CHAPTER 11 ADVANCED ALGEBRA AND GEOMETRY

FACTORS AND MULTIPLES

This lesson shows the relationship between factors and multiples of a number. In addition, it introduces prime and composite numbers and demonstrates how to use prime factorization to determine all the factors of a number.

Factors of a Number

Multiplication converts two or more factors into a product. A given number, however, may be the product of more than one combination of factors; for example, 12 is the product of 3 and 4 and the product of 2 and 6. Limiting consideration to the set of whole numbers, a **factor of a number** (call it x) is a whole number whose product with any other whole number is equal to x. For instance, 2 is a factor of 12 because $12 \div 2$ is a whole number (6). Another way of expressing it is that 2 is a factor of 12 because 12 is **divisible** by 2.

> **BE CAREFUL!**
>
> The term *factor* can mean any number being multiplied by another number, or it can mean a number by which another number is divisible. The two uses are related but slightly different. The context will generally clarify which meaning applies.

A whole number always has at least two factors: 1 and itself. That is, for any whole number y, $1 \times y = y$. To test whether one number is a factor of a second number, divide the second by the first. If the quotient is whole, it is a factor. If the quotient is not whole (or it has a remainder), it is not a factor.

Example

Which number is not a factor of 54?

A. 1 B. 2 C. 4 D. 6

The correct answer is **C.** A number is a factor of another number if the latter is divisible by the former. The number 54 is divisible by 1 because $54 \times 1 = 54$, and it is divisible by 2 because $27 \times 2 = 54$. Also, $6 \times 9 = 54$. But $54 \div 4 = 13.5$ (or 13R2). Therefore, 4 is not a factor. **See Lesson: Factors and Multiples.**

Multiples of a Number

Multiples of a number are related to factors of a number. A **multiple of a number** is that number's product with some integer. For example, if a hardware store sells a type of screw that only comes in packs of 20, customers must buy these screws in *multiples* of 20: that is, 20, 40, 60, 80, and so on. (Technically, 0 is also a multiple.) These numbers are equal to 20×1, 20×2, 20×3, 20×4, and so on. Similarly, measurements in feet represent multiples of 12 inches. A (whole-number) measurement in feet would be equivalent to 12 inches, 24 inches, 36 inches, and so on.

When counting by twos or threes, multiples are used. But because the multiples of a number are the product of that number with the integers, multiples can also be negative. For the number 2, the multiples are the set {..., −6, −4, −2, 0, 2, 4, 6,...}, where the ellipsis dots indicate that the set continues the pattern indefinitely in both directions. Also, the number can be any real number: the multiples of π (approximately 3.14) are {..., −3π, −2π, −1π, 0, 1π, 2π, 3π,...}. Note that the notation 2π, for example, means $2 \times \pi$.

The positive multiples (along with 0) of a whole number are all numbers for which that whole number is a factor. For instance, the positive multiples of 5 are 0, 5, 10, 15, 20, 25, 30, and so on. That full set contains all (whole) numbers for which 5 is a factor. Thus, one number is a multiple of a second number if the second number is a factor of the first.

Example

If a landowner subdivides a parcel of property into multiples of 7 acres, how many acres can a buyer purchase?

A. 1 B. 15 C. 29 D. 42

The correct answer is **D.** Because the landowner subdivides the property into multiples of 7 acres, a buyer must choose an acreage from the list 7 acres, 14 acres, 21 acres, and so on. That list includes 42 acres. Another way to solve the problem is to find which answer is divisible by 7 (that is, which number has 7 as a factor). **See Lesson: Factors and Multiples.**

Prime and Composite Numbers

For some real-world applications, such as cryptography, factors and multiples play an important role. One important way to classify whole numbers is by whether they are prime or composite. A **prime** number is any whole (or natural) number greater than 1 that has only itself and 1 as factors. The smallest example is 2: because 2 only has 1 and 2 as factors, it is prime. **Composite** numbers have at least one factor other than 1 and themselves. The smallest composite number is 4: in addition to 1 and 4, it has 2 as a factor.

> **BE CAREFUL!**
> Avoid the temptation to call 1 a prime number. Although it only has itself and 1 as factors, those factors are the same number. Hence, 1 is fundamentally different from the prime numbers, which start at 2.

Determining whether a number is prime can be extremely difficult—hence its value in cryptography. One simple test that works for some numbers is to check whether the number is even or odd. An **even number** is divisible by 2; an **odd number** is not. To determine whether a number is even or odd, look at the last (rightmost) digit. If that digit is even (0, 2, 4, 6, or 8), the number is even. Otherwise, it is odd. Another simple test works for multiples of 3. Add all the digits in the number. If the sum is divisible by 3, the original number is also divisible by 3. This rule can be successively applied multiple times until the sum of digits is manageable. That number is then composite.

Example

Which number is prime?

A. 6 B. 16 C. 61 D. 116

The correct answer is **C.** When applicable, the easiest way to identify a number greater than 2 as composite rather than prime is to check whether it is even. All even numbers greater than 2 are composite. By elimination, 61 is prime. **See Lesson: Factors and Multiples.**

Prime Factorization

Determining whether a number is prime, even for relatively small numbers (less than 100), can be difficult. One tool that can help both solve this problem and identify all factors of a number is **prime factorization.** One way to do prime factorization is to make a **factor tree.**

The procedure below demonstrates the process.

STEP BY STEP

Step 1. Write the number you want to factor.

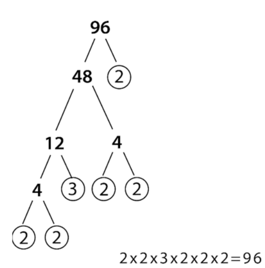
Step 2. If the number is prime, stop. Otherwise, go to Step 3.

Step 3. Find any two factors of the number and write them on the line below the number.

Step 4. "Connect" the factors and the number using line segments. The result will look somewhat like an inverted tree, particularly as the process continues.

Step 5. Repeat Steps 2–4 for all composite factors in the tree.

The numbers in the factor tree are either "branches" (if they are connected downward to other numbers) or "leaves" (if they have no further downward connections). The leaves constitute all the prime factors of the original number: when multiplied together, their product is that number. Moreover, any product of two or more of the leaves is a factor of the original number. Thus, using prime factorization helps find any and all factors of a number, although the process can be tedious when performed by hand (particularly for large numbers). Below is a factor tree for the number 96. All the leaves are circled for emphasis.



Example

Which list includes all the unique prime factors of 84?

A. 2, 3, 7 B. 3, 4, 7 C. 3, 5, 7 D. 1, 2, 3, 7

The correct answer is **A.** One approach is to find the prime factorization of 84. The factor tree shows that $84 = 2 \times 2 \times 3 \times 7$. Alternatively, note that answer D includes 1, which is not prime. Answer B includes 4, which is a composite number. Since answer C includes 5, which is not a factor of 84, the only possible answer is A. **See Lesson: Factors and Multiples.**

Let's Review!

- A whole number is divisible by all of its factors, which are also whole numbers by definition.
- Multiples of a number are all possible products of that number and the integers.
- A prime number is a whole number greater than 1 that has no factors other than itself and 1.
- A composite number is a whole number greater than 1 that is not prime (that is, it has factors other than itself and 1).
- Even numbers are divisible by 2; odd numbers are not.
- Prime factorization yields all the prime factors of a number. The factor-tree method is one way to determine prime factorization.

SOLVING QUADRATIC EQUATIONS

This lesson introduces solving quadratic equations by the square root method, completing the square, factoring, and using the quadratic formula.

Solving Quadratic Equations by the Square Root Method

A **quadratic equation** is an equation where the highest variable is squared. The equation is in the form $ax^2 + bx + c = 0$, where a is a non-zero constant and b and c are constants. There are at most two solutions to the equation because the highest variable is squared. There are many methods to solve a quadratic equation.

This section will explore solving a quadratic equation by the square root method. The equation must be in the form of $ax^2 = c$, or there is no x term.

STEP BY STEP

Step 1. Use multiplication and division properties of an equation to remove the value in front of the variable.

Step 2. Apply the square root to both sides of the equation.

Note: The positive and negative square root make the solution true. For the equation $x^2 = 9$, the solutions are −3 and 3 because $3^2 = 9$ and $(-3)^2 = 9$.

Example

Solve the equation by the square root method, $4x^2 = 64$.

A. 4 B. 8 C. ± 4 D. ± 8

The correct answer is **C.** The correct solution is ± 4. **See Lesson: Solving Quadratic Equations.**

$x^2 = 16$	Divide both sides of the equation by 4.
$x = \pm 4$	Apply the square root to both sides of the equation.

