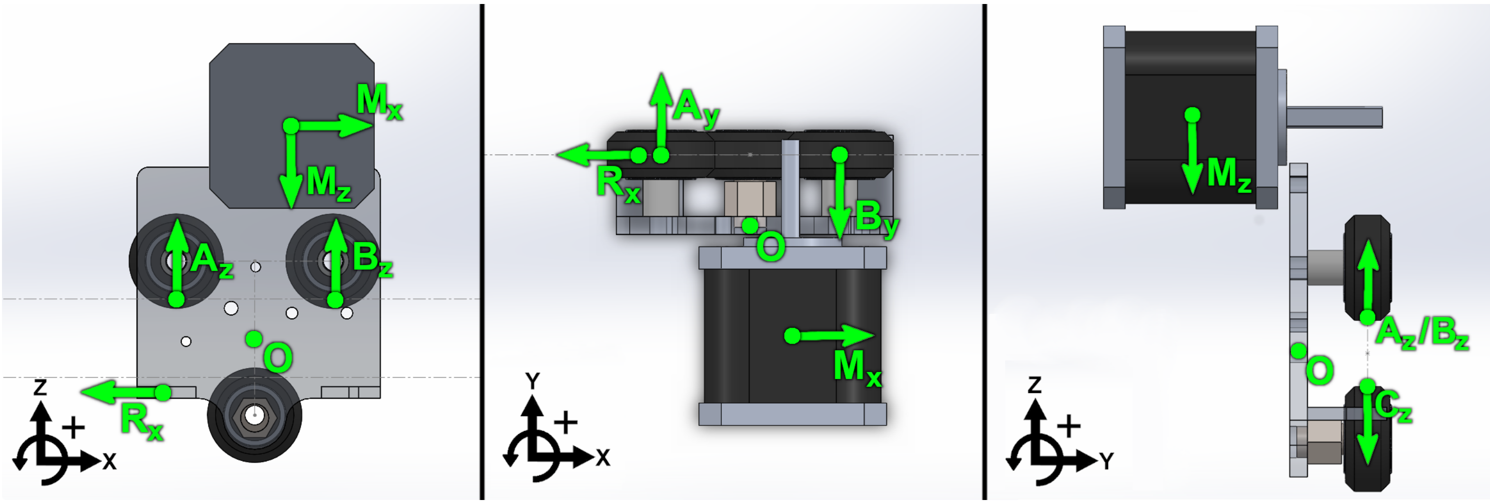
Analysis of the Mechanical Response of the MDD Assembly to Heavy Deceleration

We will use the principle of static equilibrium to determine the reaction forces created by the additional mass of the E-stepper on the X-carriage assembly.

Figures 1a, 1b, and 1c show the labeled points of interest and estimated force vectors:



Equilibrium of Forces

The wheels A, B, and C cannot exert forces in the X-direction, since they are restricted to roll in the X-axis exclusively. Assuming the conditions of static equilibrium, the remaining forces acting in the X-axis can be expressed as

$$\sum F_X = M_X - R_X = 0$$

which can be rearranged as

$$M_X = R_X$$

In the Y-axis, only the reactions of the wheels need to be considered

$$\sum F_Y = A_Y - B_Y = 0$$

In the Z-axis, the summation of the relevant forces can be expressed as

$$\sum F_Z = F_{A_Z} + F_{B_Z} - M_Z = 0$$

which can be rearranged as

$$F_{A_Z} + F_{B_Z} = M_Z$$

Equilibrium of Moments

The assumption of static equilibrium also states that the sum of moments about any point must be equal to zero. The point "O" is chosen along the midline of the X-axis beam, centered between points A and B, and directly above point C.

The sum of moments in the X-axis about this point can be expressed as

$$\sum M_{O_X} = M_Z d_{OM_Y} + A_Z d_{OA_Y} + B_Z d_{OB_Y} - C_Z d_{OC_Y} = 0$$

Similarly, the sum of moments in the Y-axis about this point can be expressed as

$$\sum M_{O_Y} = M_X d_{OM_Z} + M_Z d_{0M_X} + A_Z d_{OA_X} - B_Z d_{OB_X} + R_X d_{OR_Z} = 0$$

Finally, the sum of moments in the Z-axis about this point can be expressed as

$$\sum M_{0_Z} = M_X d_{OM_Y} + R_X d_{OR_Y} - A_Y d_{OA_X} - B_Y d_{OB_X} = 0$$

The dimensions required for these equations are derived from computer-aided assemblies, and are considered accurate to 0.1mm or better. Table X presents the component distances to point O for each key point;

Vector	— dX —	— dY —	— dZ —
OA	20.00	15.85	10.00
OB	20.00	15.85	10.00
OC	0.00	15.85	10.00
OM	9.50	24.75	54.15
OR	24.54	15.62	13.85

Substituting these values into the above equations yields

$$15.85A_Z + 15.85B_Z - 15.85C_Z + 24.75M_Z = 0$$
$$-20.00A_Z + 20.00B_Z + 13.85R_X + 54.14M_X + 9.50M_Z = 0$$
$$-20.00A_Y - 20.00B_Y + 15.62R_X + 24.75M_X = 0$$

