Fernando Zalamea, Synthetic Philosophy of Contemporary Mathematics


“[A] new synthesis of the analysis/synthesis duality is the order of the day.”

When this bold declaration suddenly emerges out of Fernando Zalamea’s quilt of academic biographies, fragmentary ontological sketches, and compilations of triadic concepts, it sticks out as the sort of thing that would be easy to champion and difficult to substantiate. What meaning could synthesis even have outside of its contrast to analysis? It is this separation and promotion of synthesis that creates some problems for Zalamea: on the one hand, he wants to distinguish contemporary mathematics as surpassing the set-theoretical limits that analytic philosophy has prescribed for itself, but the pragmatic directives that Zalamea gives himself explicitly favor the relative over the absolute, and analysis is the analysis of what is relative. There doesn’t seem to be anything particularly objectionable about the idea that synthesis orients analysis, except for the fact that Zalamea lays down a maxim affirming the “power to orient ourselves within the relative without needing to have recourse to the absolute.” If synthesis (or mathematical creativity) is separated out as that which orients analysis, is it not reduced to functioning as an absolute? Perhaps a glance at Zalamea’s adherence to pragmatic principles and how these principles cohere can offer more insight into what this synthesis of synthesis and analysis looks like for Zalamea.
He outlines this proposed synthesis in *Synthetic Philosophy of Contemporary Mathematics* as the object of a “transitory ontology” governed by Peirce’s pragmaticist maxim, which holds that knowledge, “seen as a logico-semiotic process,” should fulfill four conditions. Three of these conditions are paired with “filters,” each of which specifies a corresponding *unit of analysis* (belonging to a web of such units), while the fourth condition stipulates that these units should be synthetically recomposed by means of local *gluings*. The conditions prescribe that knowledge should have the following characteristics:

1. **Contextual** (versus absolute)
   - A web of *representations*
2. **Relational** (versus substantial)
   - A web of *relations*
3. **Modal** (versus determined)
   - A web of *modes*
4. **Synthetic** (versus analytic)

These four conditions represent a systematic analysis of three different passages from Peirce spanning 1878 to 1905, reproduced here:

Consider what effects that might conceivably have practical bearings, we conceive the object of our conception to have. Then our conception of those effects is the whole of our conception of the object. (1903)

Pragmatism is the principle that every theoretical judgment expressible in a sentence in the indicative mood is a confused form of thought whose only meaning, if it has any, lies in its tendency to enforce a corresponding practical maxim expressible as a conditional sentence having its apodosis in the imperative mood. (1878)

The entire intellectual purport of any symbol consists in the total of all general modes of rational conduct, which, conditionally upon all the possible different circumstances and desires, would ensue upon the acceptance of the symbol. (1905)

Zalamea argues that the “general modes” described in the quote from 1905 refer to a “modalization of the maxim [that] introduces into the Peircean system the problematic of the ‘interlacings’ between the possible contexts of interpretation that may obtain for a given symbol.” The other conditions of the pragmaticist maxim (context and relation) are bound to this modalization like the rings of a Borromean knot. The possible, the necessary and the actual are not just modal operators in formal logic, but also serve to delineate the major divisions of *semiotic* analysis: to know an *actual* sign, we must examine its possible contexts of interpretation and study the necessary “practical imperative consequences associated with each one of those interpretations.”

*Relations* at local and global scales become the primordial units of relevance for ontology, as reflected in Peirce’s own formalization of a logic of relations (which I will return to). Let’s look at each condition in turn
Context is connected to the issue of logical representation (linguistic or diagrammatic), which acknowledges the necessity and impossibility of expressing the absolute in relative terms—a paradox distributed across the associated subproblems of representation (fidelity, distance, reflexivity, partiality, etc.). Frege’s context principle, the foundational moment for analytic philosophy, says nothing more than that the meaning of words is context-dependent; but analytic philosophy has sutured itself to this principle for a century, sustaining itself by analyzing concepts on the model of elementary set theory. Zalamea seeks to extend the richness of mathematics beyond set theory to philosophical analysis, and this richness consists primarily in its tremendous power of creative synthesis, epitomized by the life and mathematical legacy of Alexander Grothendieck. Category theory is a framework that puts distinct branches of mathematics into communication by way of high-level abstraction (it formalizes mathematical structure in terms of objects and morphisms), and Grothendieck’s most influential contribution is his theory of sheaves, which specifies in greater detail the internal parameters of a category. Incidentally, category theory repurposes a number of mathematical concepts (‘morphism’ among them), and it is incumbent upon mathematicians to carefully discriminate between the different contexts in which these concepts occur, lest they lose track of which domain of mathematics is conditioning their thought at a given time. It is this management of the distribution of mathematical knowledges and the discovery of transits among them that David Corfield calls real mathematics, which encompasses “the warp of advanced mathematical knowledge that mathematicians encounter daily in their work, a warp that can be seen as perfectly real.”

