Loop-the-Loop

P4-2010

INSTRUCTIONAL GUIDE

With Student Activity

Contents

- Loop the Loop
- 1” neoprene rubber ball
- 1” steel ball
- Instructional Guide with Student Activity

Recommended for activities:

- Meter stick (P1-7072)

Introduction

This apparatus demonstrates phenomena of traveling upside down in a vertical circle and not falling when moving at a high enough speed. The problem requires the application of the conservation of mechanical energy with the law of centripetal force.

Background

Starting with the law of centripetal force, we need to calculate a speed at the top of the loop that produces an acceleration equal to gravity (9.81 m/s/s).

$$\frac{v^2}{r} = g \ \ therefore\ v^2 = gr$$

To find the minimal initial kinetic energy needed for the ball to move around the loop, define the zero level of potential energy to be at the bottom of the loop and make sure energy is conserved.

$$U_i + K_i = U_f + K_f$$

$$U_i = mgh, K_i = 0, while\ U_f = mg(2r)$$

Since the ball rolls while moving around the loop, and does NOT simply slide, a rotational condition must be applied to accurately determine the final kinetic energy, K_f.

$$K_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left({}^{2}\!/_{5}\,mr^2\right)\omega^2$$

Note, for a solid sphere, $I = {}^{2}\!/_{5}\,mr^2$.

$$K_f = \frac{7}{10}mv^2$$

Inserting into energy conservation equation:

$$mgh = mg(2r) + \frac{7}{10}mv^2$$

Now using the rearranged law of centripetal force:

$$mgh = mg(2r) + \frac{7}{10}mgr$$

Mass of the ball and gravity cancel out to furnish:

$$h = 2r + \frac{7}{10}r = \frac{27}{10}r$$

As seen in the above calculations the mass of the ball cancels out and therefore is not a factor.

Answers to Discussion Questions:

1. If the rubber ball were larger in diameter, would the value of *h* change? Why or why not?

   No—ball radius has no impact since it is canceled out in the rotational kinetic energy condition.

2. The steel ball and rubber ball have the same diameter. However, the steel ball has much more mass than the rubber ball. If you could get the steel ball to stop sliding on the track and *only* roll, would the value of *h* change at all? Explain why it would or would not.

   No—mass cancels out since energy is conserved.

3. If you rolled a hollow ball down the ramp, would you have to start it higher or lower?

   Higher—the moment of inertia for a hollow sphere is greater than a solid sphere, so the K_f parameter, which is directly proportional to *h* will also be greater.

4. Say you have a disk, a ring, a solid sphere, and a hollow sphere, all with the same mass. Arrange them by decreasing minimum starting height so each one would *just* make it around the loop.
   Ring > hollow sphere > disk > solid sphere

Related Products

Racing Marbles Lab (P4-1370) This discrepant event will challenge students' understanding of instantaneous and average velocity and conservation of energy. Which marble will reach the end first? Which will have the highest ending velocity?

Rotating Platform (P3-3510) The Rotating Platform can be used with hand weights to study rotational inertia, conservation of angular momentum, and action-reaction. Diameter 40cm

Newtonian Demonstrator (P1-6001) Newton's Cradle dramatizes Newton's Third Law, which states that for every action, there is an equal and opposite reaction. Use to illustrate that momentum and kinetic energy are conserved.

Activity

Part 1

First, let's make some observations.

- Roll both balls down the ramp, one at a time. What do you see and hear when each ball rolls down?
- Roll the balls down again and this time touch the underside of the track. Do you feel a difference between the two?

What types of energy are being transferred with each ball? Energy conservation is key! But don't limit your thought process to potential and kinetic energy. Remember *sound* is a form of energy, too. There is *thermal* energy as well. Where does that come from in this system?

Part 2

- First, roll the **rubber** ball down the ramp. Start so the ball is roughly at the same height as the top of the loop. At this height, the ball will not be able to travel all the way around the loop.
- Incrementally increase the starting height of the ball until it *just* makes it all the way around the loop. With a non-permanent marker, mark the spot you dropped the ball from. Remember, since the ball's height is the important measurement, mark at the bottom of the ball, not where the ball meets the track.

Now, let's see how close we can get with a mathematical model to predict the minimum starting height for the ball to make it around the loop. The first tool we need is the centripetal force equation. Centripetal force is a force that makes a body (in this case a ball) follow a curved path. The direction of this force is always directed toward the center of the curved path. In fact, "centripetal" literally means "center-seeking." The equation for centripetal force looks like this:

$$F_c = \frac{mv^2}{r}$$

Where F_c = centripetal force, m = mass of the ball, v = velocity, and r = the radius of the curved path.

