RATIO

Meaning of Ratio	Division of two quantities a and b of same units. Denoted by a:b
Inverse Ratio	b:a is inverse ratio of a:b
Compound Ratio	Compound ratio of a:b and c:d is ac:bd
Duplicate Ratio	Duplicate ratio of a:b is $a^2:b^2$
Sub-duplicate Ratio	Duplicate ratio of a:b is $\sqrt[2]{a}:\sqrt[2]{b}$
Triplicate Ratio	Triplicate ratio of a:b is $a^3:b^3$
Sub-triplicate Ratio	Triplicate ratio of a:b is $\sqrt[3]{a}:\sqrt[3]{b}$
Commensurate	If ratio can be expressed in the form of integers
Incommensurate	If ratio cannot be expressed in the form of integers
Continued Ratio	Ratio of three or more quantities e.g. a:b:c

PROPORTION

Proportion	a,b,c,d are in proportion if a:b = c:d [it is an equality of two ratios]
Term/ Proportional	first = a, second = b, third =c, fourth = d
Mean Proportional	In a continued proportion a:b=b:c, b^2=ac, b is called mean proportional
Cross Product Rule	If a:b=c:d, then ad = bc
Invertendo	If a:b=c:d, then b:a = d:c
Alternendo	If a:b=c:d, then a:c = b:d
Componendo	If a:b=c:d, then (a+b):b = (c+d):d
Dividendo	If a:b=c:d, then (a−b):b = (c−d):d
Componendo and Dividendo	If a:b=c:d, then (a+b):(a−b) = (c+d):(c−d) or (a−b):(a+b) = (c−d):(c+d)
Addendo	If a:b = c:d = e:f = ……. = k, then also (a+c+e+……):(b+d+f+….) = k

INDICES

Index / Indices	Here in 4^2, 4 is base and 2 is power or index. Plural of index is indices
Basic 1	$a^0 = 1$, any number raised to power zero equals to 1
Basic 2	$\sqrt{a} = a^{1/2}$, $\sqrt[3]{a} = a^{1/3}$
Law 1	$a^m \times a^n = a^{(m+n)}$
Law 2	$a^m / a^n = a^{(m-n)}$
Law 3	$a^{(m)^n} = a^{m\times n} = (a^m)^n$
Law 4	$(ab)^n = a^n b^n$

LOG

Basic	If 2^4=16 [2 is base, 4 is power], then $\log_2 16 = 4$ (i.e log of 16 base 2)
How to remember?	2 should be raised to what power so that it becomes 16 2 ka kitna power karne wo 16 ho jaye, ans is 4
Standard Result	$\log_a a = 1, \log_a 1 = 0$
Law 1	$\log_a(mn) = \log_a m + \log_a n$
Law 2	$\log_a(\frac{m}{n}) = \log_a m - \log_a n$
Law 3	$\log_a m^n = n \log_a m$
Change of Base	$\log_b m = \frac{\log_a m}{\log_a b}$

EQUATIONS - BASICS

Equation Means	mathematical statement of equality
Identity Equation	If equality is true for all the values of variable, ex. $2x + 3 = x + x + 3$
Conditional Equation	If the equality is true for certain value of the variable ex. $2x + 1 = 3$
Solution or Root	It is the value of variable that satisfies the equation
Degree	Highest power of variable in equation

SIMPLE EQUATION

Type	Linear equation with one unknown	Linear equation with two unknowns	Quadratic Equation	Cubic Equation
Form	$ax + b = 0$, where a and b are constants	$ax + by + c = 0$ a,b,c are constants	$ax^2 + bx + c = 0$ a,b,c are constants with a≠0	$ax^3 + bx^2 + cx + d = 0$
Degree	1 (One)	1	2	3
Roots	1 (One)	1 each for both	2 (α, β)	3
Remarks	NA	Need minimum two equations to get roots	Trial Error/ Formula based	Trial and Error
Methods for solution	NA	1. Elimination 2. Cross Multiplication	$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	NA

LINEAR EQUATIONS WITH TWO UNKNOWNS

Elimination	Eliminate one variable by algebraic operations on given equations, and then calculate the value of variable that remains. Using this value, find out the value of other root.
Cross Multiplication	$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ Solution is given by: $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

QUADRATIC EQUATION

Formula	$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Sum of Roots	$\alpha + \beta = -\frac{b}{a} = \frac{\text{coefficient of } x}{\text{coefficient of } x^2}$
Product of Roots	$\alpha \times \beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$
How to construct a quadratic equation	$x^2 -$ (sum of roots: $\alpha + \beta)x +$ Product of Roots: $\alpha \times \beta = 0$
Nature of Roots	(see table below)
Irrational Roots	If one root is $(m + \sqrt{n})$, then other root will be $(m - \sqrt{n})$

Nature of Roots

Condition	Nature of Roots
$b^2 - ac = 0$	Real and Equal (α=β)
$b^2 - ac > 0$	Real and Unequal
$b^2 - ac < 0$	Imaginary
$b^2 - ac$ is a perfect square	Real, Unequal and Rational
$b^2 - ac > 0$ but not perfect square	Real, Unequal and Irrational

