|h(x)|=0 \quad \therefore h(x)\le 0$

$\therefore$ The graph $y=h(x)$ lies under the x-axis.

$\therefore$ The graph of B) satisfies the relation of $h(x)=-|h(x)|$

A ○ B ● C ○ D ○

MVP 556 No calculator | ★★☆

The function f is defined by $f(x)=x^3+bx^2+cx+d$. If the graph of f crosses the x-axis at -2, -1 and 3, then what is the value of c?

A) -3

B) -6

C) -7

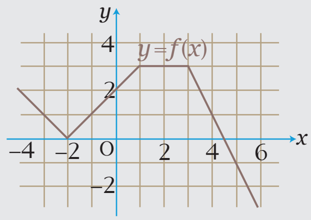
D) -9

A ○ B ○ C ● D ○

Lesson 29

Solutions p.59

MVP 557 ★☆☆



In the graph of $y=f(x)$ above, which of the following represents all values of x for which $f(x)\le 1$?

A) $x\le 3$

B) $-3\le x\le -1,\ x\ge 4$

C) $x\le -3,\ -1\le x\le 4$

D) All real values of x

A B C D

MVP 558 ★★☆

In the xy-plane, the graph of function $f(x)$ has the x-intercepts at -2 and 1 and the y-intercept at 4.

Which of the following could define $f(x)$?

A) $f(x)=2x(x+2)(x-1)$

B) $f(x)=(x-2)^2(x+1)$

C) $f(x)=2(x-2)(x+1)^2$

D) $f(x)=2(x+2)(x-1)^2$

A B C D

MVP 559 ★★☆

$$f(x)=(-2x-2)(x^2+x)$$

How many different zeros does the function $y=f(x)$ have?

A) One

B) Two

C) Three

D) Four

A B C D

MVP 560 ★★☆

For a polynomial $p(x)$, the value of $p(5)$ is 2. Which of the following must be true about $p(x)$?

A) $x-7$ is a factor of $p(x)$.

B) $p(x-2)$ is divisible by $x-5$.

C) $x-5$ is a factor of $p(x)-2$.

D) The remainder when $p(x)$ is divided by $x+5$ is 2.

A B C D

Solutions p.60

MVP 561 ★☆☆

$$p(x)=2x^4-3x^3+x-4$$

Which of the following is a factor of the polynomial of $p(x)$ above?

A) $x+1$

B) $x-1$

C) $x-2$

D) $2x+1$

A B C D

MVP 563 ★★☆

If $f(x)=3x^2+2x-1$, at what x-coordinate do the graphs of $y=f(x)-1$ and $y=f(x-1)$ intersect?

A) 0

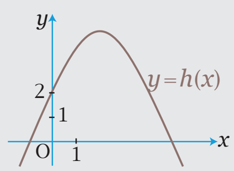
B) $\frac{1}{2}$

C) $\frac{1}{3}$

D) 1

A B C D

MVP 562 ★★★



The figure above shows the graphs of a quadratic function h whose maximum value is $h(2)$. If $h(k)=0$, which of the following could be the value of k?

A) 0

B) 1

C) 4

D) 5

A B C D

MVP 564 ★★☆

If one zero of the function of $f(x)=x^3-6x^2+3x+10$ is 5, then what is the difference of the other two zeros of $f(x)$?

A) 1

B) 2

C) 3

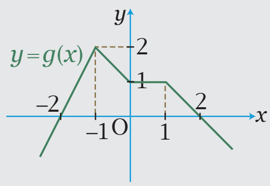
D) 4

A B C D

Lesson 29

Solutions p.60

MVP 565 ★★☆



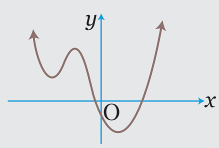
In the graph of $y=g(x)$ above, how many zeros exist in the function of $g(x)-1$?

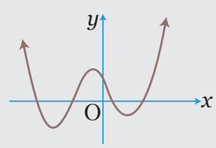
A) None
B) One
C) Two
D) Infinitely many

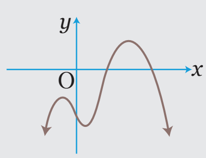
A B C D

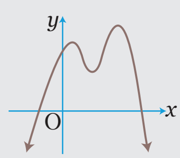
MVP 566 ★★☆

If the function $f(x)=-x^4+bx^3+cx^2+dx+e$ has two distinct positive zeros, which of the following could represent the complete graph of f in the xy-plane?

