# Chapter 17 – Theoretical Distributions

FREE FAST TRACK Lectures Every Day at 10:00 a.m.: https://www.youtube.com/watch?v=ZZpvRpkgmaE&list=PLAKrxMrPL3fwOSJWxnr8j0C9a4si2Dfdz

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Theoretical Distributions FAST TRACK Lecture 1 (Premiering on 21-06-2020 at 10:00 a.m.): https://youtu.be/4qdwY_CT-QA

Theoretical Distributions FAST TRACK Lecture 2 (Premiering on 22-06-2020 at 10:00 a.m.): https://www.youtube.com/channel/UCWNCx71oetd9G2RkMYeo1cQ

## Binomial Distribution

Binomial Distribution is used to find out the probability where the total no. of outcomes is huge. The probability is given by the following formula:

$P(x) = {}^{n}C_{x} p^{x} q^{n-x}$, for $x = 0, 1, 2, 3, \ldots, n$

Here,
$n$ = number of times the experiment is repeated
$x$ = the requirement of the question
$p$ = probability of success in each trial
$q$ = probability of failure in each trial = $1 - p$

Sometimes, $P(x)$ is also written as $f(x)$. $f(x)$ is called "Probability Mass Function".

### Conditions

Binomial distribution is applicable only if the following conditions are satisfied:

1. All the trials are independent, and
2. Each trial has only two outcomes.

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## Important Points

1. Binomial Distribution is applicable when the random variable ($x$) is discrete.
2. As $n>0$, $p,q>0$, therefore, $f(x)\geq 0$ for every $x$.

   Also, $\sum f(x)=f(0)+f(1)+f(2)+f(3)+...+f(n)=1$
3. Binomial distribution is known as biparametric distribution as it is characterised by two parameters $n$ and $p$. This means that if the values of $n$ and $p$ are known, then the distribution is known completely.
4. The mean of the binomial distribution is given by $\mu=np$.
   a. Page 3.896 – Question 93
5. Mode of a Binomial Distribution is given by $\mu_0=(n+1)p$
   a. If the value of $(n+1)p$ is an integer (i.e., without decimal part), then the binomial distribution is said to have two modes. It is called a bi-modal binomial distribution. The two modes are given by:
      i. $(n+1)p$, and
      ii. $[(n+1)p]-1$

      Page 3.900 – Question 103
   b. If the value of $(n+1)p$ is a fraction (i.e., with a decimal part), then the binomial distribution is said to have one mode. It is called a unimodal binomial distribution. Its mode is given by the largest integer contained in $(n+1)p$.

      Page 3.887 – Question 70
6. The variance of the binomial distribution is given by $\sigma^2=npq$.
   a. Variance of a binomial distribution is always less than its mean.
   b. If $p=q=0.5$, variance is the maximum, and is given by $\frac{n}{4}$.

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7. Standard Deviation of a binomial distribution is given by $\sigma=\sqrt{npq}$.

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   a. Page 3.873 – Question 43

   b. Page 3.862 – Question 20
8. Additive property of binomial distribution:
   Let *x* and *y* be two independent binomial distributions where *x* has the parameters $n_1$ and *p*, and *y* has the parameters $n_2$ and *p*. Then $(x+y)$ will be a binomial distribution with parameters $(n_1+n_2)$ and *p*.
9. Sometimes, Binomial Distribution is also written as $B(n, p)$. So, if, in a question you find something like "$X \sim B(5, 0.4)$", it means that $n = 5$, and $p = 0.4$. Here, $X$ denotes the requirement of the question.

## Poisson Distribution

Poisson Distribution is used to find out the probability where the total no. of outcomes is too huge and the probability of success is extremely small. The probability is given by the following formula:

$$P(x) = \frac{e^{-m} \times m^x}{x!}, \text{ for } x = 0, 1, 2, 3, \ldots, n$$

Here,
*e* = exponential constant = 2.71828
*m* = mean = *np*
*x* = the requirement of the question

Sometimes, $P(x)$ is also written as $f(x)$. $f(x)$ is called "Probability Mass Function".

