CONCEPT

2 ★ 3 laws of exponents(2) | EXPONENTS

Main Theory & Concept

1 Same exponents

Same exponents → Power → [(i) Keep the base (ii) Multiply the exponents]

$\underbrace{(a^m)\times(a^m)\times(a^m)\times\cdots\times(a^m)}_{(n\text{ times})} \Rightarrow (a^m)^n = a^{m\times n}$

* $\underbrace{9^3\times9^3\times9^3\times9^3}_{(4)} \Rightarrow (9^3)^4 = [(3^2)^3]^4 = 3^{2\times3\times4} = 3^{24}$

2 Power of a product

$\underbrace{(a\cdot b)\times(a\cdot b)\times(a\cdot b)\times\cdots\times(a\cdot b)}_{(n\text{ times})} \Rightarrow (a\cdot b)^n = a^n\cdot n^n$

* $\underbrace{(3\cdot4)\times(3\cdot4)\times(3\cdot4)\times(3\cdot4)}_{(4)} \Rightarrow (3\cdot4)^4 = (3\cdot2^2)^4 = 3^4\cdot2^8$

3 Power of a quotient

$\underbrace{\left(\frac{a}{b}\right)\times\left(\frac{a}{b}\right)\times\left(\frac{a}{b}\right)\times\cdots\times\left(\frac{a}{b}\right)}_{(n\text{ times})} \Rightarrow \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

* $\underbrace{\frac{8}{9}\times\frac{8}{9}\times\frac{8}{9}\times\frac{8}{9}\times\frac{8}{9}}_{(5)} = \left(\frac{8}{9}\right)^5 = \left(\frac{2^3}{3^2}\right)^5 = \frac{2^{15}}{3^{10}}$

4 Applications

Prime factorization

ⓐ (Composite number)$^n \Rightarrow$ (prime · prime · ⋯ · prime)^n

* $72^4 \Rightarrow (2^3\cdot3^2)^4 = 2^{3\cdot4}\cdot3^{2\cdot4} = 2^{12}\cdot3^8$

ⓑ (Monomial)$^n \Rightarrow$ sign^n · number^n (Coefficient) · letters^n

* $(-2x^3y^4)^5 \Rightarrow (-2)^5\cdot(x^3)^5\cdot(y^4)^5 = -32x^{15}y^{20}$

Tip from Top

1 Power of a power

ⓐ $a^{10}\times a^{10}\times a^{10}$
$\Rightarrow (a^{10})^3 = a^{10\times3} = a^{30}$

ⓑ $125^4+125^4+125^4+125^4+125^4$
$\Rightarrow 5\times125^4 = 5\times(5^3)^4$
$= 5\times5^{3\times4} = 5^{1+12} = 5^{13}$

$a^{mn} \Rightarrow (a^m)^n = (a^n)^m$
$3^{12} \Rightarrow (3^2)^6 = (3^3)^4 = (3^4)^3 = (3^6)^2$

* $a^{2x} \Rightarrow a^{2\cdot x} = (a^2)^x = (a^x)^2$, $a^{x^2} \Rightarrow a^{x\cdot x} = (a^x)^x$

2 Power of a product

$a^n\cdot b^n \Rightarrow (a\cdot b)^n$
$2^5\cdot3^5 \Rightarrow (2\cdot3)^5 = 6^5$
$(0.4)^5\times(2.5)^5$
$\Rightarrow \left(\frac{2}{5}\right)^5\cdot\left(\frac{5}{2}\right)^5 = \left(\frac{2}{5}\cdot\frac{5}{2}\right)^5 = 1^5 = 1$

3 Power of a quotient

$\frac{a^n}{b^n} \Rightarrow \left(\frac{a}{b}\right)^n$
$\frac{6^{10}}{3^{10}} \Rightarrow \left(\frac{6}{3}\right)^{10} = 2^{10} = 1024$
$333^4 \div 111^4$
$\Rightarrow \frac{333^4}{111^4} = \left(\frac{333}{111}\right)^4 = 3^4 \Rightarrow 81$

Simple Sample

1. Which of the following is equivalent to one-quarter of 8^{2n}?

(A) 8 (B) 2^{2n} (C) 4^{n-1}
(D) 4^{3n-1} (E) 8^{n-2}

2. What is the simplified value of $\frac{2222^4}{4444^2}$?

(A) 1 (B) 16 (C) $\frac{1}{4}$
(D) $\frac{1}{1111}$ (E) 1111^2

3. If $\frac{(36^6)^4}{(48^4)^3} = 81^n$, then what is the value of n?

(A) 6 (B) 7 (C) 8
(D) 9 (E) 10

Essence

1. $\frac{1}{4}\times8^{2n} = 2^{-2}\times(2^3)^{2n}$
$= 2^{-2}\times2^{3\times2n} = 2^{-2}\times2^{6n} = 2^{6n-2}$
$= 2^{2(3n-1)} = (2^2)^{3n-1} = 4^{3n-1}$
Ans (D)

2. $\frac{2222^4}{4444^2} = \frac{(2222)^2\cdot(2222)^2}{(4444)^2}$
$= \left(\frac{2222}{4444}\right)^2\cdot(2222)^2 = \left(\frac{1}{2}\right)^2\cdot2222^2$
$= \left(\frac{1}{2}\cdot2222\right)^2 = 1111^2$ Ans (E)

3. $\frac{(36^6)^4}{(48^4)^3} = \frac{((2^2\cdot3^2)^6)^4}{((2^4\cdot3)^4)^3} = \frac{(2^2\cdot3^2)^{24}}{(2^4\cdot3)^{12}}$
$= \frac{2^{48}\cdot3^{48}}{2^{48}\cdot3^{12}} = 3^{36} = (3^4)^9 = 81^9$
$\therefore n = 9$ Ans (D)

1 Power of a power ①

$\underbrace{a^m \cdot a^m \cdot a^m \cdot \cdots \cdot a^m}_{(n\text{ times})} = (a^m)^n$

$(a^m)^n \Rightarrow a^{m \times n}$

$\Rightarrow a^{mn} = (a^m)^n = (a^n)^m$

Solve for x in the equation $\dfrac{(4^3)^2 \times (4^5)^6}{(8^4)^5} = 16^x$.

(A) $x = 3$ (B) $x = 4$

(C) $x = 5$ (D) $x = 6$

(Sol) $\dfrac{(4^3)^2 \times (4^5)^6}{(8^4)^5} = \dfrac{4^{3\times2} \times 4^{5\times6}}{8^{4\times5}} = \dfrac{4^6 \times 4^{30}}{8^{20}} = \dfrac{4^{36}}{8^{20}}$

$= \dfrac{(2^2)^{36}}{(2^3)^{20}} = \dfrac{2^{2\times36}}{2^{3\times20}} = \dfrac{2^{72}}{2^{60}} = 2^{12} = (2^4)^3 = 16^x \quad \therefore x = 3$

Ans (A)

2 Power of a power ②

$a^{mn} \Rightarrow (a^m)^n = (a^n)^m$

$\Rightarrow (a^m)^n = a^{mn}$

If $2^{3x-2} = 0.002$, then what is the value of 4^x?

(A) 0.04 (B) 0.4

(C) 0.25 (D) 0.5

(Sol) Since $2^{3x-2} = 2^{3x} \div 2^2 = (2^x)^3 \div 4 = 0.002$,

$(2^x)^3 = 0.002 \times 4 = 0.008 = (0.2)^3$

$\therefore (2^x)^3 = (0.2)^3 \quad \therefore 2^x = 0.2$

$\therefore 4^x = (2^2)^x = (2^x)^2 = (0.2)^2 = 0.04$

Ans (A)

3 Power of a product

$a^m \times b^m \Rightarrow (a \times b)^m$

$\Rightarrow (ab)^n = a^n b^n$

If $1^{-3} + 2^{-3} + 3^{-3} + \cdots\cdots + 33^{-3} = A$, then what is the value of $3^{-3} + 6^{-3} + 9^{-3} + \cdots\cdots + 99^{-3}$?

(A) 9A (B) 27A

(C) $\dfrac{1}{9}$A (D) $\dfrac{1}{27}$A

(Sol) $3^{-3} + 6^{-3} + 9^{-3} + \cdots\cdots + 99^{-3}$

$= (3 \cdot 1)^{-3} + (3 \cdot 2)^{-3} + (3 \cdot 3)^{-3} + \cdots\cdots + (3 \cdot 33)^{-3}$

$= 3^{-3} \cdot 1^{-3} + 3^{-3} \cdot 2^{-3} + 3^{-3} \cdot 3^{-3} + \cdots\cdots + 3^{-3} \cdot 33^{-3}$

$= 3^{-3}(1^{-3} + 2^{-3} + 3^{-3} + \cdots\cdots + 33^{-3}) = \dfrac{1}{3^3}A = \dfrac{1}{27}A$

Ans (D)

4 Power of a quotient

$a^m \div b^m = (a \div b)^m$

$\Rightarrow \left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

What is the value of $\dfrac{\left(\frac{6}{7}\right)^4 \times \left(\frac{6}{7}\right)^4 \times \left(\frac{6}{7}\right)^4}{\left(\frac{2}{7}\right)^6 \times \left(\frac{2}{7}\right)^6}$?