A)


B)


C)


D)


A B C D

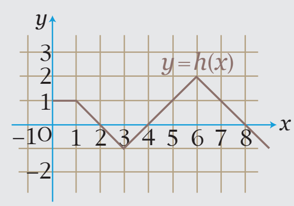
MVP 567 ★★☆

If $x+2$ is a factor of the polynomial $x^3+(k+3)x^2+3(x+1)-2k$, then what is the value of k?

A) $\frac{1}{2}$
B) $-\frac{1}{2}$
C) $\frac{3}{2}$
D) $-\frac{3}{2}$

A B C D

MVP 568 ★★★



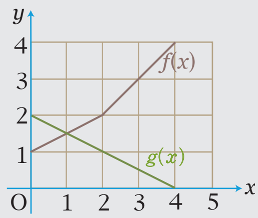
The figure above shows the graph of $y=h(x)$, where h is a function. How many distinct positive integer values of c satisfy the equation $h(c)=h(2c)$?

A) None
B) One
C) Two
D) Three

A B C D

Solutions p.61

MVP **569** ★★★



The graphs of the functions f and g are lines as shown above.

For which values of x does $f(x) \leq 2g(x)$?

A) $0 \leq x \leq 1$

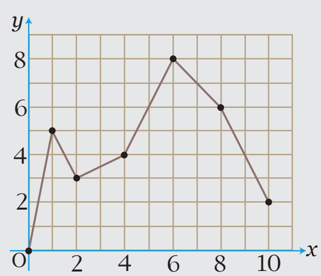
B) $0 \leq x \leq 2$

C) $2 \leq x \leq 4$

D) $3 \leq x \leq 4$

A B C D
○ ○ ○ ○

MVP **570** ★★☆



The graph of $y = g(x)$ is shown above.

If $g(1) = k$, which of the following values of x such that $g(x) > g(k)$?

A) $0 \leq x < 0.5$

B) $5 < x < 8$

C) $4.5 < x < 8.5$

D) $8 < x \leq 10$

A B C D
○ ○ ○ ○

MVP **571** ★★★

How many distinct zeros does the graph of $f(x) = x^4 - 2x^3 + 2x - 1$ have?

A) One

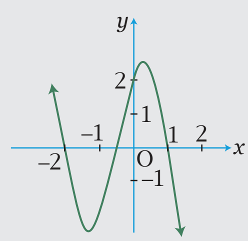
B) Two

C) Three

D) Four

A B C D
○ ○ ○ ○

MVP **572** ★★☆



The polynomial function shown in the graph crosses the x-axis at $x = -2$, $x = -\frac{1}{2}$ and $x = 1$. If the equation for this polynomial is written in the form $y = ax^3 + bx^2 + cx + d$, which of the following could be the equation?

A) $y = -2x^3 - x^2 + x + 2$

B) $y = -2x^3 + x^2 - x + 2$

C) $y = -2x^3 + 3x^2 - 3x - 2$

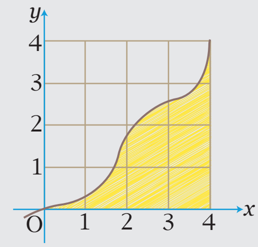
D) $y = -2x^3 - 3x^2 + 3x + 2$

A B C D
○ ○ ○ ○

Lesson 29

Solutions p.61

MVP 573 ★★★



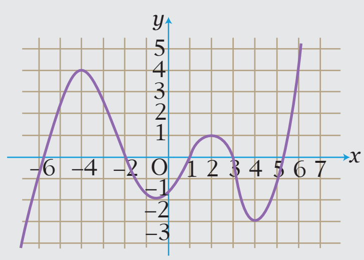
The shaded region in the figure above is bounded by the x-axis, the line $x=4$ and the graph of $y=f(x)$.
If the point (a, b) lies in the shaded region, which of the following must be true?