MATRICES

Matrix	A rectangular array of numbers (real/complex) with m rows and n columns
Order of Matrix	Order is m × n where m= no. of rows and n = no. of columns
Row Matrix	Matrix having only one row $\begin{bmatrix} 1 & 4 & 2 \end{bmatrix}$
Column Matrix	Matrix having only one column $\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$
Zero/ Null Matrix	If all the elements of matrix (any order) are zero $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
Square Matrix	If in a matrix, no. of columns = no. of rows $\begin{bmatrix} 1 & 3 \\ 9 & 2 \end{bmatrix}$
Rectangular Matrix	If in a matrix, no. of columns ≠ no. of rows $\begin{bmatrix} 1 & 3 & 2 \\ 9 & 2 & 5 \end{bmatrix}$
Leading Diagonal	Diagonal elements starting from top left to bottom right
Diagonal Matrix	A square matrix where all the elements except leading diagonal elements are zero. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$
Scalar Matrix	A diagonal square matrix where all the leading elements are equal $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
Unit Matrix	A scalar matrix whose leading diagonal elements are equal to 1 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Upper Triangle Matrix	A matrix whose all the elements below the leading diagonal are zero $\begin{bmatrix} 3 & 4 & 5 \\ 0 & 1 & 9 \\ 0 & 0 & 5 \end{bmatrix}$
Lower Triangle Matrix	A matrix whose all the elements above the leading diagonal are zero $\begin{bmatrix} 3 & 0 & 0 \\ 5 & 1 & 0 \\ 2 & 8 & 5 \end{bmatrix}$
Sub Matrix	The matrix obtained by deleting one or more rows or columns or both of a matrix is called its sub matrix.
Equal Matrices	Two matrices are are equal matrices if order of both is same and corresponding elements are same
Addition/ Subtraction	All the corresponding elements will be added/ subtracted to make a new matrix. (only possible when both matrix are of same order)
Properties of Addition/ Subtraction	**a**. A+B = B+A [Commutative], **b**. (A+B)+C = A+(B+C) [Associative], **c**. k(A+B) = kA + kB, k is constant
Multiplication	Multiplication of two matrices is possible only when no. of columns of first matrix = no. of rows of second matrix. *[To understand how to do multiplication – refer page 2.40 Example 3]*
Properties of Multiplication	**a.** In general, A×B ≠ B×A, **b**. (A×B)×C = A×(B×C) if defined, **c.** A(B+C) = AB + AC also, (A+B)C= AC+BC, **d.** if AB = AC then B≠C in general, **e.** A×O = O [O means null matrix], **f.** A×I = IA = O [I means Unit Matrix],

Transpose of a Matrix	A matrix obtained by changing rows and columns of a matrix **A** is called as Transpose Matrix of **A**. It is denoted by - **A^T or A'**
Properties of Transpose	a. A = (A')' ; b. (A+B)' = A' + B' ; c. (KA)'=K.A' ; d. (AB)'=B'×A'
Symmetric Matrix	If after transposing also there is no change in matrix. A'=A
Skew Symmetric	If after transposing a matrix, it becomes its negative. A'=−A

DETERMINANTS

Determinants	It is a valuation of a matrix using some rules. It only applies for square matrix
Denote	It is denoted by **det A** or **| A |** or **Δ**
2 × 2 Matrix	$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc)$
3 × 3 Matrix	$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2c_3-b_3c_2)-a_2(b_1c_3-b_3c_1)+a_3(b_1c_2-b_2c_1)$
Minor	M_{ij}=Minor of the element located in i^{th} row and j^{th} column. It is equal to determinant of sub matrix obtained after i^{th} row and j^{th} column
Cofactor	$C_{ij} = (-1)^{i+j} M_{ij}$
3 × 3 Formula using Cofactors	$a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13}$
Properties	a. Δ remains unaltered if its rows or columns are interchanged. b. Δ change its sign if two rows or columns interchanges c. If any two rows or columns of a determinant are identical, then Δ =0 d. If each element of matrix is multiplied by constant k , Δ will also get multiplied by k e. If some or all of the elements of a row or column of a determinant are expressed as sum, then Δ is expressed as sum of Δs f. Δ will remain same if equi-multiple of any row or column is added to each element of any row or column
Singular Matrix	if det A = 0, then singular matrix otherwise non-singular matrix
Adjoint Matrix	Adjoint of A Matrix is the transpose of the Cofactor Matrix
Inverse Matrix	If A is a square matrix, and det A ≠ 0 (non-singular), then $A^{-1} = \frac{1}{\|A\|} \times \text{Adj. } A$
Cramer's rule to find solution of linear eq. in 3 variables	$x = \frac{\Delta x}{\Delta}, y = \frac{\Delta y}{\Delta}, z = \frac{\Delta z}{\Delta}$, provided Δ≠0 [$\Delta x$ means determinant of matrix by replacing first column of matrix with RHS values of equations] See Example
Properties of Cramer's	a. If Δ≠0, the system has unique solution b. If Δ=0 and atleast one of $\Delta x, \Delta y, \Delta z$ ≠0, then system has no solution and it is inconsistent c. If Δ=0 and all of $\Delta x, \Delta y, \Delta z$ ≠0, then system may or may not have solution,. If it has solution, equations are dependent and there will be infinite no. of solutions. If it doesn't have solution, equations are inconsistent.

