ARBOR SCIENTIFIC

Ballistics Car

P3-3527

INSTRUCTIONAL GUIDE

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Background

The Ballistic Car demonstrates that the **horizontal** motion of an object is unaffected by forces which act solely in the **vertical** direction. It consists of a launcher mounted vertically on a car which propels a steel ball upward by means of a compressed spring. The car's low-friction wheel bearings allow it to coast at an essentially constant speed. If you trigger the launcher while the car is coasting, the ball rises and falls right back into the barrel.

Compare this to actions within a *moving train.* Envision a passenger inside a train throwing a ball straight upwards. The ball falls back into the passenger's hand provided the train moves at a constant speed. The Ballistics Car allows these proceedings to be watched by a stationary observer.

The purpose of the Ballistics Car is to show that the *horizontal component* of the ball's velocity must equal that of the car itself despite the large continuous changes in the *vertical component* of the ball's velocity. This explains how the ball can fall right back into the barrel of the launcher, given any initial speed of the car and any initial power settings of the launcher.

The passenger in the train is unaware of their respective movements. They see the ball move only in a vertical direction relative to their frame of reference. An observer on the ground, however, sees the ball curve in a parabola relative to his frame of reference.

At any point in a projectile's trajectory, its instantaneous velocity can be resolved into horizontal and the vertical directions, and since the horizontal component of the ball's motion stays constant while the vertical component changes due to the Earth's gravitational pull, the projectile's trajectory can be calculated and predicted. At higher speeds (speeds that cannot be attained by the Ballistics Car) the effect of air friction needs to be included since friction reduces both components of high velocity.

The motion of the ball obeys Newton's Laws of Motion due to the effect of gravity in both frames of reference. An observer cannot tell from the motion of the ball whether he is moving at any constant horizontal velocity relative to the earth.

OPERATION:

1. **First, test roll the cart, if the carts wheels rub on the frame pull the wheels away from the frame. This will free up the wheels and reduce friction**

2. **You need a level surface.**

   For this demonstration, you must run the car on a smooth, hard surface that is dust-free and horizontal to ensure a vertical launch of the ball. Use a **Bulls Eye Level** (P6-2604) to make sure your surface is level. Even floors that look horizontal to the eye can be too inaccurate for the demonstration to work. Check in both perpendicular and parallel directions to that of the car's motion.
   *Do not **drop** or allow the car to run off the edge of any table because the bearings can be damaged.*
3. **Position and shoot ball from barrel.**

   Place the ball on the piston inside the barrel. **Push down** to lock the piston in place. This is cocking the launcher.
   Place the magnet on the top of the box in the trench while you hold the trigger at the top so it will be held by the magnet. Now the trigger is ready.
   - **Grasp the cord** to the magnet in one hand. With the same hand, **push** the car away from you.

   - The ball will rise about one foot and fall back into the barrel.

4. **Try rolling the car at different speeds.**

5. If the ball is launched straight up and the car doesn't change speed very much during the flight of the ball, the ball falls right *back into the launcher*.

This confirms that the horizontal motion of the ball keeps pace with that of the car. For each locking position, the ball always rises to the same height regardless of horizontal velocity, again demonstrating the independence of the two perpendicular components.

A skeptic may claim that the car slows down while the launcher is firing because of the pull on the pin, thus invalidating the demonstration. The car does indeed slow down, but, provided the pull is directly in the line of travel and that no rocking of the car occurs, the change in speed is complete before the ball has begun its free assent. The skeptic may be convinced of this by a reversal of the motion. Pull the car toward you with a gentle pull on the cord and then give the sharp snap to pull the pin out. The same result occurs even through the car has increased its velocity.

MEASUREMENTS:

Only three measurements are needed to predict how far ahead of the barrel the ball will likely fall due to the deceleration of the car during the flight of the ball. If the position of the ball and car differ by more than a few millimeters at the end of the ball flight, the ball could not land in the barrel.

- Height H_{ball} to which ball rises above the mouth of the launcher. H_{ball} can be measured with the car stationary.
- Distance $D_{CarCoast}$ the car travels during a coasting test where wheel friction eventually drags the car to rest.
- Time $T_{CarCoast}$ taken for the car to coast down to a full stop. $T_{CarCoast}$ and $D_{CarCoast}$ must be measured together in the same trial and will be used to ascertain the deceleration of the car $a_{CarCoast}$ due to rolling friction.

Calculations

The car is slowing down because of friction in the wheel bearings. The ball in flight does not encounter that friction. How much farther, in the horizontal direction, does the ball travel than the decelerating car during the short time of flight?

Time that the ball is in the air

Measurement of the peak height of the ball H_{ball} allows you to calculate the time it will take to fall back down and thus, the total time the ball was in the air. To determine the relationship between distance and elapsed time for an object undergoing a constant gravitational acceleration **g** (which equals 9.8 m^2), use this equation:

$$H_{ball} = \frac{1}{2} g T^2$$

The time from the top of the ball's flight back down to the barrel is:

$$T = \left(\frac{2 H_{ball}}{g} \right)^{1/2}$$

The time taken to reach this height in the first place would have been the same. So, multiply the equation above by 2 to get the total the time of the ball's flight :

$$T_{ball} = 2 \left(\frac{2 H_{ball}}{g} \right)^{1/2} \quad \textbf{(Equation 1)}$$

Horizontal distance the car travels

The horizontal distance traveled by an ideal frictionless car would be given by the product of the car's initial velocity, V_0, and the time the ball was in the air **(T_{ball}).** For this real car, you need to know the rate at which its velocity decreases with time due to friction.

