# Hands 4 Building provides two minute online "watch and do", "watch and do" instructional video lessons for our Kids \& Teens products and step-by-step printed instructional lessons for our Little Hands products. These lessons are visual, sequential and easy to follow for the instructor and student. Hands 4 Building provides real-world application for math and engineering principles. Hands 4 Building supplements educational curriculum that meet the standards presented below for each grade level. 

## NGSS Standards

Grade K

Plan and conduct an investigation to compare the effects of different strengths or different directions of pushes and pulls on the motion of an object. [Clarification Statement: Examples of pushes or pulls could include a string attached to an object being pulled, a person pushing an object, a person
PS2-

S2

1. stopping a rolling ball, and two objects colliding and pushing on each other.] [Assessment Boundary:
Assessment is limited to different relative strengths or different directions, but not both at the same
time. Assessment does not include non-contact pushes or pulls such as those produced by magnets.]

Analyze data to determine if a design solution works as intended to change the speed or direction of an object with a push or a pull.* [Clarification Statement: Examples of problems requiring a solution
K-
PS2could include having a marble or other object move a certain distance, follow a particular path, and
2. knock down other objects. Examples of solutions could include tools such as a ramp to increase the speed of the object and a structure that would cause an object such as a marble or ball to turn.] [Assessment Boundary: Assessment does not include friction as a mechanism for change in speed.]

K-
ESS2-
2. 2- animals changing their environ
tree roots can break concrete.]
Farms, bridges

Use a model to represent the relationship between the needs of different plants and animals
K- (including humans) and the places they live. [Clarification Statement: Examples of relationships
ESS3- could include that deer eat buds and leaves, therefore, they usually live in forested areas; and,

1. grasses need sunlight so they often grow in meadows. Plants, animals, and their surroundings make up a system.]
Farms, bridges

K- Communicate solutions that will reduce the impact of humans on the land, water, air, and/or other
ESS3- living things in the local environment.* [Clarification Statement: Examples of human impact on
3. the land could include cutting trees to produce paper and using resources to produce bottles.

Examples of solutions could include reusing paper and recycling cans and bottles.]

## Farms, materials used to build

K-2- Ask questions, make observations, and gather information about a situation people want to ETS1-1. change to define a simple problem that can be solved through the development of a new or improved object or tool.

K-2- Develop a simple sketch, drawing, or physical model to illustrate how the shape of an object ETS1-2. helps it function as needed to solve a given problem.

K-2- Analyze data from tests of two objects designed to solve the same problem to compare the ETS1-3. strengths and weaknesses of how each performs.

## Grade1

Plan and conduct investigations to determine the effect of placing objects made with different
1- materials in the path of a beam of light. [Clarification Statement: Examples of materials could
PS4- include those that are transparent (such as clear plastic), translucent (such as wax paper), opaque
3. (such as cardboard), and reflective (such as a mirror).] [Assessment Boundary: Assessment does not include the speed of light.]
How the natural materials we use are different, reinforce learning from classroom not primary

1- Use tools and materials to design and build a device that uses light or sound to solve the problem of PS4communicating over a distance.* [Clarification Statement: Examples of devices could include a light
4. source to send signals, paper cup and string "telephones," and a pattern of drum beats.] [Assessment Boundary: Assessment does not include technological details for how communication devices work.]

Use observations of the sun, moon, and stars to describe patterns that can be predicted.
1- [Clarification Statement: Examples of patterns could include that the sun and moon appear to rise
ESS1- in one part of the sky, move across the sky, and set; and stars other than our sun are visible at night

1. but not during the day.] [Assessment Boundary: Assessment of star patterns is limited to stars being seen at night and not during the day.]
Where will the sun be and where do you build your farm

1- Make observations at different times of year to relate the amount of daylight to the time of year.
ESS1- [Clarification Statement: Emphasis is on relative comparisons of the amount of daylight in the
2. winter to the amount in the spring or fall.] [Assessment Boundary: Assessment is limited to relative amounts of daylight, not quantifying the hours or time of daylight.]
Where will the sun be and what time of year is good for a farm

K-2- Ask questions, make observations, and gather information about a situation people want to
ETS1-1. change to define a simple problem that can be solved through the development of a new or improved object or tool.

K-2- Develop a simple sketch, drawing, or physical model to illustrate how the shape of an object ETS1-2. helps it function as needed to solve a given problem.

K-2- Analyze data from tests of two objects designed to solve the same problem to compare the ETS1-3. strengths and weaknesses of how each performs.

## Grade 2

2- Plan and conduct an investigation to describe and classify different kinds of materials by their PS1- observable properties. [Clarification Statement: Observations could include color, texture, hardness, 1. and flexibility. Patterns could include the similar properties that different materials share.]