Foucault’s influence on Zalamea is evident in the latter’s insistence that real mathematics has a synthetic dynamism of its own that cannot be reduced to analytical classification, but it is also confirmed in the solitary mention he receives in the book, which emphatically separates Deleuze and Foucault from the postmodern chaff: “Leaving aside the traumatic, self-proclaimed post of their alumni, what Deleuze and Foucault teach us is how to transit, how to enter and exit modernity in many ways and from many perspectives.” Transit is the source of dynamism in mathematics and philosophy alike, and is typified by an insight that brings two or more discourses into conversation for the purpose of solving a problem that might seem to be completely unrelated to either discourse independently.
Relations can obtain globally between contexts or locally between “the fragments of necessary contradistinction”—the aforementioned “practical imperative consequences” of the interpretation of a sign. The resurgence of predicative relations as fundamental units of philosophical analysis with the rise of polyadic predicate logic, beginning with Peirce and peaking in Whitehead, is a topic to which I will devote a future post. It was a challenge to centuries-old substance metaphysics, logic in all its previous forms: Aristotelian, Stoic, Scholastic, Cartesian, etc. Relations were demoted to an Ens Minimum (as chronicled by Rodolphe Gasché in the introduction to Of Minimal Things), accidents of a lesser being that were dependent upon substance for their existence. Aquinas affirmed the objective reality of relations, as did the Cartesian coordinate system, as did Spinoza’s modes of substance, as did Kant’s transcendental categories. The vanguard of a new logical method emerged toward the end of the nineteenth century: Frege and Peirce independently developed the foundations of modern logic within six years of each other by adding to the existing Aristotelian system of logic a theory of quantification, the algebra of relations and quantified phrases (universal, existential, etc.). Frege accomplished this feat first in the 1879 publication of the Begriffsschrift, which Peirce had not read before reproducing the result in his paper “On the Algebra of Logic,” published in 1885. “Leibniz hoped for the time when two philosophers would ‘calculate’ rather than discuss the outcome of a philosophical issue,” writes Julius Rudolph Weinberg in 1936. “While many philosophers had clearly realized that propositional form and propositional inference frequently have relational structure (in contradistinction to the subject-predicate structure of the traditional form), the rigorous treatment of relational forms in terms of a logical calculus was not possible until Peirce and Schroeder’s work [on the logic of relations] had been done.” Analytic philosophy absorbed the logic of relations with a mixture of Humean nominalistic empiricism and “the Leibnizean distinction between truths of reason and truths of fact,” which is preserved in Peirce’s logic of relations insofar as it generalizes Leibniz and Kant’s formulation of the subject-predicate form to include modality. The critical reception of logicism became known as logical positivism, and spread quickly, with members of the Vienna Circle taking up academic appointments throughout Europe and North America (Schlick and Quine in the U.S., Ayer in the U.K., etc.).

Modality is a subject about which I am considerably less informed; however, I thought one of the most compelling profiles in The Synthetic Philosophy of Contemporary Mathematics was of Jean-Yves Girard, the French proof theorist who developed linear logic as a synthesis of intuitionistic logic (Peirce and Frege) and co-intuitionistic logic (exemplified by Graham Priest’s paraconsistent logic), which preserves “both the coherent dynamics of intuitionistic (and co-intuitionistic) logic, and the protegeometrical symmetry of classical logic, by symmetrically suspending the structural rules.” Zalamea also refers to linear logic as “nonperennial” and “nonidempotent”—two branches of logic so obscure that I had difficulty tracking down anything about them online. The single reference I found to nonperennial systems outside of the field of potamology, the study of rivers, was in a textbook entitled Discrete, Continuous, and Hybrid Petri Nets, which defines the conditions of a logically perennial language thusly:

Definition 3.8 Let $L_1$ be a subset of $E^*$, where $E$ is the set of events $\{E^1, E^2, \ldots\}$. Language $L_1$ is perennial if, for any word $S_a$ in $L_1$, there is a string $S_b$ such that $S_a S_b$ is a word in $L_1$ and every event in $E$ appears at least once in $S_b$.