Solving Quadratic Equations by Completing the Square

A quadratic equation in the form $x^2 + bx$ can be solved by a process known as completing the square. The best time to solve by completing the square is when the b term is even.

STEP BY STEP

Step 1. Divide all terms by the coefficient of x^2.

Step 2. Move the number term to the right side of the equation.

Step 3. Complete the square $\left(\frac{b}{2}\right)^2$ and add this value to both sides of the equation.

Step 4. Factor the left side of the equation.

Step 5. Apply the square root to both sides of the equation.

Step 6. Use addition and subtraction properties to move all number terms to the right side of the equation.

Examples

1. **Solve the equation by completing the square, $x^2 - 8x + 12 = 0$.**

A. -2 and -6 B. 2 and -6 C. -2 and 6 D. 2 and 6

The correct answer is **D.** The correct solutions are 2 and 6. **See Lesson: Solving Quadratic Equations.**

$x^2 - 8x = -12$	Subtract 12 from both sides of the equation.
$x^2 - 8x + 16 = -12 + 16$	Complete the square, $\left(-\frac{8}{2}\right)^2 = (-4)^2 = 16$. Add 16 to both sides of the equation.
$x^2 - 8x + 16 = 4$	Simplify the right side of the equation.
$(x - 4)^2 = 4$	Factor the left side of the equation.
$x - 4 = \pm 2$	Apply the square root to both sides of the equation.
$x = 4 \pm 2$	Add 4 to both sides of the equation.
$x = 4 - 2 = 2, x = 4 + 2 = 6$	Simplify the right side of the equation.

2. **Solve the equation by completing the square, $x^2 + 6x - 8 = 0$.**

A. $-3 \pm \sqrt{17}$ B. $3 \pm \sqrt{17}$ C. $-3 \pm \sqrt{8}$ D. $3 \pm \sqrt{8}$

The correct answer is **A.** The correct solutions are $-3 \pm \sqrt{17}$. **See Lesson: Solving Quadratic Equations.**

$x^2 + 6x = 8$	Add 8 to both sides of the equation.
$x^2 + 6x + 9 = 8 + 9$	Complete the square, $\left(\frac{6}{2}\right)^2 = 3^2 = 9$. Add 9 to both sides of the equation.
$x^2 + 6x + 9 = 17$	Simplify the right side of the equation.
$(x + 3)^2 = 17$	Factor the left side of the equation.
$x + 3 = \pm\sqrt{17}$	Apply the square root to both sides of the equation.
$x = -3 \pm \sqrt{17}$	Subtract 3 from both sides of the equation.

Solving Quadratic Equations by Factoring

Factoring can only be used when a quadratic equation is factorable; other methods are needed to solve quadratic equations that are not factorable.

STEP BY STEP

Step 1. Simplify if needed by clearing any fractions and parentheses.

Step 2. Write the equation in standard form, $ax^2 + bx + c = 0$.

Step 3. Factor the quadratic equation.

Step 4. Set each factor equal to zero.

Step 5. Solve the linear equations using inverse operations.

The quadratic equation will have two solutions if the factors are different or one solution if the factors are the same.

Examples

1. **Solve the equation by factoring, $x^2 - 13x + 42 = 0$.**

   A. $-6, -7$ B. $-6, 7$ C. $6, -7$ D. $6, 7$

   The correct answer is **D.** The correct solutions are 6 and 7. **See Lesson: Solving Quadratic Equations.**

$(x - 6)(x - 7) = 0$	Factor the equation.
$(x - 6) = 0$ or $(x - 7) = 0$	Set each factor equal to 0.
$x - 6 = 0$	Add 6 to both sides of the equation to solve for the first factor.
$x = 6$	
$x - 7 = 0$	Add 7 to both sides of the equation to solve for the second factor.
$x = 7$	

2. **Solve the equation by factoring, $9x^2 + 30x + 25 = 0$.**

   A. $-\frac{5}{3}$ B. $-\frac{3}{5}$ C. $\frac{3}{5}$ D. $\frac{5}{3}$

   The correct answer is **A.** The correct solution is $-\frac{5}{3}$. **See Lesson: Solving Quadratic Equations.**

$(3x + 5)(3x + 5) = 0$	Factor the equation.
$(3x + 5) = 0$ or $(3x + 5) = 0$	Set each factor equal to 0.
$(3x + 5) = 0$	Set one factor equal to zero since both factors are the same.
$3x + 5 = 0$	Subtract 5 from both sides of the equation and divide both sides of the equation by 3 to solve.
$3x = -5$	
$x = -\frac{5}{3}$	

Solving Quadratic Equations by the Quadratic Formula

Many quadratic equations are not factorable. Another method of solving a quadratic equation is by using the quadratic formula. This method can be used to solve any quadratic equation in the form $ax^2 + bx + c = 0$. Using the coefficients a, b, and c, the quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The values are substituted into the formula, and applying the order of operations finds the solution(s) to the equation.

The solution of the quadratic formula in these examples will be exact or estimated to three decimal places. There may be cases where the exact solutions to the quadratic formula are used.

KEEP IN MIND
Watch the negative sign in the formula. Remember that a number squared is always positive.

Examples

1. **Solve the equation by the quadratic formula, $x^2 - 5x - 6 = 0$.**

   A. –6 and –1 B. 6 and –1 C. –6 and 1 D. 6 and 1

   The correct answer is **B.** The correct solutions are 6 and –1. **See Lesson: Solving Quadratic Equations.**

$x = \frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-6)}}{2(1)}$	Substitute 1 for *a*, –5 for *b*, and –6 for *c*.
$x = \frac{5\pm\sqrt{25-(-24)}}{2}$	Apply the exponent and perform the multiplication.
$x = \frac{5\pm\sqrt{49}}{2}$	Perform the subtraction.
$x = \frac{5\pm7}{2}$	Apply the square root.
$x = \frac{5+7}{2}, x = \frac{5-7}{2}$	Separate the problem into two expressions.
$x = \frac{12}{2} = 6, x = \frac{-2}{2} = -1$	Simplify the numerator and divide.

2. **Solve the equation by the quadratic formula, $2x^2 + 4x - 5 = 0$.**

   A. –5.74 and –1.74 B. 5.74 and –1.74 C. –5.74 and 1.74 D. 5.74 and 1.74

   The correct answer is **C.** The correct solutions are –5.74 and 1.74. **See Lesson: Solving Quadratic Equations.**

$x = \frac{-4\pm\sqrt{4^2-4(2)(-5)}}{2(2)}$	Substitute 2 for *a*, 4 for *b*, and –5 for *c*.
$x = \frac{-4\pm\sqrt{16-(-40)}}{4}$	Apply the exponent and perform the multiplication.
$x = \frac{-4\pm\sqrt{56}}{4}$	Perform the subtraction.
$x = \frac{-4\pm7.48}{2}$	Apply the square root.
$x = \frac{-4+7.48}{2}, x = \frac{-4-7.48}{2}$	Separate the problem into two expressions.
$x = \frac{3.48}{2} = 1.74, x = \frac{-11.48}{2} = -5.74$	Simplify the numerator and divide.

Let's Review!

- There are four methods to solve a quadratic equation algebraically:
  - The square root method is used when there is a squared variable term and a constant term.
  - Completing the square is used when there is a squared variable term and an even variable term.
  - Factoring is used when the equation can be factored.
  - The quadratic formula can be used for any quadratic equation.

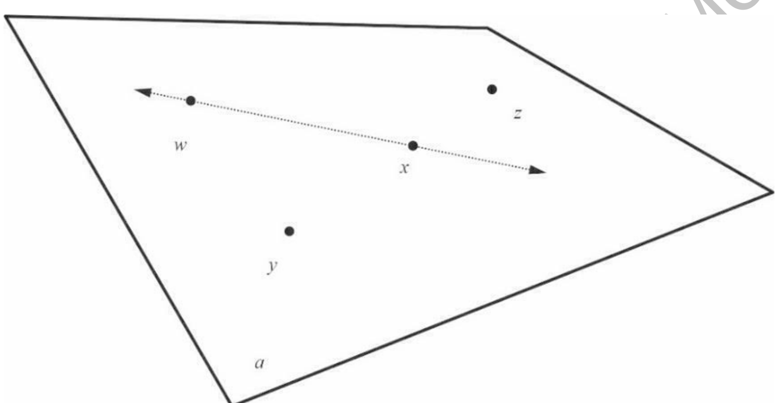
Congruence

This lesson discusses basic terms for geometry. Many polygons have the property of lines of symmetry, or rotational symmetry. Rotations, reflections, and translations are ways to create congruent polygons.

