Chapter 4 – Time Value of Money

FREE Fast Track Lectures (Every day at 10:00 a.m. on YouTube):

https://www.youtube.com/watch?v=ZZpvRpkgmaE&list=PLAKrxMrPL3fwOSJWxnr8j0C9a4si2Dfdz

Lecture 1 of Time Value of Money (Premiering on 07-06-2020 at 10:00 a.m.): https://youtu.be/LemnCsQQt3Y

Lecture 2 of Time Value of Money (Premiering on 08-06-2020 at 10:00 a.m.): https://youtu.be/PWfBodgPeVo

Lecture 3 of Time Value of Money (Premiering on 09-06-2020 at 10:00 a.m.): https://youtu.be/MBjrml3CCnk

Unit 1 – Simple Interest

Suppose you deposit ₹10,000 into a bank for 2 years. The interest rate that the bank offers is 5% p.a. After two years, you'll receive the initial amount that you invested, i.e. ₹10,000 plus interest for two years on this amount. The amount of interest will be 5% × ₹10,000 × 2 = ₹1,000. So, in effect, you'll receive ₹10,000 + ₹1,000 = ₹11,000.

Now, the initial amount that you invested, i.e. ₹10,000 is known as the **Principal**. The interest rate, i.e. 5% is known as **Rate of Interest**. The amount of interest, i.e. ₹1,000 is simply called **Interest**. The total amount received after two years is known as the **Accumulated Amount** or **Balance**.

Simple Interest

Simple Interest means the interest which is calculated only on the Principal amount, and not on the interest accrued on it. Some important formulas are given below:

1. $I = Pit$
2. $A = P + I$
   $= P + Pit$
   $= P(1 + it)$
3. $i = \dfrac{A - P}{Pt}$
4. $t = \dfrac{A - P}{Pi}$

Here,

I = Amount of Interest

P = Principal (initial value of investment)

i = Annual interest rate in decimal

t = Time in years

A = Accumulated amount (final value of investment)

Unit 2 – Compound Interest

Compound Interest

The word "compound" simply stated means "to add". Compound Interest means that interest is calculated not only on the Principal amount, but even on the interest amount accrued on it. The rate of interest in case of compound interest is usually mentioned as 5% p.a. **compounded annually**. This means that interest for every year will be added to the principal to calculate the interest for the next year. For example, if I deposit ₹10,000 into a bank for two years, the compound interest is calculated as follows:

1. For the first year: 5% on ₹10,000 for 1 year → 5% × ₹10,000 = ₹500
2. For the second year: 5% on (₹10,000 + ₹500) for 1 year → 5% × ₹10,500 = 525

Total interest received = ₹500 + ₹525 = ₹1,025. Total amount received after two years = ₹10,000 + ₹1,025 = ₹11,025. Here, we must compare it with Simple Interest. While in Simple Interest, the total interest was 5% × ₹10,000 × 2 = ₹1,000, in Compound Interest, the total interest is ₹1,025. The extra ₹25 is because of the compounding of the interest of the first year to the principal amount.

Suppose the rate of interest is 5% p.a. **compounded semi-annually**. This means that interest for every six months would be added to the principal to calculate the interest for the next six months. The interest would be calculated as follows:

1. For the first six months: 5% on ₹10,000 for 6 months → 5% × ₹10,000 × 6/12 = ₹250. Now, this ₹250 is my interest which has accrued for 6 months.
2. For the next six months: 5% on (₹10,000 + ₹250) for 6 months → 5% × ₹10,250 × 6/12 = ₹256.25.
3. For the next six months: 5% on (₹10,000 + ₹250 + ₹256.25) for 6 months → 5% × ₹10,506.5 × 6/12 = ₹262.66.
4. For the last six months: 5% on (₹10,000 + ₹250 + ₹256.25 + ₹262.66) for 6 months → 5% × ₹10,768.91 × 6/12 = ₹269.22

Total interest received = ₹250 + ₹256.25 + ₹262.66 + ₹269.22 = ₹1,038.13. Total amount received after two years = ₹10,000 + ₹1,038.13 = ₹11,038.13.

We can see that on the initial investment of ₹10,000 for two years @ 5% p.a., the interest and amount was as under:

Particulars	**Interest (₹)**	**Amount (₹)**
1. Simple Interest	1,000.00	11,000.00
2. Interest Compounded Annually	1,025.00	11,025.00
3. Interest Compounded Semi-Annually	1,038.13	11,038.13

Thus, it can be concluded that a greater frequency of compounding results in larger amount of interest.

