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Wiley's Study Guides are written by a team of highly qualified CFA charterholders and leading CFA instructors from around the globe. Our team of CFA experts work collaboratively to produce the best study materials for CFA candidates available today.

Wiley's expert team of contributing authors and instructors is led by Content Director Basit Shajani, CFA. Basit founded online education start-up Élan Guides in 2009 to help address CFA candidates' need for better study materials. As lead writer, lecturer, and curriculum developer, Basit's unique ability to break down complex topics helped the company grow organically to be a leading global provider of CFA Exam prep materials. In January 2014, Élan Guides was acquired by John Wiley & Sons, Inc., where Basit continues his work as Director of CFA Content. Basit graduated magna cum laude from the Wharton School of Business at the University of Pennsylvania with majors in finance and legal studies. He went on to obtain his CFA charter in 2006, passing all three levels on the first attempt. Prior to Élan Guides, Basit ran his own private wealth management business. He is a past president of the Pakistani CFA Society.

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Study Session 15: Fixed Income: Basic Concepts

READING 51: FIXED-INCOME SECURITIES: DEFINING ELEMENTS

LESSON 1: OVERVIEW OF A FIXED-INCOME SECURITY

Before you get into this reading, you should recognize the following relationships that will facilitate your understanding of the material. These relationships are discussed at great length in later readings:

- The higher (lower) the coupon rate on a bond, the higher (lower) its price.
- An increase (decrease) in interest rates or the required yield on a bond will lead to a decrease (increase) in price (i.e., bond prices and yields are inversely related).
- The more risky the bond, the higher the yield required by investors to purchase the bond, and the lower the bond's price.

LOS 51a: Describe basic features of a fixed-income security. **Vol 5, pp 296–302**

BASIC FEATURES OF A BOND

The important fixed-income security elements that we describe in this section include:

- The issuer of the bond.
- The maturity of the bond.
- The par value of the bond.
- The coupon rate offered on the bond and coupon payment frequency.
- The currency in which bond payments will be made to investors.

Note that in this section, we focus on "traditional" or **nonsecuritized bonds**.

Securitized bonds are created by moving some assets (e.g., residential and commercial mortgages, automobile loans, student loans, and credit card loans) into a special legal entity, which then uses those assets to guarantee a bond issue. We discuss such bonds later in the reading.

Issuer

Bond issuers can be classified based on their characteristics. Major types of issuers include:

- **Supranational organizations** (e.g., the World Bank and the International Monetary Fund).
- **Sovereign (national) governments** (e.g., the United States).
- **Nonsovereign (local) governments** (e.g., the state of Pennsylvania in the United States).
- **Quasi-government entities**, agencies owned or sponsored by governments (e.g., the postal services in many countries such as La Poste in France).
- **Companies** or **corporate issuers**, which include financial issuers (e.g., banks) and nonfinancial issuers (e.g., pharmaceuticals).

We learn more about rating agencies, credit ratings, and classification of issuers based on their credit ratings in a later reading.

Bond issuers can also be classified based on their creditworthiness as judged by **credit rating agencies**. Bonds can broadly be categorized as **investment-grade** or **noninvestment grade** (or **high-yield** or **speculative**) **bonds**.

Maturity

The **maturity date** of a bond refers to the date on which the issuer has promised to repay the entire outstanding principal on the bond. A somewhat similar (sounding) concept is the **term to maturity** or the **tenor** of the bond, which refers to the period of time remaining until the bond's maturity date.

- Fixed-income securities which, at the time of issuance, are expected to mature in 1 year or less are known as **money market securities** (e.g., commercial paper and certificates of deposits, CDs).
- Fixed-income securities which, at the time of issuance, are expected to mature in more than 1 year are referred to as **capital market securities.**
- Fixed-income securities which have no stated maturity are known as **perpetual bonds** (e.g., consols issued by the U.K. government).

Par Value

The **par value** (also known as **face value, nominal value, redemption value,** and **maturity value**) of a bond refers to the principal amount that the issuer promises to repay bondholders on the maturity date. Bond prices are usually quoted as a percentage of their par value. For example, given that a bond's par value is $1,000, a quote of 90 implies that the bond's current price is $900 (= 90% × $1,000).

- When a bond's price is above 100% of par, it is said to be trading at a **premium.**
- When a bond's price is at 100% of par, it is said to be trading at **par.**
- When a bond's price is below 100% of par, it is said to be trading at a **discount.**

Coupon Rate and Frequency

The **coupon rate** (also known as the **nominal rate**) of a bond refers to the annual interest rate that the issuer promises to pay bondholders until the bond matures. The amount of interest paid each year by the issuer is known as the **coupon,** and is calculated by multiplying the coupon rate by the bond's par value. For example, given a coupon rate of 3% for a bond with a par value of $1,000, the annual interest paid by the issuer will be $30 (= 3% × $1,000). Note that the bond indenture may call for coupon payments annually, semi-annually, quarterly, or monthly. So continuing with the previous example, if it were a semiannual-pay bond, the issuer would make $15 coupon payments every 6 months during the bond's term.

> We will describe several other coupon structures, including variable coupon rates, later in the reading.

Note that some bonds do not make any interest payments until maturity. These **zero-coupon** (or **pure discount**) bonds are issued at a discount to par value and redeemed at par (the issuer pays the entire par amount to investors at the maturity date). The difference between the (discounted) purchase price and the par value is effectively the interest on the loan.

Currency Denomination

Bonds are issued in many different currencies around the world. Bonds aimed solely at a country's domestic investors are most likely to be issued in the local currency, but if an issue aims to attract global investors, it is likely to be denominated in U.S. dollars or euros to make them more attractive for investors (especially if the local currency is relatively volatile and/or illiquid).

- **Dual-currency bonds** make coupon payments in one currency and the principal payment at maturity in another currency.
- **Currency option bonds** give bondholders a choice regarding which of the two currencies they would like to receive interest and principal payments in.

Yield Measures

Market participants use several different **yield measures** to describe a bond.

- The **current yield** or **running yield** equals the bond's annual coupon amount divided by its current price (not par value), expressed as a percentage.
    - For example, the current yield of a $1,000 par bond with a coupon rate of 5% and a current price of $980 equals 5.102% [= (50/980) × 100].
- The **yield-to-maturity (YTM)** is also known as the **yield-to-redemption** or the **redemption yield**. It is calculated as the discount rate that equates the present value of a bond's expected future cash flows until maturity to its current price.
    - Essentially, the YTM represents the internal rate of return on the bond's expected cash flows. Stated differently, the YTM represents the annual return that an investor will earn on a bond if (1) she purchases it today (for its current price), (2) holds it until maturity, and (3) reinvests all interim cash flows at the stated YTM.
    - All else being equal, a bond's yield-to-maturity is inversely related to its price.
        - Given a set of expected future cash flows, the lower (higher) the YTM or discount rate, the higher (lower) the bond's current price.

> The yield-to-maturity and several other important yield measures are discussed at length in a later reading.

LOS 51b: Describe content of a bond indenture. Vol 5, pp 303–311

The Bond Indenture

A bond is a contractual agreement between the issuer and the bondholder. The **trust deed** is the legal contract that describes the form of the bond, obligations of the issuer, and the rights of bondholders, and it is commonly referred to as the **bond indenture**. The indenture captures the following information:

- The name of the issuer.
- The principal value of the bond.
- The coupon rate.
- Dates when the interest payments will be made.
- The maturity date.
- Funding sources for the interest payments and principal repayments.
- Collaterals (i.e., assets or financial guarantees underlying the debt obligation above and beyond the issuer's promise to pay).

- Covenants (i.e., actions that the issuer is obligated to perform or prohibited from performing).
- Credit enhancements (i.e., provisions designed to reduce the bond's credit risk).

Since there are (typically) a large number of bondholders, the issuer does not get into a direct agreement with each one of them. Instead, the indenture is held by a **trustee** who, although appointed by the issuer, owes a fiduciary duty to bondholders. The trustee is primarily responsible for ensuring that the issuer complies with the obligations specified in the bond indenture. Further, in the event of a default, the trustee is responsible for calling meetings of bondholders to determine an appropriate course of action, and may also bring legal action against the issuer on behalf of bondholders.

Before deciding to invest in a particular bond instrument, investors should review the following areas.

Legal Identity of the Bond Issuer and its Legal Form

The bond indenture identifies the party that is obligated to make principal and interest payments by its legal name.

- For sovereign bonds, the legal issuer is typically the institution responsible for managing the national budget (e.g., Her Majesty's Treasury in the U.K.).
    - Note that the legal issuer may be different from the body that actually manages the bond issue process (e.g., the U.K. Debt Management Office, which is an agency of Her Majesty's Treasury).
- Corporate bonds are typically issued by the corporate legal entity (e.g., Volkswagen AG).
    - Note that bonds may be issued by the parent company or a subsidiary.
        - If issued by a subsidiary, investors must focus on the creditworthiness of the subsidiary, unless the bond is guaranteed by the parent. Oftentimes, subsidiaries carry a lower credit rating than the parent.
    - If issued by the parent company, it becomes important to analyze the assets actually held by the parent, as investors may not have recourse to assets held by subsidiaries or operating companies.
        - If bonds are issued by a parent or holding company that has no or very few assets to call in the event of default, investors face higher credit risk than if bonds were issued by one of the operating companies or subsidiaries in the group that actually hold callable assets.
- In case of securitized bonds, a separate legal entity is created (referred to as a **Special Purpose Entity** or **SPE** in the United States and **Special Purpose Vehicle** or **SPV** in Europe), and it is this entity that is obligated to repay bondholders. What happens is that the **originator** of the loans (also known as the **sponsor** or **seller** of financial assets) sells the assets (loans it has issued) to the SPE, which issues bonds (backed by those assets) to investors to come up with the funds to purchase the assets from the sponsor. The SPE's assets are the **loans** that it has bought from the sponsor, and its liabilities are the **bonds** it has sold to investors. The SPE uses the cash generated from the loans that it holds to service debt obligations owed to investors. This entire process is known as a **securitization**.

SPEs are **bankruptcy-remote vehicles** in that once the assets (loans) have been securitized (transferred to the SPE), the sponsor no longer has any ownership rights on them. Therefore, if the sponsor declares bankruptcy, its creditors have no claim on the securitized assets or their proceeds, and the SPE is able to continue to make interest and principal payments to investors using funds generated from the assets.

Source of Repayment Proceeds

The bond indenture also (usually) specifies how the issuer plans to make debt service payments (interest and principal).

- Bonds issued by supranational organizations are usually repaid through (1) proceeds from repayments of previous loans made by the organization or (2) paid-in capital from its members.
  - In some instances, national governments may guarantee certain issues.
  - Supranational organizations can generally also call on members to provide funds to repay loans.
- Sovereign bonds are backed by the "full faith and credit" of the national government.
  - Sovereign bonds issued in local currency are usually considered safer than those issued in a foreign currency, as governments have the power to raise taxes and print local currency to repay local-currency denominated loans. As a result, yields on sovereign bonds are usually lower compared to other local issuers.

Printing local currency typically does not help repay foreign currency sovereign debt as the increase in local currency supply leads to depreciation of the local currency.

- Nonsovereign government debt can usually be repaid through the following sources:
  - The general taxing authority of the issuer.
  - Cash flows from the project that the bonds were issued to finance.
  - Special taxes or fees specifically set up to make interest and principal payments.
- Corporate bond issuers typically rely on their cash flow-generating ability to repay bonds, which in turn depends on their financial strength and integrity.
  - Corporate bonds typically entail a higher level of credit risk than sovereign bonds, and therefore carry a higher yield.
- For securitized bonds, repayment depends on the cash flow generated by the underlying pool of financial assets.
  - Examples of financial assets that are usually securitized include residential and commercial mortgages, automobile loans, student loans, and credit card receivables.

Securitized bonds generally differ from corporate bonds in that they are amortizing loans (i.e., the principal amount is gradually repaid during the term of the loan instead of in one lump sum payment at maturity). We will describe amortizing bonds in detail later in the reading.

Asset or Collateral Backing

Assets pledged as collateral and financial guarantees secure a bond issue beyond the issuer's simple promise to pay. While they serve to reduce credit risk, it is important for investors to consider (1) how the specific bonds they own rank in the priority of claims in the event of default and (2) the quality of the collateral backing the bond issue.

- **Seniority ranking:** Secured bonds are backed by assets or financial guarantees to ensure debt repayment in case of default, while unsecured bonds are not protected by a pledge of any specific assets (i.e., they only have a general claim on the issuer's assets and cash flows).
    - In the event of default, unsecured bonds are repaid after secured bonds have been paid off, which makes unsecured bonds more risky, entailing a higher yield.
    - Within the general class of secured bonds, different issues have a different seniority ranking, which dictates the order of payment to secured bondholders in case of bankruptcy.
        - Senior secured debt is paid off before subordinated or junior secured debt.
    - Debentures in the U.S. are an unsecured loan certificate backed by a company's general credit. Elsewhere, debentures typically describe long-term debt securities secured by assets and paying a fixed rate of interest. Investors should review the indenture for specifics.
- **Types of collateral backing:** Bonds can also be classified based on the type of collateral.
    - Collateral trust bonds are secured by securities such as common stock, other bonds, or other financial assets.
    - Equipment trust certificates are secured by specific types of equipment or physical assets (e.g., aircraft, railroad cars, or oil rigs).
        - They are usually issued to take advantage of the tax benefits of leasing.
    - Mortgage-backed securities (MBS) are backed by a pool of mortgage loans. Cash flows generated by the pool are used to make payments on MBS.
    - Covered bonds are backed by a segregated pool of assets (known as the cover pool). One of the things that differentiates them from securitized bonds (such as MBS) is that the assets backing these bonds are still on the sponsor's balance sheet (i.e., they are not transferred to an SPE). As a result, if the sponsor defaults on covered bonds, investors have recourse to both (1) the assets held by the sponsor and (2) the cover pool. (Recall that for securitized bonds, investors only have recourse to the pool of assets, not the assets of the sponsor.) Further, if assets in the cover pool fail to generate adequate cash flow (i.e., they become nonperforming), the issuer is required to replace them with performing assets. As a result, covered bonds entail less risk and therefore offer a lower yield than securitized bonds.

Credit Enhancements

Credit enhancements are provisions that serve to reduce the credit risk of a bond issue, resulting in a lower yield.

- Internal credit enhancement focuses on the structure of the issue regarding priority of payment or the value of collateral. It includes:
  - Subordination: This refers to the allocation of cash flows among the various bond classes or tranches based on their seniority. Basically, the subordinate and junior tranches provide credit protection to the senior tranche, so in the event of default the senior tranche (the one with the highest level of seniority) has the first claim on available cash flows, and only once senior bonds have been paid off do subordinate bonds (and then junior bonds) get repaid. This type of protection is also referred to as a waterfall structure—proceeds from liquidating assets are first allocated to the more senior tranches, while losses are allocated bottom-up with the most senior tranche typically insulated from credit losses unless the total amount of loss exceeds the par amount of subordinated and junior tranches combined.
  - Overcollateralization: This occurs when the value of the collateral posted to secure an issue exceeds the par value of the securities issued. The excess collateral can be used to absorb future losses. For example, a securitized issue with a total par value of $130 million would be overcollateralized by $20m if it is backed by $150 million worth of loans.
  - Excess spread (or excess interest cash flow): A transaction can be structured in a manner such that the cash flows generated from the collateral pool exceed the amount of interest payable to bondholders so that there is a cushion for making interest payments. This excess amount is known as the excess spread, and it is sometimes deposited into a reserve account to protect against future credit losses (by being used to retire principal). This process is known as turboing.
- External credit enhancement refers to guarantees received from a third-party guarantor. Types of external credit enhancement include:
  - Surety bonds (issued by an insurance company) and bank guarantees: In the event of default, both these forms of enhancement reimburse investors for losses (up to a pre-specified maximum amount known as the penal sum).
  - Letters of credit: These are lines of credit provided by a financial institution to the issuer to reimburse any shortfalls in the cash flow generated from the collateral pool.

Note that surety bonds, bank guarantees, and letters of credit all expose investors to third party credit risk (i.e., the risk that the gurantor will not be able to fulfill its obligations). This risk can be mitigated by setting up a cash collateral account, where the issuer immediately borrows the credit-enhancement amount and invests this sum in highly rated instruments. Since the cash is actually in the hands of the issuer, a downgrade of the guarantor will not necessarily affect the rating of the bond issue.

LOS 51c: Compare affirmative and negative covenants and identify examples of each. Vol 5, pp 308–309

Covenants

Covenants are legally enforceable rules agreed upon by the borrower/issuer and lenders/investors at the time of bond issuance.

- Affirmative covenants are **requirements** placed on the issuer. They are typically administrative in nature so they do not lead to additional costs for the issuer, nor do they significantly restrict the issuer's ability to make business decisions. Examples of affirmative covenants include promises to:
  - Make timely payments to bondholders.
  - Comply with all laws and regulations.
  - Maintain the issuer's current lines of business.
  - Insure and maintain assets.
  - Pay taxes.
- Negative covenants are **restrictions** placed on the issuer. While they entail more costs than affirmative covenants and can constrain the issuer in operating the business, they protect bondholders from dilution of their claims, asset withdrawals or substitutions, and inefficient investments by the issuer. Examples of negative covenants include:
  - Restrictions on issuing additional debt so that new debt can only be issued if it can be sustained by the issuer's financial condition.
  - Negative pledges that prevent the issuer from issuing bonds with a higher rank in the priority of claims than those held by existing bondholders.
  - Restrictions on prior claims that protect unsecured bondholders by preventing the issuer from collateralizing previously uncollateralized (or unencumbered) assets.
  - Restrictions on distributions to shareholders (e.g., dividends and share repurchases) to ensure that sufficient cash is available for debt-service going forward.
  - Restrictions on asset disposals that protect bondholders from a breakup of the company.
  - Restrictions on investments that prevent the issuer from making risky or speculative investments.
  - Restrictions on mergers and acquisitions that ensure that if the issuing company is sold, the surviving company or the acquirer takes responsibility for debt service payments.

Note that the aim of negative covenants is to protect bondholders by ensuring that the issuer does not take any action that may hinder its ability to satisfy debt obligations. Bondholders should refrain from being too specific or restrictive when it comes to covenants, as such provisions may force the issuer into default even when it is avoidable.

LESSON 2: LEGAL, REGULATORY, AND TAX CONSIDERATIONS

LOS 51d: Describe how legal, regulatory, and tax considerations affect the issuance and trading of fixed-income securities. Vol 5, pp 311–316

Legal and Regulatory Considerations

There are no unified legal or regulatory requirements that apply to bond investors globally, which makes it important for investors to be aware of where a particular fixed-income security is issued and where it is traded, as they determine which laws and regulations apply.

The global bond market consists of national bond markets and the Eurobond market. A **national bond market** includes all the bonds that are issued and traded in a particular country and denominated in that country's local currency.

- Bonds issued by entities that are incorporated in that country are known as domestic bonds.
  - For example, bonds issued by Google Inc. in U.S. dollars in the United States would be classified as domestic bonds.
- Bonds issued by entities that are incorporated in another country are known as foreign bonds.
  - For example, bonds issued Toyota Motor Company in U.S. dollars in the United States would be classified as foreign bonds. Foreign bonds often receive nicknames (e.g., these bonds issued by Toyota would be known as "Yankee" bonds [a nickname for foreign bonds in the United States]).

Note that national regulators may subject resident and nonresident issuers to different requirements regarding the issuance process, level of disclosures, or restrictions imposed on the bond issuer and/or the investors who can purchase the bonds.

Eurobonds are issued and traded in the **Eurobond market.** Eurobonds refer to bonds that are denominated in a currency other than the local currency where they are issued. They may be issued in any country and in any currency (including the issuer's domestic currency) and are named based on the currency in which they are denominated. For example, Eurobonds denominated in U.S. dollars are referred to as Eurodollar bonds, and Eurobonds denominated in Japanese yen are known as Euroyen bonds.

An example of a Eurobond is a U.S. dollars-denominated bond issued by Toyota Motor Company in Australia. This bond would be a Eurodollar bond (since it is denominated in U.S. dollars).

> Note that Eurodollar bonds cannot be sold to U.S. investors at issuance as they are not registered with the U.S. Securities and Exchange Commission (SEC). Therefore, Toyota can issue a Eurodollar bond in any country, but not in the United States.

- Eurobonds are usually unsecured and are underwritten by an international syndicate.
- Generally speaking, they are less regulated than domestic and foreign bonds as they do not fall under the jurisdiction of any single country.
- Eurobonds are usually bearer bonds (i.e., no record of bond ownership is kept by the trustee). On the other hand, domestic and foreign bonds are usually registered bonds (the trustee maintains a record of bond ownership).

Global bonds are bonds that are issued simultaneously (1) in the Eurobond market and (2) in at least one domestic bond market. Simultaneously issuing bonds in different markets ensures that there is sufficient demand for the issue, and that investors are able to participate in the issue regardless of their location.

> Foreign bonds, Eurobonds, and global bonds are often referred to as international bonds.

The differences between domestic bonds, foreign bonds, Eurobonds, and global bonds are important to investors as these different types of bonds are subject to different legal, regulatory, and tax (described in the next section) requirements. There are also differences in terms of frequency of interest payments and how interest payments are calculated, both of which influence the bond's cash flows and price. Bear in mind however, that the currency denomination has a bigger influence on a bond's price than where it is issued or traded. This is because a bond's price is influenced by interest rates, and interest rates are obviously linked to the currency in which the bond is denominated.

Tax Considerations

Interest income is generally taxed at the ordinary income tax rate, which is the same rate that an individual pays tax on her wage or salary. However, municipal bonds in the United Staes are often tax-exempt, in that interest income is often exempt from federal income tax and state income tax in the state of issue. Also note that the tax status of bond income depends on where it is issued and traded. For example, interest paid on some domestic bonds may be net of income tax, while some Eurobonds make gross interest payments.

Aside from interest income, investors in bonds may also generate capital gains from selling a bond prior to maturity. In most jurisdictions, capital gains are taxed at a lower rate than interest income. Further, some countries impose a lower capital gains tax on investments held over a longer time horizon than on those held for a shorter horizon. Some countries do not tax capital gains at all.

An additional tax consideration arises when it comes to bonds issued at a discount. The difference between the par value and the original issue price is known as the **original issue discount**. In some countries (including the United Staes), a prorated portion of the original issue discount must be included in interest income each year for tax purposes, and taxes must be paid at the rate applicable on interest income. In other countries (such as Japan) the original issue discount is taxed as a capital gain when the bond eventually matures.

Finally, in some jurisdictions, investors who have purchased bonds at a premium can either (1) deduct a prorated portion of the premium paid from taxable income each year until the bond matures, or (2) declare a capital loss when the bond is eventually redeemed at maturity.

LESSON 3: STRUCTURE OF A BOND'S CASH FLOWS

LOS 51e: Describe how cash flows of fixed-income securities are structured.

Vol 5, pp 316–327

Principal Repayment Structures

How the bond issuer repays the amount borrowed is important to investors as it affects the level of credit risk faced by them. Credit risk is reduced if there are any provisions that call for periodic retirement of some of the principal amount outstanding during the term of the loan.

- A **bullet bond** is one that only makes periodic interest payments, with the entire principal amount paid back at maturity.

- An **amortizing bond** is one that makes periodic interest and principal payments over the term of the bond. A bond may be fully or partially amortized until maturity:
    - A fully amortized bond is one whose outstanding principal amount at maturity is reduced to zero through a fixed periodic payment schedule.
    - A partially amortized bond also makes fixed periodic principal repayments, but the principal is not fully repaid by the maturity date. Therefore, a balloon payment is required at maturity to repay the outstanding principal amount.

Exhibit 3-1 illustrates the payment schedules for a bullet bond, a fully amortized bond, and a partially amortized bond. All the bonds discussed here carry a par value of $1,000 and a coupon rate of 8%. They mature in 5 years and make annual coupon payments. The market interest rate has been assumed to be constant at 8%. Finally, the balloon payment at maturity for the partially amortized bond is assumed to be $200.

Exhibit 3-1: Payment Schedules for Bullet, Fully Amortized, and Partially Amortized Bonds

Bullet Bond

Year	Investor Cash Flows ($)	Interest Payment ($)	Principal Repayment ($)	Outstanding Principal at the End of the Year ($)
0	−1,000.00			1,000.00
1	80.00	80.00	0.00	1,000.00
2	80.00	80.00	0.00	1,000.00
3	80.00	80.00	0.00	1,000.00
4	80.00	80.00	0.00	1,000.00
5	1,080.00	80.00	1,000.00	0.00

- Coupon payment = 0.08 × 1,000 = $80
- Last payment = Coupon + Bullet par repayment = 80 + 1,000 = $1,080

Fully Amortized Bond

Year	Investor Cash Flows ($)	Interest Payment ($)	Principal Repayment ($)	Outstanding Principal at the End of the Year ($)
0	−1,000.00			1,000.00
1	250.46	80.00	170.46	829.54
2	250.46	66.36	184.09	645.45
3	250.46	51.64	198.82	446.63
4	250.46	35.73	214.73	231.90
5	250.46	18.55	231.90	0.00

- The annual payment is constant so it can be viewed as an annuity, and calculated as:
    - N = 5; PV = $1,000; I/Y =8; FV = 0; CPT PMT; Annual PMT = $250.46
- Interest component of Year 2 payment = 0.08 × 829.54 = $66.36

- Principal repayment component of Year 2 payment = 250.46 – 66.36 = $184.10
- Notice that the annual payment is constant, but over time the interest payment decreases (as the outstanding principal amount decreases each year) and the principal repayment increases.

Partially Amortized Bond

Year	Investor Cash Flows ($)	Interest Payment ($)	Principal Repayment ($)	Outstanding Principal at the End of the Year ($)
0	–1,000.00			1,000.00
1	216.37	80.00	136.37	863.63
2	216.37	69.09	147.27	716.36
3	216.37	57.31	159.06	557.30
4	216.37	44.58	171.78	385.52
5	*416.37	30.84	385.52	0.00

**416.37 = 200 (balloon payment) + 216.37*

The examples in this reading were created in Microsoft Excel. Numbers may differ from the results obtained using a calculator because of rounding.

- This bond can be viewed as a combination of (1) a 5-year annuity and (2) a balloon payment at maturity. The sum of the present values of these two elements is equal to the bond price of $1,000.
- The PV of bullet payment is calculated as:
  - N = 5; FV = $200; I/Y = 8; CPT PV; PV = $136.12
- Therefore, PV of 5-year annuity is calculated as:
  - 1,000 – 136.12 = $863.88
- The annuity payment can be calculated as:
  - N = 5; PV = $863.88; I/Y = 8; FV = 0; CPT PMT; Annual PMT = $216.37
- Interest component of Year 2 payment = 0.08 × 863.63 = $69.09
- Principal repayment component of Year 2 payment = 216.37 – 69.09 = $147.27
- Notice that the annual payment is constant, but over time the interest payment decreases and the principal repayment increases.
- Since the principal amount is not fully amortized, interest payments are higher for the partially amortized bond than for the fully amortized bond except for Year 1 (when they are equal).

Sinking Fund Arrangements

A **sinking fund arrangement** requires the issuer to repay a specified portion of the principal amount every year throughout the bond's life or after a specified date. For example, a $10 million issue with a term of 10 years could require the issuer to redeem bonds worth $2 million par each year starting Year 6 of the issue. Another variation of the sinking fund arrangement operates by requiring the issuer to repay a steadily increasing amount of the bond principal every year.

Callable bonds are discussed in detail later in the reading and in another reading. Reinvestment risk is discussed in detail in a later reading.

Sometimes a call provision may also be added to the bond issue. This call provision usually gives the issuer the option to repurchase bonds before maturity at the lowest of (1) market price, (2) par, and (3) a specified sinking fund price. The issuer is generally allowed to repurchase only a small portion of the bond issue, but it may sometimes make use of a doubling option (if available) to repurchase double the required number of bonds.

From the bondholders' perspective, the advantage of a sinking fund arrangement is that it reduces credit risk (principal is received over the bond's term as opposed to in a bullet payment at maturity). However, it entails two disadvantages. First, it results in reinvestment risk (i.e., the risk that investors will have to reinvest the redeemed principal at an interest rate lower than the current yield to maturity). Second, if the issue has an embedded call option, the issuer may be able to repurchase bonds at a price lower than the current market price, resulting in bondholders losing out.

Coupon Payment Structures

A coupon refers to the interest payment made by the issuer to the bondholder. Various coupon-payment structures are described below:

Floating-Rate Notes (FRN)

The coupon rate of a **FRN** has two components: a reference rate (such as LIBOR) plus a spread (also known as margin).

- The **spread** is typically fixed and expressed in basis points (bps). A basis point equals 0.01% so there are 100 bps in 1%. The spread on a FRN is determined at issuance and is based on the issuer's credit rating at issuance. The higher the issuer's creditworthiness, the lower the spread.
- The **reference rate** resets periodically based on market conditions. As the reference rate changes, the effective coupon rate on the FRN also changes.

Generally speaking, FRNs make coupon payments quarterly. So for example, consider a FRN that makes coupon payments based on 90-day LIBOR plus 20 bps (= LIBOR 90 + 0.20%). Generally speaking, current 3-month LIBOR (LIBOR-90) would be different at each coupon determination date, so this is how the coupon rate on a FRN resets periodically to reflect current market interest rates.

