Front Projection: Tessellating the Screen

By Jonathan Erland



Reprinted from the SMPTE Journal March 1986 Issue, Volume 95, Number 3

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In the fabrication of a retroreflective front projection screen for motion-picture composite photography, the variations inherent in the manufactured screen material are diminished by homogenizing. This is accomplished by dividing the stock material into small, symmetrical pieces or tiles, and then tessellating (or tiling) a substrate plane. The shape of the tile is designed to reduce to a minimum the number of conjoined segments and to entirely eliminate straight lines, thus diminishing the evidence of the mosaic pattern.

The art of motion-picture compos-ite photography was significantly advanced by the development in the late 1940s of a retroreflective screen material, commonly known as Scotchlite, manufactured by Minnesota Mining and Manufacturing Co. (3M) This material facilitated such an improvement over the preexisting methods for front projection compositing that in 1968 the Academy of Motion Picture Arts and Sciences awarded an Oscar to its creators and the developers of the process with which it is associated: Phillip V. Palmquist, Dr. Herbert Meyer, and Charles Staffell.

Front projection as a method of composite photography had its beginnings in the work of Walter Thorner around 1932,1 as part of an effort to overcome the shortcomings inherent in the rear projection process. It had been recognized quite early in the evolution of motion-picture art that the medium possessed facilities denied its progenitor, live theater. Having assimilated the fact that the camera, as the eye of the audience, was freed from the constraints of the theater seat and was able to travel not only about the stage, but, out of doors and into the open country, it was soon realized that the same freedom applied to time as well as space. Upon that realization the art and science of motion-picture visual effects was based. From that root evolved a phenomenal diversity of visual imagery, ranging from time-lapse photography of flowering plants and slow-motion hummingbirds to Star Wars.

Presented at the 126th SMPTE Technical Conference in New York (paper No. 126-90) on November 2, 1984. Jonathan Erland is affiliated with Apogee, Inc., Van Nuys, Calif. This article was received March 19, 1985. Copyright © 1986 by the Society of Motion Picture and Television Engineers, Inc.

Among the earliest applications of this new-found ability were such elementary accomplishments as filming the occupants of an automobile beyond which the exterior scenery appeared to be speeding by. The first audiences to witness such scenes were undoubtedly properly impressed by being incorporated into a mobile situation. In the main, they were sufficiently impressed that they most likely failed to notice that the scene flashing by outside the car windows was noticeably less clear and vibrant than the image they were seeing within the auto.

In any case, they were not the least aware of the elaborate staging necessary to create the vision they were witnessing. An automobile, with most of one side cut away, was positioned several feet in front of a motion-picture camera and surrounded by an assortment of lights, reflectors, and black panels or "flags." Beyond the car was a large rear projection screen, set far enough away so as not to permit the lights around the car to fall upon it. Beyond the screen, at a distance roughly equal to the distance from the camera to the screen, sat a film projector equipped with an enormously powerful arc lamp.

Between the camera and the projector were electric cables that enabled both mechanisms to operate synchronously. As the shutter opened in the camera, the shutter on the projector also opened and revealed one frame of a sequence taken in an entirely different time and place. This projected image was then visible on the screen behind the car and in full view of the camera, which duly recorded it again, combining it with the car and its occupants. Two events having occurred at different times and places had thus become a single

entity — presto, composite photography!

Thorner and his peers, however, were concerned that the background scene failed to equal the foreground in quality. The inevitable degradation of the projected image threatened to destroy the illusion they were struggling to maintain. The answer seemed to lie in eliminating the screen as such and rephotographing what amounted to an aerial image. This required a mirror instead of the screen, and a projector placed in the same position the camera occupied.

Two things were obvious: the camera and the projector could not occupy the same space and time, and the mirror would have to be concave in order to refocus the expanding image cone back to the same focal point from whence it originated. The concave mirror was, in fact, built, and the other problem was solved by the ingenious use of a partially silvered mirror, or beamsplitter, placed equidistant, and at 45°, between the camera and projector, which were situated at 90° to each other.

