

Light Box and Optical Set 2.0 P2-9580

INSTRUCTIONAL GUIDE

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Related Products

Mini Ripple Tank (PA-8638) The Mini Ripple Tank provides a simple and effective method to investigate the properties of waves. The tank has settings that allow you to adjust the wave and strobe frequency.

Introductory Optical System (92-7700) This simple but elegant Optical System is designed for basic optics experiments, and a great alternative to the traditional mounted optical benches.

Color Mixing Projector (P2-9555) Demonstrate color addition, colored shadows, and more with our color mixing projector!

Subtractive Color Theory Demonstration (P2-9565) The Subtractive Color Theory Demonstration provides students with a hands-on experience as they learn subtractive color mixing and explore color theory in a whole new way.

Specifications

Light box

- Dimensions: 15 cm x 9.5 cm x 8.8 cm (6" x 3.75" x 3.5") (mirrors closed)
- Mirror size: 9 cm x 6.4 cm (3.63" x 2.5")
- Weight: 340 g (0.75 lb)
- Lamp: 12V DC/20 W halogen lamp with adjustable holder for focus
- Built-in 12 V DC cooling fan
- Magnetic base for attachment to magnetic surfaces
- Wall-mounted power supply for 110 VAC input with 1.5 m (5') cord
- Provision for two 62 mm x 62 mm (2.38") filters in each of four apertures and a cylindrical lens at one end.

Filters & Diaphragms

- All filters and slit cards are 62 mm (2.38") wide
- 2 slit cards with narrow slits (1.28 mm/0.05" wide): one slit, two slits 28 mm (1.11") apart, three slits 14 mm (0.55") apart, five slits 8.5 mm (0.33") apart
- 1 slit card with a 9 mm (0.35") wide slit and a 15 mm (0.59") hole

- Set of 7 color filters: red, green, blue, cyan, magenta, yellow, orange
- Set of 7 color paddles: same colors as the filters

Optical Elements

- All optical elements are 23 mm (0.88") thick, have fixed magnets and whitepainted bottom surfaces
- Lenses: 75 mm (3") wide
- Rectangular: 75 mm x 50 mm (3"x2")
- Prisms: equilateral, 60 mm (2.38") right angle, 50 mm (2") 30°-60°-90°, 38 mm—62 mm—73 mm (1.5, 2.38, 2.88")
- Plane mirror: 10 cm (4") wide
- Adjustable concave/convex mirror: 135 mm (5.25") wide; approx. radii: 60 mm, 90mm, 170 mm
- Rectangular trough: 75 mm x 50 mm (3" x 2"); volume: 61 ml; refractive index of the acrylic material: 1.49

Optional White Board

- Display area: 39.5 cm x 58.5 cm (15½" x 23")
- 4 swiveling feet, each 11 cm x 3 cm (4.25" x 1.25") to allow for portrait or landscape layout

Operation:

One end of the Light Box 2.0 is for demonstrating ray optics and the other for color mixing experiments.

The "ray end" houses the collimating lens in the slot nearest the bulb and can accommodate two slit cards in the slots nearest the opening. The slots can also fit the color filters for single-color beams of light. The thumbscrew and slot on the top of the light box allow the position of the lamp to be adjusted for divergent, parallel, or convergent rays. (Figure 1)

Figure 2 shows the light box viewed from the "color mixing end." The six slots (two in front of each opening) accommodate color filters for color mixing demonstrations. The two mirrors can be adjusted to allow the colored light beams to overlap.





A built-in fan evacuates heat from the halogen lamp, although it will still become hot to the touch during operation. Oils from fingerprints on the bulb may be absorbed by the quartz of the lamp bulb and ultimately shorten the lamp's life.