Occasionally, the spread on a FRN may not be fixed, in which case the bond is known as a **variable-rate note**.

FRNs have less interest rate risk (i.e., the risk of bond price volatility resulting from changes in market interest rates) than fixed-rate bonds. This is because the coupon rate on a FRN is reset periodically and brought in line with current market interest rates (LIBOR). If market interest rates rise (fall), the periodic coupon on a FRN increases (decreases). Therefore, FRNs are preferred by investors who expect interest rates to increase. However, note that FRN investors still face credit risk. If there is a decline in the perceived credit quality of the issuer (such that the fixed spread no longer adequately compensates investors for credit risk), the price of the FRN will fall below par value.

FRNs may be structured to include a floor or a cap on the periodic coupon rate.

- A **floor** prevents the periodic coupon rate on the FRN from falling below a pre-specified minimum rate, so it benefits investors (when market interest rates fall).
- A **cap** prevents the periodic coupon rate on the FRN from rising above a pre-specified maximum rate, so it benefits the issuer (when market interest rates rise).

Note that a FRN may also be structured to have both a cap and a floor, in which, it is known as a **collared FRN**.

Finally, FRNs may also be structured such that the periodic coupon rate is inversely related to the reference rate (i.e., if the reference rate increases [decreases], the periodic coupon rate decreases [increases]). Such FRNs are known as **reverse FRNs** or **inverse floaters** and are preferred by investors who expect interest rates to decline.

Step-Up Coupon Bonds

Bear with me here. We will discuss the call feature embedded in callable bonds in detail later in the reading.

A **step-up coupon bond** (which can be fixed or floating) is one where the periodic coupon rate increases by specified margins at specified dates. Typically, the step-up coupon structure is offered with callable bonds to protect bondholders in a rising interest rate environment. If interest rates rise, it becomes increasingly unlikely that the issuer will call the bond, so the step-up feature at least allows investors to benefit from higher coupons. On the other hand, when interest rates are stable or declining, the step-up feature incentivizes issuers to call the bonds before the coupon rate steps up (increases).

However, note that despite the step-up in coupons (and the consequent increase in interest expense), the issuer may not call the bond if refinancing is less advantageous. For example, the issuer would be reluctant to refinance using the proceeds of a new bond issue if its creditworthiness has declined, such that the coupon rate on the new issue would be higher than the stepped-up coupon on the original issue. Also note that although the issuer is not required to call the bond when the coupon rate on a step-up bond increases, there is an implicit expectation from investors that it will. Failure to call the bond may be viewed negatively by market participants.

Credit-Linked Coupon Bonds

The coupon rate on **credit-linked coupon bonds** changes when the bond's credit rating changes. For example, a bond's coupon rate may be structured to increase (decrease) by a specified margin for every credit rating downgrade (upgrade) below (above) the bond's credit rating at issuance.

Credit-linked coupon bonds protect investors against a decline in the credit quality, and are therefore attractive to investors who are concerned about the future creditworthiness of the issuer. They also provide some protection against poor economic conditions, as credit ratings tend to decline during recessions. Notice that a problem with credit-linked coupon bonds is that since a rating downgrade results in higher interest payments for the issuer, it can contribute to further downgrades or even an eventual default.

Payment-in-Kind (PIK) Coupon Bonds

PIK coupon bonds allow the issuer to pay interest in the form of additional bonds instead of cash. They are preferred by issuers that are financially distressed and fear liquidity and solvency problems in the future. Investors usually demand a higher yield on these bonds to compensate them for the higher credit risk and high leverage of the issuer.

One variation of PIK coupon arrangements allow issuers to make interest payments in the form of common shares worth the amount of coupon due. A **PIK toggle note** gives the issuer the option to pay interest in cash, in kind, or a combination of the two.

Deferred Coupon Bonds (or Split Coupon Bonds)

These bonds do not pay any coupon for the first few years after issuance, but then pay a higher coupon than they normally would for the remainder of their terms. Deferred coupon bonds are usually preferred by issuers who want to conserve cash in the short run, or for project financing where cash-generation will commence after an initial development phase.

Investors are attracted to deferred coupon bonds as they are usually priced significantly below par. Further, the deferred coupon structure may help investors manage their tax liability by delaying taxes due on interest income. Note that this tax advantage is only available in certain jurisdictions.

A zero-coupon bond can be viewed as an extreme form of a deferred coupon bond. Zero-coupon bonds do not make any interest payments until maturity and are issued at a deep discount to par. Essentially, all interest payments are deferred until maturity.

Index-Linked Bonds

These are bonds whose coupon payments and/or principal repayments are linked to a specified index (such as a commodity or equity index). An example of index-linked bonds are **inflation-linked bonds** (also known as **linkers**) whose coupon and/or principal payments are linked to an index of consumer prices (e.g., Treasury Inflation Protection Securities or TIPS issued by the U.S. Government are linked to the Consumer Price Index, CPI, in the United States). Investors are attracted to inflation-linked bonds because they offer a long-term asset with a fixed **real** return that is protected against inflation risk.

National governments issue the largest amounts of inflation-linked bonds (even though they are now increasingly being offered by corporate issuers). Sovereign inflation-linked bond issuers can be grouped into three categories:

1. Countries (such as Brazil and Chile) that issue inflation-linked bonds because they are experiencing high rates of inflation and linking payments to inflation was the only option available to ensure sufficient investor appetite for the issue.
2. Countries (such as Australia and Sweden) that issue inflation-linked bonds to add credibility to the government's commitment to disinflationary policies, and to take advantage of investor demand for securities immune from inflation risk.
3. Countries (such as the United States and Canada) that are concerned about the social welfare benefits of inflation-linked securities.

The cash flows of an index-linked bond may be linked to a specified index via its interest payments, principal payments, or both.

- **Zero-coupon-indexed bonds** do not pay any coupon so only the principal repayment is linked to a specified index.
- For **interest-indexed bonds,** only coupon payments are adjusted to changes in the specified index. They repay the fixed nominal principal at maturity.
- **Capital-indexed bonds** pay a fixed coupon rate, but this rate is applied to a principal amount that is adjusted to reflect changes in the specified index. As a result, both interest payments as well as the principal repayment are adjusted for inflation.
- **Indexed-annuity bonds** are fully amortized bonds (i.e., annuity payments include both payment of interest and partial repayment of principal). The annual payment on these bonds is linked to the specified index so effectively, both interest and principal payments reflect changes in the index.

Interest-indexed bonds and capital-indexed bonds are both nonamortizing bonds.

Index-linked bonds may also be linked to an equity index. An example of such a bond is an **equity-linked note (ELN),** which is a fixed-income security whose final payment is linked to the return on an equity index. ELNs are generally principal-protected (i.e., the investor is guaranteed repayment of 100% of principal at maturity even if the value of the index has fallen since issuance). If the underlying index increases in value, investors receive an amount greater than par upon maturity. However, note that the principal payment is still subject to credit risk of the issuer, so if the issuer defaults, the investor may not receive anything even if the underlying index has increased in value.

LESSON 4: BONDS WITH CONTINGENCY PROVISIONS

LOS 51f: Describe contingency provisions affecting the timing and/or nature of cash flows of fixed-income securities and identify whether such provisions benefit the borrower or the lender. Vol 5, pp 327–333

A **contingency provision** allows for some action given the occurrence of a specified event in the future. Common contingency provisions found in a bond's indenture come under the heading of **embedded options**.

Callable Bonds

Callable bonds give the **issuer** the right to redeem (or call) all or part of the bond before maturity. This embedded option offers the issuer the ability to take advantage of (1) a decline in market interest rates and/or (2) an improvement in its creditworthiness. If interest rates decline and/or the issuer's credit rating improves, the issuer would call the outstanding issue and replace this old (expensive to pay interest on) issue with a new issue that carries a lower interest rate.

For example, assume that the market interest rate was 6% at the time of issuance and the company issues bonds carrying a coupon rate of 7% with the 100 bps spread reflecting the credit risk of the issuer. Now suppose that the market interest rate falls to 5% and the issuer's credit rating improves such that a spread of only 50 bps is appropriate to compensate investors for credit risk.

- If these bonds were callable, the issuer would call/redeem them and refinance the issue by issuing new bonds carrying a coupon rate of 5% + 0.5% = 5.5% (in line with current market interest rates and the issuer's current credit risk).
- If the bonds were not callable, the company would have continued paying a 7% coupon and would not have been able to take advantage of the decline in market interest rates and the improvement in its own creditworthiness.

While callable bonds benefit the issuer (the call option holds value to the issuer), they expose investors to a higher level of reinvestment risk than noncallable bonds. If bonds are called, bondholders would have to reinvest proceeds at the new (lower) interest rates. To compensate investors for granting the option to call the bond to the issuer and accepting potentially higher reinvestment risk, callable bonds offer a higher yield to investors and sell at lower prices compared to noncallable bonds.

From the perspective of the bondholder, she would pay less for a callable bond than for an otherwise identical noncallable bond. The difference in the value of a noncallable bond and an otherwise identical callable bond is the value of the embedded call option (which the bond holder has effectively written or sold to the issuer).

Value of callable bond = Value of noncallable bond – Value of embedded call option

Value of embedded call option = Value of noncallable bond – Value of callable bond

From the perspective of the issuer, it would have to pay more (in the form of a higher coupon or higher yield) to get investors to purchase a callable bond than an otherwise identical noncallable bond. The difference between the yield on a callable bond and the yield on a noncallable bond is the cost of the embedded option to the issuer.

Yield on callable bond = Yield on noncallable bond + Embedded call option cost in terms of yield

Embedded call option cost in terms of yield = Yield on callable bond – Yield on noncallable bond

Note that the more heavily the embedded call option favors the issuer, the lower the value of the callable bond to the investor and the higher the yield that must be offered by the issuer.

The bond indenture usually specifies the following details about the call provision:

- Call price, which is the price paid to bondholders when the bond is called.
- Call premium, which is the excess over par paid by the issuer to call the bond.
- Call schedule, which specifies the dates and prices at which the bond may be called.

Some callable bonds are issued with an initial **call protection period** (also known as **lockout period**, **cushion**, or **deferment period**), during which the issuer is prohibited from calling the bond. The bond can only be called at or after the specified call date.

A **make-whole provision** in a callable bond usually requires the issuer to pay a relatively high lump-sum amount to call the bonds. This amount is based on the present value of the remaining coupon and principal payments (that are not paid due to the bond's early redemption), based on a discount rate that adds a pre-specified spread to the YTM of an appropriate sovereign bond. The redemption value calculated in this manner generally tends to be much higher than the bond's current market price. The point of including this provision is to make the deal appear more attractive to investors as it allows for them to be handsomely compensated if the bond is called. Practically speaking however, issuers rarely invoke this provision as it is very costly. See Example 4-1.

Example 4-1: Callable bonds

A hypothetical $1,000 par 20-year bond is issued on January 21, 2013 at a price of 98.515. The issuer can call the bond in whole or in part every January 21, from 2019. Call prices at different call dates are listed below:

Year	Call Price ($)
2019	102.000
2020	101.655
2021	101.371
2022	101.095
2023	100.824
2023	100.548
2024	100.273
2025 and thereafter	100.000

1. What is the length of the call protection period?
2. What is the call premium (per bond) for the 2022 call date?
3. What type of exercise style does this callable bond illustrate?

Solution:

1. The bonds were issued in 2013 and are first callable in 2019. Therefore, the call protection period is 2019 – 2013 = 6 years.
2. Call prices are stated as a percentage of par so the call price in 2026 is $1,010.95 (= 101.095% × 1,000). The call premium is the amount paid above par by the issuer. Therefore, the call premium in 2022 is $10.95 (= 1,010.95 – 1,000).
3. The bond is callable every January 21, from 2019 until maturity. Since it is callable on specified dates following the initial call protection period the embedded option is a Bermuda call.

The following exercise styles are available for callable bonds:

- American calls (also known as continuously callable) can be called by the issuer at any time starting on the first call date.
- European calls can only be called by the issuer on the call date.
- Bermuda-style calls can be called by the issuer on specified dates following the call protection period. These dates usually coincide with coupon payment dates.

Putable Bonds

Putable bonds give **bondholders** the right to sell (or put) the bond back to the issuer at a pre-determined price on specified dates. The embedded put option offers bondholders protection against an increase in interest rates (i.e., if interest rates increase [decreasing the value of the bond], they can sell the bond back to the issuer at a pre-specified price and then reinvest the principal at [higher] newer interest rates).

From the perspective of the bondholder, she would pay more for a putable bond than for an otherwise identical nonputable bond. The difference in the value between a putable bond and an otherwise identical nonputable bond is the value of the embedded put option (which the bond holder has effectively purchased from the issuer).

Value of putable bond = Value of nonputable bond + Value of embedded put option

Value of embedded put option = Value of putable bond − Value of nonputable bond

From the perspective of the issuer, it would pay out less (in the form of a lower coupon or lower yield) on a putable bond than it would on an otherwise identical nonputable bond. The difference between the yield on a nonputable bond and the yield on a putable bond is the cost of the embedded option borne by the investor.

Yield on putable bond = Yield on nonputable bond − Embedded put option cost in terms of yield

Embedded put option cost in terms of yield = Yield on nonputable bond − Yield on putable bond

Note that the more heavily the embedded put option favors the investor, the higher the value of the putable bond to the investor and the lower the yield that must be offered by the issuer.

- Details regarding redemption dates and prices at which the bond can be sold back to the issuer are included in the bond's indenture.
- Putable bonds may give investors an opportunity to sell the bond back to the issuer once or multiple times.
    - Putable bonds that only give a single sellback opportunity are called one-time put bonds, while those that give multiple opportunities are called multiple put bonds.
    - Multiple put bonds are generally more expensive (have higher value) than one-time put bonds as they offer bondholders more flexibility.
- The exercise styles used for putable bonds are similar to those used for callable bonds.

Convertible Bonds

A **convertible bond** gives the bondholder the right to convert the bond into a pre-specified number of common shares of the issuer. Therefore, it can be viewed as a combination of a straight bond and an embedded call option on the issuer's stock. Convertible bonds may also include call or put provisions.

Convertible bonds are attractive to investors as the conversion (to equity) option allows them to benefit from price appreciation of the issuer's stock. On the other hand, if there is a decline in the issuer's share price (which causes a decline in the value of the embedded equity conversion/call option), the price of the convertible bond cannot fall below the price of an otherwise identical straight bond. Because of these attractive features, convertible bonds offer a lower yield and sell at higher prices than similar bonds without the conversion option. Note however, that the coupon rate offered on convertible bonds is usually higher than the dividend yield on the underlying equity.

Convertible bonds hold advantages for the issuer, as yields on convertible bonds are lower than yields on otherwise identical bonds without the conversion option. Further, if the conversion option is exercised, debt is eliminated. Note however, that this comes at the cost of dilution for existing shareholders.

Some terms relevant to the conversion provision are defined below:

- The conversion price is the price per share at which the convertible bond can be converted into shares.
- The conversion ratio refers to the number of common shares that each bond can be converted into. It is calculated as the par value divided by the conversion price.
    - If the par value is $1,000 and the conversion price is $20, then the conversion ratio is 50:1 or 50 (= 1,000/20) common shares per bond.
- The conversion value is calculated as current share price multiplied by the conversion ratio.
    - If the current share price is $25 and the conversion ratio is 50:1, the conversion value is $1,250 (= 25 × 50).
- The conversion premium equals the difference between the convertible bond's price and the conversion value.
    - If the convertible bond's price is $1,300 and the conversion value is $1,250, the conversion premium equals $50.
- Conversion parity occurs if the conversion value equals the convertible bond's price.
    - Continuing with our example, if the stock price were currently $26 (instead of $25), the conversion value would be $1,300 (= 26 × 50), the same as the convertible bond's price. In this case, the conversion premium equals $0. If the common share is trading below $26, the condition is below parity, while if it is trading for more than $26, the condition is above parity.

It is important that you recognize that a callable convertible bond includes (1) an embedded conversion/call option (that favors the investor) on the issuer's stock and (2) an embedded call option (that favors the issuer) on the bond itself.

Although it is common for convertible bonds to reach conversion parity before they mature, bondholders rarely exercise the conversion option, choosing to retain their bonds and receive (higher) coupon payments instead of (lower) dividend payments. As a result, issuers often embed a call option alongside the conversion option in the convertible bond, making them **callable convertible bonds**. The reason behind this is that the call option may force investors to convert their bonds into common shares when the conversion value is higher than the call price. For this reason, callable convertible bonds sell at a lower price (or offer a higher yield) than noncallable convertible bonds.

Warrants

A **warrant** is somewhat similar to a conversion option, but it is not embedded in the bond's structure. It offers the holder the right to purchase the issuer's stock at a fixed exercise price until the expiration date. Warrants are attached to bond issues as sweeteners, allowing investors to participate in the upside from an increase in share prices.

Contingent Convertible Bonds ("CoCos")

CoCos are bonds with contingent write-down provisions. They differ from traditional convertible bonds in two ways:

1. Unlike traditional convertible bonds, which are convertible at the **option** of the bondholder, CoCos convert **automatically** upon the occurrence of a pre-specified event.
2. Unlike traditional convertible bonds, in which conversion occurs if the issuer's share price **increases** (i.e., on the upside), contingent write-down provisions are convertible on the **downside**.

To understand the application of CoCos, consider a bank that is required to maintain its core equity capital above a minimum level. To ensure that it continues to adhere to capital requirements in the event of significant losses, the bank could issue CoCos that are structured to automatically convert into equity if it suffers losses that reduce its equity capital below the minimum requirement. Since CoCos are set to convert automatically, they may force holders to take losses, which is why they offer investors a higher yield than otherwise similar bonds.

Reading 52: Fixed-Income Markets: Issuance, Trading, and Funding

LESSON 1: OVERVIEW OF GLOBAL FIXED-INCOME MARKETS

LOS 52a: Describe classifications of global fixed-income markets.
Vol 5, pp 346–354

CLASSIFICATION OF FIXED-INCOME MARKETS

Fixed-income markets may be classified in the following ways:

Type of Issuer

Bond markets can be classified based on the following three types of issuers:

1. The government and government-related sector, which includes:
    - Supranational (international) organizations (e.g., the World Bank).
    - Sovereign (national) governments (e.g., the United States).
    - Non-sovereign (local) governments (e.g., the state of Pennsylvania).
    - Quasi-government entities (e.g., rail services in many countries).
2. The corporate sector, which includes financial and nonfinancial companies.
3. The structured finance (or securitized) sector, which includes bonds that are created through the securitization process.

The types of securities associated with these sectors are described in more detail later in this reading and subsequent readings.

Credit Quality

Bond markets can also be classified based on the creditworthiness of the issuer, which is reflected in credit ratings issued by credit rating agencies. Bonds with a credit rating of Baa3 or above by Moody's, or BBB– or above by S&P and Fitch are classified as investment-grade bonds. On the other hand, bonds with ratings below these levels are classified as noninvestment grade (or high-yield, speculative, or junk) bonds. This distinction is helpful for institutional investors, as they may be prohibited from investing in noninvestment-grade bonds.

Maturity

Fixed-income securities may be classified based on their original (at issuance) maturity:

- Money market securities are fixed-income securities that have a maturity of 1 year or less at the time of issuance (e.g., T-bills, commercial paper, and negotiable certificates of deposit).
- Capital market securities are fixed-income securities that have a maturity of more than 1 year at the time of issuance.

Currency Denomination

Fixed-income securities can also be classified based on the currency in which they are issued. The currency in which a bond is issued determines which country's interest rates affect its price.

Geography

Bonds may be classified on the basis of where they are issued and traded.

- Bonds that are issued in a specific country, denominated in the currency of that country, and sold in that country are classified as:
  - Domestic bonds, if they are issued by entities incorporated in that country.
  - Foreign bonds, if the issuer is domiciled in another country.

  Domestic and foreign bonds are subject to the reporting, regulatory, and tax requirements of the country they are issued in.
- A Eurobond is issued internationally (i.e., outside the jurisdiction of the country whose currency it is denominated in). Eurobonds are subject to less reporting, regulatory, and tax constraints than domestic and foreign bonds. Eurobonds are also attractive for issuers as they offer access to a larger number of global investors.

Investors also often distinguish between emerging bond markets and developed bond markets. Although emerging bond markets are much smaller, trading volumes and values have risen sharply in recent years due to increased demand from investors seeking diversification (believing that investment returns across markets are not closely correlated). Generally speaking, emerging market bonds offer higher yields than developed market bonds.

Other Classifications of Fixed-Income Markets

Fixed-income markets may also be classified based on specific characteristics of the securities. For example:

- Inflation-linked bonds offer investors protection against inflation by linking the coupon payment and/or the principal repayment to a consumer price index.
- Tax-exempt bonds, such as municipal bonds (or munis) in the United States are attractive for investors who are subject to income tax because interest income from these bonds is usually exempt from federal income tax and from state income tax (subject to some restrictions). Coupon rates on tax-exempt bonds are typically lower than those on otherwise identical taxable bonds.

Type of Coupon

Bond markets may be classified based on the type of coupon. The coupon rate for a bond (1) may be fixed throughout its term (e.g., plain-vanilla bond), or (2) may change periodically based on some reference rate (e.g., floating-rate notes).

LOS 52b: Describe the use of interbank offered rates as reference rates in floating-rate debt. **Vol 5, pp 350–351**

Reference Rates

The coupon rate of a floating-rate note (FRN) has two components: (1) a reference rate, plus (2) a spread (or margin).

1. The spread is usually constant and is set at the time of bond issuance. It is based on the issuer's creditworthiness (i.e., the higher the credit quality of the issuer, the lower the spread).

2. The reference rate resets periodically, so the coupon rate is brought in line with market interest rates each time the reference rate is reset. The reference rate is the primary driver of the bond's coupon rate.

The reference rate used for a particular floating-rate bond issue depends on where the bonds are issued and the currency denomination. An example of a reference rate is the London interbank offered rate (Libor), which is the rate at which banks can borrow unsecured funds from each other in the London interbank market for different currencies and different borrowing periods. Libor is used as the reference rate for most floating-rate bonds issued in the Eurobond market. For example, an FRN denominated in GBP that pays coupon semiannualy would typically calculate the coupon as 6-month Libor (or LIBOR-180) plus a spread.

Fixed-Income Indices

Fixed-income indices are generally used by investors (1) to describe a bond market or sector, and (2) to evaluate investment performance. Fixed-income indices usually consist of portfolios of securities reflecting a specific bond market or sector and can be price-or value-weighted.

Investors in Fixed-Income Securities

Investors in fixed-income securities include the following:

- Central banks: Central banks purchase and sell sovereign bonds issued by the national government to conduct open market operations in implementing monetary policy. They may also trade in bonds denominated in foreign currencies to manage the currency's exchange rate and the country's foreign currency reserves.
- Institutional investors: These include pension funds, hedge funds, insurance companies, foundations, endowments, banks, and sovereign wealth funds.
- Retail investors: Retail investors invest in fixed-income securities to take advantage of their relatively stable prices and steady income streams.

> While central banks and institutional investors typically invest directly in fixed-income securities, retail investors tend to invest indirectly through mutual funds and ETFs.

LESSON 2: PRIMARY AND SECONDARY BOND MARKETS

LOS 52c: Describe mechanisms available for issuing bonds in primary markets. Vol 5, pp 357–363

Primary bond markets are those in which issuers sell new bonds to investors to raise capital. On the other hand, **secondary bond markets** are those in which existing bonds are traded among investors.

Primary Bond Markets

Mechanisms for issuing bonds in primary markets differ based on the type of issuer and the type of bond issued. Bonds may be issued through a **public offering** or a **private placement**.

Public Offerings

In a **public offering** (or **public offer**), any member of the public may purchase the bonds. Bond issuers are usually assisted by **investment banks**, which provide a wide range of financial services. Mechanisms for issuing bonds in the primary market include the following:

Underwritten offerings (also referred to as a firm commitment offering): The bond issuer negotiates an offering price with the investment bank, which then guarantees the sale of the issue at that price. The risk associated with selling the bonds is therefore borne by the investment bank (also known as the underwriter). The underwriting process involves the following phases:

- Determining the funding needs of the issuer.
- Selecting the underwriter (typically an investment bank) to market and sell bonds.
- Structuring the transaction (i.e., determining the maturity date, currency denomination, expected coupon rate, and expected offering price).
- Preparing and submitting required regulatory filings, appointing a trustee, and launching the offering (typically via a press release).
- Assessing market conditions by holding discussions with anchor buyers and analyzing the grey market.
    - Anchor buyers are large institutional investors.
    - The grey market (also known as the when-issued market) is basically a forward market for bonds that are about to be issued. Trading in this market helps underwriters determine the final offer price.
- Pricing the issue accordingly to ensure that it is neither undersubscribed nor significantly oversubscribed.
- Setting the final issue price on the pricing day, which is the last day for investors to commit to purchasing bonds.
- Issuing the bond: The underwriter purchases the bond issue from the issuer, delivers the proceeds to investors, and also starts reselling the bonds through its sales network.

Best efforts offering: The investment bank only acts as a broker and tries its best to sell the bond at the negotiated offering price for a commission. The investment bank bears less risk and has less of an incentive to sell in a best efforts offering compared to an underwritten offering.

The auction process has been described in detail in the Economics section.

Auctions: Bonds are sold to investors through a bidding process, which helps in price discovery and allocation of securities. U.S. Treasuries are sold via auctions, where primary dealers are the major counterparties to the New York Fed as it conducts monetary policy through open market operations. Individuals account for a very small proportion of direct purchases of Treasury securities.

Shelf registration: Shelf registration allows certain (authorized) issuers to offer additional bonds to the general public without having to prepare a new and separate prospectus for each bond issue. Instead, there is a single prospectus, which can be used for multiple, undefined future offerings over several years. Each issue, however, must be accompanied by an announcement document describing changes to the issuer's financial condition (if any) since the filing of the master prospectus.

Since shelf issuances are subject to lower levels of scrutiny than standard public offerings, only well-established issuers with proven financial strength can make use of this facility. Further, in some jurisdictions, shelf registrations can be purchased only by qualified investors.

Private Placements

In a **private placement**, only a select group of qualified investors (typically large institutional investors) are allowed to invest in the issue.

- The bonds are neither underwritten nor registered, and can be relatively illiquid as there is usually no secondary market to trade them.
- Investors are usually able to influence the terms of the issue, so privately placed bonds typically have more customized and restrictive covenants compared to publicly issued bonds.

LOS 52d: Describe secondary markets for bonds. Vol 5, pp 358–361

Secondary markets (or **aftermarkets**) are those in which existing bonds are traded among investors. Secondary markets may be structured in the following two ways:

1. Organized exchanges: These are places where buyers and sellers can meet to arrange their trades. Buyers and sellers may come from anywhere, but transactions must be executed at the exchange in accordance with the rules and regulations of the exchange.
2. Over-the-counter (OTC) markets: In these markets, buyers and sellers submit their orders from various locations through electronic trading platforms. Orders are then matched and executed through a communications network.

The vast majority of bond trading occurs in OTC markets. Dealers quote prices at which they are willing to buy (known as the **bid**) and sell (known as the **ask**) from/to customers. The **bid-ask spread** is a commonly used measure of liquidity in the market. A bid-ask spread of 10 to 12 bps is considered reasonable.

Liquidity is defined as the ability to make trades quickly at a price close to the security's fair market value.

Settlement for government and quasi-government bonds usually occurs on a T+1 basis (one day after the transaction date), whereas settlement of corporate bonds usually occurs on a T+3 basis (three days after the transaction date). For money market trades, cash settlement (same day settlement) is also common.

LESSON 3: ISSUERS OF BONDS

LOS 52e: Describe securities issued by sovereign governments. Vol 5, pp 363–364

Sovereign Bonds

Sovereign bonds are bonds that are issued by a country's central government (or its treasury). They are issued primarily to cover expenditures when tax revenues are insufficient.

- Sovereign bonds are backed by the taxing authority of the national government.
- Sovereign bonds can be issued in the sovereign's local (domestic) currency or in a foreign currency.
    - Bonds issued in local currency typically carry a higher credit rating than those issued in a foreign currency. While both bonds are backed by the sovereign's taxing power, local currency bonds can also be serviced by printing local currency, whereas foreign currency bonds can only be serviced through foreign currency reserves (which are finite). If the sovereign prints more local currency hoping to exchange it for foreign currency (to service

foreign currency bonds), financial markets would typically be quick to recognize this, resulting in a depreciation of the local currency over time.

Secondary market trading of sovereign bonds is primarily in securities that were most recently issued (known as **on-the-run securities**). The latest sovereign bond issue for a given maturity is also referred to as a benchmark issue because it serves as a benchmark for otherwise identical bonds (in terms of maturity, coupon type and frequency, and currency denomination) issued by another type of issuer (e.g., non-sovereign, corporate). Generally speaking, as a sovereign issue ages (or becomes more seasoned) it tends to trade less frequently.

There are several types of sovereign bonds issued in the market, including fixed-rate bonds, floating-rate bonds, and inflation-linked bonds.

LOS 52f: Describe securities issued by non-sovereign governments, quasi-government entities, and supranational agencies.