Ⅰ. $a \le 4$	Ⅱ. $b \le a$	Ⅲ. $b \le f(a)$

A) Ⅰ only
B) Ⅱ only
C) Ⅰ and Ⅱ only
D) Ⅰ, Ⅱ and Ⅲ

A B C D

MVP 574 ★★☆



The figure above shows the graph of $y=f(x)$ for all values of x. If the equation $f(x)=k$ has distinct four solutions, what is the sum of all integer values of k?

A) -1
B) -2
C) 1
D) 2

A B C D

MVP 575 ★★★

$$g(x)=6x^3-x^2-x$$
$$h(x)=3x^2+x$$

The polynomials g(x) and h(x) are defined above. Which of the following polynomials is divisible by $2x+1$?

A) $g(x)$
B) $h(x)$
C) $g(x)+2h(x)$
D) $g(x)-2h(x)$

A B C D

MVP 576 ★★☆

The cost in dollars, c, of producing a custom-made T-shirt with a team logo is given by the formula $c=110+\frac{x}{2}$, where x is the number of T-shirts produced.
When every T-shirt produced is sold, the revenue from selling the customized T-shirts is given by $R=15x-\frac{x^2}{10}$.
Which one of the following would be the formula for the profit from producing and selling x T-shirts?

A) $-\frac{x^2}{10}+\frac{29}{2}x-110$
B) $-\frac{x^2}{10}+\frac{31}{2}x-110$
C) $\frac{x^2}{10}-\frac{29}{2}x+110$
D) $\frac{x^2}{10}-\frac{31}{2}x-110$

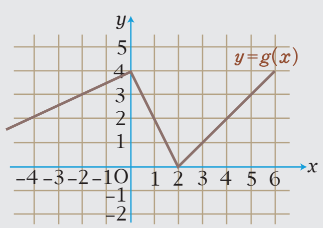
A B C D

Solutions p.62

MVP 577 ★★☆

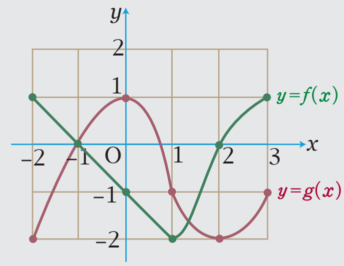
What is the x-intercept of the graph of $f(x)=x^3-5x^2+2x-10$?

MVP 578 ★★★



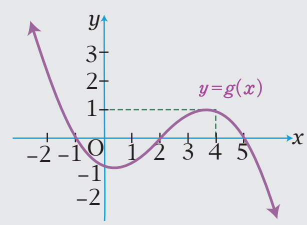
The figure above shows the graph of $y=g(x)$. If the function h is defined $h(x)=g(2x)-1$, what is the sum of all values of x such that $h(2x)=g(4)$?

MVP 579 ★★★



The functions f and g graphed above are defined for $-2 \le x \le 3$.
What is the sum of all values of x that satisfy the equation $f(x)-2g(x)=0$?

MVP 580 ★★☆



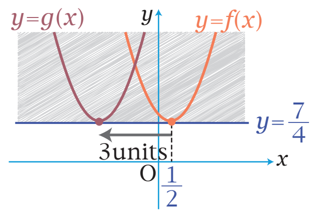
The graph above shows the function g, where $g(x)=a(x+1)(x-2)(x-5)$ for some constant a. If $g(k+3.2)=0$ and $k>0$, then what is the values of k?

551 From $f(x)=x^2-x+2=\left(x-\frac{1}{2}\right)^2+\frac{7}{4}$

$\therefore$ The vertex point of $y=f(x)$ is $\left(\frac{1}{2}, \frac{7}{4}\right)$

From $g(x)=f(x+3)$,

$y=f(x) \xrightarrow{\text{Translate left 3 units}} y=g(x)$



Since the range of $y=g(x)$ is unchanged,

$g(x)\geq\frac{7}{4}$ (← The least possible value)

Ans $\frac{7}{4}(=1.75)$

552 From the circle $x^2+y^2+2y-3=0$,

$x^2+(y^2+2y+1)-4=0 \;\; \therefore \;\; x^2+(y+1)^2=2^2$

$\therefore$ The center of the circle is $(0, -1)$ and the radius is 2.