SEQUENCE AND SERIES

<table>
<tr><td>Sequence</td><td>An ordered collection of numbers arranged as per some definite rule or pattern. $a_1, a_2, a_3, \ldots, a_n$ is a sequence if you are able to identify pattern and there by the value of a_n (n^{th} term)</td></tr>
<tr><td>Examples of Sequence</td><td>
<table>
<tr><th>Collection</th><th>Ordered</th><th>Rule/ Pattern</th><th>Conclusion</th></tr>
<tr><td>1, 4, 9, 17, 18, ……</td><td>Yes</td><td>No</td><td>Not a sequence</td></tr>
<tr><td>20, 17, 4, 3, 1, ……</td><td>Yes</td><td>No</td><td>Not a sequence</td></tr>
<tr><td>1, 4, 7, 10, 13, ……</td><td>Yes</td><td>Yes +3 on each term</td><td>Yes Sequence</td></tr>
<tr><td>20, 10, 5, 5/2, ……</td><td>Yes</td><td>Yes ÷2 on each term</td><td>Yes Sequence</td></tr>
</table>
</td></tr>
<tr><td>Terms</td><td>$a_1, a_2, a_3, \ldots, a_n$ are called as 1^{st} Term, 2^{nd} Term, 3^{rd} Term…nth term respectively</td></tr>
<tr><td>General Term</td><td>a_n is called as the n^{th} term of the sequence or General Term</td></tr>
<tr><td>Types of sequence</td><td>Finite Sequence – sequence having finite elements $\{a_i\}_{i=1}^{n}$
Infinite Sequence – sequence having infinite elements $\{a_i\}_{i=1}^{\infty}$</td></tr>
<tr><td>Series</td><td>Sum of the elements of the sequence is called as Series. $S_n = \sum_{i=1}^{n} a_i$
$S_n = a_1 + a_2 + a_3 + \cdots + a_n$
$S_1 = a_1, \quad S_2 = a_1 + a_2, \quad S_3 = a_1 + a_2 + a_3$</td></tr>
<tr><td>Arithmetic Progression (A.P.)</td><td>AP is a sequence in which each next term is obtained by adding a constant 'd' to the preceding term. This constant 'd' is called as common difference.
Let say a = first term and d = common difference, then AP can be written as –
a, a+d, a+2d, a+3d …. a+(n-1)d</td></tr>
<tr><td>Common Difference 'd'</td><td>d = any term – preceding term or $\{t_n - t_{n-1}\}$</td></tr>
<tr><td>n^{th} term of an AP</td><td>$t_n = a + (n-1)d$</td></tr>
<tr><td>Insert AMs between two numbers</td><td>If there is a problem to find out AMs between two number, consider it as an AP with first number as first term of AP and other number as last term of AP. Number of AMs required = no. of terms between first term and last term
Example: If 3 AMs between a and b is asked, form an AP as below:
a, ___ , ___ , ___ , b</td></tr>
<tr><td>Sum of first n terms of an AP</td><td>$S_n = \frac{n(a+t_n)}{2}$ or $S_n = \frac{n}{2}\{2a + (n-1)d\}$</td></tr>
<tr><td>Other Useful Formulas</td><td>
<table>
<tr><td>Sum of first n natural numbers</td><td>$\frac{n(n+1)}{2}$</td></tr>
<tr><td>Sum of first n odd numbers</td><td>n^2</td></tr>
<tr><td>Sum of squares of first n natural numbers</td><td>$\frac{n(n+1)(2n+1)}{6}$</td></tr>
<tr><td>Sum of cubes of first n natural numbers</td><td>$\left\{\frac{n(n+1)}{2}\right\}^2$</td></tr>
</table>
</td></tr>
</table>

Geometric Progression (G.P.)	GP is a sequence of terms where each term is a constant multiple of preceding term. This constant multiplier is called as common ratio. Let say *a* = first term and *r* = common ratio then GP can be written as $a, ar, ar^2, ar^3, \ldots, ar^{n-1}$
n^{th} term of a GP	$t_n = ar^{(n-1)}$
Common Ratio 'r'	$r = \frac{\text{any term}}{\text{preceding term}} = \frac{t_n}{t_{n-1}}$
Insert GMs between two numbers	If there is a problem to find out GMs between two number, consider it as a GP with first number as first term of GP and other number as last term of GP. Number of GMs required = no. of terms between first term and last term Example: If 3 GMs between a and b is asked, form an GP as below: *a, ___ , ___ , ___ , b*
Sum of first n terms of a GP	$S_n = \frac{a(1-r^n)}{(1-r)}$ when r<1, $\frac{a(r^n-1)}{(r-1)}$ when r>1
Sum of infinite GP	$S_\infty = \frac{a}{(1-r)}$ [only possible when r<1]