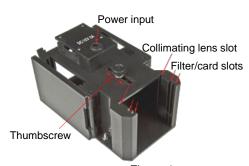


Figure 1





Figure 3

To operate the light box:

- Place the light box in the desired position. The magnetic base will hold it in place if the surface is magnetic, but a magnetic surface is not required.
- Insert any accessories required for the demonstration (cylindrical lens, diaphragms, filters, etc.) into the appropriate slots on the light box.
- Plug the light box power supply into a 110 V AC wall outlet.
- Plug the power supply adapter into the socket on the top of the light box. The lamp will illuminate and the cooling fan will spool up.
- Adjust the position of the lamp for focus and angle the mirrors as required for your demonstration.
- When the demonstration is finished, unplug the power supply from the light box and allow the lamp to cool before storing the set.

Introduction

The Light Box and Optical Set 2.0 provides a well-rounded set of tools to demonstrate and supplement the understanding of basic optics and color-mixing principles. It is most effectively used in a guided inquiry setting where students have the tools available to confirm rules of reflection and refraction, derive Snell's law, explore the fundamentals of optics such as defining the focal point, lens line, principal axis, etc. and delve into color theory. The "how to" sections in this guide provide useful setup tips and insight into the ways background knowledge of light and color can be applied practically. It is assumed that students using the Light Box have a basic working understanding of wave mechanics and how the wave theory of light applies to the electromagnetic spectrum.

Reflection

The Light Box and Optical Set helps students discover that light reflects from a mirror's surface in a systematic, predictable way. That is to say, light's angles of incidence and reflection are equal to each other when measured from the normal, the imaginary line perpendicular to the mirror's surface. We encounter mirrors every day to view an image, whether that be of ourselves or to look around a corner, or to look behind us while we're driving. A mirror's image is virtual since it is not in the "real" space; it is "behind" the mirror. But what happens when the surface isn't planar? The Light Box also provides for the investigation of reflection from curved surfaces using a flexible mirror. Concave and convex mirrors serve as a tool to extend the idea that the angle of incidence is equal to the angle of reflection, even on curved surfaces.

Setting up the Light Box for Reflection:

Set up the plane mirror on the center of the compass rose so the zero-degree line is perpendicular. This makes for a great visual for students to understand the normal. For as long as the mirror is perpendicular to the normal defined by the "zero" mark on the compass, the angles of incidence and reflection can be measured directly from the compass rose itself. The angle of incidence, often noted as θ_i , is the angle between the normal and the ray projecting from the source. Conversely, the angle of reflection, θ_r , is the angle between the normal and the ray reflected from the mirror's plane. (Figure 4a)

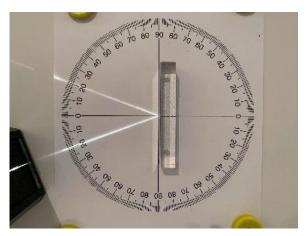


Figure 4a

Virtual images are seen "behind" the plane of the mirror and image points are on the same side as the object. They are, however, the same size as the object. Demonstrate this with a similar setup as seen to the right. Use the double-slit card and cover one of the slits with a translucent color filter. It is important to note that the beams are always parallel, indicating that the size of the image doesn't change, but the beams reverse in relation to the normal. (Figure 4b)

Concave and convex mirrors can really challenge students' understanding of reflection. The results may make sense to them fundamentally but explaining the observations will be difficult. Start by setting up the flexible mirror in the mirror holder in the first or second slot from horizontal (Figure 5a). Shine the light box with the five-slit card along the mirror's very center normal. Students should be able to notice that each ray has its own "normal" that is tangent to the edge of the mirror. In fact, the mirror has an infinite number of normal lines which all intersect at one point—the focal point (Figure 5b). This feature is indicative of a convergent mirror. The same rules apply for convex mirrors, but the focal point is actually behind the mirror. Tracing the lines and extending them back offers a clear display of divergence (Figure 5c).