Geometry Terms

The terms *point*, *line*, and *plane* help define other terms in geometry. A point is an exact location in space with no size and has a label with a capital letter. A line has location and direction, is always straight, and has infinitely many points that extend in both directions. A plane has infinitely many intersecting lines that extend forever in all directions.

The diagram shows point W, point X, point Y, and point Z. The line is labeled as $\overleftrightarrow{WX}$, and the plane is Plane A or Plane WYZ (or any three points in the plane).

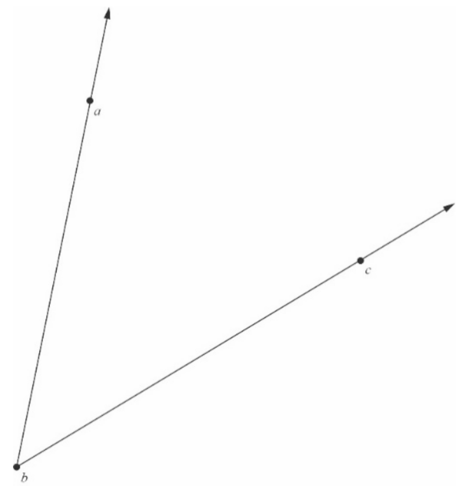


With these definitions, many other geometry terms can be defined. *Collinear* is a term for points that lie on the same line, and *coplanar* is a term for points and/or lines within the same plane. A line segment is a part of a line with two endpoints. For example, $\overline{WX}$ has endpoints W and X. A ray has an endpoint and extends forever in one direction. For example, $\overrightarrow{AB}$ has an endpoint of A, and $\overrightarrow{BA}$ has an endpoint of B. The intersection of lines, planes, segment, or rays is a point or a set of points.

Some key statements that are evident in geometry are

- There is exactly one straight line through any two points.
- There is exactly one plane that contains any three non-collinear points.
- A line with points in the plane lies in the plane.
- Two lines intersect at a point.
- Two planes intersect at a line.

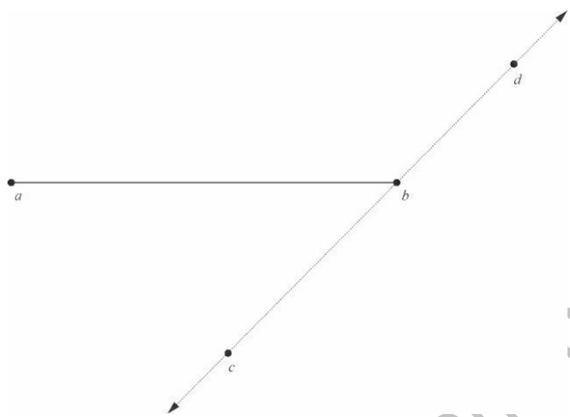
Two rays that share an endpoint form an angle. The vertex is the common endpoint of the two rays that form an angle. When naming an angle, the vertex is the center point. The angle below is named $\angle ABC$ or $\angle CBA$.



An acute angle has a measure between 0° and 90°, and a 90° angle is a right angle. An obtuse angle has a measure between 90° and 180°, and a 180° angle is a straight angle.

There are two special sets of lines. Parallel lines are at least two lines that never intersect within the same plane. Perpendicular lines intersect at one point and form four angles.

Example



> **BE CAREFUL!**
> Lines are always named with two points, a plane can be named with three points, and an angle is named with the vertex as the center point.

Describe the diagram.

A. Points A, B, C, and D are collinear.

B. Points A, C, and D are collinear.

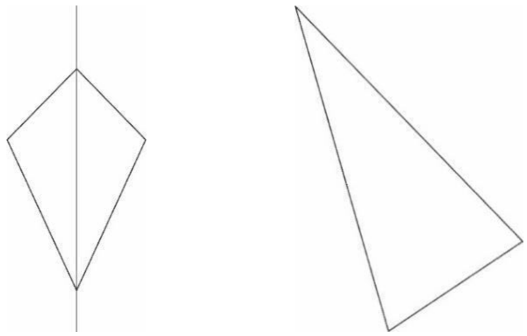
C. $\overline{CD}$ intersects $\overleftrightarrow{AB}$ at point B.

D. $\overline{AB}$ intersects $\overleftrightarrow{CD}$ at point B.

The correct answer is **D.** The correct solution is $\overline{AB}$ intersects $\overleftrightarrow{CD}$ at point B. The segment intersects the line at point B. **See Lesson: Congruence.**

Line and Rotational Symmetry

Symmetry is a reflection or rotation of a shape that allows that shape to be carried onto itself. Line symmetry, or reflection symmetry, is when two halves of a shape are reflected onto each other across a line. A shape may have none, one, or several lines of symmetry. A kite has one line of symmetry, and a scalene triangle has no lines of symmetry.



> **KEEP IN MIND**
> A polygon can have both, neither, or either reflection and rotational symmetry.

Rotational symmetry is when a figure can be mapped onto itself by a rotation about a point through any angle between 0° and 360°. The order of rotational symmetry is the number of times the object can be rotated. If there is no rotational symmetry, the order is 1 because the object can only be rotated 360° to map the figure onto itself. A square has 90° rotational symmetry and is order 4 because it can be rotated 90°, 180°, 270°, and 360°. A trapezoid has no rotational symmetry and is order 1 because it can only be rotated 360° to map onto itself.

Example

What is the rotational symmetry for a regular octagon?

A. 30° B. 45° C. 60° D. 75°

The correct answer is **B.** The correct solution is 45°. For a regular polygon, divide 360° by the eight sides of the octagon to obtain 45°. **See Lesson: Congruence.**

Rotations, Reflections, and Translations

There are three types of transformations: rotations, reflections, and translations. A rotation is a turn of a figure about a point in a given direction. A reflection is a flip over a line of symmetry, and a translation is a slide horizontally, vertically, or both. Each of these transformations produces a congruent image.

A rotation changes ordered pairs (x, y) in the coordinate plane. A 90° rotation counterclockwise about the point becomes $(-y, x)$, a 180° rotation counterclockwise about the point becomes $(-x, -y)$, and a 270° rotation the point becomes $(y, -x)$. Using the point (6, −8),

- 90° rotation counterclockwise about the origin → (8, 6)
- 180° rotation counterclockwise about the origin → (−6, 8)
- 270° rotation counterclockwise about the origin → (−8, −6)

A reflection also changes ordered pairs (x, y) in the coordinate plane. A reflection across the x-axis changes the sign of the y-coordinate, and a reflection across the y-axis changes the sign of the x-coordinate. A reflection over the line $y = x$ changes the points to (y, x), and a reflection over the line $y = -x$ changes the points to $(-y, -x)$. Using the point $(6, -8)$,

- A reflection across the x-axis → $(6, 8)$
- A reflection across the y-axis → $(-6, -8)$
- A reflection over the line $y = x$ → $(-8, 6)$
- A reflection over the line $y = -x$ → $(8, -6)$

A translation changes ordered pairs (x, y) left or right and/or up or down. Adding a positive value to an x-coordinate is a translation to the right, and adding a negative value to an x-coordinate is a translation to the left. Adding a positive value to a y-coordinate is a translation up, and adding a negative value to a y-coordinate is a translation down. Using the point $(6, -8)$,

> **KEEP IN MIND**
>
> A rotation is a turn, a reflection is a flip, and a translation is a slide.

- A translation of $(x + 3)$ is a translation right 3 units → $(9, -8)$
- A translation of $(x - 3)$ is a translation left 3 units → $(3, -8)$
- A translation of $(y + 3)$ is a translation up 3 units → $(6, -5)$
- A translation of $(y - 3)$ is a translation down 3 units → $(6, -11)$

Example

$\triangle ABC$ has points $A\,(3, -2)$, $B\,(2, -1)$, and $C\,(-1, 4)$, which after a transformation become $A'\,(2, 3)$, $B'\,(1, 2)$, and $C'\,(-4, -1)$. What is the transformation between the points?

A. Reflection across the x-axis

B. Reflection across the y-axis

C. Rotation of 90° counterclockwise

D. Rotation of 270° counterclockwise

The correct answer is **C.** The correct solution is a rotation of 90° counterclockwise because the points (x, y) become $(y, -x)$. **See Lesson: Congruence.**

Let's Review!