(A) 1 (B) 3

(C) $\dfrac{1}{3^6}$ (D) 3^{12}

(Sol) $\dfrac{\left(\frac{6}{7}\right)^4 \times \left(\frac{6}{7}\right)^4 \times \left(\frac{6}{7}\right)^4}{\left(\frac{2}{7}\right)^6 \times \left(\frac{2}{7}\right)^6} = \dfrac{\left(\left(\frac{6}{7}\right)^4\right)^3}{\left(\left(\frac{2}{7}\right)^6\right)^2} = \dfrac{\left(\frac{6}{7}\right)^{12}}{\left(\frac{2}{7}\right)^{12}} = \left(\dfrac{\frac{6}{7}}{\frac{2}{7}}\right)^{12}$

$= \left(\dfrac{6}{2}\right)^{12} = 3^{12}$

Ans (D)

pattern drill | Ace of Base

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1 Power of a power

(1) Which of the following is not equal to $(a^x)^2$?

(A) $(a^2)^x$ (B) a^{2x} (C) a^{x^2} (D) $a^x \cdot a^x$

(2) If $a^5 = 5$, then what is the value of a^{20}?

(A) 20 (B) 40 (C) 125 (D) 625

(3) Which of the following statements is true?

(A) $\frac{1}{2}(4^3)^2 = 2^6$ (B) $4^{15} = (2^5)^6$ (C) $(-4)^3 = (-2)^6$ (D) $(2^{-2})^{-3} = \frac{1}{2^6}$

(4) What is the value of n such that $\frac{((5^4)^3)^2}{5^4 \cdot 5^3 \cdot 5^2} = (5^n)^3$?

2 Power of power

(1) If $10^{100} = 100^x$, then what is the value of x?

(A) 10 (B) $\sqrt{10}$ (C) 20 (D) 50

(2) What is the positive value of x such that $5^{10} \times 5^{20} \times 5^{30} \times 5^{40} = (5^x)^x$?

(A) 120 (B) 50 (C) 10 (D) 5

(3) What is the positive value of n in the equation $8^3(4(2^3)^2)^4 = 2^n$?

(A) 21 (B) 31 (C) 41 (D) 51

(4) If $2^{3x+1} = 250$, then what is the value of 2^x?

3 Power of a product

(1) Which of the following is equivalent to $2^4 \cdot 3^4$?

(A) 6^4 (B) 5^8 (C) 6^8 (D) 6^{16}

(2) Which of the following is equivalent to $(a^2)^3 \cdot (b^3)^2$?

(A) $(a+b)^5$ (B) $(a+b)^6$ (C) $(ab)^5$ (D) $(ab)^6$

(3) What is the product of $\left(-\frac{1}{5}\right)^{25}$ and 5^{50}?

(A) -5^2 (B) -5^{25} (C) 5^{25} (D) $-\left(\frac{1}{5}\right)^{25}$

(4) What is the value of n in the equation $3^{333} \times 9^{222} \times 27^{111} = (3^n)^{111}$?

4 Power of a quotient

(1) Which of the following is equivalent to $11^n \div 33^n$?

(A) 3^n (B) $\frac{1}{3^n}$ (C) -3^n (D) $\frac{1}{3}$

(2) What is the value of $\left(\frac{a}{b}\right)^{10} \div \left(\frac{b}{a}\right)^{10}$?

(A) 0 (B) 1 (C) $\left(\frac{a}{b}\right)^{20}$ (D) $\left(\frac{b}{a}\right)^{20}$

(3) What is the value of the expression $22^{222} \div 121^{111}$?

(A) 2^{111} (B) 2^{222} (C) 11^{111} (D) 11^{222}

(4) If $0 < a < b$, which of the following is equivalent to $\frac{a^b b^a}{b^b a^a}$?

(A) $\left(\frac{a}{b}\right)^{a-b}$ (B) $\left(\frac{a}{b}\right)^{b-a}$ (C) $\left(\frac{a}{b}\right)^{\frac{b}{a}}$ (D) $(b-a)^{\frac{a}{b}}$

❶ (1) C (2) D (3) B (4) 5 ❷ (1) D (2) C (3) C (4) 5 ❸ (1) A (2) D (3) B (4) 10 ❹ (1) B (2) C (3) B (4) B

Laws of exponents (2)

① $(a^m)^n = (a^n)^m = a^{mn}$ (← Power of a power)

② $(ab)^n = a^n b^n$ (← Power of a product)

③ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ (← Power of a quotient)

spark!

Transformation of bases

▷ Change the base to the form of (prime)^n.

① $\sqrt{2}, 4, \frac{1}{8}, \sqrt[3]{16}, 32, \cdots\cdots \Rightarrow 2^n$ ② $\frac{1}{3}, \sqrt{3}, 9, 27, \sqrt[3]{81}, \cdots\cdots \Rightarrow 3^n$

③ $5\sqrt{5}, 25, \frac{1}{125}, 625, \cdots\cdots \Rightarrow 5^n$ ④ $\sqrt[5]{7}, 49, \frac{1}{343}, \cdots\cdots \Rightarrow 7^n$

CAP If $(a-2)(b-2)(c-2) = 15$, what is the ones(last) digit of the expression $\frac{((3^a)^b)^c \cdot 81^a \cdot 81^b \cdot 81^c}{(9^a)^b \cdot (9^b)^c \cdot (9^c)^a}$?

(A) 1 (B) 3 (C) 5

(D) 7 (E) 9

Accent ⓐ $(a^m)^n = a^{mn}$ ⓑ $(ab)^n = a^n b^n$ ⓒ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Simple Solution

From $(a-2)(b-2)(c-2) = 15$, $abc - 2(ab+bc+ca) + 4(a+b+c) - 8 = 15$

$\therefore abc - 2(ab+bc+ca) + 4(a+b+c) = 23$ ……①

$$\frac{((3^a)^b)^c \cdot 81^a \cdot 81^b \cdot 81^c}{(9^a)^b \cdot (9^b)^c \cdot (9^c)^a} = \frac{3^{abc} \cdot 81^{a+b+a}}{9^{ab} \cdot 9^{bc} \cdot 9^{ca}} = \frac{3^{abc} \cdot 81^{a+b+a}}{9^{ab+bc+ca}}$$

$$= \frac{3^{abc} \cdot (3^4)^{a+b+a}}{(3^2)^{ab+bc+ca}} = \frac{3^{abc} \cdot 3^{4(a+b+a)}}{3^{2(ab+bc+ca)}} = 3^{abc-2(ab+bc+ca)+4(a+b+a)}$$

From ①, $3^{abc-2(ab+bc+ca)+4(a+b+a)} = 3^{23}$

Since $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, $3^5 = 243$, ……, and $23 = 4 \times 5 + 3$,

the ones(last) digit of 3^{23} is 7. *Ans* (D)

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(1) What is the value of x in the equation $(2^{16})^{2^{16}} = 16^x$?

(A) 5 (B) 10 (C) 2^8

(D) 2^{18} (E) 2^{28}

(2) Which of the following is equivalent to $\frac{(a^x)^x}{a^3(a^x)^2}$?

(A) $\frac{1}{a^3}$ (B) a^{x-3} (C) $\left(\frac{a^x}{a}\right)^{x-3}$

(D) $\left(\frac{a^x}{a}\right)^{x+3}$ (E) $(a \cdot a^x)^{x-3}$

KNOW-HOW

Super Model

1 Power of a power ▷ $(a^m)^n = (a^n)^m = a^{mn}$

If $C = 8^{33} + 8^{-36}$ and $D = 8^{33} - 8^{-36}$, then what is the value of $C^2 - D^2$?

(A) $\frac{1}{2}$ (B) $\frac{1}{8}$

(C) $\frac{1}{32}$ (D) $\frac{1}{128}$

$C^2 - D^2 = (C + D)(C - D)$

$= (8^{33} + \cancel{8^{-36}} + 8^{33} - \cancel{8^{-36}})(\cancel{8^{33}} + 8^{-36} - \cancel{8^{33}} + 8^{-36})$

$= 2 \cdot 8^{33} \cdot 2 \cdot 8^{-36} = 2^2 \cdot (2^3)^{33} \cdot (2^3)^{-36} = 2^{2+99-108}$

$= 2^{-7} = \frac{1}{128}$

Ans (D)

2 Power of a product ▷ $(ab)^n = a^n b^n$

Which of the following is equivalent to the expression $\frac{(3^n)^n}{81}$?