While perennial languages are “deadlock-free,” the authors explain, it is not certain that non-perennial languages are as well.
In point of fact, after the sequence $S = E^1 E^2 E^1 E^3$, the marking $m = (1, 0, 1, 1, 1)$ is reached; it is a deadlock since the event $E^2$ can no longer occur.

The syntax of linear logic may be opaque, but the language of events and deadlocks resonates with Zalamea’s claim that the formulas of linear logic “work more like transient actions (and their negations as reactions) than like atemporal and ideal propositions.” This transience is exemplary of Zalamea’s vision of a transitory ontology. As a result of its action-orientedness, linear logic maintains the “profound computational dynamism” of intuitionistic and co-intuitionistic logics while substituting the asymmetrical protogeometry of these logics for the symmetrical protogeometry of classical logic. Intuitionistic logic preserves the law of non-contradiction while rejecting the law of the excluded middle, and co-intuitionistic logic preserves the excluded middle while rejecting non-contradiction, are sublated by linear logic in the form of local modal operators, which Girard describes evocatively as the “opaque modal kernels” of “essentialism.” While I could speculate at what “protogeometry” really designates, it will require another post to work out the intuitionistic overtones and their Cartesian origins, and I would like to connect it to the notion of “polysomatic form finding” as it appears in the work of Paul Schatz.

**Synthetic** logic, of which Girard’s linear logic is a prime example, treats classical logics as “no more than idealities, which can be reconstructed as limits of nonclassical perspectives that are far more real.” Incidentally, way that Zalamea illustrates the shift in perspective aligns with Giuseppe Longo’s identification of the invention of actual infinity: while the Greeks understood infinity as the potential infinity that follows from the fact that you can always add one to any number, they practiced actual infinity by constructing points where lines intersect—in this way, the point appears as a limit, even though it is not formalized as such. The notion of actual infinity fell into disuse until it was appointed a positive quality of God and absorbed into Catholic doctrine during the Italian Renaissance. While the Aristotelian account of potential
infinity was received by Medieval scholars in the form of the motto *infinitum actu non datur*, and almost universally taken for granted by the Scholastics, two stood apart: Galileo affirmed a continuum of infinite indivisibles and Leibniz stated his belief in actual infinity outright. These two dissenting voices gained purchase during the Italian Renaissance, and actual infinity became part of official church doctrine as a positive attribute of God. Giordano Bruno held that actual infinity was a worldly manifestation of God, which was, according to Longo, a precondition for the renewed mathematical engagement with infinity beginning with Leibniz and sustained through the work of Cantor, Hilbert, and many others.

Set-theoretical analysis betrays this heritage by constructing neighborhoods from *points* rather than the other way around, and so Zalamea proposes that we repeat what the Greeks achieved in practice: a synthetic understanding that no longer relies on "classical, ideal 'points', which are *never* seen in nature," one that instead operates on the "limits of real, nonclassical neighborhoods, which, by contrast, are connected to visible, physical deformations." This is the sense in which Zalamea understands the "reality" of the nonclassical perspectives he intends to synthesize: that they admit of phenomenological verification. Albert Lautman, whose work serves as a model of perspicuity and progressive thinking in the philosophy of math throughout *Synthetic Philosophy of Contemporary Mathematics*, provides the phenomenological mooring for the book alongside Maurice Merleau-Ponty. Zalamea’s remarks concerning Lautman’s call for a “rapprochement between metaphysics and mathematics” offer a clear illustration of his enthusiasm for Lautman’s core thesis:

In the Heideggerian transit between pre-ontological understanding and ontic existence, Lautman finds, in channels internal to philosophy, an important echo of his own reflections regarding the transit of the structural and the existential within modern mathematics. ... In virtue of Lautman’s synthetic perception [emphasis mine], mathematics exhibits all of its liveliness, and the nonreductive richness of its technical, conceptual and philosophical movements becomes obvious. And so the *harmonious* [emphasis original] concord of the Plural and the One, perhaps the greatest of mathematics’ ‘miracles’ shines forth.