The natural materials we use and how they are unique
2. Analyze data obtained from testing different materials to determine which materials have the

PS1- properties that are best suited for an intended purpose.* [Clarification Statement: Examples of
2. properties could include, strength, flexibility, hardness, texture, and absorbency.] [Assessment Boundary: Assessment of quantitative measurements is limited to length.]

2- Make observations to construct an evidence-based account of how an object made of a small set of PS1- pieces can be disassembled and made into a new object. [Clarification Statement: Examples of 3. pieces could include blocks, building bricks, or other assorted small objects.]

Sustainability, geometry and seeing inside, fractions

2- Compare multiple solutions designed to slow or prevent wind or water from changing the shape of ESS2- the land.* [Clarification Statement: Examples of solutions could include different designs of dikes

1. and windbreaks to hold back wind and water, and different designs for using shrubs, grass, and trees to hold back the land.]
Protect your bridge

2-ESS2- Develop a model to represent the shapes and kinds of land and bodies of water in an area.
2. [Assessment Boundary: Assessment does not include quantitative scaling in models.]

Designing or building the best bridge

K-2- Ask questions, make observations, and gather information about a situation people want to
ETS1-1. change to define a simple problem that can be solved through the development of a new or improved object or tool.

K-2- Develop a simple sketch, drawing, or physical model to illustrate how the shape of an object
ETS1-2. helps it function as needed to solve a given problem.

K-2- Analyze data from tests of two objects designed to solve the same problem to compare the ETS1-3. strengths and weaknesses of how each performs.
Compare bridges, designs for car crash stopping

## Grade 3

Plan and conduct an investigation to provide evidence of the effects of balanced and unbalanced forces on the motion of an object. [Clarification Statement: Examples could include an unbalanced
3- force on one side of a ball can make it start moving; and, balanced forces pushing on a box from
PS2- both sides will not produce any motion at all.] [Assessment Boundary: Assessment is limited to one

1. variable at a time: number, size, or direction of forces. Assessment does not include quantitative force size, only qualitative and relative. Assessment is limited to gravity being addressed as a force that pulls objects down.]

Make observations and/or measurements of an object's motion to provide evidence that a pattern can
3- be used to predict future motion. [Clarification Statement: Examples of motion with a predictable
PS2- pattern could include a child swinging in a swing, a ball rolling back and forth in a bowl, and two
2. children on a see-saw.] [Assessment Boundary: Assessment does not include technical terms such as period and frequency.]

3- Define a simple design problem that can be solved by applying scientific ideas about magnets.*
PS2- [Clarification Statement: Examples of problems could include constructing a latch to keep a door
4. shut and creating a device to keep two moving objects from touching each other.]

3- Make a claim about the merit of a design solution that reduces the impacts of a weather-related
ESS3- hazard. [Clarification Statement: Examples of design solutions to weather-related hazards could

1. include barriers to prevent flooding, wind resistant roofs, and lightning rods.]

Identify threats to stationary projects and ways to mitigate impacts

3-5- Define a simple design problem reflecting a need or a want that includes specified criteria for
ETS1-1. success and constraints on materials, time, or cost.

3-5- Generate and compare multiple possible solutions to a problem based on how well each is likely
ETS1-2. to meet the criteria and constraints of the problem.

3-5- Plan and carry out fair tests in which variables are controlled and failure points are considered to ETS1-3. identify aspects of a model or prototype that can be improved.

## Grade 4

4-
Generate and compare multiple solutions to reduce the impacts of natural Earth processes on humans.* [Clarification Statement: Examples of solutions could include designing an earthquake
ESS3 resistant building and improving monitoring of volcanic activity.] [Assessment Boundary. Assessment is limited to earthquakes, floods, tsunamis, and volcanic eruptions.]

Generate and compare multiple solutions that use patterns to transfer information.* [Clarification Statement: Examples of solutions could include drums sending coded information through sound waves, using a grid of 1 's and 0 's representing black and white to send information about a picture, and using Morse code to send text.]

## Communication as in grade 1

4- Use evidence to construct an explanation relating the speed of an object to the energy of that object. PS3- [Assessment Boundary: Assessment does not include quantitative measures of changes in the speed

1. of an object or on any precise or quantitative definition of energy.

4-
Ask questions and predict outcomes about the changes in energy that occur when objects collide. [Clarification Statement: Emphasis is on the change in the energy due to the change in speed, not on
3. the forces, as objects interact.] [Assessment Boundary: Assessment does not include quantitative measurements of energy.]