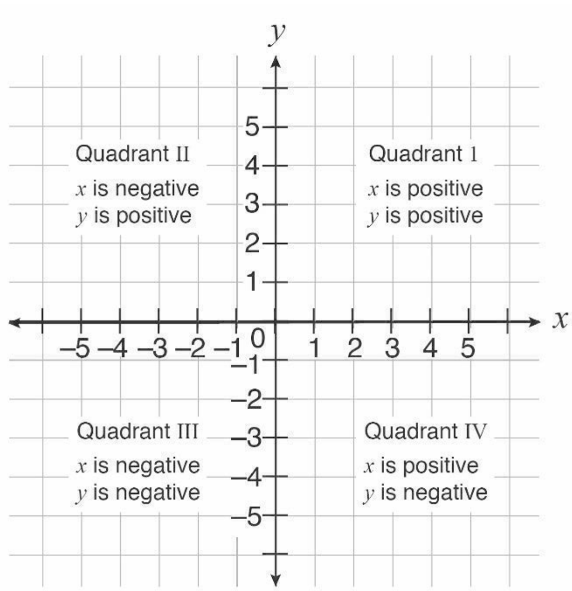
- The terms *point*, *line*, and *plane* help define many terms in geometry.
- Symmetry allows a figure to carry its shape onto itself. This can be reflectional or rotational symmetry.
- Three transformations are rotation (turn), reflection (flip), and translation (slide).

Similarity, Right Triangles, and Trigonometry

This lesson defines and applies terminology associated with coordinate planes. It also demonstrates how to find the area of two-dimensional shapes and the surface area and volume of three-dimensional cubes and right prisms.

Coordinate Plane

The **coordinate plane** is a two-dimensional number line with the horizontal axis called the **x-axis** and the vertical axis called the **y-axis**. Each **ordered pair** or **coordinate** is listed as (x, y). The center point is the origin and has an ordered pair of $(0, 0)$. A coordinate plane has four quadrants.



KEEP IN MIND

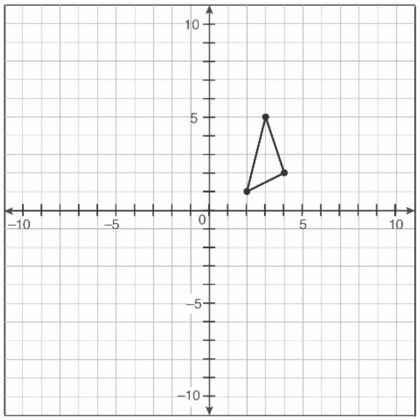
The *x*-coordinates are positive to the right of the *y*-axis. The *y*-coordinates are positive above the *x*-axis.

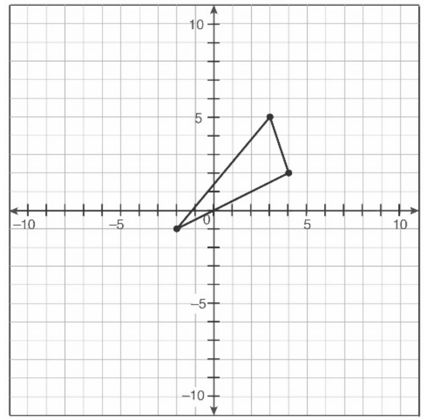
To graph a point in the coordinate plane, start with the x-coordinate. This point states the number of steps to the left (negative) or to the right (positive) from the origin. Then, the y-coordinate states the number of steps up (positive) or down (negative) from the x-coordinate.

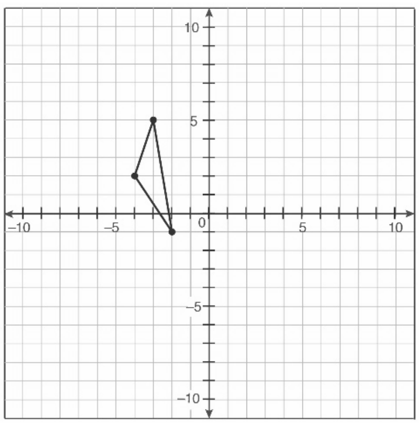
Given a set of ordered pairs, points can be drawn in the coordinate plane to create polygons. The length of a segment can be found if the segment has the same first coordinate or the same second coordinate.

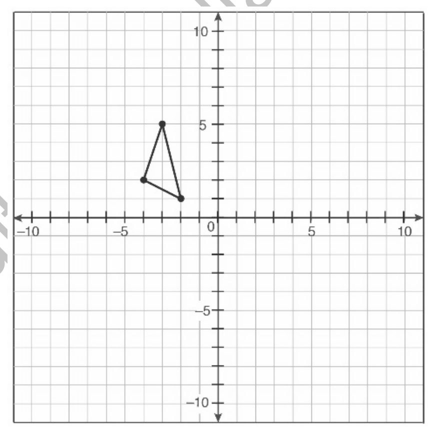
Examples

1. **Draw a triangle with the coordinates $(-2,-1), (-3,5), (-4,2)$.**

A. 

B. 

C. 

D. 

The correct answer is **C.** The first point is in the third quadrant because x is negative and y is negative, and the last two points are in the second quadrant because x is negative and y is positive. **See Lesson: Similarity, Right Triangles, and Trigonometry.**

2. **Given the coordinates for a rectangle $(4,8), (4,-2), (-1,-2)(-1,8)$, find the length of each side of the rectangle.**

   A. 3 units and 6 units

   B. 3 units and 10 units

   C. 5 units and 6 units

   D. 5 units and 10 units

The correct answer is **D.** The correct solution is 5 units and 10 units. The difference between the x-coordinates is $4-(-1)=5$ units, and the difference between the y-coordinates is $8-(-2)=10$ units. **See Lesson: Similarity, Right Triangles, and Trigonometry.**

3. **The dimensions for a soccer field are 45 meters by 90 meters. One corner of a soccer field on the coordinate plane is $(-45, -30)$. What could a second coordinate be?**

   A. $(-45, 30)$

   B. $(-45, 45)$

   C. $(-45, 60)$

   D. $(-45, 75)$

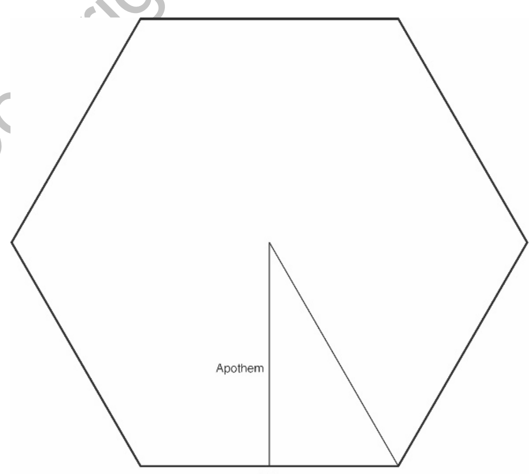
   The correct answer is **C.** The correct solution is $(-45, 60)$ because 90 can be added to the y-coordinate, $-30 + 90 = 60$. **See Lesson: Similarity, Right Triangles, and Trigonometry.**

Area of Two-Dimensional Objects

The **area** is the number of unit squares that fit inside a two-dimensional object. A unit square is one unit long by one unit wide, which includes 1 foot by 1 foot and 1 meter by 1 meter. The unit of measurement for area is units squared (or feet squared, meters squared, and so on). The following are formulas for calculating the area of various shapes.

> **BE CAREFUL!**
> Make sure that you apply the correct formula for area of each two-dimensional object.

- Rectangle: The product of the length and the width, $A = lw$.
- Parallelogram: The product of the base and the height, $A = bh$.
- Square: The side length squared, $A = s^2$.
- Triangle: The product of one-half the base and the height, $A = \frac{1}{2}bh$.
- Trapezoid: The product of one-half the height and the sum of the bases, $A = \frac{1}{2}h(b_1 + b_2)$.
- Regular polygon: The product of one-half the **apothem** (a line from the center of the regular polygon that is perpendicular to a side) and the sum of the perimeter, $A = \frac{1}{2}ap$.