Non-Sovereign Government Bonds

Non-sovereign bonds are those issued by levels of government that lie below the national level (e.g., provinces, regions, states, and cities). They are typically issued to finance public projects, such as schools and bridges. These bonds can be serviced through the following sources of income:

- The taxing authority of the local government.
- The cash flows from the project that is being financed with bond proceeds.
- Special taxes and fees established specifically for making interest payments and principal repayments.

Non-sovereign bonds are discussed in more detail in a later reading.

Generally speaking, non-sovereign bonds are of high credit quality, but they still trade at higher yields (lower prices) than sovereign bonds. The lower the credit quality and the liquidity of a non-sovereign bond relative to a sovereign bond, the greater the additional yield.

Quasi-Government Bonds

Quasi-government or **agency bonds** are issued by organizations that perform various functions for the national government, but are not actual governmental entities. These bonds are issued to fund specific financing needs. Examples of quasi-government entities include government-sponsored enterprises (GSEs) in the United States, such as Fannie Mae (that provides mortgage financing) and Sallie Mae (that provides student loans).

- Quasi-government bonds may be guaranteed by the national government, in which case they receive higher ratings and trade at lower yields than similar bonds not carrying the government guarantee.
- Generally speaking, quasi-government entities do not have direct taxing authority, so bonds are serviced with cash flows generated by the entity or from the project that is being financed by the issue.
- In some cases, quasi-government bonds may be backed by collateral.
- Historical default rates on quasi-government bonds have been extremely low.

Supranational Bonds

These bonds are issued by **supranational** (or **multilateral**) **agencies** such as the World Bank (WB) and the International Monetary Fund (IMF). Generally speaking, supranational bonds are issued as plain-vanilla bonds (though floating-rate bonds and callable bonds are also issued). They are typically highly rated and issued in large sizes (so they tend to be very liquid).

LOS 52g: Describe types of debt issued by corporations. Vol 5, pp 371–378

Corporate Debt

Bank Loans: Bilateral and Syndicated Loans

- There are two types of bank loans:
    1. A **bilateral loan** is a loan from a single bank.
    2. A **syndicated loan** is a loan from a group of lenders, or syndicate. Syndicated loans are often securitized and then sold in secondary markets to investors. Securitized instruments are discussed in detail in a later reading.
- Most bank loans are floating-rate loans, with Libor, a sovereign rate (e.g., the T-bill rate), or the prime rate serving as the reference rate.
- Bank loans can be customized (with respect to maturity and payment structure) to borrower requirements.
- Access to bank loans depends on (1) the company's financial position, and (2) market conditions and capital availability.
    1. Bank loans are the primary source of debt financing for small and medium-size companies, as well as for large companies in countries where bond markets are underdeveloped.
    2. For highly rated companies in developed markets, bank loans tend to be more expensive than issuing bonds.

Commercial Paper

- Commercial paper is an unsecured debt instrument that is popular among issuers because it is a source of flexible, readily available, and relatively low-cost financing.
- It can be used to meet seasonal demands for cash and is also commonly used to provide **bridge financing** (i.e., interim financing until long-term financing can be arranged).
- Commercial paper is usually "rolled over" by issuers. This means that companies obtain the funds to pay off maturing paper by issuing more commercial paper. In order to safeguard against **rollover risk** (the risk that the company will not be able to issue new commercial paper to replace maturing paper), issuers often maintain access to **backup lines of credit** (also referred to as **liquidity enhancement** or **backup liquidity lines**). Rollover risk may arise due to (1) market-wide events (e.g., the "freezing" of debt markets during the 2008 financial crisis) or (2) company-specific events (e.g., significant deterioration in a company's financial position may lead to a sharp increase in the required yield on new paper or to the new issue not being fully subscribed).

> Backup lines of credit typically contain a material adverse change provision which allows the bank to cancel the credit line if the financial position of the issuer deteriorates significantly.

- Terms to maturity can range from overnight to one year.
    - In the United States, because debt securities with terms that do not exceed 270 days are exempt from registration with the SEC, commercial paper is typically issued with a term of 270 days or less.
- Historically, defaults on commercial paper have been relatively rare because commercial paper has a short maturity and tends to be rolled over.
    - Each time an issuer tries to roll over paper, investors can reassess the issuer's financial position, and not subscribe to the issue if they perceive an increase in credit risk.

  - Corporate managers are wary of defaulting on issued paper because they do not want to risk losing access to such a flexible source of financing going forward.
- Most investors hold on to commercial paper until maturity, which results in very little secondary market trading in these instruments.
- Yields on commercial paper are higher than yields on short-term sovereign bonds of the same maturity because:
  - Investors in commercial paper face credit risk, while most highly rated sovereign bonds are risk-free.
  - Commercial paper markets are generally less liquid than short-term sovereign bond markets.
- In the United States, yields on commercial paper are higher than yields on municipal bonds because interest income from commercial paper is taxable, while income from municipal bonds is tax exempt.
- U.S. commercial paper (USCP) is issued as a pure discount instrument (the security is issued at a discount to par and pays the par amount at maturity, with the difference between the two being the interest paid). Euro commercial paper (ECP) is quoted on an add-on yield basis, where the interest amount is paid in addition to the par amount at maturity.

Corporate Notes and Bonds

- Corporate bonds can differ based on several characteristics. They can differ based on:
  - Coupon payment structures (e.g., fixed-rate vs. floating-rate).
  - Principal payment structures.
    - Bonds with a serial maturity structure have maturity dates that are spread out over the bond's life. A stated number of bonds mature and are paid off each year until final maturity.
    - Bonds with a term maturity structure are paid off in one lump sum payment at maturity.
    - The difference between (1) bonds with a serial maturity structure and (2) those with a term maturity structure combined with a sinking fund provision is that with a serial maturity structure bondholders know exactly which bonds will be repaid each year, while for issues with a sinking fund arrangement bonds that can be redeemed each year are designated by a random drawing.
  - Terms to maturity. Bonds with terms to maturity less than 5 years are known as short-term bonds, those with terms ranging between 5 and 12 years are known as intermediate-term bonds, and those with terms longer than 12 years are known as long-term bonds.
  - Asset or collateral backing (i.e., secured vs. unsecured).
  - Contingency provisions (e.g., embedded call, put, and conversion options).

Medium-Term Notes (MTNs)

- Medium-term notes (MTNs) are not necessarily intermediate-term securities. They are known as "medium-term" notes because when they were initially issued, they were meant to fill the funding gap between (short-term) commercial paper and long-term bonds. The MTN market can be broken down into three segments:
  1. Short-term securities that may be fixed-rate or floating-rate.
  2. Medium-to long-term securities that generally tend to be fixed-rate.

3. Structured notes, which are essentially notes combined with derivative instruments to create special features desired by certain institutional investors.

- MTNs are unique in that they are offered continuously to investors by the agent of the issuer.
  - First, the issuer provides the agent with an indication of the range of maturities that it wishes to borrow for, and also specifies the yield it is willing to offer on each maturity.
  - Investors then get in touch with the agent and provide details of the amount and maturity that they are interested in.
  - The agent then confirms the issuer's willingness to issue the desired notes and executes the transaction.
- Primary issuers of MTNs tend to be financial institutions, while pension funds, life insurance companies, and banks are among the largest buyers.
- MTNs can be customized/structured to meet investor requirements. While their customized features result in limited liquidity, MTNs offer higher yields than otherwise identical publicly traded bonds.

LESSON 4: SHORT-TERM FUNDING ALTERNATIVES AVAILABLE TO BANKS

LOS 52h: Describe short-term funding alternatives available to banks.
Vol 5, pp 382–387

Retail deposits: These include funds deposited at the bank by individual and commercial depositors into their accounts. Retail deposit accounts include:

- Checking accounts, which typically pay no interest but provide customers with transaction services and immediate access to funds.
- Money market accounts, which pay money market rates of return and offer access to funds with little or no notice.
- Savings accounts, which pay interest but are less liquid.

Short-term wholesale funds: These include:

- Central bank funds: Recall (from the Economics section) that banks in many countries are required to place a reserve balance at the central bank. The central bank funds market (the Fed funds market in the United States) is the market for reserves where banks with excess reserves are able to lend them out, and banks short of required reserves are able to borrow them. The interest rate on these borrowings is known as the central bank funds rate (Fed funds rate in the United States), and is influenced by (1) demand and supply for reserves and (2) the central bank's open market operations.
- Interbank funds: The interbank market is the market for loans and deposits between banks. Maturities can range from overnight to one year, and deposits are unsecured. At times of stress (e.g., the 2008 financial crisis), liquidity in the interbank market can dry up significantly.
- Certificates of deposit (CDs): A certificate of deposit (CD) is issued by a bank to a client when she deposits a specified sum of money for a specified maturity and interest rate.

- A non-negotiable CD is one in which the deposit and interest are paid to the initial depositor at maturity. An early withdrawal penalty is imposed if the depositor withdraws funds before the maturity date.
- A negotiable CD provides the depositor with the option to sell the CD in the open market if she wishes to liquidate it before maturity.
    - Negotiable CDs can be classified as large-denomination CDs and small-denomination CDs. Large-denomination CDs (issued in denominations of $1 million or more in the United States) are an important source of funding for banks, while small-denomination CDs are not as important.
- CDs with long-term maturities are known as **term CDs.**
- CDs are available in domestic bond markets and in the Eurobond market.
- Yields on CDs are driven by the creditworthiness of the issuing bank and (to a lesser extent) the term to maturity.

LOS 52i: Describe repurchase agreements (repos) the risks associated with them. Vol 5, pp 385–387

Repurchase and Reverse Repurchase Agreements

A **repurchase agreement** is an arrangement between two parties, where one party sells a security to the other with a commitment to buy it back at a later date for a predetermined higher price. The difference between the (lower) selling price and the (higher) repurchase price is the interest cost of the loan. Effectively, what is happening is that the seller is borrowing funds from the buyer and putting up the security as collateral.

- The annualized interest cost of the loan is called the repo rate.
- A repurchase agreement for one day is known as an overnight repo, and an agreement for a longer period is known as a term repo.
- Repo rates are usually lower than the rates that a broker or bank would charge on a margin loan.

For example, consider a $1,000 par, 4% annual-pay bond that is currently trading at $980. Party A sells this bond to Party B today for $950 with a commitment to repurchase it for $960 (repurchase price) after 60 days (repurchase date).

- The implicit interest rate for the 60-day period is calculated as $960/950 - 1 = 1.053\%$.
- The repo rate would be 1.053% for 60 days expressed as an effective annual rate.
- The percentage difference between the market value of the security and the amount of the loan is known as the repo margin or haircut. In this example, the margin equals $950/980 - 1 = -3.061\%$. The margin serves to protect the lender against a decline in the value of the collateral over the term of the repo.
- Note that interest is paid on the repurchase date, which is the date of termination of the agreement.
- Any coupon income received from the bond during the repo term belongs to the seller (borrower).
- Both parties in the repo face counterparty credit risk.
    - Suppose that the buyer (lender) is unable to deliver the collateral on the repurchase date. The seller (borrower) faces the risk that the value of the collateral has risen over the term of the repo. The seller will then be left with an amount of cash that is lower than the current market value of the security.

  - Now suppose that the seller (borrower) is unable to repurchase the collateral on the repurchase date. The buyer (lender) faces the risk that the value of the collateral (plus any income owed to the seller) has fallen over the term of the repo to a level lower than the unpaid repurchase price.
- When looking at things from the perspective of Party A (the seller or borrower), the transaction would be referred to as a repo.
- When looking at things from the perspective of Party B (the buyer or lender), the transaction would be referred to as a reverse repo.
  - Reverse repos are used to borrow securities to cover short positions. The buyer in the repo transaction immediately sells the security obtained from the seller in the repo on the open market. On the settlement date of the repo, the buyer acquires the relevant security from the open market to deliver it back to the seller. The buyer in the repo (short position on the asset) has positioned herself to profit from a decline in value of the security between the date of inception of the repo and its settlement date.
- Standard practice is to define the transaction based on the perspective of the dealer.
  - If the dealer is borrowing cash and providing collateral, the transaction is termed a repurchase agreement.
  - If the dealer is lending cash and accepting collateral, the transaction is termed a reverse repurchase agreement.

The repo rate depends on the following factors:

- The risk associated with the collateral. Repo rates increase with the level of credit risk in the collateral.
- The term of the repurchase agreement. A longer term typically entails higher repo rates.
- The delivery requirement for the collateral. Repo rates are lower when the collateral must be delivered to the lender.
- The supply and demand conditions of the collateral. The more scarce a particular piece of collateral, the lower the repo rate. This is because the borrower has an asset that lenders of cash may want for specific reasons (perhaps to short the security). Such collateral is said to be "on special" (versus "general collateral").
- The interest rates on alternative sources of financing in the money market. If rates for borrowing from other sources are higher, repo rates will also tend to be higher.

The repo margin is a function of the following factors:

- The length of the repurchase agreement. The longer the term, the higher the repo margin.
- The quality of the collateral. The higher the quality of the collateral, the lower the repo margin.
- The credit quality of the counterparty. The higher the creditworthiness of the counterparty, the lower the repo margin.
- The supply and demand conditions of the collateral. The higher the demand or the lower the supply of the collateral, the lower the repo margin.

READING 53: INTRODUCTION TO FIXED-INCOME VALUATION

LESSON 1: BOND PRICES AND THE TIME VALUE OF MONEY

LOS 53a: Calculate a bond's price given a market discount rate.
Vol 5, pp 398–402

LOS 53b: Identify the relationships among a bond's price, coupon rate, maturity, and market discount rate (yield-to-maturity). **Vol 5, pp 398–407**

Bond Pricing with a Market Discount Rate

The value or price of a bond is computed as the present value of expected future cash flows from the bond. For a plain-vanilla fixed-rate bond, the cash flows are composed of periodic coupon interest payments and principal repayment at maturity. The discount rate used to compute the present value of those cash flows is the **market discount rate** (also known as the **required yield** or the **required rate of return**), which represents the rate of return required by investors to compensate them for the perceived riskiness of the bond.

Let us use an example of a 4-year, 10% annual coupon bond with a par value of $1,000 to illustrate (1) how the value of a bond is computed and (2) some very important relationships regarding bond prices.

Scenario A:

Let's start with assuming that market discount rate for this bond equals the coupon rate offered (10%). The price of the bond can be calculated as:

FV = –$1,000; PMT = –$100; N = 4; I/Y = 10; CPT PV; PV = $1,000

$$\frac{100}{(1+0.1)^1}+\frac{100}{(1+0.1)^2}+\frac{100}{(1+0.1)^3}+\frac{100+1,000}{(1+0.1)^4}=\$1,000$$

Note that the Year 4 payment includes the final coupon payment and principal repayment.

- If the coupon rate offered on the bond equals the rate of return required by investors to compensate them for the risk inherent in the instrument, the bond will sell for its par value.

Scenario B:

Now let's tweak the assumption that the market discount rate equals 10%. Let's assume instead that the bond is less risky so investors only demand a 9% rate of return on the investment. In this case, the price of the bond can be calculated as:

FV = –$1,000; PMT = –$100; N = 4; I/Y = 9; CPT PV; PV = $1,032.40

$$\frac{100}{(1+0.09)^1}+\frac{100}{(1+0.09)^2}+\frac{100}{(1+0.09)^3}+\frac{100+1,000}{(1+0.09)^4}=\$1,032.40$$

- Notice that if the rate of return required by investors is lower (9% versus 10%) due to lower perceived risk, the value or price of the bond is higher ($1,032.40 versus $1,000).
- If the coupon rate offered on the bond is higher than the required yield, the bond will sell for a **premium**.
- Stated differently, the compensation offered to investors (the coupon) for bearing various risks inherent in the instrument is higher than the compensation required by investors for purchasing the bond (the required yield). Therefore, investors would be willing to purchase the bond at a premium to par.

Scenario C:

Finally, let's assume that the bond is actually more risky (than in Scenario A) so investors demand an 11% rate of return on the investment. In this case, the price of the bond can be calculated as:

FV = –$1,000; PMT = –$100; N = 4; I/Y = 11; CPT PV; PV = $968.98

$$\frac{100}{(1+0.11)^1}+\frac{100}{(1+0.11)^2}+\frac{100}{(1+0.11)^3}+\frac{100+1,000}{(1+0.11)^4}=\$968.98$$

- Notice that if the rate of return required by investors is higher (11% versus 10%) due to greater perceived risk, the value or price of the bond is lower ($968.98 versus $1,000).
- If the coupon rate offered on the bond is lower than the required yield, the bond will sell for a **discount**.
- Stated differently, the compensation offered to investors (the coupon) for taking on various risks inherent in the instrument is lower than the compensation required by investors for purchasing the bond (the required yield). Therefore, investors would only be willing to purchase the bond at a discount to par.

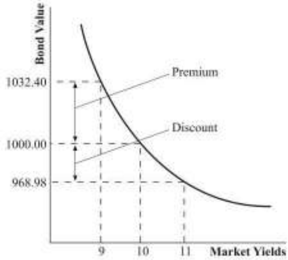
The prices calculated under these different interest rate scenarios allow us to reach the following conclusions:

- The higher the discount rate, the lower the present value of each individual cash flow, and the lower the value of the fixed income security.
- The lower the discount rate, the higher the present value of each individual cash flow, and the higher the value of the fixed income security.

Plotting the relationship between the discount rate and value of a fixed-income instrument on a graph will result in a convex curve (see Figure 1-1). This curve is known as the **price-yield profile** of a fixed income security.

Important: Notice that the *increase* in the price of the bond (from $1,000 to $1,032.40 or 3.24%) if the discount rate *decreases* by 100 basis points (bps) (from 10% to 9% in Scenario B) is *greater* than the *decrease* in the price of the bond (from $1,000 to $968.98 or 3.102%) if the discount rate *increases* by 100 bps (from 10% to 11% in Scenario C). This is a very important takeaway when it comes to bond pricing and is known as the **convexity effect.**

Figure 1-1: Price-Yield Profile



The bond that we have worked with so far (in Scenarios A, B, and C) has paid out coupon annually. While this is the norm in Europe, most Asian and North American bonds make semi-annual coupon payments. To illustrate the valuation of a bond with semi-annual coupon payments, let's start with the assumptions in Scenario B ($1,000 par, 4-year bond with a 10% coupon rate and a yield-to-maturity of 9%) but assume semi-annual instead of annual coupon payments. There will be 8 (= 4 × 2) semi-annual coupon payments on this bond and each coupon payment with be worth $50 (= 10%/2 × $1,000). Given a semiannual yield-to-maturity of 4.5% (= 9% ÷ 2), the value of the bond today can be computed as:

FV = –$1,000; PMT = –$50; N = 8; I/Y = 4.5; CPT PV; PV = $1,032.94

Notice that the value of the bond here is slightly greater than the value of the bond in Scenario B ($1,032.94 versus $1,032.40). Even though the total amount of coupon income is the same ($400), the more frequent coupon payments imply a higher present value for the bond.

Yield-To-Maturity

If the market price of the bond is known, then given the term of the bond, the coupon rate, and coupon payment frequency, we can compute its **yield-to-maturity or YTM** (also known as **redemption yield** or **yield to redemption**). The yield-to-maturity is the (uniform) interest rate that equates the sum of the present values of the bond's expected future cash flows (when discount at that rate) to its current price. It also represents the internal rate of return on the bond's cash flows given three critical assumptions:

1. The investor holds on to the bond until maturity.
2. The issuer makes all promised payments on time in their full amount.
3. The investor is able to reinvest all coupon payments received during the term of the bond at the stated yield-to-maturity until the bond's maturity date.
   See Examples 1-1 and 1-2.

Example 1-1: Computing the YTM for Semiannual-Pay Coupon Bonds

Compute the YTM of a 10-year, $1,000 par bond with an 8% coupon rate that makes semiannual coupon payments given that its current price is $925.

Solution:

PV = –$925; N = 10 × 2 = 20; PMT = $40; FV = $1,000; CPT I/Y; I/Y = 4.581%

This calculated yield of 4.581% is the yield per semiannual period or semiannual discount rate. By convention, the YTM on a semiannual coupon bond is expressed as an annualized yield by multiplying the semiannual discount rate by two. Therefore, the annualized YTM here is 4.581% × 2 = **9.16%**

Important: When calculating the value of a bond using your financial calculator, make sure that PMT and FV have the same sign, and that PV has the opposite sign. If you invest in a bond you incur an outflow up front, but you get back the coupon and principal. Alternatively, if you issue a bond you have an inflow up front, but you pay out coupon and principal.

Example 1-2: Computing the YTM for Annual-Pay Coupon Bonds

Compute the YTM for a 10-year, $1,000 par bond that pays an 8% annual coupon given that its current price is $925.

Solution:

PV = –$925; N = 10; PMT = $80; FV = $1,000; CPT I/Y; I/Y = 9.178%

This bond's yield-to-maturity equals **9.178%**.

Relationships between the Bond Price and Bond Characteristics

1. A bond's price is inversely related to the market discount rate (the inverse effect). When the discount rate increases (decreases) the price of the bond decreases (increases).
2. Given the same coupon rate and term to maturity, the percentage price change is greater in terms of absolute magnitude when the discount rate decreases than when it increases (the convexity effect).
3. For the same term to maturity, a lower coupon bond is more sensitive to changes in the market discount rate than a higher coupon bond (the coupon effect).
4. Generally speaking, for the same coupon rate, a longer term bond is more sensitive to changes in the market discount rate than a shorter term bond (the maturity effect). Note that while the maturity effect always holds for zero-coupon bonds and for bonds priced at par or premium to par, it does not always hold for long-term low coupon (but not zero-coupon) bonds that are trading at a discount.

Relationship between Price and Maturity

Now let's examine how a bond's value changes as it nears maturity. Let's calculate the values of the $1,000 par, 4-year 10% annual coupon bond as it nears maturity assuming that:

1. It was issued at a premium when market yields were 9%; and
2. It was issued at a discount when market interest rates were 11%.

1. If market interest rates remain at 9% over the term of the bond, the value of the premium bond as it nears maturity is given in the following table:

Time since Issuance (yrs)	Time to Maturity (yrs)	Price (PV of Remaining CF)
0	4	$1,032.40
1	3	$1,025.31
2	2	$1,017.59
3	1	$1,009.17
4	0	$1,000.00

2. If market interest rates remain at 11% over the bond's term, the value of the discount bond as it nears maturity is given in the following table:

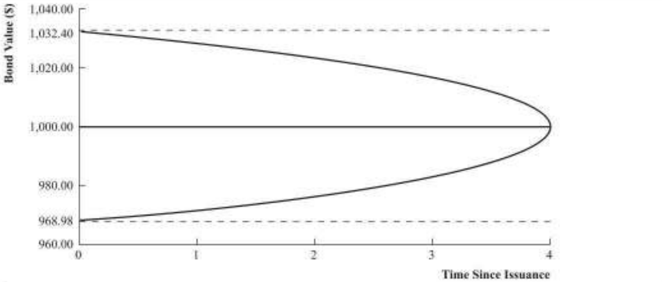
Time since Issuance (yrs)	Time to Maturity (yrs)	Price (PV of Remaining CF)
0	4	$968.98
1	3	$975.56
2	2	$982.87
3	1	$991.00
4	0	$1,000.00

From the calculations in the tables above we can conclude that if the yield remains constant:

- A *premium* bond's value *decreases* toward par as it nears maturity.
- A *discount* bond's value *increases* toward par as it nears maturity.
- A *par* bond's value remains *unchanged* as it nears maturity.

At maturity, a bond's value must equal its par value because it is the par value that is returned to investors at maturity. Therefore, over its term, a bond's value is "pulled to par" as illustrated in Figure 1-2.

Figure 1-2: Bond Value Pulled to Par



The constant-yield price trajectory shows how the price of a premium/discount code is pulled toward par as it nears maturity.

LOS 53c: Define spot rates and calculate the price of a bond using spot rates. Vol 5, pp 407–409

Pricing Bonds with Spot Rates

Under traditional methods of bond valuation, expected future cash flows are discounted at a uniform rate that reflects the risks inherent in the investment (i.e., the yield-to-maturity). The **arbitrage-free valuation approach** does not use the same rate to discount each cash flow, but uses the relevant spot rate to discount each cash flow that occurs at a different point in time. A **spot rate** (or **zero rate**) is the yield on a zero-coupon bond for a given maturity. For example, the yield on a 5-year zero coupon bond equals the 5-year spot rate and it is the rate that is used under the arbitrage-free valuation approach to discount the cash flow received from a bond at the end of Year 5.

For our purposes, there is no need to get into how spot rates are determined. Just keep in mind that the yield-to-maturity discounts each of the bond's expected cash flows at a uniform rate (i.e., the yield-to-maturity) to determine its price, while under arbitrage-free valuation, the individual discount rate that corresponds to the individual cash flow date (i.e., the spot rate) is used to determine the present value of that particular payment. Generally speaking, long-term spot rates tend to be higher than short-term spot rates as there is more risk associated with cash flows that are expected to be received further out into the future. The general formula for calculating the price of a bond given a series of spot rate is provided below:

$$PV = \frac{PMT}{(1+Z_1)^1} + \frac{PMT}{(1+Z_2)^2} + \ldots + \frac{PMT + FV}{(1+Z_N)^N}$$

where:
z_1 = Spot rate for Period 1
z_2 = Spot rate for Period 2
z_N = Spot rate for Period N

Example 1-3 illustrates how both approaches result in the same value for a particular bond.

Example 1-3: Valuing Bonds Using YTMs and Spot Rates

Suppose a $1,000 par, annual-pay bond matures in 3 years and has a coupon rate of 8%. The bond is currently selling for $1,001.34 with at a yield-to-maturity of 7.948%. The relevant spot rates are given below:

Maturity	Yield on Zero-Coupon Bond (Spot Rate)
1 year	7%
2 years	7.5%
3 years	8%

Verify that the value of the bond computed by discounting each cash flow at the relevant spot rate is the same as the value calculated by discounting all cash flows at the yield-to-maturity.

Solution:

If we discount each cash flow at the relevant spot rate, the value of the bond will equal:

$$\frac{\$80}{(1+0.07)^1}+\frac{\$80}{(1+0.075)^2}+\frac{\$1{,}080}{(1+0.08)^3}$$
$$=\$74.77+\$69.23+\$857.34=\$1{,}001.34$$

If we discount every cash flow at the yield-to-maturity, the price of the bond will equal:

N = 3; PMT = –\$80; I/Y = 7.948; FV = –\$1,000; CPT PV; PV → \$1,001.34

LESSON 2: PRICES AND YIELDS (PART I): CONVENTIONS FOR QUOTES AND CALCULATIONS

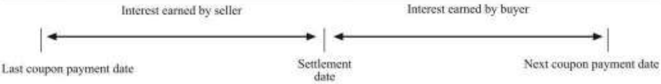
LOS 53d: Describe and calculate the flat price, accrued interest, and the full price of a bond. Vol 5, pp 409–413

Flat Price, Accrued Interest, and the Full Price

When a coupon bond is sold between coupon payment dates, we must account for the interest or coupon that the seller has earned, but not yet received. The amount that an investor/buyer should pay for a bond equals the present value of future cash flows, but the very next cash flow (the next coupon payment) includes two components:

1. Interest earned by the seller, which has been accrued from the last payment date till the transaction date. This is called **accrued interest**.
2. Interest earned by the buyer. See Figure 2-1.

Figure 2-1: Valuing a Bond between Coupon-Payment Dates



When the price of the bond is determined by calculating the present value of future cash flows as of the settlement date, the computed value is known as the **full price** (also known as the **invoice** or **dirty price**). This price includes accrued interest and reflects the amount that the buyer pays the seller. From this full price, the accrued interest is deducted to determine the **flat price** (also known as the **clean price** or **quoted price**) of the bond.

$$PV^{Full}=PV^{Flat}+AI$$

Dealers usually quote flat prices, and then if a deal is executed the accrued interest is added to the flat price to determine the full price that must be paid by the buyer to the seller on the settlement date. The reason behind quoting flat prices is to avoid misleading investors about the price trend for a bond. Given that the amount of interest accrued on the bond increases each day, full prices rise each day even if the yield-to-maturity remains constant.

Further, full prices suddenly drop at coupon payment dates when coupon is paid and accrued interest falls to zero. Quoting flat prices avoids such misrepresentation.

Also note that it is the flat price that is pulled toward par as the bond nears maturity (as shown earlier in Figure 2). The accrued interest is the part of the full price that does not depend on the yield-to-maturity. Therefore, we can say that it is the flat price that is affected by changes in the market discount rate.

Accrued interest is the seller's proportional share of the next coupon payment. It is calculated as:

$$AI = t / T \times PMT$$

where:
AI = Accrued interest.
t = Number of days from the last coupon payment to the settlement date.
T = Number of days from the last coupon payment date to the next coupon payment date.
PMT = Coupon payment.

The manner in which the number of days between two dates is calculated in the bond market depends on the day-count convention used in the particular sector of the market.