Thorner's efforts were rewarded with a substantially improved image quality, and he deserves credit for having advanced the art another step. But problems remained, principally that it was not possible to move the camera during a shot without also moving the mirror. In addition, there were difficulties in keeping stray light off the mirror, not to mention the substantial expense of building a large concave mirror initially. Consequently, Thorner's improvements were

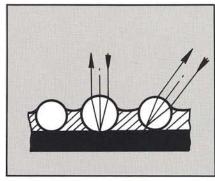


Figure 1. Illustration from Palmquist's Scotchlite patent showing how light rays impinging on the surface from various angles are retroflected along the same axis.

largely ignored by the film industry, which preferred to continue with the cumbersome, but more manageable, rear projection process.

In 1951, the Motion Picture Research Council, a now defunct and much lamented branch of the Motion Picture Producers Association, issued a report of great significance. The report, by Dr. Herbert Meyer, chief chemist, bore the title, "Front Projection Process Photography with Scotchlite."²

Meyer described the optical qualities and the application to photographic front projection of a material that had been developed by Philip Palmquist and manufactured by 3M Co. Originally intended for traffic signs and the like, Scotchlite was founded on the same principle as Thorner's mirror, but on a microscopic scale. Instead of a huge concave mirror, Palmquist used minute glass beads, each of which constituted a concave mirror and reflected back, not the whole image, but rather a tiny portion of the projected scene (Fig. 1).

There were numerous advantages to this approach versus Thorner's mirror, and these were recognized not only by Meyer but also by Will Jenkins. In 1952 and in 1953, Jenkins filed two patents detailing a plethora of different methods utilizing Scotchlite as a catadioptric front projection systems. Jenkins was followed in 1962 by two French cinematographers, Alekan and Gerard, who were somehow able to obtain a British patent for a process that, aside from a change in the refractive index of the glass from which the material was fabricated, was identical to Jenkins's.3

Meyer's original report² delineated, in a very methodical fashion, the various ways in which Scotchlite might be expected to have to perform in motion-picture production. He examined the effects of camera movements by dollying, panning, tilting, etc., as part of his tests, and concluded that, while movements calling for the camera to pivot about the nodal point of its lens, as in panning and tilting, were permissible, the system should otherwise be regarded as a stationary camera system. He did, however, observe that the process would readily lend itself to stereoscopic production.

In his conclusion, having pointed to the advantages of space and power savings, along with the disadvantages of projector noise problems, restrictions of camera movement, and the problems associated with filming through a beamsplitter, Meyers remarked that, "The fabricating of a large size screen from Scotchlite materials will introduce the problem of seams, which may require special attention."

Tessellating the Screen

Seams

Scotchlite seams have been the subject of attention for a long time. The manufacturer, 3M Co., advised initially that it would be sufficient to lay the material out in strips horizontally, in the desired lengths in shingle fashion, with each succeeding layer overlapping the previous one. The Alekan-Gerard method described butt joints with a $\frac{1}{2}$ in. wide strip covering the joint. Denys Coop, in an interview with David Samuelson published in the American Cinematographer,4 discussed this difficulty at length, concluding, "I don't think we have solved this problem yet. It is a very big problem."

It was soon apparent that the problem was somewhat more involved than merely disguising the seam itself. Owing to the critical nature of the optics involved and the vagaries of the mass-production process, there

were perceptible differences in the material, even within the same roll. Therefore, the material had not only to be joined together, but first broken up into relatively small pieces and "shuffled" in order to be re-assembled as a mosaic, thereby producing a homogeneous whole. Various methods were employed to accomplish this. Cutting the material into squares, while economical, left a checkerboard pattern that was readily discerned by eyes acculturated to straight lines and right angles. Tearing Scotchlite into irregular patches was better, but extravagantly labor-intensive and wasteful of material.

