# Chapter 7 – Sets, Relations and Functions

## *Unit 1 – Sets*

Playlist for Fast Track Lectures: https://www.youtube.com/watch?v=ZZpvRpkgmaE&list=PLAKrxMrPL3fwOSJWxnr8j0C9a4si2Dfdz

Link for Lecture 1 of Sets, Relations, and Functions: https://youtu.be/8Yfa10bWCw4

Link for Lecture 2 of Sets, Relations, and Functions: https://youtu.be/dKewi--FuLc

### Sets

A set is a collection of well-defined distinct objects. Every object of a set is called its "element". A set is usually denoted by a capital letter and its elements are denoted by small letters.

For example, $A=\{a, e, i, o, u\}$ is a set of vowels. As can be noted, the name of the set "$A$" is written in capital, while the elements inside the set, "$a$", "$e$", "$i$", "$o$", and "$u$" are written in small. Since "$a$" is an element of the set "$A$", it is written as $a \in A$, and is read as "$a$ belongs to $A$", or "$a$ is a member of $A$". Also, since "$b$" is not an element of the set "$A$", it is written as $b \notin A,$ and is read as "$b$ does not belong to $A$".

### Description of a Set

A set is generally described in either of the following two forms:

1. Roster Form or Braces Form
2. Set-Builder Form or Algebraic Form or Rule Method or Property Method

#### Roster Form or Braces Form

In this form, a list of the elements of a set is made, and then this list is put inside curly braces {}. For example, $A=\{a, e, i, o, u\}$. This is a set represented in Roster or Braces form.

Following are some other examples:

1. $B=\{2, 4, 6, 8, 10\}$ is a set of all even numbers from 2 to 10, both inclusive.

2. $C = \{pqr,\ prq,\ qrp,\ rqp,\ qpr,\ rpq\}$ is a set of all possible arrangements of the alphabets $p$, $q$, and $r$.
3. $D = \{1, 3, 5, 7, 9\}$ is a set of all odd numbers before 10.
4. $E = \{1, 2\}$ is a set of roots of the equation $x^2 - 3x + 2 = 0$.

***Notes –***

1. The order in which the elements are written in a set makes no difference. Therefore, {*a*, *e*, *i*, *o*, *u*} and {*e*, *a*, *i*, *o*, *u*} denote the same set.
2. The repetition of an element has no effect. Therefore, {1, 2, 3, 2} is the same set as {1, 2, 3}.

### Set-Builder Form or Algebraic Form or Rule Method or Property Method

In this form, the set of all even numbers from 2 to 10, both inclusive, is written as:

B = $\{x : x = 2m$ and $m$ being an integer lying in the interval $0 < m < 6\}$

It is read as "*B* is a set of *x*, such that, $x = 2m$ and *m* being an integer lying in the interval $0 < m < 6$". The colon ":" is read as "such that". Instead of the colon, sometimes "|" is also used. Therefore, the set can also be written as:

B = $\{x \mid x = 2m$ and $m$ being an integer lying in the interval $0 < m < 6\}$

It is read exactly as we discussed above. Here, "|" is read as "such that".

## Types of Sets

Following are the different types of sets:

1. Empty Set or Null Set or Void Set
2. Singleton Set
3. Finite Set
4. Infinite Set
5. Equal Sets
6. Equivalent Sets

### Empty Set or Null Set or Void Set

A set which doesn't have any element is known as an Empty Set, or a Null Set, or a Void Set. It is denoted by the Greek letter $\phi$. This letter is pronounced as phi. In Roster Form, $\phi$ is represented as {}.

A set which has at least one element is called a Non-Empty set. Therefore, the set $A = \{0\}$ is a Non-Empty set as it contains one element, i.e. 0.

***Note –*** "$\phi$", "{0}", and "0" are all different –

- $\phi$ is a set with no element at all;
- {0} is a set with the element "0"; and,
- 0 is just a number.

### Singleton Set
A set which contains only one element is called a Singleton Set. For example, $A = \{5\}$ is a singleton set whose only member/element is the number "5".

### Finite Set
A set is said to be a Finite Set if its elements can be counted. For example,

1. The set of vowels $\left[A = \{a, e, i, o, u\}\right]$ is a finite set as it has countable number of elements, i.e. 5.
2. The set of all odd numbers from 0 to 12 $\left[B = \{1, 3, 5, 7, 9, 11\}\right]$ is a finite set as it has countable number of elements, i.e. 6.

### Cardinal Number of a Finite Set
Cardinal Number of a Finite Set refers to the number of distinct elements of a finite set. For example,

1. Consider the set of all the vowels: $A = \{a, e, i, o, u\}$. The cardinal number of this set is 5, as it has 5 elements. It is represented as $n(A) = 5$.
2. Consider the set of all odd numbers from 0 to 12: $B = \{1, 3, 5, 7, 9, 11\}$. The cardinal number of this set is 6, as it has 6 elements. It is represented as $n(B) = 6$.
3. Consider the set of letters of the word "ALLOY": $C = \{A, L, L, O, Y\}$. Even though the letter "*L*" is present twice, it'll be counted only once, and therefore, the cardinal number of this set is 4. It is represented as $n(C) = 4$.