Conversion Period

The period at the end of which the interest is compounded is called the Conversion Period. In our example above, first we compounded the interest after every year, therefore, the conversion period was 1 year. After that, we compounded the interest after every six months, therefore, the conversion period was 6 months. In such a case, the number of conversion periods per year would be 2. Similarly, the table below shows the typical conversion periods that are used in the questions:

Conversion Period	**Description**	**Number of Conversion Periods in a Year**
1 Day	Compounded Daily	365
1 Month	Compounded Monthly	12
3 Months	Compounded Quarterly	4
6 Months	Compounded Semi-Annually	2
12 Months	Compounded Annually	1

Formula for Compound Interest

$$A_n = P\left(1+\frac{i}{NOCPPY}\right)^n$$

Where,

A_n = Accrued amount

P = Principal

i = Annual interest rate in decimal

$NOCPPY$ = No. of Conversion Periods Per Year

n = total conversions, i.e. $t \times NOCPPY$, where t = Time in years

Therefore, the formula can also be written as:

$$A_n = P\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}$$

Compound Interest (CI) is, therefore, given by $A_n - P$.

Therefore, $CI = \left[P\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}\right]-P$

$$= P\left[\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}-1\right]$$

Difference Between Compound Interest and Simple Interest

The difference between Compound Interest (*CI*) and Simple Interest (*SI*) on a certain sum (*P*) invested for (*t*) years at the rate (*i*) is given by the formula: $CI - SI = P\left[\left\{(1+i)^{t} - 1\right\} - it\right]$

Effective Rate of Interest

Effective Rate of Interest is denoted by the letter *E* and is calculated using the formula: $E = \left(1 + \frac{i}{NOCPPY}\right)^{t \times NOCPPY} - 1$.

The actual rate of interest given in the question (*i*) is called the Nominal Rate.

Unit 3 – Annuity and Perpetuity

Annuity

A fixed sum of money payable or receivable after every fixed period (a month, a year, etc.) for a certain number of years is called Annuity. Thus, annuity can be defined as a sequence of periodic payments (or receipts) regularly over a specified period of time.

Therefore, to be called annuity, both the following conditions must be satisfied:

1. Amount paid or received must be constant over the period of annuity, and
2. Time interval between two consecutive payments or receipts must be the same.

Annuity Regular and Annuity Due/Immediate

1. Annuity Regular – When the payments are made/received at the end of the year, it is said to be Annuity Regular.
2. Annuity Due/Immediate – When the payments are made/received in the beginning of the year, it is said to be Annuity Due/Immediate.

Future Value

When we deposit our money in a bank, or in any investment, we receive some interest. Suppose I deposit ₹10,000 today @ 10% p.a. At the end of 1 year, I will receive ₹10,000 + (10% × ₹10,000) = ₹11,000. Therefore, the **Future Value** of today's ₹10,000 is ₹11,000 when it is invested @ 10% p.a.

Future value is the cash value of an investment at some time in future compounded at some interest rate. It is very similar to the compound interest. When you calculate the Amount using the formula $A_n = P\left(1+\frac{i}{NOCPPY}\right)^n$, you are actually calculating the future value of your present investment.

The formula of Future Value can be derived simply by replacing A_n with Future Value and P with single Cash Flow. Therefore, $F.V. = C.F.\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}$.

Future Value of Annuity Regular

Annuity Regular means that the payments/receipts are made at the end of the year.

The future value can be calculated directly by using the formula:

$$A\left(n, \frac{i}{NOCPPY}\right) = A\left[\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY} - 1}{\frac{i}{NOCPPY}}\right]$$, where A = Periodic Payments.

Future Value of Annuity Due/Immediate

Annuity Due/Immediate means that the payments/receipts are made in the beginning of the year. The future value of Annuity Due/Immediate can be calculated directly by using the formula: *Future Value of Annuity Regular* × $\left(1+\frac{i}{NOCPPY}\right)$.

$$A\left(n, \frac{i}{NOCPPY}\right) = \left[A\left\{\frac{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY} - 1}{\frac{i}{NOCPPY}}\right\}\right] \times \left(1+\frac{i}{NOCPPY}\right)$$, where A = Periodic Payments.

Present Value

Present Value is simply the reverse of Future Value. When we deposit our money in a bank, or in any investment, we receive some interest. Some I deposit ₹10,000 today @ 10% p.a. At the end of 1 year, I will receive ₹10,000 + (10% × ₹10,000) = ₹11,000. Therefore, the **Present Value** of the ₹11,000 that I'll receive one year later, (i.e., in future) is ₹10,000.

We studied that future value is the cash value of an investment at some time in future compounded at some interest rate. This means that future value is tomorrow's value of today's money **compounded** at some interest rate. Similarly, present value is today's value of tomorrow's money **discounted** at some interest rate.