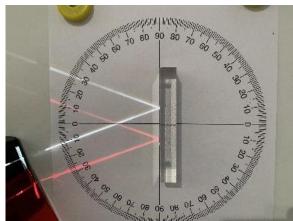


Figure 4b

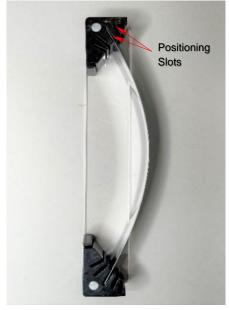


Figure 5a

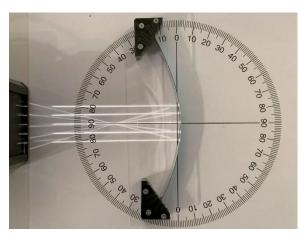


Figure 5b

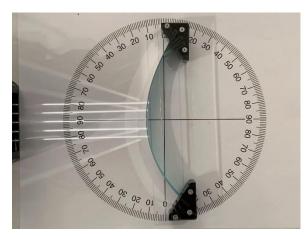


Figure 5c

Refraction

Most students are familiar with the classic pencil-in-a-beaker demonstration. There are a host of other places refraction happens, though, like in eyeglasses to refocus light onto the retina, or high-intensity car headlights, or in merely looking through a window. So common is this phenomenon that it "makes sense" to students even without conceptual understanding. The path of refracted light is also defined in terms of the incident angle, but since light passes through the barrier between two materials, there is a corresponding angle of refraction, measured from the normal. Unlike the relationship between angles of incidence and *reflection*, the angles of incidence and *refraction* are never the same, except when the incident angle is parallel to the normal. It is also important to note that the refracted ray is *always* on the opposite side from the incident ray.

The key to refraction lies in the wave behavior of light.

Familiarity with wave mechanics is essential to understanding refraction. As the wave front crosses the boundary from a less-dense medium to a more-dense medium, it slows down (Figure 6). If the light is at an angle to the boundary, one edge of the wave changes speed before the other edge. In order to maintain the integrity of the wave, the light must bend.

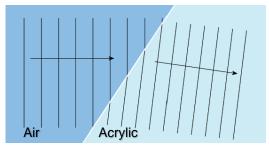


Figure 6

Index of Refraction

The slowing down and bending of the light is predictable given a few parameters. Every optically transparent material has a refractive index. The refractive index (n) is a unitless number defined as the ratio of the speed of light in a vacuum (c) to the speed of light in a particular medium (v) (known as the phase velocity). The equation for this relationship looks like this:

$$n=\frac{c}{v}$$

For example, light travels 1.33 times slower in water than it does in a vacuum, so the refractive index is 1.33.

Snell's Law

Willebrord Snellius determined that the indices of refraction can be used to describe the relationship between the angles of incidence and refraction. Snell's law (also called the law of refraction) is written like this:

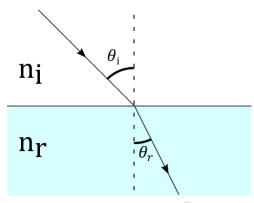


Figure 7

$$n_i \sin \theta_i = n_r \sin \theta_r$$

Critical Angle

If the angle of incidence is too great, the incident ray will reflect off the boundary between the two materials. The highest angle of incidence where light is *not reflected* is called the critical angle. In the mathematical terms of Snell's law, this occurs when the sine of the refracted angle equals 1. This of course is when the refracted angle is at 90 degrees. Rearranging Snell's law for this condition looks like this:

$$n_i \sin \theta_i = n_r \sin \theta_r$$

 $n_i \sin \theta_i = n_r \sin(90^\circ) = n_r \cdot 1$

Therefore, the critical angle can be found:

$$\theta_c = \sin^{-1}\frac{n_r}{n_i}$$

Critical angle applies in situations where the incident ray is traveling from a denser medium (higher refractive index) to a less-dense medium (lower refractive index). When the critical angle is passed, and the incident ray does not pass through the barrier but reflects back into the denser medium. This is called total internal reflection. For rays going from lower density to denser medium, the critical angle is the angle where the incident ray reflects off the surface of the denser medium, often called grazing incidence. The angle of refraction for grazing incidence from low-to-high density rays is the critical angle for total internal reflection of rays moving from high-to-low density media.