### Infinite Set
Obviously, a set whose elements can't be counted is called an Infinite Set. For example, the set of natural numbers $\{1, 2, 3, 4, 5, \ldots\}$ is an infinite set.

### Equal Sets
No rocket science here, if two sets have exactly the same elements, they are said to be Equal Sets. For example, the sets $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4\}$ are equal sets.

### Equivalent Sets
Two sets $A$, and $B$ are said to be Equivalent if their Cardinal Numbers are equal, i.e., $n(A) = n(B)$. For example, the sets $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7, 8\}$ are equivalent sets as $n(A) = n(B) = 4$.

Clearly, all equal sets are equivalent but all equivalent sets are not equal.

## Subsets

Let $A$ and $B$ be two sets. If every element of $A$ is an element of $B$, then $A$ is called a subset of $B$. For example, if $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, then every element of $A$ is an element of $B$, and hence $A$ is said to be a subset of $B$. It is written as $A \subseteq B$. Obviously, every set is a subset of itself and an empty set is a subset of every set. A subset $A$ of a set $B$ is called a proper subset if $A \neq B$. If $A$ is a proper subset of $B$, it is written as $A \subset B$.

If $A$ is a subset of $B$, it means that $B$ is the superset of $A$ and it is written as $B \supseteq A$. If $A$ is not a subset of $B$, it is written as $A \not\subset B$.

| **Symbols at a Glance** | | |
|---|---|---|
| **Description** | **Symbol** | **Meaning** |
| $\subseteq$ | Subset | If $A$ is a subset of $B$, it means that the set $A$ contains either some, or all elements of the set $B$. |
| $\subset$ | Proper Subset | If $A$ is a subset of $B$, and $A \neq B$, $A$ is said to be a proper subset of $B$. |
| $\supseteq$ | Superset | If $A$ is a subset of $B$, it means that $B$ is a superset of $A$. |
| $\not\subset$ | Not a Subset | If $A$ is not a subset of $B$, it is written as $A \not\subset B$. |

### No. of Subsets of a Given Set

Consider a set $A = \{1, 2, 3\}$. Following are its subsets: $\{1, 2, 3\}$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, $\{\}$. A set with $n$ elements has $2^n$ subsets. Therefore, the set $A$, which has 3 elements has $2^3 = 8$ subsets.

#### Questions to be Solved from Scanner

1. Page 3.419 – Question 74
2. Page 3.411 – Question 59 – Homework

The number of proper subsets of a set containing $n$ elements is $2^n - 1$. This is because $2^n$ also consists of the set itself, which is an improper subset. Therefore, 1 is removed in order to get the number of proper subsets.

#### Questions to be Solved from Scanner

1. Page 3.416 – Question 67
2. Page 3.396 – Question 30

### Power Set

Consider a set $A = \{1, 2, 3\}$. Following are its subsets: $\{1, 2, 3\}$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, $\{\}$. A set containing all the subsets of set $A$ is called the Power Set. Therefore, $P(A) = \{\{1, 2, 3\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{\}\}$.

### Points to Remember

1. Every set is a subset of itself.
2. The empty set is a subset of every set.
3. Total number of subsets of a finite set containing $n$ elements is $2^n$.
4. The set containing all the subsets of a particular set is called the power set.

## Operations on Sets

### Universal Set

In any discussion, there's always a set which contains the elements of all the sets under consideration. This set is known as the Universal Set. It is denoted by $U$ or $S$.

For example, let there be the following sets: $A = \{1, 2, 3\}$, $B = \{2, 4, 5, 6\}$, and $C = \{1, 3, 5, 7\}$. The universal set will be $U = \{1, 2, 3, 4, 5, 6, 7\}$.

### Union of Sets

If $A$ and $B$ are two sets, the union of these two sets will be a set containing the elements of both $A$ as well as $B$. It is written as $A \cup B$.

For example, if $A = \{1, 2, 3, 6\}$, and $B = \{2, 3, 4, 5\}$, then $A \cup B = \{1, 2, 3, 4, 5, 6\}$.

### Intersection of Sets

If $A$ and $B$ are two sets, the intersection of these two sets will be a set containing the elements which are present in both the sets. It is written as $A \cap B$.

For example, if $A = \{1, 2, 3, 6\}$, and $B = \{2, 3, 4, 5\}$, then $A \cap B = \{2, 3\}$.

#### Disjoint Sets

Two sets $A$ and $B$ are said to be Disjoint Sets if $A \cap B = \phi$.

For example, let $A = \{1, 2, 3\}$, and $B = \{4, 5, 6\}$. Clearly there's no common element in both the sets. Therefore, $A \cap B = \phi$. Therefore, $A$ and $B$ are disjoint sets.

#### Overlapping Sets

Two sets $A$ and $B$ are said to be Overlapping Sets if $A \cap B \neq \phi$. In other words, if the sets are not disjoint, they are overlapping.