### Questions to be Solved from Scanner

1. Page 3.862 – Question 19
2. Page 3.864 – Question 22
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13. Page 3.881 – Question 58 – Homework
14. Page 3.866 – Question 28 – Homework
15. Page 3.859 – Question 15
16. Page 3.892 – Question 87 – Homework

### Important Points

1. Poisson Distribution is applicable when the random variable ($x$) is discrete.

2. Since $e^{-m} = \frac{1}{e^m} > 0$, whatever may be the value of $m$ (>0), it follows that $f(x) > 0$ for every $x$.
   Also, $\sum f(x) = f(0) + f(1) + f(2) + f(3) + ... + f(n) = 1$.
3. Poisson distribution is known as a uniparametric distribution as it is characterised by only one parameter $m$.
4. The mean of Poisson distribution is given by $m$, i.e., $\mu = m = np$.
5. The variance of Poisson distribution is given by $\sigma^2 = m = np$.

   **Questions to be Solved from Scanner**
   a. Page 3.878 – Question 51
6. The standard deviation of Poisson distribution is given by $\sigma = \sqrt{m} = \sqrt{np}$.

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   a. Page 3.895 – Question 92
   b. Page 3.869 – Question 33 – Homework
7. Like binomial distribution, Poisson distribution could be also unimodal or bimodal depending upon the value of the parameter $m$.
   a. If $m$ is an integer, there are two modes:
      i. $m$
      ii. $m - 1$
   b. If $m$ is a fraction, the mode is given by the largest integer contained in $m$.

   **Questions to be Solved from Scanner**
   a. Page 3.870 – Question 36
8. Poisson approximation to Binomial distribution
   When $n$ is rather large and $p$ is rather small so that $m = np$ is moderate then $B(n, p) \cong P(m)$.
9. Additive property of Poisson distribution:
   Let $x$ and $y$ be two independent poisson distributions where $x$ has the parameter $m_1$, and $y$ has the parameter $m_2$. Then $(x + y)$ will be a poisson distribution with parameter $(m_1 + m_2)$.

### Applications

Poisson distribution is applied when the total number of events is quite large but the probability of occurrence is extremely small. Thus, we can apply Poisson distribution for the following cases:

1. The distribution of the no. of printing mistakes per page of a large book.
2. The distribution of the no. of road accidents on a busy road per minute.
3. The distribution of the no. of radio-active elements per minute in a fusion process.
4. The distribution of the no. of demands per minute for health centre and so on.

## Normal or Gaussian Distribution

$$P(x) = f(x) = \frac{1}{\sigma\sqrt{2\pi}} . e^{\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)}, \text{ for } -\infty < x < \infty$$

Here,
$e$ = exponential constant = 2.71828
$x$ = random variable
$\mu$ = mean of the normal random variable $x$
$\sigma$ = standard deviation of the given normal distribution

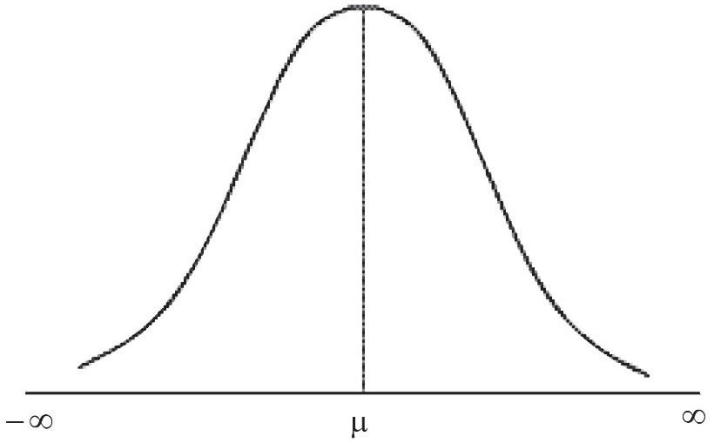
Sometimes, $P(x)$ is also written as $f(x)$. $f(x)$ is called "Probability Density Function".

## Questions to be Solved from Scanner

1. Page 3.897 – Question 95
2. Page 3.902 – Question 106
3. Page 3.876 – Question 48 – Homework

## Important Points

1. Normal Distribution is applicable when the random variable ($x$) is continuous.
2. If we plot the probability function $y = f(x)$, then the curve, known as probability curve, takes the following shape:




   The area under this curve gives us the probability.
3. The area between $-\infty$ and $\mu$ = the area between $\mu$ and $\infty$ = 0.5
4. If $\mu = 0$, and $\sigma = 1$, we have $f(z) = \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-z^2}{2}\right)}$, for $-\infty < z < \infty$.