- For government bonds, the actual/actual day-count convention is usually applied. The actual number of days is used including weekends, holidays, and weekdays.
    - For example, if the coupon payment dates for a semiannual-pay bond are May 15 and November 15, the accrued interest for settlement on June 27 would be calculated as 43 (the actual number of days from May 15 to June 27) divided by 184 (the number of days from May 15 to November 15), times the coupon payment.
- For corporate bonds, the 30/360 day-count convention is often used.
    - For example, if the coupon payment dates for a semiannual-pay bond are May 15 and November 15, the accrued interest for settlement on June 27 would be calculated as 42 (= 30 + 12) divided by 180 (= 30 × 6), times the coupon payment.

When it comes to valuing bonds between coupon payment dates, you need to understand that as time passes through the coupon period, the price of the bond increases (as the next coupon payment draws closer) at a rate equal to the discount rate per period. The full price of a fixed-rate bond between coupon payment dates can be computed as its value on the last coupon payment date multiplied by one plus the periodic discount rate (r) compounded over the period of time remaining that has elapsed since the last coupon payment (t/T):

$$PV^{Full} = PV \times (1+r)^{t/T}$$

where:
PV^{Full} = Full price of the bond (value between coupon payments)
PV = Price of bond at last/previous coupon payment date
r = Discount rate per period
t = Number of last coupon payment date to the settlement date
T = Number of days in coupon period
t/T = Fraction of coupon period that has gone by since last payment (See Example 2-1.)

Example 2-1: Computing Full Price, Accrued Interest, and Flat Price

A 5% U.S. corporate bond is priced for settlement on July 20, 2015. The bond makes semiannual coupon payments on April 21 and October 21 of each year and matures on October 21, 2021. The bond uses the 30/360 day-count convention for accrued interest.

Calculate the full price, the accrued interest, and the flat price per USD100 of par value for three stated annual yields-to-maturity: (A) 4.7%, (B) 5.00%, and (C) 5.30%.

Solution:

Given the 30/360 day-count convention, there are 89 days between the last coupon on April 21, 2015 and the settlement date on July 20, 2015 (9 days between April 21 and April 30, plus 60 days for the full months of May and June, plus 20 days in July). Therefore, the fraction of the coupon period that has gone by equals 89/180. As of the last coupon payment date, there are 6.5 years (and 13 semiannual periods) remaining until maturity.

A. Stated annual yield-to-maturity of 4.70%, or 2.35% per semiannual period:

The price as of the last coupon payment date is calculated as:

N = 13; PMT = –$2.50; FV = –$100; I/Y = 2.35%; CPT PV; PV = $101.6636

The full price on July 20 is calculated as:

$PV^{Full} = 101.6636 \times (1.0235)^{89/180} = \$102.838$

The accrued interest is calculated as:

$AI = 89/180 \times 2.50 = \$1.2361$

The flat price is calculated as:

$PV^{Flat} = 102.838 - 1.2361 = \$101.6019$

B. Stated annual yield-to-maturity of 5.00%, or 2.50% per semiannual period:

The price as of the last coupon payment date equals the bond's par value, which should be expected, as the coupon rate and the market discount rate are the same.

N = 13; PMT = – $2.50; FV = –$100; I/Y = 2.5%; CPT PV; PV = $100.00

The full price on July 20 is calculated as:

$PV^{Full} = 100.00 \times (1.025)^{89/180} = \$101.2284$

The accrued interest is calculated as:

$AI = 89/180 \times 2.5 = \$1.2361$

The flat price is calculated as:

$PV^{Flat} = 101.2284 - 1.2361 = \$99.9923$

Notice here that the flat price of the bond is lower than its par value, despite the fact that the coupon rate and the yield-to-maturity are equal. The reason for this is that the accrued interest computation does not consider the time value of money. Accrued interest is the interest earned by the seller of the bond for the time between the last coupon payment and the settlement date (in this case 1.2361 per 100 of par value). However, note that this interest income is expected to be received on the next coupon payment date so theoretically, the (true) accrued interest should actually be the present value of 1.2361 as of the settlement date. In practice however, the calculation of accrued interest neglects the time value of money. Therefore reported/computed accrued interest is slightly overstated, which results in a slightly understated flat price. Note that the full price is theoretically accurate as it is calculated as the sum of the present values of the future cash flows as of the settlement date.

C. Stated annual yield-to-maturity of 5.30%, or 2.65% per semiannual period:

The price as of the last coupon payment date is calculated as:

N = 13; PMT = –\$2.50; FV = –\$100; I/Y = 2.65%; CPT PV → PV = \$98.3685

The full price on July 20 is calculated as:

$PV^{Full} = 99.3685 \times (1.0265)^{89/180} = \$99.6488$

The accrued interest is calculated as:

$AI = 89/180 \times 2.5 = \$1.2361$

The flat price is calculated as:

$PV^{Flat} = 99.6488 - 1.2361 = \$98.4127$

Important:

- Notice that the accrued interest is the same in each case because it does not depend on the yield-to-maturity.
- Differences in the flat prices capture the differences in rates of return required by investors.

LOS 53e: Describe matrix pricing. Vol 5, pp 413–416

Matrix Pricing

Matrix pricing is a method used to estimate the market discount rate and price of bonds that are not actively traded. Essentially, prices of comparable (in terms of terms to maturity, coupon rates, and credit quality) are used to interpolate the price of the subject bond.

Example 2-2 illustrates the application of matrix pricing to compute the required yield and price of a bond that is not actively traded.

Example 2-2: Applying Matrix Pricing to Value a Bond

An analyst is trying to estimate the value of a relatively illiquid 6-year, 5% annual-pay coupon bond. She identifies two corporate bonds that have similar credit quality:

- Bond A is a 5-year, 6% annual-pay bond priced at 103.25 per 100 of par value.
- Bond B is an 8-year, 5% annual-pay bond priced at 97.75 per 100 of par value.

Estimate the price of the illiquid bond per 100 of par value.

Solution:

The first step is to determine the yields-to-maturity on the comparable bonds.

The required yield on the 5-year, 6% bond priced at 103.25 is calculated as:

N = 5; PMT = –$6; FV = –$100; PV = $103.25; CPT I/Y; I/Y = 5.244%

The required yield on the 8-year, 5% bond priced at 97.75 is calculated as:

N = 8; PMT = –$5; FV = –$100; PV = $97.75; CPT I/Y; I/Y = 5.353%

The market discount rate for the 6-year bond can be obtained using linear interpolation. The yield on the 6-year bond is estimated as the yield on the 5-year bond plus one-third of the difference between the yields on the 5-year and 8-year bonds.

$$0.05244 + \frac{(3-2)}{(5-2)} \times (0.05353 - 0.05244) = 0.0528$$

Given an estimated yield-to-maturity of 5.28%, the price of the illiquid 6-year, 5% annual-pay bond can be estimated as:

N = 6; PMT = –$5; FV = –$100; I/Y = 5.28%; CPT PV; PV = $98.59

If more than one bond is provided for a given maturity, then first calculate the average YTM for bonds with that maturity and then apply linear interpolation to estimate the yield for a maturity for which a comparable bond does not exist.

Matrix pricing can also be used when underwriting new bonds to estimate the required yield spread over the benchmark rate on the bonds to be issued.

Yield spreads and benchmarks are discussed in detail later in the reading.

- The **spread** refers to the difference between the yield-to-maturity on the new bond and the benchmark bond. It reflects the additional compensation offered by the issuer to compensate investors for the difference in credit risk, liquidity risk, and tax status of the bond relative to the benchmark.
- The **benchmark** is typically a government bond with a similar term to maturity.

Example 2-3 illustrates the use of matrix pricing to determine the required yield spread on a new bond issue.

Example 2-3: Application of Matrix Pricing to Estimate the Credit Spread on a New Issue

ABC Company is planning to issue a 5-year bond. The only other debt that the company currently has outstanding is a 2.5% annual-pay bond which is currently trading at $102.50 per 100 of par and has 4 full years to maturity (there is zero accrued interest built into its current price).

The following information is provided:

- Currently there are no outstanding government bonds with 4 years to maturity.
- The yield on a 3-year government bond is 0.85%, while the yield on a 5-year government bond is 1.55%.
- The term structure of credit spreads for bonds with the same credit rating as ABC Company suggests that 5-year spreads are around 37 bps higher than 4-year spreads.

Determine the expected market discount rate on the ABC's planned 5-year bond issue.

Solution:

The first step here is to compute the yield-to-maturity on the ABC's outstanding bonds that have 4 years remaining until maturity:

PV = $102.50; PMT = –$2.50; FV = –$100; N = 4; CPT I/Y; I/Y = 1.8459%

Next, we compute the estimated yield-to-maturity on a government bond with 4 years until maturity using the 3-year and 5-year government bond yields. This estimated yield on the 4-year government bond is then used to estimate the credit spread on ABC's 4-year bond.

Estimated 4-year government bond yield = (0.85% + 1.55%) / 2 = 1.20%

Estimated credit spread on ABC's 4-year bond = 1.8459% – 1.20% = 0.6459%

Then, we use the term structure of credit spreads for issues with a similar rating as the one in question to determine the expected spread on the 5-year bond ABC plans to issue:

Expected spread on 5-year bond = Estimated spread on 4-year bond + 0.0037
= 0.006459 + 0.0037 = 0.010159

Finally, we add the expected spread on ABC's planned 5-year issue to the 5-year government bond yield (the benchmark) to determine the expected market discount rate on the issue:

Expected discount rate on new issue = 0.0155 + 0.010159 = 0.025659 or 2.57%

The **term structure of credit spreads** shows the relationship between spreads offered on bonds with a particular credit rating over the "risk-free" yield on government bonds and terms-to-maturity.

LESSON 3: PRICES AND YIELDS (PART II): MATRIX PRICING AND YIELD MEASURES FOR BONDS

LOS 53f: Calculate and interpret yield measures for fixed-rate bonds, floating-rate notes, and money market instruments. Vol 5, pp 416–429

> We have already discussed the effective annual rate versus the stated annual rate in detail in the Quantitative Methods section. What we are doing here is using the periodicity (number of compounded periods per year) of the stated annual rate to determine the effective annual rate.

Yield Measures for Fixed-Rate Bonds

The **effective annual rate** (or **effective annual yield**) on a fixed-rate bond depends on the assumed number of periods in the year, which is known as the **periodicity of the stated annual rate** or **stated annual yield**.

Let's work with four bonds (an annual-pay bond, a semiannual-pay bond, a quarterly-pay bond, and a monthly-pay bond), each with a stated annual yield of 8% to illustrate the concept of periodicity and to highlight the difference between a stated annual yield and the effective annual yield.

An **annual-pay bond** would have a stated annual yield for periodicity of one. The stated annual yield would equal the discount rate per year.

- Effective annual yield = $(1 + 0.08)^1 - 1 = 0.08$ or 8%

A **semiannual-pay bond** would have a stated annual yield for periodicity of two. The rate per semiannual period would be computed as the stated annual yield divided by two.

- Rate per semiannual period = 0.08/2 = 0.04
- Effective annual yield = $(1 + 0.04)^2 - 1 = 0.0816$ or 8.16%

A **quarterly-pay bond** would have a stated annual yield for periodicity of four. The rate per quarter would be computed as the stated annual yield divided by four.

- Rate per quarter = 0.08/4 = 0.02
- Effective annual yield = $(1 + 0.02)^4 - 1 = 0.0824$ or 8.24%

A **monthly-pay bond** would have a stated annual yield for periodicity of 12. The rate per month would be computed as the stated annual yield divided by 12.

- Rate per month = 0.08/12 = 0.00667
- Effective annual yield = $(1 + 0.00667)^{12} - 1 = 0.083$ or 8.3%

Note that:

- The effective annual rate has a periodicity of one as it assumes only one compounding period in the year.
- Given the stated annual rate, as the number of compounding periods (periodicity) increases, the effective annual rate increases.
- Given the effective annual rate, as the number of compounding periods (periodicity) increases, the stated annual rate decreases. (This point will become clear in Example 3-1).

Important: What we refer to as stated annual rate (SAR) is referred to in the curriculum as **APR or annual percentage rate**. We stick to SAR to keep your focus on a stated annual rate versus the effective annual rate. Just remember that if you see an annual percentage rate on the exam, it refers to the stated annual rate.

In the United States, most bonds make semiannual coupon payments (periodicity equals two). The stated annual rate that has a periodicity of two is known as a **semiannual bond basis yield** or **semiannual bond equivalent yield**. The semiannual bond basis yield is calculated as the yield per semiannual period multiplied by two. To convert a stated annual rate (SAR) for M periods per year, SAR_M, to a stated annual rate for N periods per year, SAR_N, we can use the following formula:

$$\left(1+\frac{SAR_M}{M}\right)^M = \left(1+\frac{SAR_N}{N}\right)^N \quad \text{... Equation 1}$$

Example 3-1: Computing Stated Annual Yields for Different Compounding Frequencies

A 5-year, 6% semiannual-pay government bond is priced at 98 per 100 of par value. Calculate the annual yield-to-maturity stated on a semiannual bond basis (stated annual yield). Convert this stated annual yield to:

1. A stated annual yield based on annual compounding.
2. A stated annual yield based on quarterly compounding.
3. A stated annual yield based on monthly compounding.

Solution:

We first calculate the yield per semiannual period on the semiannual-pay bond.

N = 10; PMT = \$3; FV = \$100; PV = –\$98; CPT I/Y; I/Y = 3.237%

The calculated yield of 3.237% is the yield per semiannual period. This rate is multiplied by two to determine the stated annual yield or the semiannual bond basis yield:

Semiannual bond basis yield = 3.237% × 2 = 6.47%

1. To determine the stated annual yield based on annual compounding, we need to convert the stated annual yield of 6.47% from a periodicity of two to a periodicity of one.

$$\left(1+\frac{0.0647}{2}\right)^2 = \left(1+\frac{SAY_1}{1}\right)^1$$

$SAY_1 = 0.06579$ or 6.58%

The stated annual rate of 6.47% based on semiannual compounding compares to a stated annual rate of 6.58% compounded annually. Note that 6.58% (since it represents the yield based on annual compounding) is the effective annual yield on the bond.

2. To determine the stated annual yield based on quarterly compounding, we need to convert the stated annual yield of 6.47% from a periodicity of two to a periodicity of four.

$$\left(1+\frac{0.0647}{2}\right)^2=\left(1+\frac{SAY_4}{4}\right)^4$$

$SAY_4 = 0.06419$ or 6.42%

The stated annual rate of 6.47% based on semiannual compounding compares to a stated annual rate of 6.42% compounded quarterly. Notice that increasing the frequency of compounding has lowered the stated annual rate.

3. To determine the stated annual yield based on monthly compounding, we need to convert the stated annual yield of 6.47% from a periodicity of two to a periodicity of twelve.

$$\left(1+\frac{0.0647}{2}\right)^2=\left(1+\frac{SAY_{12}}{12}\right)^{12}$$

$SAY_4 = 0.06384$ or 6.38%

The stated annual rate of 6.47% based on semiannual compounding compares to a stated annual rate of 6.38% compounded monthly. Notice again that increasing the frequency of compounding has lowered the stated annual rate.

What we have effectively done in this example is computed the effective annual yield (in Part 1) and the stated annual yields (based on monthly, quarterly, and semiannual compounding) for the same bond. Notice that the effective annual yield of 6.58% (or the yield based on annual compounding) equals:

- 6.47% / 2 = 3.237% (the rate per semiannual period) compounded two times a year.
- 6.42% / 4 = 1.605% (the rate per quarter) compounded four times a year.
- 6.38% / 12 = 0.532% (the rate per month) compounded twelve times a year.

Now notice that given the effective annual rate, an increase in the number of compounding periods (periodicity) results in a lower stated annual rate. Stated differently, compounding more frequently at a lower stated annual rate corresponds to compounding less frequently at a higher stated annual rate.

Bond yields are typically quoted using the **street convention,** where the yield represents the internal rate of return on the bond's cash flows assuming all payments are made on scheduled dates regardless of whether any scheduled payment dates fall on weekends or holidays. Practically speaking, if a scheduled coupon payment date falls on the weekend or on a holiday, payment is actually made on the next business day. The **true yield** uses these actual payment dates to compute the IRR. The true yield can never be higher than the street convention yield as weekends and holidays only delay payments.

For corporate bonds, a **government equivalent yield** is sometimes quoted. This yield basically restates a yield-to-maturity based on a 30/360 day-count convention to one based on an actual/actual day-count convention. The government equivalent yield can be used to compute the spread offered on a corporate bond on top of the yield offered on a government bond (since both are based on the same, actual/actual day-count convention).

Another yield measure that is commonly seen in fixed-income markets is the **current yield** (also called the **income yield** or **interest yield**). It is calculated as the sum of coupon payments received over the year divided by the flat price.

$$\text{Current yield} = \frac{\text{Annual cash coupon payment}}{\text{Bond price}}$$

The current yield is a relatively crude measure of the rate of return to an investor because of the following reasons:

- It neglects the frequency of coupon payments in the numerator.
- It neglects any accrued interest in the denominator.
- It neglects any gains (losses) from purchasing the bond at a discount (premium) and redeeming it for par.

The **simple yield** on a bond is calculated as the sum of coupon payments received over the year plus (minus) straight line amortization of the gain (loss) from purchasing the bond at a discount (premium), divided by the flat price. See Example 3-2.

Example 3-2: Comparing the Stated YTM on Two Bonds

	Bond A	Bond B
Annual coupon rate	10%	13%
Coupon payment frequency	Semiannually	Quarterly
Years to maturity	3	3
Price (per 100 of par value)	98	105

1. Calculate the current yield and the stated annual yield-to-maturity for each of the two bonds.
2. The analyst believes that Bond B has a little more risk than Bond A. How much additional compensation, in terms of a higher yield-to-maturity, does Bond B offer relative to Bond A as compensation for additional risk?

Solution:

1. Current yield for Bond A:

   10/98 = 0.10204 or **10.204%**

   Yield-to-maturity for Bond A:

   N = 6; PMT = –\$5; FV = –\$100; PV = \$98; CPT I/Y; I/Y = 5.399%
   YTM = 5.399% × 2 = **10.798%**

Current yield for Bond B:

13 / 105 = 0.12381 or **12.381%**

Yield-to-maturity for Bond B:

N = 12; PMT = –\$3.25; FV = –\$100; PV = \$105; CPT I/Y → I/Y = 2.755%
YTM = 2.755% × 4 = **11.02%**

2. Bond A's yield-to-maturity is a stated annual rate with a periodicity of 2, while Bond B's yield-maturity is a stated annual rate with a periodicity of 4. Since the two yields are based on different periodicities, we cannot assert that the difference between the two, that is 22.2 bps (= 0.1102 – 0.10798) accurately reflects compensation for additional risk in the bond with the higher yield (Bond B).

   Converting the periodicity of Bond A stated annual yield from two to four, we obtain a stated annual yield of 10.656%.

   $(1 + 0.10798/2)^2 = (1 + SAY_4/4)^4$
   $SAY_4 = 10.656\%$

   This yield can be compared to the calculated yield-to-maturity on Bond B (with a periodicity of 4) of 11.02%. We can then say that the difference between the two, that is 36.4 bps (= 0.1102 – 0.10656) captures the additional compensation offered on Bond B for its greater risk **when yields are annualized for quarterly compounding**.

   Converting the periodicity of Bond B's stated annual yield from four to two, we obtain a stated annual yield of 11.172%.

   $(1 + 0.1102/4)^4 = (1 + SAY_2/2)^2$
   $SAY_2 = 11.172\%$

   This yield can be compared to the calculated yield-to-maturity on Bond A (with a periodicity of 2) 10.798%. We can then say that the difference between the two i.e., 37.4 bps (= 0.11172 – 0.10798) captures the additional compensation offered on Bond B for its greater risk **when yields are stated on a semiannual bond basis**.

For **callable bonds**, the yield realized by the investor will depend on whether and when (and at what call price) the bond is called. Investors compute the **yield-to-call** for each call date (based on the call price and the number of periods until the call date) and then determine the **yield-to-worst** as the worst or lowest yield among the yield-to-maturity and the various yields to call for the bond. See Example 3-3.

Example 3-3: Computing Yields-to-Call and Yield-to-Worst

Consider a 10-year, 8% annual-pay bond that is first callable in 7 years at 114 (per 100 of par value), then callable in 8 years at 105, and at par value on coupon payment dates thereafter. Given that the current price is 121 per 100 of par value, determine the yield-to-worst for the bond.

Solution:

To compute the yield-to-worst, we need to determine all the yields-to-call and the yield-to-maturity.

The yield-to-first-call in 7 years is calculated as:

N = 7; PMT = –\$8; FV = –\$114; PV = \$121; CPT I/Y; I/Y = 5.921%

The yield-to-second-call in 8 years is calculated as:

N = 8; PMT = –\$8; FV = –\$105; PV = \$121; CPT I/Y; I/Y = 5.239%

The yield-to-third-call in 9 years is calculated as:

N = 9; PMT = –\$8; FV = –\$100; PV = \$121; CPT I/Y; I/Y = 5.04%

The yield-to-maturity is calculated as:

N = 10; PMT = –\$8; FV = –\$100; PV = \$121; CPT I/Y; I/Y = 5.248%

Since yield-to-third-call is the lowest among the all the yields-to-call and the yield-to-maturity, it represents the yield-to-worst (5.04%).

While the yield-to-worst is commonly used by bond dealers and investors in fixed-rate callable bonds, a better way to evaluate investments in callable bonds is to use an **option pricing model** (which requires making an assumption regarding future interest rate volatility) to value the embedded call option. The value of the embedded call option is added to the flat price of the callable bond to determine what is known as the **option-adjusted price**. Recall that the option embedded in a callable bond favors the issuer, so it reduces the value of the bond from the investor's perspective. Therefore:

Value of callable bond = Value of non-callable bond – Value of embedded call option

Value of non-callable bond (option-adjusted price) = Flat price of callable bond + Value of embedded call option

The option-adjusted price is used to compute the **option-adjusted yield**. This yield is the required market discount rate where the price is adjusted to include the value of the embedded call option. The option adjusted yield will be lower than the yield-to-maturity on a callable bond because callable bonds offer higher yields than otherwise identical non-callable bonds to compensate investors for effectively selling the embedded call option to the issuer.

Yield Measures for Floating-Rate Notes

Interest payments on a **floating-rate note (FRN)** are not fixed. They fluctuate from period to period depending on the current level of the reference rate. The idea behind variable interest payments is to offer investors a security that has less market price risk than fixed-rate bonds when interest rates fluctuate. With fixed-rate bonds, changes in interest rates influence the market price as cash flows remain unchanged. On the other hand, with FRNs, future cash flows fluctuate with changes in interest rates.

An important thing to recognize regarding FRNs is that the effective coupon rate for a specified period is determined at the beginning of the period (calculated as the current level of the **reference rate** plus the **quoted margin**), but actually paid out at the end of the period. This payment structure is known as "in arrears."

> The most common day-count conventions for FRNs are actual/360 and actual/365.

It is important to understand the mechanics that determine the periodic coupon rate on a FRN. Assume that a FRN offers to pay interest every quarter based on 90-day LIBOR plus a quoted margin of 1%. The reference rate (LIBOR-90) used in calculating the effective coupon rate for the forthcoming 90-day period is prevailing 90-day LIBOR (at the beginning of the period). The actual payment of the coupon occurs at the end of the 90-day period (the next reset date).

For example, if LIBOR-90 on July 1, 2013 stands at 4%, and the quoted margin on a FRN is 100 bps, the issuer will make the coupon payment for the 3-month period from July 1 to September 30 at 5%. The issuer knows the coupon rate for the forthcoming 90 days at the beginning of the period (July 1), but actually makes the payment at the end of the period (September 30).

Important: When working with LIBOR, a lot of candidates forget to unannualize the coupon rate. If the effective coupon rate for a 90-day period on a loan of $1 million is 6%, the coupon payment for the period will be calculated as:

$$0.06 \times 90/360 \times \$1{,}000{,}000 = \$15{,}000$$

The **quoted margin** is the spread offered to investors on top of the reference rate to compensate them for the greater credit risk of the issuer relative to that implied by the reference rate. However, if the issuer enjoys a credit rating better than that implied by the reference rate, the issuer may be able to obtain a negative quoted margin on a FRN.

The adjustment of the periodic effective coupon rate on a FRN at every reset date in line with market interest rates (the current reference rate) results in the price of the instrument being "pulled toward par" as a reset date approaches. However, a FRN can still trade at a discount/premium if the quoted margin is different from the required margin. The **required margin** (also known as **discount margin**) refers to the yield spread above (or below) the reference rate such that the FRN trades at par at every reset rate. If the quoted margin equals the required margin, the FRN will trade at par at each reset date, but if the quoted margin is lower (greater) than the required margin, the FRN will trade at a discount (premium).

Changes in the required margin can be caused by changes in the issue's (1) credit risk, (2) liquidity, and/or (3) tax status. For example, assume that a FRN is issued at par with a coupon rate of LIBOR-90 plus 100 bps. Subsequently, if the issuer suffers a credit rating downgrade such that the required margin rises to 125 bps, the FRN will trade at a discount as the spread offered on top of the reference rate no longer compensates the investor adequately for the credit risk in the instrument.

> At issuance, the required margin typically equals the quoted margin so the FRN is issued at par.

Notice that fixed-rate and floating-rate bonds respond very similarly when it comes to credit risk.

- With fixed-rate bonds, a premium (discount) arises when the fixed coupon rate is greater (lower) than the required yield-to-maturity.
- For floating-rate bonds, a premium (discount) arises when the fixed quoted margin is greater (lower) than the required margin.

However, fixed- and floating-rate bonds respond very differently when it comes to changes in benchmark interest rates.

A simplified FRN pricing model calculates the value of a FRN at a reset date by estimating future cash flows and discounting them to the present.

- Each future coupon payment is assumed to be calculated based on the current level of the reference rate plus the quoted margin on the instrument.
- The discount rate applicable to each future payment is assumed to equal the current reference rate plus the required margin.

Example 3-4 illustrates this method.

Example 3-4: Pricing a FRN on a Reset Date

A $1,000 par quarterly-pay FRN has exactly one year to maturity. The reference rate is 90-day LIBOR and the bond carries a quoted margin of 80 bps. LIBOR-90 today (which happens to be a coupon-reset date) is 2.50%, while the current required margin on the FRN is 94 bps. Assuming a 30/360 day-count convention and evenly spaced periods, determine the current value of the FRN.

Solution:

In the simplified FRN valuation model, all future coupon payments are computed based on the current level of the reference rate (current LIBOR-90 equals 2.50%).

Coupon rate = (2.50% + 0.80%) × (90/360) = 0.825%

Coupon payment = 0.825% × $1,000 = $8.25

We value the bond based on a (constant) quarterly coupon of $8.25. The discount rate is calculated based on the current level of the reference rate (2.50%) plus the required margin (94 bps).

Discount rate per quarter = (2.50% + 0.94%) × (90/360) = 0.86%

Now we can compute the estimated value of the FRN as:

FV = –$1,000; PMT = –$8.25; N = 4; I/Y = 0.86; CPT PV; PV = $998.63

Notice that since the required margin is greater than the quoted margin, this FRN will trade at a discount.

The method illustrated in this example can also be used to estimate the required margin on a FRN given its current coupon rate and its current market price. However, note that this valuation model:

- Computes the flat price of the bond (since it does not account for any accrued interest).
- Applies the same, current level of the reference rate to compute (1) the coupon payment and (2) the discount rate in all future periods. More accurate (but complicated) FRN-pricing models use projected future reference rates to compute coupon payments, and spot rates to compute discount rates.

Yield Measures for Money Market Instruments

Money-market instruments are short-term debt securities with maturities ranging from one day (e.g., repos) to one year (e.g., bank certificates of deposit). Money market yields differ from yields in the bond market in the following three respects:

1. Bond yields-to-maturity are annualized and compounded. Money market yields are annualized but not compounded (they are stated on simple interest basis).
2. Bond yields-to-maturity can usually be calculated by applying standard time value of money analysis and using a financial calculator. Money market yields are often quoted in terms of nonstandard interest rates so users need to work with various pricing equations.
3. Bond yields-to-maturity are typically stated for a common periodicity for all terms-to-maturity. Money market instruments that have different times-to-maturity have different periodicities for the stated annual rate.

Generally speaking, money market yields are expressed on a **discount rate basis** or on an **add-on rate basis**, and can be based on a 360-day or 365-day year as you will see in the formulas that follow:

Pricing formula for money market instruments quoted on a discount rate basis:

$$PV = FV \times \left(1 - \frac{Days}{Year} \times DR\right) \quad \text{... Equation 2}$$

where:
PV = The PV or current price of the money market instrument.
FV = Future value, or the face value of the instrument at maturity.
Days = Number of days from settlement until maturity.
Year = Number of days in the year.
DR = Discount rate expressed as an annual percentage rate.

The equation above can be manipulated and stated in the following form (with DR as the subject):

$$DR = \left(\frac{Year}{Days}\right) \times \left(\frac{FV - PV}{FV}\right) \quad \text{... Equation 3}$$

Notice the following:

- The first term (Year/Days) is the periodicity of the annual rate.
- The numerator of the second term is the interest earned on the instrument.
- The interest earned is divided by the FV (which includes all earnings), not the PV (which represents the amount initially invested to purchase the instrument). Theoretically speaking, the discount rate should represent the return earned on the amount invested not on the total return at maturity. Therefore, a money market discount rate understates the rate of return to the investor and understates the cost of borrowing for the issuer (because PV is generally less than FV).