Techniques

The author contemplated borrowing a technique from fiber-optic image transmission, in which the fiber bundle is oscillated so as to blur out the bundle pattern. If this is done in synchrony at both ends of the bundle, the image remains stationary relative to the fibers, and only the bundle pattern is affected. Translating this to Scotchlite screens, however, seemed a daunting enterprise — though not too daunting to Zoran Perisic. With admirable courage, Perisic resorted to shaking a large front projection screen up and down, much as Willis O'Brien had done with small rear pro-

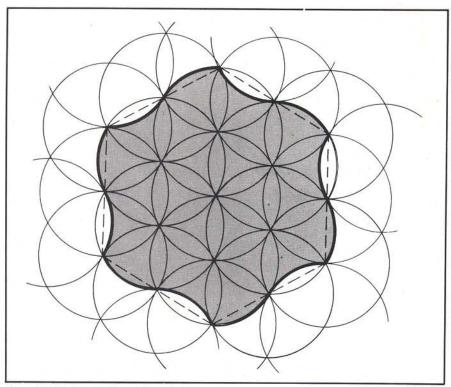


Figure 2. The Apogee tile consists of a hexagon modified by converting the straight lines into sine waves (or S bends), resulting in a symmetrical tile that will fit in any position and at any rotation.

jection screens in the '30s. The problems attendant on this approach, of course, include the necessity of preventing the vibration from affecting anything other than the screen. To do this, he was forced to go through the studio floor and sink massive concrete footings on which to support the screen structure in isolation from the rest of the building.

Clearly a passive solution was desirable. The author finally resorted to the art and science of tessellation, which explores the myriad mathematical possibilities to be found in polygons, etc. 5.6,7,8 Many kinds of polygonal shapes were examined. The hexagon emerged as the best choice, as this reduced the number of conjoined segments and limited the length of a single edge. Still, there were straight lines, and a solution was required that would remove the straight line without incurring a waste of material.

The answer was quite simple: convert the straight line to a sine wave, or in other words, to a positive and negative curve that results in an "S" bend. (Fig. 2 shows the Apogee Tile.) If this is carried out in each segment, the result is a symmetrical tile that will fit in any position and at any rotation, as shown in Fig. 2. The pattern resulting from this tessellation presents an image of a sinuous honeycomb, which

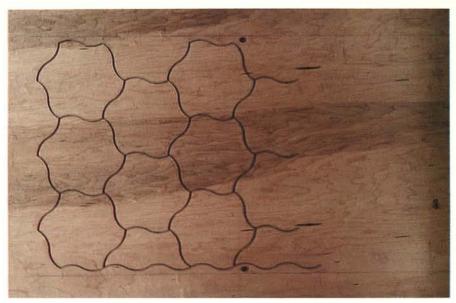


Figure 4. The steel rule die that cuts the tile pattern. This die was designed on a computer, and the data fed to a carbon dioxide laser which cut the die with formidable accuracy.

conveys the appearance of organic and indistinct blobs. (Fig. 3.)

Screen

To facilitate the construction of a screen to this design, a steel rule die was designed on a computer, and the data fed to a carbon dioxide laser which cut the die with formidable accuracy. (Fig. 4 shows the steel rule die.) The die-cutting operation pro-

duces a minimal amount of material loss along the edge of the roll (approximately 4%), and the losses of material incurred by overlapping the seams during assembly are held to 3%.

The assembly is facilitated by the use of a template that indicates the segment intersections of each tile, allowing for selvage. As seen in Fig. 5, the tiles are applied to a backing comprised of a Dacron fabric which has been laminated to a Mylar film, providing a flexible yet inelastic substrate. This backing aids in preventing the Scotchlite seams from working up into ridges, as occurs with other more elastic forms of backings.

After fabrication, the screen is photographed, using a ringlight and highcontrast film in order to exaggerate any imperfections that might exist. If any appear, these can then be corrected. In Fig. 6, on the far right, a number of tiles are shown which are made from defective Scotchlite material that will have to be replaced. The tile pattern is discernible in this illustration. For comparison, Fig. 7 shows the same screen section at normal contrast. It still may be possible to see the defective tiles, but not as easily, and the tile pattern is quite difficult to discern.