For example, let $A = \{1, 2, 3\}$, and $B = \{3, 4, 5\}$. Now, $A \cap B = \{3\}$. Therefore, $A$ and $B$ are overlapping sets.

### Difference of Sets

If $A$ and $B$ are two sets, $A - B$ will be a set containing the elements of the set $A$ which do not belong to $B$.

For example, if $A = \{2, 3, 4, 5, 6, 7\}$, and $B = \{3, 5, 7, 9, 11, 13\}$, then $A - B = \{2, 4, 6\}$; and $B - A = \{9, 11, 13\}$.

#### Questions to be Solved from Scanner

1. Page 3.418 – Question 72
2. Page 3.417 – Question 70 – Homework
3. Page 3.408 – Question 54 – Homework
4. Page 3.389 – Question 17 – Homework
5. Page 3.388 – Question 13 – Homework
6. Page 3.409 – Question 55 – Homework

### Compliment of a Set

Let $U$ be a universal set and $A$ be another set. Obviously, $A$ will be a subset of $U$. The compliment of $A$ is written as $A'$ or $A^c$ and is determined by $U - A$.

For example, if $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 5, 7\}$, and $B = \{2, 4, 6\}$; then, $A' = U - A = \{2, 4, 6, 8, 9, 10\}$; and $B' = U - B = \{1, 3, 5, 7, 8, 9, 10\}$.

## Some Rules to Remember

1. $(A \cup B)' = A' \cap B'$

2. $(A \cap B)' = A' \cup B'$

# *Unit 2 – Word Problems on Sets*

## Some Important Results on Number of Elements in Sets

If the question involves two sets *A*, and *B*, and the requirement is to find either the union, or the intersection of those two sets, use the following:

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2. $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

### Questions to be Solved from Scanner

1. Page 3.414 – Question 64
2. Page 3.412 – Question 61
3. Page 3.403 – Question 44 – Homework
4. Page 3.382 – Question 2 – Homework
5. Page 3.382 – Question 1 – Homework

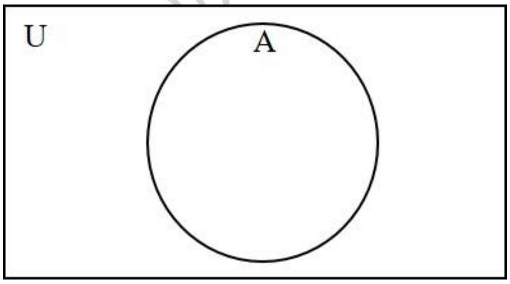
## Venn Diagrams

Venn Diagrams are used to solve word problems which involve:

1. Two sets, but the requirement is neither of the union of those sets, nor of the intersection of those sets; or
2. Three sets.

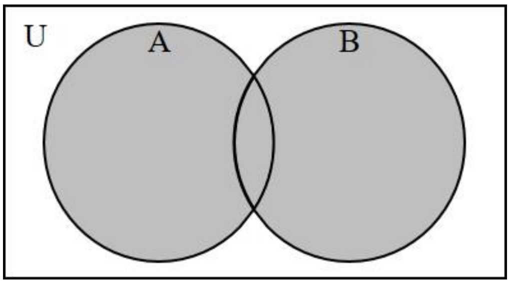
### What are Venn Diagrams?

The sets and their operations can be represented pictorially through what are called Venn Diagrams. In Venn Diagrams, a rectangle represents the universal set and all the individual sets are represented by circles. Following is the representation of a universal set and a set *A*.



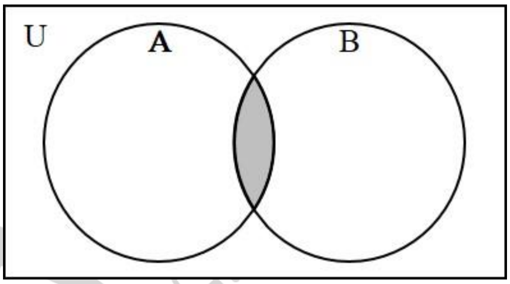

### Union of Sets

The shaded portion in the following figure represents union of sets:



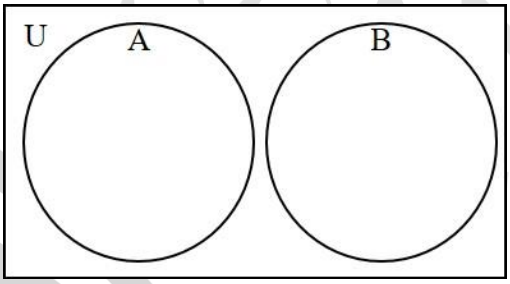

## Intersection of Sets

The shaded portion in the following figure represents intersection of sets:



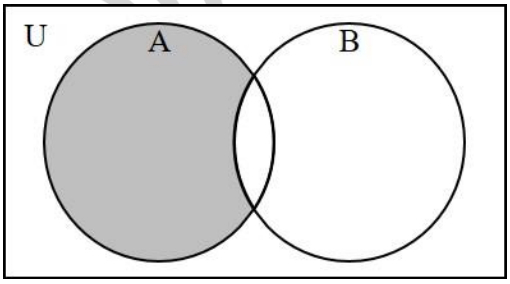

## Disjoint Sets

Following figure represents disjoint sets:

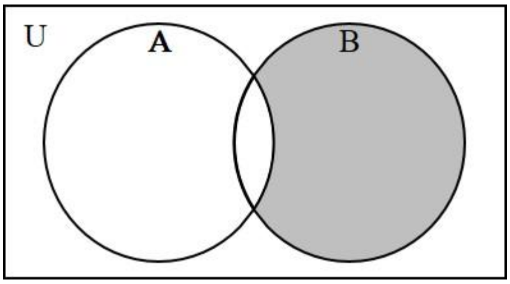



## Difference of Sets

The shaded portion in the following figure represents $A - B$:

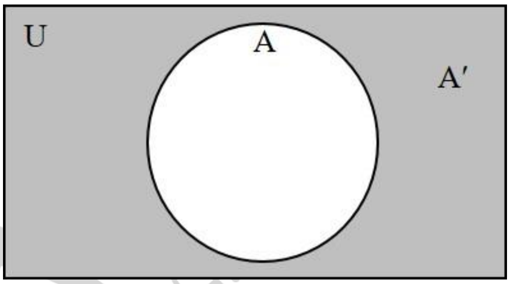



The shaded portion in the following figure represents $B - A$:




## Compliment of a Set

The shaded portion in the following figure represents A′:




## Questions to be Solved from Scanner

1. Page 3.406 – Question 52
2. Page 3.393 – Question 24 – Homework
3. Page 3.415 – Question 66
4. Page 3.401 – Question 41 – Homework
5. Page 3.398 – Question 34 – Homework
6. Page 3.386 – Question 9 – Homework
7. Page 3.384 – Question 7 – Homework
8. Page 3.383 – Question 4 – Homework
9. Page 3.391 – Question 20
10. Page 3.392 – Question 22
11. Page 3.395 – Question 29

# *Unit 3 – Relations*

## Product Sets/Cartesian Product of Sets

It is pretty simple → it's like multiplying $(x+y+z)$ and $(a+b)$. For example, let $A=\{1, 2, 3\}$, and $B=\{4, 5\}$. Then $A\times B=\{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$. The individual elements of the set $A \times B$, i.e. (1, 4), (1, 5), … are called **ordered pairs**.

### Questions to be Solved from Scanner

1. Page 3.404 – Question 47
2. Page 3.395 – Question 28 – Homework

### Number of Elements in the Cartesian Product of Two Sets

If $A$ and $B$ are two finite sets, then $n(A \times B) = n(A) \times n(B)$.

## Relations

A relation from set $A$ to set $B$ is denoted by one or more ordered pairs, where the first element of every ordered pair belongs to set $A$ and the second element of every ordered pair belongs to set $B$. We know that ordered pairs arise when two sets are multiplied. Since a relation between two sets is "one or more ordered pairs", it implies that **a relation between the sets *A* and *B* is basically a subset of the product of both the sets.**

For example, let $A=\{1, 2\}$ and $B=\{1, 2, 3, 4\}$, then,

$A\times B=\{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$.

Now, $R=\{(1, 2), (2, 4)\}$ is a relation from $A$ to $B$ since $R$ is a subset of $A \times B$.

From the above discussion, one thing is clear that a set $R$ is a relation from set $A$ to set $B$ only if $R$ is a subset of $A \times B$.

**Question 1**

If $A=\{a, b, c, d\}$, $B=\{p, q, r, s\}$, then which of the following are relations from $A$ to $B$?

1. $R_1=\{(a, p), (b, r), (c, s)\}$
2. $R_2=\{(q, b), (c, s), (d, r)\}$
3. $R_3=\{(a, p), (a, q), (d, p), (c, r), (b, r)\}$
4. $R_4=\{(a, p), (q, a), (b, s), (s, b)\}$

## Total Number of Relations

Let *A* and *B* be two non-empty finite sets consisting of *m* and *n* elements respectively. Then $A \times B$ consists of *mn* ordered pairs. So, total number of subsets of $A \times B$ is $2^{mn}$. Since each subset of $A \times B$ defines a relation from *A* to *B*, so total number of relations from *A* to *B* is $2^{mn}$.

## Questions to be Solved from Scanner

1. Page 3.417 – Question 69

## Domain and Range of a Relation

Domain is the set of all the first elements of the ordered pairs in a relation; and Range is the set of all the second elements of the ordered pairs in a relation.

For example, if $A = \{1, 3, 5, 7\}$ and $B = \{2, 4, 6, 8, 10\}$, and let $R = \{(1, 8), (3, 6), (5, 2), (1, 4)\}$ be a relation from *A* to *B*, then Domain is $\{1, 3, 5\}$ and Range is $\{8, 6, 2, 4\}$. Domain and Range are written as Dom $(R) = \{1, 3, 5\}$ and Range $(R) = \{8, 6, 2, 4\}$ respectively.