Pricing formula for money market instruments quoted on an add-on rate basis:

$$PV = \frac{FV}{\left(1 + \frac{Days}{Year} \times AOR\right)} \qquad \text{... Equation 4}$$

where:
PV = The PV, principal amount or price of the money market instrument.
FV = Future value, or the redemption amount paid at maturity including interest.
Days = Number of days from settlement until maturity.
Year = Number of days in the year.
AOR = Add-on rate, stated as an annual percentage rate.

The equation above can be manipulated and stated in the following form (with AOR as the subject):

$$AOR = \left(\frac{Year}{Days}\right) \times \left(\frac{FV - PV}{PV}\right) \qquad \text{... Equation 5}$$

Notice the following:

- The first term (Year/Days) is the periodicity of the annual rate.
- The second term is the interest earned divided by the **PV** of the investment. This makes the add-on rate a reasonable measure for the yield on a money market investment.

While market conventions differ across countries, commercial paper, U.S. T-bills, and bankers' acceptances are often quoted on a discount rate basis, and bank certificates of deposits, repos, LIBOR, and Euribor are quoted on an add-on rate basis. You should be able to notice from the equations above, that in addition to (1) differences in quoted money market rates (discount rate basis versus add-on rate basis), (2) differences in the assumed number of days in a year (360 versus 365), investment analysis for money market securities is made more difficult by the fact that the "amount" of a money market instrument quoted on a discount rate basis is typically the face value at maturity (FV), while the "amount" in an add-on rate quote is the price at issuance (PV). Therefore, in order to make money-market investment decisions, instruments must be compared on a common basis. Typically money-market yields are converted to a rate known as the **bond-equivalent yield** or **investment yield** for comparisons. The bond equivalent yield is a money-market rate stated on a **365-day year** on an **add-on rate basis**. See Example 3-5.

Example 3-5: Computing Bond Equivalent Yields

Consider the following quoted rates on four 180-day money market instruments.

Money Market Instrument	Quotation Basis	Assumed Number of Days in the Year	Quoted Rate
A	Discount Rate	360	5.15%
B	Discount Rate	365	5.30%
C	Add-On Rate	360	5.45%
D	Add-On Rate	365	5.50%

Calculate the bond equivalent yield for each instrument. Which instrument offers the investor the highest rate of return if assuming that credit risk is constant across all four?

Solution:

A. Bond A is quoted on a **discount rate** basis. We first use Equation 2 to determine PV, price per 100 of par:

$$PV = FV \times \left(1 - \frac{\text{Days}}{\text{Year}} \times DR\right)$$

$$PV = 1{,}000 \times \left(1 - \frac{180}{360} \times 0.0515\right) = 97.425$$

We then insert the PV of Bond A into Equation 5 to determine the add-on rate based on a 365-day year (i.e., the bond equivalent yield):

$$AOR = \left(\frac{\text{Year}}{\text{Days}}\right) \times \left(\frac{FV - PV}{PV}\right)$$

$$AOR = \left(\frac{365}{180}\right) \times \left(\frac{100 - 97.425}{97.425}\right) = 0.0536$$

Therefore, the bond equivalent yield for Bond A is 5.36%.

B. Bond B is also quoted on a **discount rate** basis (but assuming a 365-day year). We first use Equation 2 to determine PV, price per 100 of par:

$$PV = 100 \times \left(1 - \frac{180}{365} \times 0.053\right) = 97.386$$

We then insert the PV of Bond B into Equation 5 to determine the add-on rate based on a 365 day year (i.e., the bond equivalent yield):

$$AOR = \left(\frac{365}{180}\right) \times \left(\frac{100 - 97.386}{97.386}\right) = 0.0544$$

The bond equivalent yield for Bond B is 5.44%.

C. Bond C is quoted on **add-on rate** basis (assuming a 360-day year). In order to calculate the bond yield equivalent yield we first determine its **future value** (redemption amount) using Equation 4.

$$PV = \frac{FV}{\left(1 + \frac{Days}{Year} \times AOR\right)}$$

$$FV = 100 + \left(100 \times \frac{180}{360} \times 0.0545\right) = 102.725$$

We then use Equation 5 to determine the bond equivalent yield (based on a 365-day year).

$$AOR = \left(\frac{365}{180}\right) \times \left(\frac{102.725 - 100}{100}\right) = 0.05526$$

The bond equivalent yield for Bond C is 5.526%.

Another way to compute the bond equivalent yield for Bond C is to start with the AOR of 5.45%, which is based on a 360-day year. Notice that this rate has been computed (using Equation 5) as:

$$AOR = \left(\frac{360}{180}\right) \times \left(\frac{102.725 - 100}{100}\right) = 0.0545$$

The add-on rate based on a 360-day year only needs to be multiplied by a factor of 365/360 to get the 365-day AOR (i.e., the bond equivalent yield).

$$\frac{365}{360} \times 0.0545 = 0.05526$$

D. The quoted rate for Bond D of 5.5% is a bond equivalent yield (an add-on basis yield based on a 365-day year).

Now that we have brought the money market yields for all four instruments on an equal footing (computed their bond equivalent yields) we can make comparisons. If the risk of all four instruments is the same, Bond C offers the highest rate of return, with a bond equivalent yield of 5.526%.

Finally, suppose that an investor wants to convert money market rates to semiannual bond basis yields to compare them to yields on bonds that make semiannual coupon payments. Consider a 90-day money market instrument that offers a bond equivalent yield of 8%. This implies that the periodicity of the instrument is 365/90. We can use Equation 1 to convert a yield based on M = 365/90 to a stated annual yield based on N = 2:

$$\left(1 + \frac{0.08}{365/90}\right)^{365/90} = \left(1 + \frac{SAR_2}{2}\right)^2$$

$$SAR_2 = 0.080811$$

A stated annual rate of 8% for periodicity of 365/90 corresponds to a stated rate of 8.0811% for a periodicity of two. The difference between the two is 8.11 bps. Generally speaking, the lower the level of interest rates, the smaller the difference between the stated annual rates for any two periodicities.

LESSON 4: PRICES AND YIELDS (PART III): THE MATURITY STRUCTURE OF INTEREST RATES AND CALCULATING SPOT RATES AND FORWARD RATES

LOS 53g: Define and compare the spot curve, yield curve on coupon bonds, par curve, and forward curve. **Vol 5, pp 429–437**

The Maturity Structure of Interest Rates

A **yield curve** shows the relationship between yields-to-maturity and terms-to-maturity. Theoretically, the yields used to derive the yield curve should be for bonds with identical credit risk, liquidity risk, tax status, and currency denomination. Further, their annual rates should be quoted for the same periodicity and they should offer the same coupon so that they entail the same reinvestment risk. Practically speaking however, these assumptions rarely hold for bonds that are actually used in analyzing the **maturity structure** or **term structure of interest rates** (which describes how yields vary with terms to maturity).

The Spot Rate Curve

The ideal data set for analyzing the term structure of interest rates would be yields-to-maturity on zero-coupon government bonds or **spot rates** for a range of maturities. This data set is known as the **spot rate curve**. As we learned earlier in the reading, spot rates are yields to maturity on zero-coupon bonds. The distinguishing feature of spot rates is that they are yields that have no element of reinvestment risk. Contrast this with the yield-to-maturity on coupon bonds, which entails a significant element of reinvestment risk (as it assumes that all interim cash flows can be reinvested at the stated YTM until maturity). Because spot rates are free from reinvestment risk, using them best meets the "all other things being equal" assumption in that Treasury zero-coupon bonds or **strips** (whose yields represent spot rates for different maturities) presumably have the same currency risk, liquidity risk, credit risk, and tax status, and there is no reinvestment risk as there is no coupon. Using spot rates therefore, provides a more accurate relationship between yields and terms to maturity relative to using yields to maturity on coupon-bearing Treasuries.

Yield Curve for Coupon Bonds

A **yield curve for coupon bonds** shows the yields-to-maturity for coupon-paying bonds of different maturities. To build the Treasury yield curve, analysts use only the most recently issued and actively traded government bonds as they have similar liquidity and tax status. Even though Treasury securities are not available for every single maturity, YTMs for maturities where there are gaps can be estimated through a variety of interpolation methods, the simplest of which is **linear interpolation**. Linear interpolation evenly distributes the difference in yields over the time period between two maturities. For example, if the yield on the 7-year Treasury is 4% and that on the 10-year Treasury is 5%, linear interpolation would compute the 8-year yield to be 4.33%, and the 9-year yield to be 4.67%.

Par Curve

In addition to the yield curve on coupon bonds and the spot rate curve on zero-coupon bonds, maturity structure can be evaluated using a **par curve**, which represents a series of yields-to-maturity such that each bond trades at par. The par curve is derived from the spot rate curve, as illustrated in Example 4-1.

Example 4-1: Computing Par Rates from Spot Rates

The spot rates (expressed as effective annual rates) on government bonds are 4.75% for 1 year, and 4.86% for 2 years. Compute the 1-year and 2-year par rates.

Solution:

When trying to calculate the par rate, the aim is to solve for "PMT" which, given the expected payments on a bond and corresponding spot rates, would result in the price of the bond equaling its par value. Recall that for a bond to be trading at par, it's coupon rate must equal the yield-to maturity, so if we compute the coupon (PMT) required to force the bond to trade at par (given a series of spot rates) we can then compute the coupon rate (as PMT/100) and this rate would also represent the par rate for the corresponding horizon.

So let's start with computing the 1-year par rate.

$$100 = \frac{\text{PMT} + 100}{(1 + 0.475)^1}; \text{PMT} = 4.75$$

The 1-year par rate therefore equals 4.75%.

Next, we compute the 2-year par rate.

$$100 = \frac{\text{PMT} + 100}{(1 + 0.475)^1} + \frac{100 + \text{PMT}}{(1 + 0.486)^2}; \text{PMT} = 4.8578$$

The 2-year par rate therefore equals 4.858%

Forward Curve

A **forward market** is for future delivery, beyond the settlement horizon of the **cash market**. A **forward rate** is the interest rate on a bond or money market instrument traded in the forward market. In more simple terms, the forward rate represents the interest rate on a loan that will be originated at some point in the future. Forward rates are used to construct the **forward curve**, which represents a series of forward rates, each having the same horizon. Typically the forward curve shows 1-year forward rates stated on a semiannual bond basis.

LOS 53h: Define forward rates and calculate spot rates from forward rates, forward rates from spot rates, and the price of a bond using forward rates.

Vol 5, pp 429–437

Implied forward rates (also known as **forward yields**) can be computed from spot rates.

Forward rates can be described as the market's current estimate of future spot rates. We will also use the arbitrage principal in our derivation—two portfolios with identical cash flows and identical risks should have the same value today, all other factors constant.

Consider an investor who has a 1-year investment horizon and is faced with the following alternatives:

- Purchase a 1-year T-bill today. The 1-year spot rate today (yield on the zero-coupon 1-year T-bill) is given as 4.6%.
- Purchase a 6-month T-bill now and upon its expiration, purchase another 6-month T-bill. The 6-month spot rate today (yield on the first 6-month T-bill) is given as 4%.

Notice that in this example we work with 6-month periods. The yields on the bonds have been expressed on a semiannual bond basis yield—the semiannual rate has been multiplied by two.

The investor will be indifferent between these two options if she knows that the return from taking either option with be the same. However, she does not know the return that will be offered after 6 months on the second 6-month T-bill. We can calculate the return required (to make her indifferent between the two options) on a 6-month investment in a T-bill, 6 months from now by using the spot rates available today for the 6-month and the 1-year T-bill:

If she invests $100 today in the 6-month T-bill her return would be:

$$100 \times \left(1 + \frac{0.04}{2}\right)^1 = \$102$$

0.04/2 is the effective semiannual discount rate on the 6-month T-Bill. 0.04 is the semiannual bond basis yield.

If she invests in 1-year T-bill, she will end up with:

$$100 \times \left(1 + \frac{0.046}{2}\right)^2 = \$104.65$$

0.046/2 is the effective semiannual discount rate on the 1-year T-Bill. 0.046 is the semiannual bond basis yield.

The investor would be indifferent between the two strategies if they offer her an identical return. In order to end up with $104.65 using the rollover strategy, her return on the second 6-month T-bill must equal 5.2% expressed as a semiannual bond basis yield. This figure is calculated as:

$$102 \times \left(1 + \frac{X}{2}\right)^2 = \$104.6529$$

$$\frac{104.6529}{102} - 1 = 5.2\% \text{ expressed as a semiannual bond basis yield.}$$

> Forward rates are market estimates of future spot rates.
>
> $_1S_0$ = 1-period spot rate today (t = 0).
>
> $_xS_0$ = x-period spot rate today (t = 0).
>
> $_2f_5$ = 2-period forward rate 5 periods from today.

If the yields presented are semiannual bond basis:

$$\left(1+\frac{\text{6-mth spot rate}}{2}\right)\left(1+\frac{\text{6-mth forward rate 6 mths from now}}{2}\right)=\left(1+\frac{\text{12-mth spot rate}}{2}\right)^2$$

> **Important:** In Examples 12, 13, 14, and 15 we assume that the rates provided are effective annual interest rates.

Example 4-2: Computing Forward Rates

The current 1-year spot rate is 5%, 2-year spot rate is 5.25%, and 3-year spot rate is 5.55%. Calculate the 1-year forward rate 1 year from now and 2 years from now.

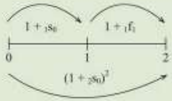
Solution:

Calculation of 1-year forward rate 1 year from today:

$$(1+{}_1s_0)(1+{}_1f_1)=(1+{}_2s_0)^2$$

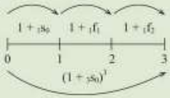
$$(1+0.05)(1+{}_1f_1)=(1+0.0525)^2$$

$${}_1f_1=\frac{1.0525^2}{1.05}-1=5.5\%$$



Calculation of 1-year forward rate 2 years from today:

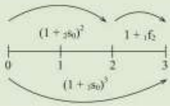
$$(1+{}_1s_0)(1+{}_1f_1)(1+{}_1f_2)=(1+{}_3s_0)^3$$



$(1+{}_1s_0)(1+{}_1f_1)$ simply equals the compounding factor for an investment for 2 years at the 2-year spot rate. The equation above can therefore be modified to:

$$(1+{}_2s_0)^2(1+{}_1f_2)=(1+{}_3s_0)^3$$

$${}_1f_2=\frac{1.0555^3}{1.0525^2}-1=6.15\%$$

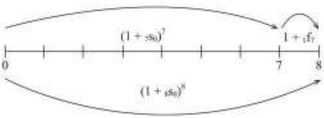


Let's summarize what we have learned so far about the relationship between multi-period spot rates and forward rates:

- $(1+{}_1s_0)(1+{}_1f_1)=(1+{}_2s_0)^2$
- $(1+{}_2s_0)^2(1+{}_1f_2)=(1+{}_3s_0)^3$

Therefore we can calculate the 1-period forward rate 7 years from now using the 8-year and the 7-year spot rates:

$$(1+{}_7s_0)^7(1+{}_1f_7)=(1+{}_8s_0)^8$$

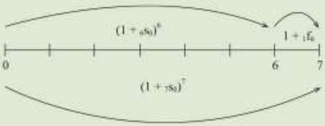


Example 4-3: Calculating Forward Rates

Calculate the 1-year forward rate 6 years from today if the 6-year spot rate is 6.25% and the 7-year spot rate is 6%.

Solution:

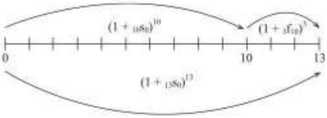
$$(1+{}_6s_0)^6(1+{}_1f_6)=(1+{}_7s_0)^7$$



$$(1+{}_1f_6)=\frac{(1.06)^7}{(1.0625)^6} => {}_1f_6=4.51\%$$

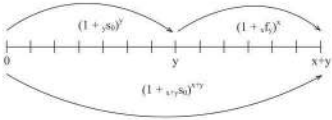
We can also calculate multi-period forward rates using multi-period spot rates. For example, we can use the following equation to calculate the 3-period forward rate 10 periods from now using the 10-year and the 13-year spot rates:

$$(1+{}_{10}s_0)^{10}(1+{}_3f_{10})^3=(1+{}_{13}s_0)^{13}$$



To calculate the x-period forward rate y periods from today, simply remember the following formula:

$$(1+{}_ys_0)^y(1+{}_xf_y)^x=(1+{}_{x+y}s_0)^{x+y}$$

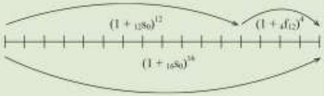


Example 4-4: Calculating Multi-Period Forward Rates

Calculate the 4-year forward rate 12 years from today if the 12-year spot rate is 4.5% and the 16-year spot rate is 4.6%.

Solution:

$$(1+{}_{12}s_0)^{12}(1+{}_4f_{12})^4=(1+{}_{16}s_0)^{16}$$



$$\left(1+{}_4f_{12}\right)^4=\frac{(1.046)^{16}}{(1.045)^{12}} => {}_4f_{12}=4.9\%$$

Even though we include the 1-year forward rate today in this example, this rate really is not a forward rate; it is simply the 1-year spot rate today.

Example 4-5: Valuing Bonds Using Forward Rates

The current 1-year forward rate is 4%, the 1-year forward rate 1 year from now is 4.25% and the 1-year forward rate 2 years from today is 4.3%. Calculate the value of a $1,000 par, annual-pay coupon bond that has a coupon rate of 4% and 3 years remaining to maturity.

Solution:

$$\frac{40}{1+{}_1f_0}+\frac{40}{(1+{}_1f_0)(1+{}_1f_1)}+\frac{40}{(1+{}_1f_0)(1+{}_1f_1)(1+{}_1f_2)}$$

$$\frac{40}{1+0.04}+\frac{40}{(1+0.04)(1+0.0425)}+\frac{1{,}040}{(1+0.04)(1+0.0425)(1+0.043)}=\$995.04$$

A forward rate can be looked upon as the marginal or incremental return from expanding the time to maturity by one more year. Assume that an investor with a 3-year investment horizon is deciding between (1) buying a 2-year bond today and reinvesting the proceeds at maturity for another year at the (then-current) 1-year rate and (2) buying a 3-year bond today.

- If the investor believes that the 1-year rate 2 years from now will be greater than the implied forward rate, she would choose the first option.
- If the investor believes that the 1-year rate 2 years from now will be less than the implied forward rate, she would choose the second option.

This example illustrates why the implied forward rate essentially represents the **breakeven reinvestment rate**. Forward rates are useful to investors in making maturity choice decisions.

Important: Please note that finance authors use different notation when it comes to forward rates. We use ${}_xf_y$, which refers to the x-period forward rate y years from today, or the interest rate on a loan that has a term of x years, where the loan will be originated y years from today. The CFA Program curriculum uses different notation. In the curriculum, the forward rate 2y5y refers to the 5-year rate 2 years into the future and 3y2y refers to the 2-year rate 3 years from now. We have stuck with our (different) notation because we feel it is easier to work with. You can work with either notation, as it won't affect your answers.

Also note that on the exam, you could be asked to work with annual rates on semiannual-pay bonds. Most of the examples we have worked with so far have used annual-pay bonds. In order to compute the implied forward rate with semiannual, pay bonds, we would use the semiannual discount rate instead of the annual rate, and use the number of semiannual periods instead of the number of annual periods (years). This is illustrated in Example 4-6.

Example 4-6: Computing Implied Forward Rates

Suppose that an investor observes these prices and yields-to-maturity on zero-coupon government bonds.

Maturity	YTM	Price (Per 100 of Par)
1 year	1.833%	$98.19
2 years	2.062%	$95.98
5 years	2.243%	$89.45

The yields-to-maturity are stated on a semiannual bond basis.

1. Compute $_1f_1$ (or 1y1y as the curriculum would put it) implied forward rates, stated on a semiannual bond basis.
2. Compute $_3f_2$ (or 2y3y as the curriculum would put it) implied forward rates, stated on a semiannual bond basis.
3. The investor has a 5-year investment horizon and is choosing between (1) buying the 2-year zero and reinvesting in another 3-year zero in 2 years' time and (2) buying and holding to maturity the 5-year zero. The investor decides to buy the 2-year bond today. Based on his decision, determine the minimum yield-to-maturity she expects on a 3-year zero two years from today.

Solution:

1. $$\left(1+\frac{_1f_0}{2}\right)^2\times\left(1+\frac{_1f_1}{2}\right)^2=\left(1+\frac{_2f_0}{2}\right)^4$$

$$\left(1+\frac{0.01833}{2}\right)^2\times\left(1+\frac{_1f_1}{2}\right)^2=\left(1+\frac{0.02062}{2}\right)^4$$

$$_1f_1=0.02291 \text{ or } 2.291\%$$

2. $$\left(1+\frac{_2f_0}{2}\right)^4\times\left(1+\frac{_3f_2}{2}\right)^6=\left(1+\frac{_5f_0}{2}\right)^{10}$$

$$\left(1+\frac{0.02062}{2}\right)^4\times\left(1+\frac{_3f_2}{2}\right)^6=\left(1+\frac{0.02243}{2}\right)^{10}$$

$$_3f_2=0.02364 \text{ or } 2.364\%$$

3. The decision to go with the 2-year bond today instead of the 5-year bond suggests that the investor believes that the 3-year forward rate 2 years from now will be greater than or equal to 2.364%.

   The $_3f_2$ implied forward rate of 2.364% is the breakeven reinvestment rate. If the investor expected the 3-year forward rate in 2 years to be less than this implied rate, she would choose to buy the 5-year zero today.

Example 4-6 is different from the earlier examples in this section because the yields presented are on a semiannual bond basis. We first compute the yield for semiannual period and then compound it over two periods to determine the effective annual yield.

LESSON 5: YIELD SPREADS

LOS 53i: Compare, calculate, and interpret yield spread measures.
Vol 5, pp 437–441

Yield Spreads over Benchmark Rates

To understand why bond prices and yields change, it is useful to separate the yield-to-maturity into two components: the **benchmark yield** and the **spread.**

> You should note that changes in macroeconomic factors can also cause spreads to narrow or widen across all issuers.

- The **benchmark yield** is the base rate and is also referred to as the **risk-free rate of return.** It captures **macroeconomic factors** such as the expected rate of inflation, currency denomination, and the impact of monetary and fiscal policy. Changes in these factors impact all bonds in the market. The benchmark can also be broken down into (1) the expected inflation rate and (2) the expected real rate.
- The **spread** refers to the difference between the yield-to-maturity on a bond and on the benchmark. It captures all **microeconomic factors** specific to the issuer, such as credit risk of the issuer, changes in the issue's credit rating, liquidity, and tax status of the bond. The spread is also known as the **risk premium** over the risk-free rate of return.

For example, consider a 5-year corporate bond that offers a yield-to-maturity of 5.50%. The benchmark bond is a 5-year U.S. Treasury, which offers a yield of 4.50%. This means that the corporate bond offers a spread of 100 bps. Now suppose that the yield on the corporate bond increases from 5.50% to 6.00%.

- If the yield on the benchmark has also increased by 50 bps, we can infer that the change in the bond's yield is caused by macroeconomic factors that affect all bond yields.
- However, if the yield on the benchmark has remained the same, we can infer that the change in the bond's yield was caused by firm-specific factors such as changes in the issuer's credit worthiness.

The benchmark varies across financial markets. Fixed-rate bonds often use the yield on a government bond with the same term to maturity as the bond being studied as the benchmark. Typically, the benchmark is the most-recently issued or **on-the-run** security. On-the-run securities are the most actively traded and have coupon rates closest to the current market yield for a given maturity. **Off-the-run** securities are more seasoned issues. Generally speaking, on-the-run issues tend to trade at slightly lower yields-to-maturity (higher price) than off-the-run issues because of (1) differences in demand and (2) differences in the cost of financing the securities in the repo market.

In the U.K., United States, and Japan, the benchmark rate for fixed-rate bonds is the government bond yield. The yield spread in basis points over an actual or interpolated government bond yield is known as the **G-spread.** On the other hand, yield spreads on euro-denominated corporate bonds use the EUR interest rate swap with the same tenor as the bond as the benchmark. The yield spread over the standard swap rate in the same currency and with the same tenor as the subject bond is known as the **I-spread** or

interpolated spread to the swap curve. Note that the government bond yield or the swap rate used as the benchmark for a specific bond will change over time as the remaining term to maturity of the bond changes. For example, when a 5-year bond is issued, the benchmark may be the 5-year government bond yield, but 2 years into the term of this bond, its G-spread will be stated relative to the 3-year government bond yield.

For floating-rate bonds, **LIBOR** is often used as a benchmark. Note that LIBOR is an interbank rate, not a risk-free rate.

Yield Spreads over the Benchmark Yield Curve

Recall that G-spreads and I-spreads are spreads on top of the Treasury yield curve and swap fixed rates respectively. Let's consider the Treasury yield curve (since you are more familiar with yield curves than swap curves at Level I). Given the term to maturity of a security, the appropriate benchmark yield-to-maturity applies the same discount rate for each cash flow. Theoretically, this method is unappealing because each cash flow received from the bond carries a different amount of risk (typically, cash flows expected to be received further out into the future entail more risk). It makes more sense to use individual spot rate rates to discount each of the bond's expected cash flows as spot rates accurately capture the risk entailed by each corresponding cash flow (i.e., for each time horizon, there is a specific spot rate unless the yield curve is flat). Therefore, practitioners tend to favor use of the **z-spread** over the G- and I-spreads. The z-spread (or **zero-volatility spread** or **static spread**) of a bond is a constant spread over the government (or interest rate swap) **spot rate curve.**

It is calculated using the following equation:

$$PV = \frac{PMT}{(1+z_1+Z)^1} + \frac{PMT}{(1+z_2+Z)^2} + \ldots + \frac{PMT+FV}{(1+z_N+Z)^N}$$

- The benchmark spot rates z_1, z_2, z_N are derived from the government yield curve (or from fixed rates on interest rate swaps).
- Z refers to the z-spread per period. It is constant for all time periods.

The z-spread is also used to calculate the **option-adjusted spread (OAS)** on a callable bond. Just like the option-adjusted yield (that we learned about earlier), the OAS is based on an option pricing model and utilizes an assumption about interest rate volatility. The OAS is calculated by subtracting the value of the embedded call option (stated in terms of bps per year) from the z-spread.

$$OAS = \text{z-spread} - \text{Option value (bps per year)}$$

Stated simply, the OAS removes the cost of the option from the z-spread, so the OAS is the spread on top of the spot rate curve that the bond would offer if it were option-free. Since the embedded call option in a callable bond favors the issuer, the OAS (the spread that an otherwise identical option-free bond would offer) is less than the z-spread. An issuer would pay out more (in terms of yield) on a callable bond than on an option-free bond. See Example 5-1.

Example 5-1: Illustrating the G-spread

A 7% annual-pay corporate bond with 2 years remaining to maturity is trading at a price of 107.75. The 2-year, 6% annual-pay government benchmark bond is trading at a price of 106.50. The 1-year and 2-year government spot rates are 1.085% and 2.67%, respectively, stated as effective annual rates.

1. Calculate the G-spread on the corporate bond.
2. Demonstrate that the z-spread is 33.5 bps.

Solution:

1. The yield-to-maturity for the corporate bond is calculated as:

   N = 2; PMT = –\$7; FV = –\$100; PV = \$107.75; CPT I/Y; I/Y = 2.953%

   The yield-to-maturity for the government benchmark bond is calculated as:

   N = 2; PMT = –\$6; FV = –\$100; PV = \$106.5; CPT I/Y; I/Y = 2.622%

   Therefore, the G-spread equals 33.1 bps (= 0.02953 – 0.02622).

2. We solve for the value of the corporate bond using $z_1 = 0.01085$, $z_2 = 0.0267$, and $Z = 0.00335$. The resulting value must equals 107.75 if the z-spread is indeed 33.5 bps.

$$= \frac{7}{(1+0.01085+0.00335)} + \frac{107}{(1+0.0267+0.00335)^2}$$
$$= \frac{7}{1.0142} + \frac{107}{(1.03005)^2}$$
$$= 107.75$$

READING 54: INTRODUCTION TO ASSET-BACKED SECURITIES

LESSON 1: INTRODUCTION, THE BENEFITS OF SECURITIZATION, AND THE SECURITIZATION PROCESS

INTRODUCTION

The focus in this reading is on fixed-income instruments created through a process known as securitization, a process where:

1. Assets (typically loans and receivables) are moved by the owner to a special legal entity.
2. The special legal entity then uses the assets as collateral to issue fixed-income securities (known as asset-backed bonds).
3. Cash flows from the collateral pool are used to make interest and principal payments on those asset-backed bonds.

The assets that are used as collateral to issue asset-backed bonds are called securitized assets and include residential mortgage loans, commercial mortgage loans, automobile loans, student loans, and credit card receivables.

The securitization process will be described in more detail later in the reading.

LOS 54a: Explain benefits of securitization for economies and financial markets. Vol 5, pp 470–472

BENEFITS OF SECURITIZATION FOR ECONOMIES AND FINANCIAL MARKETS

Before the advent of securitization, financing for most mortgages and other financial assets was provided by financial institutions (e.g., commercial banks). For investors, the only way to participate in such financings was by holding deposits, debt, or equity issued by those financial institutions. Such an arrangement posed the following problems:

- The financial institution represented an additional layer between originating borrowers and ultimate investors.
- Investors were only able to gain exposure to the bank's entire portfolio of assets; that is, they were unable to pick and choose the types of assets they desired exposure to.