TIME VALUE OF MONEY

Basics

- → The sum of money received in future is less valuable than it is today
- → Rs. 100 Note given today is more valuable than Rs. 100 note given a year later due to various reasons:

Risk Factor	Risk that payer will not give money
Liquidity Preference	Cash given today will be immediately available for spending, hence more valuable
Inflation	In general, as the time goes on purchasing power of the money gets reduced
Opportunity Cost	Cash given today could be invested to a better investment that could appreciate its value

Partied involved in Financial Transaction

Name of Parties	Treatment of Interest
Lender	Income
Borrower	Expense
Investor	Income
Investee	Expense

Simple Interest

Formula	$S.I. = \frac{P.r.t}{100}$
P	Principal means amount of money invested or loan taken
r	Rate of simple interest per annum
t	Time of loan / investment in years
Accumulated Amount under SI	Amount under SI = Principal + Simple Interest *(amount is also called as Balance)*

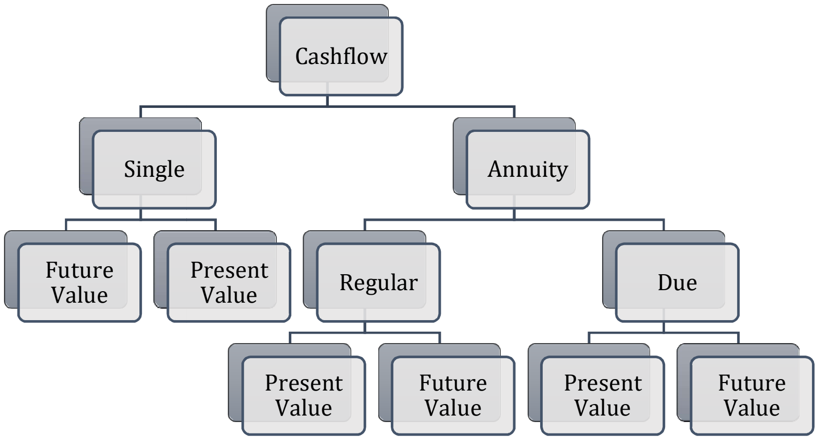
Compound Interest vs. Simple Interest

Simple Interest	Compound Interest
→ Interest earned is withdrawn every time it is earned → No re-investment of interest earned in earlier periods → Amount includes Principal and Interest on that Principal	→ Interest earned is not withdrawn till maturity → Re-investment of interest earned will be done → Amount includes Principal and Interest on that Principal and interest on interest earned in the earlier periods

Effective Rate of Interest

Meaning	The rate of interest stated in question does not always mean that effectively interest charged/ received will be same % when compared at annual level. Effectiveness depends on Compounding.
Higher the compounding for a rate of interest	Higher the effective rate for the year
Formula	$E = [(1 + i)^n - 1]$
n	here n means no. of periods in one years considering the compounding

<table>
<tr><td rowspan="9">Compound Interest</td><td>Compounding Frequency and Conversion Periods</td><td>It means no. of times interest is compounded in a year or no. of conversions in a year. Compounding means calculation of interest by bank. For e.g.
<table>
<tr><th>Conversion Period</th><th>Compounding Frequency</th></tr>
<tr><td>Yearly</td><td>1</td></tr>
<tr><td>Half-yearly</td><td>2</td></tr>
<tr><td>Quarterly</td><td>4</td></tr>
<tr><td>Monthly</td><td>12</td></tr>
<tr><td>Daily</td><td>365</td></tr>
</table>
While calculating compound interest, we need to adjust interest rate and time period using compounding frequency.</td></tr>
<tr><td>Formula for Accumulated Amount of CI</td><td>$\boldsymbol{A = P(1+i)^n}$</td></tr>
<tr><td>A</td><td>Accumulated amount as per CI</td></tr>
<tr><td>P</td><td>Principal means amount of money invested or loan taken</td></tr>
<tr><td>i</td><td>Interest rate (adjusted as per compounding) e.g.
If rate of interest given is *r=10%* and if compounding is half-yearly, $i = \frac{10\%}{2} = 5\% = 0.05$</td></tr>
<tr><td>n</td><td>It means no. of periods (not necessarily no. of years). It depends on type of compounding. E.g. if compounding is quarterly and *t = 2 years,* it means we will have 2×4 = 8 no. of periods. *n=8*</td></tr>
<tr><td>Shortcut in calculator to calculate amount</td><td>Example: P=1000, *i* = 10%, n=3 then
Calculator Steps: *Write P i.e* [1000] *then press*
[+] [10] [%] [+] [10] [%] [+] [10] [%] *(three times because n=3)*</td></tr>
<tr><td>Direct Formula of Amount in Calculator</td><td>Example: P=1000, *i* = 10% = 0.1, n=3 then
Calculator Steps: [1 + 0.1] [×] [=] [=] *(first equal will be considered as power 2, second as 3 and so on)* [×] 1000 (Principal)</td></tr>
<tr><td>How to calculate CI?</td><td>$A = P + CI \Rightarrow CI = A - P$
$CI = P(1+i)^n - P$
$CI = P[(1+i)^n - 1]$</td></tr>
<tr><td>Annuity</td><td colspan="2">
<table>
<tr><td>Definition</td><td>→ Sequence of periodic payments (installment)
→ Same amount
→ Regularly
→ For a specified period of time</td></tr>
<tr><td>Annuity Regular</td><td>Installment commencing from the end of the period</td></tr>
<tr><td>Annuity Due</td><td>Installment commencing from the beginning of the period</td></tr>
</table></td></tr>
<tr><td>Future Value</td><td colspan="2">Future value is the cash value of an investment at some time in the future. It is tomorrow's value of today's money compounded at the rate of interest.</td></tr>
<tr><td>Present Value</td><td colspan="2">Present value is today's value of tomorrow's money discounted at the interest rate.</td></tr>
</table>