Variations on the Process

We will return now to Herbert Meyer's very astute observation with respect to Scotchlite's stereoscopic capabilities. Meyer recognized that while Scotchlite functioned in many

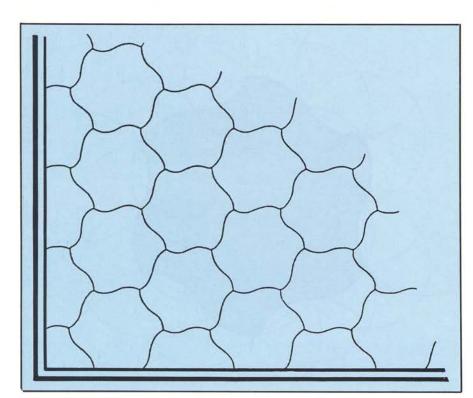


Figure 3. Apogee tile pattern. Resulting from tessellating with the Apogee tile, this pattern presents an image of a sinuous honeycomb made up of organic and indistinct blobs.

respects like Thorner's mirror (it returned a light ray to its focal point of origin), it differed radically in that it could serve the same function for a multiplicity of focal points. Therefore, for example, a camera utilizing a stereoscopic apparatus providing for two adjacent points of view could utilize two projectors, supplying the left and right images of a stereoscopic plate, respectively situated to the right and left of the camera, and utilizing separate beamsplitters. Thus, the camera would record a composite image in which both the foreground and background merged together in all three dimensions.

Meyer's insightful observation, made over 30 years ago, has gone largely unexploited by the film industry. An effort towards a 3-D application of Scotchlite was made on behalf of Paramount by Apogee's John Dykstra sometime around 1975, before the days of Star Wars. Dykstra's experiments were aimed at an amusement park application and, although he demonstrated the feasibility of the process, Paramount failed to pursue it. Currently, Zoran Perisic has an interesting motion-picture application under development. Some of the features of Peresic's new system include its ability to work in any format from 2-perf through 8, either over and under or side by side. He is particularly pleased with his viewfinder that allows the cinematographer to see a three-dimensional image on the groundglass.

Other sophisticated embellishments on the basic front projection technique, however, have been brought into daily practice, three of which will be briefly discussed here.

Introvision

In 1981, the motion picture Outland introduced a company called Introvision to the visual effects community. John Eppolito had produced a system of front projection that allowed characters to appear not only in front of, but behind, sections of the front projection plate. On that film, and subsequently several others, expensive sets were replaced by paintings or models that actors could actually inhabit. Perhaps the most exemplary application of this technique occurred in the television film, Inside The Third Reich, in which actors portraying Adolf Hitler and Albert Speer were seen touring the recently completed Reichstag. Since

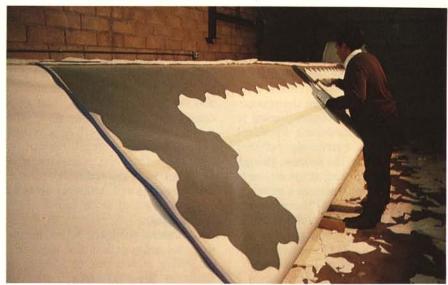


Figure 5. Screen fabrication. Applied to a flexible but inelastic backing comprised of laminated Dacron and Mylar, the tiles are positioned with the aid of a template that indicates the segment intersections (allowing for a 3% overlap).

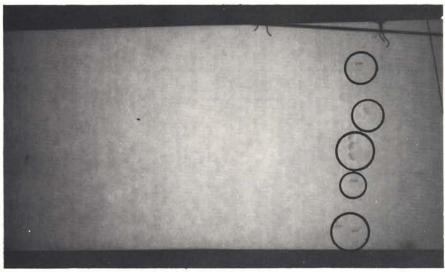


Figure 6. High-contrast photo of screen. When complete, the screen is photographed on high-contrast film to exaggerate any imperfections. Shown on the far right of this frame are some defective tiles that will have to be replaced.