Examples

1. **A trapezoid has a height of 3 centimeters and bases of 8 centimeters and 10 centimeters. Find the area in square centimeters.**

   A. 18 B. 27 C. 52 D. 55

   The correct answer is **B.** The correct solution is 27. Substitute the values into the formula and simplify using the order of operations, $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(3)(8 + 10) = \frac{1}{2}(3)(18) = 27$ square centimeters. **See Lesson: Similarity, Right Triangles, and Trigonometry.**

2. **A regular decagon has a side length of 12 inches and an apothem of 6 inches. Find the area in square inches.**

   A. 120 B. 360 C. 720 D. 960

   The correct answer is **B.** The correct solution is 360. Simplify using the order of operations, $A = \frac{1}{2}ap = \frac{1}{2}(6)(12(10)) = 360$ square inches. **See Lesson: Similarity, Right Triangles, and Trigonometry.**

3. **Two rectangular rooms need to be carpeted. The dimensions of the first room are 18 feet by 19 feet, and the dimensions of the second room are 12 feet by 10 feet. What is the total area to be carpeted in square feet?**

   A. 118 B. 236 C. 342 D. 462

   The correct answer is **D.** The correct solution is 462. Substitute the values into the formula and simplify using the order of operations, $A = lw + lw = 18(19) + 12(10) = 342 + 120 = 462$ square feet. **See Lesson: Similarity, Right Triangles, and Trigonometry.**

4. **A picture frame is in the shape of a right triangle with legs 12 centimeters and 13 centimeters and hypotenuse of 17 centimeters. What is the area in square centimeters?**

   A. 78 B. 108 C. 117 D. 156

   The correct answer is **A.** The correct solution is 78. Substitute the values into the formula and simplify using the order of operations, $A = \frac{1}{2}bh = \frac{1}{2}(12)(13) = 78$ square centimeters. **See Lesson: Similarity, Right Triangles, and Trigonometry.**

Surface Area and Volume of Cubes and Right Prisms

A three-dimensional object has length, width, and height. **Cubes** are made up of six congruent square faces. A **right prism** is made of three sets of congruent faces, with at least two sets of congruent rectangles.

BE CAREFUL!

Surface area is a two-dimensional calculation, and volume is a three-dimensional calculation.

The **surface area** of any three-dimensional object is the sum of the area of all faces. The formula for the surface area of a cube is $SA = 6s^2$ because there are six congruent faces. For a right rectangular prism, the surface area formula is $SA = 2lw + 2lh + 2hw$ because there are three sets of congruent rectangles. For a triangular prism, the surface area formula is twice the area of the base plus the area of the other three rectangles that make up the prism.

The **volume** of any three-dimensional object is the amount of space inside the object. The volume formula for a cube is $V = s^3$. The volume formula for a rectangular prism is the area of the base times the height, or $V = Bh$.

Examples

1. **A cube has a side length of 5 centimeters. What is the surface area in square centimeters?**

   A. 20 B. 25 C. 125 D. 150

   The correct answer is **D.** The correct solution is 150. Substitute the values into the formula and simplify using the order of operations, $SA = 6s^2 = 6(5^2) = 6(25) = 150$ square centimeters. **See Lesson: Similarity, Right Triangles, and Trigonometry.**

2. **A cube has a side length of 5 centimeters. What is the volume in cubic centimeters?**

   A. 20 B. 25 C. 125 D. 180

   The correct answer is **C.** The correct solution is 125. Substitute the values into the formula and simplify using the order of operations, $V = s^3 = 5^3 = 125$ cubic centimeters. **See Lesson: Similarity, Right Triangles, and Trigonometry.**

3. **A right rectangular prism has dimensions of 4 inches by 5 inches by 6 inches. What is the surface area in square inches?**

   A. 60 B. 74 C. 120 D. 148

   The correct answer is **D.** The correct solution is 148. Substitute the values into the formula and simplify using the order of operations, $SA = 2lw + 2lh + 2hw = 2(4)(5) + 2(4)(6) + 2(6)(5) = 40 + 48 + 60 = 148$ square inches. **See Lesson: Similarity, Right Triangles, and Trigonometry.**

4. **A right rectangular prism has dimensions of 4 inches by 5 inches by 6 inches. What is the volume in cubic inches?**

   A. 60 B. 62 C. 120 D. 124

   The correct answer is **C.** The correct solution is 120. Substitute the values into the formula and simplify using the order of operations, $V = lwh = 4(5)(6) = 120$ cubic inches. **See Lesson: Similarity, Right Triangles, and Trigonometry.**

Let's Review!

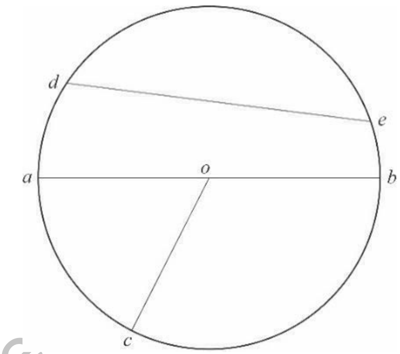
- The coordinate plane is a two-dimensional number line that is used to display ordered pairs. Two-dimensional shapes can be drawn on the plane, and the length of the objects can be determined based on the given coordinates.
- The area of a two-dimensional object is the amount of space inside the shape. There are area formulas to use to calculate the area of various shapes.
- For a three-dimensional object, the surface area is the sum of the area of the faces and the volume is the amount of space inside the object. Cubes and right rectangular prisms are common three-dimensional solids.

CIRCLES

This lesson introduces concepts of circles, including finding the circumference and the area of the circle.

Circle Terminology

A **circle** is a figure composed of points that are equidistant from a given point. The **center** is the point from which all points are equidistant. A **chord** is a segment whose endpoints are on the circle, and the **diameter** is a chord that goes through the center of the circle. The **radius** is a segment with one endpoint at the center of the circle and one endpoint on the circle. **Arcs** have two endpoints on the circle and all points on a circle between those endpoints.



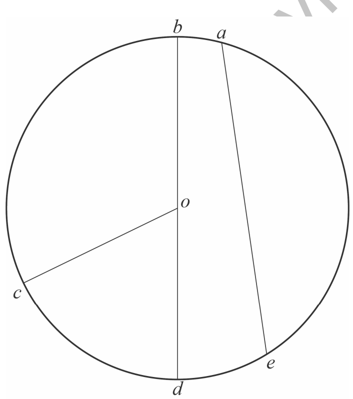
In the circle at the right, O is the center, $\overline{OC}$ is the radius, $\overline{AB}$ is the diameter, $\overline{DE}$ is a chord, and $\widehat{AD}$ is an arc.

KEEP IN MIND

The radius is one-half the length of the diameter of the circle.

Example

Identify a diameter of the circle.



A. $\overline{BD}$ B. $\overline{OC}$ C. $\overline{DO}$ D. $\overline{AE}$

The correct answer is **A.** The correct solution is $\overline{BD}$ because points B and D are on the circle and the segment goes through the center O. **See Lesson: Circles.**

Circumference and Area of a Circle

The **circumference** of a circle is the perimeter, or the distance, around the circle. There are two ways to find the circumference. The formulas are the product of the diameter and pi or the product of twice the radius and pi. In symbol form, the formulas are $C = \pi d$ or $C = 2\pi r$.

> **BE CAREFUL!**
> Make sure that you apply the correct formula for circumference and area of a circle.

The **area** of a circle is the amount of space inside a circle. The formula is the product of pi and the radius squared. In symbol form, the formula is $A = \pi r^2$. The area is always expressed in square units.

Given the circumference or the area of a circle, the radius and the diameter can be determined. The given measurement is substituted into the appropriate formula. Then, the equation is solved for the radius or the diameter.

Examples

1. **Find the circumference in centimeters of a circle with a diameter of 8 centimeters. Use 3.14 for π.**

   A. 12.56 B. 25.12 C. 50.24 D. 100.48

   The correct answer is **B.** The correct solution is 25.12 because $C = \pi d \approx 3.14(8) \approx 25.12$ centimeters. **See Lesson: Circles.**

2. **Find the area in square inches of a circle with a radius of 15 inches. Use 3.14 for π.**

   A. 94.2 B. 176.63 C. 706.5 D. 828.96

   The correct answer is **C.** The correct solution is 706.5 because $A = \pi r^2 \approx 3.14(15)^2 \approx 3.14(225) \approx 706.5$ square inches. **See Lesson: Circles.**

3. **A circle has a circumference of 70 centimeters. Find the diameter to the nearest tenth of a centimeter. Use 3.14 for π.**

   A. 11.1 B. 22.3 C. 33.5 D. 44.7

   The correct answer is **B.** The correct solution is 22.3 because $C = \pi d; 70 = 3.14d; d \approx 22.3$ centimeters. **See Lesson: Circles.**

4. **A circle has an area of 95 square centimeters. Find the radius to the nearest tenth of a centimeter. Use 3.14 for π.**

   A. 2.7 B. 5.5 C. 8.2 D. 10.9

   The correct answer is **B.** The correct solution is 5.5 because $A = \pi r^2; 95 = 3.14r^2; 30.25 = r^2;\ r \approx 5.5$ centimeters. **See Lesson: Circles.**

Finding Circumference or Area Given the Other Value

Given the circumference of a circle, the area of the circle can be found. First, substitute the circumference into the formula and find the radius. Substitute the radius into the area formula and simplify.