Single cash flow	Meaning	Payment / Receipt one time at the beginning. No other payment/ receipt till maturity
	Formula of Future Value	$FV = PV\,(1+i)^n$
	Formula for Present Value	$PV = \frac{FV}{(1+i)^n}$
	Remark	Both the above formulas are similar to formula of Amount of compound interest. Principal is taken as PV and Amount is taken FV
Future value of Annuity	Formula for FV of Annuity Regular	$FVA = A_I \times [FVAF(n,i)]$ $FVA = A_I \left[\frac{(1+i)^n - 1}{i}\right]$ A_I = amount of installment or Annuity
	Formula for FV of Annuity Due	$FVA\ Due = FVA \times (1+i)$ Calculate FVA regular normally and then multiply it by $(1+i)$

Present Value of Annuity	Formula for PV of Annuity Regular	$PVA = A_I \times [PVAF(n,i)]$ $$PVA = A_I\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$ or $$PVA = \frac{A_I}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$ A_I = amount of installment or Annuity
	Formula for PV of Annuity Due	PVA Regular for one shorter period + Initial Cashflow
	Calculator Trick of PVAF (Present Value Annuity Factor)	$\boxed{1+i}\ \boxed{\div}\ \boxed{=}\ \boxed{=}$ … … n $times$ $\boxed{GT}$

Applications of Time Value of Money

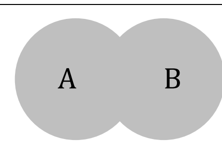
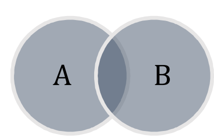
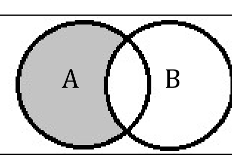
Particulars	Application	Remark
Sinking Fund	Future Value of Annuity is the amount which is required in future and annuity amounts are the regular savings required for creation of fund	Sinking fund means a fund created for specific purpose where a big amount of money is required at any specific point in future. An annuity is set aside and invested so that it will mature on that specific date giving the required amount.
Leasing	Present Value of Annuity (Lease Rentals) are compared with Asset Cash down price	**Lessor**: Owner of Asset, who gives asset on rent. Lease Rentals are income for Lessor **Lessee**: User of the asset who has taken asset on rent. Lease Rentals are expense for Lessee
Capital Expenditure or Investment Decision	Present value of savings and benefits are compared with purchase value of asset, to decide whether asset to purchase or not	**Capital Expenditure**: Expenditure on capital assets in anticipation of future benefits **Future Benefits**: Contribution from sales and other benefits or savings derived from a capital investment
Valuation of Bond	Present value of interest income and maturity value is compared with the issue price of bond	**Bond**: It is a debt security. Type of loan taken by company from public. Like debentures **Face Value**: Value written on the document of bond. This value is used to calculate Interest Amount **Issue Price**: Actual payment made to purchase the bond **Maturity value**: Amount to be received on redemption or maturity of bond

Perpetuity

Meaning	An annuity that continues till infinite period of time is called as Perpetuity.
Formula Perpetuity	Present Value of Perpetuity = $\frac{A_I}{i}$
Formula Growing Perpetuity	Present Value of Growing Perpetuity = $\frac{A_I}{(i-g)}$ *g is constant growth rate*

Net Present Value	NPV = Present Value of Cash Inflows – Present Value of Cash Outflows If NPV ≥ 0, accept the proposal, If NPV < 0, reject the proposal
Nominal Rate of Return	Real Rate of Return = Nominal Rate of Return – Rate of Inflation
CAGR	Compounded Annual Growth rate is the interest rate we used in Compound Interest. It is used to see returns on investment on yearly basis