The point we need to focus on to make sure the ball gets around the loop completely is at the very top of the loop. When the ball has *just enough* energy to make it around the loop, the centripetal force on the ball is the same as the force of gravity. So *only at the top of the loop,* we can define centripetal force like this:

$$\frac{mv^2}{r} = mg \;\; therefore \; v^2 = gr$$

Next, recall the Conservation of Energy equation:

$$U_i + K_i = U_f + K_f$$

Now, by defining the zero-level for potential energy to be at the bottom of the loop, we can simplify our conservation equation to solve for kinetic energy at the bottom of the loop (K_f). Why is that important? I thought we were talking about the *top* of the loop before! Well, if the ball is *just* going to make it around the loop, the *kinetic energy* at the bottom of the loop will get transferred into *potential energy* by the time it gets to the top of the loop.

That means we can define the other terms in the energy conservation equation as follows: U_i = mgh,

K_i=0, and U_f=mg(2r). For the final kinetic energy (at the bottom of the loop), the equation looks something like this:

$$K_f = \frac{1}{2}mv^2$$

Alright. Now we have all the variables in the conservation of energy equation and we have the centripetal force equation. Two equations, one unknown, let's solve for the starting height!

$$mgh + 0 = mg(2r) + \frac{1}{2}mv^2$$

After everything cancels out, we're left with $h = 2r + \frac{1}{2}r$

- With a meter stick, ruler, or tape measure, find the inside diameter of the loop. Divide by 2 to find the radius.
- Use the radius to solve for *h* in the equation above.
- Set a meter stick flat at the base of the loop, between the base of the ramp and the "exit" run. Make sure the end of the meter stick extends below the starting ramp of the Loop-the-Loop.
- Now, measure vertically for *h*. Where the ramp intersects your calculated value for *h* on the measuring device is the minimum height required for the ball to travel around the loop.
- Run the ball down the ramp and see what happens!

Part 3

Well, that didn't work. But why not?

Let's take a look at the terms for conservation of energy. U_i simply has two constants (mass and gravity) and our unknown (h). There's not much to go wrong here. The same goes for K_i. The ball isn't moving, so K_i must be zero. The final potential energy, when the ball is at the top of the loop, is just three constants—mass, gravity, and the radius of the loop. The must lie within the equation for K_f, the term for final kinetic energy.

In fact, there is something *missing* from the K_f equation! We calculated the *translational* kinetic energy, but the ball is **rolling** down the ramp, too. There is a simple fix for this. We just need to add in a rotational condition into the final kinetic energy equation:

$$K_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Remember that rotational inertia, *I*, for a sphere equals $^2/_5\, mr^2$, so

$$K_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(^2/_5\, mr^2\right)\omega^2$$

Which leaves the complete version of K_f define as:

$$K_f = \frac{7}{10}mv^2$$

After again inserting this value into the equation for energy conservation, *h* is calculated:

$$h = 2r + \frac{7}{10}r$$

- Use the radius to solve for *h* in the new equation above.
- Now, measure vertically for *h* with the new value. Where the ramp intersects your calculated value for *h* on the measuring device is the minimum height required for the ball to travel around the loop.
- Are you close to the mark you made when you first rolled the ball down the ramp in Part 1?
- Run the ball down the ramp and see what happens!

Part 4

Using the same value of *h* where the rubber ball was just able to make it around the loop, try out the steel ball. Does the steel ball get all the way around the loop? Why or why not?

Discuss what might be the cause of your results with a classmate.

- With a marker, draw a line around the circumference of the steel ball.
- Roll it down the ramp again. Do you see anything that might impact a part of the conservation of energy equation?
- Roll the ball down again, but this time *listen* to the noise the ball makes against the metal track.
- Hold the ball between your index finger and thumb, and slide the ball over the flat part of the track. Do you recognize the sound?

The steel ball is actually *sliding* down the track instead of just rolling. But what difference does that make? Once again, let's look at the K_f equation:

$$K_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Remember that pesky rolling condition we had to add to get the correct value for *h*? Well, what do you think happens to the kinetic energy if the ball is rolling a little bit but mostly sliding?

That's right! The rotational kinetic energy is decreased, and the overall final kinetic energy isn't enough to send the ball around the loop!

Discussion

Talk about the following questions with your classmates and write down your answers on another sheet of paper.

1. If the rubber ball were larger in diameter, would the value of *h* change? Why or why not?

2. The steel ball and rubber ball have the same diameter. However, the steel ball has much more mass than the rubber ball. If you could get the steel ball to stop sliding on the track and *only* roll, would the value of *h* change at all? Explain why it would or would not.

3. If you rolled a hollow ball down the ramp, would you have to start it higher or lower?

4. Say you have a disk, a ring, a solid sphere, and a hollow sphere, all with the same mass. Arrange them by decreasing minimum starting height so each one would *just* make it around the loop