Setting up the Light Box for Refraction:

The single-slit card will be most useful for Snell's law activities since the angle of incidence can be easily dialed-in and the angle of refraction can be measured.

First, line up the beam with one of the center lines so that the refracted angles can be easily noticed. Then, have students experiment with the rectangular prism. Note that the beam leaving the block is parallel to the incident ray. Keep this observation in mind for when triangular prisms and lenses come into play! The reason for this is because the ray passes through two parallel sides of the rectangular prism. (Figure 8)

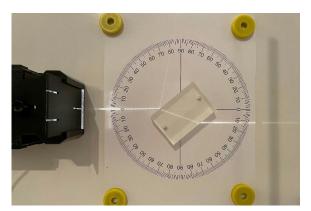


Figure 8

Determining Index of Refraction

Next, use the semi-circular prism to measure the angles of incidence and refraction. After collecting several data points, students can calculate the index of refraction for the acrylic prism.

To do this, line up the ray from the light box with the center line on the compass. Then, place the flat side of the semi-circular prism along the line perpendicular to the incident ray, facing the light box. Slide the prism up and down until the ray leaving the prism also runs through the zero mark (Figure 9a). This ensures the prism is centered on the compass rose.

Now, rotate the light box to any angle and record the angles of incidence and refraction from the compass. Since the prism is semi-circular and the incident ray passes through the center of the flat side, the angle of refraction can be read outside of the prism. The refracted ray passes through to air parallel with the normal of the curved surface, and so does not refract a second time (Figure 9b).

Calculating the index of refraction for the acrylic prism with the measurements taken from Figure 9b looks like this:

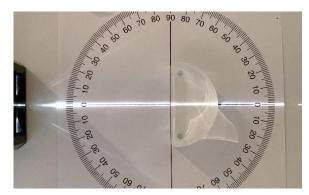


Figure 9a



Figure 9b

$$n_i \sin \theta_i = n_r \sin \theta_r$$

$$1.00 \sin(35^\circ) = n_r \sin(23^\circ)$$

$$n_r = \frac{\sin(35^\circ)}{\sin(23^\circ)} = 1.48$$

The literature value for the index of refraction of acrylic is 1.49.

Measuring Critical Angle

Center the semi-circular prism again, but this time with the flat face away from the light box. Again, ensure the prism is centered by shining a beam down one axis of the compass rose and placing the prims in its path, adjusting as needed for the beam to cross both zero marks. (Figure 10a)

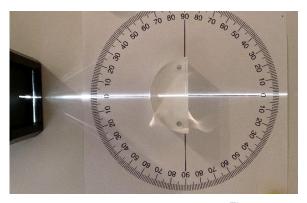


Figure 10a

Move the light box around and watch as the refracted ray (acrylic to air) moves farther away from the normal. Make sure the ray is always focused on the center of the compass rose. Since the acrylic prism has imperfections and the incident ray has a width (not a theoretically ideal ray), the phenomenon of total internal reflection will gradually appear (Figure 10b). This photo shows both reflection and refraction. Most eyeglasses have special coatings on the surfaces to reduce the internal reflections.

