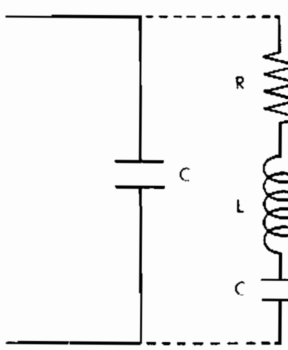
TECHNICAL BULLETIN NO. 05

CAPACITORS...PF? DF? Q?

In any electrical device, including capacitors, when we apply a certain amount of total power (energy) into the device, we get a lesser amount out of it. The difference between the amounts in and out is "lost" or used within the device and is referred to as the "power loss." If we now divide this "power loss" by the input power, the resulting ratio figure is the "power factor" of the device. The "power factor" (PF) then is a direct measure of the "inefficiency" of a capacitor, in that it supplies us with a measuring tool to determine how much of the total power supplied to a capacitor is used by the capacitor itself and therefore not available to do otherwise "useful" work.

Before proceeding further, a review of some fundamental concepts relative to the application of a sine wave voltage to a capacitor will be helpful.

Although a capacitor is primarily capacitive in nature, it does possess very small distributed amounts of resistive and inductive elements. These distributed resistor and inductor elements can be lumped into a single value for computation purposes as shown in the following equivalent circuit.

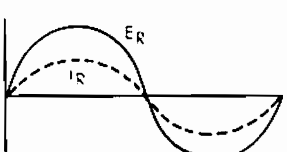


R=Series resistance (ohms)

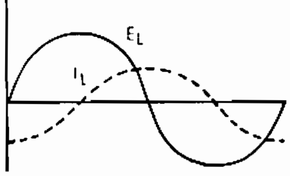
L=Inductance (henries)

C=Capacitance (farads)

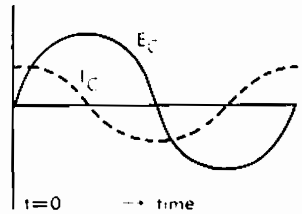
What we have then is a circuit containing all three primary elements of resistance, inductance, and capacitance. This means that the application of a sine wave voltage to the capacitor will set up a fundamental vector relationship between the voltages and currents in each element as follows:



Resistive Element
(very low value in capacitors)
E_R=Resistive voltage
I_R=Resistive current
I_R in phase with E_R



Inductive Element
(extremely low value in capacitors)
E_L=Inductive voltage
I_L=Inductive current
I_L lagging E_L by 90° in time



Capacitive Element
(extremely large in capacitors)
E_C=Capacitive voltage
I_C=Capacitive current
I_C leading E_C by 90° in time

Note also that since R, L, and C are in series in our circuit, the circuit current (I) is the same current that passes through all elements (in magnitude) and therefore:

$$I=I_R=I_L=I_C \text{ (Magnitude)}$$

With the element currents out of phase with the voltages, Ohm's Law must be modified to use impedances and reactances in addition to resistance. That is—

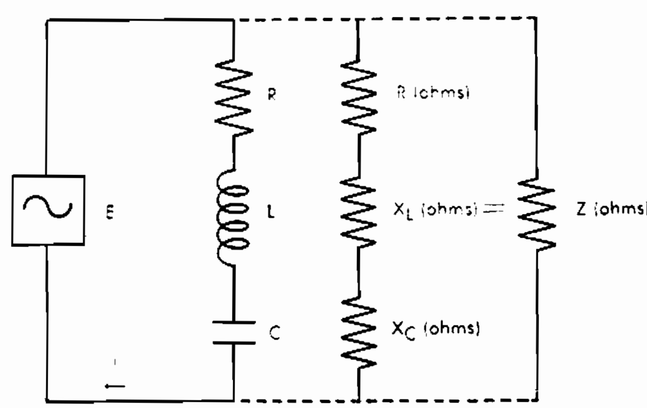
$E_R=IR$

$E_L=IX_L$ where $X_L=2\pi fL$ (Inductive reactance in ohms)

$E_C=IX_C$ where $X_C=\frac{1}{2\pi fC}$ (Capacitive reactance in ohms)