I. $\left(\frac{3^n}{9}\right)^{n+2}$ II. $3^{(\frac{n}{2})^2}$ III. $(9 \cdot 3^n)^{n-2}$

(A) I only (B) II only

(C) I and II only (D) I and III only

$\frac{(3^n)^n}{81} = \frac{3^{n^2}}{3^4} = 3^{n^2-4} = 3^{(n+2)(n-2)} = (3^{n+2})^{n-2}$

$= (3^2 \cdot 3^n)^{n-2} = (9 \cdot 3^n)^{n-2}$ (← III)

$= (3^{n-2})^{n+2} = \left(\frac{3^n}{3^2}\right)^{n+2} = \left(\frac{3^n}{9}\right)^{n+2}$ (← I)

and $3^{(\frac{n}{2})^2} = 3^{\frac{n^2}{4}} = \sqrt[4]{3^{n^2}} = \sqrt[4]{(3^n)^n}$

Ans (D)

3 Power of a quotient ▷ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Which of the following is equivalent to the expression $\frac{x^3(x^{a-b-1})^2}{x^{1+2b}}$?

(A) x^{2a} (B) $x^2 \cdot x^a$

(C) $\left(\frac{x^a \cdot x^a}{x^b}\right)^2$ (D) $\left(\frac{x^a}{x^b \cdot x^b}\right)^2$

$\frac{x^3(x^{a-b-1})^2}{x^{1+2b}} = \frac{x^3 \cdot x^{2(a-b-1)}}{x^{1+2b}} = \frac{x^{3+2(a-b-1)}}{x^{1+2b}}$

$= \frac{x^{2a-2b+1}}{x^{1+2b}} = x^{2a-2b+1-(1+2b)} = x^{2a-4b}$

$= x^{2(a-2b)} = (x^{a-2b})^2 = (x^a \div x^{2b})^2$

$= \left(\frac{x^a}{x^{2b}}\right)^2 = \left(\frac{x^a}{x^b \cdot x^b}\right)^2$

Ans (D)

4 Change of a power ▷ $x^n = y \Rightarrow x = y^{\frac{1}{n}}$

If $\frac{3}{a} + \frac{4}{b} = \frac{12}{c}$ and $16^a = 27^b = x^c$, then what is the positive integer value of x?

(A) 6 (B) 12

(C) 18 (D) 36

From $16^a = 27^b = x^c$, $(2^4)^a = (3^3)^b = x^c$

$\therefore 2^{4a} = 3^{3b} = x^c \quad \therefore \begin{cases} 2 = x^{\frac{c}{4a}} \cdots\cdots ① \\ 3 = x^{\frac{c}{3b}} \cdots\cdots ② \end{cases}$

From ① × ②, $2 \times 3 = x^{\frac{c}{4a} + \frac{c}{3b}} = x^{\frac{c}{12}(\frac{3}{a} + \frac{4}{b})}$

Since $\frac{3}{a} + \frac{4}{b} = \frac{12}{c}$, $2 \times 3 = x^{\frac{\cancel{c}}{\cancel{12}} \cdot \frac{\cancel{12}}{\cancel{c}}} \quad \therefore x = 6$

Ans (A)

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SHOWCASE

1 If $a^x = b$, $b^x = c$ and $c^x = d$, then what is the value of $\left((a^{x-\sqrt{2}})^x\right)^{x+\sqrt{2}}$?

(A) $\frac{d}{a}$ (B) $\frac{d}{b^2}$ (C) $\frac{2d}{c}$

(D) cd (E) cd^2

I sea!

$$\left((a^{x-\sqrt{2}})^x\right)^{x-\sqrt{2}} = a^{(x-\sqrt{2})\cdot x\cdot(x+\sqrt{2})} = a^{x(x^2-2)} = a^{x^3-2x} = a^{x^3} \div a^{2x}$$
$$= ((a^x)^x)^x \div (a^x)^2 = (b^x)^x \div b^2 = (c^x) \div b^2 = d \div b^2 = \frac{d}{b^2}$$

Ans (B)

2 What is the value of n which satisfies the expression $128^5 \div 32^n = 8^5$?

(A) 3 (B) 4 (C) 5

(D) 6 (E) 7

3 For positive values a, b, c, ·····, y and z, $\sqrt{a}+\sqrt{a}+\sqrt{a}=\sqrt{b}$, $\sqrt{b}+\sqrt{b}+\sqrt{b}=\sqrt{c}$, $\sqrt{c}+\sqrt{c}+\sqrt{c}=\sqrt{d}$, ····· and $\sqrt{y}+\sqrt{y}+\sqrt{y}=\sqrt{z}$. Express z in terms of a.

(A) $z = 3^{25}a$ (B) $z = 3^{50}a$ (C) $z = 3^{100}a$

(D) $z = (3a)^{25}$ (E) $z = (3a)^{50}$

4 If $3^x = 6$ and $6^x = 18$, then what is the value of 3^{x^2+2x-3} ?

(A) 24 (B) 36 (C) 48

(D) 60 (E) 72

5 If $5^{10} = A$, express $\frac{15^{30}}{45^{15}}$ in terms of A.

(A) $\frac{A^2}{2}$ (B) $\frac{3A}{2}$ (C) $A\sqrt{A}$

(D) $\sqrt[3]{A}$ (E) $\frac{A^2}{3}$

6 Which of the following is equivalent to the expression $\frac{4^{\pi-\sqrt{3}}\cdot 10^{\pi+2\sqrt{3}}}{5^{2\pi+2\sqrt{3}}}$?

(A) $(0.4)^\pi$ (B) $(0.8)^\pi$ (C) $(1.6)^\pi$

(D) $(2.4)^\pi$ (E) $(3.2)^\pi$

7 Solve for x in the equation $2^{16^x} = 16^{2^x}$

(A) $x = \frac{2}{3}$ (B) $x = \frac{3}{4}$ (C) $x = \frac{3}{2}$

(D) $x = -2$ (E) $x = 2$

JUMP

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1 If $f(x) = x^x$, which of the following expressions represents $f(f(x))$?

(A) x^{x^2}

(B) x^{x^x}

(C) $x^{x^{x+1}}$

(D) $x^{(2x)^x}$

(E) $x^{(x+1)^2}$

2 Simplify the expression of $\left(\frac{4}{\sqrt{3}}a^{\frac{7}{4}}b^{-\frac{2}{5}}c^{\frac{1}{2}}\right)^3 \times \left(-\frac{3}{2}a^{\frac{5}{4}}b^{\frac{7}{5}}c^{\frac{1}{6}}\right)^3$

(A) $4\sqrt{3}a^6bc^2$

(B) $8\sqrt{3}a^6bc^2$

(C) $-8\sqrt{3}a^9b^3c^2$

(D) $-24\sqrt{3}a^9b^3c^2$

(E) $72a^{12}b^9c^3$

3 What is the value of $(0.\overline{4})^{-100} \cdot (2.25)^{-99}$?

(A) $\frac{2}{9}$

(B) $\frac{4}{9}$

(C) $\frac{9}{4}$

(D) $-\frac{4}{9}$

(E) $-\frac{9}{4}$

4 If $m = 2n + 3$, which of the following is equivalent to $\frac{(3^m)^m \cdot (81^n)^n}{(81^m)^n}$?

(A) 3^3

(B) $(3^3)^2$

(C) $(3^3)^3$

(D) $(3^3)^4$

(E) $(3^3)^5$

5 What is the value of x that satisfies the equation?

(1) $(3^{12})(2^{15}) = 8 \cdot 6^x$

(2) $2^x = \frac{16^5}{4^3 \cdot 8^4}$

(3) $(2x)^{10} = 12^5 \cdot x^5$

6 (1) What is the value of $\frac{\sqrt{2^7 + 4^3 + 8^2}}{\sqrt[3]{15 \cdot 48 \cdot 300}}$ in simplest form?

(2) What is the value of $\sqrt{\frac{32^{28} + 8^{40} + 4^{80}}{32^{26} + 8^{50} + 4^{55}}}$?

7 (1) If $S = 3^{15} + 9^9 + 27^6 + 81^3$ is divisible by 3^k, what is the greatest integer value of k?

(2) What is the value of n which satisfies $5^{1001}(3^{1002} - 3^{1000}) = n \times 15^{1000}$?

8 What is the value of the expression $\left((2+\sqrt{5})^{99} + (2-\sqrt{5})^{99}\right)^2 - \left((2+\sqrt{5})^{99} - (2-\sqrt{5})^{99}\right)^2$?

9 If $x = t^{\frac{1}{t-1}}$ and $y = t^{\frac{t}{t-1}}$ where $t > 0$ and $t \neq 1$, what is the relation between x and y?

All-Round Checks

EXAMINATION

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1 What is the value of the expression $2^{10} \cdot 5^7 \cdot 7 - 2^{11} \cdot 5^7 - 2^8 \cdot 3 \cdot 5^8$?