When the critical angle has been reached, some aberration can be seen at 90 degrees from the normal (Figure 10c). This is the theoretical maximum angle of refraction as defined by Snell's law, restated from above:

$$n_{acrylic} \sin \theta_{acrylic} = n_{air} \sin(90^\circ) = n_{air} \cdot 1$$

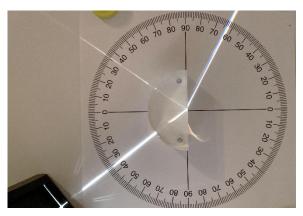


Figure 10b

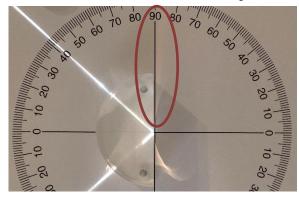


Figure 10c

Since the index of refraction for acrylic has already been determined, this equation can be filled in and rearranged to give:

$$n_{acrylic} \sin \theta_{acrylic} = n_{air} \cdot 1$$

$$\sin \theta_{acrylic} = \frac{1.00 \cdot 1}{1.49}$$

$$\theta_{acrylic} = \sin^{-1} \left(\frac{1.00}{1.49}\right) = 42.2^{\circ}$$

This lines up nicely with the measured angle in Figure 10c.

Prisms

Prisms are powerful optical tools which allow light to be directed with great precision. High-end telescopes and microscopes often use prisms instead of mirrors or direct light sources to refract and reflect light from their observations into lenses for magnification.

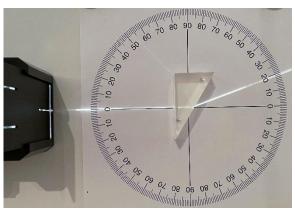


Figure 11

With the single-slit card installed in the light box, line up the 30-60-90 prism so one of its long faces is perpendicular to the incoming light beam (Figure 11). In other words, align the beam with the normal of one face. Notice the light does not refract as it enters the prism along the normal line but does when it exits the prism at an incident angle of 30° (determined from the 30-60-90 geometry).

Again, with the single-slit card installed in the light box, line up the equilateral prism so one of its faces is perpendicular to the vertical center line of the compass. This time, aim the beam at an incident angle and watch as the light refracts twice—once at an incident angle entering the prism, and again leaving the prism. (Figure 12)

Using Snell's law to determine the ultimate angle of refraction from acrylic to air takes a few more steps and some geometry knowledge. Figure 13 helps specify the angles in question:

$$n_{air} \sin \theta_A = n_{acrylic} \sin \theta_B$$

1. 00 sin 35° = 1. 49 sin θ_B
 $\theta_B = 22.6^\circ$
 $\theta_C = 90^\circ - \theta_B = 67.4^\circ$

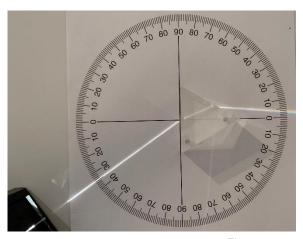
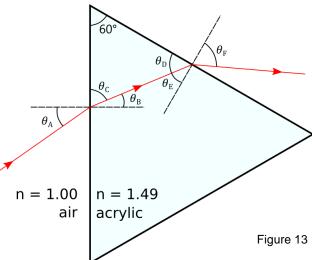


Figure 12



Since the inside angles of a triangle add to 180°, and therefore each inside angle of an equilateral triangle is 60°:

$$\theta_D = 180^{\circ} - 60^{\circ} - \theta_C = 52.6^{\circ}$$

Therefore, the incident angle from acrylic to air again, θ_E , is the complimentary angle to θ_D :

$$\theta_E = 90 - \theta_D = 37.4^{\circ}$$

Using Snell's law again for the transition from the prism back to air furnishes:

$$n_{acrylic} \sin \theta_E = n_{air} \sin \theta_F$$

1.49 sin 37.4° = 1.00 sin θ_F
 $\theta_F = 64.8^\circ$

Now, replace the equilateral prism with the semi-circular prism with the flat side facing toward the light box. Change out the single-slit card for the triple-slit card. Again, make sure the prism is centered by moving the prism so the center beam passes through the prism and remains on the horizontal axis. Notice how the top and bottom beams refract only once as they enter the prism along the normal line. When they come out, they refract toward the center beam and cross at the same point. This is called the focal point. (Figure 14a)

Finally, rotate the semi-circular prism 180° and again align the flat side with the vertical axis, repeating the centering process. Notice how the top and bottom beams now refract twice, as in the equilateral prism above. Again, because both the top and bottom beams enter at the same distance from the center of the prism, they refract equally and meet at the focal point. (Figure 14a)

Figure 14c Demonstrates a vivid display of convergence by removing the slit card altogether.