Reverse the process to find the circumference given the area. First, substitute the area into the area formula and find the radius. Substitute the radius into the circumference formula and simplify.

> **BE CAREFUL!**
> Pay attention to the details with each formula and apply them in the correct order.

Examples

1. **The circumference of a circle is 45 inches. Find the area of the circle in square inches. Round to the nearest tenth. Use 3.14 for π.**

   A. 51.8 B. 65.1 C. 162.8 D. 204.5

   The correct answer is **C.** The correct solution is 162.8.
   $C = 2\pi r; 45 = 2(3.14)r; 45 = 6.28r; r \approx 7.2$ inches. $A = \pi r^2 \approx 3.14(7.2)^2 \approx 3.14(51.84) \approx 162.8$ square inches. **See Lesson: Circles.**

2. **The area of a circle is 60 square centimeters. Find the circumference of the circle in centimeters. Round to the nearest tenth. Use 3.14 for π.**

   A. 4.4 B. 13.8 C. 19.1 D. 27.6

   The correct answer is **D.** The correct solution is 27.6.
   $A = \pi r^2; 60 = 3.14r^2; 19.11 = r^2; r \approx 4.4$ centimeters. $C = 2\pi r; C = 2(3.14)4.4 \approx 27.6$ centimeters. **See Lesson: Circles.**

Let's Review!

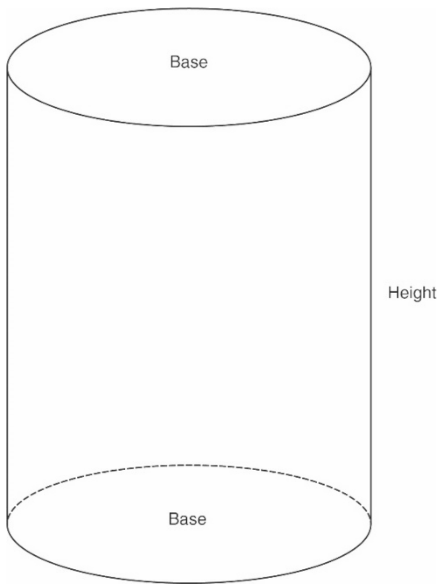
- Key terms related to circles are *radius, diameter, chord,* and *arc*. Note that the diameter is twice the radius.
- The circumference or the perimeter of a circle is the product of pi and the diameter or twice the radius and pi.
- The area of the circle is the product of pi and the radius squared.

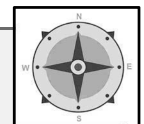
MEASUREMENT AND DIMENSION

This lesson applies the formulas of volume for cylinders, pyramids, cones, and spheres to solve problems.

Volume of a Cylinder

A **cylinder** is a three-dimensional figure with two identical circular bases and a rectangular lateral face.





KEEP IN MIND

The volume of a cylinder can be expressed in terms of π, and the volume is measured in cubic units.

The volume of a cylinder equals the product of the area of the base and the height of the cylinder. This is the same formula used to calculate the volume of a right prism. In this case, the area of a base is a circle, so the formula is $V = Bh = \pi r^2 h$. The height is the perpendicular distance between the two circular bases.

Example

Find the volume of a cylinder in cubic centimeters with a radius of 13 centimeters and a height of 12 centimeters.

A. 156π B. 312π C. $1{,}872\pi$ D. $2{,}028\pi$

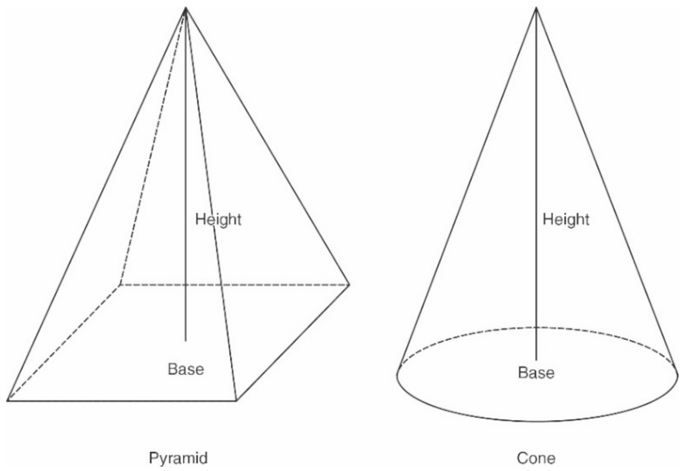
The correct answer is **D.** The correct solution is $2{,}028\pi$. Substitute the values into the formula and simplify using the order of operations, $V = \pi r^2 h = \pi 13^2(12) = \pi(169)(12) = 2{,}028\pi$ cubic centimeters. **See Lesson: Measurement and Dimension.**

Volume of a Pyramid and a Cone

A **pyramid** is a three-dimensional solid with one base and all edges from the base meeting at the top, or apex. Pyramids can have any two-dimensional shape as the base. A **cone** is similar to a pyramid, but it has a circle instead of a polygon for the base.

BE CAREFUL!

Make sure that you apply the correct formula for area of the base for a pyramid.



The formula for the volume of a pyramid is similar to a prism, $V = \frac{1}{3}Bh$ where B is the area of the base. The base is a circle for a cone, and the formula for the volume is $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$.

Examples

1. **A regular hexagonal pyramid has base with side lengths of 5 centimeters and an apothem of 3 centimeters. If the height is 6 centimeters, find the volume in cubic centimeters.**

   A. 90 B. 180 C. 270 D. 360

   The correct answer is **A.** The correct solution is 90. Substitute the values into the formula and simplify using the order of operations, $V = \frac{1}{3}Bh = \frac{1}{3}\left(\frac{1}{2}ap\right)h = \frac{1}{3}\left(\frac{1}{2}(3)(30)\right)6 = 90$ cubic centimeters. **See Lesson: Measurement and Dimension.**

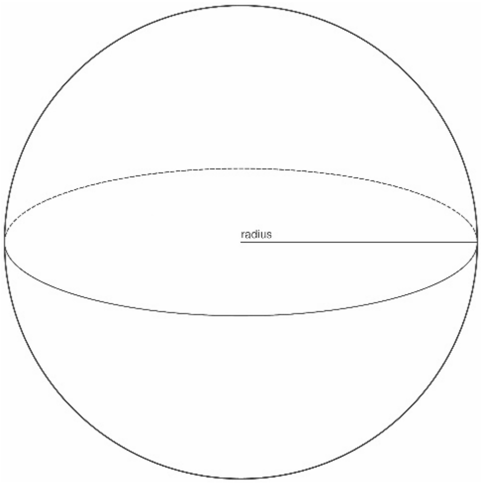
2. **A cone has a radius of 10 centimeters and a height of 9 centimeters. Find the volume in cubic centimeters.**

   A. 270π B. 300π C. 810π D. 900π

   The correct answer is **B.** The correct solution is 300π. Substitute the values into the formula and simplify using the order of operations, $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi 10^2(9) = \frac{1}{3}\pi(100)(9) = 300\pi$ cubic centimeters. **See Lesson: Measurement and Dimension.**

Volume of a Sphere

A **sphere** is a round, three-dimensional solid, with every point on its surface equidistant to the center. The formula for the volume of a sphere is represented by just the radius of the sphere. The volume of a sphere is $V = \frac{4}{3}\pi r^3$. The volume of a hemi (half) of a sphere is $V = \left(\frac{1}{2}\right)\frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$.



BE CAREFUL!

The radius is cubed, not squared, for the volume of a sphere.

Example

A sphere has a radius of 3 centimeters. Find the volume of a sphere in cubic centimeters.

A. 18π B. 27π C. 36π D. 45π

The correct answer is **C.** The correct solution is 36π. Substitute the values into the formula and simplify using the order of operations, $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi 3^3 = \frac{4}{3}\pi(27) = 36\pi$ cubic centimeters.

See Lesson: Measurement and Dimension.

Let's Review!

- The volume is the capacity of a three-dimensional object and is expressed in cubic units.
- The volume formula for a cylinder is the product of the area of the base (which is a circle) and the height of the cylinder.
- The volume formula for a pyramid or cone is one-third of the product of the area of the base (a circle in the case of the cone) and the height of the pyramid or cone.
- The volume formula for a sphere is $V = \frac{4}{3}\pi r^3$.