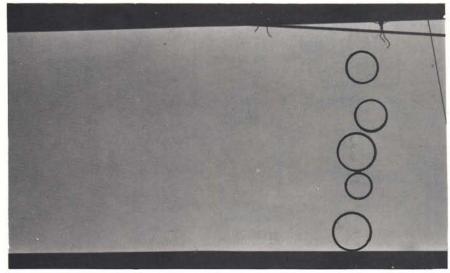


Figure 7. Normal-contrast photo of screen. The same section of screen as in Fig. 6 photographed on normal-contrast color stock. The defective tiles are now barely perceptible.

the Reichstag was destroyed during the war, and it would have prohibitive, if not impossible, to have reproduced it as a set, this ingenious process permitted the inclusion of a sequence which would surely have otherwise been cut from the production. It is a very difficult process to master, and Introvision's President Tom Naud is extremely reticent about describing the process, though he has indicated that, at some time in the future, he may present a paper on this topic at an SMPTE conference. In the meantime, the fundamentals involved in Introvision may be observed in the works of Eugene Schuftan9 and Will Jenkins.3

In Jenkins' Patent Illustration 3 (Fig. 8), he describes how the spent beam from the projector (i.e., the light that has passed through the beamsplitter, as opposed to that reflected towards the screen directly in front of the camera) passes through a

lens in addition to through a matte, then to another front projection screen, and back to the camera. The primary beam, meanwhile, has been reflected by the beamsplitter towards the primary front projection screen, and this beam also passes through a countermatte on the way to the screen. The camera will now record an entire field of front projection, some of it from the primary screen and some from the secondary screen, and the actors will at various times be either in front of or behind the projected image.

In Jenkins' visionary patent, he anticipated being able to also move the respective mattes during a shot, so that an actor could be seen to enter the set through a door that existed only in the projected plate. He subsequently would be able to retrace his steps over the same terrain, although rather than disappearing again, he would now be seen in front of the door.

MASK
SCOTCH LIGHT
SCREEN

MASK
PROJECTOR

CONDENSER

CAMERA

Figure 8. Jenkins' patent Illustration 3. In Jenkins' visionary patent, he anticipated being able to place actors either in front of or behind portions of the projected plate.

John Eppolito, Tom Naud, and the talented Bill Mesa have evolved a sophisticated state-of-the-art process based on these earlier efforts. In addition to Introvision, in Europe there is a dual screen front projection system known as the J. J. Composite Screen Front Projection System which was granted a British patent in 1972. 10 This is a very competent and capable-sounding system, and worthy of serious study.

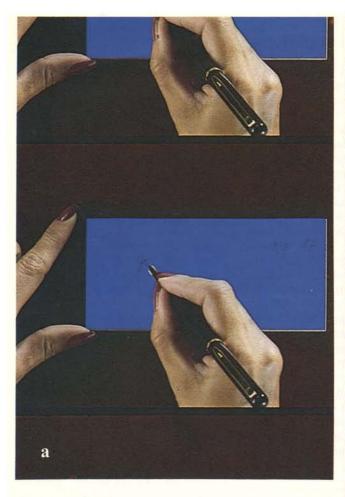
Zoptics

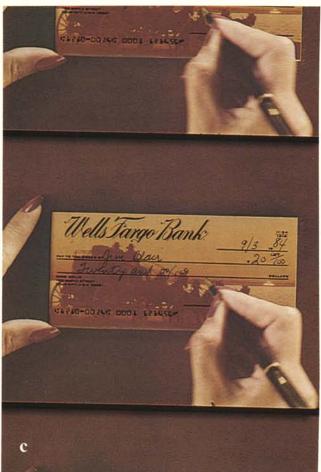
The next front projection process we will look at is Zoptics. In this instance, the invention depended only marginally on the work of Jenkins et al. Zoran Perisic reasoned that if a zoom lens was mounted on the projector, a similar zoom lens was mounted on the camera, and both were linked together and synchronized, then a subject interposed between the screen and the camera could be made to appear to move towards the camera. (If desirable, at great speed and also, if desirable, apparently unsupported.) In short, the subject could fly. And that is exactly how Superman was made to fly. For this novel process, Perisic was presented with a Technical Achievement Award by the Academy of Motion Picture Arts and Sciences, and was granted a patent for it in July 1978.11