Apply scientific ideas to design, test, and refine a device that converts energy from one form to another.* [Clarification Statement: Examples of devices could include electric circuits that convert electrical energy into motion energy of a vehicle, light, or sound; and, a passive solar heater that converts light into heat. Examples of constraints could include the materials, cost, or time to design the device.] [Assessment Boundary: Devices should be limited to those that convert motion energy to electric energy or use stored energy to cause motion or produce light or sound.]
From battery to movement, from movement to something else

3-5- Define a simple design problem reflecting a need or a want that includes specified criteria for ETS1-1. success and constraints on materials, time, or cost.

3-5- Generate and compare multiple possible solutions to a problem based on how well each is likely ETS1-2. to meet the criteria and constraints of the problem.

3-5- Plan and carry out fair tests in which variables are controlled and failure points are considered to ETS1-3. identify aspects of a model or prototype that can be improved.

## Grade 5

5- Make observations and measurements to identify materials based on their properties. [Clarification
PS1- Statement: Examples of materials to be identified could include baking soda and other powders,
3. metals, minerals, and liquids. Examples of properties could include color, hardness, reflectivity,
electrical conductivity, thermal conductivity, response to magnetic forces, and solubility; density is not intended as an identifiable property.] [ssment Boundary: Assessment does not include density or distinguishing mass and weight.]
How are natural materials we use different?

5- Support an argument that the gravitational force exerted by Earth on objects is directed down.
[Clarification Statement: "Down" is a local description of the direction that points toward the center

1. of the spherical Earth.] [Assessment Boundary: Assessment does not include mathematical representation of gravitational force.]

3-5- Define a simple design problem reflecting a need or a want that includes specified criteria for ETS1-1. success and constraints on materials, time, or cost.

3-5- Generate and compare multiple possible solutions to a problem based on how well each is likely ETS1-2. to meet the criteria and constraints of the problem.

3-5- Plan and carry out fair tests in which variables are controlled and failure points are considered to ETS1-3. identify aspects of a model or prototype that can be improved.

## MIDDLE SCHOOL

Apply Newton's Third Law to design a solution to a problem involving the motion of two colliding MS- objects.* [Clarification Statement: Examples of practical problems could include the impact of
PS2- collisions between two cars, between a car and stationary objects, and between a meteor and a space

1. vehicle.] [Assessment Boundary: Assessment is limited to vertical or horizontal interactions in one dimension.]

Plan an investigation to provide evidence that the change in an object's motion depends on the sum of the forces on the object and the mass of the object. [Clarification Statement: Emphasis is on
MS- balanced (Newton's First Law) and unbalanced forces in a system, qualitative comparisons of
PS2- forces, mass and changes in motion (Newton's Second Law), frame of reference, and specification
2. of units.] [Assessment Boundary: Assessment is limited to forces and changes in motion in onedimension in an inertial reference frame and to change in one variable at a time. Assessment does not include the use of trigonometry.]

Ask questions about data to determine the factors that affect the strength of electric and magnetic forces. [Clarification Statement: Examples of devices that use electric and magnetic forces could
MS- include electromagnets, electric motors, or generators. Examples of data could include the effect of
PS2- the number of turns of wire on the strength of an electromagnet, or the effect of increasing the
3. number or strength of magnets on the speed of an electric motor.] [Assessment Boundary: Assessment about questions that require quantitative answers is limited to proportional reasoning and algebraic thinking.]
Change battery strength, motor strength

Construct and interpret graphical displays of data to describe the relationships of kinetic energy to MS- the mass of an object and to the speed of an object. [Clarification Statement: Emphasis is on PS3- descriptive relationships between kinetic energy and mass separately from kinetic energy and speed.

1. Examples could include riding a bicycle at different speeds, rolling different sizes of rocks downhill, and getting hit by a wiffle ball versus a tennis ball.]

Develop a model to describe that when the arrangement of objects interacting at a distance changes, different amounts of potential energy are stored in the system. [Clarification Statement: Emphasis is on relative amounts of potential energy, not on calculations of potential energy. Examples of objects
MS- within systems interacting at varying distances could include: the Earth and either a roller coaster
PS3- cart at varying positions on a hill or objects at varying heights on shelves, changing the
2. direction/orientation of a magnet, and a balloon with static electrical charge being brought closer to a classmate's hair. Examples of models could include representations, diagrams, pictures, and written descriptions of systems.] [Assessment Boundary: Assessment is limited to two objects and electric, magnetic, and gravitational interactions.]
Elevator, tilt and spin, merry go round, car crash

Construct, use, and present arguments to support the claim that when the kinetic energy of an object MS- changes, energy is transferred to or from the object. [Clarification Statement: Examples of empirical
PS3- evidence used in arguments could include an inventory or other representation of the energy before
5. and after the transfer in the form of temperature changes or motion of object.] [Assessment Boundary: Assessment does not include calculations of energy.]