## Different Types of Relations

Following are the various types of relations:

1. Identity Relation
2. Reflexive Relation
3. Symmetric Relation
4. Transitive Relation
5. Equivalence Relation
6. Inverse Relation

## Identity Relation

If every element in a set is related *only to itself*, such a relation is known as an identity relation. For example, let $A = \{1, 2, 3\}$, and $R = \{(1, 1), (2, 2), (3, 3)\}$. Now, *R* is an identity relation because every element of the set *A* is related only to itself.

Mathematically, it can be represented as:

A relation *R* on the set *A* is an identity relation if and only if $R = \{(a, a) : a \in A\}$.

## Reflexive Relation

A reflexive relation is very similar to identity relation with only one difference, i.e., in addition to there being all the elements related to themselves, there may also be some other elements present in the relation. For example, let $A = \{1, 2, 3\}$, and $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$. Now, *R* is a reflexive relation because in addition to all the elements being related to themselves, some elements are related to some other elements as well. The only condition is that both the elements of the ordered pairs should belong to the same set *A*.

Mathematically, it can be represented as:

A relation *R* on the set *A* is a reflexive relation if $(a, a) \in R$ for all $a \in A$.

## Questions to be Solved from Scanner

1. Page 3.399 – Question 37

### Symmetric Relation
Consider the following example: Ayushman Khurana is the brother of Aparshakti Khurana, and therefore, obviously, Aparshakti Khurana is the brother of Ayushman Khurana. Such a relation is known as a symmetric relation. So, in a relation $R$, if $(a, b) \in R$, and therefore, $(b, a)$ also belongs to $R$, such a relation is known as a symmetric relation.

Mathematically, it can be represented as:

A relation $R$ on the set $A$ is a symmetric relation if $(a, b) \in R \Rightarrow (b, a) \in R$.

### Questions to be Solved from Scanner
1. Page 3.396 – Question 31

### Transitive Relation
Consider the following example: Salman Khan is the brother of Arbaz Khan, and Arbaz Khan is the brother of Sohail Khan; so, obviously, Salman Khan is the brother of Sohail Khan, right! This is called transitive relation. So, in a relation $R$, if $(a, b) \in R$ and $(b, c) \in R$, and therefore, $(a, c)$ also belongs to $R$, such a relation is known as a transitive relation.

Mathematically, it can be represented as:

A relation $R$ on the set $A$ is a symmetric relation if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.

### Questions to be Solved from Scanner
1. Page 3.404 – Question 48

### Equivalence Relation
A relation which is reflexive, symmetric, as well as transitive, is known as an equivalence relation, or simply an equivalence. Consider a set of straight lines. The relation "is parallel to" is:

- A reflexive relation, as every straight line is parallel to itself;
- A symmetric relation, as if a line $A$ is parallel to another line $B$, then obviously, the line $B$ is parallel to the line $A$;
- A transitive relation, as if a line $A$ is parallel to another line $B$, and the line $B$ is parallel to another line $C$, then obviously, the line $A$ is parallel to the line $C$.

Therefore, this relation is an equivalence relation.

### Inverse Relation
An example of an inverse relation is given below:

Consider the set $A = \{1, 2, 3\}$, and the relation $R = \{(1, 2), (2, 2), (3, 1), (3, 2)\}$. Here, Dom $(R) = \{1, 2, 3\}$, and Range $(R) = \{2, 1\}$. Now the inverse of this relation $(R^{-1}) = \{(2, 1), (2, 2), (1, 3), (2, 3)\}$. Therefore, to make an inverse relation, simply interchange the position of elements in all the ordered pairs. Now, here, Dom $(R^{-1}) = \{2, 1\}$, and Range $(R^{-1}) = \{1, 2, 3\}$.

From the above discussion, it can be concluded that Dom $(R)$ = Range $(R^{-1})$; and Range $(R)$ = Dom $(R^{-1})$.

## Questions to be Solved from Scanner

1. Page 3.419 – Question 73

# *Unit 4 – Functions*

## Introduction

Consider the following linear equation in two variables:

$y = 2x + 3$

Now, you may easily find out the value of *y* for any value of *x*.

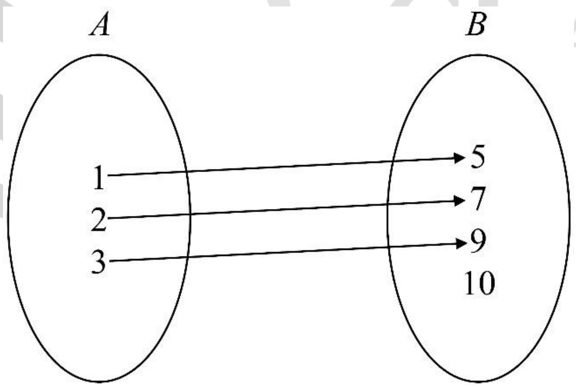
For example, for $x = 1$, $y = 2 \times 1 + 3 = 5$

for $x = 2$, $y = 2 \times 2 + 3 = 7$

for $x = 3$, $y = 2 \times 3 + 3 = 9$, and so on…

Here, in this equation "$y = 2x + 3$", *y* is nothing but a function of *x*. In terms of functions, it is written as: $f(x) = 2x + 3$.