Securitization solves these problems, and provides the additional benefits:

- It removes the layer between borrowers and investors. As a result of the reduced role of the financial intermediary (financial institutions, including banks), the cost paid by borrowers is reduced and at the same time the return realized by investors is improved.
- It allows investors to have a stronger legal claim on the collateral pool of assets.
- Investors can pick and choose the types of securities they want to invest in (in terms of interest rate and credit risk).
- Financial intermediaries are able to originate more loans (by using financing provided by outside investors to originate loans) than they would be able to if they were only able to issue loans that they could finance themselves. In other words,

securitization allows banks to originate, monitor, and collect loans beyond what they could do if they were limited to their own deposits and capital. This results in an improvement in their profitability.

- The increase in the total supply of loanable funds benefits organizations (governments and companies) that need to borrow.
- Since securitized bonds are sold in the public market, they enjoy much better liquidity (lower liquidity risk) than the original loans on bank balance sheets. Further, financial markets are made more efficient.
- Securitization encourages innovation in investment products, which can offer investors access to (otherwise directly unavailable) assets that match their risk, return, and maturity profiles. For example, a pension fund looking to invest in long-term assets can invest (directly) in long-term housing loans via residential mortgage-backed securities, as opposed to investing (indirectly) in bonds or stocks issued by banks.
- Even large investors, who may be able to purchase real estate loans, automobile loans, or credit card loans directly, would prefer to invest in asset-backed bonds since they would not be required to originate, monitor, and collect payments from the underlying loans themselves (the securitization process takes care of all this).
- ABS offer companies an alternative means of raising finance that can be considered alongside bond and equity issuance.

Note that along with these benefits, securitization brings many risks. We will describe many of these risks later in the reading.

LOS 54b: Describe securitization, including the parties involved in the process and the roles they play. Vol 5, pp 472–481

THE SECURITIZATION PROCESS

An Example of a Securitized Transaction

Let's work with a hypothetical example to illustrate the securitization process.

Assume that ABC Company is in the business of manufacturing and selling motor vehicles. While some of its sales are made for cash, most sales are made on installment sales contracts where ABC advances loans to customers to finance their purchases. These loans are fixed-rate, full-amortizing loans that have a term of 48 months (four years). ABC receives a monthly payment (that consists of principal and interest) from each loan until it is paid off (fully amortized), while the vehicles serve as collateral against the loans. If a buyer/borrower is unable to make the scheduled monthly payment, ABC can take over ownership of the motor vehicle and sell it to recover the principal amount outstanding.

ABC's credit department decides to whom loans should be advanced and at what terms based on certain criteria (e.g., creditworthiness of purchaser). These criteria are known as underwriting standards. Since ABC is extending the loan, it is referred to as the originator of the loan. ABC's credit department also services the loans that are made. In other words, it collects payments from borrowers, issues notifications to borrowers who have not made scheduled payments, and recovers and disposes of the collateral (vehicles) in case of default. Note that even though this is the case in our example, it is not necessary that the originator of the loan also performs the role of the servicer.

Currently, ABC has 100,000 auto loans worth $1 billion of par value outstanding. These loans are classified as assets (since they are amounts owed to the company) on its balance sheet. ABC wants to raise $1 billion for its expansion program and decides to securitize these loans to raise the funds instead of issuing corporate bonds (due to the potentially lower costs of securitization). The first thing that ABC must do in order to securitize these assets is sell the loans (receivables) to a special purpose vehicle (SPV) or special purpose entity (SPE). The SPV takes legal ownership of the loans and pays ABC $1 billion (the amount it wanted to raise) for them. The SPV obtains these funds by issuing asset-backed securities (ABS) collateralized by the $1 billion pool of loans.

In the prospectus for these asset-backed securities, the SPV is referred to as the issuer or the trust. The seller of the loans, ABC, is referred to as the seller or depositor. It is important for you to understand that each individual loan issued by ABC (the depositor/seller) is collateralized by a motor vehicle, while each ABS issued by the SPV (issuer/trust) is collateralized by cash flows generated from the pool of loans. Figure 1-1 illustrates the securitization described above, while Table 1-1 lists the roles performed by the three primary parties in a securitization.

Figure 1-1: Securitization Illustration for ABC Company

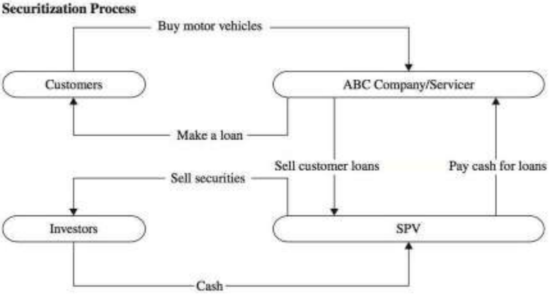


Table 1-1: Parties to the Securitization

Party	Description	Party in Illustration
Seller	Originates the loans and sells loans to the SPV	ABC Company
Issuer/Trust	The SPV that buys the loans from the seller and issues the asset-backed securities	SPV
Servicer	Services the loans	Servicer

Payments received from the collateral (loans) are used to pay servicing fees, other administrative fees, and principal and interest payments to ABS holders. The priority and amount of payments to be distributed to the various classes/tranches of security holders is referred to as the structure's waterfall.

The different types of ABS classes/tranches are described later in the reading.

In addition to the (1) seller/depositor (ABC), (2) the servicer (also ABC in this example), and the (3) issuer/trust (SPV), there are several other parties involved in a securitization:

All parties other than the seller and the issuer, including the servicer when it is different from the seller, are known as third parties to the transaction.

- Attorneys prepare legal documents, including:
  - The purchase agreement between the seller (ABC) and the SPV that includes representations and warranties made by the seller regarding the assets sold.
  - A document that describes the structure's waterfall.
  - The servicing agreement between the SPV and the servicer.
- An independent accounting firm verifies the accuracy of all numerical information in the prospectus and issues a comfort letter for the securitization.
- The trustee or trustee agent (typically a financial institution) holds the assets in a trust, holds the payments due to bondholders until they are paid, and provides remittance reports to bondholders.
- The underwriter markets the securities to investors and handles the logistics of the transaction.
- Rating agencies determine the level of credit enhancement required for each bond class to obtain the credit rating desired by the issuer.
- Guarantors play a very important role for securitized instruments. They are discussed later.

Key Role of the Special Purpose Vehicle

In order for a securitization to serve its purpose, the SPV must be a bankruptcy-remote entity. That is, its obligations remain secure even if the parent company goes bankrupt. We illustrate the key role played by the SPV using our example with ABC and the SPV.

The credit spread that ABC would be required to pay if it were to issue corporate bonds would depend on its perceived credit risk, which would be reflected in the credit rating assigned to it by credit rating agencies (S&P, Moody's, and Fitch). Suppose that the credit rating assigned to ABC is BB, so the credit spread that ABC would be required to pay on any unsecured bonds that it were to issue would be commensurate with the spread required of an entity rated BB. ABC could try to lower its cost of borrowing by collateralizing the bonds with the auto loans it has issued, thereby issuing secured bonds. Practically speaking, however, this exercise (of putting up the auto loans as collateral) will have a negligible impact on the interest rate ABC would have to pay on the proposed bonds. We will shortly illustrate why this is the case.

Now suppose that ABC sells the loans to an SPV in an arm's-length transaction, and the SPV then issues securities to raise the required $1 billion (instead of using the auto loans as collateral for a secured corporate bond issue). Once the auto loans have been sold to the SPV, the SPV takes full legal ownership of the receivables. If ABC were to subsequently go bankrupt, its creditors would have no recourse to those auto loans (as they are legally owned by the SPV). Essentially, when the SPV issues bonds backed by cash flows from the pool of auto loans, the evaluation of credit risk of the issue (and hence its credit rating and required credit spread) is based solely on the collectability of those receivables independent ABC's credit rating. Typically, because of the securitization, the (high) quality of the collateral, and the capital structure of the SPV, a company can raise funds at a lower funding cost by issuing asset-backed bonds than by issuing corporate bonds.

A fair question at this stage would be how bonds issued in a securitization can have a higher credit rating (and a lower funding cost) than secured corporate bonds directly issued by the originator if they are both collateralized by the same assets (pool of auto loans). In order to understand this, we must first become familiar with the rule of absolute priority.

The absolute priority rule refers to the principle that in case of default, senior secured creditors are repaid before subordinated creditors receive anything, and all creditors are repaid before equity holders receive anything. The thing is that the absolute priority rule generally holds in liquidations, but not in a reorganization, where it is possible for the actual outcome regarding distributions to various classes of bondholders and equity holders to be markedly different from the terms stated in the debt agreement. This means that even if there is collateral backing a corporate bond, there is no assurance that the rights of senior secured bondholders will be respected, and this is why collateral backing for a corporate bond does not result in a significant reduction in the credit spread.

However, when it comes to securitization via a bankruptcy-remote SPV, bankruptcy of a company does not affect the SPV, so the rules set forth in the securitization's prospectus regarding how credit losses will be absorbed by each bond class are generally adhered to. Going back to our example, creditors who hold bonds issued by ABC would have no legal claim on the cash flows generated from the securitized pool of auto loans (as they were sold to the SPV). The cash flows from the pool of auto loans are completely and legally independent of anything that happens to ABC. The security holders face credit risk only to the extent that the borrowers of the auto loans that the SPV has purchased from ABC default on those loans.

The absolute priority rule, the priority of claims, and recovery rates for various classes of bonds in a liquidation and reorganization are discussed in Fixed Income.

Finally, note that not all countries have a legal framework and a concept of trust law that is as well developed as in the United States. Therefore, investors should be wary of legal considerations that apply in countries where they purchase asset-backed bonds.

LESSON 2: RESIDENTIAL MORTGAGE LOANS AND RESIDENTIAL MORTGAGE-BACKED SECURITIES (RMBS)

LOS 54d: Describe types and characteristics of residential mortgage loans that are typically securitized. Vol 5, pp 481–485

Please note: Throughout this reading, the term "mortgage-backed securities," or MBS, refers to securities backed by high-quality real estate mortgages, and the term "asset-backed securities," or ABS, refers to securities backed by other types of assets.

RESIDENTIAL MORTGAGE LOANS

A mortgage loan is a borrowing that is secured by some form of real estate. In case the borrower (usually someone wishing to buy a home) fails to make her mortgage payments, the lender (usually a bank or mortgage company) has the right to seize the property (foreclose on the loan) and recover the amount due by selling it.

Typically, the amount of the loan is less than the property's purchase price. The difference between the purchase price and the amount borrowed equals the down payment made by the borrower. Upon initiation of the mortgage loan, the borrower's equity equals the down payment, but over time, the borrower's equity changes as a result of (1) changes in the market value of the property and (2) payment of periodic mortgage payments (that include a principal component).

The ratio of the purchase price to the amount of the mortgage loan is known as the loan-to-value (LTV) ratio. The lower (higher) the LTV ratio, the higher (lower) the borrower's equity, the less (more) likely the borrower is to default, and the more (less) protection the lender has for recovering the amount loaned in case the borrower defaults.

In the sections that follow, we discuss the five primary specifications of mortgage design.

Maturity

The term of a mortgage loan refers to the number of years to maturity. In the United States, the term of mortgage loans typically varies between 15 and 30 years, while in Europe the term varies between 20 and 40 years. In Japan, terms can extend to as many as 100 years.

Interest Rate Denomination

The interest rate on a mortgage loan is called the mortgage rate or contract rate. Typically, mortgage rates are specified in one of the following forms:

Fixed-rate: The mortgage rate is fixed over the term of the loan.

Adjustable-rate mortgage (ARM) or variable-rate mortgage: The mortgage rate is reset periodically (daily, weekly, monthly, or annually).

- In an index-referenced adjustable-rate mortgage, the reset rate may be based on some reference rate or index.
- In a reviewable ARM, the mortgage rate is determined at the lender's discretion.

Note that ARMs typically have clauses placing caps on (1) the maximum interest rate change at any reset date and (2) the maximum interest rate that can be charged at any point during the mortgage's entire term.

Initial period fixed rate: The mortgage rate is fixed initially for a specified period and is then adjusted to either (1) a new fixed rate, in which case it is known as a rollover or renegotiable mortgage, or (2) a variable rate, in which case it is known as a hybrid mortgage.

Convertible: The mortgage rate is initially either fixed or adjustable, with the borrower having an option to convert the mortgage into a fixed rate or adjustable rate for the remaining term of the mortgage.

Amortization Schedule

Most mortgages around the world are structured as amortizing loans, where the principal amount outstanding declines over time as monthly payments (that consist of both an interest component and a principal repayment component) come in. Mortgage loans can be structured as fully amortizing loans or partially amortizing loans (where the last payment is referred to as a balloon payment).

In certain countries, mortgages are structured as interest-only mortgages, where no principal repayment is specified for a number of years. Other types of mortgage loans call for no principal repayment through the entire term of the mortgage, so the balloon payment at maturity equals the original loan amount. These mortgages are known as interest-only lifetime mortgages or bullet mortgages.

Prepayments and Prepayment Penalties

A mortgage loan may allow the borrower to prepay (make a principal repayment that exceeds the scheduled repayment for the month) a portion, or the entire amount, of the outstanding mortgage principal at any point during the term of the mortgage. This option is known as a prepayment option or early retirement option. Certain mortgages (known as prepayment penalty mortgages) may stipulate that the borrower pay some sort of penalty if she prepays within a certain time period following inception or the mortgage (this time period can also extend through the entire term of the mortgage). Since borrowers can accelerate payments on the mortgage loan, the cash flows from the loan cannot be known with certainty, and this risk faced by lenders is known as prepayment risk. Homeowners usually prepay their mortgages by refinancing their loans when interest rates fall. The owner of the mortgage loan receives her principal ahead of schedule and can reinvest the proceeds at current (lower) rates. The prepayment penalty is meant to compensate the lender for this loss.

Prepayment risk affects all mortgage loans, not just level-payment, fixed-rate, fully amortized mortgage loans.

Rights of the Lender in a Foreclosure

In the United States, residential mortgages are typically structured as nonrecourse loans, while in Europe they are primarily structured as recourse loans. If the borrower defaults, resulting in foreclosure:

- In a recourse loan, the lender has a claim against the borrower if the proceeds from sale of the property fall short of the mortgage balance outstanding.
- In a nonrecourse loan, the lender does not have such a claim against the borrower if the proceeds from sale of the property fall short of the mortgage balance outstanding.

Whether a lender has recourse to the borrower in case of default is an important determinant of the probability of default on a mortgage loan. With a nonrecourse loan, if the market value of the property declines below the mortgage amount outstanding, the borrower has an incentive not to repay the loan even if she has the funds available (as she would end up paying more than the current value of the property). This is known as a strategic default. In the case of a recourse loan, the borrower will be less likely to strategically default because the lender can come after her other income/assets to recover the shortfall.

LOS 54c: Describe typical structures of securitizations, including credit tranching and time tranching. Vol 5, pp 477–478

LOS 54e: Describe types and characteristics of residential mortgage-backed securities, including mortgage pass-through securities and collateralized mortgage obligations, and explain the cash flows and risks for each type. Vol 5, pp 487–492

LOS 54f: Define prepayment risk and describe the prepayment risk of mortgage-backed securities. Vol 5, pg 484

RESIDENTIAL MORTGAGE-BACKED SECURITIES

In the United States, residential mortgage-backed securities are divided into two sectors:

- Agency RMBS, which are issued by (1) federal agencies (e.g., Ginnie Mae, which is a federally related institution) and (2) quasi-government entities (e.g., Freddie Mac and Fannie Mae, which are government-sponsored enterprises, or GSEs).

- There is no credit risk for agency RMBS issued by Ginnie Mae as they are backed by the full faith and credit of the U.S. government.
- There is minimal credit risk for agency RMBS issued by GSEs as they are guaranteed by the GSEs themselves.
- Note that mortgage loans issued by GSEs must satisfy specific underwriting standards established by various government agencies to qualify for the collateral pool backing agency RMBS issued by GSEs.

- Nonagency MBS, which are issued by private entities.
  - They typically come with credit enhancements (described later) to reduce credit risk.
  - Note that there are no restrictions on the types on nonagency mortgage loans that can be used to back nonagency RMBS.

Mortgage Pass-Through Securities

A mortgage pass-through security is created when shares or participation certificates in a pool of mortgage loans are sold to investors.

Cash Flow Characteristics

The cash flow collected from the collateral pool includes scheduled principal and interest payments, and prepayments. However, the amount and timing of cash flows paid to investors in the pass-through securities are different from those of the cash flows collected from the collateral pool of mortgages.

Payments made by borrowers pass through the government agency (and on to investors) net of servicing and guaranteeing fees. Therefore, the pass-through rate (coupon rate on the pass-through security, which is net interest/net coupon) is lower than the mortgage rate on the underlying pool of mortgages.

Note that the mortgage loans that are securitized to create pass-through securities do not all carry the same mortgage rate and maturity. Therefore, a weighted average coupon (WAC) rate and weighted average maturity (WAM) are calculated to describe the pool of mortgages that serves as collateral for the pass-through securities.

- The weighted average coupon (WAC) rate is calculated by weighting the mortgage rate of each loan in the pool by its percentage of mortgage balance outstanding relative to the total mortgage balance outstanding for all mortgages in the pool.
- The weighted average maturity (WAM) is calculated by weighting the number of months to maturity of each loan in the pool by its percentage of mortgage balance outstanding relative to the total mortgage balance outstanding for all mortgages in the pool.

Conforming versus Nonconforming Loans

The underlying loans for agency RMBS must be conforming mortgages. That is, they must meet specified underwriting standards relating to (1) the maximum loan-to-value (LTV) ratio, (2) the loan documentation required, (3) whether insurance is required, and (4) the maximum loan amount.

If a loan is a nonconforming mortgage (i.e., it does not satisfy the underwriting standards mentioned earlier), it can only be used as collateral for privately issued mortgage pass-through securities. These nonagency RMBS are issued by thrift institutions, commercial banks, and private conduits.

Measures of Prepayment Rate

Since the mortgage loans that are securitized to create pass-through securities entail prepayment risk, the pass-through securities themselves also expose investors to prepayment risk. In describing prepayments, market participants refer to a prepayment rate or prepayment speed in terms of a monthly measure known as the single monthly mortality (SMM) rate or its corresponding annualized rate, the conditional prepayment rate (CPR).

The SMM equals the amount of prepayment for a month as a percentage of the mortgage balance that was available to be prepaid that month. Note that the mortgage balance available to be prepaid during a particular month equals the mortgage balance available to be repaid that month (mortgage balance outstanding at the beginning of the month) adjusted for the scheduled principal payment. The SMM for month t is calculated as:

$$\text{SMM}_t = \frac{\text{Prepayment in month } t}{\text{Beginning mortgage balance for month } t - \text{Scheduled principal payment in month } t}$$

Market participants prefer to define prepayment rates in terms of the annualized CPR. A CPR of 6% means that, ignoring scheduled principal repayments, approximately 6% of the mortgage balance outstanding at the beginning of the year will be prepaid by the end of the year.

In the United States, prepayment rates over the term of mortgage securities are described in terms of a prepayment pattern or benchmark introduced by the Public Securities Association (PSA). The PSA prepayment benchmark is defined in terms of a monthly series of CPRs. The benchmark assumes that prepayment rates are relatively low during the early years of the term of a mortgage, but gradually increase and eventually level off at higher rates (once the loans are seasoned). The 100 PSA benchmark assumes the following:

- A CPR of 0.2% in Month 1.
- The CPR increases by 0.2% per year every month for the next 30 months until it reaches 6% per year.
- A CPR of 6% for the remaining term.

100 PSA

- If $t < 30$, then CPR = 6% − (t/30).
- If $t \geq 30$, then CPR = 6%.
- t = Number of months that have passed since mortgage origination.

Note: If the mortgage pool consists of loans that had an original term of 360 months (30 years) and if the weighted average maturity of the pool is currently 357 months, the mortgage pool is seasoned three months. Therefore, in determining prepayments for the next month, the CPR (and corresponding SMM) for Month 4 (NOT Month 1) is applicable.

Slower or faster prepayment speeds are described in terms of percentages of 100 PSA. For example, 75 PSA implies three quarters of the CPR of the 100 PSA benchmark, while 250 PSA implies 2.5 times the CPR of the 100 PSA benchmark.

Cash Flow Construction

Table 2-1 shows the cash flows to bondholders for selected months on a hypothetical pass-through security with the following assumed features:

- The par value of the collateral pool of mortgages is $400 million.
- All the mortgages are fixed-rate, level-payment, fully amortizing loans.
- The WAC rate for the pool is 8.125%.
- The WAM for the pool is 357 months.
- The pass-through rate (net of servicing and other fees) is 7.5%.
- The prepayment rate is 165 PSA.

Table 2-1: Monthly Cash Flows for Hypothetical Mortgage Pass-Through Security

Month	Outstanding Balance	Total Principal	Net Interest	Cash Flow
1	400,000,000	709,923	2,500,000	3,209,923
2	399,290,077	821,896	2,495,563	3,317,459
3	398,468,181	933,560	2,490,426	3,423,986
4	397,534,621	1,044,822	2,484,591	3,529,413
5	396,489,799	1,155,586	2,478,061	3,633,647
80	207,862,347	2,050,422	1,299,140	3,349,562
81	205,811,925	2,032,197	1,286,325	3,318,522
82	203,779,729	2,014,130	1,273,623	3,287,753
98	173,617,879	1,745,550	1,085,112	2,830,662
99	171,872,329	1,729,979	1,074,202	2,804,181
100	170,142,350	1,714,544	1,063,390	2,777,934
101	168,427,806	1,699,243	1,052,674	2,751,917

Table 2-1: *(continued)*

Month	Outstanding Balance	Total Principal	Net Interest	Cash Flow
176	75,390,685	861,673	471,192	1,332,865
177	74,529,012	853,813	465,806	1,319,619
178	73,675,199	846,023	460,470	1,306,493
179	72,829,176	838,300	455,182	1,293,482
356	299,191	150,389	1,870	152,259
357	148,802	148,802	930	149,732

Note: You do not need to know how to derive the cash flows in the table above (it is a fairly complicated exercise). We have inserted this table here as the values it contains will be used to derive Table 2-3 in Example 2-1.

Weighted Average Life

For nonamortizing bonds (or bullet bonds) we know that, for a given coupon rate, the longer the maturity of the bond, the greater its interest rate risk (duration). For mortgage-backed securities, principal payments (scheduled payments and prepayments) are made over the term of the security. When evaluating the interest rate risk of an MBS, using its legal maturity (the date at which the last payment is due) is inappropriate, as it does not reflect the actual pattern of principal payments. For example, it would be incorrect to assume that a 30-year corporate bond and a 30-year MBS with the same coupon rate carry the same interest rate risk.

Therefore, market participants use average life (the weighted average time it will take for all the principal payments [i.e., scheduled repayments and projected prepayments] to be received) as a measure of the interest rate risk of a mortgage-backed security. The average life of an MBS depends on the assumed prepayment speed: the higher the prepayment speed assumed, the shorter the average life of the mortgage-backed security.

> Duration can also be used to evaluate the interest rate risk of an MBS.

Contraction Risk and Extension Risk

An investor in a mortgage pass-through faces uncertainty regarding the amount and timing of payments, as they depend on the speed at which prepayments actually come in. This uncertainty is referred to as prepayment risk, which encompasses contraction risk and extension risk.

Contraction risk occurs when interest rates fall. There are two adverse consequences of lower interest rates for pass-through investors.

> The response of callable bond prices to a decline in interest rates has been discussed in "Fixed-Income Securities: Defining Elements" and is revisited later in "Understanding Fixed-Income Risk and Return."

- Option-free bond prices increase when interest rates fall. However, for mortgage loans, the issuer (homeowner) has the right to prepay and can easily do so by refinancing her mortgage. This option to prepay is similar to the call option granted to the issuer of a callable bond. When interest rates fall, it becomes feasible for the mortgage issuer/borrower to prepay (just like it becomes feasible for an issuer

to call a callable bond), so the upside potential of the pass-through security is limited. It experiences price compression at low interest rates and exhibits negative convexity.
- To make things worse, when interest rates fall, refinancing activity typically increases and leads to an increase in prepayments, reducing or shortening the average life of the pass-through. The (higher-than-expected) cash flows from the pass-through must then be reinvested at current (lower) rates.

Extension risk occurs when interest rates rise.

- The price of a pass-through (just like the price of any bond) will decline when interest rates increase.
- To make things worse, refinancing activity and prepayment rates slow down when interest rates rise, increasing or lengthening the average expected life of the pass-through. Consequently, a greater-than-anticipated amount remains invested in the pass-through at the coupon rate of the instrument, which is lower than current interest rates.

COLLATERALIZED MORTGAGE OBLIGATIONS

The fact that mortgage pass-through securities entail significant uncertainty regarding the amount and timing of their cash flows (contraction and extension risk) makes them undesirable for institutional investors interested in matching the cash flows from their investment portfolios with the expected maturity of their liabilities and obligations. Collateralized mortgage obligations (CMOs) redistribute the cash flows from mortgage pass-through securities into packages/classes/tranches with different risk exposures to prepayment risk. The risk/return characteristics and exposures to prepayment risk of the various CMO tranches are different from those of the underlying mortgage pass-through securities, which makes them attractive as investments to different types of investors.

An important thing to bear in mind is that for CMOs, the mortgage-related products from which cash flows are obtained are considered the collateral (i.e., a pool of mortgage pass-through security serves as the collateral). Recall that for mortgage pass-through securities, a pool of mortgage loans serves as collateral.

The major varieties of CMO structures are described next.

Sequential-Pay Tranches

In this CMO structure, each class/tranche of bonds is retired sequentially in a predetermined order. An example of the rules for monthly distribution of principal payments (scheduled payments plus prepayments) would be:

- Distribute all principal payments to Tranche A until it is fully paid off.
- Once Tranche A is paid off, distribute all principal payments to Tranche B until it is fully paid off.
- Once Tranche B is paid off, distribute all principal payments to Tranche C until it is fully paid off.
- And so on.

Note that the rules for distributing coupon interest are different from the rules for distributing principal payments (scheduled amortization and prepayments). Each tranche is paid coupon interest each month at its stated coupon rate based on its principal amount outstanding at the beginning of the month. The principal payment pattern for each of the tranches is not known, as it depends on the actual prepayment rate of the collateral. Therefore, PSA rate assumptions are made to project cash flows for each tranche. Example 2-1 provides an illustration of a sequential-pay CMO structure with four tranches. This comprehensive example will serve to clarify all the features of sequential-pay CMOs that have been described.

Example 2-1: Monthly Cash Flows on Sequential-Pay CMO

A sequential-pay CMO structure is backed by $400m worth of mortgage pass-through securities. The pass-through coupon rate is 7.5%, the weighted average coupon is 8.125%, and the weighted average maturity is 357 months. Table 2-2 provides par values, coupon rates, and payment rules for the sequential-pay tranches.

Note that the total par value of the four tranches equals the par value of the collateral pass-through securities ($400m). The cash flows on the collateral for selected months were provided in Table 2-1. Further note that in our example, the coupon rate on the tranches equals the pass-through coupon rate (7.5%). Typically, however, a different coupon rate is offered on each tranche.

Table 2-2: Four-Tranche Sequential Pay Structure

Tranche	Par Amount ($)	Coupon Rate (%)
A	194,500,000	7.5
B	36,000,000	7.5
C	96,500,000	7.5
D	73,000,000	7.5
Total	400,000,000	

Payment Rules

- For payment of monthly coupon interest: Disburse monthly coupon interest to each tranche on the basis of the amount of principal outstanding for the tranche at the beginning of the month.
- For disbursement of principal payments: Disburse principal payments to Tranche A until it is fully paid off. After Tranche A is fully paid off, distribute principal payments to Tranche B until it is fully paid off. After Tranche B is fully paid off, distribute principal payments to Tranche C until it is fully paid off. After Tranche C is completely paid off, disburse principal payments to Tranche D until it is fully paid off.

Demonstrate the distribution of monthly cash flows from the pool of pass-through securities to the four tranches of the CMO assuming that prepayments come in at 165 PSA.

Solution:

Table 2-3 lists monthly cash flows for selected months for all four tranches **assuming 165 PSA**.