If the center $(0, -1)$ is translated 1 unit up, the new center is $(0, 0)$.

If the radius of 2 is increased by 50%,

the new radius is $2\left(1+\frac{50}{100}\right)=2\cdot\frac{3}{2}=3$

$\therefore$ The equation of the resulting circle is $x^2+y^2=3^2$

From $x^2+y^2+ax+by=c$,

$a=0$, $b=0$ and $c=3^2=9$

$\therefore a+b+c=0+0+9=9$ **Ans** 9

553 $y=x^2 \xrightarrow{\text{Reflect the } x\text{-axis}} -y=x^2 \;\; \therefore y=x^2$

$\xrightarrow{\text{1 unit left, 5 units up}} y-5=-(x+1)^2$

$\therefore y=-(x+1)^2+5=-x^2-2x+4$

From $\begin{cases} y=x^2 \\ y=-x^2-2x+4, \end{cases}$

$x^2=-x^2-2x+4 \quad \therefore 2x^2+2x-4=0$

$\therefore x^2+x-2=0 \quad \therefore (x+2)(x-1)=0$

$\therefore x=-2, 1$

$\therefore$ The intersection points of the two parabolas are $P(-2, 4)$ and $Q(1, 1)$

$\therefore$ The distance of two points of intersection P and Q is

$d=\sqrt{(1-(-2))^2+(1-4)^2}=\sqrt{3^2+(-3)^2}$

$=\sqrt{18}=3\sqrt{2}. \quad \therefore d^2=18$ **Ans** 18

554 From $f(x)=f(x+5)$, $y=f(x)$ is a periodic function with a period of 5.

In the graph, $y=f(x)$ has four different roots $\left(x=\frac{1}{3}, \frac{3}{2}, \frac{7}{3}, \frac{7}{2}\right)$ where $0\leq x\leq 5$.

$\therefore (3\times4+3=)15$ x-values satisfy $f(x)=0$ where $0\leq x\leq 18$. **Ans** 15

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556 Since the function $f(x)$ crosses the x-axis at -2, -1 and 3, $f(x)$ has $x+2$, $x+1$ and $x-3$ as its factors.

$\therefore f(x)=(x+2)(x+1)(x-3)$

$=(x^2+3x+2)(x-3)=x^3-7x-6$

$=x^3+bx^2+cx+d$

$\therefore b=0$, $c=-7$ and $d=-6$ **Ans** C)

557 From $f(x)\leq 1$, $y=f(x)$ is below the graph of $y=1$.

In the graph, $f(x)\leq 1$ when $-3\leq x\leq -1$ and $x\geq 4$. **Ans** B)

558 Since the function $f(x)$ has x-intercepts at -2 and 1, f has $x+2$ and $x-1$ as its factors.

Since $f(x)$ has y-intercept at 4, $f(0)=4$

$\therefore f(x)=2(x+2)(x-1)^2$ $[=2x^3+\cdots\cdots+4]$ could be the function. **Ans** D)

559 From $f(x)=(-2x-2)(x^2+x)$,

$f(x)=-2(x+1)\cdot x(x+1)=-2x(x+1)^2$

$\therefore y=f(x)$ has two different zeros such as $x=0, -1$. (←Two zeros) **Ans** B)

560 Since $p(5)=2$, the remainder when $p(x)$ is divided by $x-5$ is 2.

$\therefore x-5$ is a factor of $p(x)-2$. **Ans** C)

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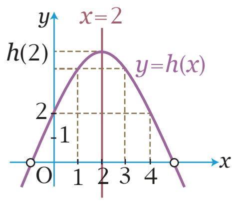
561 From $p(x)=2x^4-3x^3+x-4$,
$p(-1)=2(-1)^4-3(-1)^3+(-1)-4$
$=2+3-1-4=0$
$\therefore$ $p(x)$ has a factor of $x+1$. **Ans** A)

562 Since $h(2)$ is the maximum value in the graph, the axis of symmetry is $x=2$.
Since $h(1)=h(3)>2$,
$h(0)=h(4)=2$,
$h(-1)=h(5)$ could be the value of zero.