Reverse Front Projection

Finally, we shall look at one more application of the remarkable characteristics of Scotchlite. Not long ago, Apogee was asked to produce a commercial for Wells Fargo Bank. The commercial depicts a blank check in the process of being filled out by a well-manicured female hand with highly polished fingernails, wielding a beautiful pen with a gleaming golden nib (Fig. 9a). As the check is being completed, the graphics denoting the bank are wiped onto the check (Fig. 9b). By the completion of the check writing, a still background scene depicting a stagecoach has been processed onto the check (Fig. 9c). On completion, the check is torn from the checkbook, and the background scene springs to life as the stagecoach rides off into the sunset (Fig. 9d).

On the face of it, this would appear to be a simple bluescreen composite. However, the obvious choice of using a conventional transmission bluescreen as the background for the check surface would present blue spill





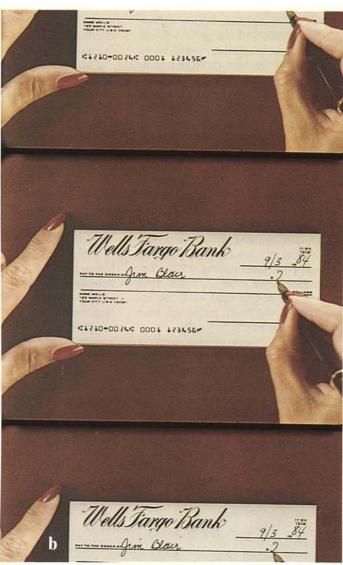




Figure 9a-d. Reverse front projection bluescreen element from a Wells Fargo Bank commercial. Note the absence of blue spill on fingernails and pen nib.

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problems due to those highly reflective fingernails and the golden nib of the pen. An alternative would be to front-project the blue through a window to a Scotchlite panel situated beyond the check. However, we see the pen actually writing on an animation cell resting on the glass window; there would be reflections from these to contend with.

Another approach would be to use Scotchlite as the check surface, relying on the writing pen to "turn off" the screen wherever it wrote. Unfortunately, the foreground lighting would contaminate the screen too badly to permit this solution, so it was necessary to place the Scotchlite beyond the window after all, and to find some other way to deal with the reflections of the front projected blue light from the animation cell.

Apogee's director of photography for this commercial, John Sullivan, and I wrestled with this problem until, in a moment of inspiration, Sullivan proposed that we move the front projection beamsplitter and the projector behind the window along with the Scotchlite (Fig. 10). This was an emi-

nently satisfactory solution, and the result was the superb composite seen in Fig. 9d.

The solution to this problem draws attention to an inherent, though not generally well understood faculty of Scotchlite — it is, in a sense, an optical condenser. Unlike a plane mirror (from which a reflected diverging cone of light will continue to diverge), the reflected diverging cone returns from Scotchlite as a converging one. Therefore, when a beamsplitter is interposed into the beam, the cone is turned and can be delivered to any desired point.

Armed with this insight, it is possible to evolve from front projection to reverse front projection. Along the way, certain benefits are acquired: reflections from foreground subjects are eliminated; contamination of the screen by the foreground lighting is eliminated; the critical lineup to prevent the camera seeing the shadow cast by the foreground subject is eliminated, as there is no shadow cast on the screen; and glass objects and water reproduce better, as the projected ray of light no longer has to pass

through them on its way to the screen, only en route to the camera. This technique, therefore has application in the filming of closeups, whether for bluescreen or front projection compositing, in which the camera-to-subject distance is less than the minimum required to prevent contamination of the subject by the projector and excessive fringing at the screen.

Figure 11 shows how this is accomplished. In this illustration, refinements are evident that overcome some historical front projection problems. Backscattered light from a foreground subject is no longer able to travel from the subject to the front projection screen and back again to the camera (producing a halo), and ambient light contamination of the screen has been eliminated. This is the result of: (a) replacing the screen in front of the camera with black velvet; and (b) changing the reflection/ transmission ratio of the beamsplitter from 50/50 to 8/92. This relative inefficiency of the beamsplitter produces what, for want of a better description, we call the "diode effect." Thus, in one direction, the projected light is effectively delivered to the camera through the system, while in the other direction, the backscattered and ambient light is absorbed by the system.