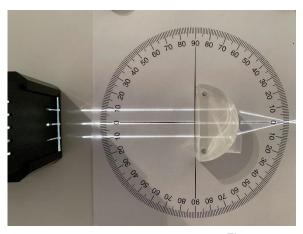


Figure 14a

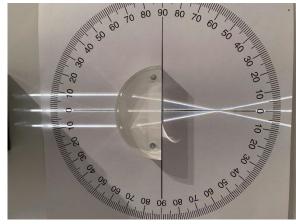


Figure 14b

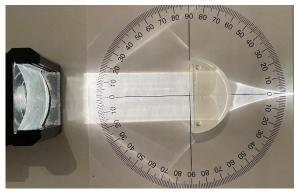
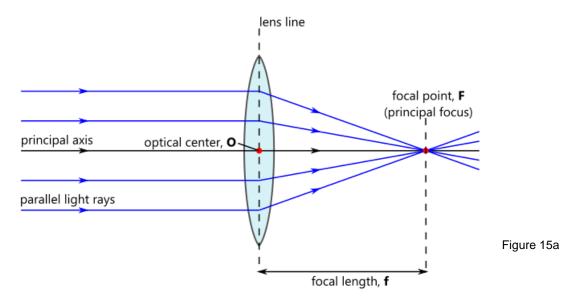


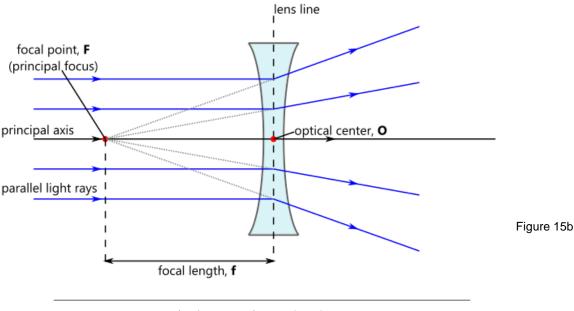
Figure 14c

Lenses

Although an extension of refraction, the study of lenses deserves more in-depth focus. Lenses are an amazing application of refraction and make use of certain geometries to converge or diverge light in a number of ways. A convex lens refracts the light that passes through it in such a way that all parallel light rays converge at a single point, which is why a convex lens is frequently called a "converging lens" (Figure 15a). That point of convergence is called the focal point of the lens, and it is determined by the curved edges' radius of curvature. (Note, light rays actually bend at the front and back surfaces of the lens.



In contrast to converging lenses, the purpose of diverging lenses is to spread light out. These are shaped with concave edges, and so are also called concave lenses (Figure 15b). You can also define a focal point for this type of lens by tracing the diverging rays "behind" the lens to an intersection point.



Conventionally, the focal length of a converging lens is denoted with a positive sign (+), whereas the focal length of a diverging lens is denoted with a negative sign (-). Also by convention, rays are drawn to the center of the lens (the lens line) and diffracted from there. These types of lenses are usually referred to as "thin lenses" since they're theoretically have zero thickness. In reality, there is some aberration seen at the focal point of most lenses since they are not perfectly formed and the rays passing through them go through more material toward the center than the edges.