SET

Set means	Collection of well-defined distinct objects. It is usually denoted by capital letter
Element	Each object of set is called as element. It is usually denoted by small letter
Braces Form	When set shown as a list of elements within braces { } e.g. A = {1,3,5,7}
Descriptive Form	Set can be presented in statement form e.g. A = set of first four odd numbers
Set-Builder or Algebraic form	Here Set is written in the algebraic form in this format – $\{x: x$ satisfies some properties or rule$\}$. The method of writing this form is called as Property or Rule method
Belongs to	It is denoted by '∈', **a ∈ A** means that element **a** is one of the element of Set A. ∉ used for do not belongs to.
Subset	Set A is a subset of Set B if all the elements of Set A also exist in Set B. It is denoted as - A⊂B
Proper Subset	A is a proper subset of B if A is a subset of B and A≠B
Improper Subset	Two equal sets are improper subsets of each other
Null Set	A set having no elements is called as Null or Empty Set. It is denoted by **ϕ**
No. of subsets	Formula: no. of subsets = 2^n, no. of proper subsets = 2^n-1
Intersection denoted by **[A∩B]**	Intersection set of A and B is a set that contains common elements between both of the sets
Union denoted by **[A∪B]**	Union set of A and B is a set that contains all the elements contained in both the sets without repeating the common elements
Universal Set	The set which contains all the elements under consideration in a particular problem is called the universal set generally denoted by **S**
Complement Set	A complement set of set P is a set that contains all the elements contained in the universe other than elements of P. It is denoted by **P' or P^c**
Set (A-B)	A-B is a set that contains elements of A other than those which are common in A and B. [A-B = A - A∩B]
De Morgan's Law	1. (P∪Q)' = P'∩Q' 2. (P∩Q)' = P'∪Q'
Venn Diagrams	Universal Set:  Union Set A∪B:  Intersection Set A∩B:  Set A-B: 
2 sets – Formula	n(A∪B) = n(A) + n(B) – n(A∩B)
3 sets – Formula	n(A∪B∪C) = n(A) + n(B) + n(C) – n(A∩B) –n(B∩C) –n(C∩A) + n(A∩B∩C)

<table>
<tr><td>Venn Diagram related some basics</td><td><table>
<tr><td>A or B , atleast A or B, either A or B</td><td>$A \cup B$</td></tr>
<tr><td>A and B, Both A and B</td><td>$A \cap B$</td></tr>
<tr><td>Only A means</td><td>$A - B$</td></tr>
<tr><td>Only B means</td><td>$B - A$</td></tr>
<tr><td>Neither A nor B</td><td>$(A \cup B)'$</td></tr>
</table></td></tr>
<tr><td>Cardinal Number</td><td>No. of distinct elements contained in a finite Set A is called Cardinal Number.
For Set A = {4,6,8,3}, cardinal no. n(A) = 4</td></tr>
<tr><td>Equivalent Set</td><td>Two sets A and B are equivalent sets if n(A) = n(B)</td></tr>
<tr><td>Power Set</td><td>Collection of all possible subsets of a given set A is called Power set of Set A. It is denoted by P(A)</td></tr>
<tr><td>Ordered Pair</td><td>Pair of two elements both taken from different Sets. E.g. if $a \in A$ and $b \in B$ then ordered pair is (a,b) where first element will always from A and second always from B in every pair</td></tr>
<tr><td>Product of Sets</td><td>Also called as Cartesian Product. If A and B are two non-empty sets, then set of all the ordered pairs such that $a \in A$ and $b \in B$ is called as Product Set. It is denoted by A×B. [A×B = {(a:b): a∈A and b∈B}]</td></tr>
<tr><td>Why Product?</td><td>n(A×B) = n(A) × n(B) i.e. cardinal no. of product set is equal to product of cardinal no. of each set</td></tr>
</table>

FUNCTION

<table>
<tr><td>Relation</td><td>Any subset of product set is called A×B is said to define relation from A to B. It's any collection of ordered pairs taken from a product set.</td></tr>
<tr><td>Function (set based definition)</td><td>A relation where no ordered pairs have same first elements is called Function. First element of the ordered should not be repeated in the relation set. (a,b) all a should be unique for different values of b</td></tr>
<tr><td>Function (non set based definition)</td><td>A rule which associate all elements of A to B is called function from A to B. It is denoted by $f: A \rightarrow B$ or $f(x)$ of B</td></tr>
<tr><td>Image, Pre-image</td><td>$f(x)$ is called the image of x and x is called the pre-image of $f(x)$
Pre-image is input and Image is output</td></tr>
<tr><td>Domain, Co-domain, Range</td><td>Let $f: A \rightarrow B$, then A is called domain of f and B is called the co-domain of f.
Set of all the images (contained in B) of pre-images taken from A is called Range. Domain is a set of all pre-images and Range is a set of all images. Also Range is a subset of Co-domain.</td></tr>
<tr><td>Types of Functions</td><td><table>
<tr><td>One-One Function</td><td>Let $f: A \rightarrow B$, if different elements in A have different images in B then f is one-one or injective function or one-one mapping</td></tr>
<tr><td>Onto Function</td><td>Let $f: A \rightarrow B$, if every element in B has at least one pre-image in A, then f is an onto or surjective function</td></tr>
<tr><td>Into Function</td><td>Let $f: A \rightarrow B$, if even a single element in B is not having pre-image in A, then it is said to be into function</td></tr>
<tr><td>Bijection Function</td><td>If a function is both one-one and onto it is called as Bijection Function</td></tr>
<tr><td>Identity Function</td><td>If domain and co-domain are same then function is identity function Let $f: A \rightarrow A \text{ and } f(x) = x$</td></tr>
<tr><td>Constant Function</td><td>If all pre-images in A will have a single constant value in B then the function is constant function</td></tr>
</table></td></tr>
<tr><td>Equal Function</td><td>Two functions f and g are said to be equal function if both have same domain and same range</td></tr>
<tr><td>Inverse Function</td><td>Let $f: A \rightarrow B$, is a one-one and onto function. Every value of x (preimage)will</td></tr>
</table>