(A) 0
(B) 5^4
(C) 10^7
(D) 10^8
(E) 10^9

2 If $\left(\frac{2}{x^2} + \frac{x^2}{2}\right)^3 = 0$, then what is the value of x^{12}?

(A) 8
(B) –8
(C) 16
(D) 64
(E) –64

3 If $x > 0$, which of the following is equal to the expression $(4^x)(8^x)$?

(A) $2^5 \times 2^x$
(B) $(2^x)^5$
(C) $(2^6)^x$
(D) 2^{5x^2}
(E) 2^{6x^2}

4 If $a > 0$ and $b > 0$, $\sqrt{\frac{a}{b}\sqrt{\frac{b}{a}\sqrt{\frac{a}{b}}}} = \left(\frac{a}{b}\right)^k$. What is the value of k?

(A) $\frac{3}{8}$
(B) $\frac{5}{8}$
(C) $\frac{7}{8}$
(D) $-\frac{5}{8}$
(E) $-\frac{7}{8}$

5 If $A = 4^{2^{2^2}}$, $B = ((4^2)^2)^2$ and $C = (4^2)(4^2)(4^2)$, $\frac{A}{BC} = 2^p$. What is the value of p?

(A) 0
(B) 1
(C) 2
(D) 4
(E) 8

6 Which of the following is equivalent to $(x^{x^{x+3}})(x^{2 \cdot x^{x+2}})$?

(A) $(x^{x+1})^{x+2}$
(B) $(x^{x+2})^{x-2}$
(C) $(x^{x-2})^{x+2}$
(D) $(x^{x+2})^{x^{x+2}}$
(E) $(x^{x+3})^{x^{x+2}}$

7 If $A = 1^{-5} + 2^{-5} + 3^{-5} + 4^{-5} + 5^{-5} + \cdots\cdots$ and $B = 1^{-5} + 3^{-5} + 5^{-5} + 7^{-5} + \cdots\cdots$, then what is the value of $\frac{B}{A}$?

(A) $\frac{4}{5}$
(B) $\frac{5}{4}$
(C) $\frac{15}{16}$
(D) $\frac{31}{32}$
(E) $\frac{32}{31}$

8 If $z^x = y^{2x}$ and $2^z = 2(4^x)$, then what is the possible ordered pair for (x, y)?

(A) (1, 3)
(B) (2, –5)
(C) (3, 7)
(D) (4, –3)
(E) (5, 5)

9 Which of the following is equivalent to the expression $\frac{2^{42}}{5^{21}-5^{20}}$?

(A) $(0.2)^{20}$
(B) $(0.4)^{20}$
(C) $(0.6)^{20}$
(D) $(0.8)^{20}$
(E) $(1.25)^{20}$

10 What is the value of $\left((-0.008)^{-0.\overline{6}}\right)^{-0.5}$?

(A) 2
(B) 0.2
(C) –0.2
(D) 0.5
(E) –0.5

11 What is the value of $\left(\frac{8}{4^{\sqrt{3}}}\right)^{3+\sqrt{2}}$?

(A) 4
(B) 8
(C) 16
(D) $\frac{1}{8}$
(E) $\frac{1}{16}$

12 What is the value of the expression $\left(\frac{3}{4}\right)^{-98}\cdot\left(\frac{8}{9}\right)^{-99}\div\left(\frac{2}{3}\right)^{-100}$?

(A) 2
(B) $\frac{1}{2}$
(C) 4
(D) $\frac{1}{4}$
(E) 8

13 If $75^x = \frac{1}{5}$ and $3^y = 25$, what is the value of $\frac{1}{x}+\frac{1}{y}$?

(A) 2
(B) –2
(C) 3
(D) –3
(E) 4

14 Which of the following is equivalent to $\frac{333^{120}}{999^{90}}$?

(A) $\left(\frac{37}{3}\right)^{30}$
(B) $\left(\frac{3}{37}\right)^{30}$
(C) $\left(\frac{3}{37}\right)^{60}$
(D) $\left(\frac{37}{9}\right)^{60}$
(E) $\left(\frac{9}{37}\right)^{60}$

15 What is the value of $(1+\sqrt{2})^{97}(1-\sqrt{2})^{98}(3-2\sqrt{2})^{99}(3+2\sqrt{2})^{100}$?

(A) $1+\sqrt{2}$
(B) $1-\sqrt{2}$
(C) $3-\sqrt{2}$
(D) $3+\sqrt{2}$
(E) $7-4\sqrt{2}$

16 If $a^{2x} = \sqrt{2}-1$, then what is the value of the expression $\frac{a^{3x}+a^{-3x}}{a^x+a^{-x}}$?

(A) $2\sqrt{2}-1$
(B) $2\sqrt{2}+1$
(C) $\sqrt{2}+1$
(D) $2-2\sqrt{2}$
(E) $2\sqrt{2}-2$

17 What is the value of n in the equation?

(1) $(8^{225}+8^{225})(8^{225}+8^{225})(8^{225}+8^{225}) = 2^n$

(2) $2^{8^{n-1}} = 4^{4^{n+1}}$

18 (1) If $4^x = 25$, what is the value of 2^{3x+4}?

(2) If $2b^2 = 6$, what is the value of $(b^5)^2$?

19 (1) What is the positive integer N for which $22^7 \times 55^7 = 10^7 \times N^7$?

(2) For all real values of n, what is the value of a in the equation $4 \times 5^{n-1} \times (3^n + 3^{n+2}) = a \times 15^n$?

20 (1) For positive integers x, y, and z, $2^x = 8^y = 32^z$. What is the value of $\frac{x}{y} + \frac{x}{z}$?

(2) Solve for x and y in the system of equations $\begin{cases} 2^x \times 4^{y-1} = 1 \\ 81^x \div 3^{y+1} = 3 \times 27^y \end{cases}$

21 What is the difference of two values of x which satisfy the equation $\frac{(8^x)^3}{2(4^x)^x} = 256$?

22 If two solutions of the equation $3x^2 - 6x + 2 = 0$ are m and n, then what is the value of $(8^m)^m \cdot (8^n)^n$?

23 (1) If $3^a = 6$ and $9^{a+b} = 48$, then what is the value of 3^{a+2b}?

(2) If $5^a = 10$ and $5^b = 40$, then what is the value of $\frac{5^{2a}}{5^{\frac{2}{3}(b-1)}}$?

24 (1) If $a^{2x} = A$, then $\frac{a^{3x} + a^{-3x}}{a^x - a^{-3x}} = A + \frac{c}{A-1}$. what is the value of c?

(2) If $3^{2x} = A$, express $\frac{3^{5x} + 3^{3x} + 3^x + 3^{-x}}{3^{7x} - 3^{-x}}$ in terms of A.

22. (1) $3^{x+1}-3^{x-1}=3^{x-1}(3^2-1)=3^{x-1}\cdot 8$

$\therefore 8\cdot 3^{x-1}=648 \quad \therefore 3^{x-1}=81$

$\therefore 3^{x-1}=3^4 \quad \therefore x-1=4 \quad \therefore x=5$

(2) $(6.25\times 10^{-3})\times(1.6\times 10^{11})$

$=(6.25\times 10^{-3})\times(16\times 10^{10})$

$=(6.25\times 16)\times(10^{-3}\times 10^{10})$

$=100\times(10^{-3+10})=10^2\times 10^7=10^9$

Ans (1) 5 (2) 10^9

23. $\dfrac{2^{n+4}-2(2^n)}{2(2^{n+3})}=\dfrac{2^{n+4}-2\cdot 2^n}{2\cdot 2^{n+3}}=\dfrac{2^{n+4}-2^{n+1}}{2^{n+4}}$

$=1-\dfrac{1}{2^3}=1-\dfrac{1}{8}=\dfrac{7}{8}$ Ans $\dfrac{7}{8}$

24. Since $f(x)=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}=\dfrac{e^x-\frac{1}{e^x}}{e^x+\frac{1}{e^x}}=\dfrac{e^{2x}-1}{e^{2x}+1}$,

$f(a)=\dfrac{e^{2a}-1}{e^{2a}+1}=\dfrac{1}{2} \quad \therefore 2(e^{2a}-1)=e^{2a}+1$

$\therefore 2\cdot e^{2a}-2=e^{2a}+1 \quad \therefore e^{2a}=3$ ……①

$f(b)=\dfrac{e^{2b}-1}{e^{2b}+1}=\dfrac{1}{3} \quad \therefore 3(e^{2b}-1)=e^{2b}+1$

$\therefore 3\cdot e^{2b}-3=e^{2b}+1 \quad \therefore 2e^{2b}=4$

$\therefore e^{2b}=2$ ……②

From ① and ②, $f(a+b)=\dfrac{e^{2(a+b)}-1}{e^{2(a+b)}+1}$

$=\dfrac{e^{2a}\cdot e^{2b}-1}{e^{2a}\cdot e^{2b}+1}=\dfrac{3\cdot 2-1}{3\cdot 2+1}=\dfrac{6-1}{6+1}=\dfrac{5}{7}$