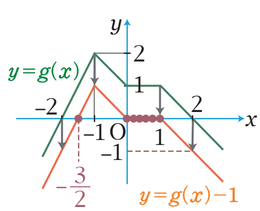


Ans D)

563 From $f(x)=3x^2+2x-1$,
$f(x)-1=3x^2+2x-2$
and $f(x-1)=3(x-1)^2+2(x-1)-1$
$=3(x^2-2x+1)+2x-2-1=3x^2-4x$
From $3x^2+2x-2=3x^2-4x$, $6x=2$
$\therefore x=\dfrac{1}{3}$
$\therefore$ The x-coordinate of the intersection point of two graphs is $\dfrac{1}{3}$. **Ans** C)

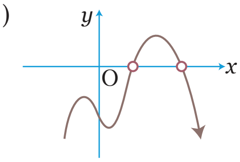
564 Since one zero of the function $f(x)=x^3-6x^2+3x+10$ is 5,
$f(x)$ has a factor of $x-5$.
$\therefore f(x)=(x-5)(x^2-x-2)$
$=(x-5)(x-2)(x+1)$
$\therefore x=5$, 2 and -1
$\therefore$ The difference of the other two zeros is $(2-(-1)=)$ 3. **Ans** C)

565 In the graph,
$g(x)-1=0$ where
$x=-\dfrac{3}{2}$ and
$0\le x\le 1$



$\therefore$ The function of $y=g(x)-1$ has infinitely many zeros. **Ans** D)

566 Since the coefficient of the 4th degree term is negative, the graph is finally downward.
Since the function has two distinct positive zeros, the graph intersects the x-axis at two points as shown below.
C)

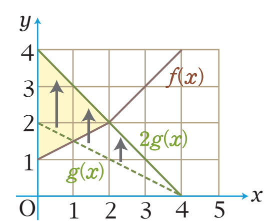


Ans C)

567 Since $x+2$ is a factor of the polynomial $x^3+(k+3)x^2+3(x+1)-2k$,
$x=-2$ is a solution of the equation.
$\therefore$ $(-2)^3+(k+3)(-2)^2+3(-2+1)-2k=0$
$\therefore$ $-8+4(k+3)-3-2k=0$
$\therefore$ $-8+4k+12-3-2k=0$ $\therefore$ $2k=-1$
$\therefore$ $k=-\dfrac{1}{2}$ **Ans** B)

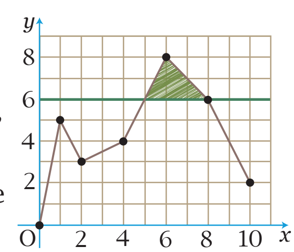
568 In the graph of $y=h(x)$,
i) $f(2)=f(4)=0$ where $c=2$
ii) $f(4)=f(8)=0$ where $c=4$
From i) and ii), $c=2$ and 4
[← Two values of c] **Ans** C)

569 In the graph,
$f(x)\le 2g(x)$
where $0\le x\le 2$.



Ans B)

570 In the graph,
$g(1)=5$ $\therefore$ $k=5$
From $g(x)>g(5)$,
where $g(5)=6$
$\therefore$ $g(x)>6$ where
$5<x<8$



Ans B)

571 From $f(x)=x^4-2x^3+2x-1$,

$$\begin{aligned}f(x)&=(x^4-1)-2x(x^2-1)\\&=(x^2-1)(x^2+1)-2x(x^2-1)\\&=(x^2-1)(x^2-2x+1)\\&=(x-1)(x+1)(x-1)^2=(x-1)^3(x+1)\end{aligned}$$

$\therefore$ $f(x)$ has distinct two zeros such as 1 and -1. **Ans** B)

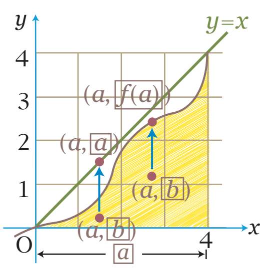
572 Since the x-intercepts of the graph is $x=-2$, $x=-\frac{1}{2}$ and $x=1$, the equation has factors of $x+2$, $2x+1$ and $x-1$.