Apply scientific principles to design a method for monitoring and minimizing a human impact on the environment.* [Clarification Statement: Examples of the design process include examining
MS- human environmental impacts, assessing the kinds of solutions that are feasible, and designing and
ESS3- evaluating solutions that could reduce that impact. Examples of human impacts can include water
3. usage (such as the withdrawal of water from streams and aquifers or the construction of dams and levees), land usage (such as urban development, agriculture, or the removal of wetlands), and pollution (such as of the air, water, or land).]
farms

MS-
Define the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles and potential impacts on people and the natural environment that may limit possible solutions.

MS- Evaluate competing design solutions using a systematic process to determine how well they ETS1-2. meet the criteria and constraints of the problem.

Analyze data from tests to determine similarities and differences among several design solutions
ETS1-3. to identify the best characteristics of each that can be combined into a new solution to better meet the criteria for success.

MS- Develop a model to generate data for iterative testing and modification of a proposed object, ETS1-4. tool, or process such that an optimal design can be achieved.

## Common Core Math

## Grade K

## Know number names and the count sequence.

CCSS.Math.Content.K.CC.A. 2
Count forward beginning from a given number within the known sequence (instead of having to begin at 1 ) CCSS.Math.Content.K.CC.A. 3
Write numbers from 0 to 20 . Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

## Count to tell the number of objects

## CCSS.Math.Content.K.CC.B. 4

Understand the relationship between numbers and quantities; connect counting to cardinality.
CCSS.Math.Content.K.CC.B.4.a
When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
CCSS.Math.Content.K.CC.B.4.b
Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
CCSS.Math.Content.K.CC.B.4.c
Understand that each successive number name refers to a quantity that is one larger.
CCSS.Math.Content.K.CC.B. 5
Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

## Compare numbers

CCSS.Math.Content.K.CC.C. 6
Identify whether the number of objects in one group is greater than, less than, or
equal to the number of objects in another group, e.g., by using matching and counting strategies. ${ }^{1}$

## Understand addition, and understand subtraction

CCSS.Math.Content.K.OA.A. 1
Represent addition and subtraction with objects, fingers, mental images, drawings ${ }^{1}$, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
CCSS.Math.Content.K.OA.A. 2
Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
CCSS.Math.Content.K.OA.A. 3
Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5=2+3$ and $5=4+1$ ).
CCSS.Math.Content.K.OA.A. 5
Fluently add and subtract within 5.

## Work with numbers 11-19 to gain foundations for place value

## CCSS.Math.Content.K.NBT.A. 1

Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as $18=10+8$ ); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

## Describe and compare measurable attributes

CCSS.Math.Content.K.MD.A. 1
Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
CCSS.Math.Content.K.MD.A. 2
Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.

## Classify objects and count the number of objects in each category

CCSS.Math.Content.K.MD.B. 3
Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. ${ }^{1}$

## Identify and describe shapes

CCSS.Math.Content.K.G.A. 1
Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
CCSS.Math.Content.K.G.A. 2
Correctly name shapes regardless of their orientations or overall size.
CCSS.Math.Content.K.G.A. 3
Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

## Analyze, compare, create, and compose shapes

## CCSS.Math.Content.K.G.B. 4

Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
CCSS.Math.Content.K.G.B. 5
Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
CCSS.Math.Content.K.G.B. 6
Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"

## Grade 1

## Represent and solve problems involving addition and subtraction

CCSS.Math.Content.1.0A.A. 1
Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. ${ }^{1}$
CCSS.Math.Content.1.0A.A. 2
Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 , e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

## Understand and apply properties of operations and the relationship between addition and subtraction

CCSS.Math.Content.1.0A.B. 3
Apply properties of operations as strategies to add and subtract. ${ }^{2}$ Examples: If $8+3$ $=11$ is known, then $3+8=11$ is also known. (Commutative property of addition.) To
add $2+6+4$, the second two numbers can be added to make a ten, so $2+6+4=2+10$ = 12. (Associative property of addition.)
CCSS.Math.Content.1.OA.B. 4
Understand subtraction as an unknown-addend problem. For example, subtract 10 8 by finding the number that makes 10 when added to 8 .

## Add and subtract within 20

CCSS.Math.Content.1.0A.C. 5
Relate counting to addition and subtraction (e.g., by counting on 2 to add 2 ).
CCSS.Math.Content.1.0A.C. 6
Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8+6=8+2+4=10$ $+4=14$ ); decomposing a number leading to a ten (e.g., 13-4=13-3-1=10-1= 9 ); using the relationship between addition and subtraction (e.g., knowing that $8+4$ $=12$, one knows 12-8=4); and creating equivalent but easier or known sums (e.g., adding $6+7$ by creating the known equivalent $6+6+1=12+1=13$ ).