Ans $\dfrac{5}{7}$

2★3 laws of exponents(2)

P. 95 pattern drill

❶ (1) $(a^x)^2=(a^x)(a^x)=a^{2x}=(a^2)^x$

$*\ a^{x^2}=(a^x)^x$ Ans (C)

(2) $a^{20}=a^{5\times 4}=(a^5)^4=5^4=625$ Ans (D)

(3) (A) $\dfrac{1}{2}(4^3)^2=\dfrac{1}{2}\cdot 4^{3\times 2}=\dfrac{1}{2}\cdot 4^6=2^{-1}\cdot(2^2)^6$

$=2^{-1+12}=2^{11}\neq 2^6$

(B) $4^{15}=(2^2)^{15}=2^{2\times 15}=2^{30}=(2^5)^6$

(C) $(-4)^3=-4^3=-(2^2)^3=-2^6\neq(-2)^6$

(D) $(2^{-2})^{-3}=2^{(-2)\times(-3)}=2^6\neq\dfrac{1}{2^6}$

Ans (B)

(4) $\dfrac{((5^4)^3)^2}{5^4\cdot 5^3\cdot 5^2}=\dfrac{5^{4\times 3\times 2}}{5^{4+3+2}}=\dfrac{5^{24}}{5^9}=5^{15}$

$=(5^5)^3=(5^n)^3 \quad \therefore n=5$ Ans 5

❷ (1) From $10^{100}=100^x$, $10^{100}=(10^2)^x$

$\therefore 10^{100}=10^{2x} \quad \therefore 2x=100 \quad \therefore x=50$

Ans (D)

(2) $5^{10}\times 5^{20}\times 5^{30}\times 5^{40}=5^{10+20+30+40}=5^{100}$

and $(5^x)^x=5^{x\cdot x}=5^{x^2}$

From $5^{100}=5^{x^2}$, $x^2=100 \quad \therefore x=\pm 10$

$\therefore$ The positive value of $x=10$

Ans (C)

(3) $8^3(4(2^3)^2)^4=(2^3)^3(2^2(2^3)^2)^4=2^9(2^2\cdot 2^6)^4$

$=2^9(2^8)^4=2^9\cdot 2^{32}=2^{41} \quad \therefore n=41$

Ans (C)

(4) Since $2^{3x+1}=2^{3x}\cdot 2^1=2\cdot(2^x)^3=250$,

$(2^x)^3=125=5^3 \quad \therefore 2^x=5$ Ans 5

❸ (1) $2^4\cdot 3^4=(2\cdot 3)^4=6^4$ Ans (A)

(2) $(a^2)^3\cdot(b^3)^2=a^6\cdot b^6=(ab)^6$ Ans (D)

(3) $\left(-\dfrac{1}{5}\right)^{25}\cdot 5^{50}=\left(-\dfrac{1}{5}\right)^{25}\cdot 5^{25+25}$

$=\left(-\dfrac{1}{5}\right)^{25}\cdot 5^{25}\cdot 5^{25}=\left(-\dfrac{1}{5}\times 5\right)^{25}\cdot 5^{25}$

$=(-1)^{25}\cdot 5^{25}=-5^{25}$ Ans (B)

(4) $3^{333}\times 9^{222}\times 27^{111}=(3^3\times 9^2\times 27)^{111}$

$=(3^3\cdot(3^2)^2\cdot 3^3)^{111}=(3^{3+4+3})^{111}=(3^{10})^{111}$

$= (3^n)^{111} \quad \therefore n = 10$ Ans 10

❹ (1) $11^n \div 33^n = \frac{11^n}{33^n} = \left(\frac{11}{33}\right)^n = \left(\frac{1}{3}\right)^n = \frac{1}{3^n}$

Ans (B)

(2) $\left(\frac{a}{b}\right)^{10} \div \left(\frac{b}{a}\right)^{10} = \frac{\left(\frac{a}{b}\right)^{10}}{\left(\frac{b}{a}\right)^{10}} = \left(\frac{\frac{a}{b}}{\frac{b}{a}}\right)^{10}$

$= \left(\frac{a^2}{b^2}\right)^{10} = \left(\frac{a}{b}\right)^{20}$

Ans (C)

(3) Since $22^{222} \div 121^{111} = \frac{22^{222}}{121^{111}} = \frac{22^{222}}{(11^2)^{111}}$

$= \frac{22^{222}}{11^{222}} = \left(\frac{22}{11}\right)^{222} = 2^{222}$

Ans (B)

(4) $\frac{a^b b^a}{b^b a^a} = \left(\frac{a}{b}\right)^b \cdot \left(\frac{b}{a}\right)^a = \left(\frac{a}{b}\right)^b \cdot \left(\frac{a}{b}\right)^{-a}$

$= \left(\frac{a}{b}\right)^{b-a}$

Ans (B)

P.96 training

1. From $(2^{16})^{2^{16}} = 16^x$, $(2^{2^4})^{2^{16}} = (2^4)^x$

$\therefore 2^{2^{20}} = (2^4)^x \quad \therefore 2^{2^{20}} = 2^{4x} \quad \therefore 2^{20} = 4x$

$\therefore 2^{20} = 2^2 \cdot x \quad \therefore x = 2^{20-2} = 2^{18}$

Ans (D)

2. $\frac{(a^x)^x}{a^3(a^x)^2} = \frac{a^{x^2}}{a^3 \cdot a^{2x}} = \frac{a^{x^2}}{a^{2x+3}} = a^{x^2-2x-3}$

$= a^{(x-3)(x+1)} = (a^{x+1})^{x-3} = (a \cdot a^x)^{x-3}$

Ans (E)

P.98 the melting zone

2. From $128^5 \div 32^n = 8^5$, $(2^7)^5 \div (2^5)^n = (2^3)^5$

$\therefore 2^{35} \div 2^{5n} = 2^{15} \quad \therefore 2^{35-5n} = 2^{15}$

$\therefore 35 - 5n = 15 \quad \therefore 5n = 20 \quad \therefore n = 4$

Ans (B)

3. Since $3\sqrt{a} = \sqrt{b}$, $b = 9a$

Since $3\sqrt{b} = \sqrt{c}$, $c = 9b = 9(9a) = 9^2 a$

Since $3\sqrt{c} = \sqrt{d}$, $d = 9c = 9(9^2 a) = 9^3 a$

.....

$\therefore 3\sqrt{y} = \sqrt{z} \quad \therefore z = 9y = \cdots\cdots = 9^{25} a$

$\therefore z = (3^2)^{25} a = 3^{50} a$

Ans (B)

4. $3^{x^2+2x-3} = 3^{x^2} \times 3^{2x} \div 3^3 = (3^x)^x \times (3^x)^2 \div 27$

$= 6^x \times 6^2 \div 27 = \frac{18^2 \cdot 36^{12}}{27^3} = 2 \cdot 12 = 24$

Ans (A)

5. $\frac{15^{30}}{45^{15}} = \frac{15^{30}}{(3 \cdot 15)^{15}} = \frac{15^{30}}{3^{15} \cdot 15^{15}} = \frac{15^{15}}{3^{15}}$

$= \left(\frac{15}{3}\right)^{15} = 5^{15} = (5^{10})^{\frac{3}{2}} = A^{\frac{3}{2}} = A^{1+\frac{1}{2}} = A\sqrt{A}$

Ans (C)

6. $\frac{4^{\pi-\sqrt{3}} \cdot 10^{\pi+2\sqrt{3}}}{5^{2\pi+2\sqrt{3}}} = \frac{10^{\pi+2\sqrt{3}} \cdot 4^{\pi-\sqrt{3}}}{5^{\pi+2\sqrt{3}} \cdot 5^{\pi}}$

$= \left(\frac{10}{5}\right)^{\pi+2\sqrt{3}} \frac{4^{\pi-\sqrt{3}}}{5^{\pi}} = 2^{\pi+2\sqrt{3}} (2^2)^{\pi-\sqrt{3}} \cdot \frac{1}{5^{\pi}}$

$= 2^{\pi+2\sqrt{3}} 2^{2\pi-2\sqrt{3}} \cdot \frac{1}{5^{\pi}} = 2^{3\pi} \cdot \frac{1}{5^{\pi}} = \frac{8^{\pi}}{5^{\pi}} = (1.6)^{\pi}$

Ans (C)

7. From $2^{16^x} = 16^{2^x}$, $2^{16^x} = (2^4)^{2^x} \quad \therefore 2^{16^x} = 2^{4 \cdot 2^x}$

$\therefore 16^x = 4 \cdot 2^x \quad \therefore (2^4)^x = 2^2 \cdot 2^x \quad \therefore 2^{4x} = 2^{2+x}$

$\therefore 4x = 2 + x \quad \therefore 3x = 2 \quad \therefore x = \frac{2}{3}$

Ans (A)