If $x\to\infty$, then $y=-\infty$ $\quad\therefore a<0$

$$\begin{aligned}\therefore y&=-(x+2)(2x+1)(x-1)\\&=-(2x^2+5x+2)(x-1)\\&=-(2x^3+3x^2-3x-2)\end{aligned}$$

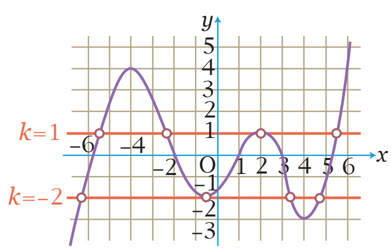
$\therefore y=-2x^3-3x^2+3x+2$ **Ans** D)

573



In the graph, the given statements Ⅰ, Ⅱ and Ⅲ must be true. **Ans** D)

574



In the graph, $y=f(x)$ and $y=k$ have different four intersection points, $k=-2$ and $k=1$

$\therefore$ The sum of all values of k is $(-2+1=)$ -1. **Ans** A)

575 $g(x)=6x^3-x^2-x=x(6x^2-x-1)$

$\quad=x(3x+1)(2x-1)$ ……①

$h(x)=3x^2+x=x(3x+1)$ ……②

From ① and ②, $g(x)=h(x)(2x-1)$

$\therefore g(x)+2h(x)=h(x)(2x-1)+2h(x)$

$\quad=h(x)(2x-1+2)=(2x+1)h(x)$

$\therefore g(x)-2h(x)=(2x-1)h(x)-2h(x)$

$\quad=h(x)(2x-1-2)=(2x-3)h(x)$

$\therefore g(x)+2h(x)$ is divisible by $2x+1$.

Ans C)

576 Since the Profit is 「(Revenue)−(Cost)」,

$$p(x)=15x-\frac{x^2}{10}-\left(110+\frac{x}{2}\right)$$

$$=-\frac{x^2}{10}+\frac{29}{2}x-110$$ **Ans** A)

577 Since the x-intercept of $f(x)$ is the value of x such that $f(x)=0$,

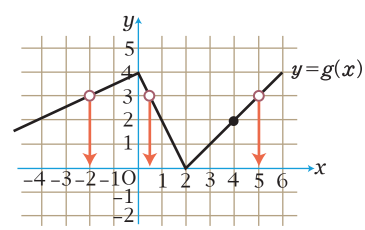
$x^3-5x^2+2x-10=0$

Since the possible rational roots of the equation are $x=\pm(1, 2, 5, 10)$, if $x=5$,

$5^3-5\cdot5^2+2\cdot5-10=125-125+10-10=0$

$\therefore x=5$ **Ans** 5

578



In the graph, $g(4)=2$

From $h(2x)=g(4)$, $h(2x)=2$

Since $h(2x)=g(2\cdot2x)-1=g(4x)-1$,

$g(4x)-1=2\quad\therefore g(4x)=3$

In the graph, $4x=-2$, $\frac{1}{2}$ and 5

$\therefore x=-\frac{1}{2}$, $\frac{1}{8}$ and $\frac{5}{4}$

$\therefore$ The sum of all values of x is

$\left(-\frac{1}{2}+\frac{1}{8}+\frac{5}{4}=\right)\frac{7}{8}$ **Ans** $\frac{7}{8}(=0.875)$

Lesson 29

579 From the graphs of f and g,

x	$f(x)$	$g(x)$	$f(x)-2g(x)$
-2	1	-2	$1-2(-2)=5$
-1	0	0	$0-2\cdot 0=0$
0	-1	1	$(-1)-2\cdot 1=-3$
1	-2	-1	$-2-2(-1)=0$
2	0	-2	$0-2(-2)=4$
3	1	-1	$1-2(-1)=3$