## Work with addition and subtraction equation

CCSS.Math.Content.1.0A.D. 7
Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? $6=6,7=8-1,5+2=2+5,4+1=5+2$. CCSS.Math.Content.1.0A.D. 8
Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8+?=11,5=-3,6+6=$.

## Understand place value

CCSS.Math.Content.1.NBT.B. 2
Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
CCSS.Math.Content.1.NBT.B.2.a
10 can be thought of as a bundle of ten ones - called a "ten."
CCSS.Math.Content.1.NBT.B.2.b
The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
CCSS.Math.Content.1.NBT.B.2.c
The numbers $10,20,30,40,50,60,70,80,90$ refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
CCSS.Math.Content.1.NBT.B. 3
Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols $>,=$, and $<$.

## Use place value understanding and properties of operations to add and subtract

CCSS.Math.Content.1.NBT.C. 4
Add within 100 , including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10 , using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
CCSS.Math.Content.1.NBT.C. 5
Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
CCSS.Math.Content.1.NBT.C. 6
Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

## Measure lengths indirectly and by iterating length units

CCSS.Math.Content.1.MD.A. 1
Order three objects by length; compare the lengths of two objects indirectly by using a third object.
CCSS.Math.Content.1.MD.A. 2
Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps

## Represent and interpret data

CCSS.Math.Content.1.MD.C. 4
Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

9 Reason with shapes and their attributes.
CCSS.Math.Content.1.G.A. 1
Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
CCSS.Math.Content.1.G.A. 2
Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-
circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. ${ }^{1}$ CCSS.Math.Content.1.G.A. 3
Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

## Grade 2

1 Represent and solve problems involving addition and subtraction.
CCSS.Math.Content.2.0A.A. 1
Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. ${ }^{1}$

2 Add and subtract within 20.
CCSS.Math.Content.2.0A.B. 2
Fluently add and subtract within 20 using mental strategies. ${ }^{2}$ By end of Grade 2, know from memory all sums of two one-digit numbers.

3 Work with equal groups of objects to gain foundations for multiplication. CCSS.Math.Content.2.0A.C. 3
Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2 s ; write an equation to express an even number as a sum of two equal addends.
CCSS.Math.Content.2.0A.C. 4
Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

4 Use place value understanding and properties of operations to add and subtract.
CCSS.Math.Content.2.NBT.B. 9
Explain why addition and subtraction strategies work, using place value and the properties of operations. ${ }^{1}$

5 Measure and estimate lengths in standard units.
CCSS.Math.Content.2.MD.A. 1
Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
CCSS.Math.Content.2.MD.A. 2
Measure the length of an object twice, using length units of different lengths for the
two measurements; describe how the two measurements relate to the size of the unit chosen.
CCSS.Math.Content.2.MD.A. 3
Estimate lengths using units of inches, feet, centimeters, and meters.
CCSS.Math.Content.2.MD.A. 4
Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

6 Relate addition and subtraction to length.
CCSS.Math.Content.2.MD.B. 5
Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. CCSS.Math.Content.2.MD.B. 6
Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers $0,1,2, \ldots$, and represent whole-number sums and differences within 100 on a number line diagram.

7 Represent and interpret data.
CCSS.Math.Content.2.MD.D. 9
Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
CCSS.Math.Content.2.MD.D. 10
Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems ${ }^{1}$ using information presented in a bar graph.

8 Represent and interpret data.
CCSS.Math.Content.2.MD.D. 9
Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
CCSS.Math.Content.2.MD.D. 10
Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems ${ }^{1}$ using information presented in a bar graph.

## Grade 3

1 Represent and solve problems involving multiplication and division. CCSS.Math.Content.3.OA.A. 1
Interpret products of whole numbers, e.g., interpret $5 \times 7$ as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as $5 \times 7$.

CCSS.Math.Content.3.OA.A. 2
Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.
CCSS.Math.Content.3.OA.A. 3
Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. ${ }^{1}$

2 Understand properties of multiplication and the relationship between multiplication and division.
CCSS.Math.Content.3.OA.B.5
Apply properties of operations as strategies to multiply and divide. ${ }^{2}$ Examples: If $6 \times 4=24$ is known, then $4 \times 6=24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5=15$, then $15 \times 2=30$, or by $5 \times 2=10$, then $3 \times 10=30$. (Associative property of multiplication.) Knowing that $8 \times 5=40$ and $8 \times 2=16$, one can find $8 \times 7$ as $8 \times(5+2)=(8 \times 5)+(8 \times 2)=40+16=56$.
(Distributive property.)
CCSS.Math.Content.3.OA.B. 6
Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8 .