CHAPTER 11 ADVANCED ALGEBRA AND GEOMETRY PRACTICE QUIZ 1

1. **Half of a circular garden with a radius of 11.5 feet needs weeding. Find the area in square feet that needs weeding. Round to the nearest hundredth. Use 3.14 for π.**

   A. 207.64 B. 415.27

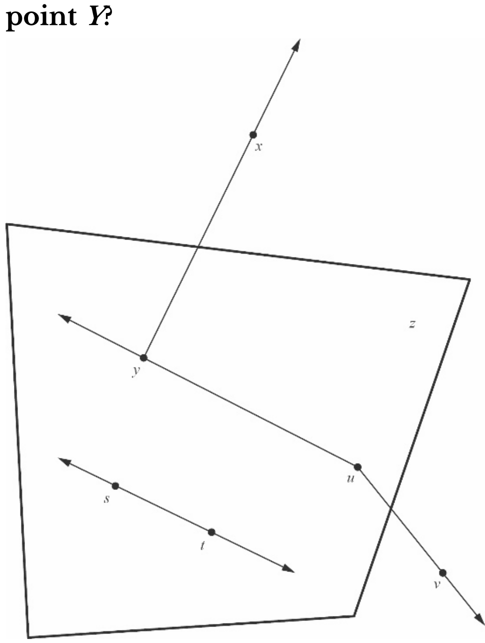
   C. 519.08 D. 726.73

2. **The area of a circle is 18 square inches. Find the circumference of the circle to the nearest tenth of an inch. Use 3.14 for π.**

   A. 2.4 B. 7.5

   C. 15.1 D. 30.1

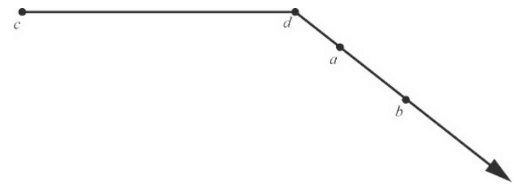
3. **What are the rays that intersect at point *Y*?**



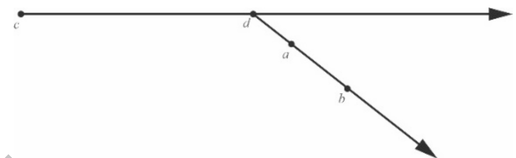
   A. $\overrightarrow{YX}$ and $\overrightarrow{UY}$ B. $\overrightarrow{YX}$ and $\overrightarrow{YU}$

   C. $\overrightarrow{XY}$ and $\overrightarrow{UY}$ D. $\overrightarrow{XY}$ and $\overrightarrow{YU}$

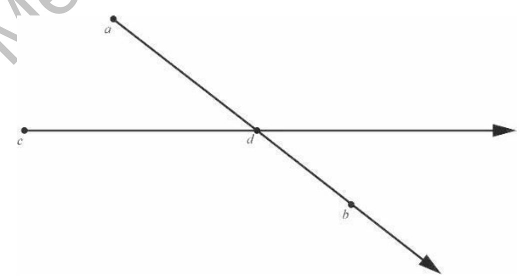
4. **Select the drawing of $\overrightarrow{AB}$ and $\overrightarrow{CD}$ intersecting at *D*.**



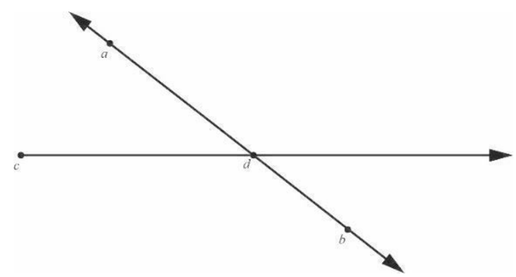
   A.



   B.



   C.



   D.

5. **Which statement best describes the multiples of a whole number?**

   A. The multiples of a whole number exclude 0.

   B. The multiples of a whole number are integers.

   C. The multiples of a whole number are all positive.

   D. The multiples of a whole number are all negative.

6. **How many whole-number factors does a prime number have?**

   A. 0

   B. 1

   C. 2

   D. Not enough information

7. **A rectangular pyramid has a height of 7 meters and a volume of 112 cubic meters. Find the area of the base in square meters.**

   A. 16 B. 28

   C. 42 D. 48

8. **Find the volume in cubic centimeters of a square pyramid with a side length of 4 centimeters and a height of 12 centimeters.**

   A. 48 B. 64

   C. 144 D. 192

9. **A box in the shape of a right rectangular prism has dimensions of 6 centimeters by 7 centimeters by 8 centimeters. What is the volume in cubic centimeters?**

   A. 280 B. 336

   C. 560 D. 672

10. **A stop sign is a regular octagon with an area of 27,000 square centimeters and an apothem of 90 centimeters. What is the length in centimeters of one side?**

    A. 38 B. 75

    C. 300 D. 600

11. **Solve the equation by the quadratic formula, $x^2 + 10x + 8 = 0$.**

    A. −0.88 and 9.13

    B. 0.88 and −9.13

    C. −0.88 and −9.13

    D. 0.88 and 9.13

12. **Solve the equation by the square root method, $2x^2 = 162$.**

    A. ±8 B. ±9

    C. ±10 D. ±11

Chapter 11 Advanced Algebra and Geometry Practice Quiz 1 – Answer Key

1. A. The correct solution is 207.64 because $A = \frac{1}{2}\pi r^2 \approx \frac{1}{2}(3.14)(11.5)^2 \approx \frac{1}{2}(3.14)(132.25) \approx 207.64$ square feet. **See Lesson: Circles.**

2. C. The correct solution is 15.1.

$A = \pi r^2; 18 = 3.14r^2; 5.73 = r^2; r \approx 2.4$ centimeters. $C = 2\pi r; C = 2(3.14)2.4 \approx 15.1$ centimeters.
See Lesson: Circles.

3. A. The correct solution is $\overrightarrow{YX}$ and $\overrightarrow{UY}$ because these rays intersect at point Y.
See Lesson: Congruence.

4. C. The two rays intersect at point D. **See Lesson: Congruence.**

5. B. Any product of a whole number and an integer is a multiple of that number. Note that the multiples of a whole number and another whole number must be integers because they are the products of two whole factors. Changing the sign of these products does not change their status as integers. Therefore, the multiples of a whole number are all integers. An alternative approach to this question is elimination: the products of 9 and 0, 1, and −1 are 0, 9, and −9; therefore, answers A, B, and C are false. **See Lesson: Factors and Multiples.**

6. C. Recall that a number is prime if it only has 1 and itself as factors. Both must be whole; therefore, a prime number has two whole-number factors. **See Lesson: Factors and Multiples.**

7. D. The correct solution is 48. Substitute the values into the formula, $112 = \frac{1}{3}B(7)$ and simplify the right side of the equation, $112 = \frac{7}{3}B$. Multiply both sides of the equation by the reciprocal, $B = 48$ square meters. **See Lesson: Measurement and Dimension.**

8. B. The correct solution is 64. Substitute the values into the formula and simplify using the order of operations, $V = \frac{1}{3}Bh = \frac{1}{3}s^2h = \frac{1}{3}(4^2)12 = \frac{1}{3}(16)(12) = 64$ cubic centimeters. **See Lesson: Measurement and Dimension.**

9. B. The correct solution is 336. Substitute the values into the formula and simplify using the order of operations, $V = lwh = 6(7)(8)$ cubic centimeters. **See Lesson: Similarity, Right Triangles, and Trigonometry.**

10. B. The correct solution is 75. Substitute the values into the formula, $27{,}000 = \frac{1}{2}(90)p$ and simplify using the order of operations, $27{,}000 = 45p$. Divide both sides of the equation by 45 to find the perimeter, $p = 600$ centimeters. Divide the perimeter by 8 to find the length of 75 centimeters for each side. **See Lesson: Similarity, Right Triangles, and Trigonometry.**

11. C. The correct solutions are −0.88 and −9.13. **See Lesson: Solving Quadratic Equations.**

$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(8)}}{2(1)}$	Substitute 1 for *a*, 10 for *b*, and 8 for *c*.
$x = \frac{-10 \pm \sqrt{100-32}}{2}$	Apply the exponent and perform the multiplication.
$x = \frac{-10 \pm \sqrt{68}}{2}$	Perform the subtraction.
$x = \frac{-10 \pm 8.25}{2}$	Apply the square root.
$x = \frac{-10+8.25}{2}, x = \frac{-10-8.25}{2}$	Separate the problem into two expressions.
$x = \frac{-1.75}{2} = -0.88, x = \frac{-18.25}{2} = -9.13$	Simplify the numerator and divide.