	give unique image $f(x)$ using *f.* If there is a function that takes value of images as input and gives pre-images as output, such function is called inverse function. It is denoted as $f^{-1}: B \rightarrow A$.
Composite Function	A function of function is called composite function. Example: if f *and* g are functions, then $f[g(x)]$ *and* $g[f(x)]$ are composite functions. Also called as fog *or* gof

RELATION

Relations	Any subset of product set is called A×B is said to define relation from A to B. It's any collection of ordered pairs taken from a product set.	
Domain and Range	If R is a relation from A to B, then set of all first elements of ordered pairs is domain and set of all second elements of ordered pairs is range.	
Types of Relation	Reflexive	If S is a universal set, S = {a,b,c …} then R is a relation from S to S. If this R contains all the ordered pairs in the form (a,a) in S×S, then it is a reflexive relation
	Symmetric	If (a,b) ∊ R, then if (b,a) ∊ R then R is called Symmetric
	Transitive	If (a,b) ∊ R and also (b,c) ∊ R, then if (a,c) ∊ R such relation is Transitive. [if in a relation only (a,b) is present but (b,c) is not present we will consider it as transitive relation]
Equivalence Relation	If a relation is Reflexive, Transitive and Symmetric as well, then it is called as Equivalence Relation	

Permutations and Combinations

Fundamental Principles of Counting	Multiplication Rule AND → Multiply: If one thing can be done in 'm' ways and when it has been done, another thing can be done in 'n' different ways then the total number of ways of doing **both the things simultaneously = m × n** Addition Rule OR → Add: If two alternative jobs can be done in 'm' and 'n' way respectively then **either of the two jobs** can be done in **(m+n) ways**
Factorial	It is written as n! or $n\rfloor = n(n-1)(n-2)\ldots 3\times 2\times 1$ 0! = 1, 1! = 1, 2! = 2×1, 3! = 3×2×1, 4! = 4×3×2×1
Permutations means	It is the ways of **arranging or selecting** things from a group of things with due regard being paid **to order** of the arrangement or selection.
Basic Example 1	**Arranging** three persons A,B,C for a group photograph can be done as {ABC, ACB, BAC, BCA, CAB, CBA}, thus total no. of ways is 6
Basic Example 2	**Selecting** two persons as Winner and Runner-up for a contest having 4 participants P,Q,R,S can be done as {PQ, PR, PS, QP, QR, QS, RP, RQ, RS, SP, SQ, SR}, thus total no. of ways is 12 (here in the set of arrangement first element is winner and second is runner up)
Theorem for Permutations	The number of permutations of n things chosen r at a time is given by ${}^nP_r = \frac{n!}{n-r!}$ *or* $n(n-1)(n-2)\ldots(n-r+1)$
Basic Example 3	${}^5P_3 = \frac{5!}{(5-3)!} = \frac{5\times4\times3\times2\times1}{2\times1} = 5\times4\times3 = 60$ Or simply here r = 3, so do reverse multiplication of 5 up to three terms so it will be 5×4×3 = 60
Use of Theorem	We are able to find no. of ways manually also *(as done in Basic Example 1 and 2)* but that is easy for lower values of n and r. When there is a higher value of n, manually creating the set of arrangements will be tedious which requires the need of this theorem. Check Basic Example 1 and Example 2 using theorem
Why 0! = 1	${}^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!}$ also, ${}^nP_n = n!$, thus $\frac{n!}{0!} = n!$, $0! = \frac{n!}{n!} = 1$
Special Formula	$(n+1)! - n! = n.n!$ (for proof – refer Example 10 Study Mat Page 5.6)
Question Patterns with remarks	(see table below)

Question Patterns with remarks

Type	Remark
Calculate No. of words using letters of a particular word	Simple ${}^nP_r$ Note: Meaning of words has no relevance
Group Photograph	${}^nP_n$
Rank Awards first, second, third etc.	${}^nP_r$ here r is no. of ranks
Theorem based questions, calculation of n or r with the given data	Directly apply theorem
Selection of different unique designations/ positions from a group of persons	${}^nP_r$ here r is no. of unique designations/ positions