Figure 12 depicts another application derived from these characteristics. This was inspired in large part by a television newsroom situation where the news item being reported is to be composited onto a screen beyond the newscaster. Since the television cameras employed for newscasts invariably have teleprompters attached to them which utilize beamsplitters, it is clear that conventional front projection is out of the question.

As is currently practiced with chroma key, the newscaster sits in front of a blue-painted backing. Consequently the picture is liable to blue spill, especially if the newscaster has blonde hair or, worse, is wearing silver lamé (Fig. 13). It is the object of reverse front projection to correct these deficiencies, as shown in Fig. 12. The screen or flat behind the performer is replaced by a chamber (approximately cubic) the dimensions of which are governed by the height and width of the replaced screen or flat (up to approximately 4 ft \times 6 ft). The chamber has an opening in the face corresponding to the replaced flat. Another opening is in the upper horizontal surface,

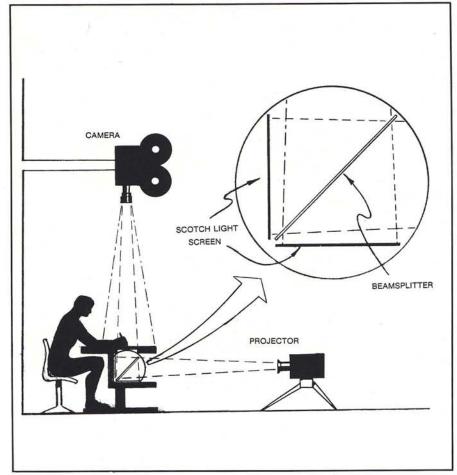


Figure 10. Rig that produced Wells Fargo commercial shown in Fig. 9a-d.

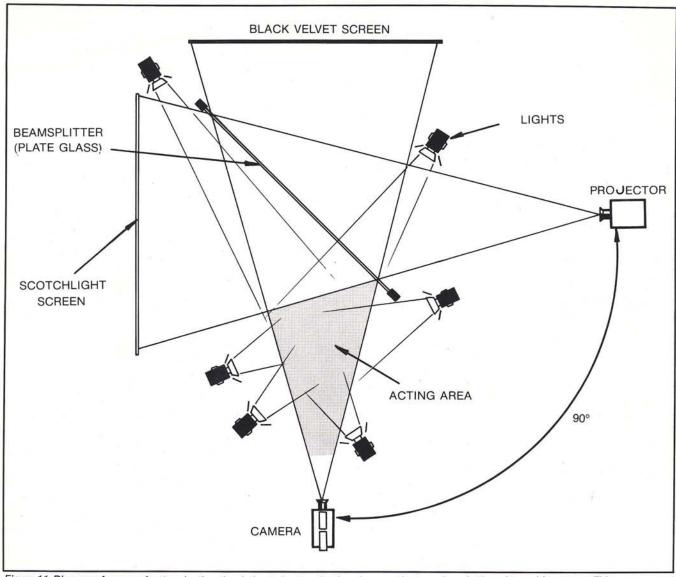


Figure 11. Diagram of reverse front projection. A solution to front projection close-up photography, whether plate or bluescreen. This arrangement eliminates the front projected blue contamination, as well as fringing and halo problems.

or ceiling, of the chamber. The back surface and the bottom surface of the chamber are coated with a retroreflective screen material.

In the illustration, a plane drawn through the chamber diagonally, from the forward upper corner (or ceiling) to the lower rear corner, is a partially silvered membrane, possibly of glass, plastic film, or similar material, to serve as a beamsplitter. A projection lamphouse is situated above the upper opening in the chamber. (It is understood that this is in a schematic sense, as in reality, this would include beam-folding mirrors.) The location above the chamber of the projector is equal to the distance from the retroreflective screen on the back surface of the chamber to the camera recording the scene. Of course, these elements could be rearranged in some other manner. The projector is

equipped with a lamp and filtering system to emit a preselected primary color; the projected beam is governed by optical elements to correspond approximately to the viewing angle of the camera optics; and the camera is situated at the focal point of the projected beam.