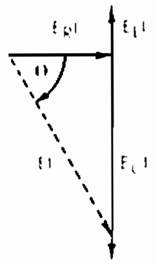
$E=IZ$ where Z=Total impedance of R, X_L, X_C combination in ohms

The power relationship of our circuit is:



E'=total circuit power (θ)
E_R'=Resistive power (0°)
E_L'=Inductive power (−90°)
E_C'=Capacitive power (+90°)

Drawing the vector relationship, our power equation is:



$$(EI)^2=(E_RI)^2+(E_CI-E_LI)^2$$

$$(IZI)^2=(IRI)^2+(IX_CI-IX_LI)^2$$

$$(I^2Z)^2=(I^2R)^2+(I^2X_C-I^2X_L)^2$$
$$=I^2R^2+I^2(X_C-X_L)^2$$

$$I^2Z^2=I^2[R^2+(X_C-X_L)^2]$$

Therefore: $Z^2=R^2+(X_C-X_L)^2$ and $Z=\sqrt{R^2+(X_C-X_L)^2}$

POWER FACTOR

The "power loss" within the capaitor is that portion of the total power applied to the capacitor that is not stored by the capacitor on an instantaneous basis. As the current passes through the series resistance element, it generates heat and this then is the "power loss" we are concerned with. It should be noted here that all energy losses (due to leads, dielectric polarization, connections, and eddy currents in the electrode material) are taken into account by the "**equivalent** series resistance" element. Analysis of the power equation shows:

$(EI)^2=(E_RI)^2+(E_CI-E_LI)^2$

substituting: $(I^2Z)^2=(I^2R)^2+(I^2X_C-I^2X_L)^2$

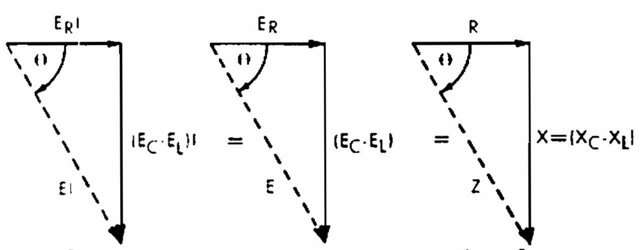
Power in = Power losses + power out

And therefore the power factor by definition is:

$$PF=\frac{\text{Power Losses}}{\text{Power In}}=\frac{(I^2R)^2}{(I^2Z)^2}=\frac{E_RI}{EI}=\cos\theta$$

(from the vector diagram)

Note also that the following vector relationships hold true:



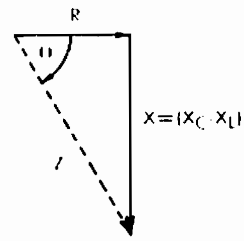
Power Diagram — Voltage Diagram — Impedance Diagram

$$PF=\cos\theta=\frac{E_RI}{EI}=\frac{E_R}{E}=\frac{R}{Z}$$

We see then that in its simplest form the power factor is the ratio of the equivalent series resistance to the impedance of the device.

Dissipation Factor

With the exception of electroytic and large power capacitors, most wound dielectric type units utilize a ratio figure known as the "dissipation factor" (DF) instead of the Power Factor (PF). By definition, this DF is the ratio of the equivalent series resistance to the reactance. For comparison purposes we see that:



$$PF=\frac{R}{Z}=\cos\theta$$

$$DF=\frac{R}{X}=\cot\theta$$

We can readily see that if X and Z are practically identical (which would be the case when θ approaches 90°), then the PF and DF would also be practically identical.

For any given unit, an analysis of the divergence error in the equation DF=PF shows:

$$DF=\frac{R}{X} \qquad PF=\frac{R}{Z}$$

$$\therefore R=(DF)X \qquad \therefore R=(PF)Z$$

And: $(DF)X=(PF)Z=(PF)\left(\sqrt{R^2+X^2}\right)=(PF)\left(\sqrt{(DF)^2X^2+X^2}\right)$

$$(DF)X=(PF)\left(X\sqrt{(DF)^2+1}\right)$$

$$(DF)=(PF)\left(\sqrt{(DF)^2+1}\right)$$

Squaring: $(DF)^2=(PF)^2\{(DF)^2+1\}=(PF)^2(DF)^2+(PF)^2$

Rearranging: $(DF)^2-(DF)^2(PF)^2=(PF)^2$

$$(DF)^2[1-(PF)^2]=(PF)^2$$

$$(DF)^2=\frac{(PF)^2}{[1-(PF)^2]}$$

And finally: $DF=\dfrac{PF}{\sqrt{1-(PF)^2}}$

Computations:

PF	DF	Divergence Error
0	0	0
.001	.0010000005	.00005 %
.01	.0100005	.005 %
.10	.1005	.5 %
.20	.204	2.0 %

We see that for all values of DF=.10 (10% DF) or less, the error in the assumption DF= PF will be .5% or less.

The reason for using DF as a parameter instead of PF is that determination of PF values requires much more complicated equipment and procedures compared to the simple comparison technique used for measuring DF.

DF then is a convenient, somewhat artifical method by which the "inefficiency" of a capacitor can be noted.

Q (Factor of Merit)

The "Q" or "Factor of Merit" of a device is also an artifical measurement that will conveniently allow notation ot the "inefficiency" (or power losses) of that device.

By definition, Q is the ratio of the reactance to the series resistance. This then shows the following relationship:

$$Q = \frac{X}{R} = \frac{1}{DF}$$

or: Q is the reciprocal of the DF.

For instance:

$$DF = 1.0\% \quad \text{then} \quad Q = \frac{1}{.01} = 100$$

Common usage pretty much boils down to this:

1. "Power Factor" is used for capacitors when the PF is 10% or greater.
2. "Dissipation Factor" is used when the PF is less than 10%.
3. "Q" is occasionally used for capacitors. It is widely used for inductors and total ciruits.

TECHNICAL BULLETIN NO. 06

CAPACITORS... DISSIPATION FACTOR

Whenever power (energy) in the form of voltage times current is applied to a capacitor, part of that total power is used or "lost" within the capacitor itself. The ratio of this "power loss" to the total power supplied is the "power factor" (PF) of the capacitor. This PF figure then is a measurement factor for rating the "inefficiency" of the power transfer capabilites of the capacitor.

For those capacitors where the PF figure is .1 (10%) or less, a ratio figure known as the "dissipation factor" (DF) is more commonly used. The reason for this usage of the DF figure is simply a convenience that takes advantage of the fact that DF measurements on a capacitor are much simpler and easier to make on standard capacitance bridges than the determination of PF.

The relationships between PF and DF, and the factors that are concerned in these figures are delineated in the following: (AC voltage applied)

where;

R=equivalent series resistance (ohms)

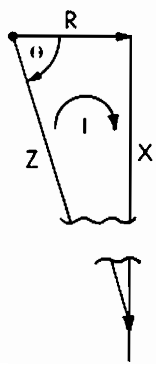
$X = (X_C - X_L)$ = total reactance (ohms)

$X_C = \frac{1}{2\pi fC}$ = capacitive reactance (ohms)

C=capacitance (farads)

f=frequency (Hz)

$X_L = 2\pi fL$ = inductive reactance (ohms)



L=inductance (henries)

$Z^2 = R^2 + X^2$ = impedance (ohms)

I=current (amperes)

θ=phase angle (radians or degrees)

Ohm's Law equations are:

$$E = IZ$$
$$E_X = IX$$
$$E_R = IR$$

where: E_R=series resistance voltage

E_X=reactance voltage

E–circuit voltage

So: $$PF = \frac{\text{Power Loss}}{\text{Total Power}} = \frac{E_R I}{EI} = \frac{I^2 R}{I^2 Z} = \frac{R}{Z} = \cos\theta$$

The cos θ and cot θ approach convergence as θ approaches 90°. At 90°, both cos θ and cot θ = 0 and Z = X.

From cos θ = 0 to cos θ = .1 (10%), the divergence error of the equation cos θ = cot θ goes from 0 to .5% error.

∴ For values of PF = cos θ = .1 (10%) or less, we equate cos θ and cot θ

Thus: $PF = \cos\theta \simeq \cot\theta = \frac{R}{X} = DF$