If we consider a set $A = \{1, 2, 3\}$, and a set $B = \{5, 7, 9, 10\}$ and prepare a relation "$y = 2x + 3$", it'll be the set $R = \{(1, 5), (2, 7), (3, 9)\}$. Now this set *R* is known as a function which represented as: $f(x) = \{(x, y) : y = 2x + 3\}$. The pictorial representation is given below:

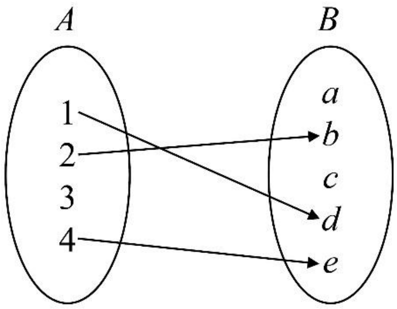



The values 5, 7, and 9 are known as the **images**, and the values 1, 2, and 3 are known as the **pre-images** of the function $f(x)$.

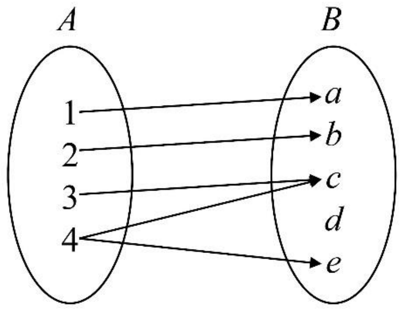
We can see from the above discussion that for any value of *x*, there is always a unique value of *y*. This is the basic definition of function. A function is a relation, wherein, for every value of *x*, there is a unique value of *y*. In other words, a relation is said to be a function when **both** the following conditions are satisfied:

1. **All** the values of set *A* have an image in set *B*, and
2. All the values of set *A* have **a single** image in set *B*.

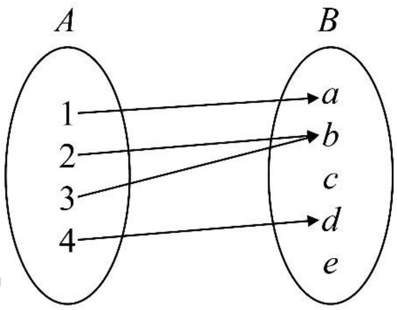
Consider the following diagrams:



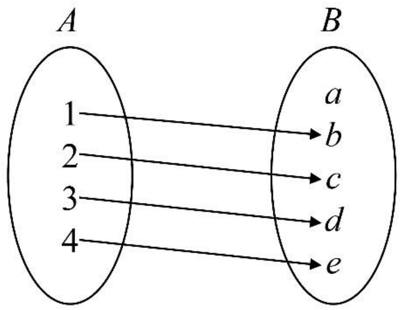

$f_1$




$f_2$




$f_3$




$f_4$

$f_1$ is not a function from *A* to *B* because the first condition, i.e. "All the values of set *A* should have an image in set *B*" is not satisfied. This is so, because the value "3" in set *A* does not have any image in set *B*.

$f_2$ is not a function from *A* to *B* because the second condition, i.e. "All the values of set *A* should have **a single** image in set *B*" is not satisfied. This is so, because the value "4" in set *A* has two images in set *B*, i.e. "*c*" and "*e*".

$f_3$ is a function from *A* to *B* because both the conditions are satisfied. Please note that it doesn't matter that the elements "2", as well as "3" from set *A* have the same image "*b*" in set *B*. All that matters is that the element "2" has only one image, i.e. "*b*", and the element "3" also has only one image, i.e. "*b*". It doesn't matter that the image for both the elements is the same.

$f_4$ is a function from *A* to *B* because both the conditions are satisfied.

Note – A function from set *A* to set *B* is also known as mapping from set *A* to set *B*, and is represented as $f : A \to B$.

## Questions to be Solved from Scanner

1. Page 3.420 – Question 76
2. Page 3.418 – Question 71 – Homework
3. Page 3.406 – Question 51 – Homework
4. Page 3.388 – Question 14 – Homework
5. Page 3.402 – Question 43
6. Page 3.395 – Question 27

7. Page 3.392 – Question 23
8. Page 3.421 – Question 78 – Homework
9. Page 3.389 – Question 16 – Homework
10. Page 3.389 – Question 15 – Homework
11. Page 3.385 – Question 8 – Homework
12. Page 3.400 – Question 39 – Homework
13. Page 3.402 – Question 42 – Homework
14. Page 3.405 – Question 49 – Homework

## Domain, Co-Domain, and Range of a Function

Let $f : A \to B$, then set *A* is known as the Domain of the function, set *B* is known as the Co-Domain of the function and the set of all the image elements is known as the Range of the function.