The Lensmaker's Equation

The lensmaker's equation relates the focal length of a thin lens to the index of refraction and radii of curvature of its surfaces. The general equation looks like this:

$$\frac{1}{f} = (n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

Where f is the focal length of the lens, n is the index of refraction, R_1 is the radius of the side of the lens closest to the light source, and R_2 is the radius of the side of the lens facing away from the light source. The Cartesian sign conventions for the radii of curvature are:

Convex (converging) –
$$R_1 > 0$$
, $R_2 < 0$ Concave (diverging) – $R_1 < 0$, $R_2 > 0$

Setting up the Light Box for Lenses:

The purpose of using lenses with the Light Box is to develop a basic knowledge of how the principles of refraction are applied in lenses and introduce conventional terminology used in the further study of lenses and ray optics. Make sure the light box and lenses are used on a surface that can be drawn on, such as a whiteboard. Even better if it is magnetic!

To start out, have students draw a principal axis. Using the 3- or 5-slit card, center the middle beam of the light box on the principal axis. Mark a perpendicular line on the principal axis. This will be the lens line. (Figure 16a)

Proceed to set the double convex lens on the lens line. Mark the focal point with another vertical line. (Figure 16b)

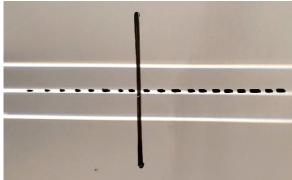


Figure 16a

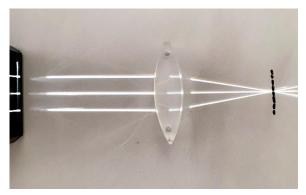


Figure 16b

Remove the lens and measure the distance between the lens line and the focal point. This is the experimental focal length of the converging lens (Figure 16c). From here, the radius of the lens can be calculated with the lensmaker's equation. It is important to note that in this case, both sides of the lens have the same radius. In cases where radii are different, one radius must be known to determine the other.

$$\frac{1}{f} = (n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\frac{1}{8.5 \, cm} = (1.49 - 1) \left[\frac{1}{R_1} - \frac{1}{-R_2} \right]$$

Note, R_1 is positive and R_2 is negative due to the sign convention mentioned previously.

$$\frac{1}{8.5 cm} = (0.49) \left[2 \cdot \frac{1}{R} \right]$$
$$R \approx 8.33 cm$$

Repeat the first step, but this time set the diverging lens on the lens line. With the lens in place, trace the diverging rays with a straight edge. Then, remove the lens and replace the straight edge on one of the divergent rays. Trace it back until it crosses the principal axis (Figure 17). Repeat for the other diverging rays. Once again, measure between the lens line and the focal point along the principal axis. The radius of curvature is the same for the converging and diverging lenses, so the measurement should be equal in the opposite direction.

The lensmaker's equation for this model is:

$$\frac{1}{f} = (n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$
$$\frac{1}{8.5 \, cm} = (1.49 - 1) \left[\frac{1}{-R_1} - \frac{1}{R_2} \right]$$

This time, R_1 is negative and R_2 is positive due to the sign convention mentioned previously.

$$\frac{1}{8.5 cm} = (0.49) \left[-2 \cdot \frac{1}{R} \right]$$
$$R \approx -8.33 cm$$

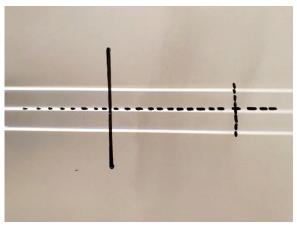


Figure 16c

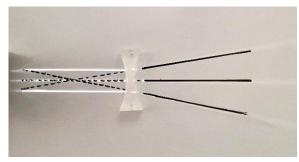


Figure 17

Color

The spectrum of visible light consists of a broad range of wavelengths and frequencies along the electromagnetic spectrum. On the other hand, our perception of color is really quite complex. We have color receptors called cones in our retina. Some cones fire with red light, some with green light, and some are sensitive to blue light. The firing rate of these various cones determines the individual colors that we perceive. This model of R, G, B is used in color cameras, TVs, etc. It is amazing that by varying the intensities of only these 3 colors that you can create thousands of colors!"