P.99 JUMP

1. From $f(x) = x^x$,

$f(f(x)) = f(x^x) = (x^x)^{x^x} = x^{x \cdot x^x} = x^{x^{x+1}}$

Ans (C)

2. $\left(\frac{4}{\sqrt{3}} a^{\frac{7}{4}} b^{-\frac{2}{5}} c^{\frac{1}{2}}\right)^3 \times \left(-\frac{3}{2} a^{\frac{5}{4}} b^{\frac{7}{5}} c^{\frac{1}{6}}\right)^3$

$= \left(\frac{4}{\sqrt{3}} a^{\frac{7}{4}} b^{-\frac{2}{5}} c^{\frac{1}{2}} \times \left(-\frac{3}{2}\right) a^{\frac{5}{4}} b^{\frac{7}{5}} c^{\frac{1}{6}}\right)^3$

$= \left(\frac{4}{\sqrt{3}} \left(-\frac{3}{2}\right) a^{\frac{7}{4}+\frac{5}{4}} b^{-\frac{2}{5}+\frac{7}{5}} c^{\frac{1}{2}+\frac{1}{6}}\right)^3$

EXPONENTS

$= \left(-2\sqrt{3}a^3bc^{\frac{2}{3}}\right)^3 = -8\cdot 3\sqrt{3}a^{3\times 3}b^3c^{\frac{2}{3}\times 3}$

$= -24\sqrt{3}a^9b^3c^2$

Ans (D)

3. $(0.\overline{4})^{-100}\cdot(2.25)^{-99} = \left(\frac{4}{9}\right)^{-100}\cdot\left(2\frac{1}{4}\right)^{-99}$

$= \left(\frac{4}{9}\right)^{-100}\cdot\left(\frac{9}{4}\right)^{-99} = \left(\frac{4}{9}\right)^{-99}\cdot\left(\frac{4}{9}\right)^{-1}\cdot\left(\frac{9}{4}\right)^{-99}$

$= \left(\frac{4}{9}\cdot\frac{9}{4}\right)^{-99}\cdot\left(\frac{4}{9}\right)^{-1} = 1^{-99}\cdot\frac{9}{4} = 1\cdot\frac{9}{4} = \frac{9}{4}$

Ans (C)

4. $\frac{(3^m)^m\cdot(81^n)^n}{(81^m)^n} = \frac{3^{m^2}\cdot((3^4)^n)^n}{((3^4)^m)^n} = \frac{3^{m^2}\cdot 3^{4n^2}}{3^{4mn}}$

$= \frac{3^{m^2+4n^2}}{3^{4mn}} = 3^{m^2-4mn+4n^2} = 3^{(m-2n)^2} = 3^{3^2} = (3^3)^3$

Ans (C)

5. (1) $(3^{12})(2^{15}) = (3^{12})(2^{12})\cdot 2^3 = 2^3(3\cdot 2)^{12}$

$= 8\cdot 6^{12} = 8\cdot 6^x \quad \therefore x = 12$

(2) $2^x = \frac{16^5}{4^3\cdot 8^4} = \frac{(2^4)^5}{(2^2)^3\cdot(2^3)^4} = \frac{2^{20}}{2^6\cdot 2^{12}}$

$= \frac{2^{20}}{2^{6+12}} = \frac{2^{20}}{2^{18}} = 2^2 \quad \therefore x = 2$

(3) From the equation $(2x)^{10} = 12^5\cdot x^5$,

$2^{10}\cdot x^{10} = (2^2\cdot 3)^5\cdot x^5$

$\therefore 2^{10}\cdot x^{10} = 2^{10}\cdot 3^5\cdot x^5 \quad \therefore x^{10} = 3^5\cdot x^5$

$\therefore x^5 = 3^5 \quad \therefore x = 3$

Ans (1) 12 (2) 2 (3) 3

6. (1) $\frac{\sqrt{2^7+4^3+8^2}}{\sqrt[3]{15\cdot 48\cdot 300}} = \frac{\sqrt{2^7+(2^2)^3+(2^3)^2}}{\sqrt[3]{(3\cdot 5)(3\cdot 2^4)(2^2\cdot 3\cdot 5^2)}}$

$= \frac{\sqrt{2^7+2^6+2^6}}{\sqrt[3]{2^6\cdot 3^3\cdot 5^3}} = \frac{\sqrt{2^6(2+1+1)}}{\sqrt[3]{(2^2\cdot 3\cdot 5)^3}}$

$= \frac{\sqrt{2^6\cdot 2^2}}{2^2\cdot 3\cdot 5} = \frac{\sqrt{2^8}}{2^2\cdot 3\cdot 5} = \frac{2^4}{2^2\cdot 3\cdot 5}$

$= \frac{2^2}{3\cdot 5} = \frac{4}{15}$

(2) $\sqrt{\frac{32^{28}+8^{40}+4^{80}}{32^{26}+8^{50}+4^{55}}}$

$= \sqrt{\frac{(2^5)^{28}+(2^3)^{40}+(2^2)^{80}}{(2^5)^{26}+(2^3)^{50}+(2^2)^{55}}}$

$= \sqrt{\frac{2^{140}+2^{120}+2^{160}}{2^{130}+2^{150}+2^{110}}}$

$= \sqrt{\frac{2^{120}(2^{20}+1+2^{40})}{2^{110}(2^{20}+2^{40}+1)}} = \sqrt{\frac{2^{120}}{2^{110}}}$

$= \sqrt{2^{120-110}} = \sqrt{2^{10}} = (2^{10})^{\frac{1}{2}} = 2^5 = 32$

Ans (1) $\frac{4}{15}$ (2) 32

7. (1) $S = 3^{15}+9^9+27^6+81^3$

$= 3^{15}+(3^2)^9+(3^3)^6+(3^4)^3$

$= 3^{15}+3^{18}+3^{18}+3^{12}$

$= 3^{12}(3^3+3^6+3^6+1)$

Since $3^3+3^6+3^6+1$ is not a multiple of 3, the greatest integer value of k is 12.

(2) $5^{1001}(3^{1002}-3^{1000})$

$= 5\times 5^{1000}(3^{1000}(3^2-1))$

$= 5\cdot(3^2-1)\cdot 5^{1000}\cdot 3^{1000} = 5\cdot 8\cdot(5\cdot 3)^{1000}$

$= 40\times 15^{1000} = n\times 15^{1000} \quad \therefore n = 40$

Ans (1) 12 (2) 40

8. If $(2+\sqrt{5})^{99} = A$ and $(2-\sqrt{5})^{99} = B$,

$(A+B)^2-(A-B)^2$

$= (A^2+2AB+B^2)-(A^2-2AB+B^2)$

$= 4AB = 4(2+\sqrt{5})^{99}\cdot(2-\sqrt{5})^{99}$

$= 4[(2+\sqrt{5})(2-\sqrt{5})]^{99} = 4(-1)^{99} = 4(-1) = -4$

Ans -4

9. From $x = t^{\frac{1}{t-1}}$ and $y = t^{\frac{t}{t-1}}$,

$\frac{y}{x} = \frac{t^{\frac{t}{t-1}}}{t^{\frac{1}{t-1}}} = t^{\frac{t}{t-1}-\frac{1}{t-1}} = t^{\frac{t-1}{t-1}} = t$

From $y = t^{\frac{t}{t-1}} = \left(t^{\frac{1}{t-1}}\right)^t = x^t = x^{\frac{y}{x}}$, $y = x^{\frac{y}{x}}$

$\therefore y^x = (x^{\frac{y}{x}})^x \quad \therefore y^x = x^y$

Ans $y^x = x^y$

P.100 E·X·A·M·I·N·A·T·I·O·N

1. $2^{10}\cdot 5^7\cdot 7 - 2^{11}\cdot 5^7 - 2^8\cdot 3\cdot 5^8$
$= 2^8\cdot 5^7(2^2\cdot 7 - 2^3 - 3\cdot 5)$
$= 2^8\cdot 5^7(28 - 8 - 15)$
$= 2^8\cdot 5^7\cdot 5 = 2^8\cdot 5^8 = (2\cdot 5)^8 = 10^8$ Ans (D)

2. From $\left(\frac{2}{x^2} + \frac{x^2}{2}\right)^3 = 0$, $\frac{2}{x^2} + \frac{x^2}{2} = 0$
$\therefore \frac{2}{x^2} = -\frac{x^2}{2}$ $\therefore x^4 = -4$
$\therefore x^{12} = (x^4)^3 = (-4)^3 = -64$ Ans (E)

3. $(4^x)(8^x) = (4\cdot 8)^x = 32^x = (2^5)^x = (2^x)^5$ Ans (B)