Table 2-3: Selected Monthly Cash Flows for Sequential-Pay CMO in Example 2-1 (Assuming 165 PSA)

	Tranche A			Tranche B			Tranche C			Tranche D		
Month	Balance ($)	Principal ($)	Interest ($)	Balance ($)	Principal ($)	Interest ($)	Balance ($)	Principal ($)	Interest ($)	Balance ($)	Principal ($)	Interest ($)
1	194,500,000	709,923	1,215,625	36,000,000		225,000	96,500,000		603,125	73,000,000		456,250
2	193,790,077	821,896	1,211,188	36,000,000		225,000	96,500,000		603,125	73,000,000		456,250
3	192,968,181	933,560	1,206,051	36,000,000		225,000	96,500,000		603,125	73,000,000		456,250
4	192,034,621	1,044,822	1,200,216	36,000,000		225,000	96,500,000		603,125	73,000,000		456,250
5	190,989,799	1,155,586	1,193,686	36,000,000		225,000	96,500,000		603,125	73,000,000		456,250
80	2,362,347	2,050,422	14,765	36,000,000		225,000	96,500,000		603,125	73,000,000		456,250
81	311,925	311,926	1,950	36,000,000	1,720,271	225,000	96,500,000		603,125	73,000,000		456,250
82				34,279,729	2,014,130	214,248	96,500,000		603,125	73,000,000		456,250
98				4,117,879	1,745,550	25,737	96,500,000		603,125	73,000,000		456,250
99				2,372,329	1,729,979	14,827	96,500,000		603,125	73,000,000		456,250
100				642,350	642,350	4,015	96,500,000	1,072,194	603,125	73,000,000		456,250
101							95,427,806	1,699,243	596,424	73,000,000		456,250
176							2,390,685	861,673	14,942	73,000,000		456,250
177							1,529,012	853,813	9,556	73,000,000		456,250
178							675,199	675,199	4,220	73,000,000	170,824	456,250
179										72,829,176	838,300	455,182
356										299,191	150,389	1,870
357										148,802	148,802	930

Each tranche receives interest each month (coupon rate × balance outstanding at the beginning of the month). The aggregate amount of interest paid out to the tranches for any particular month equals the total amount of interest collected from the collateral pool during that month (since the pass through rate equals the coupon rate). For example, total interest collected from the collateral in Month 5 equals $2,478,061 (see Table 2-3), and the total amount of interest paid out to the four tranches during Month 5 also equals $2,478,061 ($1,193,686 + $225,000 + $603,125 + $456,250; see Table 2-3).

Step 1: Initially all principal payments (scheduled amortization + prepayments) received from the pass-through securities are forwarded to Tranche A. For example, the entire principal payment received from the collateral pool in Month 1 (i.e., $709,923; see Table 2-1) goes to Tranche A.

Step 2: Tranche A is fully paid off in Month 81. The remaining principal payments received during Month 81 ($1,720,271) are forwarded to Tranche B. All principal payments going forward are distributed to Tranche B until it is fully paid off.

Step 3: Tranche B is fully paid off in Month 100. The remaining principal payments received during Month 100 ($1,072,194) are forwarded to Tranche C. All principal payments going forward are distributed to Tranche C until it is fully paid off.

Step 4: Tranche C is fully paid off in Month 178. The remaining principal payments received during Month 178 ($170,824) are forwarded to Tranche D. All principal payments going forward are distributed to Tranche D until it is fully paid off.

Step 5: Tranche D is fully paid off in Month 357.

Notice how:

- Tranche B receives only interest payments (no principal payments) until Tranche A is paid off.
- Tranche C receives only interest payments (no principal payments) until Tranche B is paid off.
- Tranche D receives only interest payments (no principal payments) until Tranche C is paid off.

The principal pay-down window or principal window for a tranche is the time period between the first and last principal payments to that tranche. The principal pay-down windows for the four sequential-pay tranches are listed in Table 2-4. Note that the figures in this table are based on the 165 PSA prepayment assumption.

Table 2-4: Principal Pay-Down Windows for Sequential-Pay CMO Tranches Assuming 165 PSA

Tranche	Principal Paid During	Principal Pay-Down Window
A	Month 1 to Month 81	81 months
B	Month 81 to Month 100	20 months
C	Month 100 to Month 178	79 months
D	Month 178 to Month 357	180 months

Note that although the payment rules for distribution of cash flows from the collateral pass-through securities are known, the actual amount of cash flows that will come in is uncertain (it depends on the actual speed of prepayments). The 165 PSA assumption only allows us to project expected, not actual, cash flows.

Now let's describe what we have accomplished by "repackaging" the mortgage pass-through securities into sequential-pay CMO tranches. Table 2-5 lists the average lives of the pass-through and the four sequential-pay CMO tranches that we created from the pass-through securities under a variety of prepayment speed assumptions.

Table 2-5: Average Life for Collateral and CMO Tranches under Different Prepayment Assumptions

	Average Life (in Years)				
(PSA)	Collateral	Tranche A	Tranche B	Tranche C	Tranche D
50	15.11	7.48	15.98	21.02	27.24
100	11.66	4.90	10.86	15.78	24.58
165	8.76	3.48	7.49	11.19	20.27
200	7.68	3.05	6.42	9.60	18.11
300	5.63	2.32	4.64	6.81	13.36
400	4.44	1.94	3.70	5.31	10.34
500	3.68	1.69	3.12	4.38	8.35
600	3.16	1.51	2.74	3.75	6.96
700	2.78	1.38	2.47	3.30	5.95

Notice that:

- The four sequential-pay tranches have average lives that are both longer and shorter than the collateral pass-through securities. The average life of the collateral pool assuming 165 PSA is 8.76 years.
- Tranche A is given priority in the distribution of principal payments (has the shortest average life) and therefore offers the most protection against extension risk. This protection comes from the other three tranches in the CMO structure (Tranches B, C, and D) that consequently have higher extension risk.
- Tranche D has the longest average life and offers the most protection against contraction risk. This protection comes from the other three tranches in the structure (Tranches A, B, and C) that face higher contraction risk.
- This exercise of redistributing prepayment risk by creating different bond classes with differing exposures to prepayment risk is known as time tranching. More formally, time tranching occurs when prepayment risk (or the timing of distribution of cash flows) is redistributed among the various classes of ABS.
- Notice how the securities created come closer to satisfying the asset/liability needs of institutional investors. Short-term investors would be drawn toward Tranche A, while longer-term investors would find Tranche D most attractive.
- There is still considerable variation in the average lives of the tranches depending on prepayment speeds. This variation is mitigated in the PAC structure described next.

We have not illustrated the derivation of the numbers used to calculate these average lives, nor have we performed these calculations, as this is not required. We are just using the average life numbers for illustrative purposes.

Planned Amortization Class (PAC) Tranches

PAC bonds were introduced in the bond market to improve upon sequential-pay structures (that entail significant variation in average lives depending on realized prepayment patterns) and to offer investors greater protection from prepayment risk (both contraction and extension risk). PAC bonds bring increased predictability of cash flows, as they specify a repayment schedule that will be satisfied as long as actual prepayments realized from the collateral fall within a predefined band.

We illustrate the characteristics of a PAC bond through a CMO structure that has one PAC tranche and one support tranche. First, a lower and upper PSA prepayment rate assumption is made for the PAC tranche. This is known as the initial PAC collar or initial PAC band. Principal payments are calculated for each month under the lower PSA prepayment assumption and the upper PSA prepayment assumption, and investors in the PAC tranche are promised a minimum monthly principal payment equal to the **lower** of the principal payments under the two PSA assumptions. The greater certainty in payments for the PAC tranche comes at the expense of greater uncertainty for the support or companion tranche, which absorbs the prepayment risk. Note that the support tranche provides **two-sided protection** (i.e., protection against extension **and** contraction risk) to the PAC tranche.

Example 2-2: PAC Tranches

In this example, $400m worth of mortgage pass-through securities carrying a coupon rate of 7.5% (that are backed by a pool of mortgage loans with a weighted average coupon of 8.125% and a weighted average maturity of 357 months) serve as collateral for a planned amortization class (PAC) CMO structure with one PAC tranche and one support tranche. The PAC tranche has an initial PAC collar of 90 PSA and 300 PSA.

Tranche	Par Amount ($)	Coupon Rate (%)
PAC	243,800,000	7.5
Support	156,200,000	7.5
Total	400,000,000	

Payment Rules

- For payment of monthly coupon interest: Disburse monthly coupon interest to each tranche on the basis of the amount of principal outstanding for the tranche at the beginning of the month.
- For disbursement of principal payments: Disburse principal payments to the PAC tranche based on its schedule of principal repayments. The PAC tranche has priority with respect to current and future principal payments to satisfy the schedule. Any excess principal payments in a month over the amount necessary to satisfy the schedule for the PAC tranche are paid to the support tranche. When the support tranche is completely paid off, all principal payments are to be made to the PAC tranche regardless of the schedule.

Table 2-6 lists the average lives of the PAC and support tranches under a variety of prepayment rate assumptions.

Table 2-6: Average Lives for PAC and Support Tranches under Various Prepayment Speeds

Prepayment Rate (PSA)	PAC Bond (P)	Support Bond (S)
0	15.97	27.26
50	9.44	24
90	7.26	20.06
100	7.26	18.56
150	7.26	12.57
165	7.26	11.16
200	7.26	8.38
250	7.26	5.37
300	7.26	3.13
350	6.56	2.51
400	5.92	2.17
450	5.38	1.94
500	4.93	1.77
700	3.7	1.37

Notice the following:

- Over the collateral's life, if prepayments occur at any rate that falls within the initial PAC collar (90300 PSA), the average life of the PAC tranche will be known with certainty (7.26 years).
- Over the collateral's life, if actual prepayments occur at a rate that is slower than the lower PSA band (90 PSA), the average life of the PAC tranche extends, but that of the support tranche extends more substantially (as it carries more extension risk). The PAC tranche has a priority claim against principal payments, so principal payments to the support tranche are deferred until the PAC tranche receives its scheduled principal payments. This reduces extension risk for the PAC tranche.
- Over the collateral's life, if actual prepayments occur at a rate that is faster than the upper PSA band (300 PSA), the average life of the PAC tranche contracts, but that of the support tranche contracts more substantially (as it carries more contraction risk as well). All excess principal payments are absorbed by the support tranche until it is fully paid off. Once this occurs, all principal payments will be distributed to the PAC tranche, which would then be referred to as a broken or busted PAC.
- The average life of the support tranche fluctuates more wildly than that of the PAC tranche. Therefore, support tranches entail the greatest prepayment risk.
- The greater the par value of the support tranche relative to that of the PAC tranche, the greater the prepayment protection for the PAC, the lower the PAC's average life variability, and the greater the support tranche's average life variability.

CMO structures can also be created with more than one PAC tranche. Each PAC tranche has its own schedule of payments, with each tranche being paid off in sequence.

Floating-Rate Tranches

There is often demand for tranches that carry a floating rate. Even though the collateral for CMOs carries a fixed rate, it is possible to create a floating-rate tranche (along with an inverse-floater tranche) from any of the fixed-rate tranches in a CMO structure. Since the floater varies positively with interest rates, and the inverse floater varies negatively with interest rates, they offset each other.

NONAGENCY RESIDENTIAL MORTGAGE-BACKED SECURITIES

As mentioned earlier, nonagency RMBS are not guaranteed by the government or a GSE, which makes the evaluation of credit risk an important consideration when investing in them. Within the nonagency RMBS sphere, market participants identify two types of transactions based on whether the mortgage loans in the collateral pool are prime or subprime loans. Prime loans are those advanced to borrowers with high credit quality and substantial equity in the underlying property, whereas subprime loans refer to loans advanced to borrowers with lower credit quality or loans that do not have first lien on the underlying property (i.e., the current potential lender has a subordinate claim on the underlying property).

While nonagency RMBS share many similarities with agency RMBS in terms of features and structures, the following two complementary mechanisms are typically required in structuring nonagency RMBS:

1. Cash flows are distributed according to a set of rules that determines the distribution of interest and principal payments to tranches with varying levels of priority/seniority.
2. There are also rules for allocating realized losses, with senior tranches having a priority claim over payments and subordinated tranches absorbing losses.

When it comes to forecasting cash flows on nonagency MBS, investors must make assumptions regarding (1) the default rate for the collateral and (2) the recovery rate (in case of default, repossession and subsequent sale of a recovered property typically generate cash flows for payments to bondholders).

In order to obtain favorable credit ratings, nonagency RMBS (as well as other types of asset-backed bonds that are discussed later in this reading) are credit enhanced. The amount of credit enhancement built into an issue depends on the credit rating desired by the issuer.

Internal Credit Enhancements

Internal credit enhancements include:

- Reserve funds:
    - Cash reserve funds: The entity seeking to raise the funds deposits some of the proceeds of sale of the loan pool with the SPV. This cash can be used to pay for potential future losses.
    - Excess spread accounts: The excess spread or cash that remains after the payment of net coupon, servicing fees, and all other expenses is kept in reserve to pay for future credit losses on the collateral. For example, if the

interest rate paid by borrowers in the loan pool is 5.75%, and 0.75% is paid for servicing fees while 4.75% is the interest rate paid to bond classes, the excess spread equals 5.75% (0.75% + 4.75%) = 0.25% or 25 bps.

Note that if any funds remain in the reserve account after paying off the last bond class in the securitization, the cash is returned to the residual owner of the SPV.

- Overcollateralization: This refers to a situation where the value of the collateral exceeds the par value of the securities issued by the SPV. The amount of overcollateralization (excess collateral) can be used to absorb future losses. For example, a securitization backed by $350m worth of loans against which securities with a total par value of $325m are issued is overcollateralized by $25m. Note that the amount of overcollateralization for an issue changes over time due to (1) defaults, (2) amortization, and (3) prepayments.
- Senior/subordinate structure: This type of structure has senior bond classes and subordinate bond classes (also known as non-senior/junior bond classes). The structure basically provides credit tranching as the subordinate bond classes provide credit protection to the senior classes. More formally, credit tranching occurs when credit risk (or risk of loss from default) is redistributed among the various classes of ABS. To understand how the senior-subordinate structure works, consider the following ABS structure:

Bond Class	**Par Value ($ millions)**	**Initial Pass-Through Rate (%)**
A (Senior)	250	3
B (Subordinate)	60	3.5
C (Subordinate)	15	4.2
Total	325	

The first $15m of losses are absorbed by Class C. Losses in excess of $15m are absorbed by Class B until total losses exceed $75m. Class C provides credit enhancement not only to Class A (the senior bond class), but also to Class B (the other subordinate bond class). Class A only suffers from credit losses once the total amount of credit losses exceeds $75m.

The issue with the senior-subordinate structure is that the level of credit protection provided by subordinate classes changes over time due to voluntary prepayments and defaults. To protect investors in the senior bond classes, a shifting interest mechanism is added to the structure. This mechanism locks out subordinated classes from receiving payments for a period of time if the collateral performs poorly, thereby ensuring that the level of credit protection enjoyed by the senior classes does not deteriorate over time.

External Credit Enhancements

External credit enhancements are third-party guarantees for payments to security holders should the issuer not be able to meet payment requirements. Monoline insurers are the most common third-party financial guarantors. Their business is restricted to providing guarantees for financial products (such as municipal securities and ABS). However, the popularity of monoline insurers has declined in recent years following the financial difficulties and downgrades of major monoline insurers in the aftermath of the financial crisis of 2007 to 2009.

LESSON 3: COMMERCIAL MORTGAGE-BACKED SECURITIES (CMBS) AND NON-MORTGAGE ASSET-BACKED SECURITIES (ABS)

LOS 54g: Describe the characteristics and risks of commercial mortgage-backed securities. Vol 5, pp 500–504

COMMERCIAL MORTGAGE-BACKED SECURITIES (CMBS)

Credit Risk

CMBS are backed by a pool of commercial mortgage loans on income-generating properties (e.g., apartments, warehouses, shopping centers, etc.). In the United States (and many other countries), commercial mortgage loans are nonrecourse loans (i.e., if there is a default and proceeds from sale of the property are insufficient for repayment, the lender has no recourse to the borrower for the unpaid amount). As a result, evaluation of credit risk for commercial mortgage loans requires examining the income-generating capacity and value of each property on a stand-alone basis.

Two measures are commonly used to evaluate the potential credit performance of a commercial property:

- The debt-service coverage (DSC) ratio is used to evaluate the adequacy of income generated from the property to service the loan. It is calculated as net operating income (NOI) divided by debt service. NOI is calculated as rental income minus cash operating expenses and a noncash replacement reserve that reflects depreciation of the property over time.
    - A ratio greater than 1 means that cash flow from the property covers debt servicing costs adequately.
    - The higher the ratio, the lower the credit risk.
- The loan-to-value ratio equals the loan amount divided by the appraised value of the property.
    - The lower the ratio, the lower the credit risk.

Basic CMBS Structure

- A credit-rating agency determines the level of credit enhancement required for the issuer to attain the credit rating desired by the issuer. For example, if the DSC and LTV ratios are unable to warrant the desired credit rating, the issuer may use subordination to attain the desired credit rating.
- Different bond classes are created, with each class having a different priority on cash flows. The various bond classes are retired sequentially with all principal repayments, prepayments, and proceeds from default (from selling repossessed properties) being used to repay the highest-rated tranche first.
- Losses from loan defaults are charged against the lowest-rated tranche first. If this tranche is not rated by credit-rating agencies, it is known as the first-loss piece, residual tranche, or equity tranche. The equity tranche typically has no specific interest rate (as it is the residual tranche). Investors price it based on the expected residual rate of return. Actual returns can be better or worse than expected depending on actual future interest rate movements and actual defaults.
- Interest payments are made to all tranches.

Two characteristics that are usually specific to CMBS are call protection and balloon maturity provisions.

1. Call protection

Typically, CMBS investors have significant call protection, which actually results in these securities trading more like corporate bonds than like RMBS. Call protection can come at (1) the loan level or (2) the structural level.

Call protection at the loan level can come in the following forms:

- Prepayment lockouts, which prohibit any prepayments during a specified period of time.
- Defeasance, which occurs when the borrower, instead of prepaying the loan, provides funds to the servicer to invest in a portfolio of Treasuries whose cash flows replicate those of the loan assuming no prepayments. Upon completion of the defeasance period, these securities are liquidated to pay off the loans. Note that there is no cash distribution to bondholders when the loan is defeased. Further, the credit quality of the deal improves as cash flows from Treasuries replace expected cash flows from the commercial loan pool as collateral. The cost of creating such a portfolio is the cost of defeasing the loan that must be paid by the issuer.
- Prepayment penalties are levied upon borrowers if they wish to refinance their loans.
- Yield maintenance charges are intended to compensate the lender for interest lost due to prepayments. These charges make lenders indifferent to the timing of prepayments, as they are always "made whole."

Call protection at the structure level comes from credit tranching. Senior tranches are given priority on principal payments, while principal losses from defaults impact the lowest-rated tranches first.

2. Balloon maturity provisions

Most commercial loans that back CMBS have balloon maturity provisionsthat is, they require a significant amount of principal to be repaid at maturity (as opposed to during the term of the loan). This exposes investors to balloon risk, as there will be a "default event" if the borrower is unable to generate the entire (significant) amount of funds required to retire the balloon balance by either (1) refinancing the loan or (2) selling the property. In the context of CMBS transactions, balloon risk is more like extension risk, as there is usually a workout period during which the borrower and lender try to modify the original terms of the loan to ensure eventual repayment. Note that the interest rate charged over the workout period (known as default interest rate) may be higher than the interest rate on the original loan.

LOS 54h: Describe types and characteristics of non-mortgage asset-backed securities, including the cash flows and credit risk of each type.

Vol 5, pp 504–508

NON-MORTGAGE ASSET-BACKED SECURITIES

Non-mortgage assets that have been used as collateral in securitizations include auto loan and lease receivables, credit card receivables, personal loans, and commercial loans.

Generally speaking, the collateral backing asset-backed securities can be classified as either amortizing or nonamortizing assets.

Amortizing loans

- The periodic cash flows include interest payments, principal repayments (in accordance with an amortization schedule), and (if allowed) prepayments.
- Examples include residential mortgage loans and automobile loans.

Nonamortizing loans

- These only require a monthly minimum payment with no scheduled principal payment.
- If the payment received is less than the interest amount due, the shortfall is added to the outstanding loan balance.
- If the payment is greater than the interest amount due, the excess serves to reduce the outstanding loan balance.
- Since there is no scheduled principal payment amount, the concept of prepayment does not apply to nonamortizing assets.
- Examples include credit card receivables.

The type of collateral (amortizing versus nonamortizing assets) has a significant effect on the structure of the securitization.

- When amortizing assets are securitized, the total face value (total amount outstanding) declines over time due to scheduled repayments and prepayments, and the number of outstanding loans (composition of the collateral) declines as a result of (1) defaults and (2) full principal repayments or full amortizations.
- On the other hand, securitizations of nonamortizing loans usually take the form of a revolving structure, where the composition of the collateral can change over the term of the securities. This is because principal repayments can be either (1) reinvested by purchasing additional loans or (2) passed on to security holders.
  - During an initial lockout or revolving period (which immediately follows the origination of the transaction), all principal repayments are reinvested in additional loans with a principal amount equal to the total principal amount received. While this can result in a smaller total number of individual loans comprising the collateral, the total face value outstanding remains the same.
  - During the principal amortization period (which follows the lockout period) principal repayments are not used to purchase additional loans, but are distributed to security holders.

We now describe the securitization of the two most popular non-mortgage assets: (1) auto loan receivables (that are amortizing loans) and (2) credit card receivables (that are nonamortizing loans).

Auto Loan Receivable–Backed Securities

These securities are backed by auto loan and lease receivables. Cash flows for auto loan–backed securities consist of regularly scheduled monthly interest and principal payments and prepayments. Prepayments result from the following:

- Sales and trade-ins requiring full payoff of the loan.
- Repossession and subsequent resale of the automobile.
- Proceeds from insurance claims arising from loss or destruction of the vehicle.
- Cash payments to save interest costs.
- Refinancing of the loan at lower interest rates.

Generally speaking, auto loan–backed securitizations come with some form of credit enhancement. Typically, the securitizations involve a senior/subordinate structure (to provide credit protection to the senior tranches). Further, some securitizations also involve reserve accounts, overcollateralization, and excess interest on the receivables.

Credit Card Receivable–Backed Securities

Credit cards may be issued by banks, retailers, and travel and entertainment companies. When a purchase is made on a credit card, the credit card company (the credit card issuer or lender) effectively extends a loan to the card holder (borrower) equal to the cost of purchase. In return, the card holder agrees to repay the amount borrowed along with any applicable finance charges. These receivables for the credit card company are used as collateral for credit card receivable–backed securities.

- The cash flow from a pool of credit card receivables includes (1) finance charges collected (interest charges on the unpaid balance after the grace period), (2) fees (for late payments and membership), and (3) principal repayment.
- Interest payments are made to security holders periodically (monthly, quarterly, or semiannually). The interest rate may be fixed or floating (typically with no cap).

As mentioned earlier, credit card receivable–backed securities are nonamortizing securities. During the revolving period (also known as the lockout period) any principal repayments from the pool of receivables are used to purchase additional receivables to maintain the size of the pool. During this period, the cash flow passed on to security holders consists only of (1) finance charges and (2) fees collected from the pool of receivables. Principal repayments are passed on to security holders only once the principal-amortizing period sets in.

Even though receivables in a revolving structure may not be prepaid (since there is no concept of prepayment here), all the bonds may be retired early if an early or rapid amortization provision is triggered. An example of a trigger for early amortization is poor performance of the collateral. In such a situation, principal repayments received from the collateral pool during the lockout period would be passed on to security holders to pay down principal, not to purchase additional loans.

LESSON 4: COLLATERALIZED DEBT OBLIGATIONS (CDOS)

LOS 54i: Describe collateralized debt obligations, including their cash flows and risks. Vol 5, pp 508–511

A collateralized debt obligation (CDO) is a security that is backed by a diversified pool of securities that may include:

- Corporate bonds and emerging market bonds (collateralized bond obligations or CBOs).
- ABS, RMBS, and CMBS (structured finance CDOs).
- Leveraged bank loans (collateralized loan obligations or CLOs).
- Credit default swaps and other structured securities (synthetic CDOs).

Structure of a CDO Transaction

- Like an ABS, a CDO also involves the creation of an SPV.
- A CDO manager (also known as collateral manager) is responsible for managing the collateral portfolio of assets (consisting of debt obligations).
- The funds used to purchase the collateral assets are raised from the issuance of bonds to investors. Bond classes may include senior bonds, mezzanine bonds (with credit ratings between senior and subordinated bonds), and subordinated bonds (also known as the residual or equity class).
- Restrictive covenants are placed on the manager to ensure that the credit ratings assigned to the various tranches at issuance are maintained during the term of the CDO.
- Cash flows from the underlying portfolio of assets include (1) coupon interest payments, (2) proceeds from maturing assets, and (3) proceeds from sale of assets. These cash flows are used to make interest and principal payments to the various bond classes.
- From the asset manager/issuer's perspective, the aim is to earn a rate of return on the collateral pool of assets that is higher than the interest costs of bonds issued. This excess return accrues to the equity holders and the CDO manager. Effectively, issuing a CDO is like undertaking a leveraged transaction where the idea is to use borrowed funds (raised from bonds issued) to generate a return that exceeds the funding cost.
- From the investors perspective, each class of bonds entails a different level of risk. Senior/mezzanine bond investors may be able to earn a potentially higher return than on a comparably rated corporate bond by gaining exposure to debt products that they may not otherwise be able to purchase. Equity investors can earn an equity-type return by taking the (higher) risk associated with the subordinated class.
- Certain restrictions are placed on the manager (via various tests and limits) to ensure that the senior bond classes are adequately protected and the ratings issued to the bond classes are maintained. Failure to meet these tests may trigger an immediate payoff to the senior bond classes until the tests are satisfied. This payoff would have the effect of deleveraging the CDO (as the asset manager's reliance on its cheapest source of funding [i.e., senior bonds] would be reduced).

The example in the next section will make the cash flows and credit risks of a CDO clearer.

Illustration of a CDO Transaction

Example 4-1: A Hypothetical CDO Transaction

Consider a $50 million CDO with the following structure:

Tranche	Par Value ($)	Coupon Rate
Senior	40,000,000	LIBOR + 50 basis points
Mezzanine	5,000,000	5-year Treasury rate plus 100 basis points
Subordinate/Equity	5,000,000	

The collateral backing the CDO is composed of corporate bonds with a par value of $50million. They mature in five years and offer a coupon rate equal to the 5-year Treasury rate plus 200 basis points (bps).

Notice that the senior tranche (which comprises the largest portion of the total issue) is variable-rate, while the collateral consists of fixed-rate assets. In order to eliminate

An interest rate swap is simply an agreement between two parties to exchange periodic interest payments based on a certain notional amount.

this mismatch, the asset manager enters into an interest rate swap with the following characteristics:

- A notional principal of $40 million.
    - The notional principal on the swap is $40m (not $50m) because the floating-rate exposure of the issuer is limited to the par value of the senior tranche.
- The fixed-rate payer (the asset manager) will make payments at a rate equal to the 5-year Treasury rate plus 50 basis points.
- The floating-rate payer (the swap counterparty) will make payments at LIBOR.

Assume that the 5-year Treasury rate at the time when the CDO is issued equals 5%. In this case:

Interest received by the asset manager on the collateral is calculated as:

$50,000,000 × (0.05 + 0.02) = $3,500,000

Interest paid to the senior CDO tranche is calculated as:

$40,000,000 × (LIBOR + 50 bps)

Interest paid to the mezzanine tranche is calculated as:

$5,000,000 × (0.05 + 0.01) = $300,000

Interest paid by the asset manager on the interest rate swap as the fixed-rate payer is calculated as:

$40,000,000 × (0.05 + 0.005) = $2,200,000

Interest received by the asset manager on the interest rate swap as the floating-rate receiver is calculated as:

$40,000,000 × (LIBOR)

Total interest payments received by the asset manager are calculated as the sum of floating-rate receipts from the swap and the interest received on the collateral:

Interest from collateral + Interest from swap counterparty
= $3,500,000 + $40,000,000 × (LIBOR)

Total interest paid out by the asset manager is calculated as the sum of payments to the senior and mezzanine CDO tranches and the fixed-rate payments on the swap:

Interest to senior tranche + Interest to mezzanine tranche + Fixed-rate interest on swap
= $40,000,000 × (LIBOR + 50 bps) + $300,000 + $2,200,000
= $2,500,000 + $40,000,000 × (LIBOR + 50 bps)

Netting the interest payments coming in and going out, we have:

$$\$3{,}500{,}000 + \$40{,}000{,}000 \times (\text{LIBOR}) - [\$2{,}500{,}000 + \$40{,}000{,}000 \times (\text{LIBOR} + 50 \text{ bps})]$$
$$= \$1{,}000{,}000 - (\$40{,}000{,}000 \times 50 \text{ bps})$$
$$= \$800{,}000$$

Asset management fees and other charges are deducted from the net interest earned by the CDO to determine the amount earned by the equity tranche. For example, if these charges amount to $100,000, then the annual return on the equity tranche equals:

$$\frac{\$800{,}000 - \$100{,}000}{\$5{,}000{,}000} \times 100 = 14\%$$

Do note, however, that we have made several simplifying assumptions in this example, including:

- There are no defaults. If there were defaults, there would be a risk that the manager may not be able to pay off the senior and/or mezzanine tranches, resulting in a loss. Investors in the equity tranche risk the loss of their entire investment.
- The collateral is composed of bonds that are all noncallable. Therefore, the coupon rate does not decline from securities being called.
- Practically speaking, the asset manager must start returning principal to the senior and mezzanine tranches once the reinvestment period is over (i.e., the securities issued do not have bullet maturities like the example assumed). Therefore, the interest rate swap must be structured in a manner that accounts for the amortization of principal over time.