3 Multiply and divide within 100.
CCSS.Math.Content.3.OA.C. 7
Fluently multiply and divide within 100 , using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5=40$, one knows $40 \div 5=8$ ) or properties of operations. By the end of Grade 3 , know from memory all products of two one-digit numbers.

4 Solve problems involving the four operations, and identify and explain patterns in arithmetic.
CCSS.Math.Content.3.OA.D. 8
Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. ${ }^{3}$
CCSS.Math.Content.3.OA.D. 9
Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

5 Develop understanding of fractions as numbers.
CCSS.Math.Content.3.NF.A. 1
Understand a fraction $1 / b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a / b$ as the quantity formed by $a$ parts of size $1 / b$.
CCSS.Math.Content.3.NF.A. 3
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. CCSS.Math.Content.3.NF.A.3.a
Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
CCSS.Math.Content.3.NF.A.3.d
Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>,=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

## 6 Solve problems involving measurement and estimation.

CCSS.Math.Content.3.MD.A. 2
Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms $(\mathrm{kg})$, and liters (l). ${ }^{1}$ Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. ${ }^{2}$

7 Represent and interpret data.
CCSS.Math.Content.3.MD.B. 3
Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets. CCSS.Math.Content.3.MD.B. 4
Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units - whole numbers, halves, or quarters.

## 8 Geometric measurement: understand concepts of area and relate area to multiplication

 and to addition.CCSS.Math.Content.3.MD.C. 5
Recognize area as an attribute of plane figures and understand concepts of area measurement.
CCSS.Math.Content.3.MD.C.5.a
A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
CCSS.Math.Content.3.MD.C.5.b
A plane figure which can be covered without gaps or overlaps by $n$ unit squares is said to have an area of $n$ square units.
CCSS.Math.Content.3.MD.C. 6
Measure areas by counting unit squares (square cm , square m , square in, square ft , and improvised units).
CCSS.Math.Content.3.MD.C. 7
Relate area to the operations of multiplication and addition.
CCSS.Math.Content.3.MD.C.7.a
Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
CCSS.Math.Content.3.MD.C.7.b
Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
CCSS.Math.Content.3.MD.C.7.c
Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths $a$ and $b+$ $c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
CCSS.Math.Content.3.MD.C.7.d
Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

## 9 Geometric measurement: recognize perimeter.

CCSS.Math.Content.3.MD.D. 8
Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters

## 10 Reason with shapes and their attributes. <br> CCSS.Math.Content.3.G.A. 1

Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
CCSS.Math.Content.3.G.A. 2
Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as $1 / 4$ of the area of the shape

## Grade 4

1 Generate and analyze patterns.
CCSS.Math.Content.4.OA.C. 5
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

## 2 Use place value understanding and properties of operations to perform multi-digit arithmetic.

CCSS.Math.Content.4.NBT.B. 4
Fluently add and subtract multi-digit whole numbers using the standard algorithm.
CCSS.Math.Content.4.NBT.B.5
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
CCSS.Math.Content.4.NBT.B. 6
Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

## 3 Extend understanding of fraction equivalence and ordering. CCSS.Math.Content.4.NF.A. 1

Explain why a fraction $a / b$ is equivalent to a fraction $(n \times a) /(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
CCSS.Math.Content.4.NF.A. 2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1 / 2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>,=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

## 4 Build fractions from unit fractions.

CCSS.Math.Content.4.NF.B. 3
Understand a fraction $a / b$ with $a>1$ as a sum of fractions $1 / b$.
CCSS.Math.Content.4.NF.B.3.a
Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

CCSS.Math.Content.4.NF.B.3.b
Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
Examples: $3 / 8=1 / 8+1 / 8+1 / 8 ; 3 / 8=1 / 8+2 / 8 ; 21 / 8=1+1+1 / 8=8 / 8+8 / 8+1 / 8$.
CCSS.Math.Content.4.NF.B.3.d
Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. CCSS.Math.Content.4.NF.B. 4
Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. CCSS.Math.Content.4.NF.B.4.c
Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $3 / 8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

## 5 Solve problems involving measurement and conversion of measurements. CCSS.Math.Content.4.MD.A. 1

Know relative sizes of measurement units within one system of units including $\mathrm{km}, \mathrm{m}, \mathrm{cm} ; \mathrm{kg}, \mathrm{g} ; \mathrm{lb}, \mathrm{oz} . ; \mathrm{l}$, ml ; hr , min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs $(1,12),(2,24),(3,36), \ldots$
CCSS.Math.Content.4.MD.A. 2
Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. CCSS.Math.Content.4.MD.A. 3
Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

## 6 Represent and interpret data.

CCSS.Math.Content.4.MD.B. 4
Make a line plot to display a data set of measurements in fractions of a unit ( $1 / 2,1 / 4,1 / 8$ ). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