4. From $\sqrt{\frac{a}{b}\sqrt{\frac{b}{a}\sqrt{\frac{a}{b}}}} = \left(\frac{a}{b}\right)^k$,
$\left(\frac{a}{b}\left(\frac{b}{a}\left(\frac{a}{b}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(\frac{a}{b}\left(\left(\frac{a}{b}\right)^{-1}\cdot\left(\frac{a}{b}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$
$= \left(\frac{a}{b}\left(\left(\frac{a}{b}\right)^{-\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(\frac{a}{b}\left(\frac{a}{b}\right)^{-\frac{1}{4}}\right)^{\frac{1}{2}} = \left(\left(\frac{a}{b}\right)^{\frac{3}{4}}\right)^{\frac{1}{2}}$
$= \left(\frac{a}{b}\right)^{\frac{3}{8}}$ $\therefore k = \frac{3}{8}$ Ans (A)

5. $A = 4^{2^{2^2}} = 4^{2^4} = 4^{16}$, $B = ((4^2)^2)^2 = 4^{2\cdot 2\cdot 2} = 4^8$
and $C = (4^2)\cdot(4^2)\cdot(4^2) = (4^2)^3 = 4^6$
$\therefore \frac{A}{BC} = \frac{4^{16}}{4^8\cdot 4^6} = \frac{4^{16}}{4^{14}} = 4^2 = (2^2)^2 = 2^4 = 2^p$
$\therefore p = 4$ Ans (D)

6. $(x^{x^{x+3}})(x^{2\cdot x^{x+2}}) = (x^{x^x\cdot x^3})(x^{2\cdot x^{x+2}})$
$= x^{x\cdot x^x\cdot x^2\cdot x^{2\cdot x^{x+2}}} = x^{x^{x+2}\cdot(x+2)} = (x^{x+2})^{x^{x+2}}$ Ans (D)

7. $A = 1^{-5} + 2^{-5} + 3^{-5} + 4^{-5} + 5^{-5} + \cdots\cdots$①
$B = 1^{-5} + 3^{-5} + 5^{-5} + 7^{-5} + \cdots\cdots$②
From ① − ②,
$A - B = 2^{-5} + 4^{-5} + 6^{-5} + \cdots\cdots$
$= (2\cdot 1)^{-5} + (2\cdot 2)^{-5} + (2\cdot 3)^{-5} + \cdots\cdots$
$= 2^{-5}(1^{-5} + 2^{-5} + 3^{-5} + \cdots\cdots) = \frac{1}{32}\cdot A$
From $A - B = \frac{1}{32}A$, $\frac{31}{32}A = B$ $\therefore \frac{B}{A} = \frac{31}{32}$
Ans (D)

8. Since $z^x = y^{2x} = (y^2)^x$, $z = y^2$ ……①
Since $2^z = 2(4^x) = 2(2^{2x}) = 2^{2x+1}$,
$z = 2x + 1$ ……②
From ① and ②, $2x + 1 = y^2$
$\therefore$ If $x = 4$, $y^2 = 9$ $\therefore y = \pm 3$
$\therefore$ The possible ordered pair for (x, y)
$= (4, 3)$, and $(4, -3)$. Ans (D)

9. $\frac{2^{42}}{5^{21} - 5^{20}} = \frac{(2^2)^{21}}{5^{20}(5-1)} = \frac{4^{21}}{4\cdot 5^{20}} = \frac{4^{20}}{5^{20}} = \left(\frac{4}{5}\right)^{20}$
$= (0.8)^{20}$ Ans (D)

10. $((-0.008)^{-0.\overline{6}})^{-0.5} = \left(\left(-\frac{8}{1000}\right)^{-\frac{6}{9}}\right)^{-\frac{1}{2}}$
$= \left(\left(-\frac{2}{10}\right)^3\right)^{-\frac{2}{3}}\right)^{-\frac{1}{2}} = \left(\left(-\frac{1}{5}\right)^{-2}\right)^{-\frac{1}{2}} = \left(\left(\frac{1}{5}\right)^{-2}\right)^{-\frac{1}{2}}$
$= \frac{1}{5} = 0.2$ Ans (B)

11. $\left(\frac{8}{4^{\sqrt{3}}}\right)^{3+2\sqrt{3}} = \left(\frac{2^3}{2^{2\sqrt{3}}}\right)^{3+2\sqrt{3}} = (2^{3-2\sqrt{3}})^{3+2\sqrt{3}}$
$= 2^{9-12} = 2^{-3} = \frac{1}{8}$ Ans (D)

12. $\left(\frac{3}{4}\right)^{-98}\cdot\left(\frac{8}{9}\right)^{-99} \div \left(\frac{2}{3}\right)^{-100}$
$= \left(\frac{4}{3}\right)^{98}\cdot\left(\frac{9}{8}\right)^{99} \div \left(\frac{3}{2}\right)^{100}$
$= \left(\frac{4}{3}\right)^{98} \times \left(\frac{9}{8}\right)^{99} \times \left(\frac{2}{3}\right)^{100}$
$= \left(\frac{4}{3}\right)^{98}\cdot\left(\frac{9}{8}\right)^{98}\cdot\left(\frac{2}{3}\right)^{98}\cdot\frac{9}{8}\cdot\left(\frac{2}{3}\right)^2$
$= \left(\frac{\cancel{4}}{\cancel{3}}\cdot\frac{\cancel{9}}{\cancel{8}}\cdot\frac{\cancel{2}}{\cancel{3}}\right)^{98}\cdot\frac{\cancel{9}}{8}\cdot\frac{4}{\cancel{9}} = \frac{4}{8} = \frac{1}{2}$
Ans (B)

13. i) $75^x = \frac{1}{5} \Rightarrow 75 = \left(\frac{1}{5}\right)^{\frac{1}{x}}$ $\therefore 5^{-\frac{1}{x}} = 75$
$\therefore 5^{\frac{1}{x}} = \frac{1}{75}$

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ii) $3^y = 25 \Rightarrow 3 = (25)^{\frac{1}{y}} \quad \therefore 5^{\frac{2}{y}} = 3$

From i) and ii), $5^{\frac{1}{x}} \times 5^{\frac{2}{y}} = \frac{1}{75} \times 3$

$\therefore 5^{\frac{1}{x}+\frac{2}{y}} = \frac{1}{25} \quad \therefore 5^{\frac{1}{x}+\frac{2}{y}} = 5^{-2} \quad \therefore \frac{1}{x} + \frac{1}{y} = -2$

Ans (B)

14. $\frac{333^{120}}{999^{90}} = \frac{(3 \cdot 111)^{120}}{(9 \cdot 111)^{90}} = \frac{3^{120} \cdot 111^{120}}{9^{90} \cdot 111^{90}}$

$= \frac{3^{120} \cdot 111^{120}}{(3^2)^{90} \cdot 111^{90}} = \frac{3^{120} \cdot 111^{120}}{3^{180} \cdot 111^{90}} = \frac{111^{30}}{3^{60}}$

$= \frac{111^{30}}{(3^2)^{30}} = \left(\frac{111}{9}\right)^{30} = \left(\frac{37}{3}\right)^{30}$

Ans (A)

15. $(1+\sqrt{2})^{97}(1-\sqrt{2})^{98}(3-2\sqrt{2})^{99}(3+2\sqrt{2})^{100}$

$= ((1+\sqrt{2})(1-\sqrt{2}))^{97}(1-\sqrt{2})$

$\cdot((3-2\sqrt{2})(3+2\sqrt{2}))^{99}(3+2\sqrt{2})$

$= (1^2-(\sqrt{2})^2)^{97}(1-\sqrt{2}) \cdot (3^2-(2\sqrt{2})^2)^{99}(3+2\sqrt{2})$

$= (1-2)^{97}(1-\sqrt{2}) \cdot (9-8)^{99}(3+2\sqrt{2})$

$= (-1)^{97}(1-\sqrt{2}) \cdot 1^{99}(3+2\sqrt{2})$

$= -(1-\sqrt{2})(3+2\sqrt{2}) = -(3+2\sqrt{2}-3\sqrt{2}-2(\sqrt{2})^2)$

$= -(3-\sqrt{2}-4) = 1+\sqrt{2}$

Ans (A)

16. $\frac{a^{3x}+a^{-3x}}{a^x+a^{-x}} = \frac{(a^{3x}+a^{-3x}) \cdot a^x}{(a^x+a^{-x}) \cdot a^x} = \frac{a^{4x}+a^{-2x}}{a^{2x}+1}$

$= \frac{(a^{2x})^2 + \frac{1}{a^{2x}}}{a^{2x}+1} = \frac{(\sqrt{2}-1)^2 + \frac{1}{\sqrt{2}-1}}{(\sqrt{2}-1)+1}$

$= \frac{2-2\sqrt{2}+1+\frac{\sqrt{2}+1}{2-1}}{\sqrt{2}} = \frac{3-2\sqrt{2}+\sqrt{2}+1}{\sqrt{2}}$

$= \frac{4-\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}-2}{2} = 2\sqrt{2}-1$

Ans (A)