The effect brought about by this arrangement is that the beam of light seems to have originated from the camera lens. As such, it is a coherent beam, a single ray of which can be traced out to the retroreflective screen and back again, unless an object interposed between the camera and the screen interrupts the ray. By contrast, with a diffuse light source, rays of light are emanating in all directions. A ray of light can proceed from a point on the screen to a point on the foreground subject, then reflected to the camera, where it is recorded as

blue spill on the subject (Fig. 13.)

The function of the retroreflective screen is to turn a diverging cone of light (emitted from the projector) into a converging one, with a focal point equal to the distance from the projector to the retroreflecting screen (Fig. 10). Or, as pointed out earlier, the Scotchlite is serving as a very large condenser. The function of the beamsplitter is to change the direction of the beam and to direct it to the nodal point of the camera lens. Approximately 40% of the rays emanating from the retroreflective screen will travel to the camera lens, approximately 40% will return to the projector, and the remainder will be lost to absorption at the beamsplitter and retroreflective screen surfaces.

Other applications could be enumerated, among them close-ups, portrait photography, and product pho-

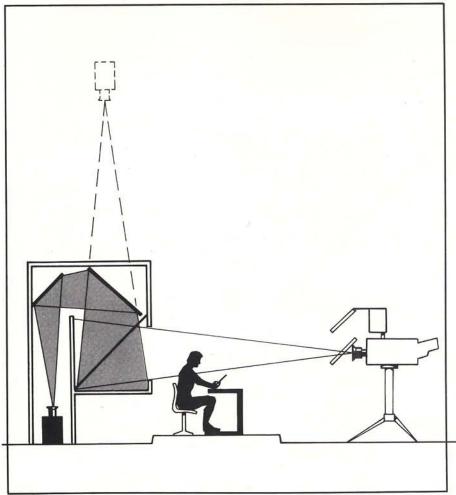


Figure 12. Reverse front projection as it might be applied to a newsroom chroma-key situation, permitting the simultaneous use of a teleprompter on the camera. Folding the reverse front projection beam would allow for a fairly compact fixture.

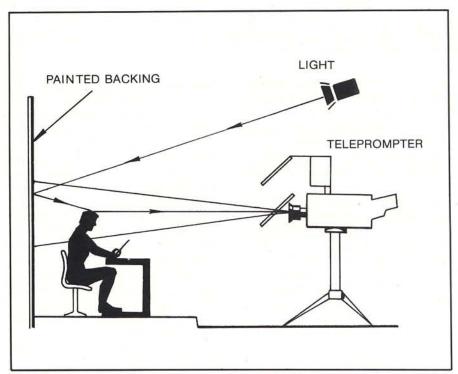


Figure 13. Diagram of diffuse transmission screen. With a diffuse transmission screen, light emanates in all directions. Some light rays can proceed from a point on the screen to a point on the foreground subject, and from there to the camera to be recorded as blue spill.

tography, in which front projected light would be a problem. Assuming all elements are critically aligned, an image may be projected in place of a primary color, and in that arrangement a variety of animation stand applications spring readily to mind. Where previously an image was projected from below the artwork via a set of condensers which could rarely exceed about 15 in. in diameter, now an image plane of, say, 5×7 ft can be used. Some Zoptic applications could also benefit from this approach. At the moment, the restricting element is the size of available beamsplitter material.

The Apogee front projection screen design, which won an Academy Technical Achievement Award in 1985 for the author and Robert Bealmear, has been granted a patent and is available from Apogee. The front projection tiles comprised of Scotchlite are also patented, and are available from Apogee either loose or made up into screens.

Conclusion

In summary, we feel the improvements introduced in the fabrication of front projection screens will enhance the effectiveness of this technique and serve to expand the applications for its use. Despite the fact that Scotchlite has been around for over 30 years, we are far from closing the book on what can be achieved with this remarkable material. As has been demonstrated over the years, problems provide opportunities for insight and occasionally marvelous revelations.

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