Given the previously established relationships

$$M_X = R_X$$
$$F_{A_Y} - F_{B_Y} - F_{C_Y} = 0$$
$$F_{A_Z} + F_{B_Z} = M_Z$$

the following system of equations can be assembled:

$$0.00A_Y + 15.85A_Z + 0.00B_Y + 15.85B_Z - 15.85C_Z + 0.00R_X + 0.00M_X + 24.75M_Z = 0$$
$$0.00A_Y - 20.00A_Z + 0.00B_Y + 20.00B_Z + 0.00C_Z + 13.85R_X + 54.14M_X + 9.50M_Z = 0$$
$$-20.00A_Y + 0.00A_Z - 20.00B_Y + 0.00B_Z + 0.00C_Z + 15.62R_X + 24.75M_X + 0.00M_Z = 0$$
$$0.00A_Y + 0.00A_Z + 0.00B_Y + 0.00B_Z + 0.00C_Z + 1.00R_X - 1.00M_X + 0.00M_Z = 0$$
$$1.00A_Y + 0.00A_Z - 1.00B_Y + 0.00B_Z + 0.00C_Z + 0,00R_X + 0.00M_X + 0.00M_Z = 0$$
$$0.00A_Y + 1.00A_Z + 0.00B_Y + 1.00B_Z + 0.00C_Z + 0.00R_X + 0.00M_X - 1.00M_Z = 0$$

which can also be expressed as

$$\begin{pmatrix} 0 & 15.85 & 0 & 15.85 & -15.85 & 0 & 0 & 24.75 \\ 0 & -20 & 0 & 20 & 0 & 13.85 & 54.14 & 9.50 \\ -20 & 0 & -20 & 0 & 0 & 15.62 & 24.75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} A_Y \\ A_Z \\ B_Y \\ B_Z \\ C_Z \\ R_X \\ M_X \\ M_Z \end{pmatrix} = 0$$

With eight unknown values and only six equations, this system cannot be solved. However, we have not yet considered the actual forces applied to this system.

The standard "42-40" NEMA17 stepper motor provided by Creality, also known as the "JK42HS40-1704XA", has a specified stall torque of 0.4 N-m. The attached 16-tooth GT2 pulley has a pitch diameter of 8mm, and therefore a pitch radius of 4 mm. Assuming no compliance within the GT2 belt between the pulley and the assembly, the tension applied at point R is equal to the tangential force generated at the pulley, which can be calculated as

$$F_T = R_X = \frac{\tau_{motor}}{r_{pulley}}$$

$$R_X = \frac{(0.4\,N)}{(0.004\,mm)} = 100\,N$$

Finally, we can calculate the gravitational force acting on the mass of the motor. The aforementioned ”42-40” stepper motor has a specified mass of 0.28 kg. Therefore,

$$M_Z = m \times g = (0.28\,kg) \times (9.81\,\frac{m}{s^2}) = 2.75\,N$$

Adding these values to our system of equations yields

$$\begin{pmatrix} 0 & 15.85 & 0 & 15.85 & -15.85 & 0 & 0 & 24.75 \\ 0 & -20 & 0 & 20 & 0 & 13.85 & 54.14 & 9.50 \\ -20 & 0 & -20 & 0 & 0 & 15.62 & 24.75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_Y \\ A_Z \\ B_Y \\ B_Z \\ C_Z \\ R_X \\ M_X \\ M_Z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 100 \\ 2.75 \end{pmatrix}$$

Now our system of equations can be solved. The row-reduced echelon form of this system is calculated as

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_Y \\ A_Z \\ B_Y \\ B_Z \\ C_Z \\ R_X \\ M_X \\ M_Z \end{pmatrix} = \begin{pmatrix} 100.925 \\ 172.003 \\ 100.925 \\ -169.253 \\ 7.044 \\ 100.00 \\ 100.00 \\ 2.75 \end{pmatrix}$$

The total force acting on wheels A, B, and C can be calculated as

$$F_{A_{total}} = \sqrt{F_{A_Y}^2 + F_{A_Z}^2} = \sqrt{(100.925\,N)^2 + (172.003\,N^2)} = 199.426\,N$$

$$F_{B_{total}} = \sqrt{F_{B_Y}^2 + F_{B_Z}^2} = \sqrt{(100.925\,N)^2 + (169.253\,N^2)} = 197.059\,N$$

$$F_{C_{total}} = F_{C_Z} = 7.044\,N$$

Given that the elastic modulus (or the amount of pressure required per unit compression distance) of acetals (ie Delrin) is approximately 2.8 GPa, or 2800 Newtons per square millimeter, these forces are orders of magnitude too small to produce any meaningful compression at the point of contact between the wheels and the rail. This validates the assumption that the system is rigid, and therefore no deflection or rotation of the X-carriage is possible. Given the significant differences between the calculated forces and those required for any deformation, no further investigation is merited.