7 Geometric measurement: understand concepts of angle and measure angles. CCSS.Math.Content.4.MD.C. 5
Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
CCSS.Math.Content.4.MD.C. 6
Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

8 Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
CCSS.Math.Content.4.G.A. 1
Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
CCSS.Math.Content.4.G.A. 2
Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the
presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
CCSS.Math.Content.4.G.A. 3
Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

## Grade 5

1 Analyze patterns and relationships.
CCSS.Math.Content.5.OA.B. 3
Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0 , and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

2 Use equivalent fractions as a strategy to add and subtract fractions.
CCSS.Math.Content.5.NF.A. 2
Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2 / 5+1 / 2=3 / 7$, by observing that $3 / 7<1 / 2$.

3 Apply and extend previous understandings of multiplication and division. CCSS.Math.Content.5.NF.B. 3
Interpret a fraction as division of the numerator by the denominator $(a / b=a \div b)$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?
CCSS.Math.Content.5.NF.B. 4
Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
CCSS.Math.Content.5.NF.B.4.a
Interpret the product $(a / b) \times q$ as a parts of a partition of $q$ into $b$ equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show (2/3) $\times 4=8 / 3$, and create a story context for this equation. Do the same with $(2 / 3) \times(4 / 5)=8 / 15$. (In general, $(a / b) \times(c / d)=$ $a c / b d$.)
CCSS.Math.Content.5.NF.B.4.b
Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
CCSS.Math.Content.5.NF.B. 5
Interpret multiplication as scaling (resizing), by:

CCSS.Math.Content.5.NF.B.5.a
Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
CCSS.Math.Content.5.NF.B.5.b
Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a / b=(n \times a) /(n \times b)$ to the effect of multiplying $a / b$ by 1 .
CCSS.Math.Content.5.NF.B. 6
Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
CCSS.Math.Content.5.NF.B. 7
Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. ${ }^{1}$
CCSS.Math.Content.5.NF.B.7.a
Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $(1 / 3) \div 4$, and use a visual fraction model to show the quotient. Use the
relationship between multiplication and division to explain that $(1 / 3) \div 4=1 / 12$ because $(1 / 12) \times 4=1 / 3$. CCSS.Math.Content.5.NF.B.7.b
Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div(1 / 5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div(1 / 5)=20$ because $20 \times(1 / 5)=4$.
CCSS.Math.Content.5.NF.B.7.c
Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $1 / 2 \mathrm{lb}$ of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

## 4 Represent and interpret data.

CCSS.Math.Content.5.MD.B. 2
Make a line plot to display a data set of measurements in fractions of a unit ( $1 / 2,1 / 4,1 / 8)$. Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

## 5 Geometric measurement: understand concepts of volume.

CCSS.Math.Content.5.MD.C. 3
Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
CCSS.Math.Content.5.MD.C.3.a
A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
CCSS.Math.Content.5.MD.C.3.b
A solid figure which can be packed without gaps or overlaps using $n$ unit cubes is said to have a volume of $n$ cubic units.
CCSS.Math.Content.5.MD.C. 4
Measure volumes by counting unit cubes, using cubic cm , cubic in, cubic ft , and improvised units.
CCSS.Math.Content.5.MD.C. 5
Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
CCSS.Math.Content.5.MD.C.5.a
Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

CCSS.Math.Content.5.MD.C.5.b
Apply the formulas $V=l \times w \times h$ and $V=b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. CCSS.Math.Content.5.MD.C.5.c
Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

## 6 Classify two-dimensional figures into categories based on their properties. CCSS.Math.Content.5.G.B. 3 <br> Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. <br> CCSS.Math.Content.5.G.B. 4 <br> Classify two-dimensional figures in a hierarchy based on properties.

## Grade 6

1 Understand ratio concepts and use ratio reasoning to solve problems.
CCSS.Math.Content.6.RP.A. 1
Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."
CCSS.Math.Content.6.RP.A. 2
Understand the concept of a unit rate $\mathrm{a} / \mathrm{b}$ associated with a ratio $\mathrm{a}: \mathrm{b}$ with $\mathrm{b} \neq 0$, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $\$ 75$ for 15 hamburgers, which is a rate of $\$ 5$ per hamburger." ${ }^{1}$
CCSS.Math.Content.6.RP.A. 3
Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
CCSS.Math.Content.6.RP.A.3.a
Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
CCSS.Math.Content.6.RP.A.3.b
Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
CCSS.Math.Content.6.RP.A.3.c
Find a percent of a quantity as a rate per 100 (e.g., $30 \%$ of a quantity means $30 / 100$ times the quantity); solve problems involving finding the whole, given a part and the percent.
CCSS.Math.Content.6.RP.A.3.d
Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

2 Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
CCSS.Math.Content.6.NS.A. 1
Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $(2 / 3) \div(3 / 4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2 / 3) \div(3 / 4)=8 / 9$ because $3 / 4$ of $8 / 9$ is $2 / 3$. (In general,
$(a / b) \div(c / d)=a d / b c$.$) How much chocolate will each person get if 3$ people share $1 / 2 \mathrm{lb}$ of chocolate equally? How many 3/4-cup servings are in $2 / 3$ of a cup of yogurt? How wide is a rectangular strip of land with length $3 / 4$ mi and area $1 / 2$ square mi? .