12. B. The correct solution is ±9. **See Lesson: Solving Quadratic Equations.**

$x^2 = 81$	Divide both sides of the equation by 2.
$x = \pm 9$	Apply the square root to both sides of the equation.

CHAPTER 11 ADVANCED ALGEBRA AND GEOMETRY PRACTICE QUIZ 2

1. A half circle has an area of 50 square inches. Find the radius to the nearest tenth of an inch. Use 3.14 for π.

A. 1.4 B. 2.8

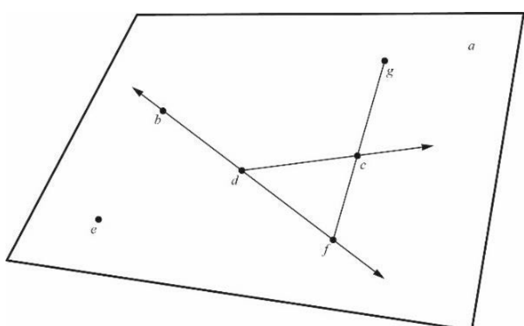
C. 4.2 D. 5.6

2. The bottom of a plastic pool has an area of 64 square feet. What is the radius to the nearest tenth of a foot? Use 3.14 for π.

A. 2.3 B. 4.5

C. 6.9 D. 10.2

3. What points in the diagram are collinear?



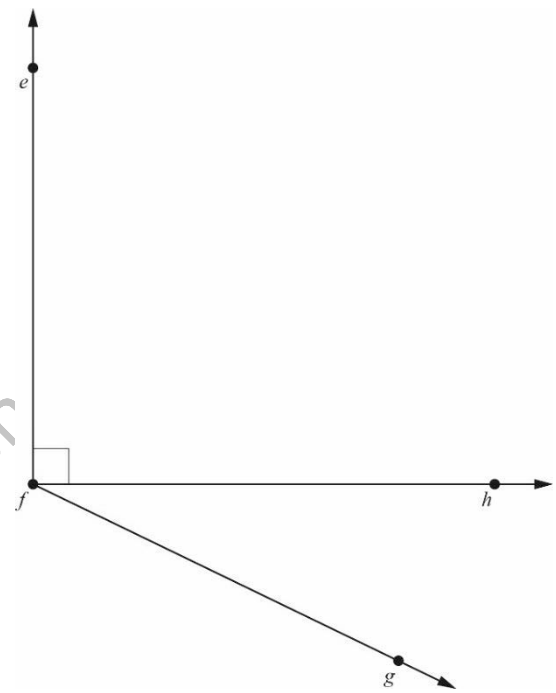
A. Points *D*, *C*, and *F*

B. Points *B*, *D*, and *F*

C. Points *B*, *C*, and *E*

D. Points *B*, *C*, and *D*

4. Name the right angle in the diagram.



A. $\angle EHF$ B. $\angle EFG$

C. $\angle EFH$ D. EGF

5. Which type of number is always composite?

A. The product of two prime numbers

B. The product of two whole numbers

C. The product of 0 and a prime number

D. The product of 1 and a whole number

6. How many unique prime factors does 75 have?

A. 1
B. 2
C. 3
D. 4

7. A jar of salsa has a diameter of 12 centimeters and a height of 10 centimeters. There are 4 centimeters of salsa left in the jar. How much salsa was used if the jar was originally filled to the top? State the answer in cubic centimeters in terms of π.

A. 216π
B. 360π
C. 864π
D. $1{,}440\pi$

8. A cone has a radius of 4 centimeters and a height of 9 centimeters. Find the volume in cubic centimeters.

A. 16π
B. 32π
C. 48π
D. 64π

9. Three vertices of a parallelogram are $(8, 5), (-2, 5), (-1, 1)$. What is the fourth coordinate?

A. $(9, 1)$
B. $(8, 1)$
C. $(9, -1)$
D. $(8, -1)$

10. A cube has a surface area of 54 square feet. What is the side length in feet?

A. 2
B. 3
C. 4
D. 5

11. Solve the equation by any method, $x^2 + 16x + 33 = 0$.

A. $-8 \pm \sqrt{31}$
B. $8 \pm \sqrt{31}$
C. $-8 \pm \sqrt{33}$
D. $8 \pm \sqrt{33}$

12. Solve the equation by any method, $6x^2 + 19x + 10 = 0$.

A. $\frac{5}{2}$ and $\frac{2}{3}$
B. $\frac{5}{2}$ and $-\frac{2}{3}$
C. $-\frac{5}{2}$ and $\frac{2}{3}$
D. $-\frac{5}{2}$ and $-\frac{2}{3}$

CHAPTER 11 ADVANCED ALGEBRA AND GEOMETRY PRACTICE QUIZ 2 – ANSWER KEY

1. D. The correct solution is 5.6 because $A = \frac{1}{2}\pi r^2; 50 = \left(\frac{1}{2}\right)3.14r^2; 50 = 1.57r^2; 31.85 = r^2;\ r \approx$ 5.6 inches.
See Lesson: Circles.

2. B. The correct solution is 4.5 because $A = \pi r^2; 64 = 3.14r^2; 20.38 = r^2;\ r \approx 4.5$ feet.
See Lesson: Circles.

3. B. The correct solution is points *B*, *D*, and *F* because these points are line $\overleftrightarrow{BF}$.
See Lesson: Congruence.

4. C. The correct solution is $\angle EFH$ because the vertex of the right angle is *F* and the other two points are *E* and *H*. **See Lesson: Congruence.**

5. A. The product of two prime numbers has those two prime numbers as factors, and because a prime number is greater than or equal to 2, the product must be at least 4. Therefore, its factors must include numbers other than itself and 1—specifically, the two prime numbers. **See Lesson: Factors and Multiples.**

6. B. The prime factorization—for example, using a factor tree—shows that 75 has the prime factors 3, 5, and 5, since 3 × 5 × 5 = 75. Because 5 repeats, 75 has only two unique prime factors. **See Lesson: Factors and Multiples.**

7. A. The correct solution is 216π. The radius is one-half of the diameter, 6 centimeters. The height of the used salsa is 10 – 4, or 6 centimeters. Substitute the values into the formula and simplify using the order of operations, $V = \pi r^2 h = \pi 6^2(6) = \pi(36)(6) = 216\pi$ cubic centimeters.
See Lesson: Measurement and Dimension.

8. C. The correct solution is 48π cubic centimeters. Substitute the values into the formula and simplify using the order of operations, $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4^2)(9) = \frac{1}{3}\pi(16)(9) = 48\pi$ cubic centimeters. **See Lesson: Measurement and Dimension.**

9. A. The correct solution is $(9, 1)$ because this point shows a parallelogram with a base length of 10 units. **See Lesson: Similarity, Right Triangles, and Trigonometry.**

10. B. The correct solution is 3. Substitute the values into the formula $54 = 6s^2$. Solve the equation by dividing both sides of the equation by 6 and applying the square root, $9 = s^2; s = 3$ feet. **See Lesson: Similarity, Right Triangles, and Trigonometry.**

11. A. The correct solutions are $-8 \pm \sqrt{31}$. The equation can be solved by completing the square.
See Lesson: Solving Quadratic Equations.

$x^2 + 16x = -33$ Subtract 33 from both sides of the equation.

$x^2 + 16x + 64 = -33 + 64$ Complete the square, $\left(\frac{16}{2}\right)^2 = 8^2 = 64$.

Add 64 to both sides of the equation.

$x^2 + 16x + 64 = 31$ Simplify the right side of the equation.

$(x + 8)^2 = 31$ Factor the left side of the equation.

$x + 8 = \pm\sqrt{31}$ Apply the square root to both sides of the equation.

$x = -8 \pm \sqrt{31}$ Subtract 8 from both sides of the equation.

12. D. The correct solutions are $-\frac{5}{2}$ and $-\frac{2}{3}$. The equation can be solved by factoring. **See Lesson: Solving Quadratic Equations.**

$(2x + 5)(3x + 2) = 0$ Factor the equation.

$(2x + 5) = 0$ $(3x + 2) = 0$ Set each factor equal to 0.

$2x + 5 = 0$ Subtract 5 from both sides of the equation and divide both sides of the equation by 2 to solve.

$2x = -5$

$x = -\frac{5}{2}$

$3x + 2 = 0$ Subtract 2 from both sides of the equation and divide both sides of the equation by 3 to solve.

$3x = -2$

$x = -\frac{2}{3}$