17. (1) $(8^{225}+8^{225})(8^{225}+8^{225})(8^{225}+8^{225})$

$= (8^{225}+8^{225})^3 = (2 \cdot 8^{225})^3 = (2 \cdot (2^3)^{225})^3$

$= (2 \cdot 2^{675})^3 = (2^{676})^3 = 2^{2028} = 2^n$

$\therefore n = 2028$

(2) From $2^{8^{n-1}} = 4^{4^{n+1}}$, $2^{8^{n-1}} = (2^2)^{4^{n+1}}$

$\therefore 2^{(2^3)^{n-1}} = 2^{2 \cdot (2^2)^{n+1}} \quad \therefore 2^{2^{3n-6}} = 2^{2^{1+2(n+1)}}$

$\therefore 3n-6 = 2n+3 \quad \therefore n = 9$

Ans (1) 2028 (2) 9

18. (1) Since $4^x = (2^2)^x = 2^{2x} = (2^x)^2 = 5^2$, $2^x = 5$

$\therefore 2^{3x+4} = 2^{3x} \cdot 2^4 = 16 \times (2^x)^3 = 16 \times 5^3$

$= 16 \times 125 = 2000$

(2) From $2b^2 = 6$, $b^2 = 3$

$\therefore (b^5)^2 = (b^2)^5 = 3^5 = 243$

Ans (1) 2000 (2) 243

19. (1) $22^7 \times 55^7 = (22 \times 55)^7$

$= (2 \times 11 \times 5 \times 11)^7$

$= [(2 \times 5) \times (11 \times 11)]^7 = (10 \times 121)^7$

$= 10^7 \times 121^7 = 10^7 \times N^7 \quad \therefore N = 121$

(2) From $4 \times 5^{n-1} \times (3^n + 3^{n+2})$

$= 4 \cdot 5^{n-1} \cdot (3^n + 3^2 \cdot 3^n)$

$= 4 \cdot 5^{n-1} \cdot 3^n (1+3^2)$

$= 4 \cdot 5^{n-1} \cdot 3^n \cdot 10 = 4 \cdot (2 \cdot 5) \cdot 5^{n-1} \cdot 3^n$

$= 4 \cdot 2 \cdot (5^n \cdot 3^n) = 8 \times (5 \cdot 3)^n = 8 \times 15^n$

$= a \times 15^n \quad \therefore a = 8$

Ans (1) 121 (2) 8

20. (1) From $2^x = 8^y = 32^z$, $2^x = (2^3)^y = (2^5)^z$

$\therefore 2^x = 2^{3y} = 2^{5z} \quad \therefore x = 3y = 5z$

From $x = 3y$, $\frac{x}{y} = 3$ ……①

From $x = 5z$, $\frac{x}{z} = 5$ ……②

From ① and ②, $\frac{x}{y} + \frac{x}{z} = 3 + 5 = 8$

(2) From $2^x \times 4^{y-1} = 1 \quad \therefore 2^x \times (2^2)^{y-1} = 1$

$\therefore 2^x \cdot 2^{2y-2} = 2^0 \quad \therefore 2^{x+2y-2} = 2^0$

$\therefore x + 2y - 2 = 0$ ……①

From $81^x \div 3^{y+1} = 3 \times 27^y$,

$(3^4)^x \div 3^{y+1} = 3 \times (3^3)^y \quad \therefore 3^{4x-(y+1)} = 3^{1+3y}$

$\therefore 4x - y - 1 = 1 + 3y \quad \therefore 4x - 4y - 2 = 0$

$\therefore 2x - 2y - 1 = 0$ ……②

From ① + ②, $3x - 3 = 0 \quad \therefore x = 1$ ……③

From ③ → ①, $1 + 2y - 2 = 0 \quad \therefore 2y = 1$

$\therefore y = \frac{1}{2} \qquad \therefore (x, y) = \left(1, \frac{1}{2}\right)$

Ans (1) 8 (2) $(x, y) = \left(1, \frac{1}{2}\right)$

21. From the equation $\frac{(8^x)^3}{2(4^x)^x} = 256$,

$\frac{((2^3)^x)^3}{2((2^2)^x)^x} = \frac{(2^{3x})^3}{2 \cdot (2^{2x})^x} = 2^8 \quad \therefore \frac{2^{9x}}{2 \cdot 2^{2x^2}} = 2^8$

$\therefore 2^{9x-(1+2x^2)} = 2^8 \quad \therefore 9x - 1 - 2x^2 = 8$

$\therefore 2x^2 - 9x - 9 = 0 \quad \therefore (2x - 3)(x - 3) = 0$

$\therefore x = \frac{3}{2}, 3$

$\therefore$ The difference of two roots is $3 - \frac{3}{2} = \frac{3}{2}$

Ans $\frac{3}{2}$

22. From the equation $3x^2 - 6x + 2 = 0$,

$\begin{cases} m + n = -\frac{-6}{3} = 2 \\ mn = \frac{2}{3} \end{cases}$

$\therefore (8^m)^m \cdot (8^n)^n = 8^{m^2} \cdot 8^{n^2} = 8^{m^2+n^2} = 8^{(m+n)^2-2mn}$

$= 8^{2^2-2\cdot\frac{2}{3}} = 8^{4-\frac{4}{3}} = 8^{\frac{8}{3}} = (2^3)^{\frac{8}{3}} = 2^8 = 256$

Ans 256

23. (1) $\begin{cases} 3^a = 6 \cdots\cdots① \\ 9^{a+b} = 48 \Rightarrow (3^2)^{a+b} = 3^{2a+2b} = 48 \cdots\cdots② \end{cases}$

From ② ÷ ①, $3^{2a+2b-a} = 48 \div 6$

$\therefore 3^{a+2b} = 8$

(2) $\frac{5^{2a}}{5^{\frac{2}{3}(b-1)}} = \frac{(5^a)^2}{5^{(b-1)\frac{2}{3}}} = \frac{(5^a)^2}{\left(5^b \div 5^1\right)^{\frac{2}{3}}}$

$= \frac{10^2}{\left(\frac{5^b}{5}\right)^{\frac{2}{3}}} = \frac{100}{\left(\frac{40}{5}\right)^{\frac{2}{3}}} = \frac{100}{8^{\frac{2}{3}}} = \frac{100}{(2^3)^{\frac{2}{3}}}$

$= \frac{100}{2^2} = \frac{100}{4} = 25$

Ans (1) 8 (2) 25

24. (1) $\frac{a^{3x} + a^{-3x}}{a^x - a^{-3x}} = \frac{(a^{3x} + a^{-3x}) \cdot a^{3x}}{(a^x - a^{-3x}) \cdot a^{3x}} = \frac{a^{6x} + 1}{a^{4x} - 1}$

$= \frac{(a^{2x})^3 + 1}{(a^{2x})^2 - 1} = \frac{A^3 + 1}{A^2 - 1} = \frac{(A+1)(A^2 - A + 1)}{(A+1)(A-1)}$

$= \frac{A^2 - A + 1}{A - 1} = A + \frac{1}{A - 1} = A + \frac{c}{A - 1}$

$\therefore c = 1$

(2) $\frac{3^{5x} + 3^{3x} + 3^x + 3^{-x}}{3^{7x} - 3^{-x}}$

$= \frac{(3^{5x} + 3^{3x} + 3^x + 3^{-x}) \cdot 3^x}{(3^{7x} - 3^{-x}) \cdot 3^x}$

$= \frac{3^{6x} + 3^{4x} + 3^{2x} + 1}{3^{8x} - 1}$

$= \frac{(3^{2x})^3 + (3^{2x})^2 + 3^{2x} + 1}{(3^{2x})^4 - 1}$

$= \frac{A^3 + A^2 + A + 1}{A^4 - 1} = \frac{A^2(A+1) + (A+1)}{(A^2+1)(A^2-1)}$

$= \frac{(A+1)(A^2+1)}{(A^2+1)(A+1)(A-1)} = \frac{1}{A-1}$

Ans (1) 1 (2) $\frac{1}{A-1}$

3★1 feature

P. 107 pattern drill

❶ (1) Since x is an integer, $2x$ is an even integer.

Ans (C)

(2) Since $n - 2$ is an odd integer,
$(n - 2) + 2 = n$ is the next larger odd integer.

Ans (B)

(3) Since $n - 1$ is a multiple of 3,
$(n - 1) \pm 3$, $(n - 1) \pm 6$, $(n - 1) \pm 9$, ……
are the multiples of 3.
$\therefore n + 2$, $n - 4$, $n + 5$, $n - 7$, $n + 8$, $n - 10$,
…… are the multiples of 3.
$\therefore n - 6$ is not a multiple of 3. Ans (A)

(4) The average of four consecutive odd integers is $\frac{n + (n+2) + (n+4) + (n+6)}{4}$

$= \frac{4n + 12}{4} = n + 3$ Ans (B)

❷ (1) The number of eggs in x dozen is $12 \times x = 12x$.

Ans (C)

(2) Since the speed of the train is 60 miles per hour, the training is traveling.

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