3 Solve real-world and mathematical problems involving area, surface area, and volume. CCSS.Math.Content.6.G.A. 1
Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
CCSS.Math.Content.6.G.A. 2
Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V=l w h$ and $V=b h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
CCSS.Math.Content.6.G.A. 3
Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
CCSS.Math.Content.6.G.A. 4
Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

## Grade 7

1 Analyze proportional relationships and use them to solve real-world and mathematical problems.
CCSS.Math.Content.7.RP.A. 1
Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $1 / 2$ mile in each $1 / 4$ hour, compute the unit rate as the complex fraction ${ }^{1 / 2} /{ }_{1 / 4}$ miles per hour, equivalently 2 miles per hour.
CCSS.Math.Content.7.RP.A. 2
Recognize and represent proportional relationships between quantities.
CCSS.Math.Content.7.RP.A.2.a
Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
CCSS.Math.Content.7.RP.A.2.b
Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
CCSS.Math.Content.7.RP.A.2.c
Represent proportional relationships by equations. For example, if total cost tis proportional to the number $n$ of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t=p n$.
CCSS.Math.Content.7.RP.A. 3
Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

2 CCSS.Math.Content.7.NS.A. 3
Solve real-world and mathematical problems involving the four operations with rational numbers. ${ }^{1}$

3 Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
CCSS.Math.Content.7.EE.B. 3
Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $\$ 25$ an hour gets a $10 \%$ raise, she will make an additional $1 / 10$ of her salary an hour, or $\$ 2.50$, for a new salary of $\$ 27.50$. If you want to place a towel bar 9 3/4 inches long in the center of a door that is $271 / 2$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
CCSS.Math.Content.7.EE.B. 4
Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
CCSS.Math.Content.7.EE.B.4.a
Solve word problems leading to equations of the form $p x+q=r$ and $p(x+q)=r$, where $p, q$, and $r$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm . Its length is 6 cm . What is its width?
CCSS.Math.Content.7.EE.B.4.b
Solve word problems leading to inequalities of the form $p x+q>r$ or $p x+q<r$, where $p, q$, and $r$ are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $\$ 50$ per week plus $\$ 3$ per sale. This week you want your pay to be at least $\$ 100$. Write an inequality for the number of sales you need to make, and describe the solutions.

## 4 Draw construct, and describe geometrical figures and describe the relationships

 between them.CCSS.Math.Content.7.G.A. 1
Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
CCSS.Math.Content.7.G.A. 2
Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. CCSS.Math.Content.7.G.A. 3
Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids

5 Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
CCSS.Math.Content.7.G.B. 4
Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
CCSS.Math.Content.7.G.B. 5
Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
CCSS.Math.Content.7.G.B. 6
Solve real-world and mathematical problems involving area, volume and surface area of two- and threedimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

## Grade 8

1 Understand the connections between proportional relationships, lines, and linear equations.
CCSS.Math.Content.8.EE.B. 5
Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
CCSS.Math.Content.8.EE.B. 6
Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a nonvertical line in the coordinate plane; derive the equation $\mathrm{y}=\mathrm{mx}$ for a line through the origin and the equation $y=m x+b$ for a line intercepting the vertical axis at $b$.

## 2 CCSS.Math.Content.8.EE.C.8.c

Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

3 Understand congruence and similarity using physical models, transparencies, or geometry software.
CCSS.Math.Content.8.G.A. 1
Verify experimentally the properties of rotations, reflections, and translations:
CCSS.Math.Content.8.G.A.1.a
Lines are taken to lines, and line segments to line segments of the same length.
CCSS.Math.Content.8.G.A.1.b
Angles are taken to angles of the same measure.
CCSS.Math.Content.8.G.A.1.c
Parallel lines are taken to parallel lines.
CCSS.Math.Content.8.G.A. 2
Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
CCSS.Math.Content.8.G.A. 3
Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
CCSS.Math.Content.8.G.A. 4
Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
CCSS.Math.Content.8.G.A. 5
Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so

## 4 Understand and apply the Pythagorean Theorem.

CCSS.Math.Content.8.G.B. 7
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
CCSS.Math.Content.8.G.B. 8
Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