   The random variable $z$ is known as standard normal variate (or variable) or standard normal deviate. It is given by $z = \frac{x-\mu}{\sigma}$.
5. Normal distribution is bell shaped.
6. It is unimodal.
7. The normal distribution is known as biparametric distribution as it is characterised by two parameters $\mu$ and $\sigma^2$. Once the two parameters are known, the normal distribution is completely specified.
8. Since the normal distribution is symmetrical about its mean ($\mu$), Mean = Median = Mode.
9. Mean Deviation = $0.8\sigma$.
10. Quartile Deviation = $0.675\sigma$.

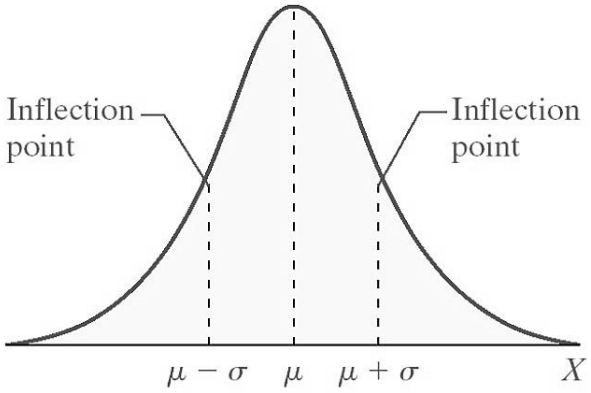
**Questions to be Solved from Scanner**

a. Page 3.878 – Question 52

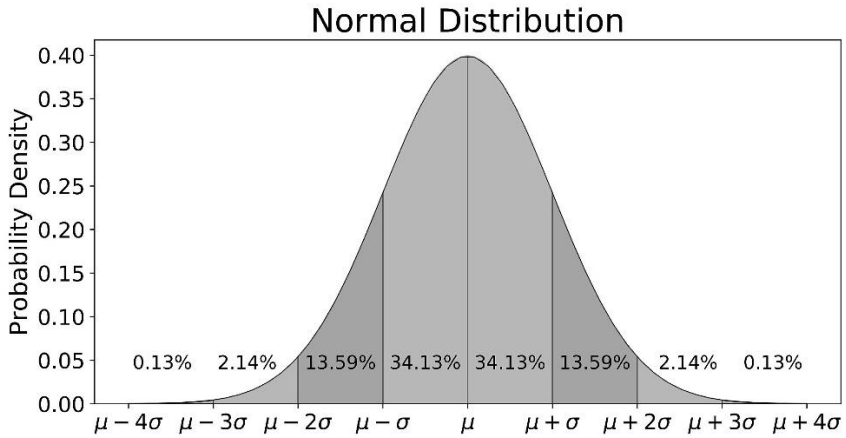
11. $Q_1$ and $Q_3$ are equidistant from the median, therefore,
    a. $Q_1 = \mu - 0.675\sigma$, and
    b. $Q_3 = \mu + 0.675\sigma$.

    **Questions to be Solved from Scanner**
    a. Page 3.894 – Question 88
    b. Page 3.854 – Question 5
12. Median $- Q_1 = Q_3 -$ Median.

    **Questions to be Solved from Scanner**
    a. Page 3.896 – Question 94
13. The normal distribution is symmetric about $x = \mu$. Therefore, its skewness is zero, i.e., the curve is neither tilted towards right (negatively skewed), nor towards left (positively skewed).
14. Points of inflexion – A normal curve has two inflexion points, i.e., the points where the curve changes its shape from concave to convex, and from convex to concave. These two points are given by:
    a. $x = \mu - \sigma$, and
    b. $x = \mu + \sigma$




    **Questions to be Solved from Scanner**
    a. Page 3.872 – Question 40
15. In a normal distribution, $\mu \pm 1\sigma$ covers 68.27% of area, $\mu \pm 2\sigma$ covers 95.45% of area, and $\mu \pm 3\sigma$ covers 99.73% of area.



**Questions to be Solved from Scanner**

a. Page 3.900 – Question 101
b. Page 3.899 – Question 100
c. Page 3.891 – Question 93 – Homework
d. Page 3.871 – Question 38

16. Under a normal distribution, the area enclosed between mean ($\mu$) and $1\sigma$ is 0.34135; mean and $2\sigma$ is 0.47725; and mean and $3\sigma$ is 0.49865.
17. In case of normal distribution
    a. Highest Value = Mean + Half of Range, and
    b. Lowest Value = Mean – Half of Range
18. Normal Distribution with $X = 0$, and $\sigma = 1$ is known as Standard Normal Distribution.
19. The height of normal curve is maximum at the Mean Value.