For example, consider a set *A* = {1, 2, 3}, and a set *B* = {5, 7, 9, 10}.
The set for the function $f(x) = 2x + 3$ would be $f = \{(1, 5), (2, 7), (3, 9)\}$.
Now, $Dom(f) = \{1, 2, 3\}$; $Co-Dom(f) = \{5, 7, 9, 10\}$; $Range(f) = \{5, 7, 9\}$.

### Questions to be Solved from Scanner

1. Page 3.418 – Question 68
2. Page 3.403 – Question 45 – Homework
3. Page 3.396 – Question 32 – Homework

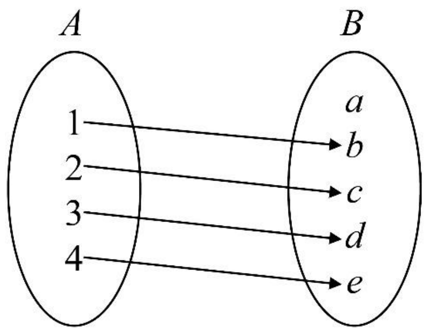
## Various Types of Functions

Following are the various types of functions:

1. One-One Function
2. Many-One Function
3. Onto or Surjective Functions
4. Into Functions
5. Bijection Function
6. Identity Function
7. Constant Function
8. Equal Functions
9. Composite Functions
10. Inverse of a Function

### One-One Function

Let there be two non-empty sets – *A* and *B*. A function *f* is said to be a one-one function from set *A* to set *B* if different elements of set *A* have different images in set *B*. Following is the diagrammatic representation of a one-one function.

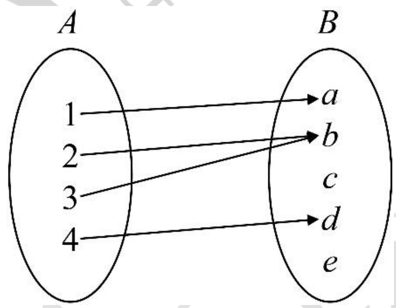



For example, let $A$ = {4, 5, 6}, $B$ = {9, 10, 11, 12}. Let $f$ = {(4, 9), (5, 10), (6, 12)}. Here, $f$ is a one-one function as different elements in set $A$ have different images in set $B$.

One-one functions are also known as injective functions.

## Many-One Function

Let there be two non-empty sets – $A$ and $B$. A function $f$ is said to be a many-one function from set $A$ to set $B$ if two or more distinct elements of set $A$ have the same image in set $B$. Following is the diagrammatic representation of a many-one function.




For example, let $A$ = {–4, 4, 5}, $B$ = {16, 25}. Consider the rule $f(x) = x^2$. Now, $f(-4) = (-4)^2 = 16$; $f(4) = (4)^2 = 16$; $f(5) = (5)^2 = 25$. Therefore, $f$ = {(–4, 16), (4, 16), (5, 25)}. Here, since two distinct elements of set $A$, i.e. "–4" and "4" have the same image "16" in set $B$, this is a many-one function.

## Onto or Surjective Functions

Let there be two non-empty sets – $A$ and $B$. A function $f$ is said to be an onto function from set $A$ to set $B$ if every element in set $B$ has at least one pre-image in set $A$.

For example, let $A$ = {6, 7, 8}, $B$ = {9, 10}, and let $f$ = {(6, 9), (7, 10), (8, 10)}. Here, since every element of $B$ has at least one pre-image in set $A$, $f$ is an onto function.

## Questions to be Solved from Scanner

1. Page 3.414 – Question 65
2. Page 3.405 – Question 50 – Homework
3. Page 3.403 – Question 46 – Homework

## Into Functions

Let there be two non-empty sets – $A$ and $B$. A function $f$ is said to be an into function from set $A$ to set $B$ if one or more elements in set $B$ do not have even a single pre-image in set A.

### Questions to be Solved from Scanner

1. Page 3.393 – Question 25
2. Page 3.410 – Question 57

### Bijection Function

A function which is one-one and onto is known as a Bijective Function. A bijective function is also known as one-to-one correspondence.

### Identity Function

Let *A* be a non-empty set. An identity function is defined as follows:

$$f : A \to A : f\left(x\right) = x \ \forall \ x \in A$$

Therefore, it is a one-to-one onto function with domain *A* and range *A*.

An identity function is denoted by the letter *I*.

If you think about it, it is exactly like an identity relation.

### Constant Function

Let there be two non-empty sets – *A* and *B*. A function *f* is said to be a constant function from set *A* to set *B* if every element in set *A* has the same image element in set *B*.

For example, let *A* = {1, 2, 3}, *B* = {5, 7, 9}. Let $f : A \to B : f\left(x\right) = 5$ for all $x \in A$. Here, $f\left(1\right) = 5;\ f\left(2\right) = 5;\ f\left(3\right) = 5$. Therefore, *f* = {(1, 5), (2, 5), (3, 5)}. Clearly, all the elements of *A* have the same image in set *B*. Therefore, this is a constant function.