The formula for the present value can be derived from the formula of Amount that we studied in Compound Interest as follows:

$$A_n = P\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}$$

$$\Rightarrow P = \frac{A_n}{\left(1+\frac{i}{NOCPPY}\right)^{t\times NOCPPY}}$$

Present Value of Annuity Regular

Annuity Regular means that the payments/receipts are made at the end of the year.

Present Value of Annuity Regular = Annuity × Sum of Discounting Factors

$$\text{Present Value of Annuity Regular} = \text{Annuity} \times \frac{(Factor)^n - 1}{(\text{Discount Rate}) \times (Factor)^n}$$

$$\text{Present Value of Annuity Regular (P.V.)} = \text{Annuity} \times \frac{\left(1+\frac{i}{NOCPPY}\right)^{t \times NOCPPY} - 1}{\left(\frac{i}{NOCPPY}\right) \times \left(1+\frac{i}{NOCPPY}\right)^{t \times NOCPPY}}.$$

Note – The Present Value of Annuity is sometimes also denoted by *V*.

Present Value of Annuity Due/Immediate

P.V. of Annuity Due/Immediate = Initial Cash Payment/Receipt + P.V. of Annuity Regular (for n – 1 periods)

$n = t \times NOCPPY$

Applications of Future Value and Present Value

Following are some of the applications of future value and present value:

1. Sinking Fund
2. Leasing
3. Capital Expenditure (Investment Decision)
4. Net Present Value (NPV)
5. Valuation of Bond

Let's look at these applications one by one.

Sinking Fund

Sinking Fund is a fund created for a specified purpose. Some amount is deposited in this fund regularly over a period of time at a specified interest rate. Interest is compounded at the end of every period. We can clearly calculate the future value to find out how much balance the fund would have at the end of the period.

Leasing

Leasing in layman's terms means taking some asset on rent. The party who lends the asset is called the "lessor", and the party who borrows the asset is called the "lessee". You'll study about leases in detail later in CA Intermediate and CA Final. Obviously, the lessor would charge some rent on the asset lent by him. This rent is known as "Lease Rental". By using the concepts of Present Value and Future Value, we'll see whether leasing is preferable for the company or not. Following examples will make things clear.

Capital Expenditure (Investment Decision)

We have studied in Accounts that an expenditure which results in benefit for more than one year is known as a Capital Expenditure. Usually, a Capital Expenditure results in a huge amount of Outflow. However, there's anticipation of periodic inflows as well. These inflows would obviously last till the life of the capital expenditure. In order to find out whether a capital expenditure is beneficial or not, we compare the outflow that occurs today with the present value of all the future inflows. If the present value of the inflows exceeds the outflow, the capital expenditure is said to be beneficial. Following examples will make things clear:

Net Present Value (NPV)

This is similar to what we studied above in Capital Expenditure (Investment Decisions). The only difference is, that while in Capital Expenditure (Investment Decisions), we used to just see whether the present value of future cash flows is exceeding our initial investment or not, here, in Net Present Value, we're actually going to find out how much does the present value of future cash inflows exceed our initial investment. The difference between the present value of future cash flows and the initial cash outflow is known as the Net Present Value (NPV). Needless to say, if NPV is +ve, it is beneficial to take the project, whereas, if NPV is -ve, it is worthless to take the project.

Net Present Value = P.V. of Cash Inflows – Initial Cash Outflow

Sometimes, it may so happen, that a project requires not just the initial investment, but also some additional investment in the future. In such a case, we not only take the P.V. of the Cash Inflows, but we also calculate the P.V. of the Cash Outflows, and then compare the same.

Net Present Value = P.V. of Cash Inflows – P.V. of Cash Outflows

Valuation of Bond

A Bond is a financial instrument similar to a debenture. When you purchase a debenture, you pay a certain amount of money, and you receive interest periodically from it. Similarly, Bond is also a financial instrument, containing a fixed percentage of interest. Bonds are generally issued for a fixed term longer than one year. After the specified duration, the bond is redeemed.

Perpetuity

Perpetuity is simply an annuity that lasts forever. For e.g., if I receive ₹10,000 at the end of every year for the rest of my life, we'll call this perpetuity. The present value of perpetuity is calculated by using the formula: $\dfrac{A}{\frac{i}{NOCPPY}}$. Here, A is the payment or receipt each period.

Perpetual Growth or Growing Perpetuity

We studied perpetuity where we said that if I want ₹10,000 every year for the rest of my life, this is called perpetuity. On the other hand, if I want ₹10,000 at the end of the first year, ₹15,000 at the end of the second year, ₹20,000 at the end of the third year, ₹25,000 at the end of the fourth year, and so on for the rest of my life, it is called Perpetual Growth. The present value of Growing Perpetuity is calculated by the formula: $\frac{A}{i-g}$. Here, A is the periodic payment or receipt; i is the annual rate of interest in decimal; and g is the annual growth rate in decimal.