Since rolling friction is a constant force, you can assume the velocity of the car decreases at a uniform rate: the acceleration $a_{CarCoast}$ is *negative and constant*. With the car coasting on level ground at some low initial velocity (V_0), we measure the time **($T_{CarCoast}$)** and distance traveled **($D_{CarCoast}$)** as friction drags the car's velocity to zero. This is written as:

$V_{final} = V_0 + at$ $\qquad$ $0 = V_0 + a_{CarCoast} T_{CarCoast}$

So: $V_0 = - a_{CarCoast} T_{CarCoast}$ $\qquad$ **(Equation 2)**

Using the formula for distance traveled during constant acceleration: $\quad d = vt + 1/2\, at^2$

We get: $\quad D_{CarCoast} = V_0 T_{CarCoast} + \frac{1}{2} a_{CarCoast} T_{CarCoast}^2$ $\qquad$ **(Equation 3)**

where $a_{CarCoast}$ has a negative value.

Using Equation 2 we substitute for V_0 in Equation 3 to get:

$D_{CarCoast} = (-a_{CarCoast} T_{CarCoast}) T_{CarCoast} + \frac{1}{2} a_{CarCoast} (T_{CarCoast})^2$

So, $D_{CarCoast} = -\frac{1}{2} a_{CarCoast} T_{CarCoast}^2$

Therefore: $a_{CarCoast} = \frac{-2 D_{CarCoast}}{T_{CarCoast}^{2}}$ **(Equation 4)**

Flight of the projectile

The measurements of $D_{CarCoast}$ and $T_{CarCoast}$ are used to find the (negative) acceleration of the car. We can now compare motion of the ball and the decelerating launcher during the short time of flight.

While the ball is in the air for T_{ball} seconds, the ball travels a horizontal distance $D_{ball} = V_0 T_{ball}$. The launcher, however, travels a shorter distance during the **time the ball is in the air** (T_{ball}) calculated as follows:

$D_{barrel} = V_0\, T_{ball} + \frac{1}{2}\, a_{CarCoast}\, T_{ball}^{2}$

The ball travels further than the barrel. The further amount of travel is delta $\triangle$:

$\triangle = D_{ball} - D_{barrel}$

$= (V_0\, T_{ball}) - (V_0\, T_{ball} + \frac{1}{2}\, a_{CarCoast}\, T_{ball}^{2})$

$= (V_0\, T_{ball}) - V_0\, T_{ball} - \frac{1}{2}\, a_{CarCoast}\, T_{ball}^{2}$

$= -\frac{1}{2}\, a_{CarCoast}\, T_{ball}^{2}$

Substituting for $a_{CarCoast}$ from Equation 4 and substituting Equation 1 for T_{ball} gives us an answer in terms of our three easily measured quantities: H_{ball}, $T_{CarCoast}$ & $D_{CarCoast}$:

$\triangle = -\frac{1}{2}\, [a_{CarCoast}]\, [T_{ball}]^{2}$

$= -\frac{1}{2}\, \frac{-2 D_{CarCoast}}{T_{CarCoast}^{2}} \left[\frac{2\; 2 H_{ball}}{g}\right] \left(\left[\, ^{1/2}\quad ^{2} \,\right] \right)$

$= \frac{D_{CarCoast}}{T_{CarCoast}^{2}} \left[\frac{2^{2} * 2 H_{ball}}{g}\right]$

$= \frac{D_{CarCoast}}{T_{CarCoast}^{2}} \left[\frac{8 H_{ball}}{g}\right]$

$= \frac{8\, D_{CarCoast}\, H_{ball}}{T_{CarCoast}^{2}\, g}$

Typical values of $D_{CarCoast}$ = 2.0 meters, H_{ball} = 0.27meters, $T_{CarCoast}$ = 12 seconds yields:

$\triangle = \frac{8 \times 2.0m \times 0.27m}{(12sec)^{2} \times 9.8m/sec^{2}} = 0.003m = 3mm$

Therefore as long as friction is low enough to give similar values of delta $\triangle$ (which are small compared with the barrel mouth) the demonstration is valid. The Ballistics Car is a real world case which shows how the horizontal motion of an object is unaffected by vertical motion.

Concepts

Teaching with the Ballistics Car is a curriculum fit for *Physics Sequence; Motion & Force. Unit: Causes of Motion.* It will help you cover the following concepts:

- Scalar vs vector quantities
- Velocity as a vector
- Independence of horizontal and vertical components of motion
- Projectile motion
- Newton's laws of motion
- Equations of motion and calculation of variables using them
- Acceleration due to gravity

Related Products

Monkey & Hunter Set (P4-1965) Analysis of the projectile path of the bullet and the monkey's freefall shows that the bullet will hit the target. But are your students still unconvinced? Show them!

Vertical Acceleration Demonstrator (P3-3520) The Vertical Acceleration Demonstrator illustrates a concept that is crucial for understanding projectile motion: that the acceleration due to gravity only affects an object's vertical motion.

Mini Projectile Launcher (94-1970) This simple but precise launcher is versatile and great for indoor classroom use with projectile motion studies! The Mini Projectile Launcher projects 16 mm steel balls at ranges suitable for use on the benchtop or from the bench to the floor.