ARBOR SCIENTIFIC

SpillNot

P4-2500

INSTRUCTIONAL GUIDE

Contents

- SpillNot
- Instructional Guide

Recommended for activities:

- Beaker

Background

Newton's Laws are essential in explaining how the SpillNot prevents liquids spilling while walking or running. Walking with a cup held firmly in your hand subjects the cup and its liquid to constant lateral accelerations with each step. The resulting starting and stopping motion causes the liquid in the cup to slosh back and forth. The behavior of the liquid in the cup is simply obeying Newton's law of Inertia: objects at rest try to stay at rest and objects in motion try to stay in motion. In the case of the liquid in the cup, the fluid's surface goes from being horizontal and parallel with the rim of the cup to rising up the walls of the cup at an angle. The angle increases with acceleration until it reaches the rim of the cup and spills over. In the mathematical language of physics, the angle θ between the surface of the liquid and the horizontal plane is represented as:

$$\theta = \tan^{-1} a/g$$

where 'a' is the acceleration of the cup and 'g' is the acceleration due to gravity. This rise and fall of the liquid in the cup is further amplified when the frequency of steps, while walking, resonates with the natural frequency of the back and forth sloshing of the liquid in the cup. So how does the SpillNot prevent spilling?

Suppose a cup with its liquid contents is to be carried on the SpillNot rather than directly by the cup's handle. The SpillNot's handle is a flexible strap, which is above and directly in line with the cup's center of mass. When holding the cup on the SpillNot at rest, the upward force applied to the handle is equal to the weight of the cup and the SpillNot. If a person abruptly applies an additional horizontal force to the top part of the SpillNot handle, the cup and SpillNot will lag behind the handle due to its greater inertial mass and will appear to swing back at an angle. As the hand holding the handle of the SpillNot continues to accelerate forward, the tension in the flexible handle of the SpillNot is now applying a force angled upward and horizontal. Now both the cup and its liquid content are tilted at the same angle preventing the liquid from spilling.

Activities

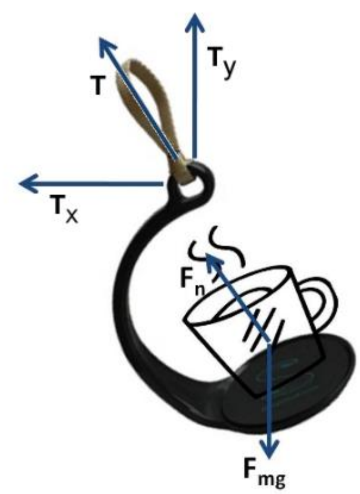
Two Examples of Centripetal Forces Using the SpillNot

In the first example, the SpillNot is held over the head and swung in a horizontal circle. In the second, the SpillNot is held in front and made to swing in a vertical circle. You might want to use a plastic cup with colored water on the SpillNot for these demonstrations. Even without the calculations, the different role that tension plays in contributing to the central force is an important distinction to make.

Horizontal Swing

When held over the person's head while swinging the SpillNot and its cup, the flexible handle will cause both to swing out at an angle depending on the speed the swing. At the point where the flexible handle attaches, there are two forces acting on the SpillNot. The first is the force of gravity (mg) pulling down on the SpillNot and the cup and the second is the tension (T) in the flexible handle, which is pulling up at an angle. Keeping the speed constant in a circle requires an inward or centripetal force. In this case, the centripetal force is not a specific force like gravity or friction but another name for the net force causing centripetal acceleration. The cause of the net force in this case is the horizontal component of the tension in the SpillNot handle.

As the diagram shows, the tension component in the horizontal direction (T_x) is directed toward the center of the circular path of the SpillNot. The upward, vertical component of the tension (T_y) balances the gravitational pull (mg). Also shown in the diagram, are the individual forces on the cup, where the force normal (F_n) replaces the tension for the whole apparatus.



The centripetal acceleration (a_c) can be represented by Newton's second law as:

$$a_c = T_x/m \quad \text{or} \quad a_c = T \cdot \cos\theta\,/m$$

Here, 'T' is the force of tension exerted by the handle on the SpillNot, 'm' is the mass of the SpillNot and cup, and "θ" is the angle of swing from the horizontal.

Further analysis of the motion can be made to calculate the angle (θ). Measuring the radius of the circle for the SpillNot and timing one revolution, one can calculate the centripetal acceleration (a_c). With this value we can calculate the angle (θ) using the horizontal component of the tension (T_x) and vertical component (T_y), where (T_y) is equal to the weight of the SpillNot and cup (mg). This agrees with the formula given earlier: $\theta = \tan^{-1} ma_c/mg$ or $\theta = \tan^{-1} a_c/g$.

Vertical Swing

In the second demonstration of centripetal forces, the SpillNot is swung in a vertical circle. In this case, the centripetal force keeping the SpillNot moving in a circle is the result of the vector addition of the tension (T) in the handle and the gravitational force (mg). However, unlike the horizontal swing, the speed of the swing must be fast enough to produce a centripetal acceleration equal to or greater than gravity (g) or approximately $10 m/s^2$. Since the force of tension is constantly changing during the swing, we will consider the points at the top and bottom of the swing where tension is at its minimum and maximum. At the top of the swing, both the tension in the SpillNot handle and the force of gravity (mg) are directed downward and combine together to produce the centripetal force. At the minimum speed to make it completely around the top, the tension approaches zero with gravity providing all the centripetal force at the very top.

At the bottom of the swing, however, tension in the handle is at its maximum. Here the centripetal force is equal to the force of tension minus the force of gravity (mg).
The minimum speed needed to complete the vertical circle can be calculated using the acceleration of gravity (g) for the centripetal acceleration at the top of the circle, where gravity alone provides the force towards the center of the circle. Using the equation for centripetal acceleration ($a_c = v^2/r$) and substituting the value 'g' for 'ac', we get the equation

$$v = \sqrt{g \cdot r}$$

In this case, 'g' is approximately $10 m/s^2$ and 'r' is the radius of the circle.

Related Products

Flying Pig (P4-2165) Add some excitement to your lessons on circular motion with the Flying Pig. Suspended from the ceiling, his wings flap and carry him in a regular circle.

Exploring Newton's First Law: Inertia Kit (P6-7900) Students investigate inertia by observing a marble's motion around a specially designed circular track.

Inertia Apparatus (P3-3524) Do you know what will happen when the spring is released? Test students' understanding of inertia and Newton's First Law in a memorable way!