To say that the Plural and the One coexist harmoniously requires a great deal of qualification—for Badiou, whom Zalamea also cites as a practitioner of synthetic thought, the individual and the collective are constituted politically in the form of militancy; and if ‘concord’ is to be taken in the aesthetic sense, then it would be forged, according to Heidegger, in *strife*: "strife arrives at its high point in the simplicity of intimacy that the unity ... comes about in the instigation of strife." Lautman and Zalamea are clearly more concerned with *resolution*—with synthesis—and it is for this reason, I think, that a Leibnizean optimism presides over much of their book. They both propose that the idiosyncratic creativity of mathematics cannot be separated out from its products, echoing Leibniz’s organic conception of logic, according to which it is regarded as both a finite part of philosophy (with its own objects and methods) and an *instrument* of philosophy (in the sense that it supplies other branches of philosophy (ethics, metaphysics, etc.) with tools for analyzing their own respective objects.

Boethius, making the same argument 1200 years prior, used the analogy of an eye in his commentaries on Porphyry—as an *organ* of the body, it is both a part of the body and an aid to its orientation (i.e. an *instrument* of the body). While their approach consists in a “Phenomenology of Mathematical Creativity” (the title of chapter ten of *Synthetic Philosophy of Contemporary Mathematics*), the topic could just as easily be addressed using the conceptual vocabulary of organic philosophy. And, in fact, it has been.
Since mathematics is suspended between the poles (Zalamea calls them *adjunctions*) of composition and decomposition, integration and differentiation, universalization and particularization—synthesis and analysis, respectively, in their many forms—Lautman emphasizes the practical necessity of having *mixtures* that can, according to Zalamea, “serve as support structures for an extended reason (‘reasonability’).” These mixtures act as “relays in the transmission of information,” mediating the transitions to new mathematical discoveries by imitating the structure of more fundamental mathematical domains and providing the raw materials “for the structuration of higher domains”—it is both part of and an instrument of mathematical reason. But it is clear, by Zalamea’s invocation of Pierre Francastel’s notion of the “relay” as a “juncture of great value; one where the perceived, the real and the imaginary are conjugated,” that he (Zalamea) considers this ideal process to be continuous with real organic processes. Francastel writes:

The plastic sign, by being a place where elements proceeding from these three categories encounter and interfere with one another, is neither expressive (imaginary and individual) nor representative (real and imaginary), but also [sic] figurative (unity of the laws of the brain’s optical activity and those of the techniques of elaboration of the sign as such). The figurative dimension of the plastic sign, according to Francastel, is its *reflexive unity* in accounting for the material conditions of its own articulation. [Speaking of reflexivity: Barry Sandywell’s *The Beginnings of European Theorizing: Reflexivity in the Archaic Age* just arrived—I hope to post about it sometime in the foreseeable future.] This is perhaps why Zalamea insists that “mixture” should be understood as *sunthesis* (as composition, which is reversible), as opposed to *sunchasis* (as fusion, which is usually irreversible). According to Lautman, composition, in contrast to the Aristotelian/set-theoretical logic of genera and species, aligns with “the Platonic method of division ... for which the unity of Being is a unity of composition and a point of departure for the search for the principles that are united in the ideas.” This search is, for Zalamea, guided by the pragmaticist maxim, a “minimal instrumentalium” that furnishes philosophy and mathematics with consciousness of multiplicity; he expresses it in the technical vocabulary of category theory, writing, “pragmatics aims to reintegrate the differential fibers of the world, *explicitly inserting the broad relational and modal spectrum of fibers into the investigation as a whole.*” The unity of composition and the reflexive point of departure of analysis (i.e. “the search for the principles that are united in the ideas”)—the part and the instrument, again—are recast in phenomenological terms by Zalamea in an attempt to capture the “indispensable creative ‘impulse’ to which Valéry refers”; but the same creative impulse orients vitalist and organicist philosophy (Bergson, Whitehead, Deleuze). Zalamea argues that the transitory character of mathematics—the dynamic activity underlying mathematical creativity and discovery—is a powerful engine of abstraction, in the same way it is for Deleuze a profoundly virtualizing language, but Deleuze is never considered in much detail; however, the repeated description of the dynamics of mathematics as “pendular” cannot help but evoke what Deleuze writes of the body without organs:

It is in the BwO that the organs enter into the relations of composition called the organism. ... It is the BwO that is stratified. It *swings* between two poles, the surfaces of stratification into which it is recoiled ... and the plane of consistency in which it unfurls and opens to experimentation. If the BwO is a limit, if one is forever attaining it, it is because behind each stratum, encasted in it, there is always another stratum.
Deleuze explicitly cites Merleau-Ponty as an influence on his concept of the fold, which is intended to overcome phenomenological intentionality as an overly empirical determination of the subject (cf. Kant’s reply to Humean empiricism); by contrast, Zalamea explicitly reads Merleau-Ponty as belonging to a Platonic tradition including Lautman and Badiou—a dynamic Platonism, he specifies, but a Platonism nonetheless. This, in addition to his profound sensitivity to the question of value, is what distinguishes Deleuze from Zalamea; but both thinkers are compatible in the main, which confirms the purpose of Synthetic Philosophy of Contemporary Mathematics: to introduce the analytical faithful to ways of thinking that are respectful of continuity. While Zalamea promotes a synthetic philosophy to complement contemporary mathematics, the book does not offer much in the way of a verifiably rigorous demonstration of the synthetic powers of mathematics when applied to the problems of philosophy; for this reason, it seems more appropriate to classify Zalamea as a neocartesian thinker (alongside Badiou). His work offers a grander suite of mathematical models for philosophy to assimilate, an expanded toolbox enabling a more sophisticated analysis of reality at precisely specified degrees of abstraction—the bit from Plato’s Phaedrus about carving nature at its joints springs to mind—but for all that it offers relatively little that could convince analytic philosophers to abandon set theory, which they of course privilege for its utility in formalizing sentential logic. Zalamea gestures toward higher-order logics (like linear logic) without clarifying their specific advantages when applied to the kinds of questions that have occupied analytic philosophy.

For continental philosophy, on the other hand, the primer in mathematical arcana has already provoked a reevaluation of the potential gains that feminist theory stands to make by allying itself with queerish (‘paracohherent’) logics and the technical vocabulary of mathematics, for example. In the following passage, Laboria Cuboniks amplifies the contempt Zalamea expresses for postmodernity and quite clearly accepts his alternative:

Like engineers who must conceive of a total structure as well as the molecular parts from which it is constructed, XF emphasises the importance of the mesopolitical sphere against the limited effectiveness of local gestures, creation of autonomous zones, and sheer horizontalism, just as it stands against transcendent, or top-down impositions of values and norms. The mesopolitical arena of xenofeminism’s universalist ambitions comprehends itself as a mobile and intricate network of transits between these polarities. As pragmatists, we invite contamination as a mutational driver between such frontiers.

Their conception of pragmatism is evidently more practical than it is technical (concerned as it is with political “effectiveness”), and more opportunistic than formalistic. It is an attempt to think the synthesis of the forms of political synthesis (multitudes, groups-in-fusion, etc.) and analysis (demographics, classes, etc.)—but what is the nature of this synthesis? It appears to be nothing more than a muddy, “mesopolitical” middle ground, and something of an end in itself: “We are adamantly synthetic, unsatisfied by analysis alone.” But what substantive reason is there to privilege synthesis over analysis? It’s almost like saying, “I am adamantly awake, unsatisfied by sleep alone.” The quality of your waking hours are roughly correlated to the quality of your sleep, but the significance of their contrast becomes increasingly apparent if you have been favoring one over the other, even as the two states begin to blur together in a uniform mesosleep. A synthesis is, in some sense, only as good as what it synthesizes, so it makes little sense to fancy it over analysis—except, perhaps,
insofar as there is some specific issue with the balance of analysis and synthesis, but this would have to be discerned analytically. The target of Zalamea’s and the Xenofeminist critique is analytic philosophy’s refusal to structure their discipline around a more supple mathematical formalism than set theory; but to accuse it of an outright refusal of synthesis is an unwarranted oversimplification. If analytic philosophy is politically regressive or stylistically monotonous, this is due primarily to its content: ahistorical laws, liberal-democratic institutions, and ideal language situations. While these may be more amenable to logical decomposition than historical a prioris, liminal political spaces and linguistic pragmatics, they are not the ‘natural’ objects of analysis, but are instead determined by a wider range of sociopolitical factors. The synthetic ‘moments’ in analytic philosophy, furthermore, may be suppressed (as Sebastian Rödl argues in Categories of the Temporal) in the form of a “grammar” (Wittgenstein) or “style” (McDowell) that falls outside of the deductive order; nonetheless, the acknowledgment of something beyond schematization requires an intuition for synthesis, even if the elaboration of the attendant synthetic concepts is arguably lacking in philosophic rigor.