On the table above, $f(x)-2g(x)=0$ where $x=-1$ and 1

$\therefore$ The sum of all values of x such that $f(x)-2g(x)=0$ is $(-1)+1=0$

Ans 0

580 In the graph of $g(x)=a(x+1)(x-2)(x-5)$, the x-intercepts are $x=-1$, 2 and 5.

$\therefore\ g(-1)=g(2)=g(5)=0$

From $g(k+3.2)=0$, $k+3.2=-1$, 2 and 5

$\therefore\ k=-4.2,\ -1.2$ and 1.8

Since $k>0$, $k=1.8$ Ans $1.8\left(=\frac{9}{5}\right)$

Lesson 30

Lesson 30 PASSPORT TO ADVANCED MATH
Ratiomal / Radical Equations

582 From the equation $\sqrt{x+8}+\sqrt{x-8}=8$,

$(\sqrt{x+8}+\sqrt{x-8})^2=8^2$

$\therefore\ x+8+2\sqrt{(x+8)(x-8)}+x-8=64$

$\therefore\ 2x+2\sqrt{x^2-64}=64\quad \therefore\ \sqrt{x^2-64}=32-x$

$\therefore\ x^2-64=((32-x)^2=)1024-64x+x^2$

$\therefore\ 64x=(1024+64=)1088\quad \therefore\ x=17$

$\therefore\ \sqrt{5x-4}=\sqrt{5\cdot 17-4}=\sqrt{85-4}=9$

Ans A)

583 From the equation $\frac{64}{x-8}+7=11$,

$\frac{64}{x-8}=4\quad \therefore\ \frac{16}{x-8}=1\quad \therefore\ x-8=16$

$\therefore\ x=24$

$\therefore\ \frac{64}{x+8}+7=\frac{64}{24+8}+7=\frac{64}{32}+7=2+7=9$

Ans D)

584 From the equation $\frac{1}{y}-\frac{1}{y+1}=\frac{1}{y+3}$,

$\frac{y+1-y}{y(y+1)}=\frac{1}{y+3}\quad \therefore\ \frac{1}{y(y+1)}=\frac{1}{y+3}$

$\therefore\ y^2+y=y+3\quad \therefore\ y^2=3\quad \therefore\ y=\pm\sqrt{3}$

Since $y>0$, $y=\sqrt{3}$ Ans B)

585 From $\frac{2d^2-d-10}{d^2+7d+10}=\frac{d^2-4d+3}{d^2+2d-15}$,

$\frac{(2d-5)(d+2)}{(d+2)(d+5)}=\frac{(d-1)(d-3)}{(d+5)(d-3)}$

$\therefore\ \frac{2d-5}{d+5}=\frac{d-1}{d+5}\quad \therefore\ 2d-5=d-1\ \therefore\ d=4$

Ans B)

586 From $\frac{2x+1}{2x-5}-\frac{2x+3}{2x-7}=0$, $\frac{2x+1}{2x-5}=\frac{2x+3}{2x-7}$

$\therefore\ (2x+1)(2x-7)=(2x+3)(2x-5)$

$\therefore\ 4x^2-12x-7=4x^2-4x-15\quad \therefore\ 8x=8$

$\therefore\ x=1$ Ans D)

587 From $\frac{x}{2x-1}-\frac{2}{4x^2-1}=\frac{3}{2x+1}$,

$\frac{x}{2x-1}-\frac{2}{(2x+1)(2x-1)}=\frac{3}{2x+1}$.

$\therefore\ x(2x+1)-2=3(2x-1)$

$\therefore\ 2x^2+x-2=6x-3\quad \therefore\ 2x^2-5x+1=0$

Since $x\neq\pm\frac{1}{2}$, the sum of two values of x is $\left(-\frac{-5}{2}=\right)\frac{5}{2}$. Ans D)

588 From $\frac{2}{x+1}-\frac{x}{3}=1$, $\frac{2\cdot 3-x(x+1)}{3(x+1)}=1$

$\therefore\ 6-x^2-x=3x+3\quad \therefore\ x^2+4x-3=0$

$\therefore\ x=-2\pm\sqrt{2^2-1\cdot(-3)}=-2\pm\sqrt{7}$

$\therefore\ x_1+x_2=(-2+\sqrt{7})+(-2-\sqrt{7})=-4$

$\left[\leftarrow \frac{-4}{1}=-4\right]$ Ans D)

589 From $\frac{1}{x-3}\leq 1$, $(x-3)^2\cdot\frac{1}{x-3}\leq 1\cdot(x-3)^2$

$\therefore\ (x-3)\leq(x-3)^2\ (x\neq 3)$

$\therefore\ (x-3)^2-(x-3)\geq 0$

$\therefore\ (x-3)(x-3-1)\geq 0\ \therefore\ (x-3)(x-4)\geq 0$