Mixing different colors of light is known as additive color mixing. In contrast, subtractive color mixing creates different colors when additional colors of the spectrum are subtracted or absorbed by a medium, allowing a limited range of light to reflect off of the medium. The light that is reflected is the color detected by the eye. Primary colors for subtractive color mixing are cyan, magenta, and yellow. With the addition of black—or key—the CYMK color model is typically used for inks, paints and printing, including photographic printing.

Setting up the Light Box for Color Mixing:

For color mixing activities with the light box, a dark room is required for best results. It is also helpful to have the light box set up on a white background for the truest color interpretation. An angled piece of cardstock is included and can be mounted to a magnetic whiteboard with the circular magnets. This gives an angled field ideal for viewing color mixing.

White light can be fairly well approximated using three sources of light—one in the red-yellow region, the yellow-green region, and the blue-violet regions. These are defined as "primary colors". The color triangle to the right (Figure 18) displays the relationship between the primary colors and how they combine to form the secondary colors of cyan, magenta, and yellow. These colors will be covered more in subtractive color mixing.

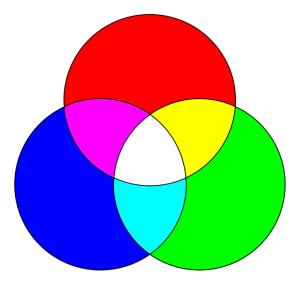


Figure 18

Unfold the mirrors from the sides of the color-mixing end of the light box. Next, select the red, green, and blue transparent color filters and install them in the light box in the same way as the slit cards as seen in Figure 19a. Of course, the three primary colors are present, but only magenta and yellow are present of the secondary colors. As expected, when all three colors are mixed equally, a white triangle appears.

Now, switch the red and green color filters (Figure 19b). The yellow appears at the same place since the red and blue are combining there. Where there was magenta, cyan is now present. Regardless of the positioning of the three primary colors, the white triangle is still present.

With a small piece of paper such as a sticky note, create an obstruction in each of the fields of colored light. Although the shadow just behind the obstruction will be black (an interesting observation in itself), the shadows will reveal the component colors of the field of light the obstruction is in. Notice how the primary and secondary colors appear when the obstruction is placed in the white light. Red is not present, however, due to its positioning in the light box. (Figure 19c)

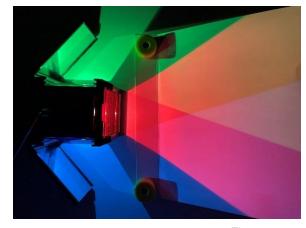


Figure 19a



Figure 19b

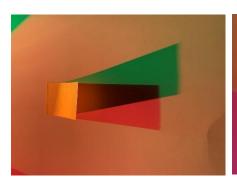






Figure 19c

Unlike "white light", the absence of reflected light, black, cannot so easily be approximated. Even most "black" pigments reflect some incident rays. When our eyes see these rays, we cannot discern a color on the spectrum and call it black. For demonstrations of color subtraction, the observation resultant from light shining on objects from the opposite side of the subtractive color triangle (Figure 20) will appear colorless but not black. It will most likely look like light shining on a black object.

To demonstrate color subtraction with the light box, first install the cylindrical collimating lens in the ray-end of the light box. Then, insert the wide-slit card in the slot nearest the lens, leaving the forward-most slot open for the color filters. Figure 21 shows magenta filtered light shining on a green color paddle. See how the light reflecting off of the color paddle seems to be colorless, as if white light were shining off a back paddle.

This can be repeated for any combination of colors with varying results. The biggest challenge for students is predicting which color combinations will result in the production of a certain color.

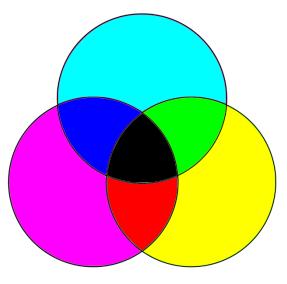


Figure 20



Figure 21