Note – The range set of a constant function is a singleton set.

### Questions to be Solved from Scanner

1. Page 3.399 – Question 36

### Equal Functions

Let there be two functions *f* and *g*. They will be said to be equal to each other when both the following conditions are satisfied:

1. Both *f* and *g* have the same domain, and
2. $f\left(x\right) = g\left(x\right)$ for all the values of *x*.

For example, let *A* = {1, 2}, *B* = {3, 6}; $f : A \to B : f\left(x\right) = x^2 + 2$ and $g : A \to B : g\left(x\right) = 3x$.

Then, obviously, *f* and *g* have the same domain. Also, $f\left(1\right) = 1^2 + 2 = 3;\ f\left(2\right) = 2^2 + 2 = 6$, and, $g\left(1\right) = 3 \times 1 = 3;\ g\left(2\right) = 3 \times 2 = 6$. Therefore, we find that $f\left(1\right) = g\left(1\right)$, and $f\left(2\right) = g\left(2\right)$. Since both the conditions of equality of two functions are satisfied, $f = g$.

## Composite Functions

Till now, we've studied functions such as $f(x)=x+1$, or $f(x)=x^2+2$, or $f(x)=2x$, and so on. In all such functions, we simply had to put the value of *x* to find the value of the function. For example, find the value of the function $f(x)=x+1$ for $x=3$. You would simply put the value of *x* as "3", and solve it as follows: $f(3)=3+1=4$. Easy!

Now, consider the following:

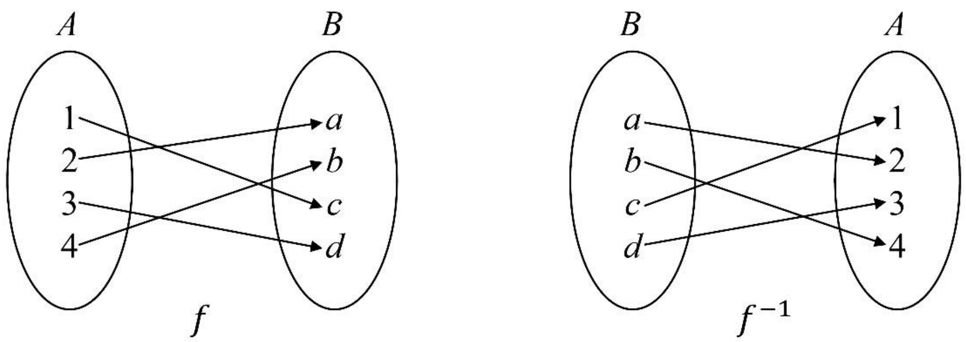
Let there be a function $f(x)=x+1$, and another function $g(x)=2x+2$. Find the value of $f(g(x))$ for $x=3$. Confused? It's pretty simple actually! The question wants us to calculate $f(g(3))$. Therefore, instead of putting the value of *x* as "3" in $f(x)=x+1$, we simply have to put the value of *x* as "$g(3)$" in $f(x)=x+1$. For this, first we'll calculate $g(3)$. $g(3)=(2\times3)+2=8$; now we'll put this value 8 in $f(x)=x+1 \Rightarrow f(8)=8+1=9$. Thus, we have arrived at the value of $f(g(x))$ for $x=3$, or $f(g(3))$. It is also written as $fog(3)$. Such functions are known as Composite Functions.

## Questions to be Solved from Scanner

1. Page 3.408 – Question 53
2. Page 3.419 – Question 75
3. Page 3.413 – Question 63
4. Page 3.412 – Question 62 – Homework
5. Page 3.401 – Question 40 – Homework
6. Page 3.399 – Question 38 – Homework
7. Page 3.394 – Question 26 – Homework
8. Page 3.392 – Question 21 – Homework
9. Page 3.383 – Question 5 – Homework
10. Page 3.383 – Question 3 – Homework

## Inverse of a Function

Let there be two non-empty sets – *A* and *B*. Let there be a one-one onto function $f: A \to B$. The inverse of this function $\left(f^{-1}\right)$ is given as $f^{-1}: B \to A$. Following diagram will make it clear:




Notes –

1. A function is invertible only if it is one-one onto.
2. If $f$ is one-one onto, then $f^{-1}$ is also one-one onto.

Algorithm for finding out the inverse of a function:

| | |
|---|---|
| Step 1 – | Write the function in the form of an equation, substituting $y$ in place of $f(x)$. |
| Step 2 – | Rearrange the terms so that $x$ comes on the LHS. |
| Step 3 – | Substitute $f^{-1}(x)$ in place of $x$, and $x$ in place of $y$. |

## Questions to be Solved from Scanner

1. Page 3.421 – Question 79
2. Page 3.410 – Question 58 – Homework
3. Page 3.398 – Question 35 – Homework
4. Page 3.387 – Question 11 – Homework
5. Page 3.386 – Question 10 – Homework
6. Page 3.384 – Question 6 – Homework