Circular Permutations	Above discussion was relevant for things that are arranged in a row. However when the things are arranged in a circle, the permutation is termed as circular.
Theorem: Circular Permutations	The number of circular permutations of n different things chosen all at a time is **(n–1)!**

Standard Results

number of ways of arranging n persons along a round table so that no person has the same two neighbors is	$\frac{1}{2}(n-1)!$
the number of necklaces formed with n beads of different colors	$\frac{1}{2}(n-1)!$

Permutation with Restrictions

Note: These two theorems are useful for formula based questions. For practical questions we will use logic. (explained in example)

Theorem 1	Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is $^{(n-1)}P_r$
Theorem 2	Number of permutations of r objects out of n distinct objects when a particular object is always included in any arrangement is $^{(n-1)}P_{(r-1)}$

Some tips useful while solving problems having restrictions

Requirement of Que.	Tips
Calculate permutation when two or more objects are always together	In that case consider that group of objects as 1 object for the purpose of ^nP_r formula, then multiply factorial of no. of objects in the group
Calculate permutation when two or more objects will never come together	Step 1: Calculate the no. of ways without restriction using ^nP_r Step 2: Calculate Permutation of 2 or more thing always together (as per above point) Step 3: Result of Step 1 – Result of Step 2
When there are two types of objects and ask is to calculate the ways in which no two objects of one the category will be together	In that case, that particular group of objects can be arranged in the alternate places as a neighbor of each object of other category Refer Example 10 Study Mat Page 5.13SS

Standard Results

Permutations when some of the things are alike, taken all at a time	$p = \frac{n!}{n_1! \times n_2! \times n_3!}$
Permutations when each thing may be repeated once, twice, upto r times in any arrangement.	n^r

<table>
<tr><td>Combinations</td><td>The number of ways in which smaller or equal number of things are arranged or selected from a collection of things where the order of selection or arrangement is not important, are called combinations. It is just a GROUPING</td></tr>
<tr><td>Basic Example 1</td><td>Grouping of two persons out of three persons A,B,C for a group photograph can be done as {AB, BC, AC}, thus total no. of ways is 3. Here AB and BA are same group and will be counted once only, even though the sequence is not same. Sequence has no relevance while finding combinations.</td></tr>
<tr><td>Basic Example 2</td><td>Selection of persons for a committee of 2 out of total 4 applicants P,Q,R,S can be done in {PQ, QR, RS, PS, PR, QS} – total 6 ways. Here we used combinations because in the committee of two there is no designations all are same so sequence of selection does not matter.</td></tr>
<tr><td>Theorem of Combinations</td><td>$${}^{n}C_{r} = \frac{n!}{r!(n-r)!} \text{ or } {}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!}$$</td></tr>
<tr><td>Standard Results</td><td>$${}^{n}C_{0} = 1 \text{ , } {}^{n}C_{n} = 1$$</td></tr>
<tr><td>Complimentary Combinations</td><td>${}^{n}C_{r} = {}^{n}C_{(n-r)}$ example: ${}^{5}C_{3} = {}^{5}C_{2}$</td></tr>
<tr><td>Special Formulas</td><td>$${}^{n+1}C_{r} = {}^{n}C_{r} + {}^{n}C_{r-1}$$
<u>Memorize:</u>
Combination of (n+1) things when one thing is always included [${}^{n}C_{r}$]+
Combination of (n+1) things when one thing is always excluded [${}^{n}C_{r-1}$]</td></tr>
<tr><td>Permutation Special formula</td><td>$${}^{n}P_{r} = {}^{n-1}P_{r} + r.{}^{n-1}P_{r-1}$$
Memorize in the same way as above</td></tr>
<tr><td>Standard Results</td><td><table><tr><td>Combinations of n different things taking some or all of n things at a time</td><td>$\mathbf{2^{n}}$ – 1 [1 is subtracted because we are removing all rejection case]</td></tr></table></td></tr>
<tr><td>Question Patterns with remarks</td><td><table>
<tr><th>Type</th><th>Remark</th></tr>
<tr><td>Different pocker hands in a pack of cards</td><td>When we play Poker, Teen Patti etc. only group of 5 cards, sequence in which it is picked does not matter hence we take combinations</td></tr>
<tr><td>Formation of triangles when vertices (corner points) are given</td><td>We need three vertices to make a triangle. Now with group of three points to make a triangle and sequence of points does not matter, hence will use combination. Example: Using eight points how many triangle can be formed - ${}^{8}C_{3} = 56$</td></tr>
<tr><td>No. of ways of invitation</td><td>Here also sequence does not matter, hence will use combination</td></tr>
<tr><td>Selection of color balls from box</td><td>Here combination is used assuming that balls are of identical color</td></tr>
<tr><td>No. of ways of forming words from n letter taking few letters and the letter are not unique</td><td>Refer Example 6 – Page 5.25 Study Mat</td></tr>
<tr><td>Number of diagonals of a polygon</td><td>${}^{n}C_{2} - n$, here n means no. of side of polygon (refer Q.10 Exercise 5C)</td></tr>
</table></td></tr>
</table>