In short, Zalamea is correct that there is more to a thing than its parts and that we should consider other compositions, but his critique of analytic philosophy qua analytic does not adequately consider the forms of synthesis specific to analytic philosophy. Moreover, the methods of analytic philosophy are presumed to dictate its objects—while the methods and the objects do indeed appear to be complicit or mutually reinforcing in some sense, Zalamea does not consider their synthetic connection as it obtains for analytic philosophy. Does analytic philosophy proceed analytically because it fails to recognize the creative powers of synthesis? Possibly—but it seems more likely that propositional calculus is chosen for practical reasons: it is easier and more reliable, by and large, to anticipate a thing’s effects by breaking it into parts. By this measure, by entreating analytic philosophy to move beyond set theory, Zalamea does not account for or take seriously its reasons for pursuing set-theoretical logic. It may well be that they do so for the purposes of simplification, and this simplification may in fact work contrary to their own stated intentions (i.e. they become simplistic); but this should be argued, not presupposed, and analysis itself should not be painted as inherently oversimplifying or oversimplified. Sensitivity to complexity and parsimony of expression, in fact, make for a compelling pair of terms—a polarity—whose transits should be carefully examined by synthetic philosophy according to Zalamea’s own standards.

None of the preceding has really touched on the obvious problem of Zalamea’s rally to embrace a synthesis of analysis and synthesis: how can two terms, defined in opposition to one another, be sublated under one of those terms? In Zalamea’s subsequent book, Peirce’s Logic of Continuity, the synthesis appears to be reified as an articulated pendulum or a boundary, the object of horotic thought: “The horotic pendulum, a double transit of information, lies in the very bottom of intuitionistic processes.” I will return to this in a future post, but the key question is this: does the boundary or the pendulum amount to a philosophical absolute, in breach of the first dictum of the pragmaticist maxim? Or does a contaminated mathematics actually contribute (nontrivially) to a revivified philosophy of relations? Does it offer tools that can furnish us with a compelling new approach to questions of value, or is it itself valued (by Xenofeminism, for instance) for its abstractness alone, or for the empty promise of encoding politically revolutionary praxis? My intent here has been simply to explore how Synthetic Philosophy of Contemporary Mathematics builds itself around a Peircean core of pragmatic principles and where it is situated with respect to the analytic
and continental traditions, and to suggest that synthesis is not as foreign an operation to analytic philosophy as Zalamea and Laboria Cuboniks insinuate. But for all that I am actually very sympathetic to Zalamea’s mission to make the outcomes of contemporary mathematics accessible to philosophy without being (as far as I can tell) reductive. While I am convinced that there is a more specific point of divergence between his task and the task of analytic philosophy than their respective comportments toward synthesis, the dexterity of thought required to traverse the islands of mathematics serves as a valuable model for interdisciplinary philosophy.

2 Ibid., 112.
3 Ibid., 115.
4 Ibid., 114.
5 Ibid., 24.
6 Ibid., 366.
7 Ibid., 115.
10 Ibid., 45.
11 Ibid., 315.
14 Ibid., 315.
15 Ibid., 56-57.
18 Ibid., 61-62.
19 Ibid., 61n.
20 Ibid., 348.
21 Albert Lautman, qtd. in Zalamea, *Synthetic Philosophy of Contemporary Mathematics*, 63-64.