The major difference between an ABS and a CDO is that in an ABS the cash flows from the collateral pool are used to pay off bondholders without the active management of collateral. In a CDO, the manager buys and sells debt obligations (assets) to (1) generate the cash flow required to repay bondholders and to (2) earn a competitive return for the equity tranche.

Study Session 16: Fixed Income: Analysis of Risk

Reading 55: Understanding Fixed-Income Risk and Return

LESSON 1: SOURCES OF RISK

LOS 55a: Calculate and interpret the sources of return from investing in a fixed-rate bond. Vol 5, pp 526–532

SOURCES OF RETURN

There are three sources of return on a fixed-rate bond:

1. Receipt of promised coupon and principal payments.
2. Reinvestment of coupon payments.
3. Potential capital gains/losses if the bond is sold prior to maturity.

In this reading, we will assume that issuers always make promised coupon and principal payments, so our focus here is not on credit risk. Instead, we focus on the impact of a change in interest rates, which affects (1) income from reinvestment of coupon payments and (2) the market price of the bond if sold prior to maturity.

If a bond is purchased at a premium/discount, it adds another dimension to the total rate of return:

- A **discount** bond offers a coupon rate that is *lower* than the required rate of return, so amortization of the discount (as the bond's carrying value is pulled **up** to par as it nears maturity) serves to enhance the return to bring it in line with the market discount rate.
- A **premium** bond offers a coupon rate that is *higher* than the required rate of return, so amortization of the premium (as the bond's carrying value is pulled **down** to par as it nears maturity) serves to lower the return to bring it in line with the market discount rate.

We will now go through a series of examples to demonstrate the effects of changes in interest rates on total rates of return realized by two investors:

- Investor A is a buy-and-hold investor who holds on to the bond until maturity.
- Investor B is a short-term investor who sells the bond prior to maturity.

In Examples 1-1 and 1-2, interest rates are unchanged, but the investors have different investment horizons.

Example 1-1

Investor A (the buy-and-hold investor) purchases a 10-year, 8% annual-pay bond at a price of 85.503075 (per 100 of par). The investor holds the bond until maturity and is able to reinvest all coupon payments received during the term of the bond at the YTM at issuance.

1. Compute the total amount of reinvestment income earned by Investor A.
2. Determine her realized rate of return on the investment.

Solution:

1. In order to calculate the amount of reinvestment income earned over the bond's term, we first need to calculate the YTM at issuance (which is the rate at which coupon payments are reinvested):

   $N = 10$; PMT = \$8; PV = –\$85.503075; FV = \$100; CPTI / Y; I / Y = 10.40%

   Next, we must compute the future value of the ten \$8 coupon payments that are received over the bond's term if they are invested until maturity at the bond's YTM (10.40%):

   I / Y = 10.40; PMT = \$8; N = 10; PV = 0; CPT FV; FV = \$129.970678

   We can now compute total reinvestment income as the difference between the FV of this annuity (calculated as \$129.970678) and the total amount of coupon income (10 × \$8 = \$80) received from the bond.

   Reinvestment income = \$129.970678 – \$80 = \$49.970678

   The investor's total return equals the sum of the reinvestment income, coupon payments and principal redemption at maturity.

   Total return = \$49.970678 + \$80 + \$100 = \$229.970678

2. The realized rate of return is calculated as:

$$\$85.503075 = \frac{\$229.970678}{(1+r)^{10}} = \$1,000$$

$$r = 0.1040 \text{ or } 10.40\%$$

Think of this reinvestment income as the "interest-on-interest" income from compounding.

Takeaway: This example illustrates that the YTM represents the investor's realized rate of return on a bond if the following three conditions hold:

1. The bond is held till maturity.
2. The issuer does not default.
3. All coupon payments can be reinvested through the term of the bond at a rate that equals the bond's YTM at issuance.

Example 1-2

Investor B (the short-term investor) purchases a 10-year, 8% annual-pay bond at a price of 85.503075 (per 100 of par) implying a YTM of 10.40%. The investor holds the bond for 4 years, after which she sells the bond at a price that entails a YTM of 10.40% (the YTM at date of sale is the same as the YTM at issuance). She is also able to reinvest the coupons at 10.40%.

1. Compute the total amount of reinvestment income earned by Investor B.
2. Determine her realized rate of return on the investment.

Solution:

1. The future value of the four \$8 coupon payments that are received over the investment horizon if they are invested until maturity at 10.40% (YTM at issuance) is calculated as:

   I/Y = 10.40; PMT = \$8; N = 4; CPT FV; FV = \$37.347111

   Total reinvestment income is calculated as:

   Reinvestment income = \$37.347111 – (\$8 × 4) = \$5.347111

   The bond's selling price at the end of Year 4 is calculated as the PV of expected future payments:

   N = 6; I/Y = 10.40; PMT = \$8; FV = \$100; CPT PV; PV = \$89.66877

   The investor's total return is calculated as the sum of the selling price, coupon receipts and reinvestment income:

   Total return = \$89.66877 + \$32 + \$5.347111 = \$127.015881

2. The realized rate of return is calculated as:

   $$\$85.503075 = \frac{\$127.015881}{(1+r)^4}$$

   r = 0.1040 or 10.40%

Investor B's **horizon yield** (or annualized holding period rate of return) in this example is 10.40%.

Takeaway: This example illustrates that the horizon yield equals the YTM at issuance if the following two conditions hold:

1. The bond is sold at its **carrying value** (i.e., at a price that lies on its **constant-yield price trajectory**).
2. All coupon payments can be reinvested at a rate that equals the bond's YTM at issuance until date of sale.

Carrying value refers to the value of a bond (at any time between the purchase date and maturity date) that entails the same YTM as when the bond was purchased. The carrying value reflects amortization of any premium/discount since the time of purchase. To facilitate your understanding of carrying value of a bond, think back to the FRA section where we described the effective interest method of determining the carrying value of a liability.

Going back to Example 1-2, the carrying value of the bond at the end of Year 1 equals \$86.395394 (N = 9; I/Y = 10.40; PMT = \$8; FV = \$100; CPT PV), which means that discount amortization for Year 1 equals \$0.892319 (= 86.395394 – 85.503075). By the end of Year 4 (time of sale) the carrying value of the bond has risen to \$89.66877. This increase in the value of the bond is a movement along the constant-yield price trajectory (YTM = 10.40%).

Capital gains/losses arise if a bond is sold at a price different from its carrying value. In Example 1-2, at the end of Year 4, the bond is sold at a price that equals its carrying value ($89.66877, priced to yield 10.40%), so there is no capital gain/loss.

In Examples 1-3 and 1-4, we illustrate the impact on our investors' horizon yields if interest rates rise by 100 bps (i.e., the market discount rate increases from 10.40% to 11.40%).

Example 1-3

Investor A (the buy-and-hold investor) purchases a 10-year, 8% annual-pay bond at a price of 85.503075 (per 100 of par). After she purchases the bond, but before she receives the first coupon payment, interest rates rise to 11.40%.

1. Compute the future value (FV) of reinvested coupons (coupon receipts plus reinvestment income).
2. Determine her realized rate of return on the investment.

Solution:

1. The future value of the ten $8 coupon payments that are received over the investment horizon if they are invested until maturity at 11.40% is calculated as:

   $I/Y = 11.40;\ PMT = \$8;\ N = 10;\ PV = 0;\ CPT\ FV;\ FV = \$136.380195$

2. The realized rate of return is calculated as:

   $$\$85.503075 = \frac{\$136.380195 + \$100}{(1+r)^{10}}$$

   $r = 0.1070$ or 10.70%

The buy-and-hold investor benefits from the higher coupon reinvestment rate as her realized rate of return *rises* by 30 bps from 10.40% (in Example 1-1) to 10.70%. Note that there is no capital gain or loss since the investor holds the bond until maturity.

Example 1-4

Investor B (the short-term investor) purchases a 10-year, 8% annual-pay bond at a price of 85.503075 (per 100 of par). After she purchases the bond, but before she receives the first coupon payment, interest rates rise to 11.40%. The investor sells the bond at the end of Year 4 (when it is priced to yield 11.40%).

1. Compute the future value (FV) of reinvested coupons (coupon receipts plus reinvestment income).
2. Determine her realized rate of return on the investment.

Solution:

1. The future value of the four $8 coupon payments that are received over the investment horizon if they are invested until the end of Year 4 at 11.40% is calculated as:

   $$I / Y = 11.40;\ PMT = \$8;\ PV = 0;\ N = 4;\ CPT\ FV;\ FV = \$37.89972$$

   The selling price of the bond (at the end of Year 4) is calculated as the present value of remaining payments:

   $$N = 6;\ I / Y11.40;\ PMT = \$8;\ FV = \$100;\ CPT\ PV;\ PV = \$85.780408$$

2. The realized rate of return is calculated as:

   $$\$85.503075 = \frac{37.899724 + \$85.780408}{(1+r)^4}$$
   $$r = 0.0967 \text{ or } 9.67\%$$

The short-term investor (Example 1-4) has a *lower* rate of return now that interest rates have gone up compared to her return if interest rates were to remain unchanged (Example 1-2) –9.67% vs. 10.40%. Even though the FV of reinvested coupons goes up ($37.899724 here vs. 37.347111 in Example 1-2) with the increase in interest rates, this benefit is more than offset by the resulting capital loss of $3.888362 (= $89.66877 – $85.780408) as the bond is sold for less than its carrying value.

Examples 1-5 and 1-6 assume that interest rates fall by 100 bps from 10.40% to 9.40%.

Example 1-5

Investor A (the buy-and-hold investor) purchases a 10-year, 8% annual-pay bond at a price of 85.503075 (per 100 of par). After she purchases the bond, but before she receives the first coupon payment, interest rates decline to 9.40%.

1. Compute the future value (FV) of reinvested coupons (coupon receipts plus reinvestment income).
2. Determine her realized rate of return on the investment.

Solution:

1. The future value of the ten $8 coupon payments that are received over the investment horizon if they are invested until maturity at 9.40% is calculated as:

   $$I / Y = 9.40;\ PMT = \$8;\ PV = 0;\ N = 10;\ CPT\ FV;\ FV = \$123.888356$$

2. The realized rate of return is calculated as:

   $$\$85.503075 = \frac{\$123.888356 + \$100}{(1+r)^{10}}$$
   $$r = 0.1010 \text{ or } 10.10\%$$

The buy-and-hold investor suffers from the lower coupon reinvestment rate as her realized rate of return *falls* by 30 bps from 10.40% to 10.10%. Note that there is no capital gain or loss since the investor holds the bond until maturity.

Example 1-6

Investor B (the short-term investor) purchases a 10-year, 8% annual-pay bond at a price of 85.503075 (per 100 of par). After she purchases the bond, but before she receives the first coupon payment, interest rates fall to 9.40%. The investor sells the bond at the end of Year 4 (when it is priced to yield 9.40%).

1. Compute the future value (FV) of reinvested coupons (coupon receipts plus reinvestment income).
2. Determine her realized rate of return on the investment.

Solution:

1. The future value of the four \$8 coupon payments that are received over the investment horizon if they are invested until the end of Year 4 at 9.40% is calculated as:

   $I/Y = 9.40$; $PMT = \$8$; $N = 4$; CPT FV; $FV = \$36.801397$

   The selling price of the bond (at the end of Year 4) is calculated as:

   $N = 6$; $I/Y = 9.40$; $PMT = \$8$; $FV = \$100$; CPT PV; $PV = \$93.793912$

2. The realized rate of return is calculated as:

   $$\$85.503075 = \frac{\$36.801397 + \$93.793912}{(1+r)^4}$$

   $r = 0.1117$ or 11.17%

The short-term investor has a higher rate of return, now that interest rates have gone down compared to her return; if interest rates were to remain unchanged – 11.17% here vs. 10.40% in Example 1-2. Even though the FV of coupon payments goes down (\$36.801397 here vs. 37.347111 in Example 1-2) with the decrease in interest rates, this loss is more than offset by the resulting capital gain of \$4.125142 (= \$93.793912 – \$89.66877) as the bond is sold for more than its carrying value.

It is very important that you understand the following:

- In all our examples:
  - **Interest income** for the investor is the return associated with the passage of time. It includes (1) coupon receipts, (2) reinvestment income, and (3) amortization of any discount (premium) from purchasing the bond at a price less (greater) than its par value.
  - A **capital gain (loss)** to the investor is associated with a change in value of the security caused by a change in the yield-to-maturity (market discount rate).

- In practice, the calculation of interest income and capital gains/losses reported in financial statements depends on financial and tax accounting rules.
- The investment horizon is an important factor in determining fixed-income risk and return. There are two types of **interest rate risk**, which offset each other:
    - **Reinvestment risk**. The future value of any interim cash flows received from a bond (these could be coupon payments as well as principal repayments on amortizing bonds) increases when interest rates rise and decreases when interest rates decline.
    - **Market price risk**. The selling price of a bond (at any point during its term or before maturity) decreases when interest rates rise and increases when interest rates decline.

Reinvestment risk matters more to a long-term investors (such as the buy-and-hold investor in our examples), while market price risk matters more to short-term investors. Therefore, two investors who are holding the same bond can have different exposures to interest rate risk depending on their individual investment horizons.

LESSON 2: INTEREST RATE RISK ON FIXED-RATE BONDS

LOS 55b: Define, calculate, and interpret Macaulay, modified, and effective durations. Vol 5, pp 534–544

LOS 55c: Explain why effective duration is the most appropriate measure of interest rate risk for bonds with embedded options. Vol 5, pp 541–544

Duration

Duration measures the sensitivity or responsiveness of a bond's **full price** (including accrued interest) to changes in **its yield-to-maturity** (or market discount rate). When measuring duration, we assume that all other variables that influence a bond's price, including its time to maturity, remain constant. There are several types of duration. Broadly speaking, duration can be classified as:

- **Yield duration**, which measures the responsiveness of a bond's price with respect to **its own yield-to-maturity**. Yield duration statistics include Macaulay duration, modified duration, money duration, and the price value of a basis point.
- **Curve duration**, which measures the responsiveness of a bond's price with respect to **a benchmark yield curve** (e.g., government yield curve, forward curve, or government par curve). Coupon duration statistics include effective duration.

Macaulay Duration

Macaulay duration represents the weighted average of the time it would take to receive all the bond's promised cash flows, where the weights are calculated as the present value of each cash flow divided by the bond's **full price**. See Examples 2-1 and 2-2.

Example 2-1: Calculating Macaulay Duration (for Annual-Pay Bond)

Consider a newly-issued 10-year, 8% annual-pay bond priced at 85.503075 per 100 of par value to yield 10.40%. Calculate the bond's Macaulay duration.

Solution:

Period (1)	Cash Flow (2)	Present Value (3)	Weight (4)	Period × Weight (5)
1	8	7.24638	0.08475	0.08475
2	8	6.56375	0.07677	0.15353
3	8	5.94542	0.06953	0.20860
4	8	5.38535	0.06298	0.25194
5	8	4.87803	0.05705	0.28525
6	8	4.41851	0.05168	0.31006
7	8	4.00227	0.04681	0.32766
8	8	3.62525	0.04240	0.33919
9	8	3.28374	0.03840	0.34564
10	108	40.15439	0.46963	4.69625
		85.50307	**1.00000**	**7.00288**

Calculations:

- Column 2: Annual coupon payment = 8% × \$100 = \$8
- Column 3 contains the present value of each cash flow.
    - The PV of the Year 10 cash flow (\$108) is calculated as 108 / $(1.104)^{10} = 40.154389$
- Column 4 shows the weight (i.e., the share of total market (present) value corresponding to each cash flow).
    - The weight of the Year 10 payment is calculated as 40.154389 / 85.503075 = 0.46963
- Column 5 multiplies the number of periods to receipt of the cash flow (Column 1) by the weight (Column 4). The sum of Column 5 (7.0029) is the Macaulay duration of this 10-year, 8% annual-pay bond.

Example 2-2: Calculating Macaulay Duration (for Semiannual-Pay Bond between Coupon Dates)

Consider a 7-year, 5% semiannual-pay bond that matures on September 15, 2020, which is purchased on January 3, 2014. The coupon payments are made on March 15 and September 15 each year. The yield-to-maturity is 5.00% quoted on a street-convention semiannual bond basis. Compute the bond's Macaulay duration at time of purchase.

Solution:

Period (1)	Time to Receipt (2)	Cash Flow (3) ($)	Present Value (4)	Weight (5)	Time × Weight (6)
1	0.4000	2.5	2.475429	0.02439	0.0097561
2	1.4000	2.5	2.415053	0.02380	0.0333135
3	2.4000	2.5	2.356149	0.02321	0.0557160
4	3.4000	2.5	2.298682	0.02265	0.0770058
5	4.4000	2.5	2.242616	0.02210	0.0972240
6	5.4000	2.5	2.187918	0.02156	0.1164101
7	6.4000	2.5	2.134555	0.02103	0.1346024
8	7.4000	2.5	2.082492	0.02052	0.1518381
9	8.4000	2.5	2.031700	0.02002	0.1681530
10	9.4000	2.5	1.982146	0.01953	0.1835816
11	10.4000	2.5	1.933801	0.01905	0.1981576
12	11.4000	2.5	1.886635	0.01859	0.2119134
13	12.4000	2.5	1.840620	0.01814	0.2248803
14	13.4000	102.5	73.624790	0.72542	9.7206330
			101.492586	**1.00000**	**11.383185**

Calculations:

- Column 1: There are 14 semiannual periods remaining until maturity (including the current period).
- Column 2 contains the number of semiannual periods to receipt of cash flow.
    - Payment 1 will be received in 1 – 108/180 = 0.4 semiannual periods, Payment 2 will be received in 2 – 108/180 = 1.4 semiannual periods, and so on.
- Column 3: The coupon payment per semiannual period is $2.50.
- Column 4: The annual yield-to-maturity is 5.00% so the yield per semiannual period is 2.50%. When this yield per semiannual period is used to compute the present value of each cash flow, the full price of the bond equals 101.492586 (sum of Column 4).
- Column 5 contains the weights (the shares of the full price corresponding to each cash flow).
- Column 6: The values in this column are obtained by multiplying the values in Column 2 by those in Column 5.

Macaulay duration equals the sum of the values in Column 6. However, note that the value obtained here (11.383185) is in terms of number of semiannual periods. Similar to coupon rates and yields-to-maturity, duration statistics are also annualized in practice. This is done by dividing the duration per period by the number of periods in the year. Therefore, in this example annualized Macaulay duration equals 11.383185 / 2 = 5.691592 years.

Macaulay duration can also be computed using the following closed-form formula:

$$\text{MacDur} = \left\{\frac{1+r}{r} - \frac{1+r+[N\times(c-r)]}{c\times\left[(1+r)^N-1\right]+r}\right\} - (t/T) \qquad \text{... (Equation 1)}$$

where:
c = Coupon rate per period (PMT/FV)

Note that Equation 1 uses the YTM *per period*, coupon rate *per period*, the number of *periods* until maturity, and the fraction of the current *period* that has elapsed. The number attained from applying the formula gives us the Macaulay duration **in terms of periods**, which can be annualized by dividing it by the number of periods in a year. See Example 2-3.

Example 2-3: Calculating Macaulay Duration Using the Closed-Form Formula

Determine the Macaulay duration of the fixed-rate bond in Example 2-2 using the general closed-form formula.

Solution:

$$\text{MacDur} = \left\{\frac{1+r}{r} - \frac{1+r+[N\times(c-r)]}{c\times[(1+r)^N-1]+r}\right\} - (t/T)$$

r = 0.025, c = 0.025, N = 14, and t/T = 108/180

Therefore,

$$\text{MacDur} = \left\{\frac{1+0.025}{0.025} - \frac{1+0.025+[14\times(0.025-0.025)]}{0.025\times[(1+0.025)^{14}-1]+0.025}\right\} - (108/180)$$

MacDur = 11.383185 semiannual periods or 5.691592 years

Macaulay duration is typically not used as a measure of the interest rate sensitivity of a bond's price (since better measures such as modified duration exist). However, it does have has some useful applications, including measurement of the **duration gap**, which is discussed towards the end of the reading.

Modified Duration

Modified duration is calculated by dividing Macaulay duration by one plus the yield per period. See Example 2-4.

$$\text{ModDur} = \frac{\text{MacDur}}{1+r}$$

Example 2-4: Calculating Modified Duration

Determine the annual modified duration of the fixed-rate bond in Example 2-2 given that its Macaulay duration is 11.383185 semiannual periods.

Solution:

$$\text{ModDur} = \frac{\text{MacDur}}{1+r}$$

$$\text{ModDur} = \frac{11.38315}{1+(0.05/2)} = 11.1055146 \text{ semiannual periods}$$

The annualized modified duration of the bond is **5.552773** (= 11.1055146/2).

Note that we can also calculate the annualized modified duration of the bond as the annualized Macaulay duration divided by 1 plus the yield per semiannual period:

$$\text{ModDur}_{\text{ANN}} = \frac{\text{MacDur}_{\text{ANN}}}{1+r}$$

$$\text{ModDur}_{\text{ANN}} = \frac{5.691592}{1+(0.05/2)} = 5.552773$$

Modified duration has a very important application in risk management. It can be used to estimate the percentage price change for a bond in response to a change in its yield-to-maturity.

$$\%\Delta PV^{\text{Full}} \approx -\text{AnnModDur} \times \Delta\text{Yield}$$

Note that the formula above estimates the percentage price change in the **full price** of a bond, and uses the **annual** modified duration and the **annual** yield-to-maturity. Also, bond prices are *inversely* related to changes in yields, which explains the minus sign in the formula. See Example 2-5.

Example 2-5: Using Modified Duration to Estimate the Change in a Bond's Price

Given an annualized modified duration of 5.552773 for the bond in Example 2-2, estimate the change in value of the bond in response to (1) a 100 bp increase in yields and (2) a 100 bp decrease in yields.

Solution:

If the yield-to-maturity increases by 100 bps (to 6.00%), the estimated loss in the value will be:

$$\%\Delta PV^{\text{Full}} = -5.552773 \times 0.0100 = -0.05552773 \text{ or } -5.55\%$$

If the yield-to-maturity decreases by 100 bp (to 4.00%), the estimated loss in the value will be:

$$\%\Delta PV^{\text{Full}} = -5.552773 \times -0.0100 = 0.05552773 \text{ or } 5.55\%$$

Notice that the absolute value of the percentage change in the price of the bond is the same for either an increase or a decrease in yields (5.55%). Modified duration only provides a linear estimate of the change in the price of a bond in response to a change in yields. It provides good estimates for bond prices in response to relatively small changes in yields, but its estimating accuracy fades with larger changes in yields as the curvature (convexity) of the price-yield profile becomes more pronounced. Recall that given the coupon rate and term to maturity, the percentage increase in the price of a bond in response to a decrease in yields is *greater* than the percentage decrease in its price in response to an equivalent increase in yields. Later in this reading, we will study the **convexity adjustment** to account for the asymmetric response of bond prices to changes in yields.

If Macaulay duration is not already known, annual modified duration can be estimated using the following formula:

$$\text{ApproxModDur} = \frac{(PV_{-}) - (PV_{+})}{2 \times (\Delta\text{Yield}) \times (PV_0)} \quad \text{... (Equation 2)}$$

This approximation basically estimates the slope of the line tangent to the price-yield profile for a bond at a particular yield level, as illustrated in Figure 2-1 (that follows Example 2-6). The value for approximate modified duration obtained by applying this formula gives us the percentage change in the price of a bond in response to a 100-bp (1%) change in yields. The percentage price change if yields were to change by 50 basis points would be half the figure obtained from applying the formula. Also note that the change in YTM (Δ Yield) must be entered as a decimal (not as a percentage) in Equation 2.

Example 2-6: Estimating Modified Duration

Using the bond that we worked with in Example 2-2, estimate its modified duration using the approximation formula. Compute PV_{-} and PV_{+} by decreasing and increasing yields by 5 bps.

Solution:

Given that the yield-to-maturity is 5.00%, the full price (PV0) of a 7-year, 5% semiannual-pay corporate bond (maturing on September 15, 2020 and settling on January 3, 2014) is calculated as:

$N = 14$; $PMT = -\$2.50$; $FV = -\$100$; $I/Y = 2.5\%$; CPT PV $\rightarrow$ $PV = \$100$

$PV_0 = \$100 \times (1.025)^{108/180} = \mathbf{\$101.492586}$

In order to compute PV_{+}, we raise the annual yield-to-maturity by 5 bps (from 5.00% to 5.05%). The yield per semiannual period therefore rises from 2.50% to 2.525%. PV_{+} is calculated as:

$N = 14$; $PMT = -\$2.50$; $FV = -\$100$; $I/Y = 2.525\%$; CPT PV $\rightarrow$ $PV = \$99.7082$

$PV_{+} = \$99.7082 \times (1.02525)^{108/180} = \mathbf{\$101.211273}$

In order to compute PV_-, we lower the annual yield-to-maturity by 5 bps (from 5.00% to 4.95%). The yield per semiannual period therefore falls from 2.50% to 2.475%. PV_- is calculated as:

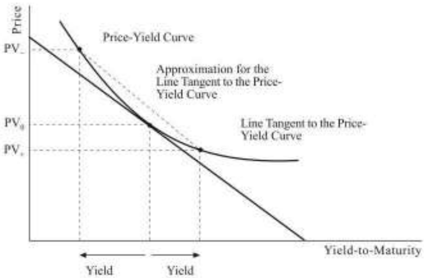
N = 14; PMT = –$2.50; FV = –$100; I/Y = 2.475%; CPT PV → PV = $100.2928

$PV_- = \$100.2928 \times (1.02475)^{108/180} = \mathbf{\$101.774839}$

Now that we have computed PV_0, PV_+ and PV_- we can estimate the duration of the bond given $\Delta\text{Yield} = 0.0005$ as:

$$\text{ApproxModDur} = \frac{\$101,774839 - \$101.211273}{2 \times 0.0005 \times \$101.492586} = \mathbf{5.55278}$$

Figure 2-1: Approximate Modified Duration



Important: Even though we computed the values of PV_+ and PV_- by moving yields up and down by 5 bps, the way that the modified duration formula is structured dictates that the value obtained for duration approximates the change in the value of a bond **for a 1% or 100-bp change in yields**. Therefore, we would interpret our value of modified duration as follows: If the market discount rate were to shift (up or down) by 1%, the estimated change in the value of the bond would be 5.55% (in the other direction).

Notice that our approximate value for modified duration, 5.55278% (using Equation 2 in Example 2-6) is very close to the actual value of 5.552773% (computed earlier in Example 2-4).

We can also use the approximate modified duration (ApproxModDur) to estimate Macaulay duration (ApproxMacDur) by applying the following formula:

$$\text{ApproxMacDur} = \text{ApproxModDur} \times (1 + r) \quad \text{... (Equation 3)}$$

Note that both Equation 2 and Equation 3 produce results for *annualized* modified and Macaulay durations. The frequency of payments and the periodicity of the yield-to-maturity are included in bond price calculations.

Effective Duration

Another approach for measuring a bond's interest rate risk is to compute **effective duration** (also known as **OAS duration**), which measures the sensitivity of a bond's price to a change in **the benchmark yield curve**.

$$\text{EffDur} = \frac{(PV_-) - (PV_+)}{2 \times (\Delta\text{Curve}) \times (PV_0)} \quad \text{... (Equation 4)}$$

Note that:

- Effective duration is a **curve duration** statistic that measures interest rate risk in terms of a change in the benchmark yield curve (ΔCurve).
- Modified duration is a **yield duration** statistic that measures interest rate risk in terms of a change in the bond's own yield-to-maturity (ΔYield).

Effective duration is the appropriate measure of risk **for bonds with embedded call options**, including callable bonds (and mortgage-backed bonds). For such bonds, future cash flows are uncertain as they depend on the path taken by interest rates in the future. Generally speaking, if interest rates fall (rise) there is a greater (lower) likelihood that the issuer will call the bonds. Since future cash flows are contingent on future interest rates, there is no well-defined internal rate of return or yield-to-maturity on these bonds. Therefore, yield duration statistics (i.e., modified and Macaulay duration) do not apply. Effective duration, being a curve duration statistic, is the appropriate duration measure here.

In case you were wondering, the sensitivity of a callable bond's price to a change in the yield-to-worst is also not an appropriate measure of duration either.

Option-pricing models are used to determine PV_- and PV_+, the inputs for computing effective duration for a callable bond. Note that these pricing models make use of several inputs, including (1) the length of the call protection period, (2) the schedule of call prices and call dates, (3) an assumption regarding credit spreads on top of benchmark yields, (4) an assumption about future interest rate volatility, and (5) the level of market interest rates (e.g., the government par curve). In order to compute PV_- and PV_+, the first four are held constant and then we change the fifth (by a specific number of bps). See Example 2-7.

Example 2-7: Computing Effective Duration

A callable bond is currently trading at 101.05 per 100 of par. When the government par curve is raised by 25 bps, the value of the bond falls to 99.04, and when it is lowered by 25 bps the value of the bond rises to 102.87. Compute the effective duration of this bond.

Solution:

- $PV_0 = 101.05$
- $PV_- = 102.87$
- $PV_+ = 99.04$
- $\Delta\text{Curve} = 0.0025$

$$\text{EffDur} = \frac{102.87 - 99.04}{2 \times 0.0025 \times 101.05} = 7.5804$$